Vortices in Disks and Proto-Planet Migration

Hui Li (LANL)

Collaborators:

Josef Koller (LANL/Rice), D. Lin (UCSC)

B. Wendroff (LANL), E. Liang (Rice)

Migration due to disk-planet tidal interaction

- Previous talks for review: ……
- Focus on the potential vorticity (p.v.) evolution
- Possibility and role of secondary instabilities in the co-orbital region

Potential Vorticity ζ

- Potential Vorticity (PV or vortensity):

$$\zeta\,\hat{\mathbf{z}} = \frac{(\nabla\times\mathbf{v})_z}{\Sigma}$$

- PV along streamlines for inviscid flow:

$$\frac{D(\zeta\,\hat{\mathbf{z}})}{Dt} = (\zeta\,\hat{\mathbf{z}}\cdot\nabla)\mathbf{v} + \frac{1}{\Sigma^3}\nabla\Sigma\times\nabla p$$

  - If barotropic p=p(Σ), baroclinic term

$$\nabla\Sigma\times\nabla p = 0$$

  - and in 2D ($\hat{\mathbf{z}}\cdot\nabla = 0$):

$$\frac{D\zeta}{Dt} \equiv \mathbf{0}$$

Break the conservation of Potential Vorticity

$$\frac{D\zeta}{Dt} = \textbf{viscosity} + \textbf{shocks} + \textbf{non-adiabatic forcing}$$

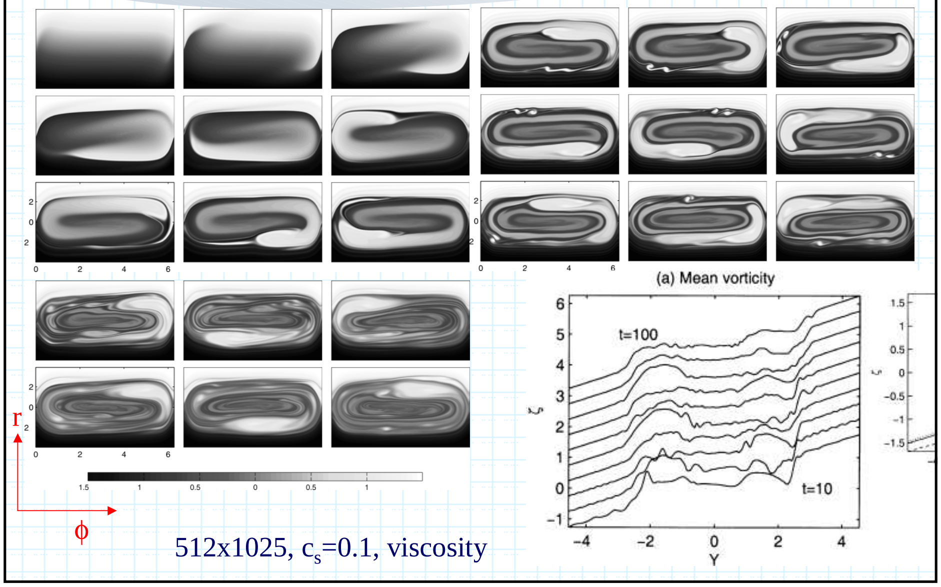
- Viscosity: either imposed or numerical
- Spiral shocks, cutting through the whole disk
- Switch-on of the planet mass

Problem Setup

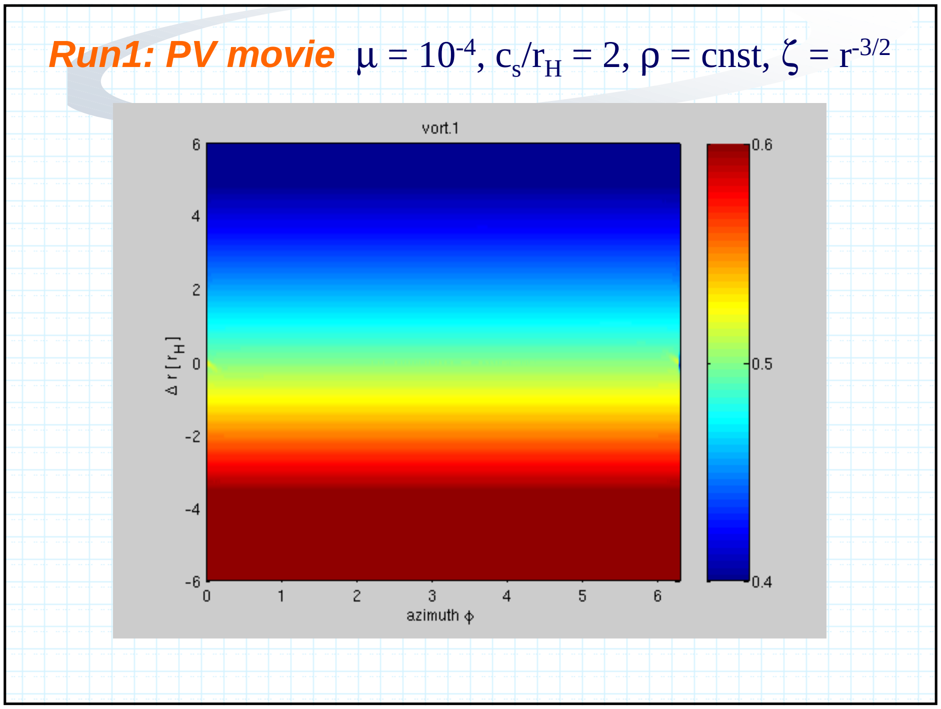
- 2D disk size: $0.4 < r / r_p < 2.0$
- Isothermal: $c_s / v_{\phi 0} = 0.05$, initially in force equilibrium
- Initial: a) $\rho = cnst,\ v_\phi = r^{-1/2},\ \zeta = r^{-3/2}$; **run1**
  b) $\rho = r^{-3/2},\ v_\phi = r^{-1/2},\ \zeta = cnst$; **run2, 3**
- Planet mass: $\mu = 10^{-4}$, no gaps, gradually "turned-on", on fixed orbits, $e=0$

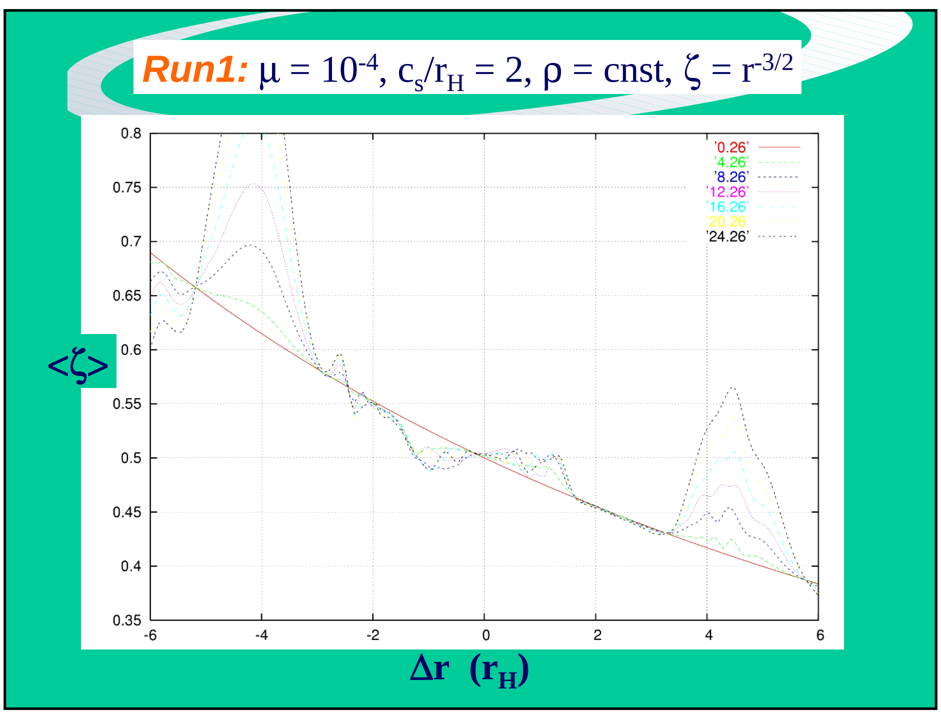
- Resolution: typically (n_r, n_φ)=(300,1200) up to (600x2400)
- Hybrid: Lax-Wendroff + Lax-Friedrich (for shocks)
- Masset's FARGO for fast integration
- No explicit artificial viscosity → inviscid limit
- In co-rotating frame where star and planet are fixed

