ASTEROIDS, KBOS AND OTHER DEBRIS IN PLANETARY SYSTEMS

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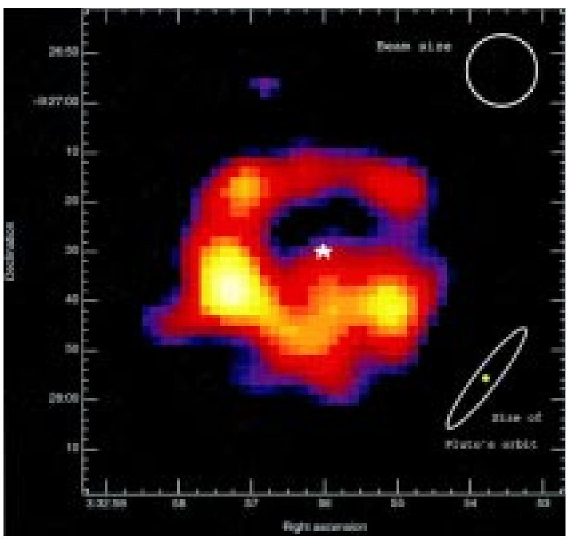
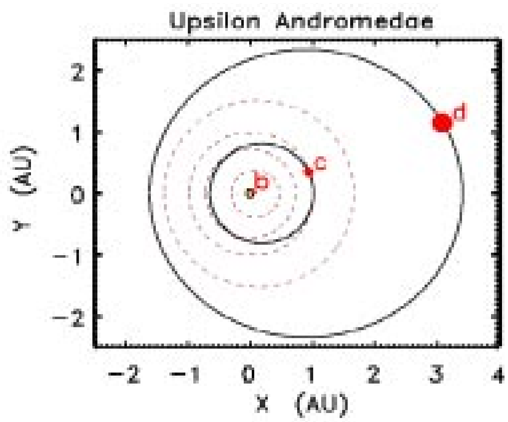
1. Locating debris in exo-planetary systems

2. Chaotic diffusion of resonant Kuiper Belt objects

@KITP: Planet Formation Conference, March 15-19, 2004


Beam size
Size of
Pluto's orbit


Upsilon Andromedae
Y (AU)
X (AU)
2
1
0
−1
−2
−2
−1
0
1
2
3
4
b
c
d

Orbital stability criteria

1 Planet System

- Hill stability: $|\Delta a|/a_{\rm p} > 2.4(m_{\rm p}/m_\star)^{\frac{1}{3}} \simeq 3R_{\rm H}$
- Resonance overlap: $|\Delta a|/a_{\rm p} > 1.5(m_{\rm p}/m_\star)^{\frac{2}{7}}$
- Eccentric planet: $r \ni (q - 3R_{\rm H}, Q + 3R_{\rm H})$

2 Planet System

- Direct numerical integration of test particles
- Patchwork of 1-planet criteria
- Secular stability analysis
  - Laplace-Lagrange secular theory for planets: linear modes
    - e.g., g_5, g_6 for Sun-Jupiter-Saturn system
  - Locate secular resonances of test particles
  - Nonlinear saturation of test particle eccentricity
    - resonance Hamiltonian, phase space pictures
    - Initially circular orbits are unstable near $g_0 \approx g_j$
    - maximum eccentricity excitation, $e_{\rm max} \propto |\mathbf{E}^{(j)}|$,
    - depends strongly on a, weakly on m_1/m_2.

Secular Resonance Hamiltonian

toy model: planet in eccentric, precessing orbit

$$H_{\rm sec} = -\frac{m_{\rm p}}{a_p}\Big\{A_0(\alpha) + A(\alpha)e^2 + B(\alpha)e^4 - C(\alpha)ee_{\rm p}\cos(\varpi_{\rm p} - \varpi)\Big\}. \quad (1)$$

in canonical variables, $-\varpi$ and $J = \sqrt{a}(1 - \sqrt{1-e^2})$,

$$H_{\rm sec} = -\frac{m_{\rm p}}{a_m}A_0(\alpha) - g_0 J + \beta J^2 + \varepsilon\sqrt{2J}\cos(\varpi - \varpi_{\rm p}), \quad (2)$$

