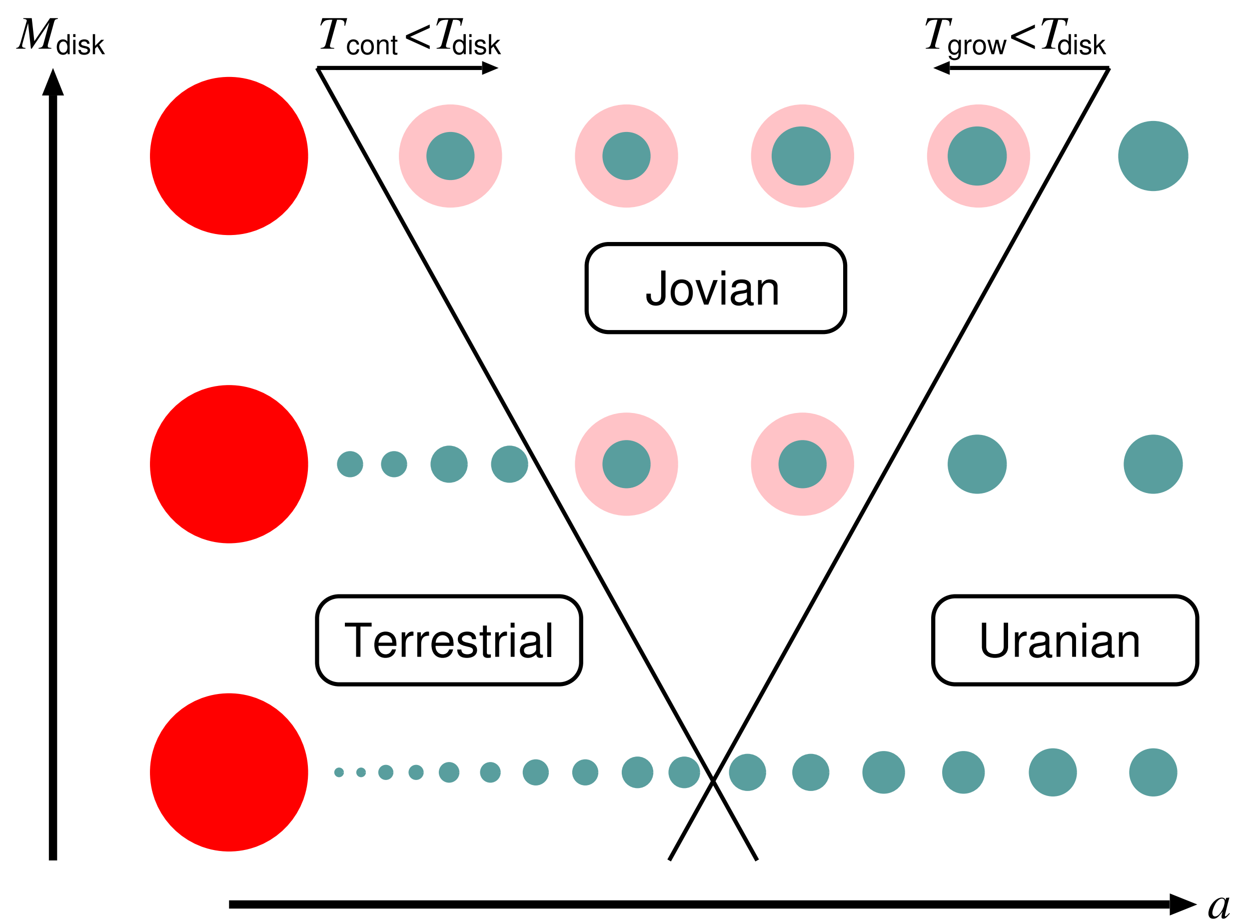
Oligarchic Growth of Protoplanets and Diversity of Planetary Systems



Eiichiro Kokubo
National Astronomical Observatory

Shigeru Ida
Tokyo Institute of Technology

Outline

- Basic Concepts of Oligarchic Growth
  - Slowdown of runaway growth (Ida & Makino 1993)
  - Orbital repulsion (Kokubo & Ida 1995)
  - Application to solar system (Kokubo & Ida 1998,2000)
- Application to General Planetary Systems
  - Diversity of protoplanet systems
  - Habitat segregation of planets
  - Diversity of planetary systems (Kokubo & Ida 2002)

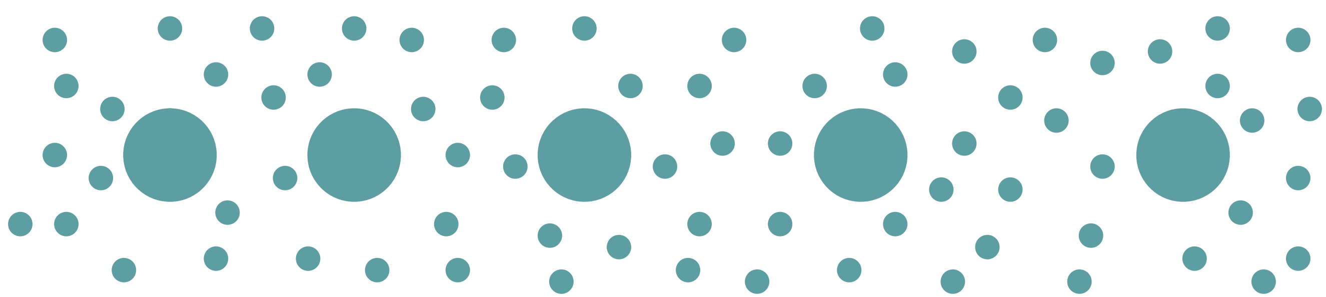
Oligarchic Growth of Protoplanets

- Multiple similar-sized protoplanets grow in the orderly mode

Slowdown of runaway growth

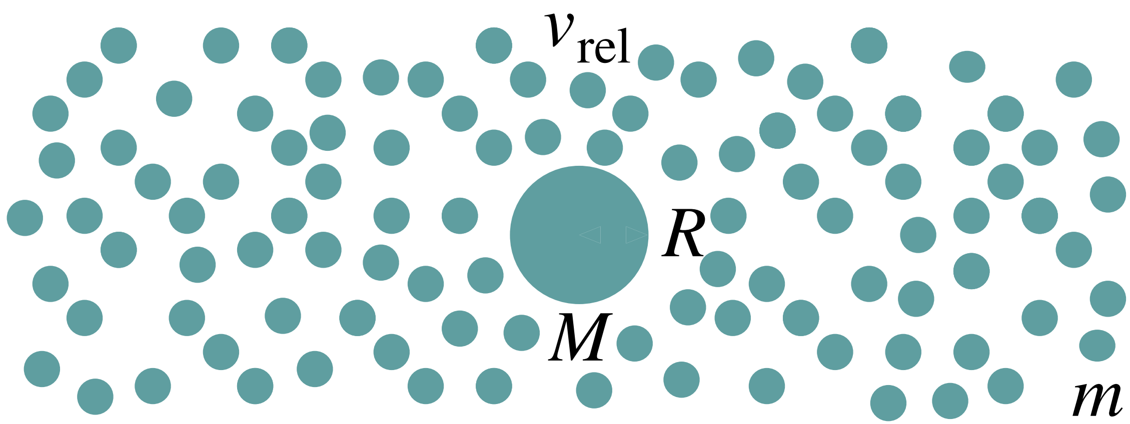
- Orbital separations of protoplanets are proportional to their Hill radii

Orbital repulsion of protoplanets



Orbital Separation $\simeq 10 r_{\mathrm{H}} = 10 \left(\frac{2M}{3M_{\odot}} \right)^{1/3} a$

Slowdown of Runaway Growth

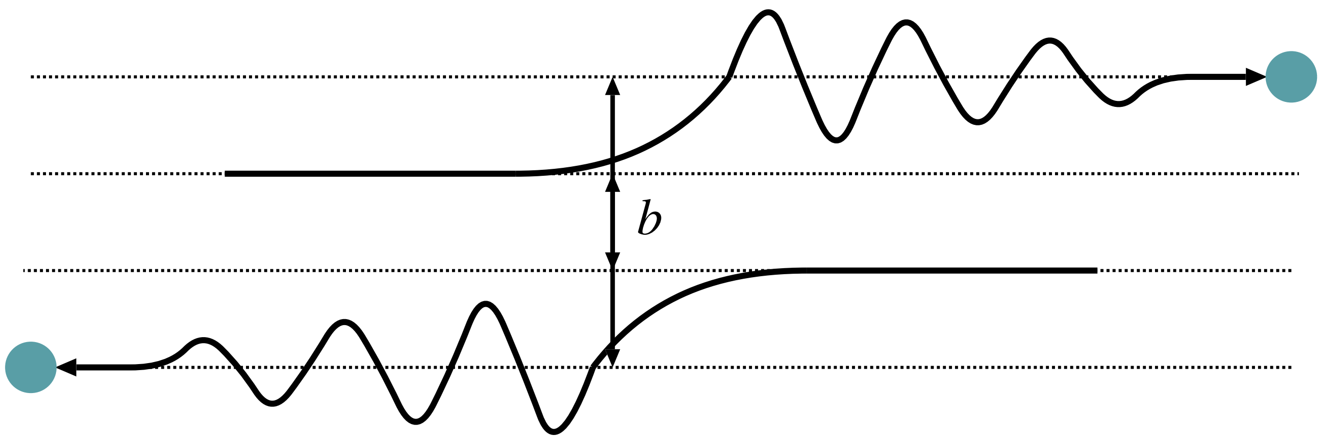


Heating of planetesimals by protoplanets

$$M/m \gtrsim 100 \Longrightarrow v_{\rm rel} \propto M^{1/3}$$

$$\frac{1}{M}\frac{dM}{dt} \propto M^{1/3} v_{\rm rel}^{-2} \propto M^{-1/3} \Rightarrow \text{orderly growth}$$