Balmforth & Korycansky'01:
Flow inside the critical annual region of corotation resonance

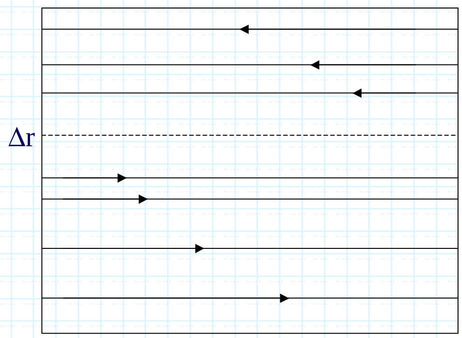


512x1025, c_s=0.1, viscosity

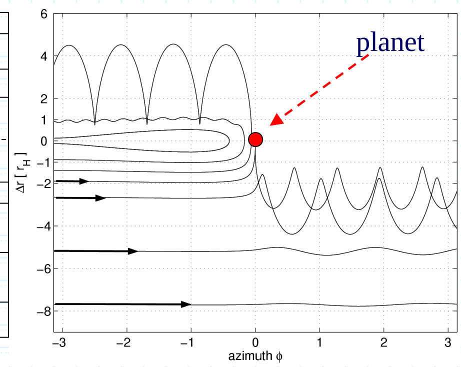

Run1: PV movie μ = 10^-4, c_s/r_H = 2, ρ = cnst, ζ = r^-3/2
vort.1
Δ r [r_H]
6
4
2
0
-2
-4
-6
0 1 2 3 4 5 6
azimuth φ
0.6
0.5
0.4


Run1: μ = 10^-4, c_s/r_H = 2, ρ = cnst, ζ = r^-3/2
<ζ>
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
-6 -4 -2 0 2 4 6
'0.26'
'4.26'
'8.26'
'12.26'
'16.26'
'20.26'
'24.26'
Δr (r_H)

Joining flow-lines by turning on planet

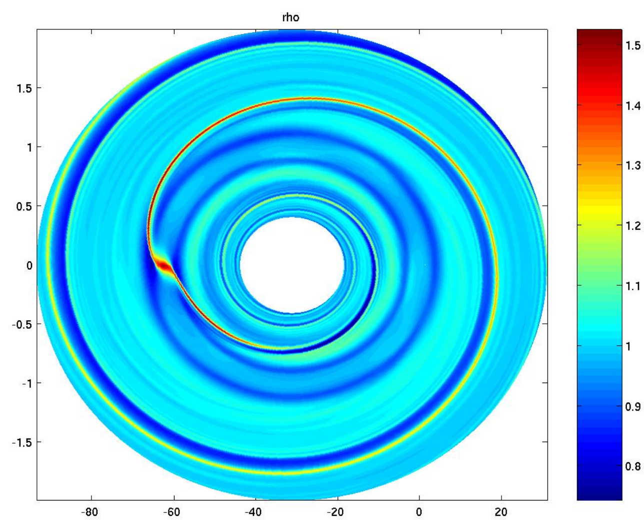


Keplerian Rotation



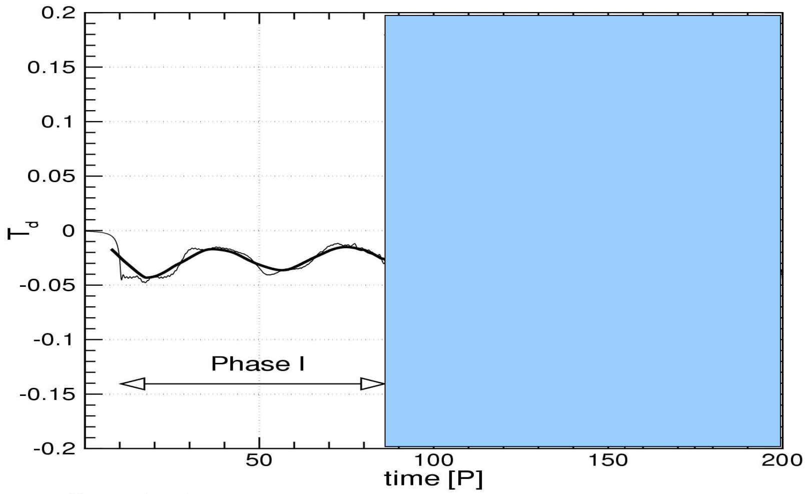
$$r_H/r_p = (\mu/3)^{1/3},\ \mu = M_p/M_*$$

Run2: $\mu = 10^{-4}$, $r_H = 0.032$, c_s=0.05, $\rho = r^{-3/2}$, ζ = cnst



Shown: surface density $\times\ r^{3/2}$

Torque on the planet



Koller, Li, Lin'03

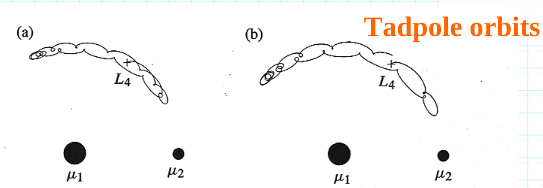


Fig. 3.16. Two examples of tadpole orbits librating about the L_4 equilibrium point (denoted by a cross and located at $x_0 = 1/2 - \mu_2$, $y_0 = \sqrt{3}/2$) for $\mu_2 = 0.001$. The masses μ_1 and μ_2 are denoted by the filled circles. (a) The starting conditions are $x = x_0 + 0.0065$, $y = y_0 + 0.0065$ with $\dot{x} = \dot{y} = 0$ and the orbit is followed for 15 orbital periods of μ_2. (b) The starting conditions are $x = x_0 + 0.008$, $y = y_0 + 0.008$ with $\dot{x} = \dot{y} = 0$ and the orbit is followed for 15.5 orbital periods of μ_2.

