Dispersion effects in light propagation in fiber gratings

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Carlos Martel (martel@fmetsia.upm.es)
Depto. Fundamentos Matemáticos
E.T.S.I. Aeronáuticos, Universidad Politécnica de Madrid
28040 Madrid, Spain

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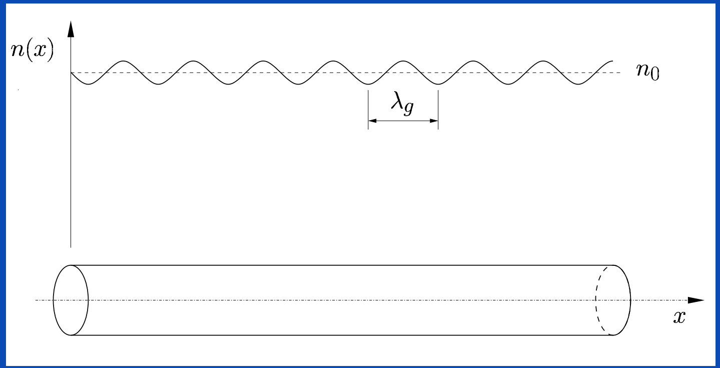
Light propagation in a fiber grating

Fiber grating



1D Nondimensional Maxwell-Lorentz Equations (MLE):

$$\frac{\partial B}{\partial t} = \frac{\partial E}{\partial x}$$
$$\frac{\partial D}{\partial t} = \frac{\partial B}{\partial x}$$
$$D = E + P$$

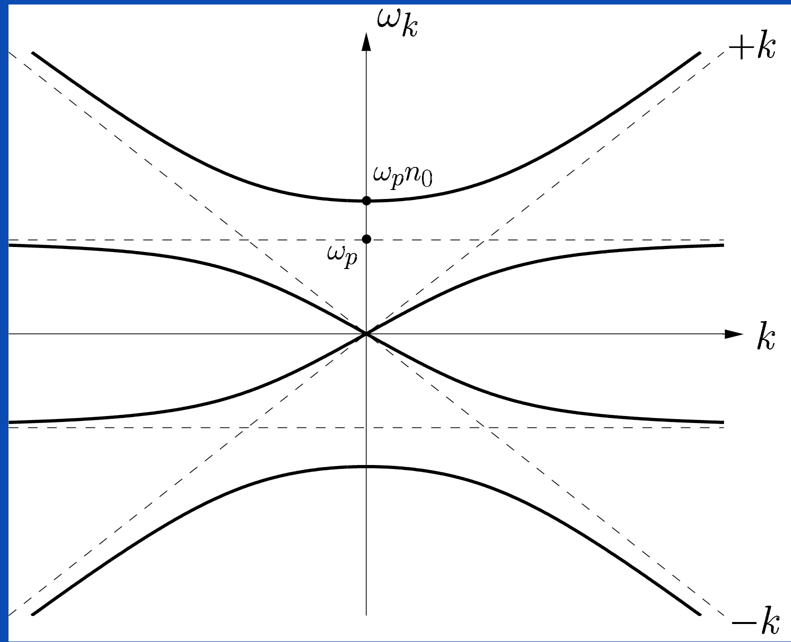
$$\omega_p^{-2}\frac{\partial^2 P}{\partial t^2} + (1 - 2\Delta n \cos(2x))P - P^3 = (n_0^2 - 1)E$$

$$\Delta n \ll 1 \quad L \gg 1 \qquad |E|, |P|, |B|, |D| \ll 1 \qquad (t \to -t,\, t \to t + c,\, x \to -x)$$