where

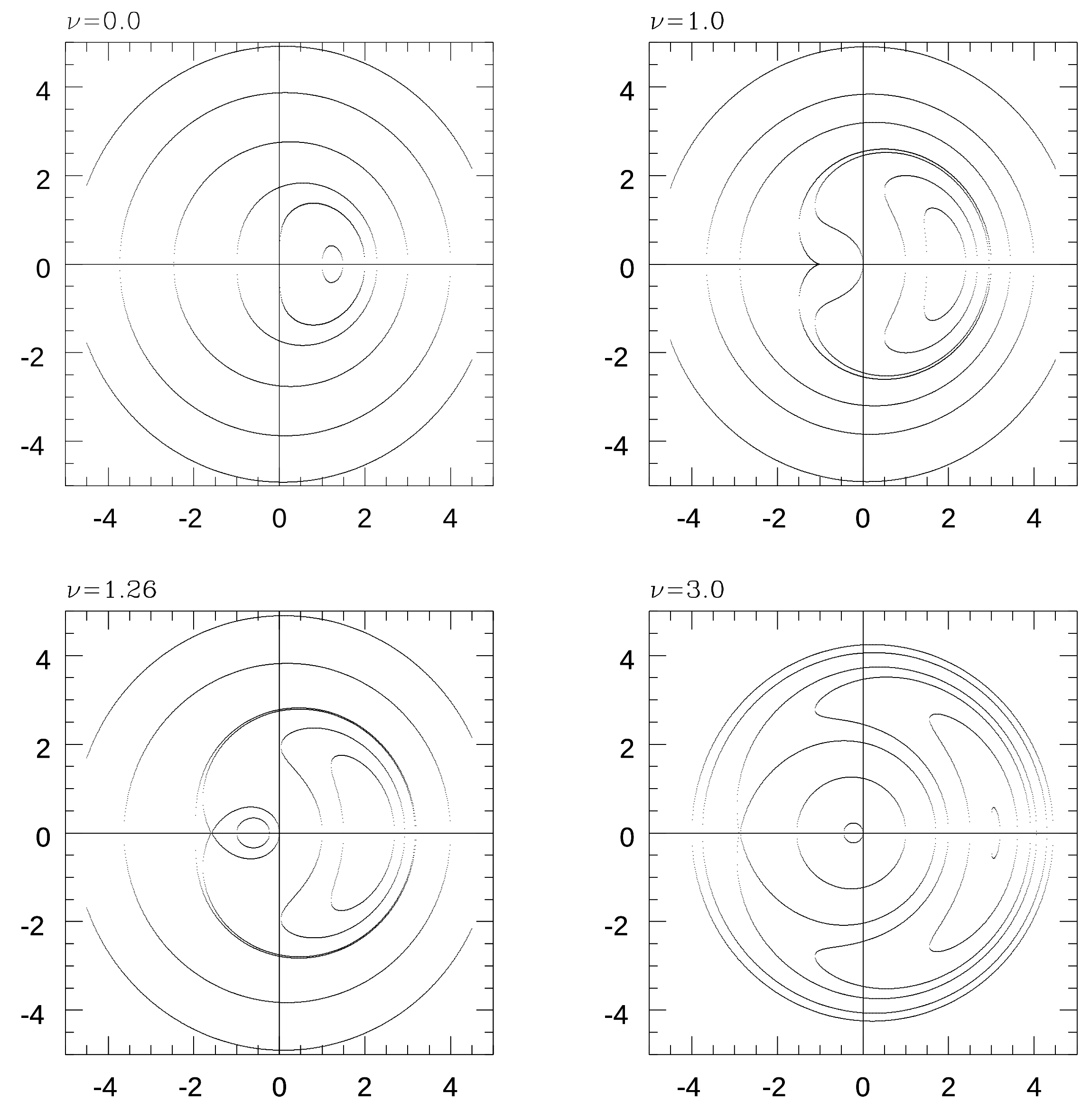
$$g_0 = \frac{2A(\alpha)}{a^{\frac{1}{2}}a_m}m_{\rm p}, \qquad \beta = \frac{A(\alpha) - 4B(\alpha)}{aa_m}m_{\rm p}, \qquad \varepsilon = \frac{C(\alpha)}{a^{\frac{1}{4}}a_m}m_{\rm p}e_{\rm p}. \quad (3)$$

For initially circular test particle orbits, the maximum perturbation occurs at

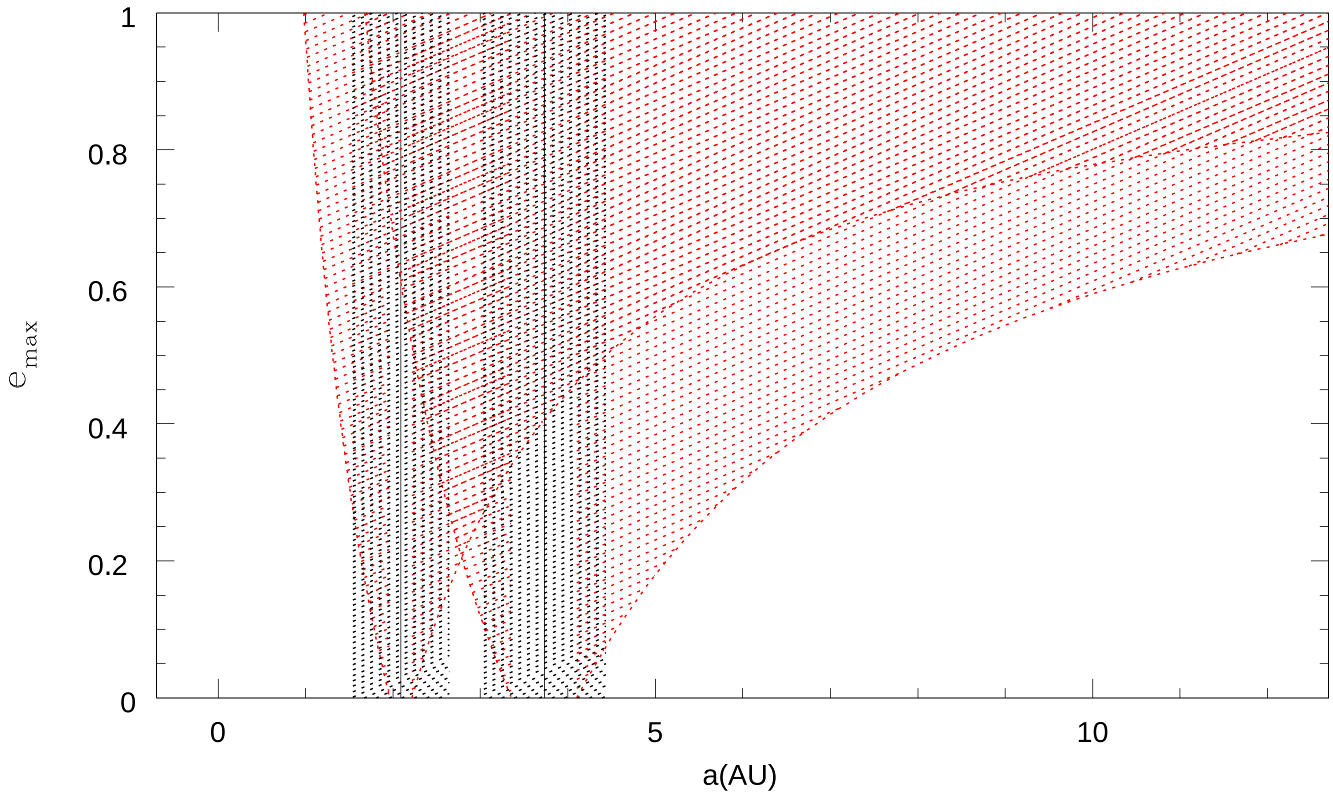
$$g_0 = g_{\rm p} + 3(\beta\varepsilon^2/2)^{1/3},$$