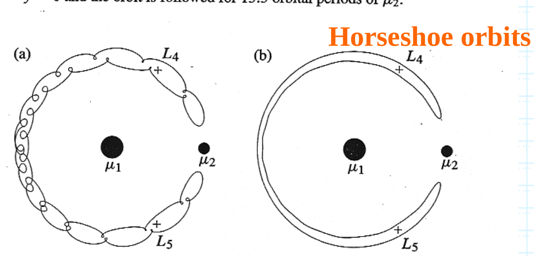


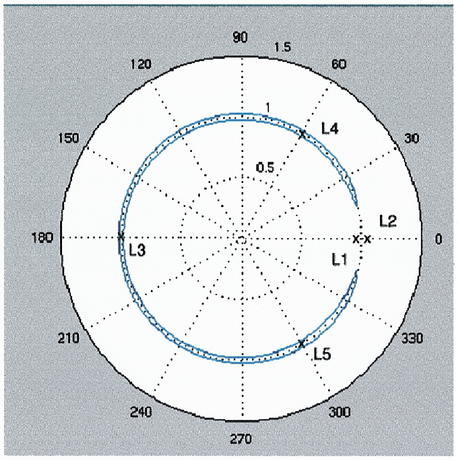
Fig. 3.17. Two examples of near-periodic horseshoe orbits librating about the L_4 equilibrium point for $\mu_2 = 0.000953875$, taken from data given by Taylor (1981). (a) The starting conditions are $x = -0.97668$, $y = \dot{x} = 0$, $\dot{y} = -0.06118$. (b) The starting conditions are $x = -1.02745$, $y = \dot{x} = 0$, $\dot{y} = 0.04032$.

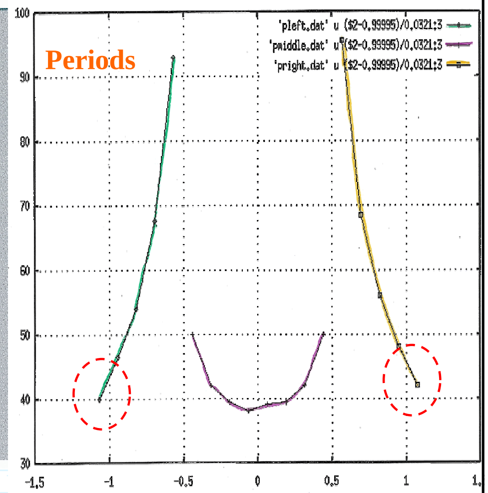
Restricted 3 Body Simulations

- Solve equation of motion in co-rotating system
- Fourth order Runge-Kutta scheme with adaptive time control

Murray and Dermott (1999)

