The Final Chapter of Planet Formation

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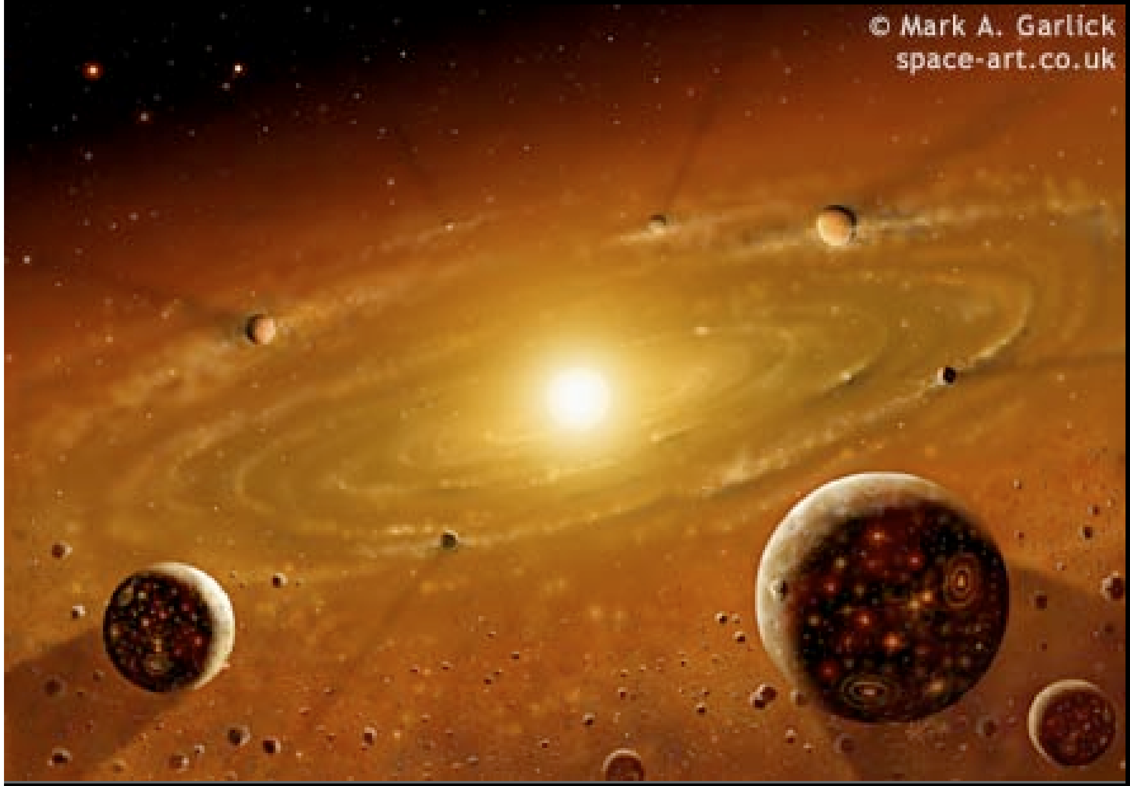