and the maximum amplitude is given by

$$J_{\rm max} = 2\Big|\frac{2\varepsilon}{\beta}\Big|^{\frac{2}{3}} \qquad \text{or} \qquad e_{\rm max} \approx 2\Big|\frac{2C}{A - 4B}e_{\rm p}\Big|^{\frac{1}{3}}. \quad (4)$$

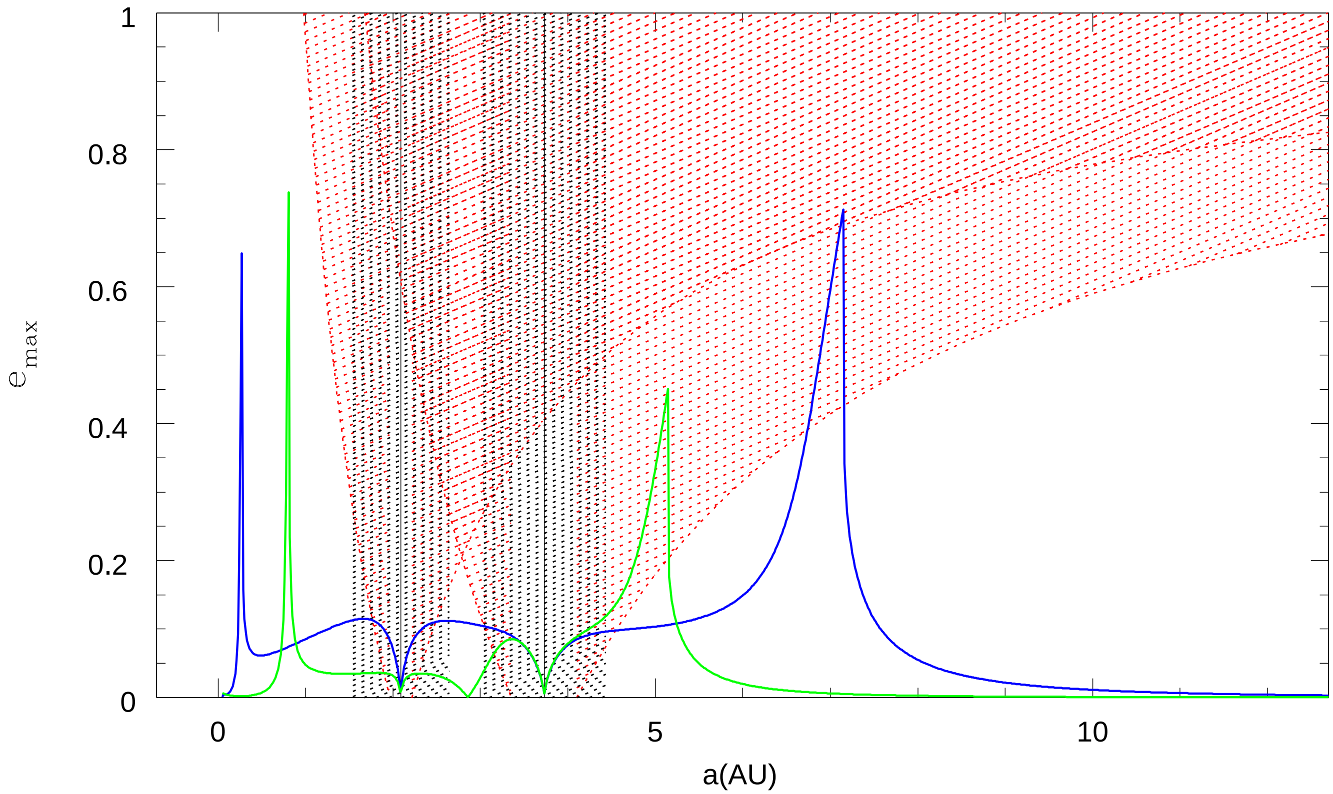

ν=0.0
ν=1.0
ν=1.26
ν=3.0
4
2