Periods of Particles

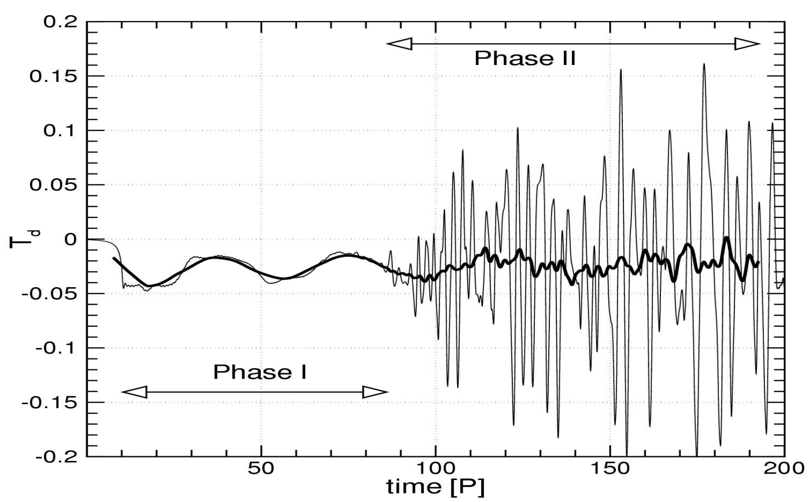




Libration period: ~40 orbital period

Distance to L4

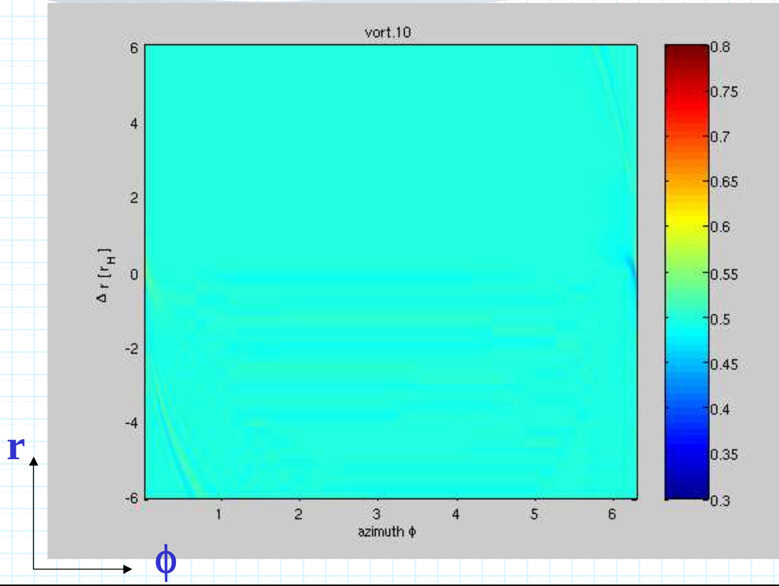
Surprise

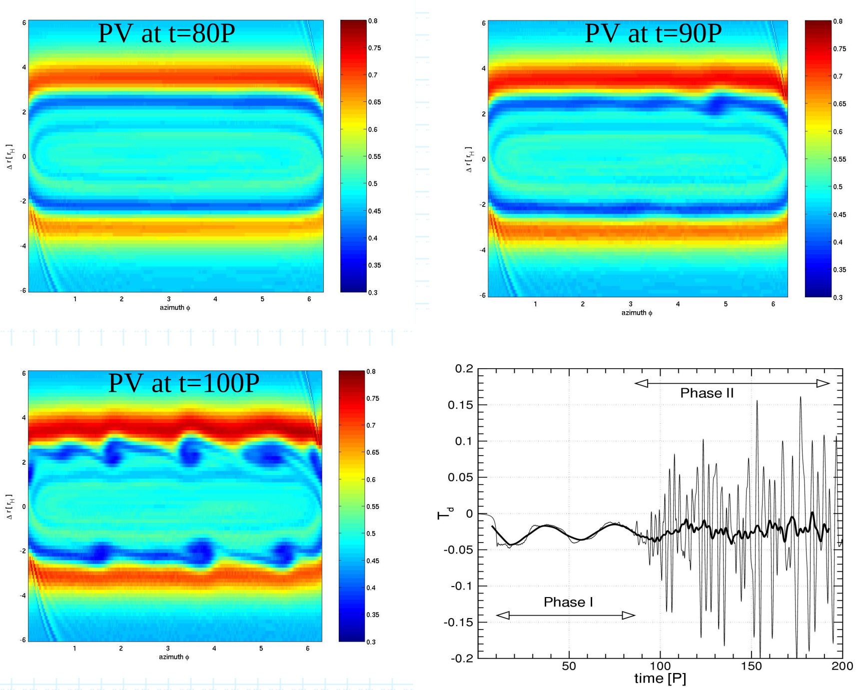


Koller, Li, Lin'03

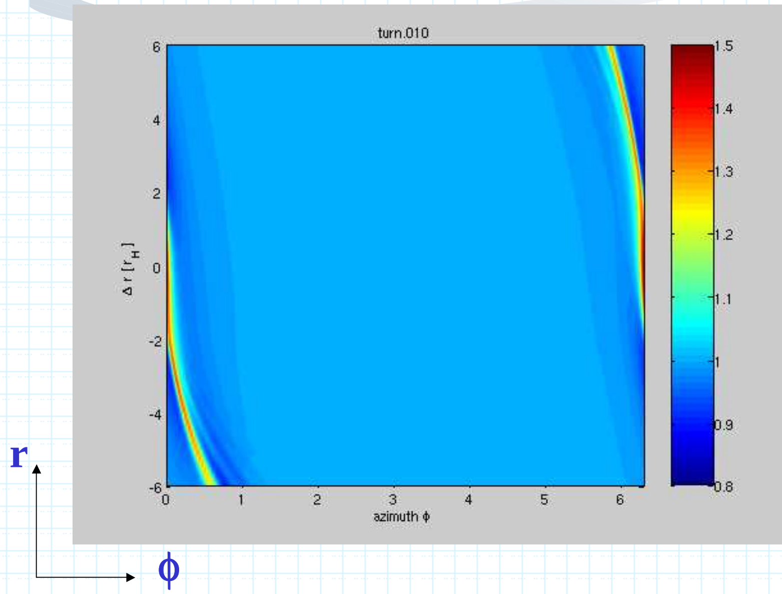
Run2: PV movie

$\mu=10^{-4}$, $c_s/r_H = 1.55$, $\rho = r^{-3/2}$, $\zeta =$ cnst





Run2: Density Movie

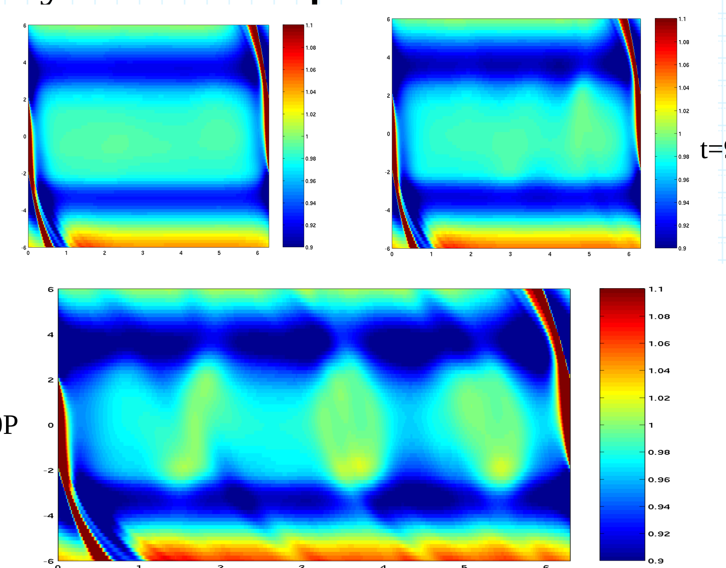


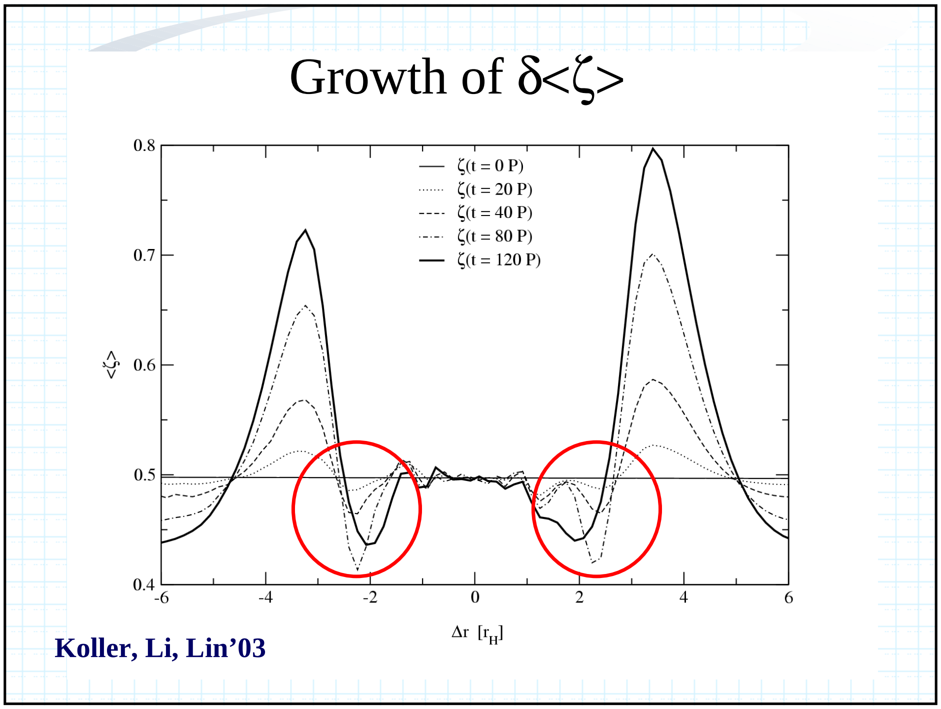
Density Evolution $\rho \times r^{3/2}$

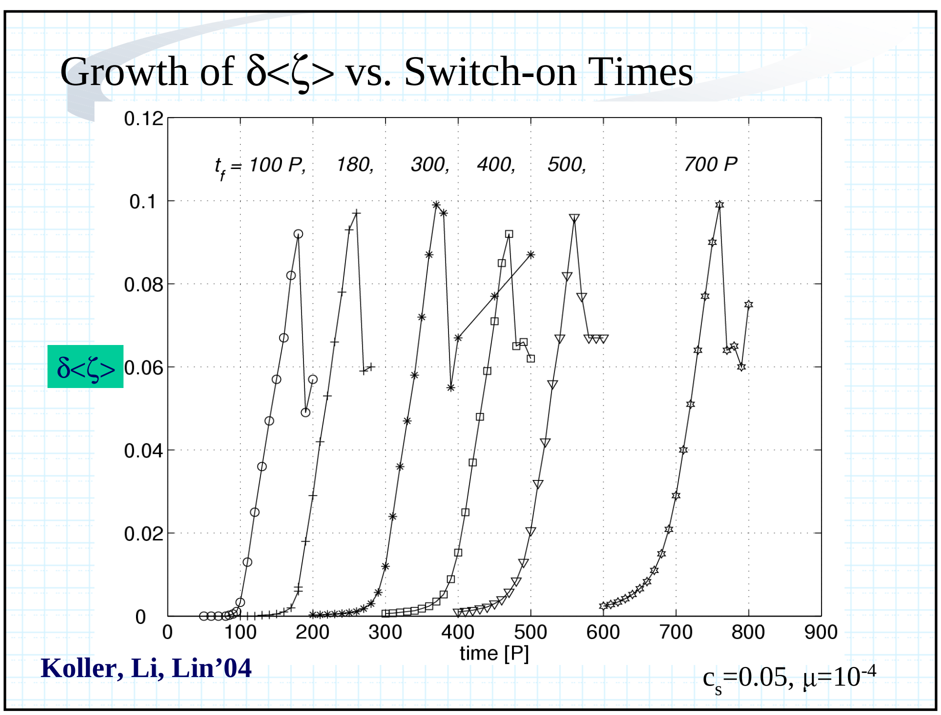
t=80P

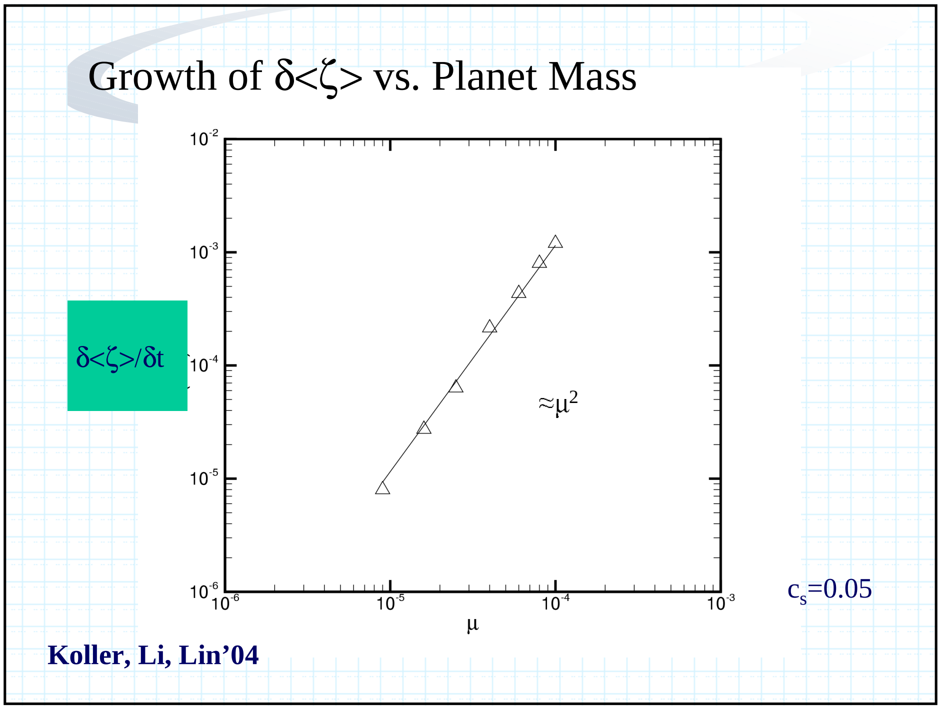