Linear propagation characteristics

$$\left\{ \begin{array}{c} E(x,t) \\ B(x,t) \\ D(x,t) \\ P(x,t) \end{array} \right\} = \left\{ \begin{array}{c} E_k \\ B_k \\ D_k \\ P_k \end{array} \right\} A \;\; e^{ikx+i\omega_k t} + \text{c.c.}$$

$$(\Delta n = 0,\; k \rightarrow -k,\; \omega \rightarrow -\omega)$$



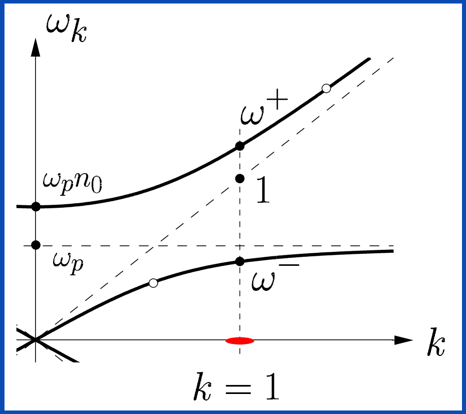
Grating effect

$$\sim \Delta n(e^{i2x} + e^{-i2x})(Ae^{ikx+i\omega_k t} + \text{c.c}) \rightarrow \Delta n A e^{(\pm 2+k)x+i\omega_k t} \quad (\text{resonant } k = \pm 1)$$

Weakly nonlinear description

$$\left\{\begin{array}{c} E(x,t) \\ B(x,t) \\ D(x,t) \\ P(x,t) \end{array}\right\} = V(A^+(x,t)\ e^{ix+i\omega_k t} + A^-(x,t)e^{-ix+i\omega_k t} + \text{c.c.}) + \cdots$$

$$\cdots \ll |A^{\pm}_{xx}| \ll |A^{\pm}_x| \ll |A^{\pm}| \ll 1, \quad \cdots \ll |A^{\pm}_t| \ll |A^{\pm}| \ll 1 \quad \text{and} \quad \Delta n \ll 1$$



$$A^+_t = \quad v_g A^+_x + idA^+_{xx} + iw\Delta n A^- + i\alpha A^+(|A^+|^2 + 2|A^-|^2) + \ldots$$
$$A^-_t = -v_g A^-_x + idA^-_{xx} + iw\Delta n A^+ + i\alpha A^-(|A^-|^2 + 2|A^+|^2) + \ldots$$

- transport (v_g) and dispersion (d).
-$k \neq \pm 1$ NLS, $\tilde{x} = x + v_g t$.

$$L \gg 1, \qquad A^+(x+L) = A^+(x,t), \quad A^-(x+L) = A^-(x,t) \qquad (L = 2\pi m)$$

Amplitude equations

$$\tilde{x} \sim x/L, \quad \tilde{t} \sim t/L, \quad |\tilde{A}^{\pm}|^2 \sim |A^{\pm}|^2/L, \quad L \gg 1 \qquad (\sigma = \frac{1}{2})$$

$$A_t^+ = \quad A_x^+ + i\varepsilon A_{xx}^+ + i\kappa A^- + iA^+(\sigma|A^+|^2 + |A^-|^2)$$

$$A_t^- = -A_x^- + i\varepsilon A_{xx}^- + i\kappa A^+ + iA^-(\sigma|A^-|^2 + |A^+|^2)$$

$$A^{\pm}(x+1,t) = A^{\pm}(x,t)$$

- $\kappa \sim \Delta n L \sim 1$
- $|\varepsilon| \sim \frac{1}{L} \ll 1, \qquad \varepsilon = 0$ NLCME (CW,Gap Solitons,...).
- $|\varepsilon| \sim 1$ Champneys et al. PRL (1998), J. Phys. A (1999), J.Schöllmann et al. PRE (2000).
- Singular limit $\varepsilon \longrightarrow 0.|\varepsilon| \sim \frac{1}{L} \ll 1$, positive (negative) for $\omega^+(\omega^-)$.
- $|A_x^{\pm}| \gg \varepsilon|A_{xx}^{\pm}|$, $\delta x \gg |\varepsilon|$ (slow modulation)

$\delta x \sim 1$ (transport) $\qquad \delta x \sim \sqrt{|\varepsilon|} \gg |\varepsilon|$ (dispersive scales)