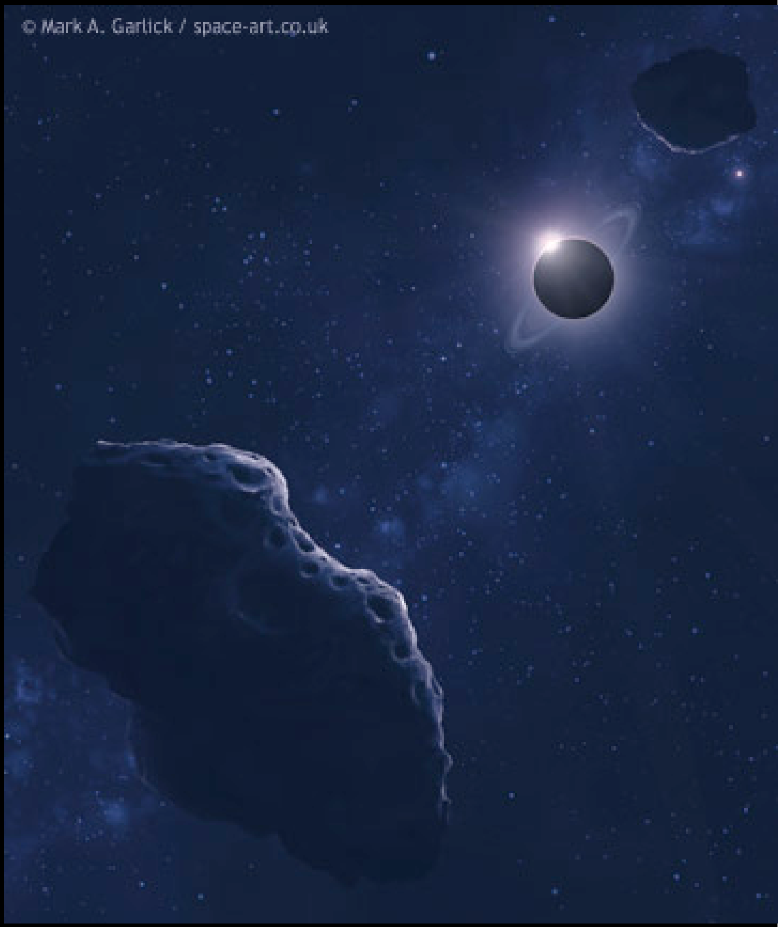

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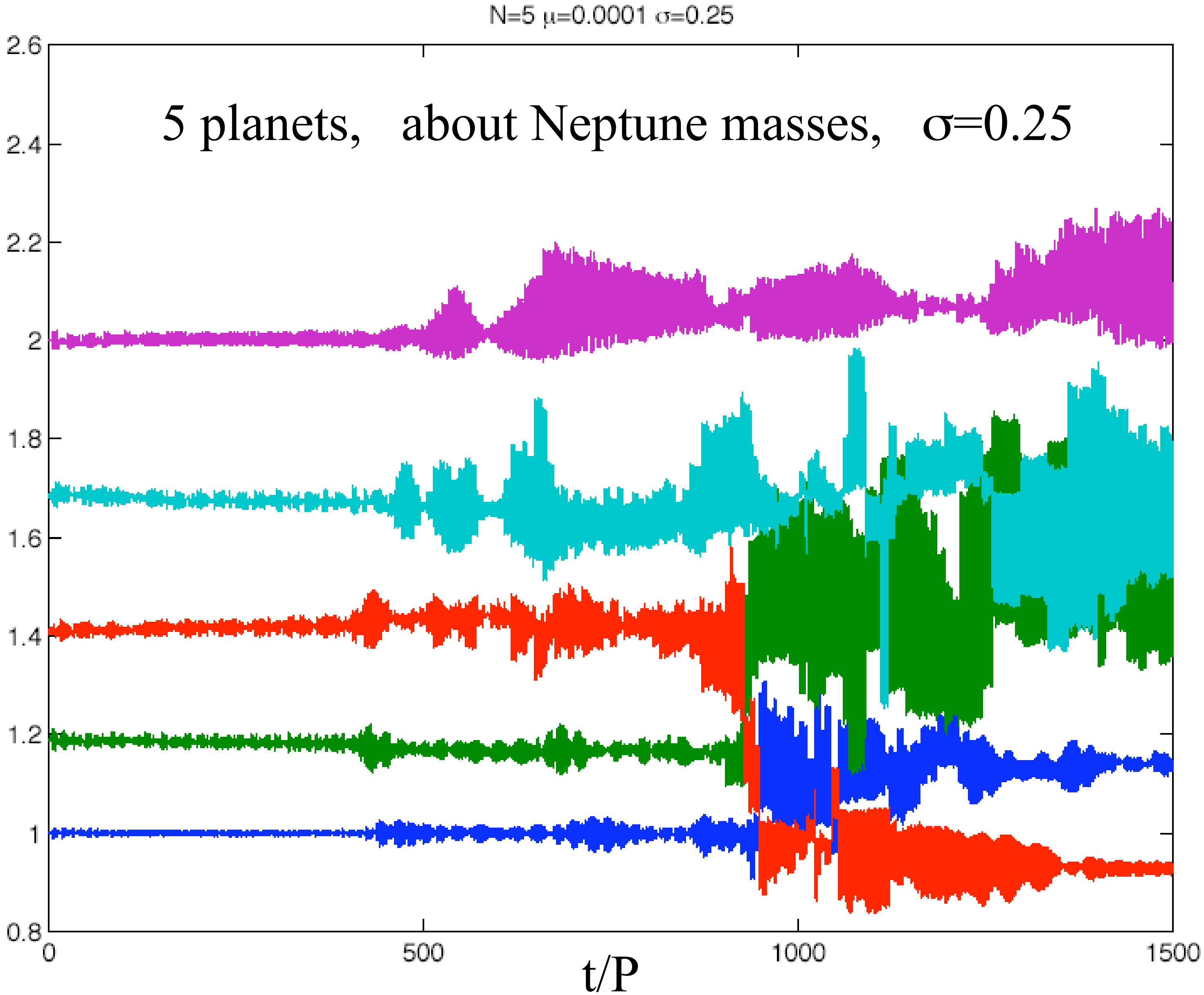
Questions

- When did oligarchy end?
- What set the number of solar system planets?
- How long did it take them to form?
- Why are their orbits circular and coplanar?

Answers

- When did oligarchy end?
  - When $\Sigma = \sigma$!! GENERAL RESULT
- What set the number of solar system planets?
  - Stability: No wide range chaos.
- How long did it take them to form?
  - About 100 Myr at a~1AU. Faster at a~30AU !
  - More time until ejection for Uranus-Neptune.
- Why are their orbits circular and coplanar?
  - Velocity damping by small bodies.

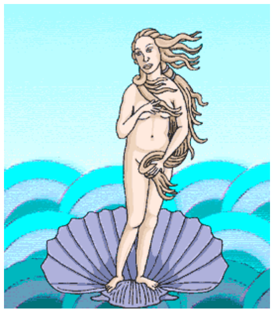
Preliminary Simulation



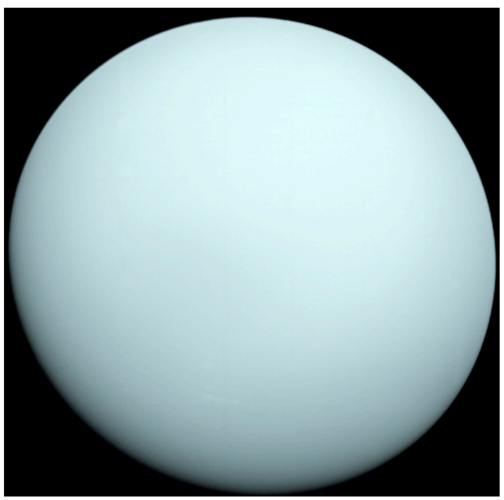
Collide or Eject?

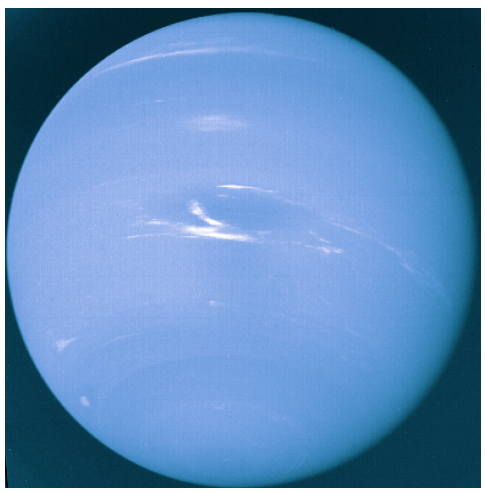
$$\frac{V_{esc}}{V_{orbit}} \sim \alpha^{-3/2}\left(\frac{\sigma}{\rho a}\right)^{1/3} \sim \begin{cases} 0.16 & a = 1\mathrm{AU} \\ 3 & a = 25\mathrm{AU} \end{cases}$$

$$V_{esc} = V_{orb} \quad at \quad a \sim 3AU$$



Collide

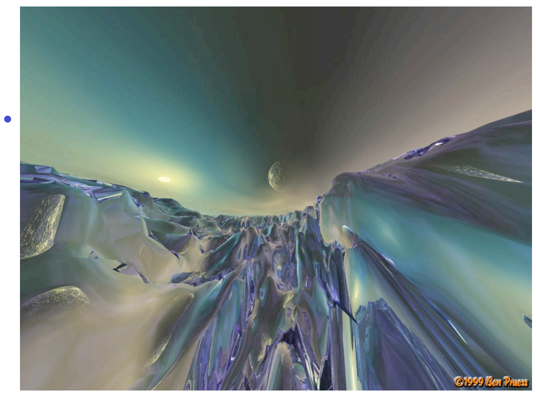




Eject

More Implications

- Small bodies:
  - Inner planetary region:
    - Most will be accreted.
    - MMSN is sufficient!
  - Outer solar system: ~5 MMSN to form Neptune.
- Large ejected bodies:
  - ~3 bodies >M_{earth} could be in Oort cloud.
  - Origin in outer planetary system
  - Survival depends on their long term stability and solar environment.