t=90P

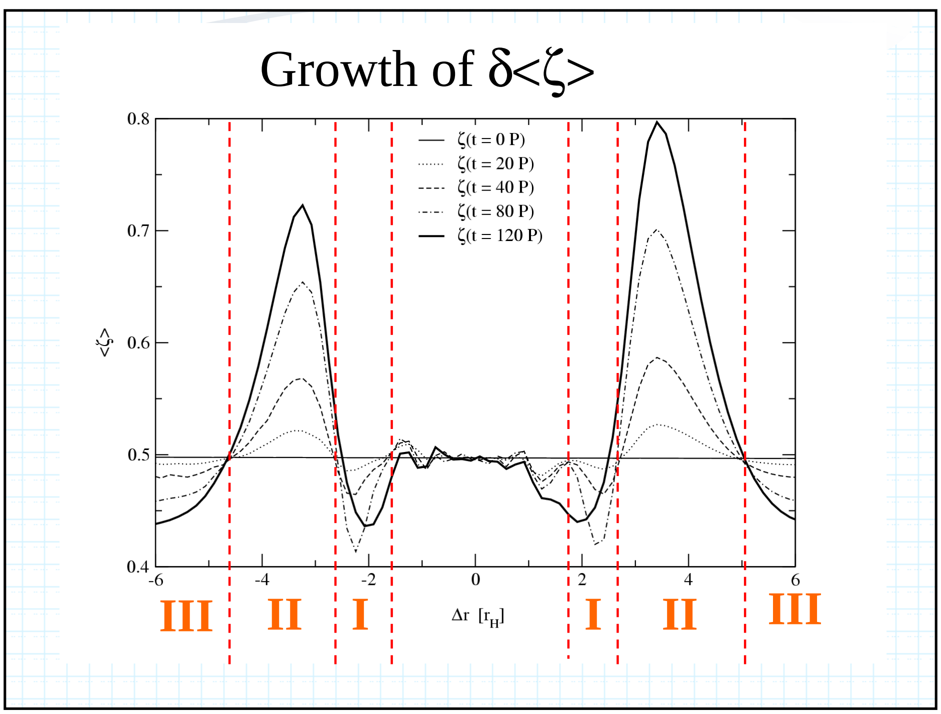
t=100P



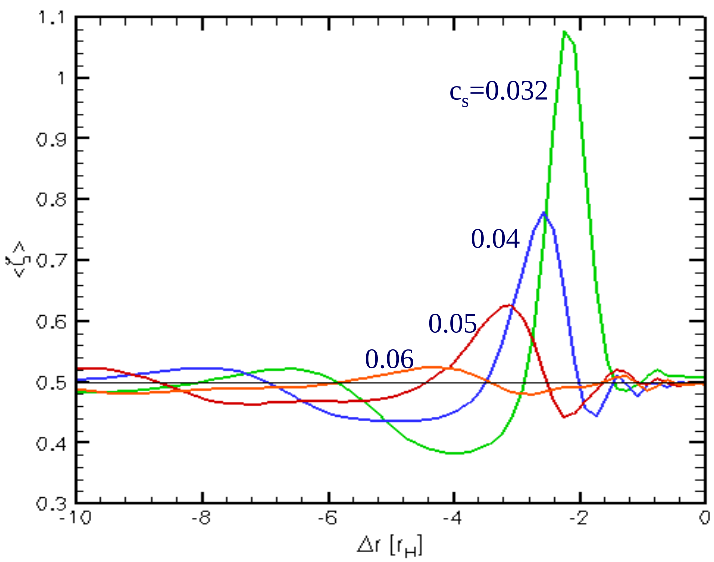

Growth of δ<ζ>
ζ(t = 0 P)
ζ(t = 20 P)
ζ(t = 40 P)
ζ(t = 80 P)
ζ(t = 120 P)
<ζ>
Δr [r_H]
Koller, Li, Lin'03


Growth of δ<ζ> vs. Switch-on Times
t_f = 100 P, 180, 300, 400, 500, 700 P
δ<ζ>
time [P]
Koller, Li, Lin'04
c_s=0.05, μ=10^-4

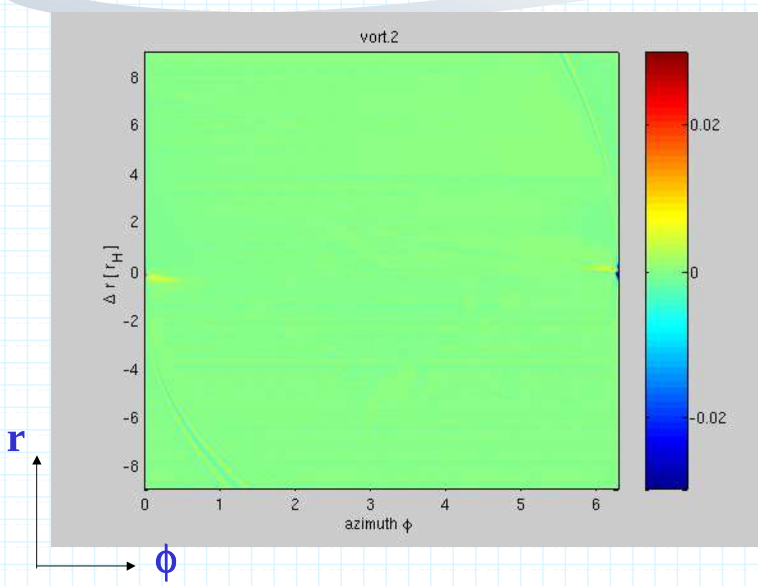

Growth of δ<ζ> vs. Planet Mass
δ<ζ>/δt
≈μ²
μ
c_s=0.05
Koller, Li, Lin'04


Growth of δ<ζ>
ζ(t = 0 P)
ζ(t = 20 P)
ζ(t = 40 P)
ζ(t = 80 P)
ζ(t = 120 P)
<ζ>
Δr [r_H]
III
II
I
I
II
III