Continuous wave solutions

CW:

$$A^+ = A^+_{cw} = \rho \cos\theta \, e^{i\alpha t + imx}$$

$$A^- = A^-_{cw} = \rho \sin\theta \, e^{i\alpha t + imx}$$

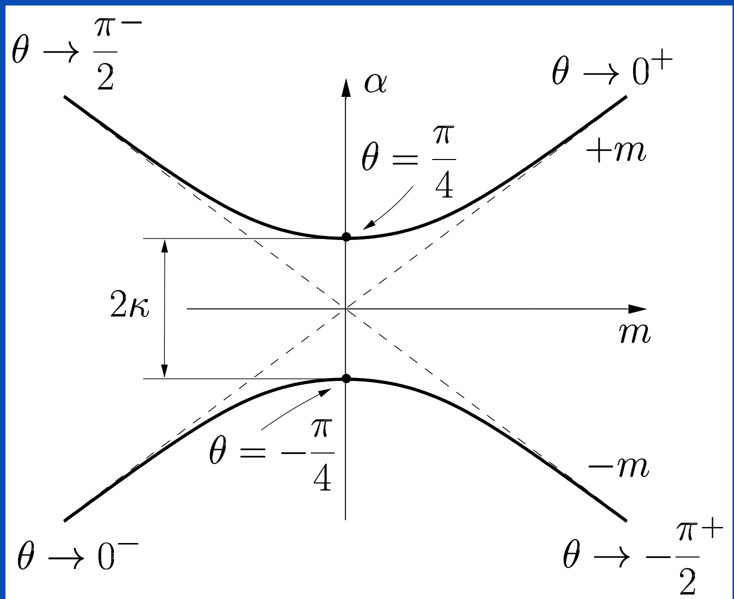
$$\rho > 0 \quad \theta \in]-\tfrac{\pi}{2}, 0[\cup]0, \tfrac{\pi}{2}[$$

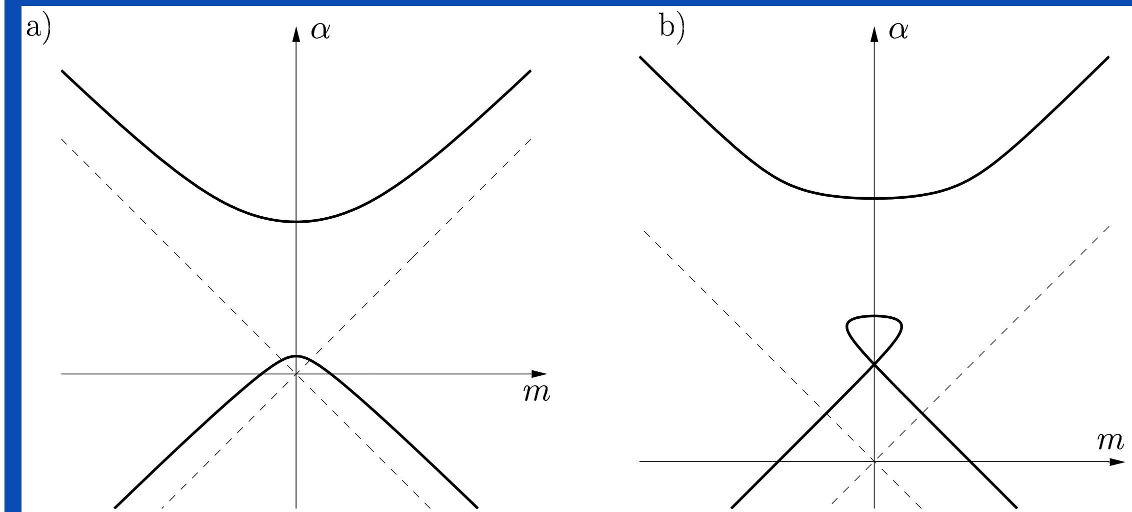
$$\alpha = \frac{\kappa}{\sin 2\theta} + \frac{\sigma+1}{2}\rho$$