- Giant impacts:
  - Expected internal to 3AU.
  - Our moon

Orbital Regularization

- Eccentricity decays due to leftover small bodies.
  - Initial timescale = ejection (outer) or collision (inner) timescale
- Gas effects?
  - Could have helped in cooling the small bodies during oligarchy.
  - Unlikely to be present at the end stages
    - 100Myr for inner solar system
    - 1Gyr after ejection in outer solar system
  - Must rely on small bodies.
- Residual mass (of small bodies) during regularization?
  - Of order the initial mass in outer solar system
  - Perhaps somewhat smaller in inner solar system (delicate balance between accretion and shattering).

Gap Formation

- Once e decays ($e<\Delta a/a$) planet disk torque open gaps.
- Size of gap:

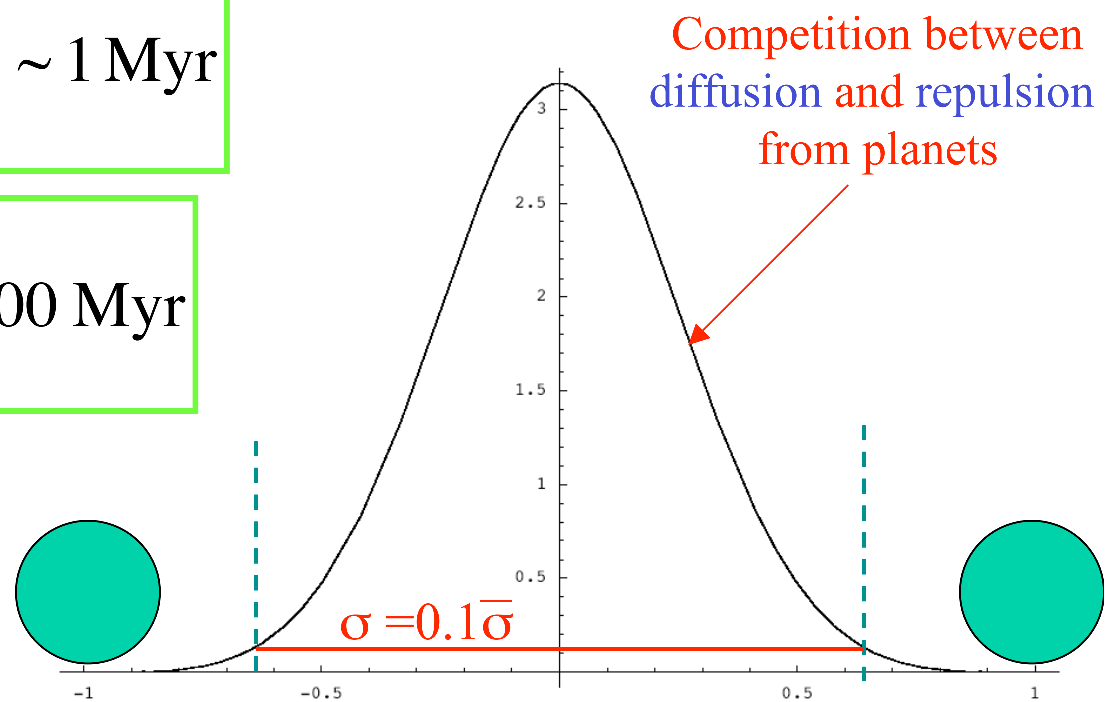
$$\frac{x}{a} \sim \left(\frac{M_{Planet}}{M_{Sun}}\right)^{2/5} (T\Omega)^{1/5} \approx \left(\frac{T}{1\text{ Gyr}}\right)^{1/5}$$

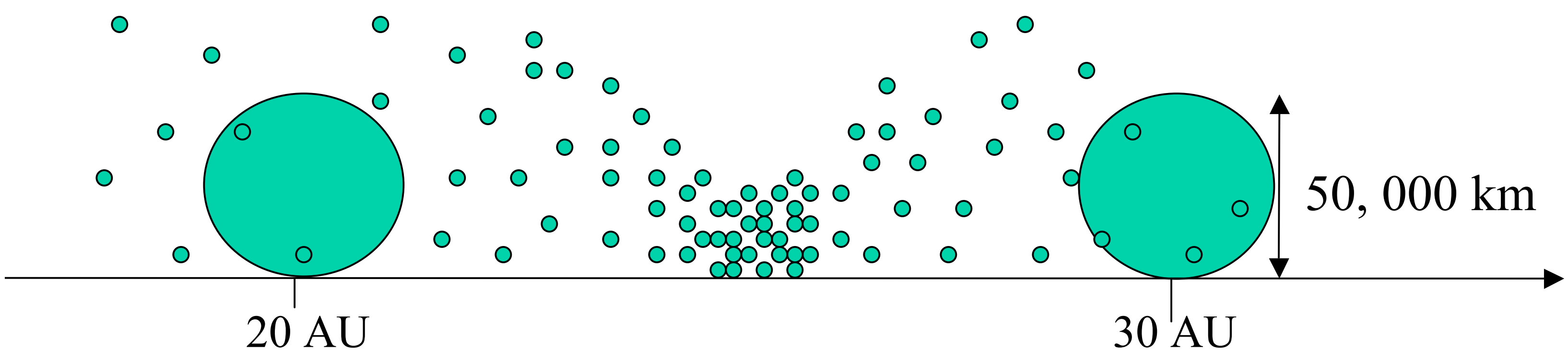
- Clean-sharp or dirty-smooth gaps?
- Could we have accretion through the gaps?