Different c_s



Run3: PV movie μ=10^{-4}, c_s/r_H = 2 , ζ = cnst



PV Changes due to shocks Region II & III

- Vorticity jump across shock: Truesdell (‘52), Lighthill (’57), Hayes(’57),…, Kevlahan(’97)

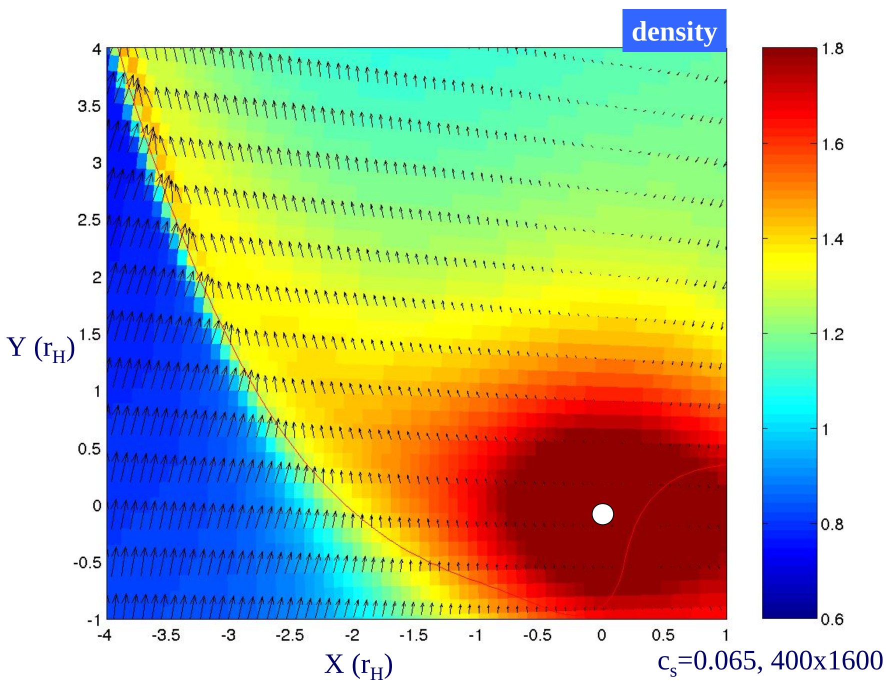
$$\delta\omega = \frac{q^2}{1+q}\frac{\partial Cr}{\partial S} + baroclinic \quad term \; + q\,\omega_a$$

$$where \quad q = \frac{\rho_b}{\rho_a} - 1, \;\; Cr = -V_\perp \,(in \;\; shock \;\; frame\,)$$

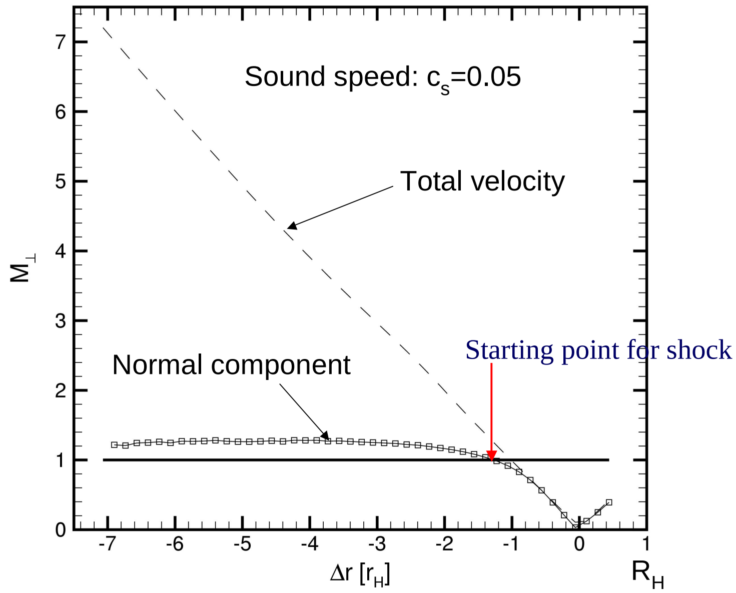
- For isothermal shock, potential vorticity jump:

$$\delta\varsigma = \frac{1}{\rho_b}\frac{q^2}{1+q}\;\frac{\partial Cr}{\partial S}$$