Orbital Repulsion



Protoplanet-Protoplanet Scattering

- increase b and e

Dynamical Friction from Planetesimals

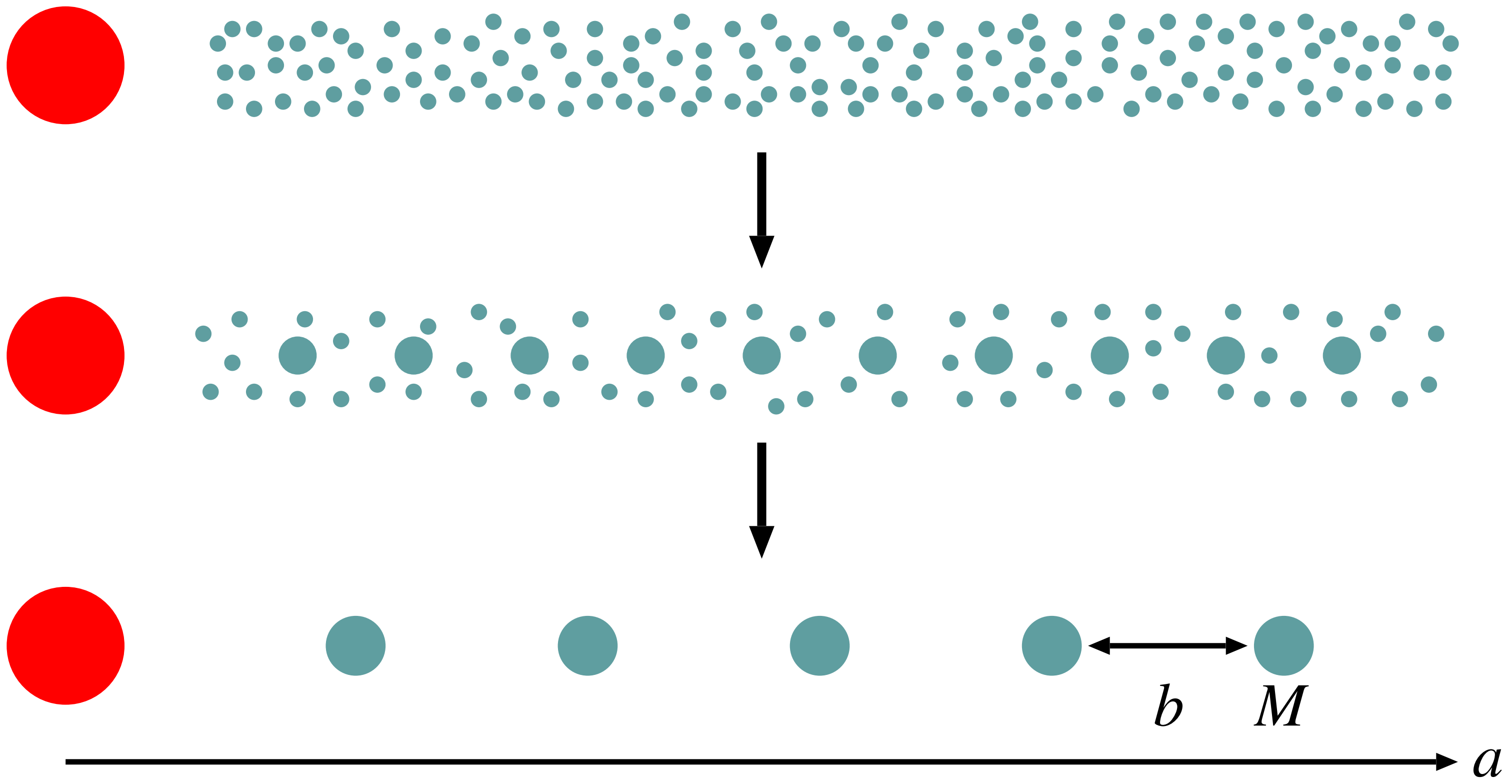
- reduction of e

Orbital Repulsion

- expansion of b keeping nearly circular orbits
- b is proportional to r_{H}

Characteristics of Protoplanets

Planetesimal distribution $\Sigma \propto \Sigma_1 a^{-\alpha}$



isolation mass $M \propto \Sigma_1^{3/2} a^{(3/2)(2-\alpha)}$

orbital separation $b \propto \Sigma_1^{1/2} a^{(1/2)(4-\alpha)}$

growth time $T_{\rm grow} \propto M^{1/3} \Sigma_1^{-1} a^{1/2+\alpha}$

Characteristics of Protoplanets

Terrestrial Region

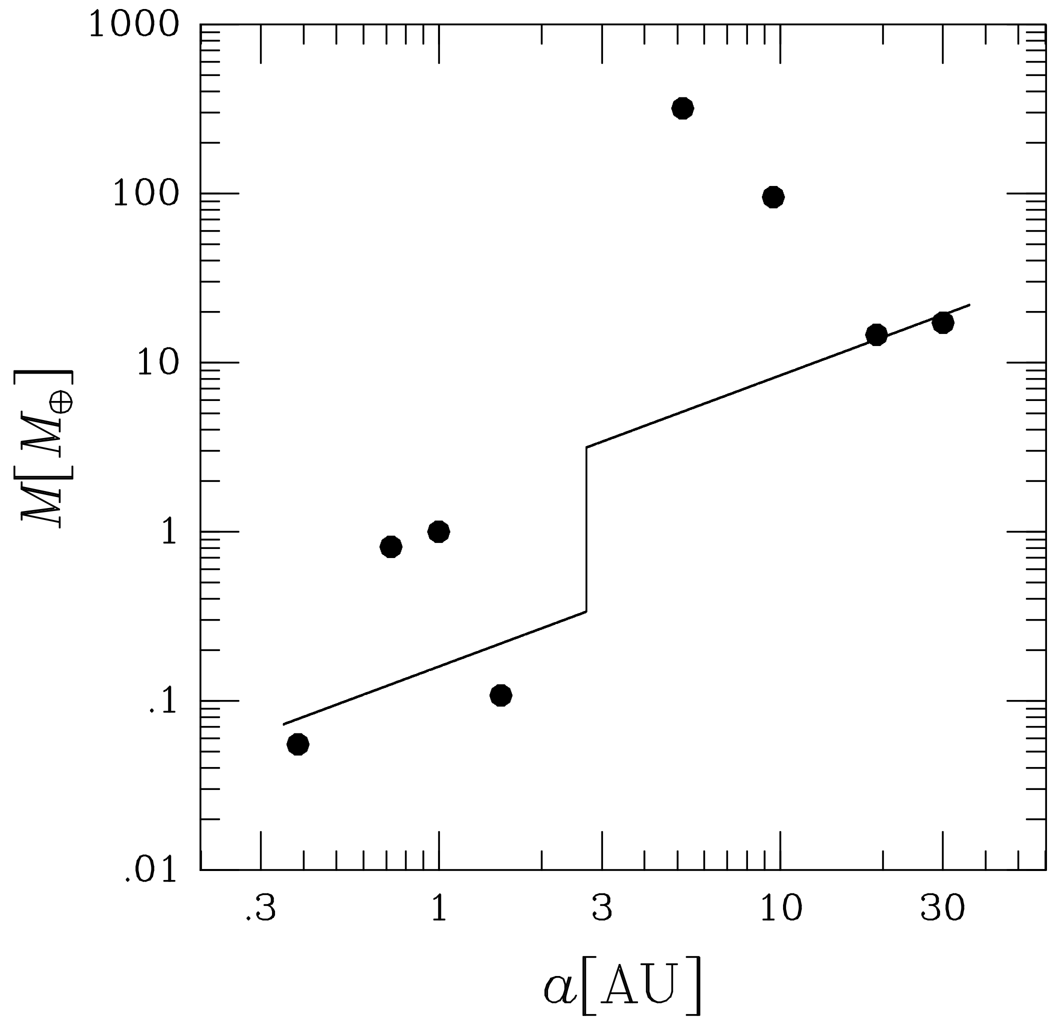
$\sim 0.1 M_{\oplus} <$ terrestrial planets

Jovian Region

$\sim 10 M_{\oplus} \ll$ Jovian planets

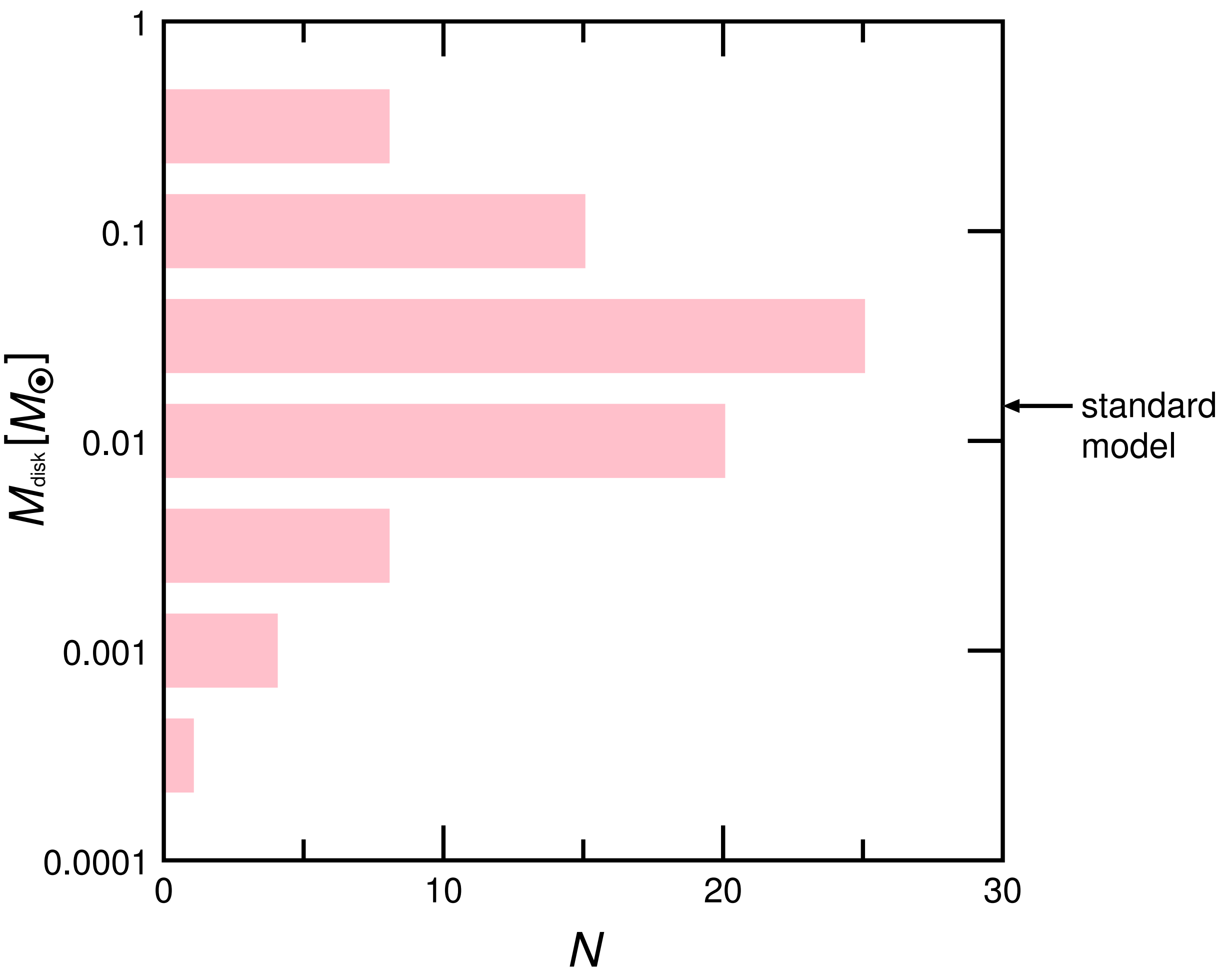
Uranian Region

$\sim 10 M_{\oplus} \simeq$ Uranian planets



Observation of Protoplanetary Disks

Protoplanetary Disks in Taurus and Ophiuchus
(Beckwith & Sargent 1996)



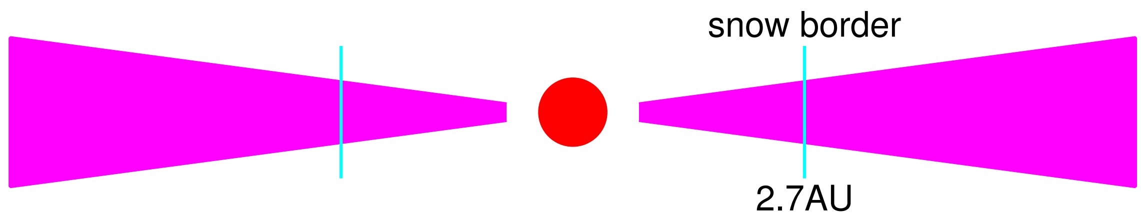
Disk mass

$M_{\rm disk} \sim 10^{-3}$-$10^{-1} M_\odot$

Disk profile

?