Smooth Gaps?

$$T_{\text{diffuse}} \sim \Omega^{-1}\left(\frac{M_{sun}}{M_{Neptune}}\right)^{2}\left(\frac{\Delta a}{a}\right)^{5} \sim 1\,\text{Myr}$$

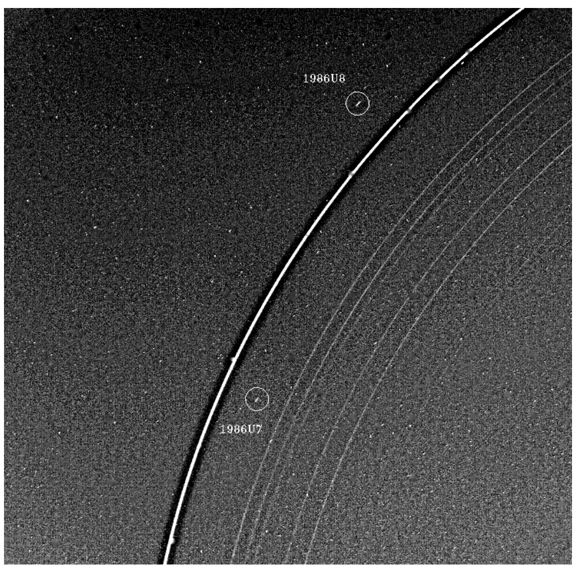
$$T_{accretion} \sim T_{\text{isolation}}\left(\frac{\Delta a}{R_H}\right)^{\sim 4} \sim 100\,\text{Myr}$$



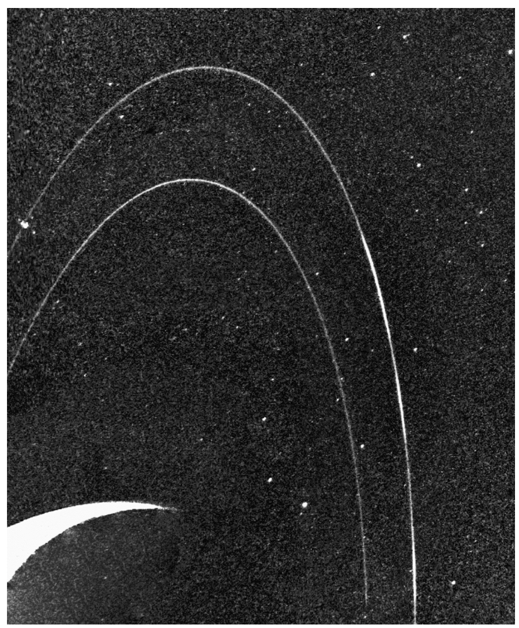


Sharp Gaps?

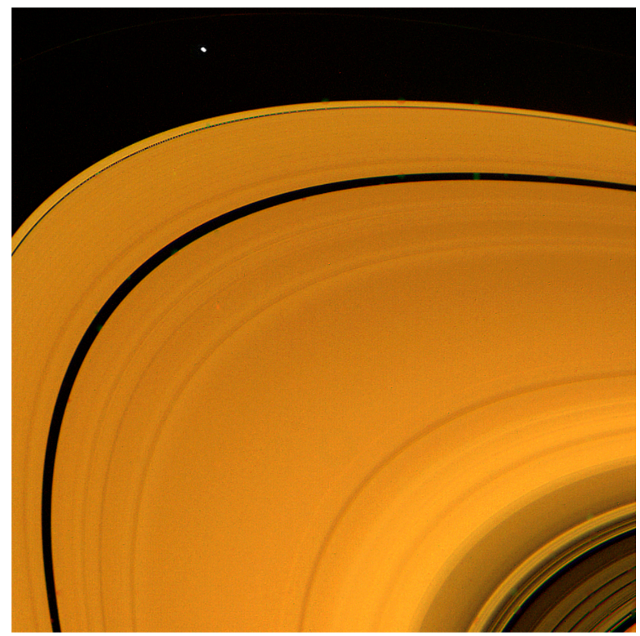
- Most of the mass in small bodies.
- Optical depth is high.
- Negative “viscosity”
- Angular momentum diffuses inward.