0
-2
-4
-4 -2 0 2 4

47 Ursae Majoris system



Mar 15 15:16:12 2004

47 Ursae Majoris system



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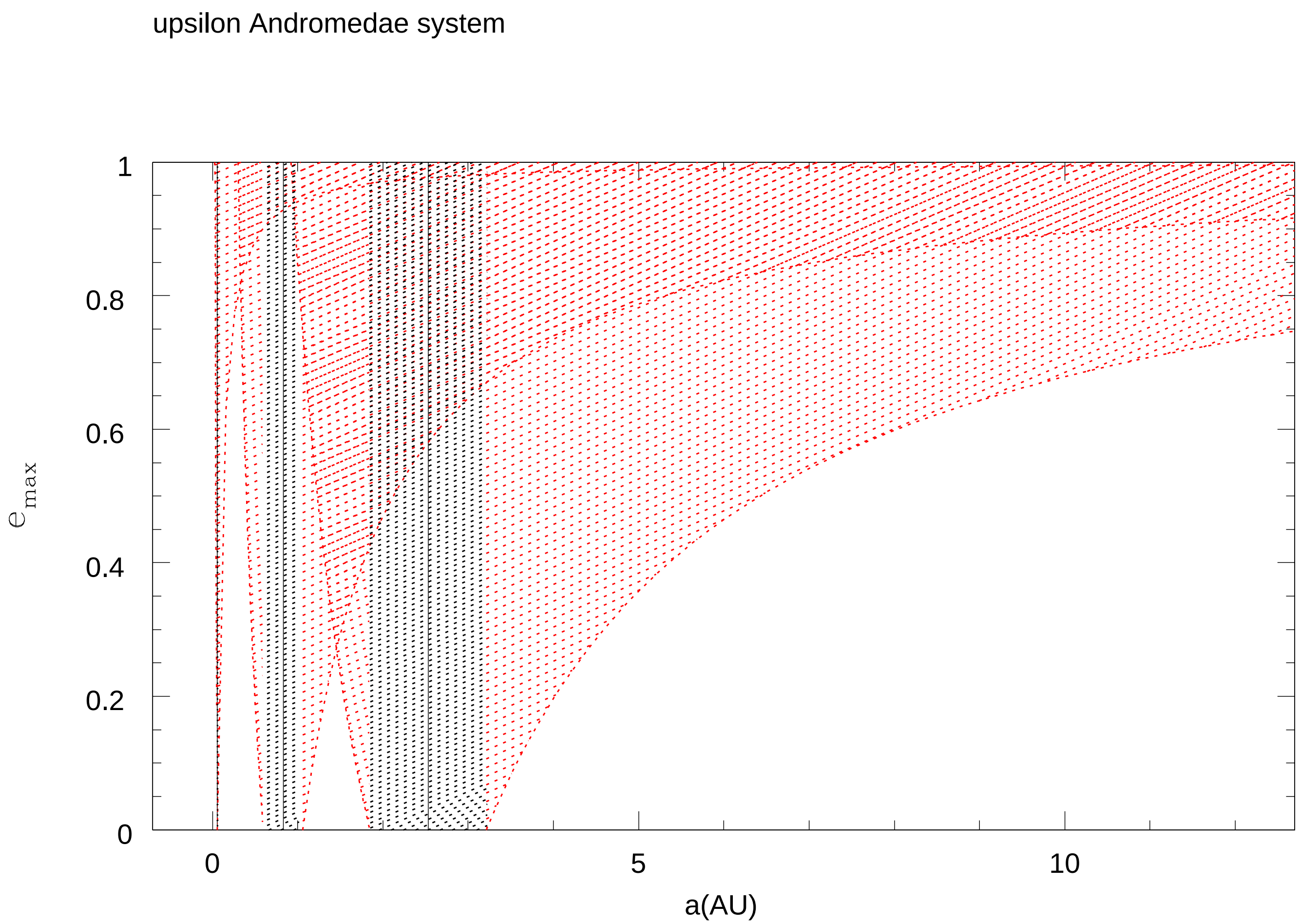

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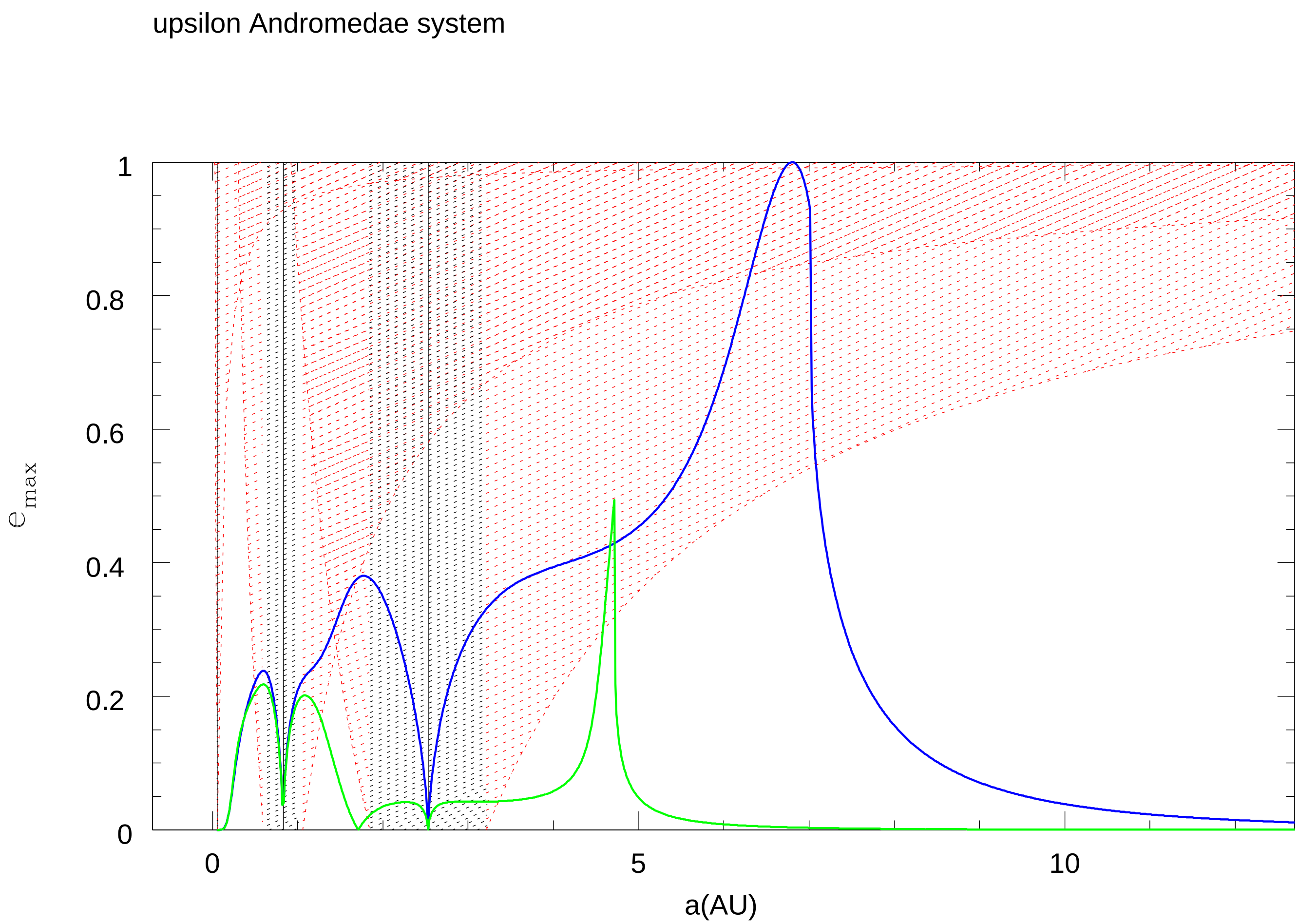