Disk Model



Power-Law Disk with Solar Composition

Solid

$$\Sigma_{\rm solid} = f_{\rm ice}\Sigma_1 \left(\frac{a}{1{\rm AU}}\right)^{-\alpha} \ [{\rm gcm}^{-2}]$$

$$f_{\rm ice} = \begin{cases} 1.0 & a < a_{\rm snow} \\ 4.2 & a > a_{\rm snow} \end{cases}$$

Gas

$$\Sigma_{\rm gas} = f_{\rm gas}\Sigma_1 \left(\frac{a}{1{\rm AU}}\right)^{-\alpha} \ [{\rm gcm}^{-2}]$$

$$f_{\rm gas} = 240$$

Snow Border

$$a_{\rm snow} = 2.7{\rm AU} \ \ {\rm for} \ \ T = 280\left(\frac{a}{1{\rm AU}}\right)^{-1/2}{\rm K}$$

$a < a_{\rm snow}$ rocky dust (planetesimals)

$a > a_{\rm snow}$ icy dust (planetesimals)

Oligarchic Growth Model

Assumptions

- Orbital separation of protoplanets $\tilde{b} \simeq 10$
- Accretion in gas
- No migration of planetesimals and protoplanets
- Accretion efficiency 100%

Disk Model

$$\Sigma_{\rm solid} = f_{\rm ice}\Sigma_1 \left(\frac{a}{1{\rm AU}}\right)^{-\alpha} \ [{\rm gcm}^{-2}]$$

$$\Sigma_{\rm gas} = f_{\rm gas}\Sigma_1 \left(\frac{a}{1{\rm AU}}\right)^{-\alpha} \ [{\rm gcm}^{-2}]$$

Oligarchic Growth Model

Isolation Mass of Protoplanets

$$M_{\rm iso} \simeq 0.16 \left(\frac{\tilde{b}}{10}\right)^{3/2} \left(\frac{f_{\rm ice}\Sigma_1}{10}\right)^{3/2} \left(\frac{a}{1{\rm AU}}\right)^{(3/2)(2-\alpha)} \left(\frac{M_*}{M_\odot}\right)^{-1/2} M_\oplus$$

Growth Time of Protoplanets

$$T_{\rm grow} \simeq 1.7\times 10^5 \left(\frac{\langle\tilde{e}^2\rangle_{\rm eq}^{1/2}}{6}\right)^{2} \left(\frac{M}{10^{26}{\rm g}}\right)^{1/3} \left(\frac{f_{\rm ice}\Sigma_1}{10}\right)^{-1} \left(\frac{a}{1{\rm AU}}\right)^{1/2+\alpha} \left(\frac{M_*}{M_\odot}\right)^{-1/6} [{\rm year}]$$

Equilibrium Eccentricity of Planetesimals

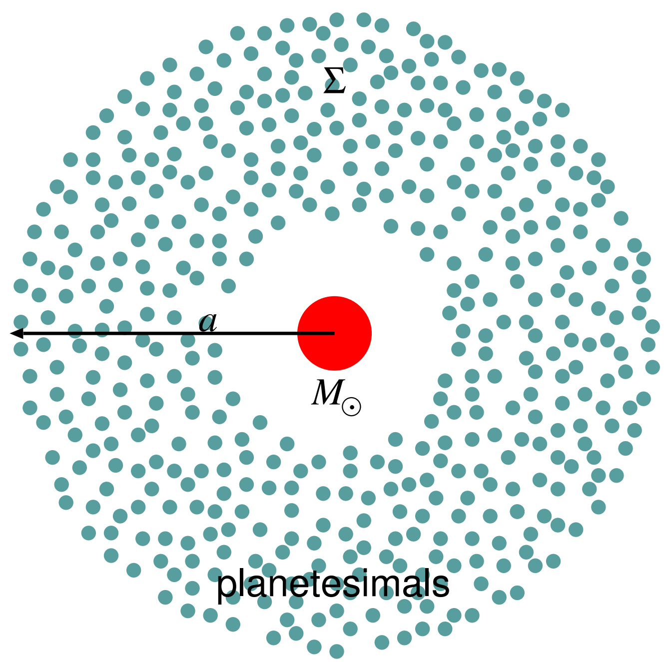
$$\langle\tilde{e}^2\rangle_{\rm eq}^{1/2} \simeq 5.6 \left(\frac{m}{10^{23}{\rm g}}\right)^{1/15} \left(\frac{\tilde{b}}{10}\right)^{-1/5} \left(\frac{f_{\rm gas}}{240}\right)^{-1/5} \left(\frac{\Sigma_1}{10}\right)^{-1/5} \left(\frac{a}{1{\rm AU}}\right)^{(1/5)\alpha+1/20}$$

N-Body Simulation

Equation of Motion

$$\frac{d\boldsymbol{v}_i}{dt} = -GM_\odot \frac{\boldsymbol{x}_i}{|\boldsymbol{x}_i|^3} + \sum_{j \neq i}^{N} G m_j \frac{\boldsymbol{x}_j - \boldsymbol{x}_i}{|\boldsymbol{x}_j - \boldsymbol{x}_i|^3} + \boldsymbol{f}_{\rm col}$$

Initial Condition



$$N = 5000\text{-}10000$$

$$\Sigma = \Sigma_1 \left(\frac{a}{1\mathrm{AU}}\right)^{-\alpha} \mathrm{gcm}^{-2}$$

$$\Sigma_1 = 1, 10, 100$$

$$\alpha = 1/2, 3/2, 5/2$$

$$0.5\mathrm{AU} \leq a \leq 1.5\mathrm{AU}$$

Method of Calculation

4th-Order Hermite Integrator + HARP/GRAPE

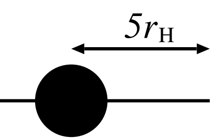
(perfect accretion + f-fold radius)

Results of Standard Disk

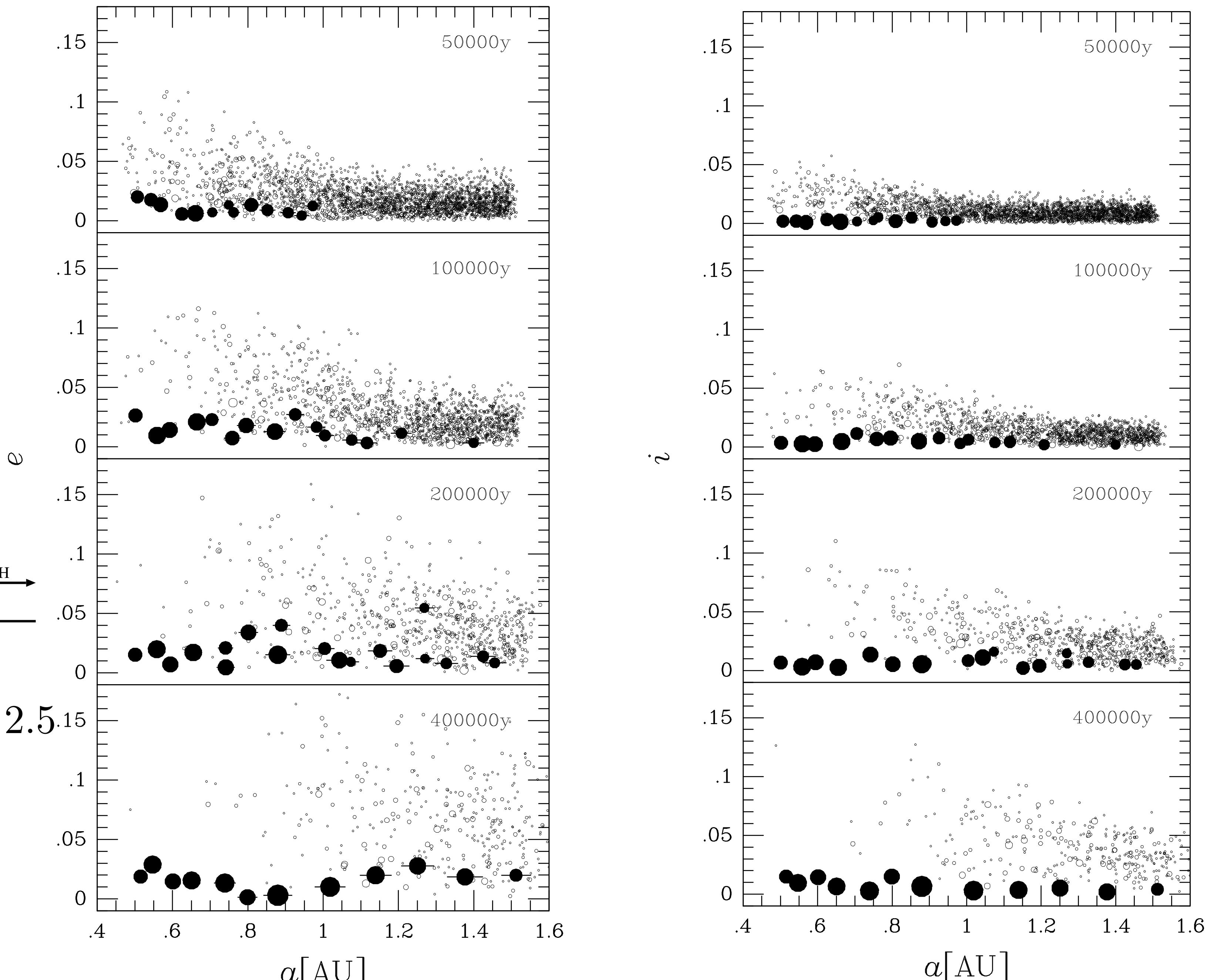
$_1 = 10$

$= 3/2$

$= 6$



13.7 ± 2.5

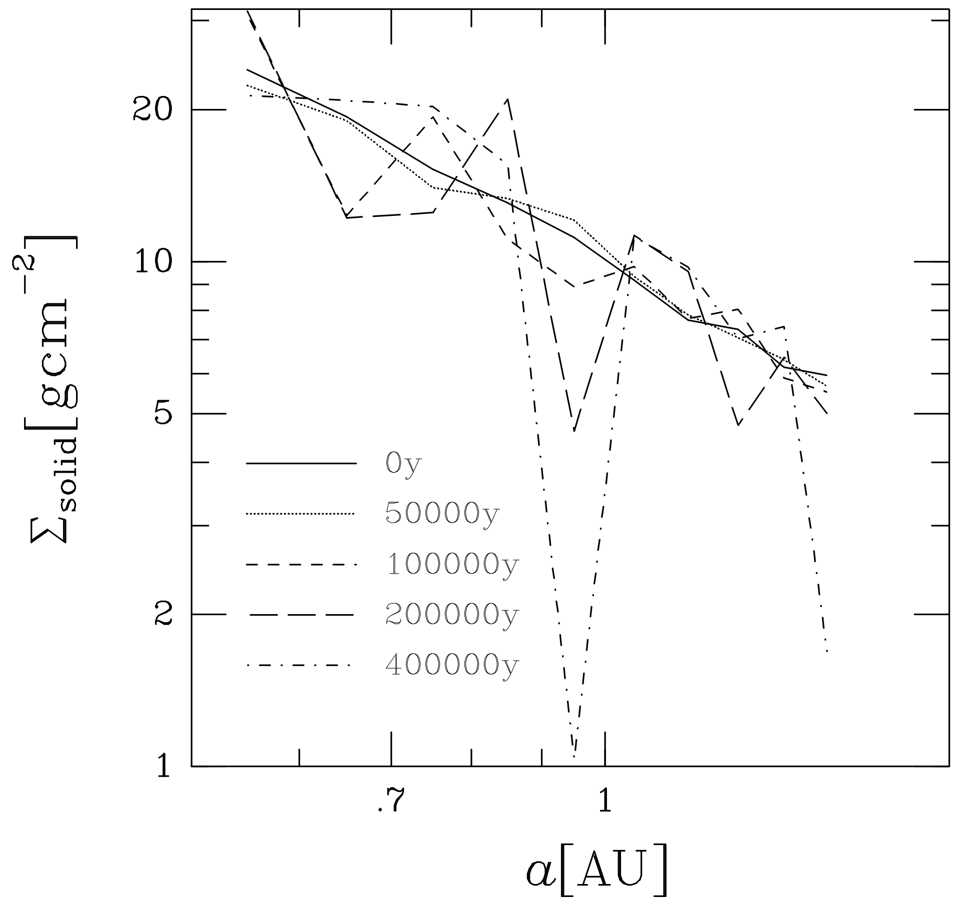


Results of Standard Disk

$\Sigma_1 = 10$

$\alpha = 3/2$

$f = 6$



Diffusion Time Scale of a Planetesimal Disk

$$T_{\rm diff} \simeq 0.63\times10^9 \left(\frac{\Delta a}{1{\rm AU}}\right)^2 \left(\frac{e}{0.01}\right)^2 \left(\frac{m}{10^{24}{\rm g}}\right)^{-1} \left(\frac{\Sigma}{10{\rm gcm}^{-2}}\right)^{-1} \left(\frac{a}{1{\rm AU}}\right)^{-5/2} \left(\frac{M_*}{M_\odot}\right)^{3/2} \ [{\rm year}]$$