$$m = (\frac{\kappa}{\sin 2\theta} - \frac{\sigma-1}{2}\rho^2)\cos 2\theta$$

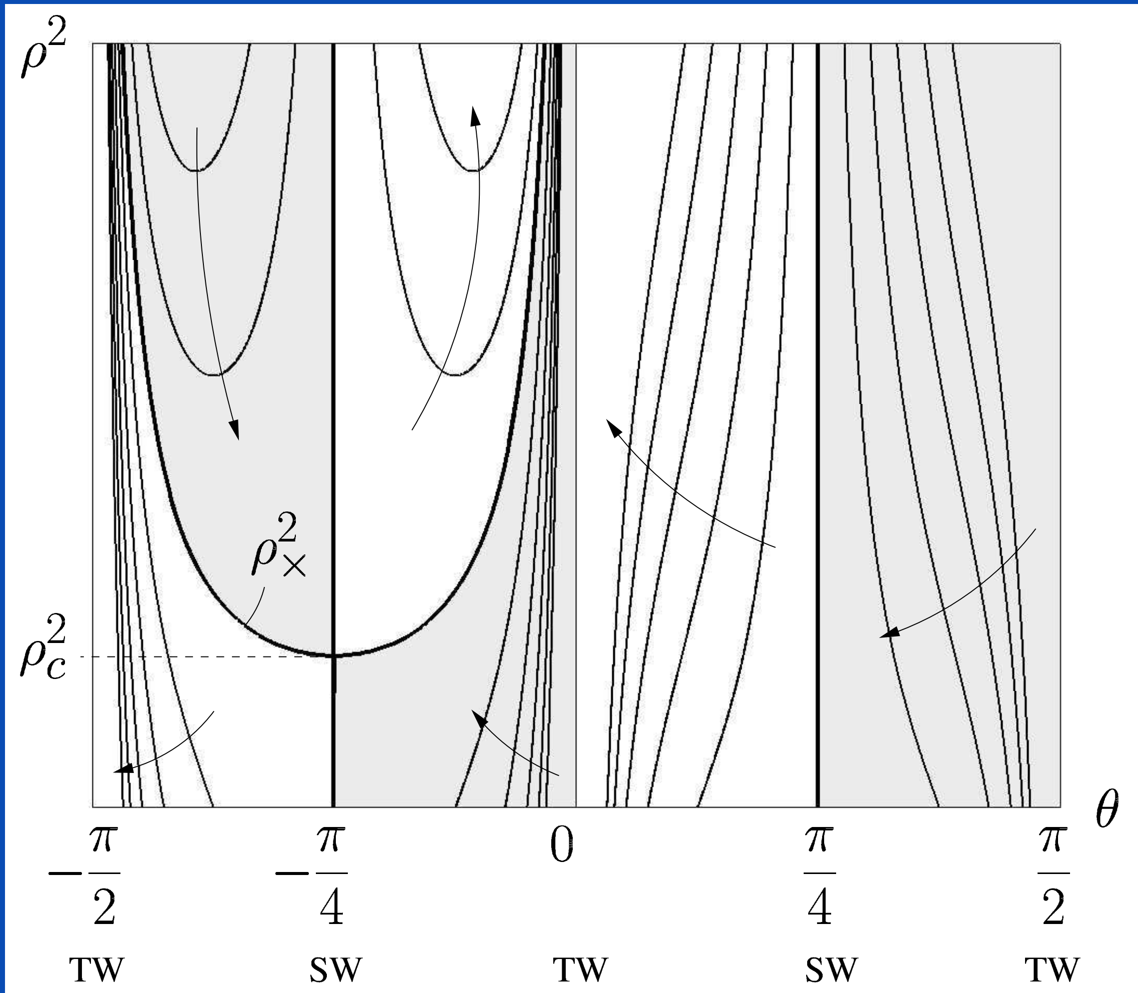
linear $(\rho = 0)$