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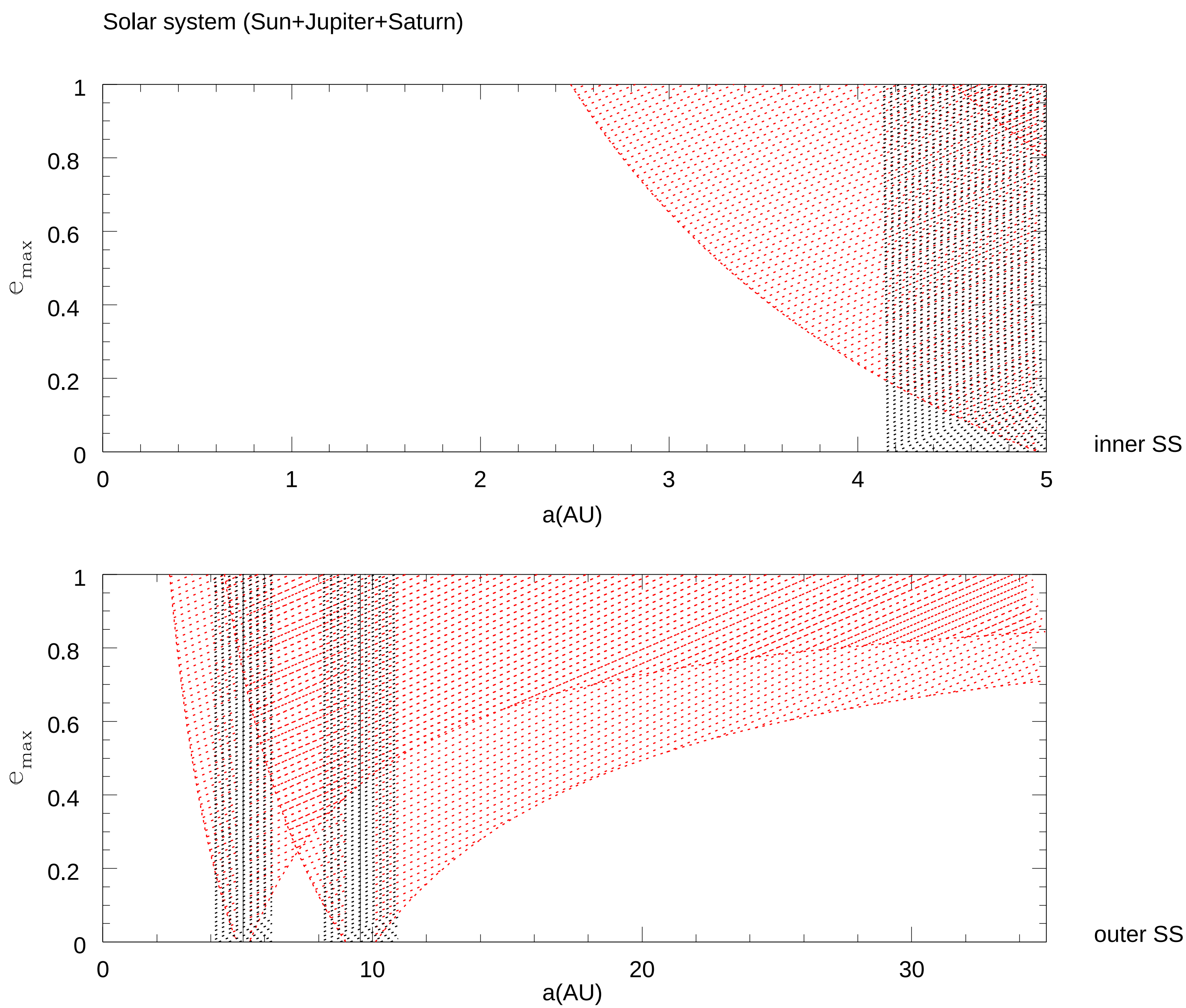