Neptune



Uranus



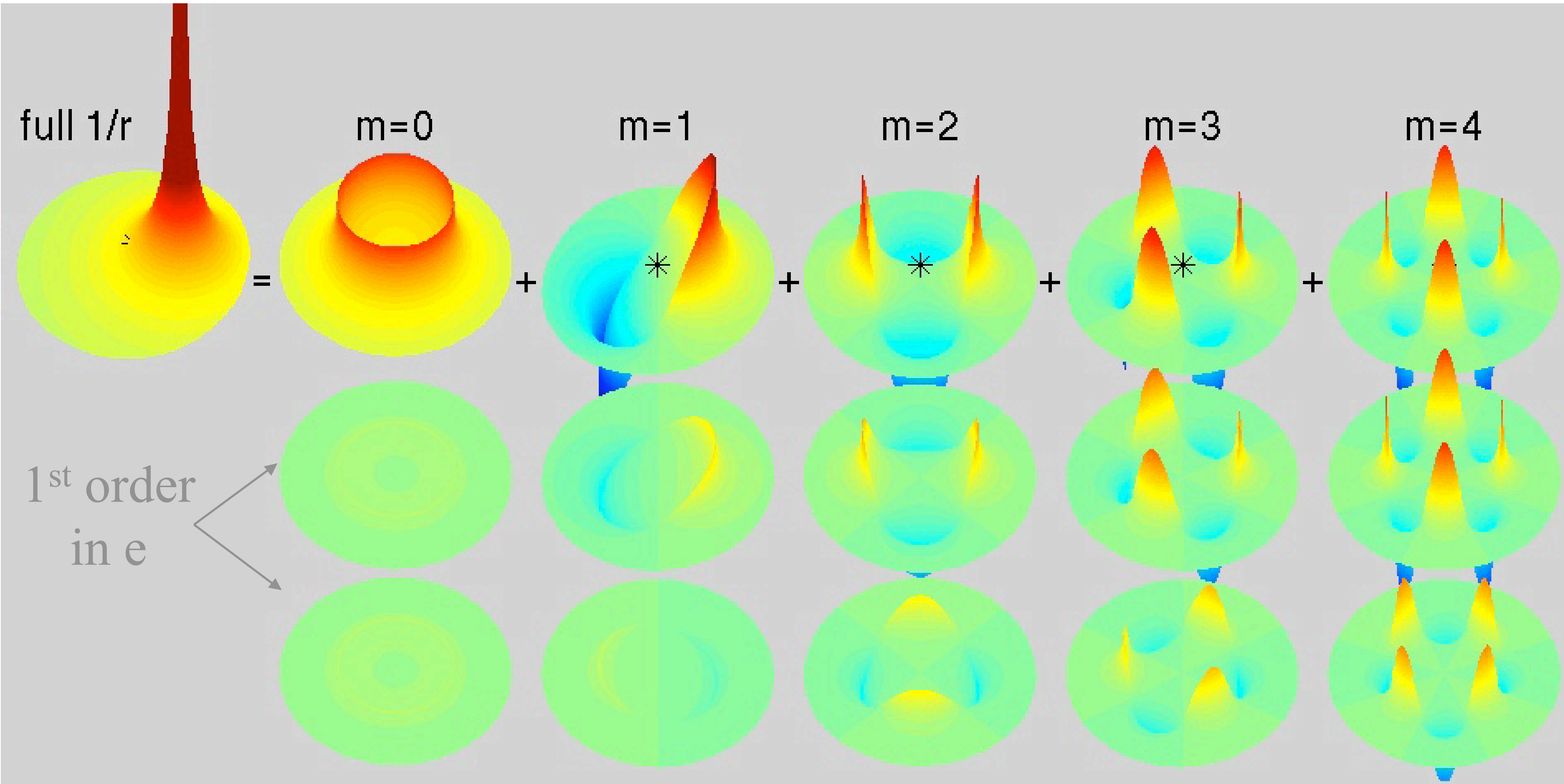
Saturn

Eccentricity evolution in clean gaps

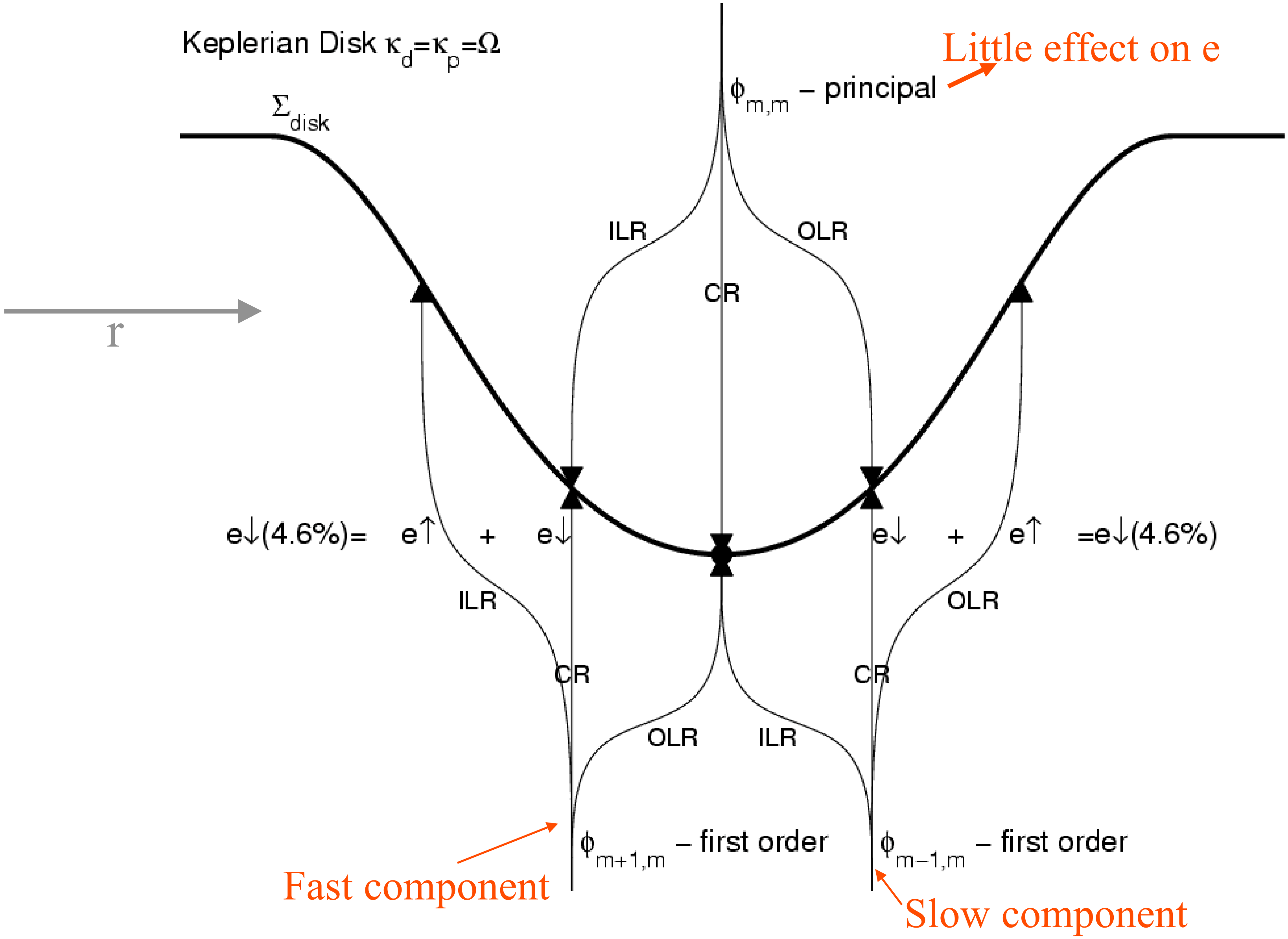
- Similar question to exoplanets.

- BUT DIFFFERENT ANSWER!