$T_{\rm diff} \gg T_{\rm grow} \Rightarrow \Sigma$ **hardly changes**

Results of Standard Disk

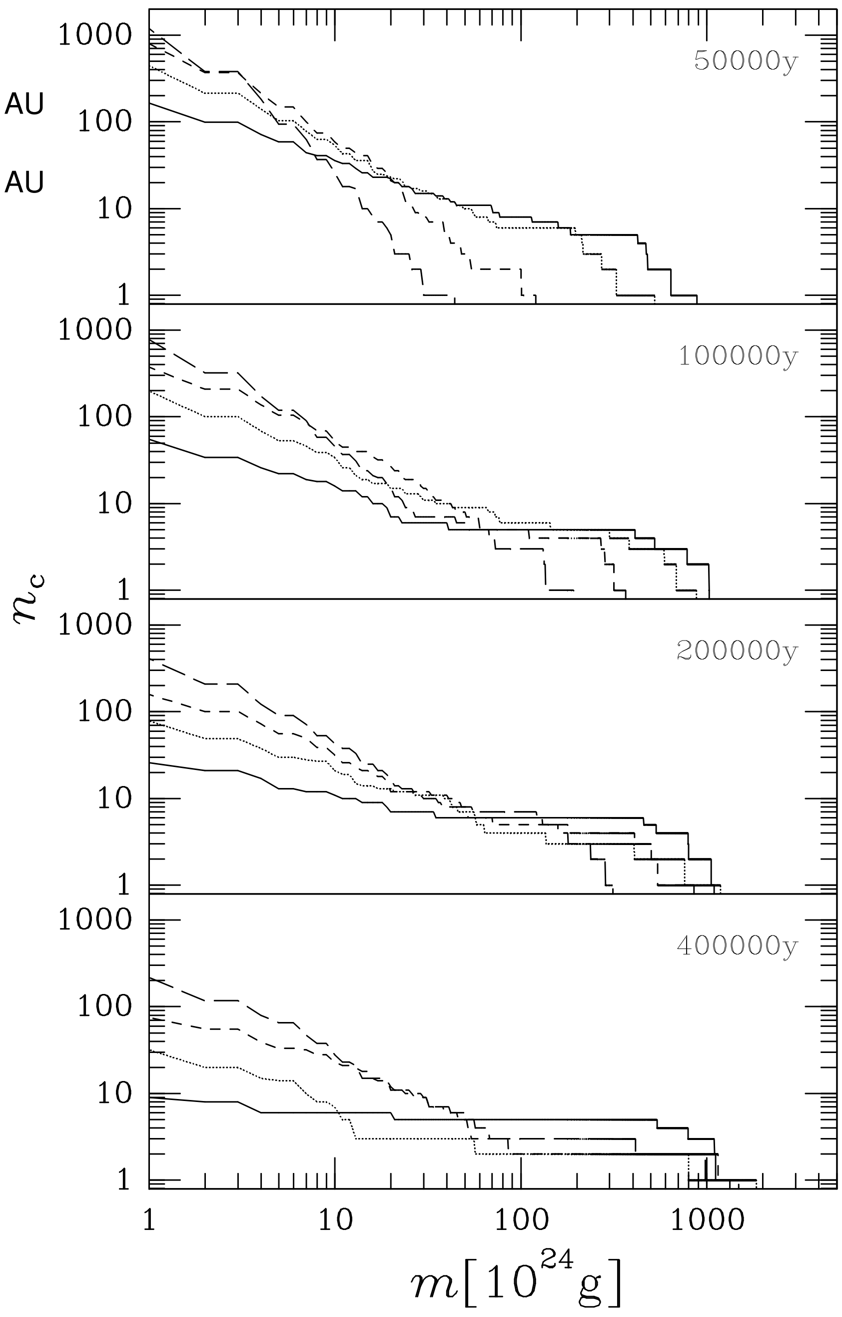
-0.75 AU ········ 0.75-1.00 AU

1.00-1.25 AU — — — — 1.25- AU

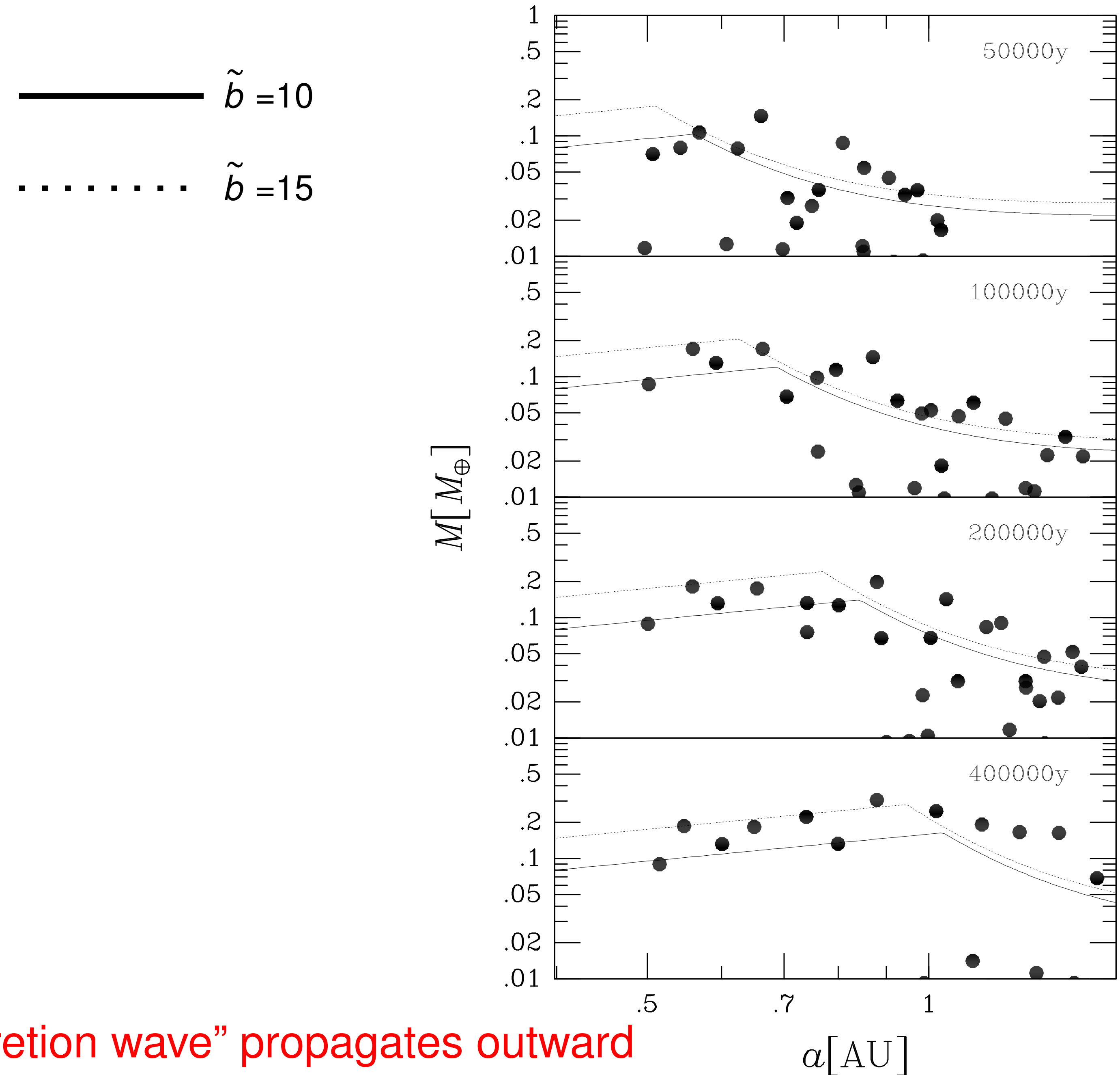
$\Sigma_1 = 10$

$\alpha = 3/2$

$f = 6$



Results of Standard Disk

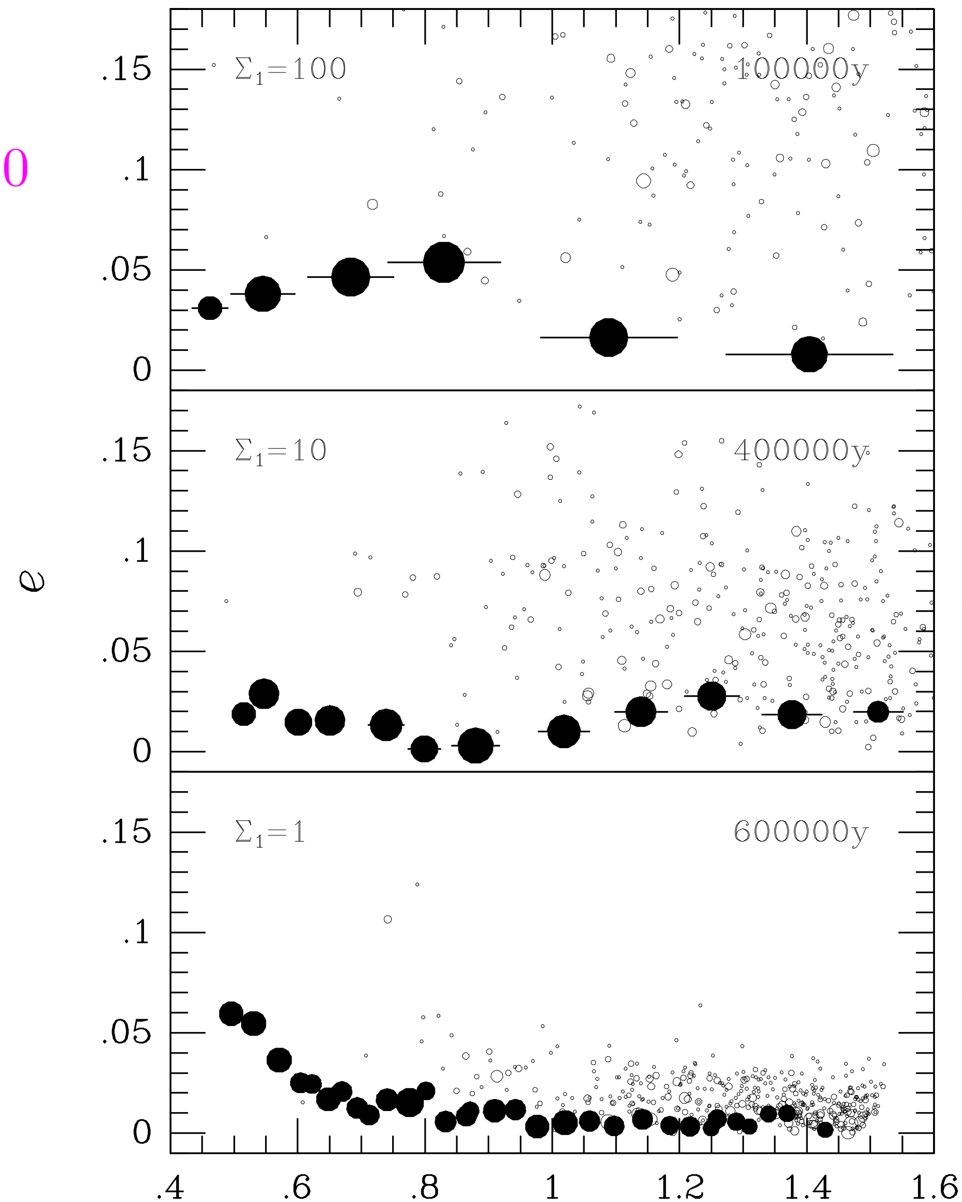


ccretion wave" propagates outward

Disk Mass Dependence

$\alpha = 3/2$

$\Sigma_1 = 1, 10, 100$



$\alpha[\mathrm{AU}]$

$\tilde{b} = 11.5 \pm 1.7$

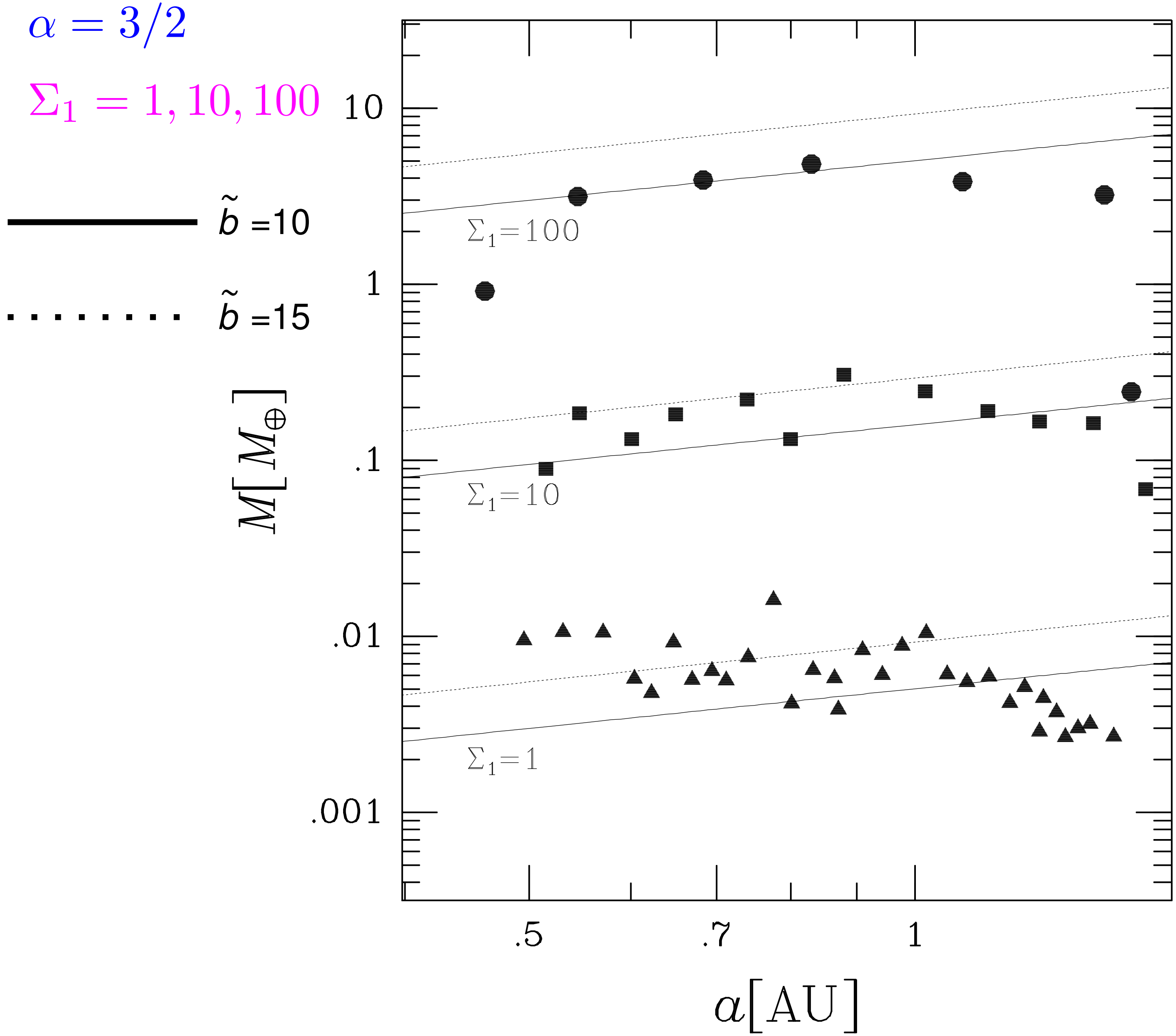
$\tilde{b} = 13.7 \pm 2.5$

$\tilde{b} = 15.1 \pm 5.1$

Disk Mass Dependence

$\alpha = 3/2$

$\Sigma_1 = 1, 10, 100$

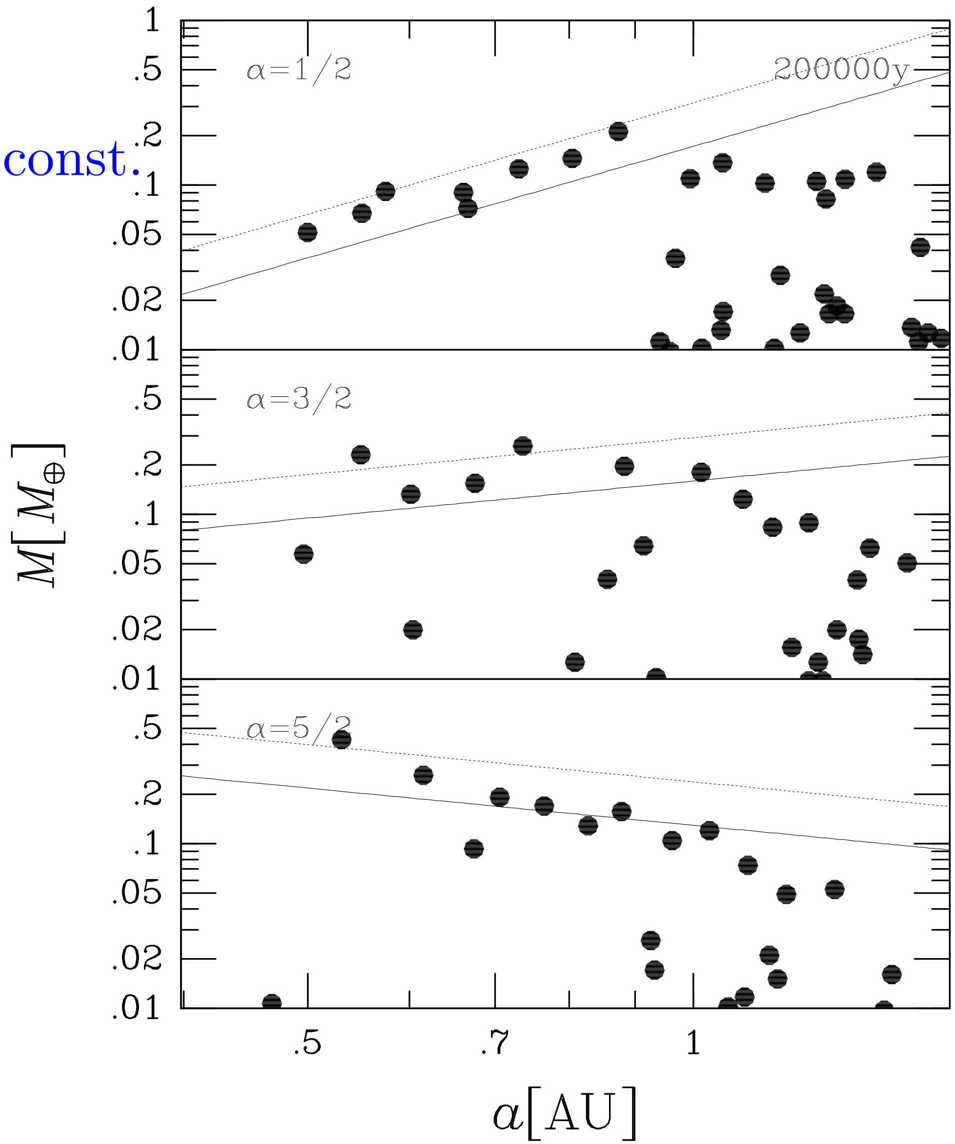


$M_{\rm iso} \propto \Sigma_1^{3/2},\ N \propto \Sigma_1^{-1/2}$

Disk Profile Dependence

$$_{\rm disk} = \int_{a_{\rm in}}^{a_{\rm out}} 2\pi a \Sigma da = \text{const.}$$

$$= 1/2, 3/2, 5/2$$



$\tilde{b}$ =10

$\tilde{b}$ =15

$$M_{\rm iso} \propto a^{(3/2)(\alpha-2)}$$

Diversity of Protoplanet Systems

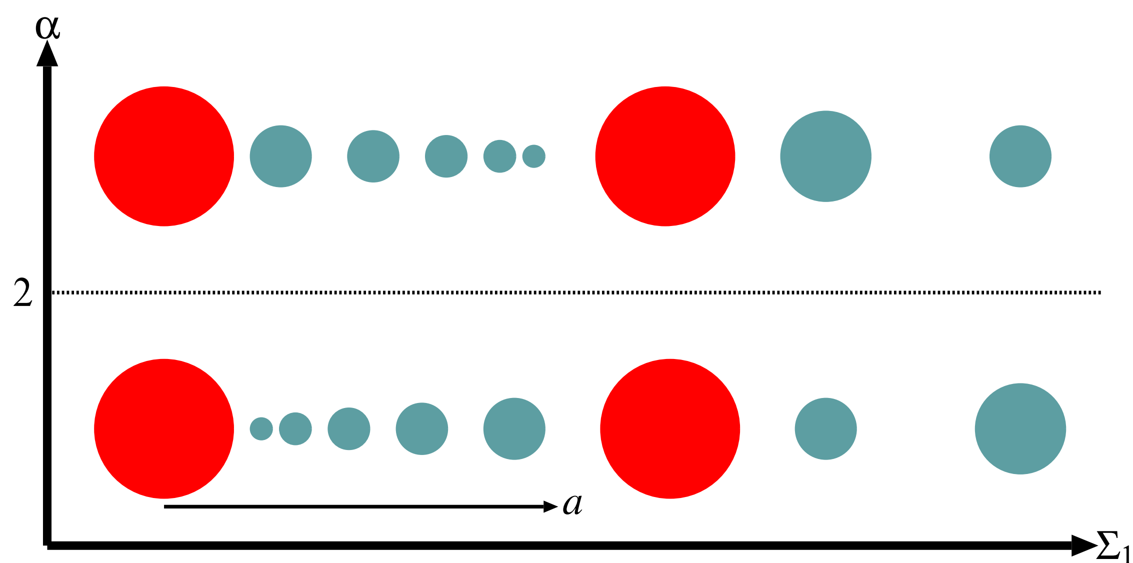
Planetesimal Disk

$$\Sigma \propto \Sigma_1 a^{-\alpha}$$