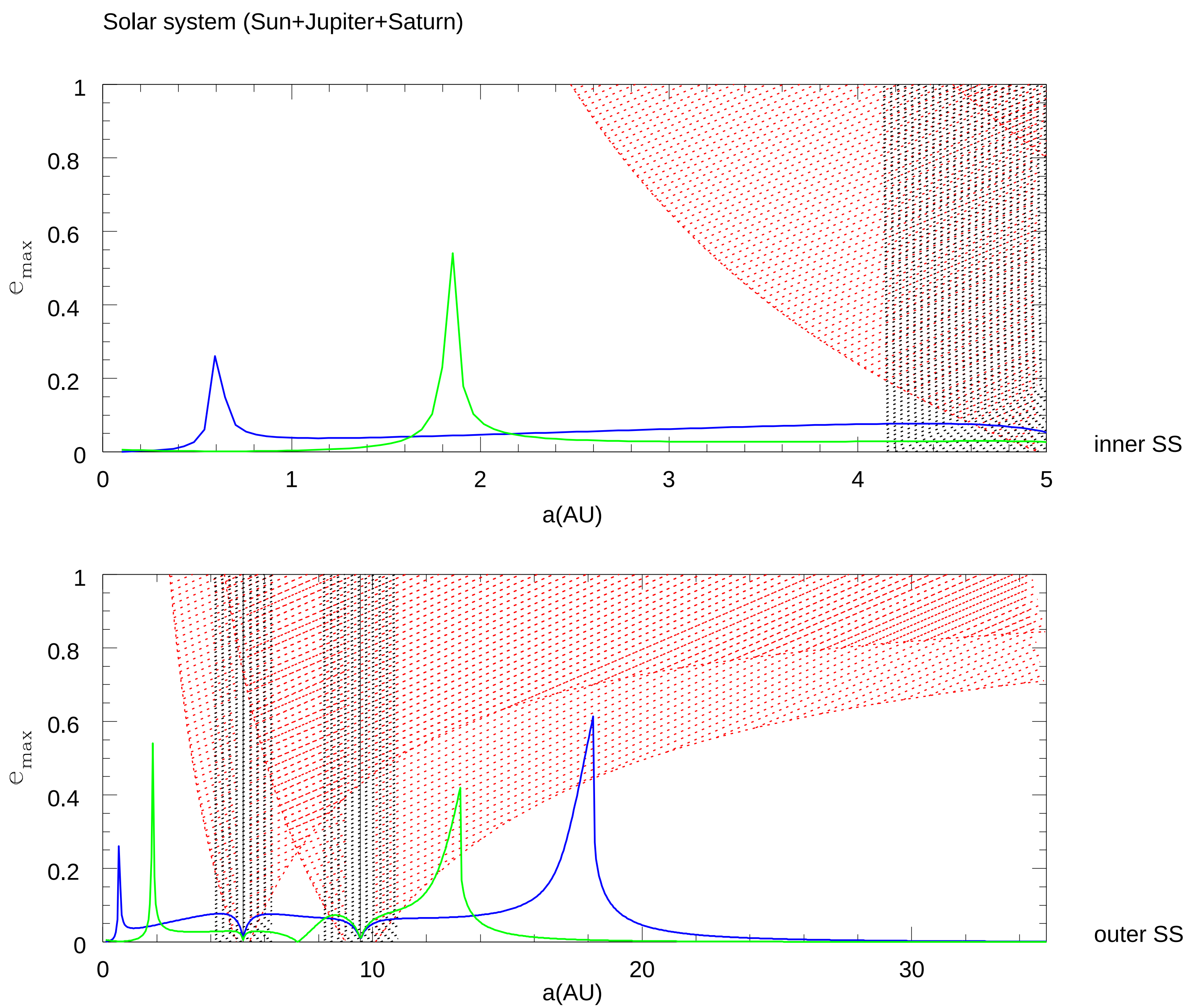
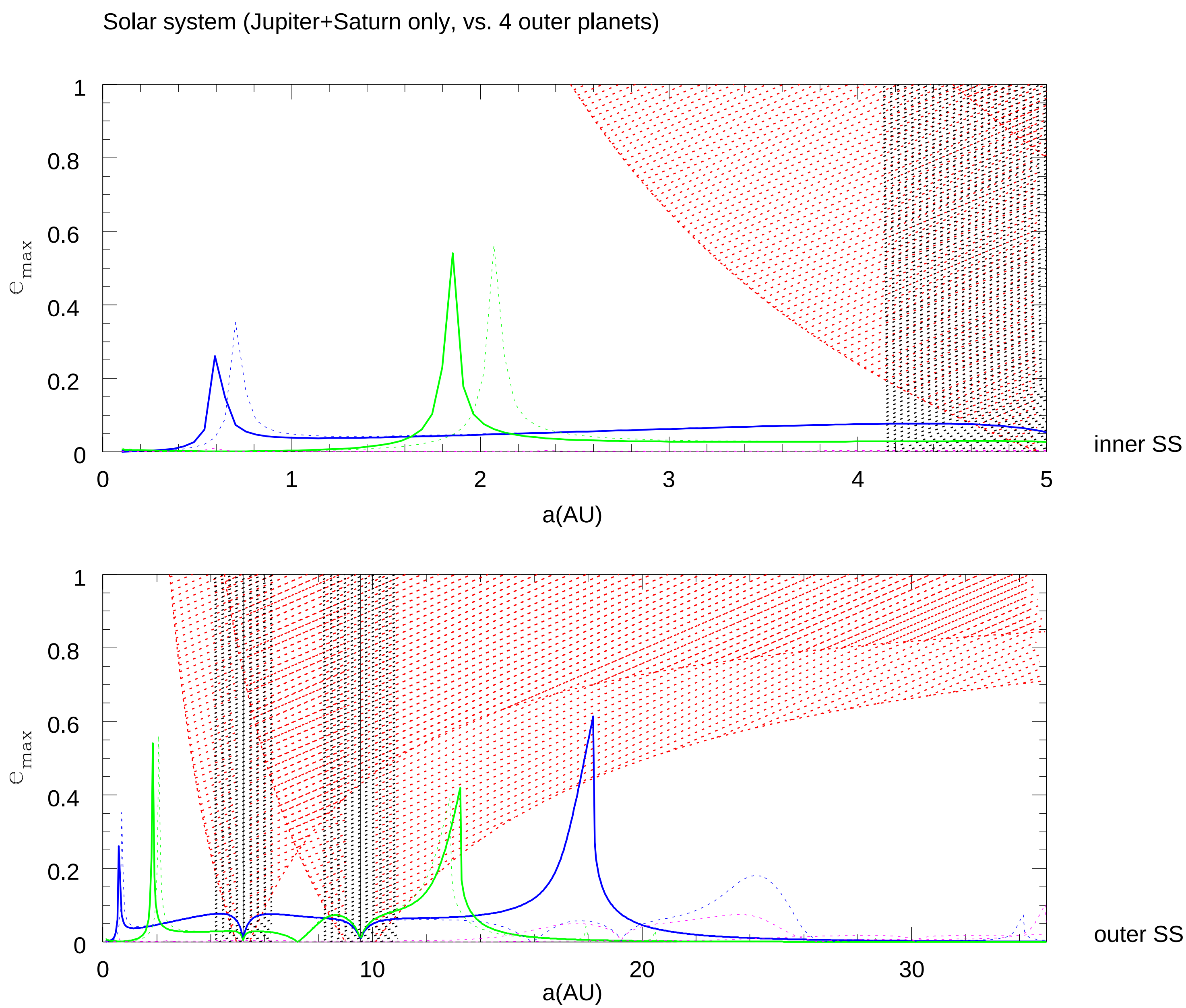