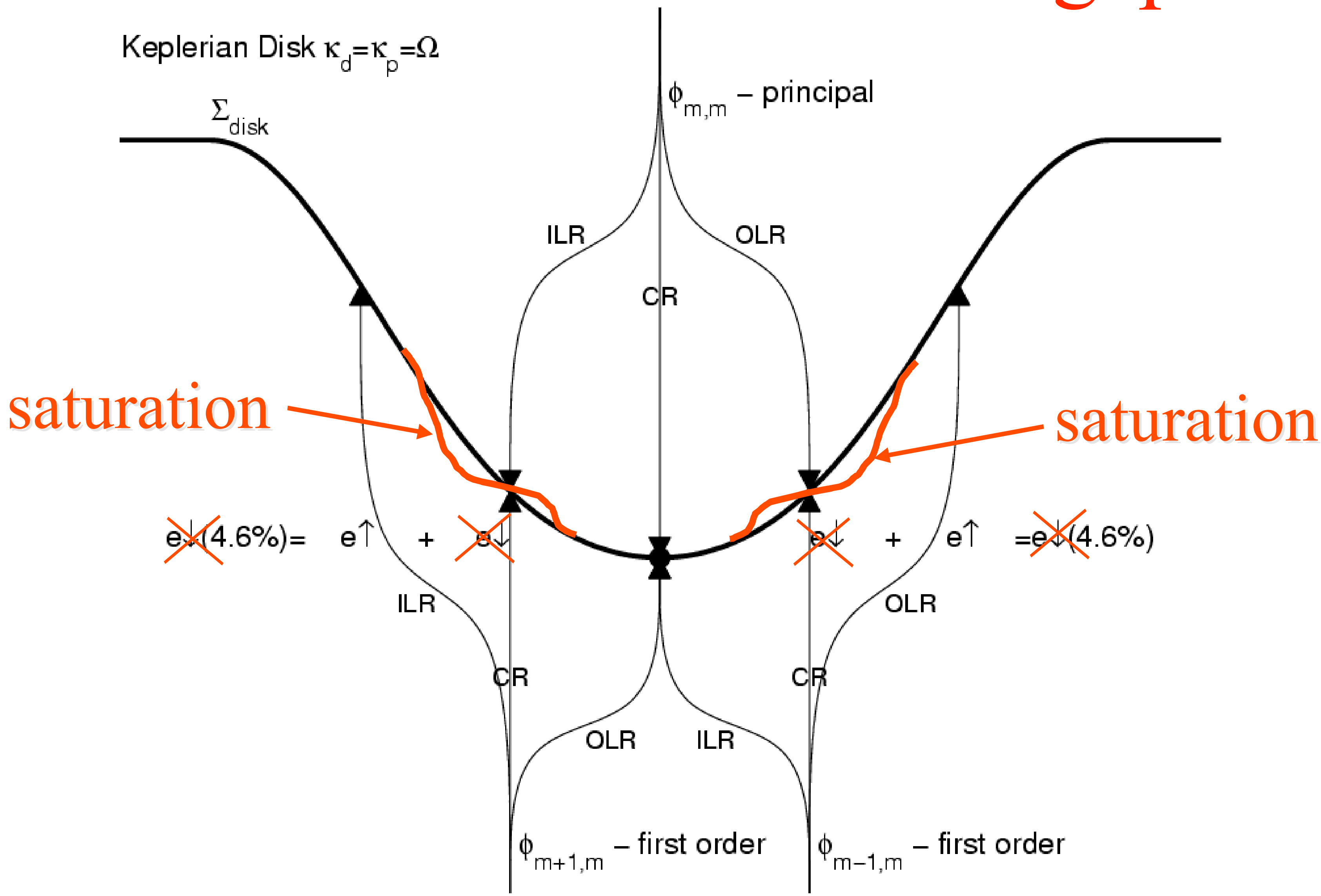
e=0.2 expansion in m *& e*



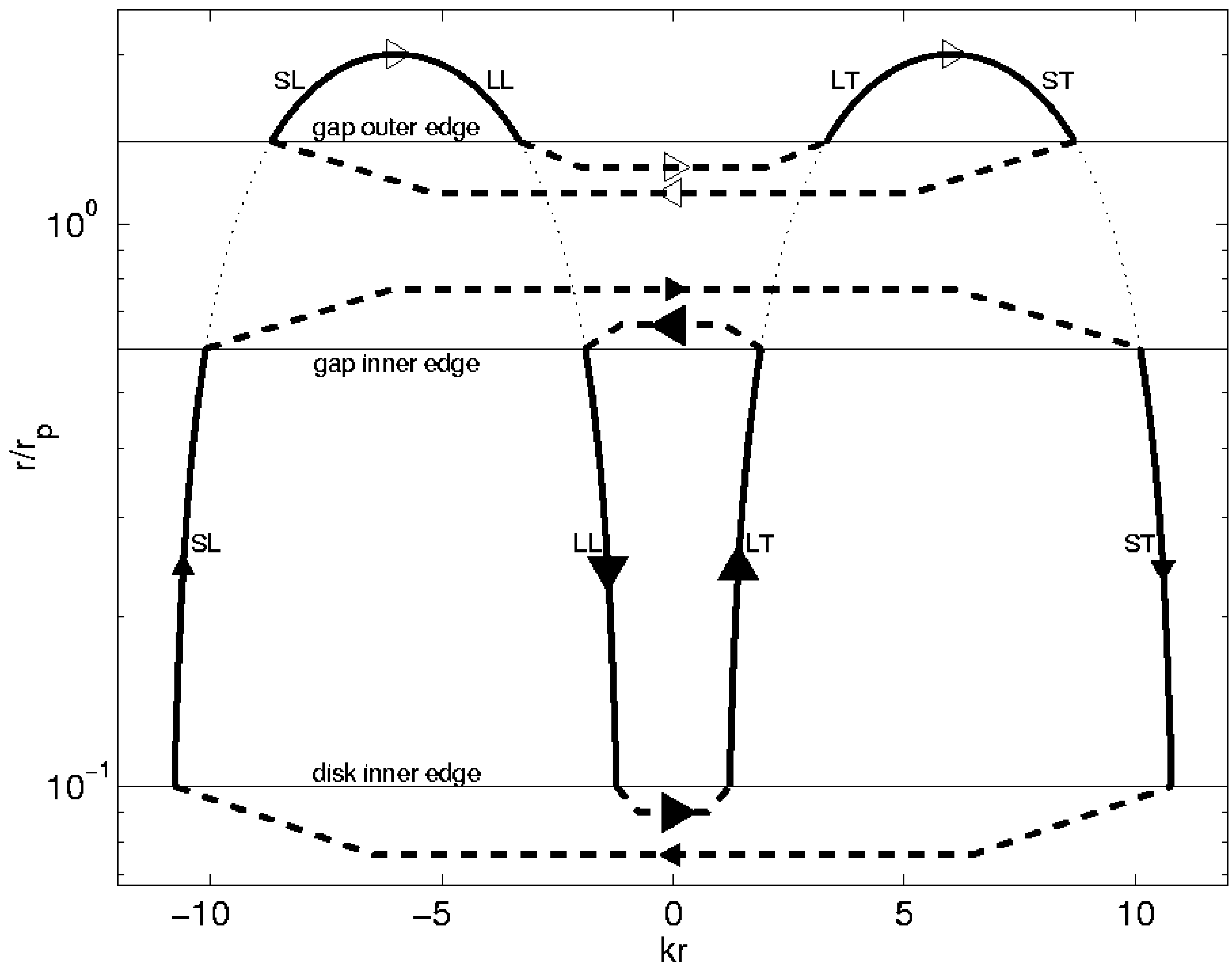
Planet disk interactions in gaps



Planet disk interaction in gaps



Apsidal Wave With Self Gravity



Eccentricity evolution in clean gaps

- Similar question to exoplanets.
- BUT DIFFFERENT ANSWER!