$$\Longrightarrow$$

Protoplanet System

$$M \propto \Sigma_1^{3/2} a^{-(3/2)\alpha+3}$$
$$b \propto \Sigma_1^{1/2} a^{-(1/2)\alpha+2}$$
$$N \propto \Sigma_1^{-1/2} a^{(1/2)\alpha-2}$$



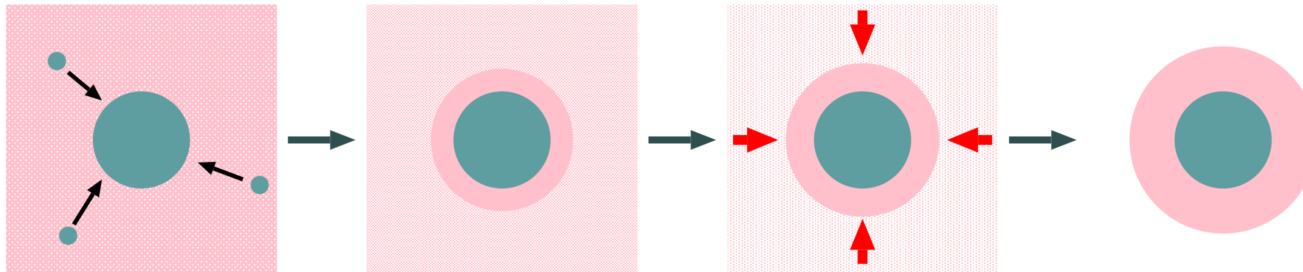
Σ_1-dependence

$$\frac{dN}{d\Sigma_1} < 0$$

α-dependence

$$\frac{dM}{da} > 0 \ (\alpha < 2), \quad \frac{dM}{da} < 0 \ (\alpha > 2)$$

Conditions for Jovian Planet Formation



Life Time of Gas Disk

$$T_{\rm disk} \sim 10^{6\text{-}8}{\rm year}$$

Contraction Time Scale of Gas

$$T_{\rm cont} \sim 10^{8\text{-}9}\left(\frac{M_{\rm iso}}{M_{\oplus}}\right)^{-5/2}{\rm year}$$

Ikoma et al. (2000)

Conditions for Gas Giant Formation

$T_{\rm grow} < T_{\rm disk} \cap T_{\rm cont} < T_{\rm disk} \Longrightarrow$ **Limited disk range for jovian planets**

Conditions for Jovian Planet Formation

$$T_{\rm cont} < T_{\rm disk} \; (\alpha = 3/2) \Longrightarrow$$

$$a > a_{\rm gas}^{\rm min} \simeq 12 f_{\rm ice}^{-2} \left(\frac{T_{\rm disk}}{10^8 {\rm year}} \right)^{-8/15} \left(\frac{\tilde{b}}{10} \right)^{-2} \left(\frac{\Sigma_1}{10} \right)^{-2} {\rm AU}$$