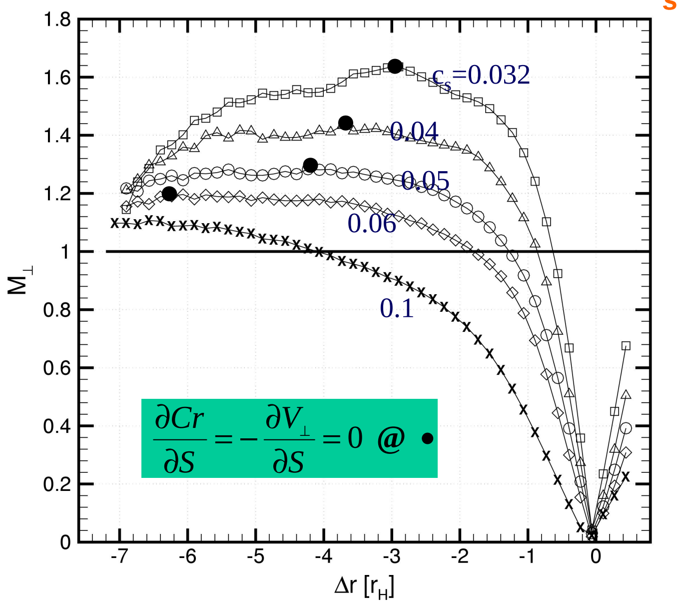
density_factor   curvature_factor



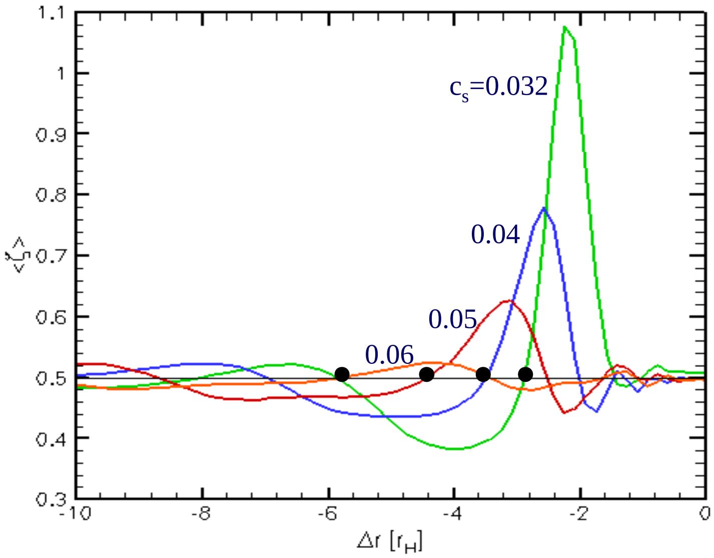
Shock Structure



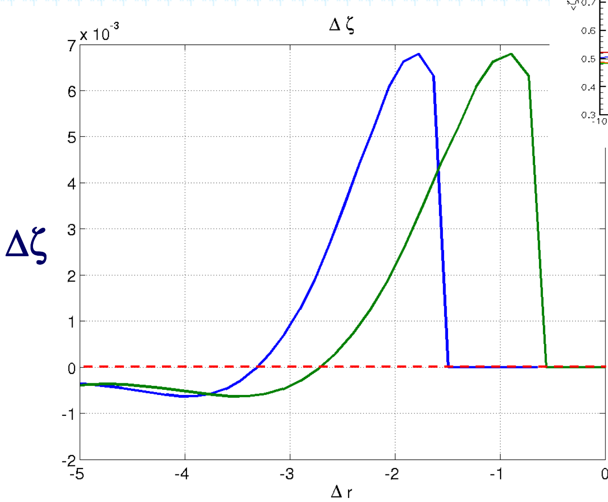
Curvature Factor for different c_s

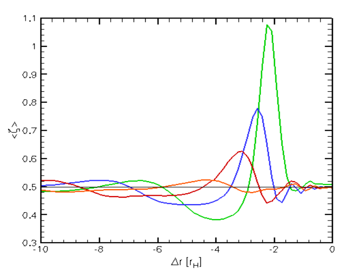


Different c_s

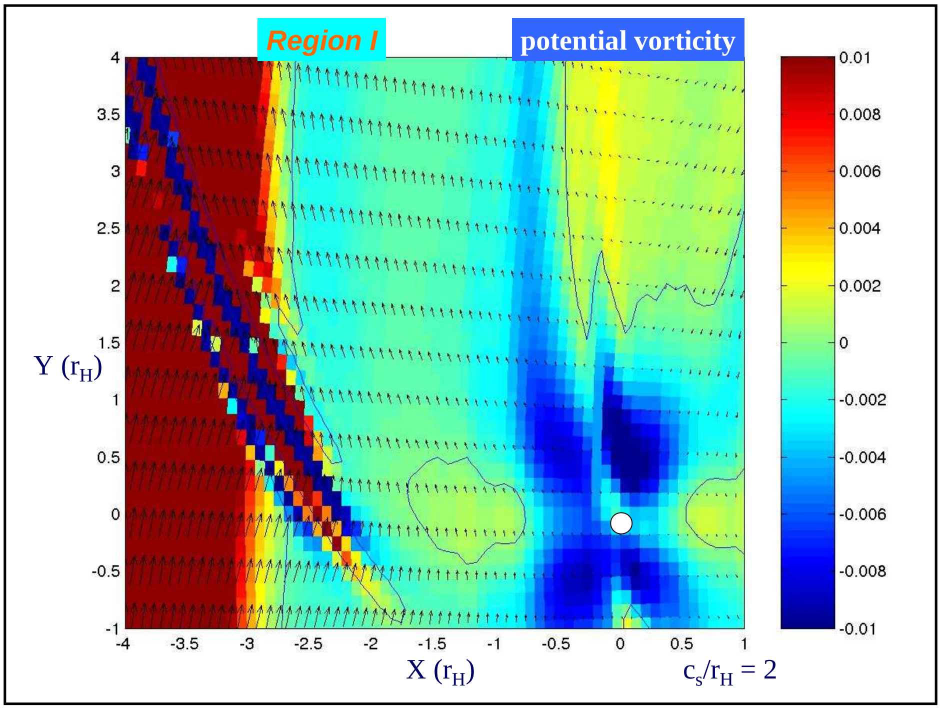


Combined with density factor…





c_s=0.032

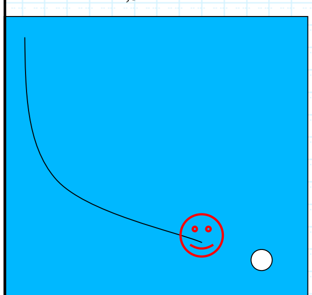


Region I : Uncertain

Vorticity Jump : $\delta\omega = [u_{s,n} - u_{n,s}]$

$[u_{s,n}]$ *by jump in s – component* : $\rho\left(\frac{D\mathbf{u}}{Dt}\right) = -\nabla p + \mathbf{F}_{ext}$

$[u_{n,s}]$ *by s-derivative of R-H condition:* $[P + \rho u_n^2]$



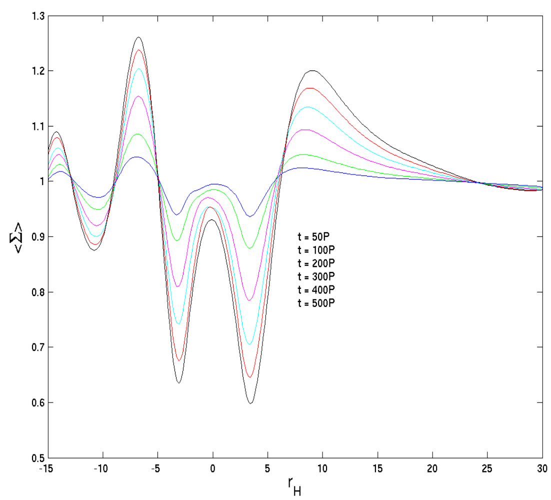
Two complications:

- **at the "tip" of shock, not clear how to apply the calculations.**
- **F_{ext} needs to be included.**

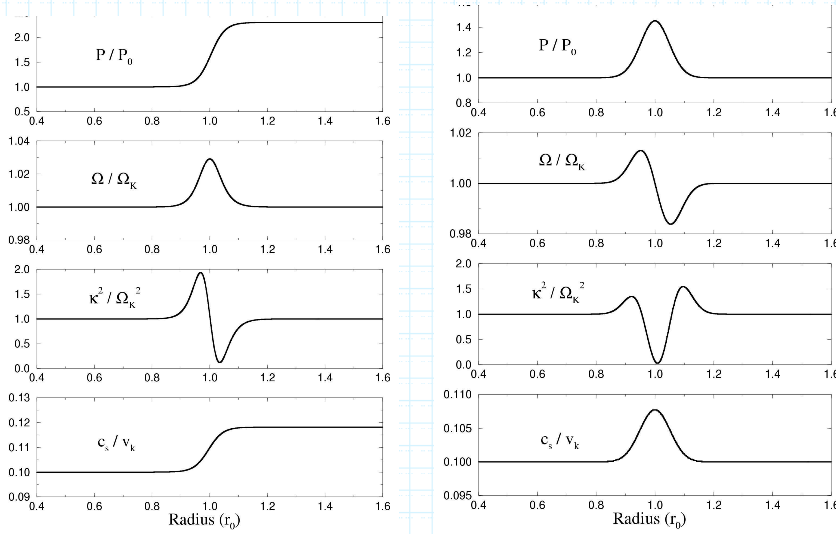
Secondary Instability

- Shocks generate/destroy p.v.
- Formation of inflexion points in p.v. profile
  e.g., "dips", "gaps", "bumps" can be unstable!
- Global non-axisymmetric instability
  (e.g., Papaloizou & Pringle instability'85,
  Rossby Vortex Instability, Li et al.'00,01)
- Perform linear theory analysis:
  1) threshold;
  2) fast growth rates;
  3) vortex as the non-linear outcome.