- Corotation resonance saturates.
- Relative importance of Apsidal waves:

$$\alpha_{SS}\left(\frac{x}{a}\right)^3\left(\frac{\Omega a}{u}\right)^2 \approx \begin{cases} 10^5 & \text{planetesimal disks} \\ 0.1 & \text{gaseous disk} \end{cases}$$

10^2 - 10^4

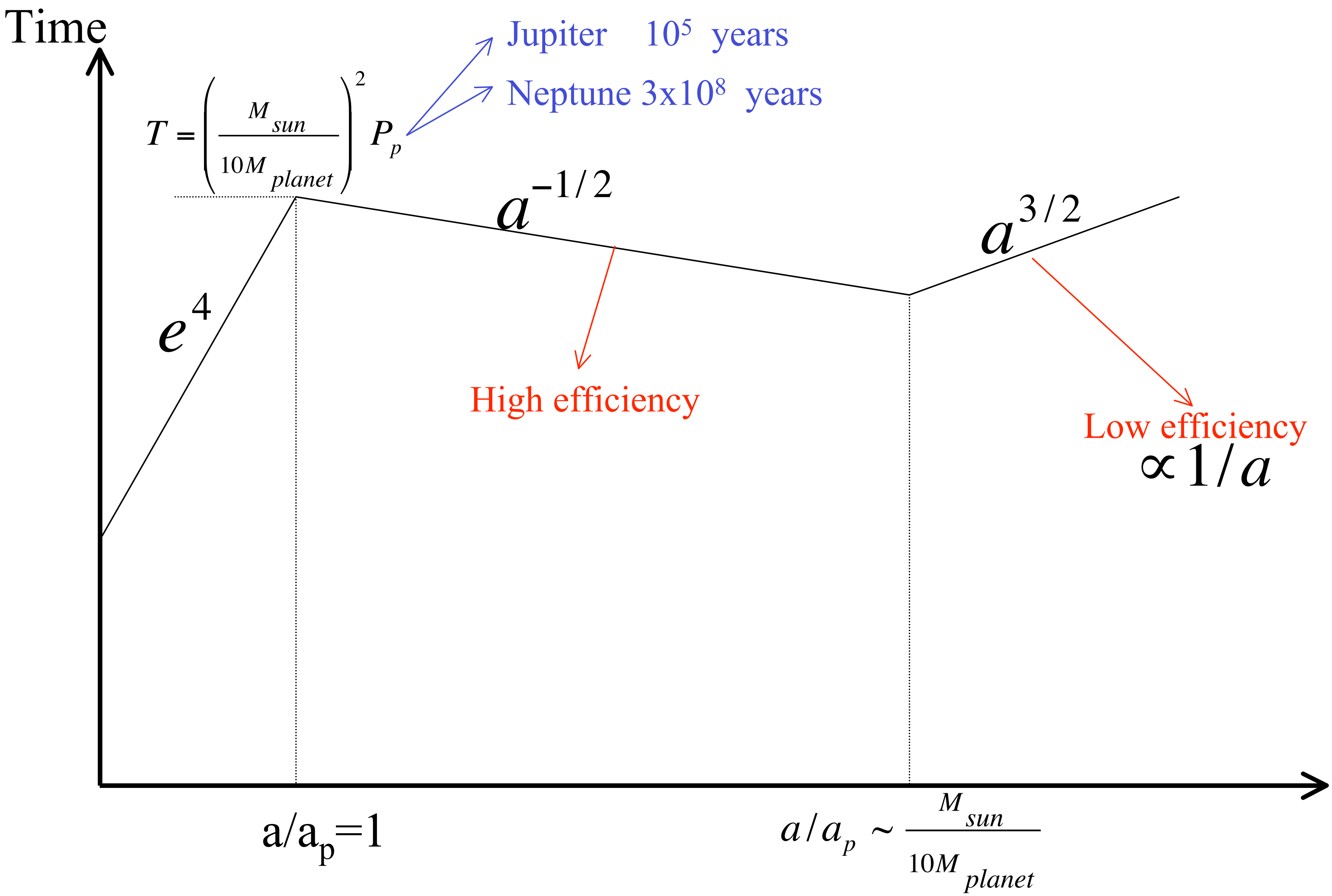
Clean Up?