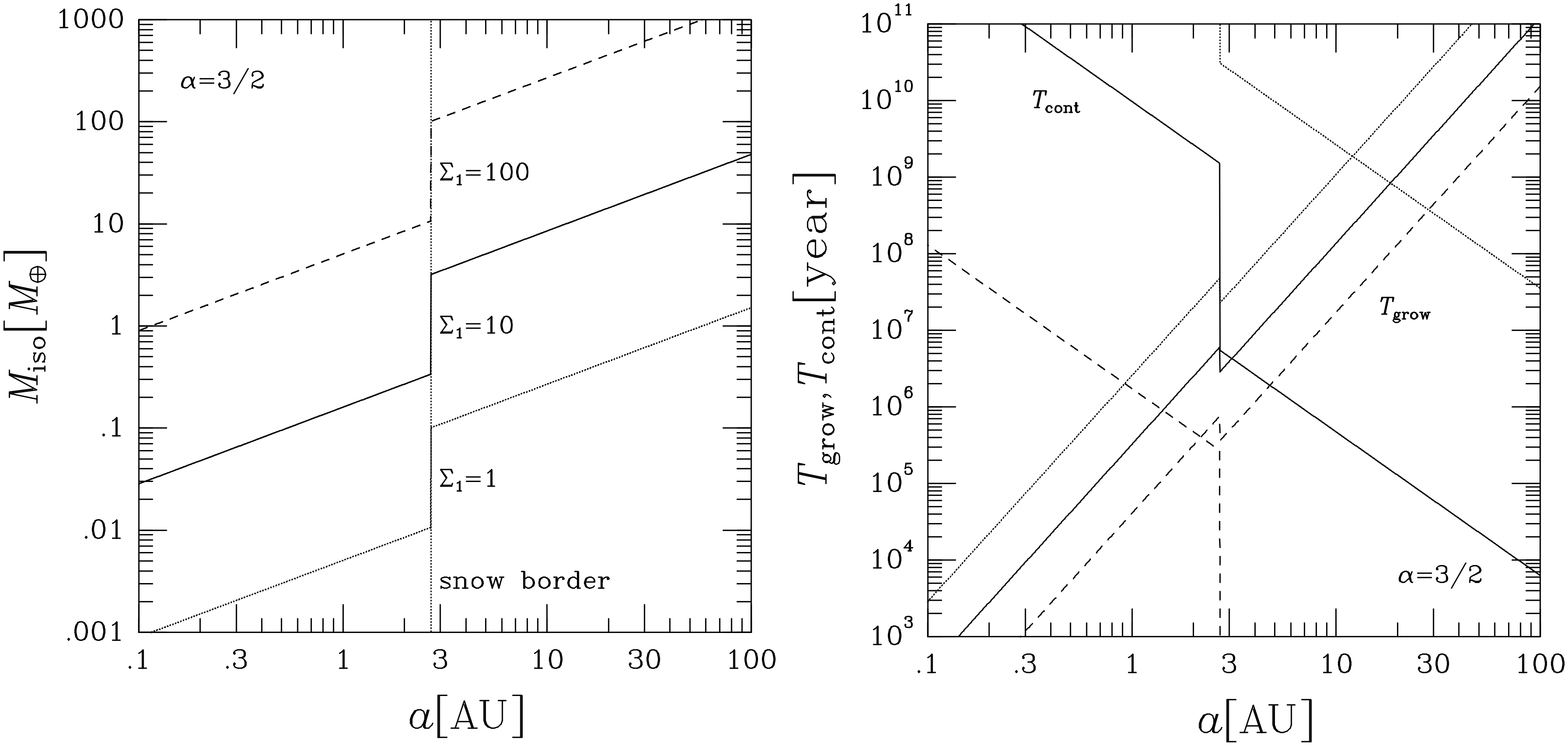
$$T_{\rm grow} < T_{\rm disk} \; (\alpha = 3/2) \Longrightarrow$$

$$a < a_{\rm gas}^{\rm max} \simeq 5.5 f_{\rm ice}^{10/59} \left(\frac{T_{\rm disk}}{10^8 {\rm year}} \right)^{20/59} \left(\frac{\tilde{b}}{10} \right)^{-2/59} \left(\frac{\Sigma_1}{10} \right)^{10/59} {\rm AU}$$

Jovian Range	$a_{\rm gas}^{\rm min} \lesssim a \lesssim a_{\rm gas}^{\rm max}$
Terrestrial Range	$a_{\rm gas}^{\rm min} \lesssim a \lesssim a_{\rm gas}^{\rm max} \cap a \lesssim a_{\rm snow}$
Uranian Range	$a_{\rm gas}^{\rm min} \lesssim a \lesssim a_{\rm gas}^{\rm max} \cap a \gtrsim a_{\rm snow}$