Solar system (Sun+Jupiter+Saturn)
e_{max}
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
a(AU)
inner SS
e_{max}
0
0.2
0.4
0.6
0.8
1
0
10
20
30
a(AU)
outer SS

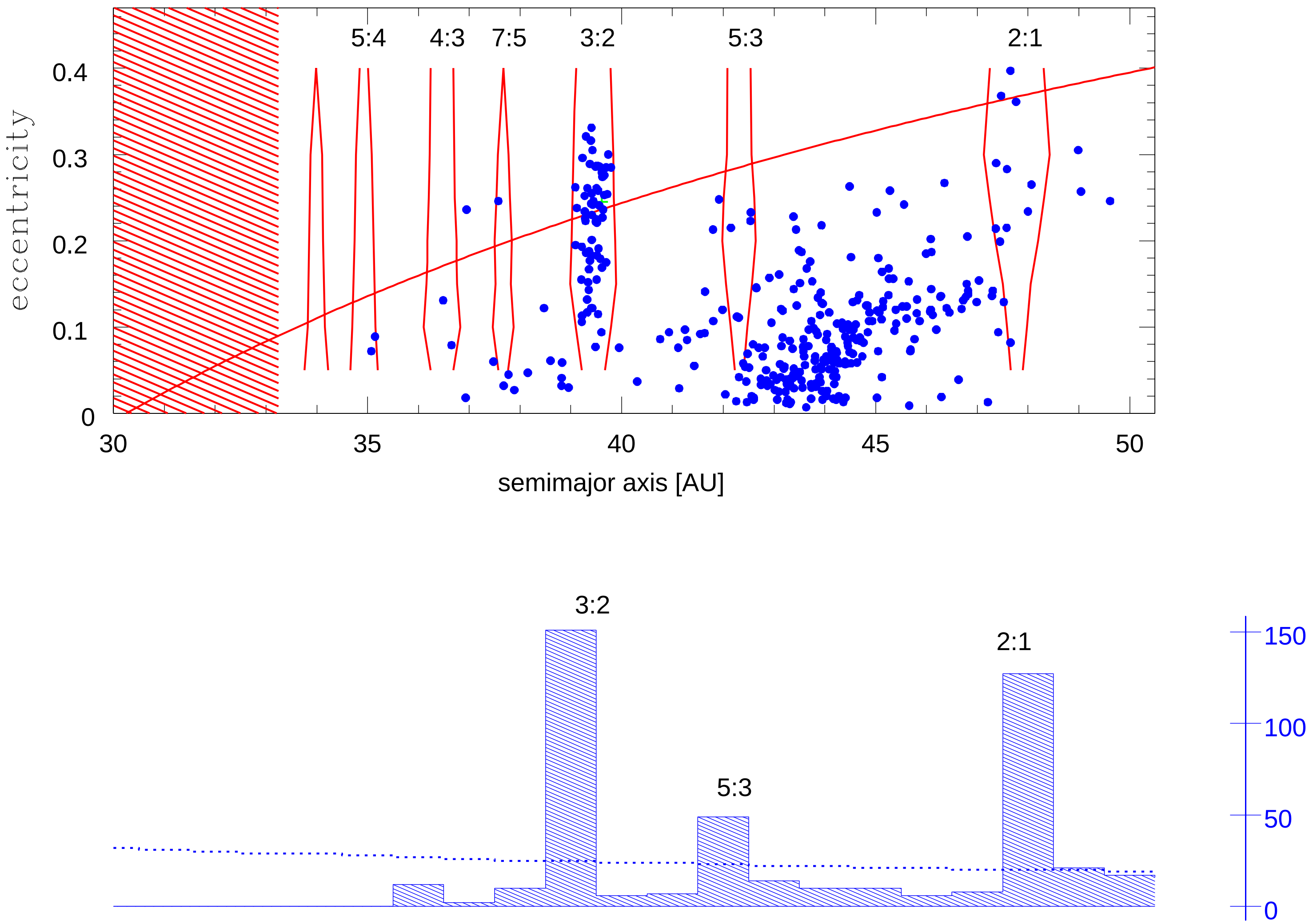

Solar system (Sun+Jupiter+Saturn)
e_{max}
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
a(AU)
inner SS
e_{max}
1
0.8
0.6
0.4
0.2
0
0
10
20
30
a(AU)
outer SS