- Tight limits on residual mass.
  - For a>30 AU, $M_{disk}<10M_{earth}(a/100)^3$ Halley's 10-86
- If all bodies are very small (as we suggest):
  - Gaps are clean - no accretion.
  - No ejection.
  - Cannot form the Oort cloud.
- What size is small?
  - S_{cross} no collision in planet crossing time.

$$s_{cross} \approx \frac{\sigma}{\rho}\left(\frac{M_{sun}}{M_{planet}}\right)^2\left(\frac{x}{a}\right)^5 \approx \begin{cases} 60\,\mathrm{km} & a \sim 1\mathrm{AU} \\ 1\,\mathrm{km} & a \sim 30\,\mathrm{AU} \end{cases}$$

$$s_{eject} \approx \frac{\sigma}{\rho}\left(\frac{M_{sun}}{M_{planet}}\right)^2 \approx 100\,\mathrm{km}, \quad a \sim 30\,\mathrm{AU}$$

Ejection & Oort Cloud



Gravitational Disk Instabilities

- In very cold disk:

Largest unstable wavelength

$$R_{\max} \sim \alpha^{-2} \frac{\sigma}{\rho} \approx \begin{cases} 2km & a \sim 1au \\ 100km & a \sim 30au \end{cases}$$

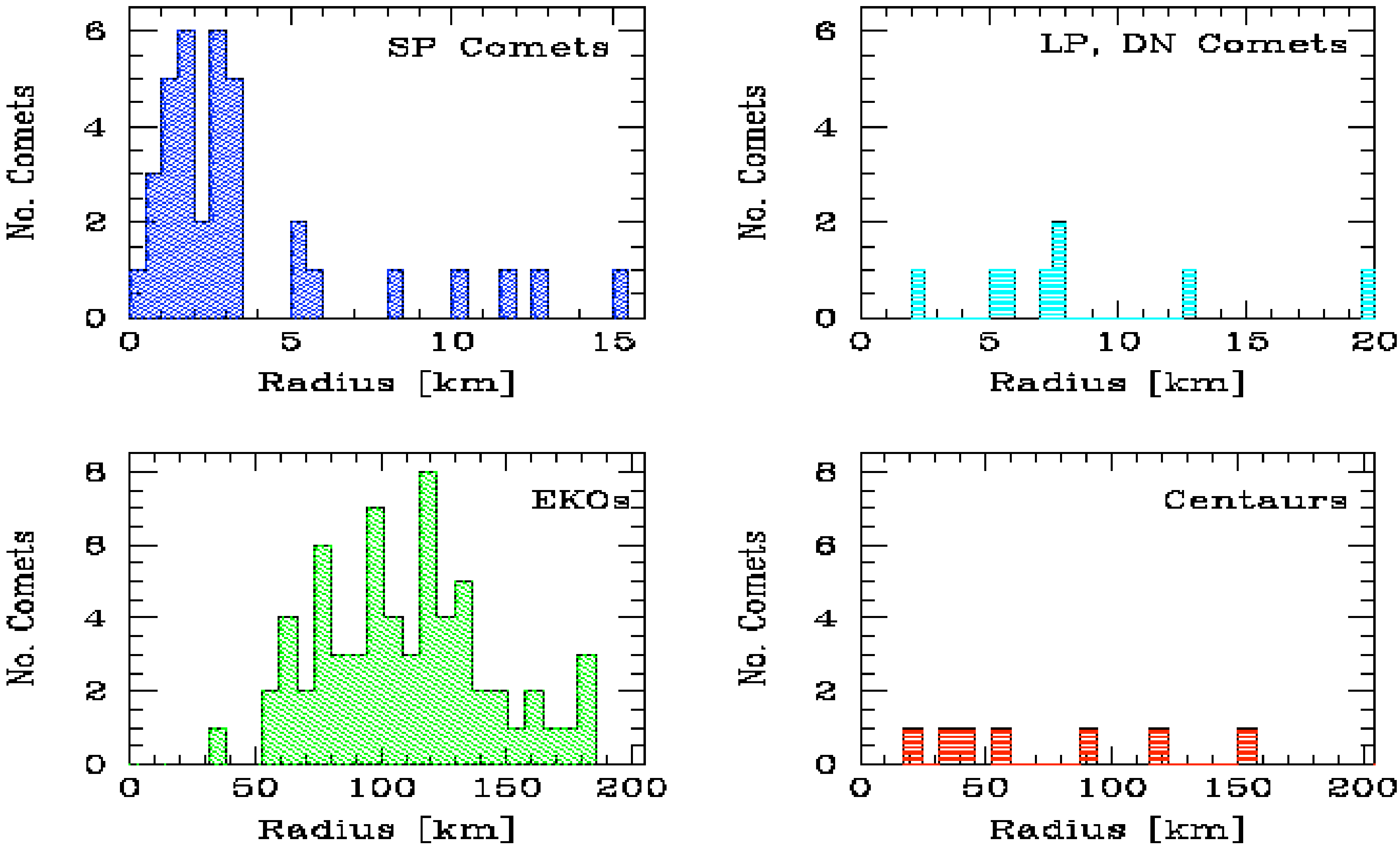
Quick collapse

No angular momentum lost

Energy dissipates by collisions

$$S_{*} \sim \alpha^{-3/2} \frac{\sigma}{\rho} \approx \begin{cases} 0.05\ km & a \sim 1au \\ 2\ km & a \sim 30au \end{cases}$$

Sizes of comets



Meech, Hainaut and Marsden 2001

Setup for instabilities - what is s

- From Q~1:

$$u_{stab} \approx \frac{\pi G \sigma}{\Omega} \approx \begin{cases} 10 \quad \text{cm/s} & a \sim 1AU \\ 200 \ \text{cm/s} & a \sim 30AU \end{cases}$$

- Required size for setup of such velocities:

$$s_{stab} \approx \frac{x}{a}\frac{\sigma}{\rho} \approx \begin{cases} 0.2 \ \text{cm} & a \sim 1AU \\ 0.5 \ \text{cm} & a \sim 30AU \end{cases}$$

- Required size for u~v_H

$$s_{u=v_H} \approx \alpha^{3/2}\left(\frac{\sigma a}{\rho}\right)^{1/2} \approx \begin{cases} 10 \ \text{m} & a \sim 1AU \\ 1 \quad \text{m} & a \sim 30AU \end{cases}$$

Summary

Oligarchy ends when $\Sigma = \sigma$

With small bodies, last stage of isolation is fast:

- Could be 0.1Myr for Uranus and Neptun
- Even faster for earth.

Ejection (outer) or collisions (inner). Then,

Orbital regularization by small bodies

Regularization works also in clean gaps - unlike exoplanets

Difficult (but possible) to get rid of small bodies

Gravitational instability - key player:

- Reforms 1km size bodies.
- Sets minimum to velocity dispersion.
- May set minimum to size of particles.

Most likely - ejection by Jupiter.

Cannot form Oort cloud directly out of U-N region

- Total mass too high - collision will happen first.
- 1km size can be ejected out of Jupiter-Saturn without collisions.

Unlikely to have sub-km bodies in Oort cloud $\Sigma \propto s^1$