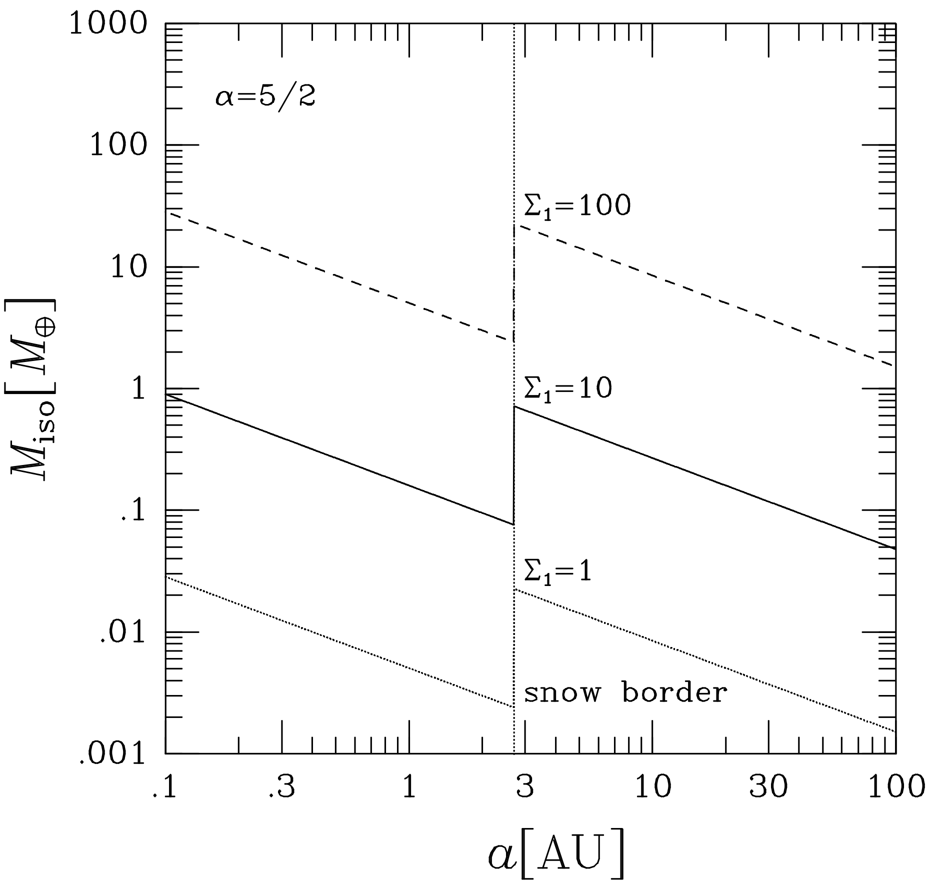
Isolation Mass and Timescales

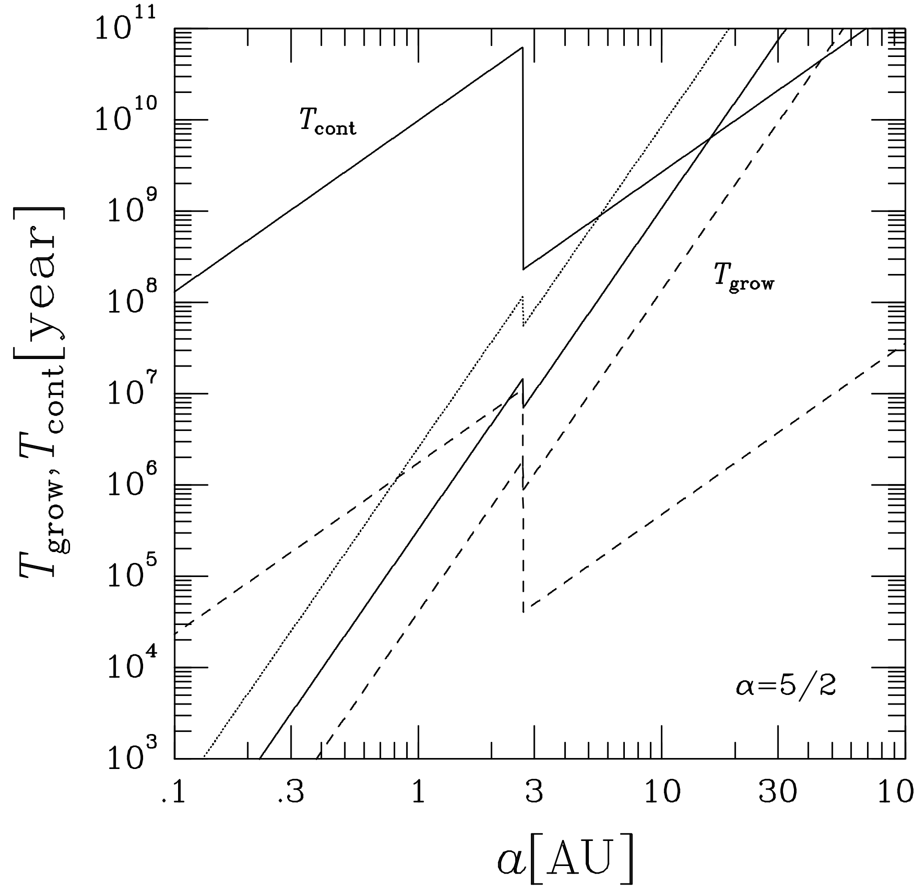
$\alpha = 3/2$



Isolation Mass and Timescales

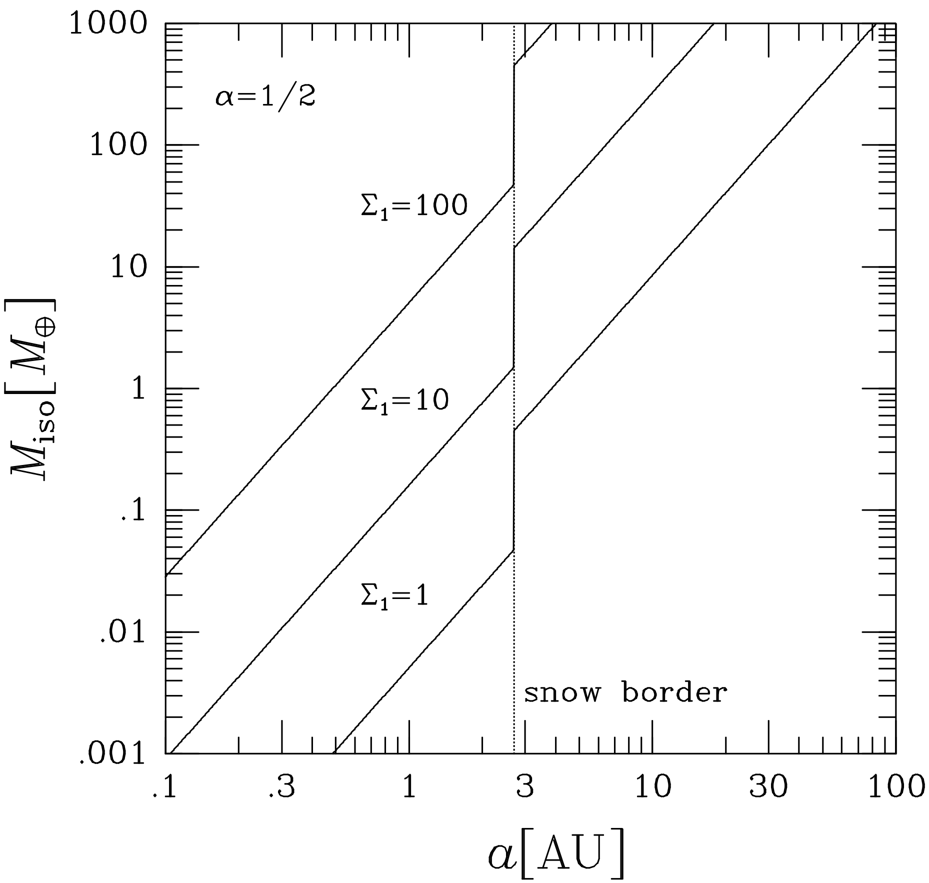
$\alpha = 5/2$

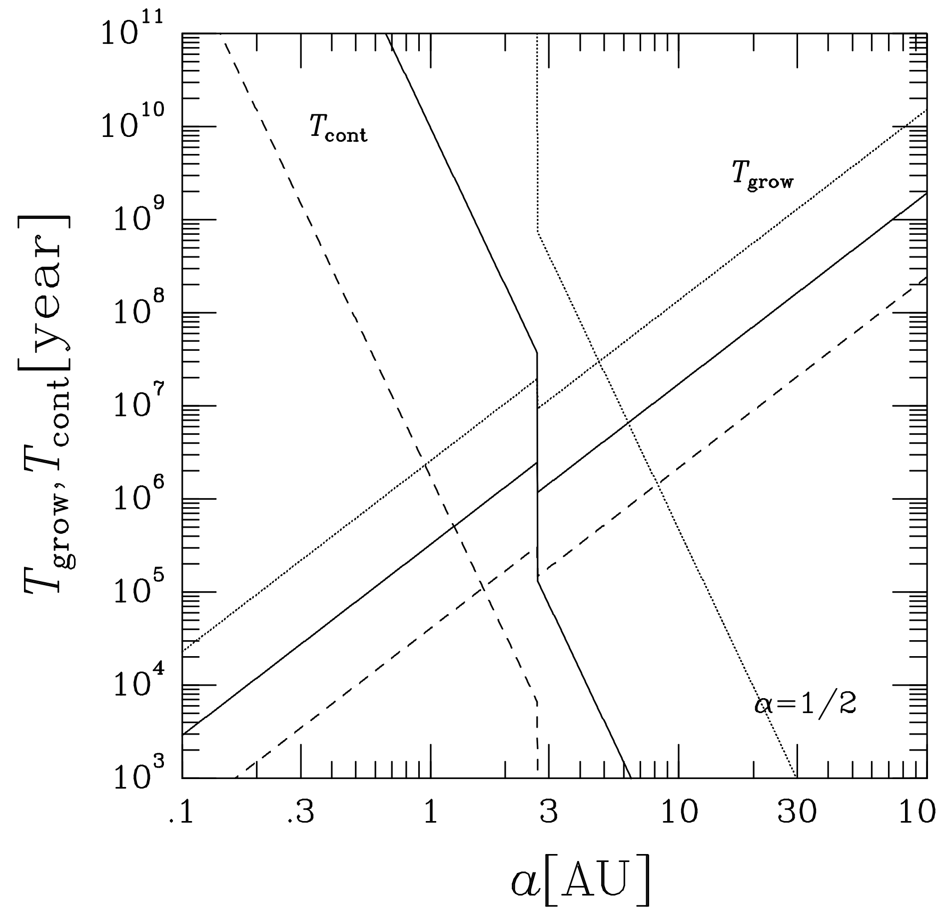




Isolation Mass and Timescales

$\alpha = 1/2$



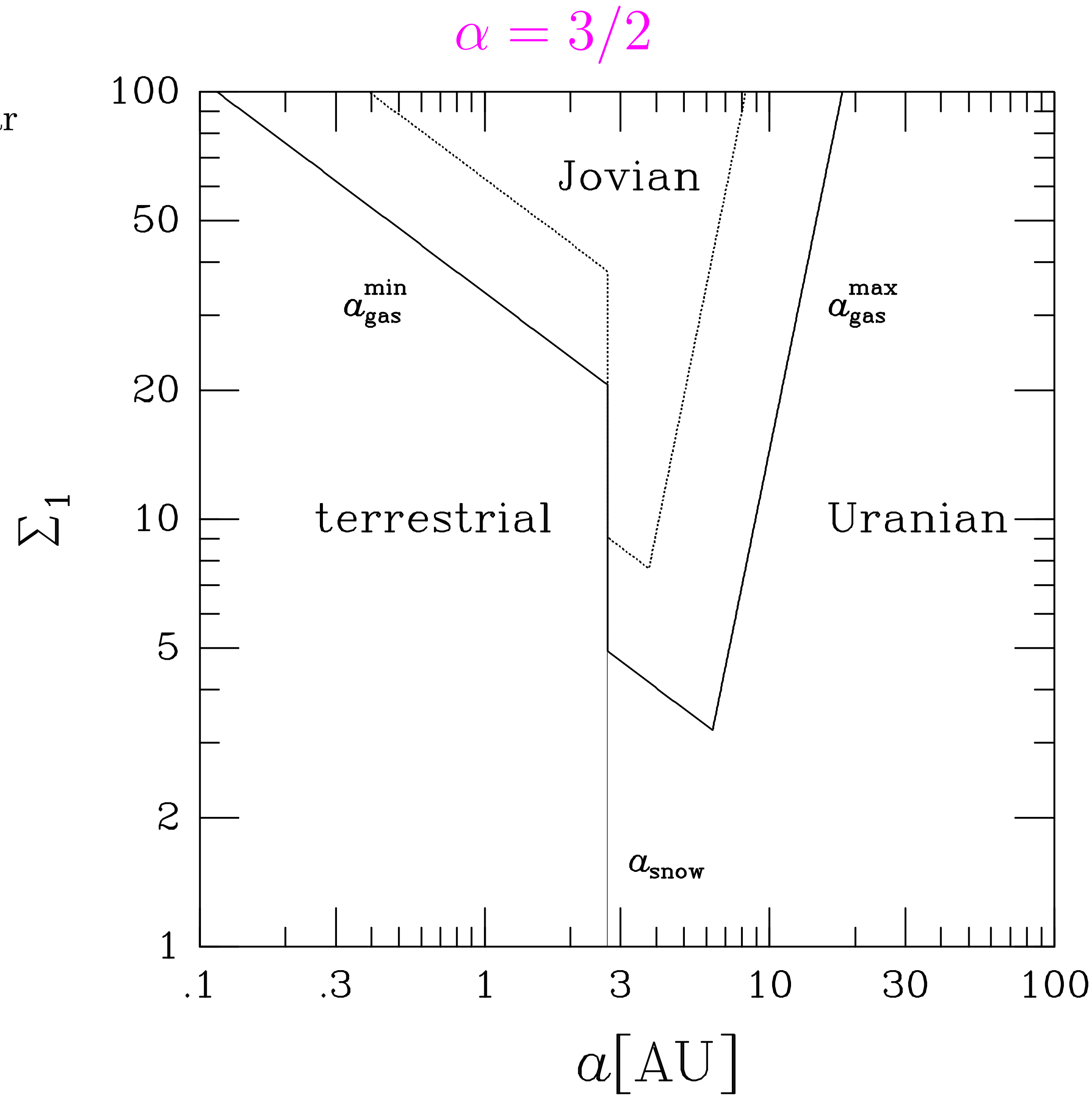


Habitat Segregation of Planets

$$T_{\rm cont} = 10^8 \left(\frac{M_{\rm iso}}{M_\oplus}\right)^{-5/2} \text{year}$$

$$T_{\rm disk} = \begin{cases} 10^7\text{year} & \cdots\cdots \\ 10^8\text{year} & \text{———} \end{cases}$$

$$M_* = M_\odot$$

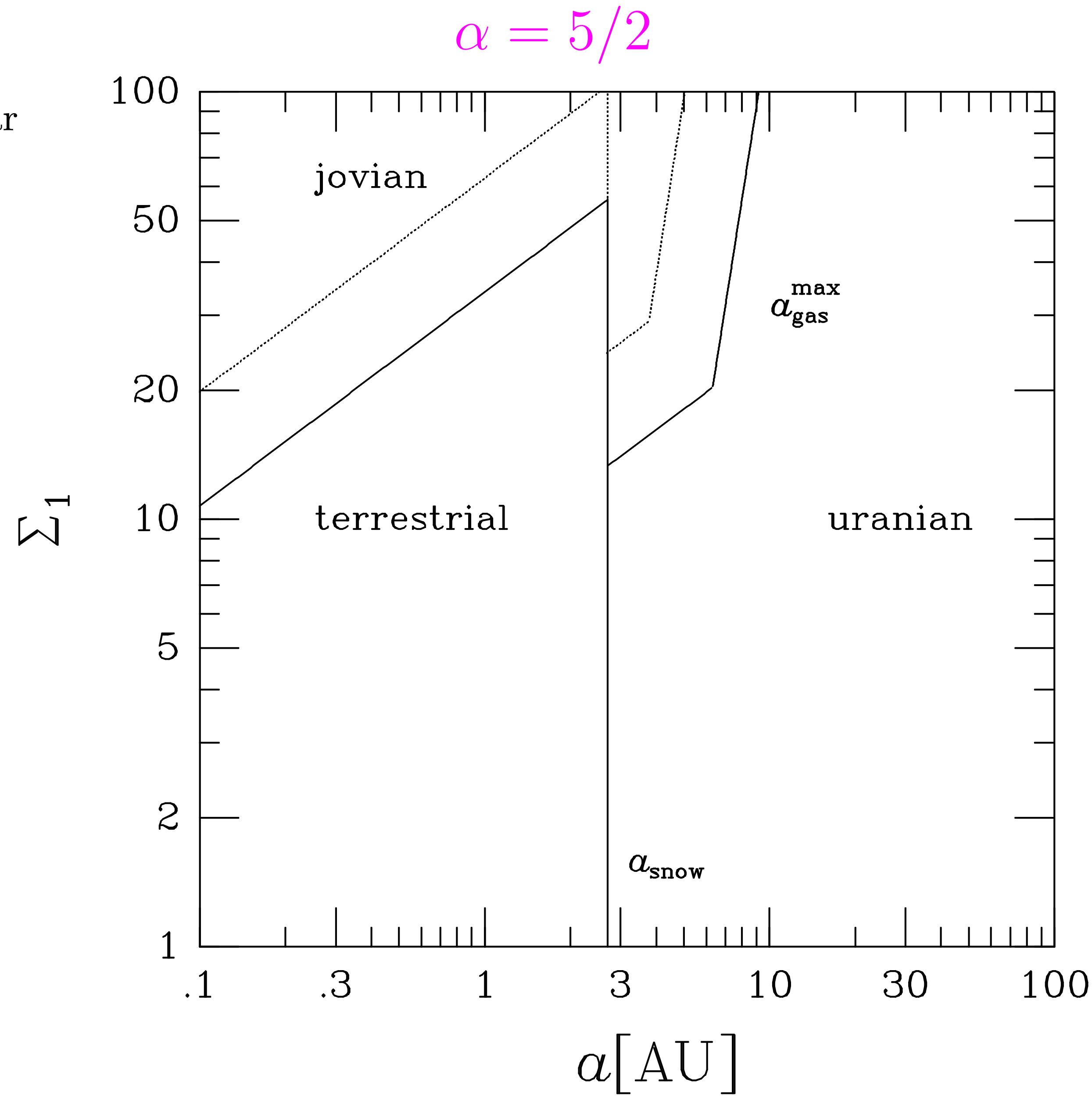


Habitat Segregation of Planets

$$T_{\rm cont} = 10^8 \left(\frac{M_{\rm iso}}{M_\oplus}\right)^{-5/2} \text{year}$$

$$T_{\rm disk} = \begin{cases} 10^7\text{year} & \cdots\cdots \\ 10^8\text{year} & \text{———} \end{cases}$$