Solar system (Jupiter+Saturn only, vs. 4 outer planets)
e_{max}
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
a(AU)
inner SS
e_{max}
1
0.8
0.6
0.4
0.2
0
0
10
20
30
a(AU)
outer SS

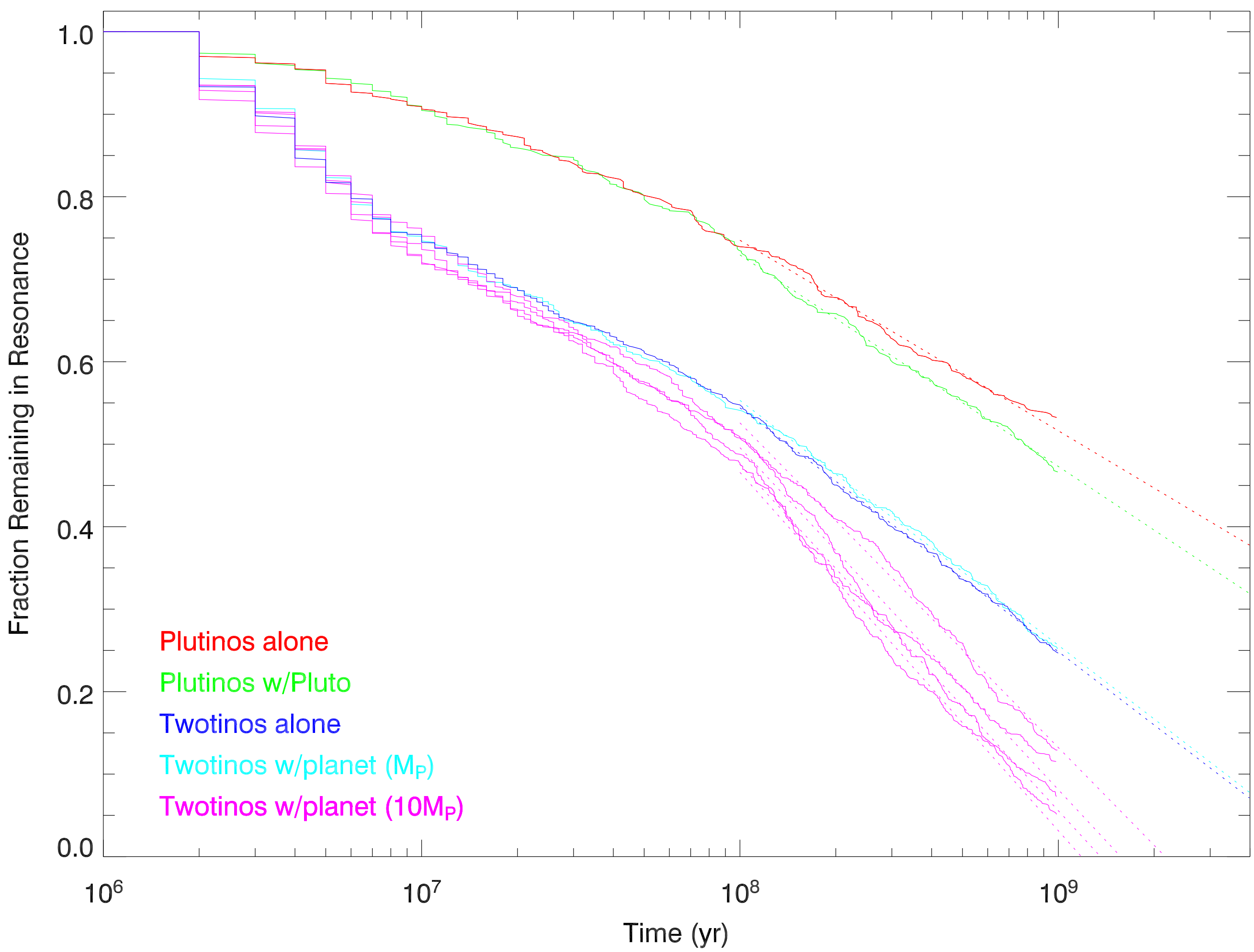
Possible debris locations in:

- 47 Ursae Majoris (two jovian-mass planets)
  - 0.3-0.6 AU
  - 1.0-1.5 AU
  - outside 7.5 AU
- υ Andromedae (three jovian-mass planets)
  - between 0.06 and 0.3 AU
  - outside 8 AU
- Sun–Jupiter–Saturn system
  - inward of 0.5 AU
  - between 0.8 and 1.6 AU
  - between 2.0 and 4.3 AU
  - outside 18 AU

Orbital Distribution of Kuiper Belt objects



Planet Migration and Resonance Sweeping of the Kuiper Belt


Fraction Remaining in Resonance
Time (yr)
1.0
0.8
0.6
0.4
0.2
0.0
10^6
10^7
10^8
10^9
Plutinos alone
Plutinos w/Pluto
Twotinos alone
Twotinos w/planet (M_P)
Twotinos w/planet ($10M_P$)

Conclusion: **Assuming equal initial populations of Plutinos and Twotinos, 4 Gyr of dynamical diffusion reduces the ratio of Twotinos:Plutinos to < 0.2.**

Table 1. **Fate of Resonance Escapees**

	Plutinos	Twotinos
Avg. lifetime	150 My	270 Myr
lifetime $>$ 1 Gy	5%	29–36%
Reach inner SS	$\sim$ 25%	$\sim$ 20%