Density Profile

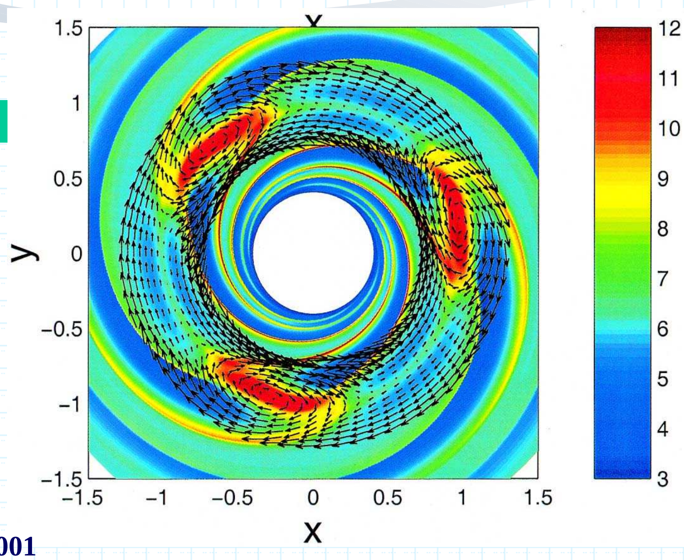


Non-axisymmetric Global Instability (Li et al. 2000, 2001)

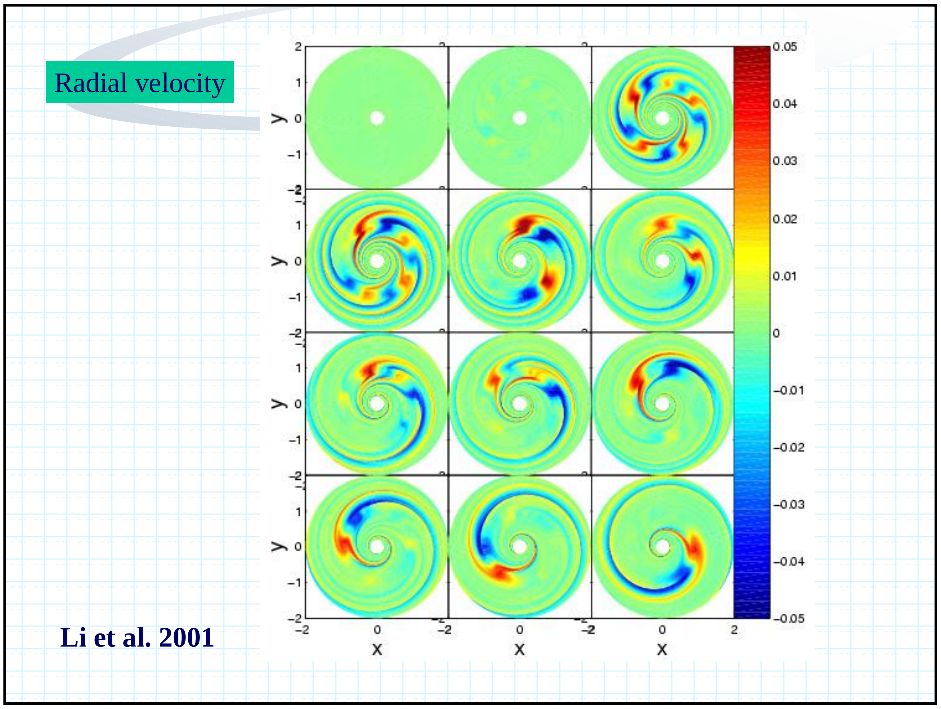


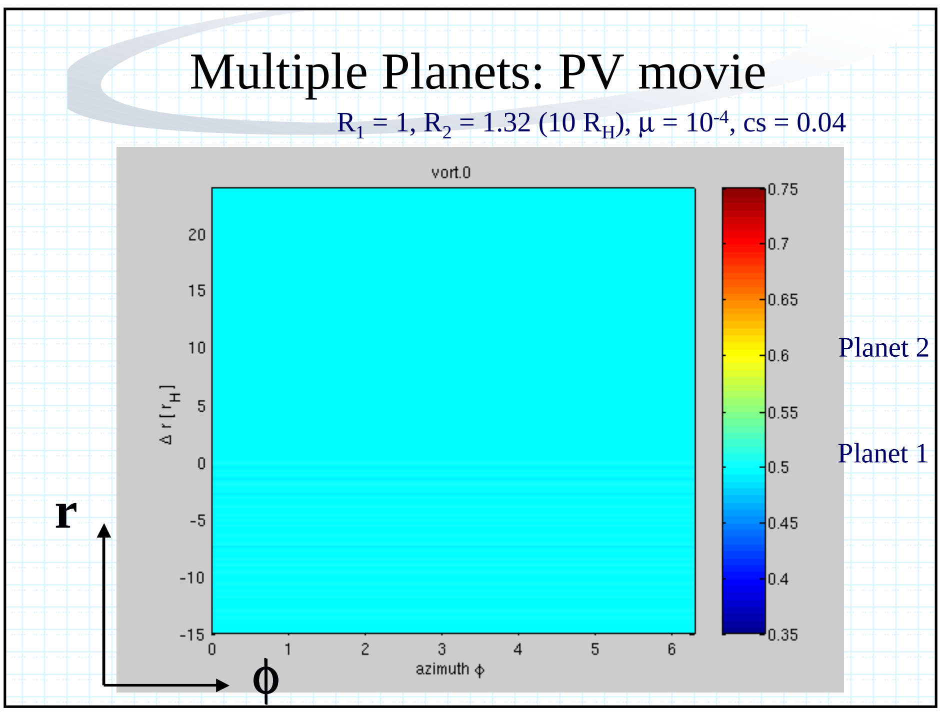
Rossby Vortices

Pressure

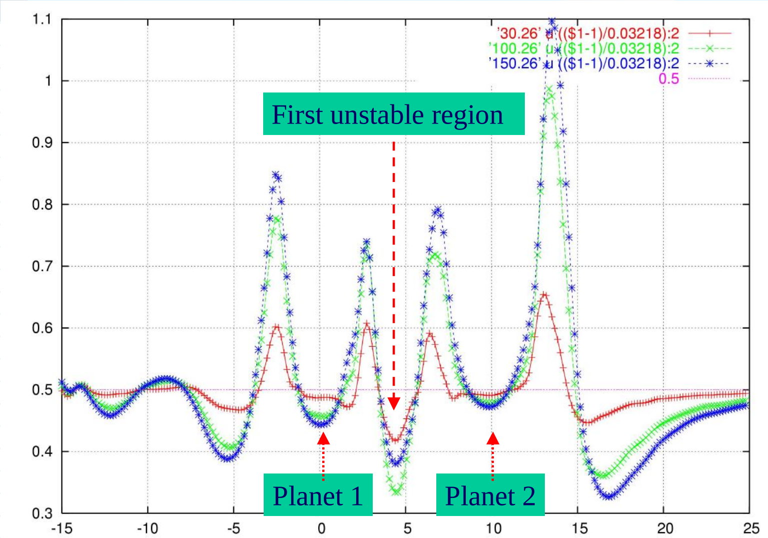


Li et al. 2001


Radial velocity
Li et al. 2001
y
x
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05


Multiple Planets: PV movie
$R_1 = 1$, $R_2 = 1.32$ (10 R_H), $\mu = 10^{-4}$, cs = 0.04
vort.0
Δ r [r_H]
azimuth φ
Planet 2
Planet 1
r
φ

<PV> for two planets



Summary

- Low viscosity simulation show:
  - non-conservation of PV due to shocks
  - developing inflection points in PV profile, which eventually de-stablize the flow
  - vortices as non-linear outcome
  - torque changes significantly due to “collisions” between vortices and planet, implication for migration and eccentricity driving

- Future work:
  - Migrating planets, eccentricity driving
  - Multiple planets