$$M_* = M_\odot$$

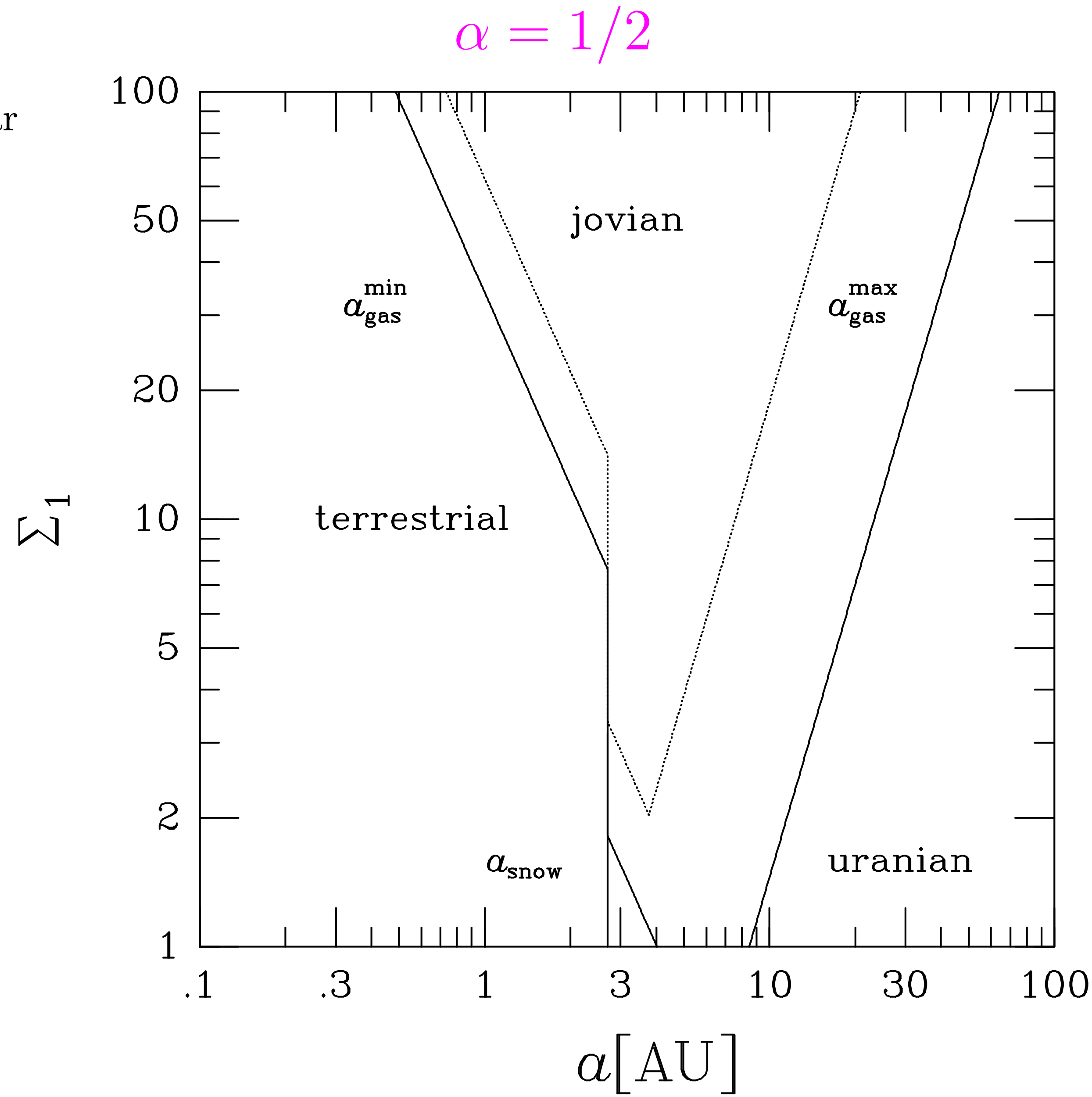


Habitat Segregation of Planets

$$T_{\rm cont} = 10^8 \left(\frac{M_{\rm iso}}{M_\oplus} \right)^{-5/2} \text{year}$$

$$T_{\rm disk} = \begin{cases} 10^7 \text{year} & \cdots\cdots \\ 10^8 \text{year} & \text{———} \end{cases}$$

$$M_* = M_\odot$$



Diversity of Planetary Systems

Disk Mass Dependence ($\alpha = 3/2$)

Light Disk ($\Sigma_1 \lesssim 3$)

Many relatively small terrestrial and uranian planets

Massive Disk ($\Sigma_1 \gtrsim 30$)

Several jovian and uranian planets

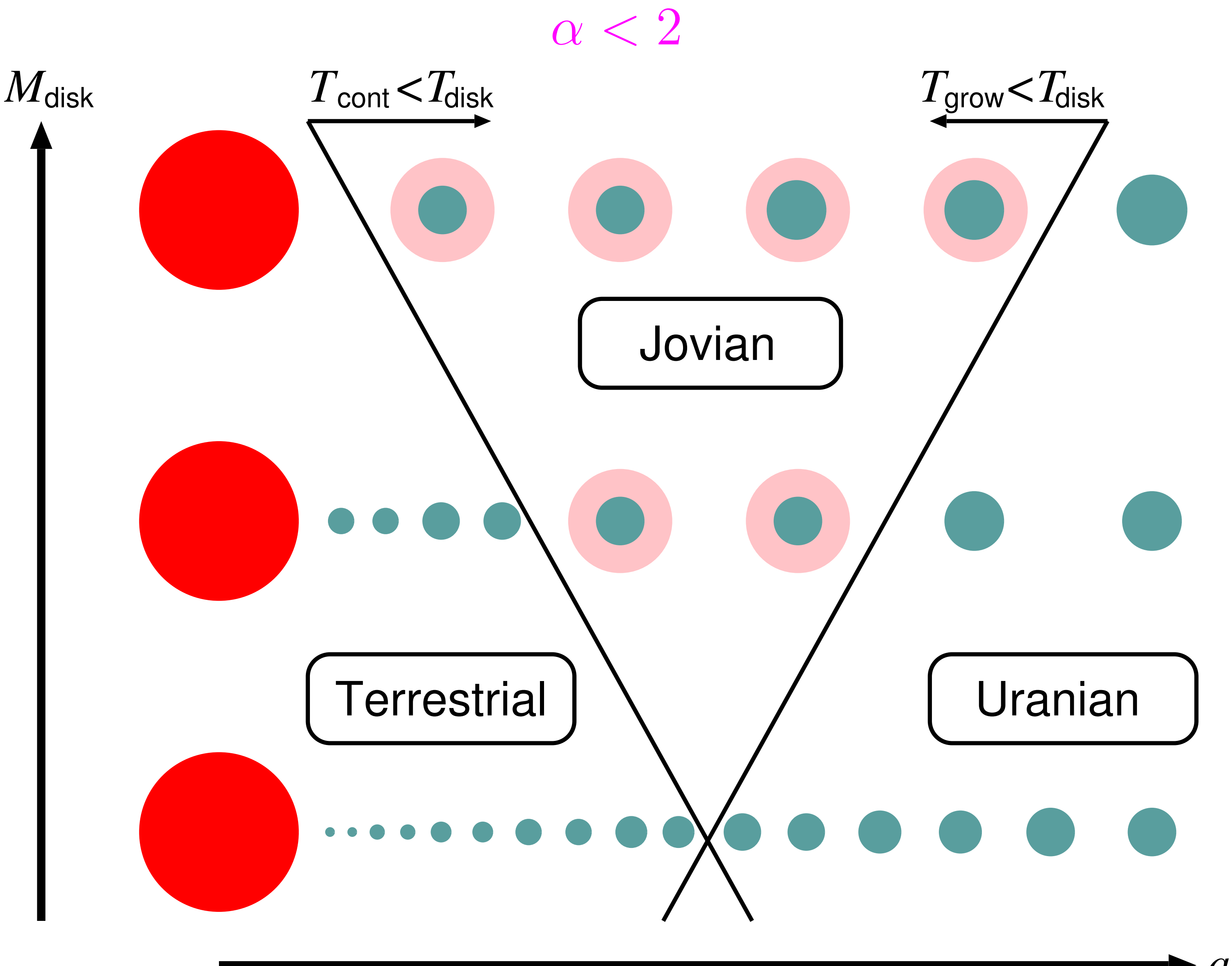
orbital instability $\Longrightarrow$ eccentric planets

interaction with gas disk $\Longrightarrow$ hot jupiters

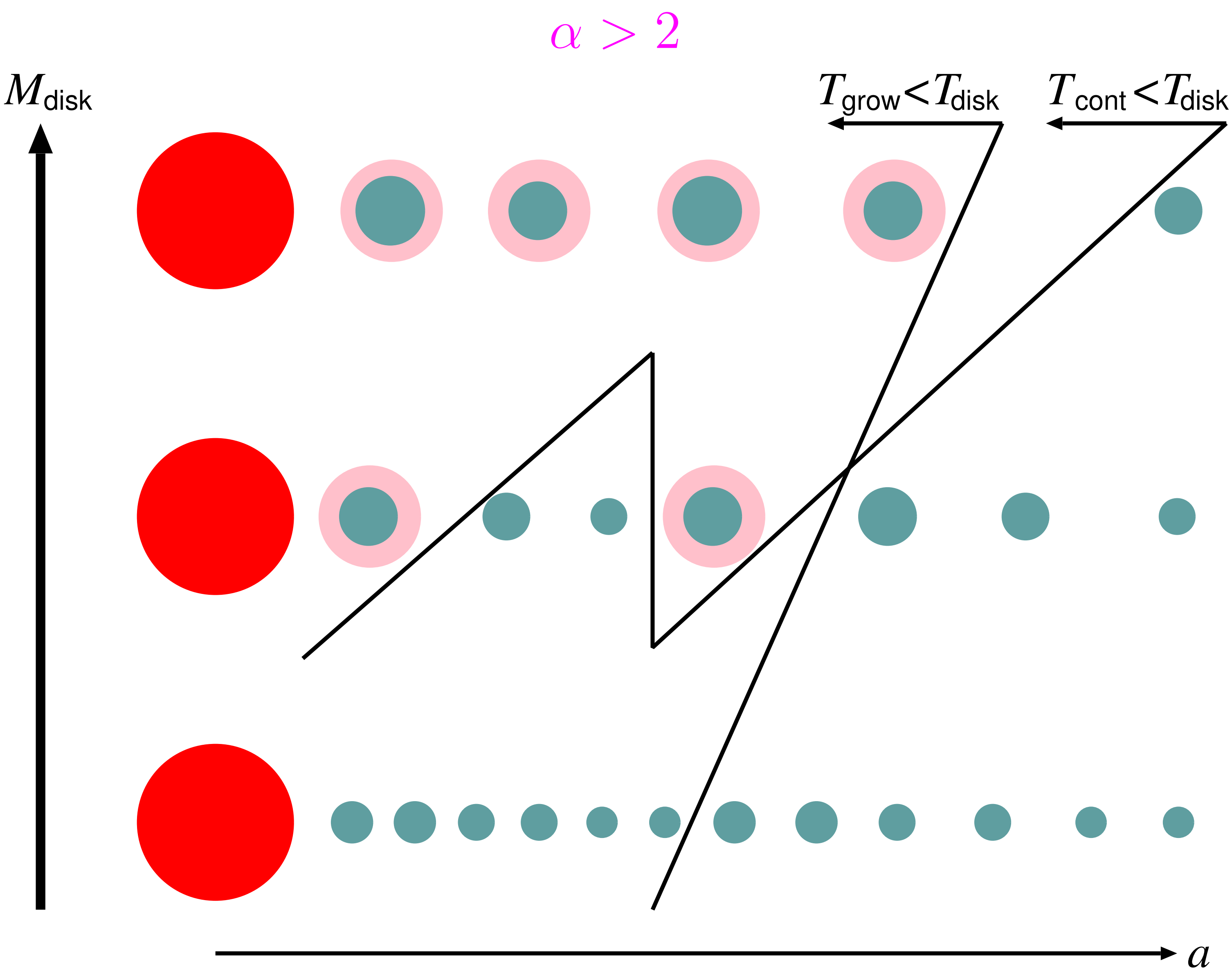
Medium Disk ($\Sigma_1 \simeq 10$)

Solar system-like planetary system

Diversity of Planetary Systems



Diversity of Planetary Systems



Future Work

Terrestrial Region

- Number, mass, and orbital elements (habitability)?
- Effect of Jovian planets?

Jovian Region

- Number and mass?
- Orbital stability?

Uranian Region

- Migration?
- Effect of Jovian planets?

Oligarchic Growth Model

- Migration of Protoplanets
- Migration of Planetesimals (Thommes et al. 2003)

Summary

Oligarchic Growth of Protoplanets

planetesimal disk $\Longrightarrow$ protoplanet system

+

Formation of Jovian Planets

protoplanet $\Longrightarrow$ Jovian planet

$\Downarrow$

Diversity of Planetary Systems

- Habitat segregation of planets
- Dependence on disk mass and profile