nonlinear $(\rho \neq 0)$ **a)** $\rho < \rho_c$ **b)** $\rho > \rho_c$



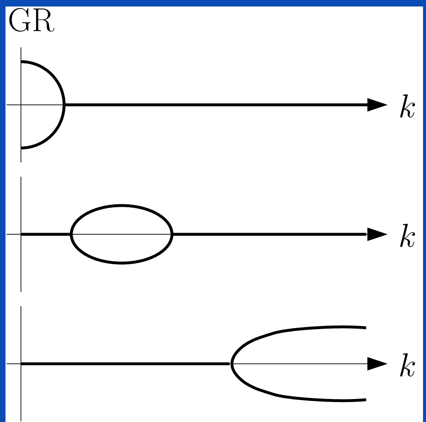


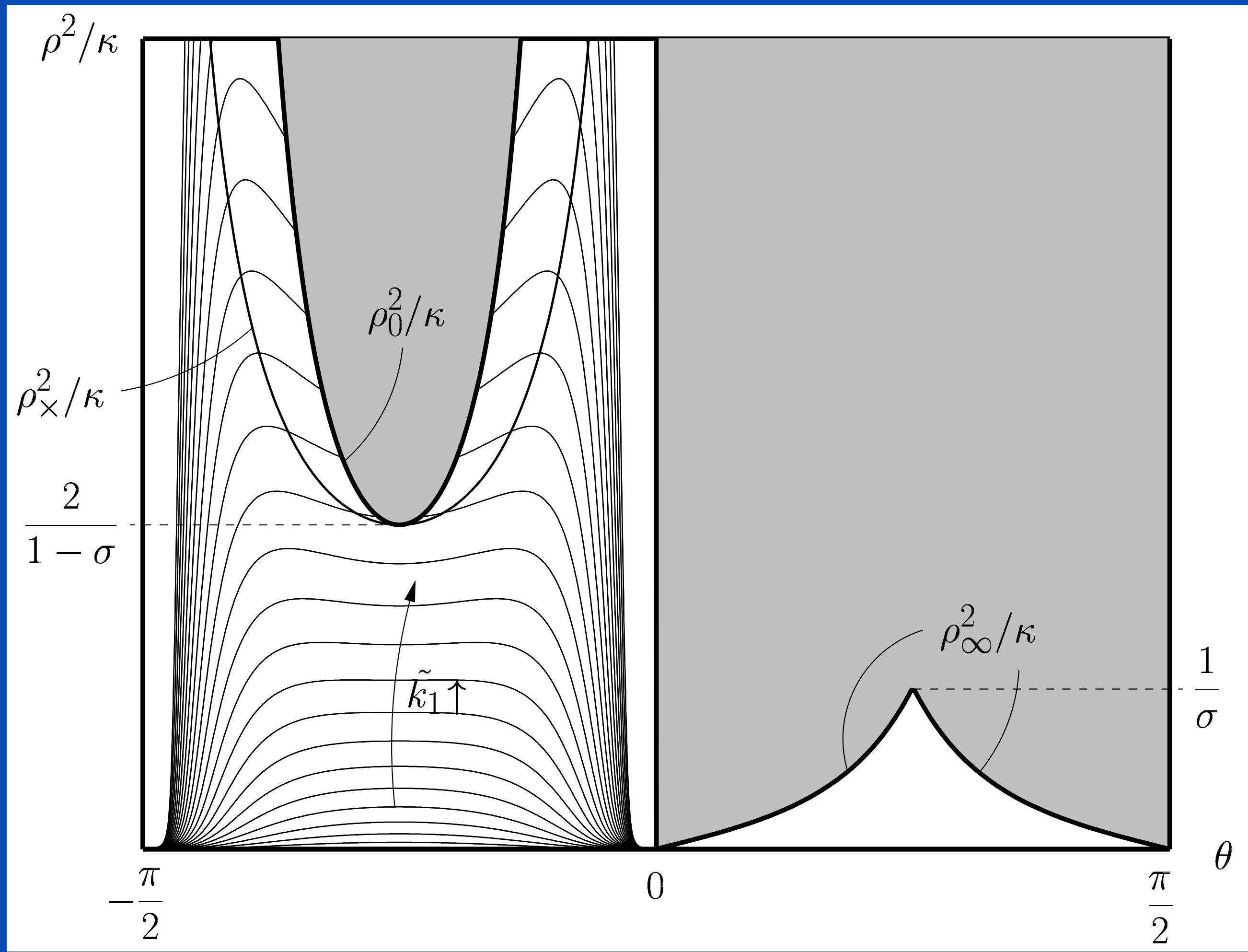
Continuous wave solutions



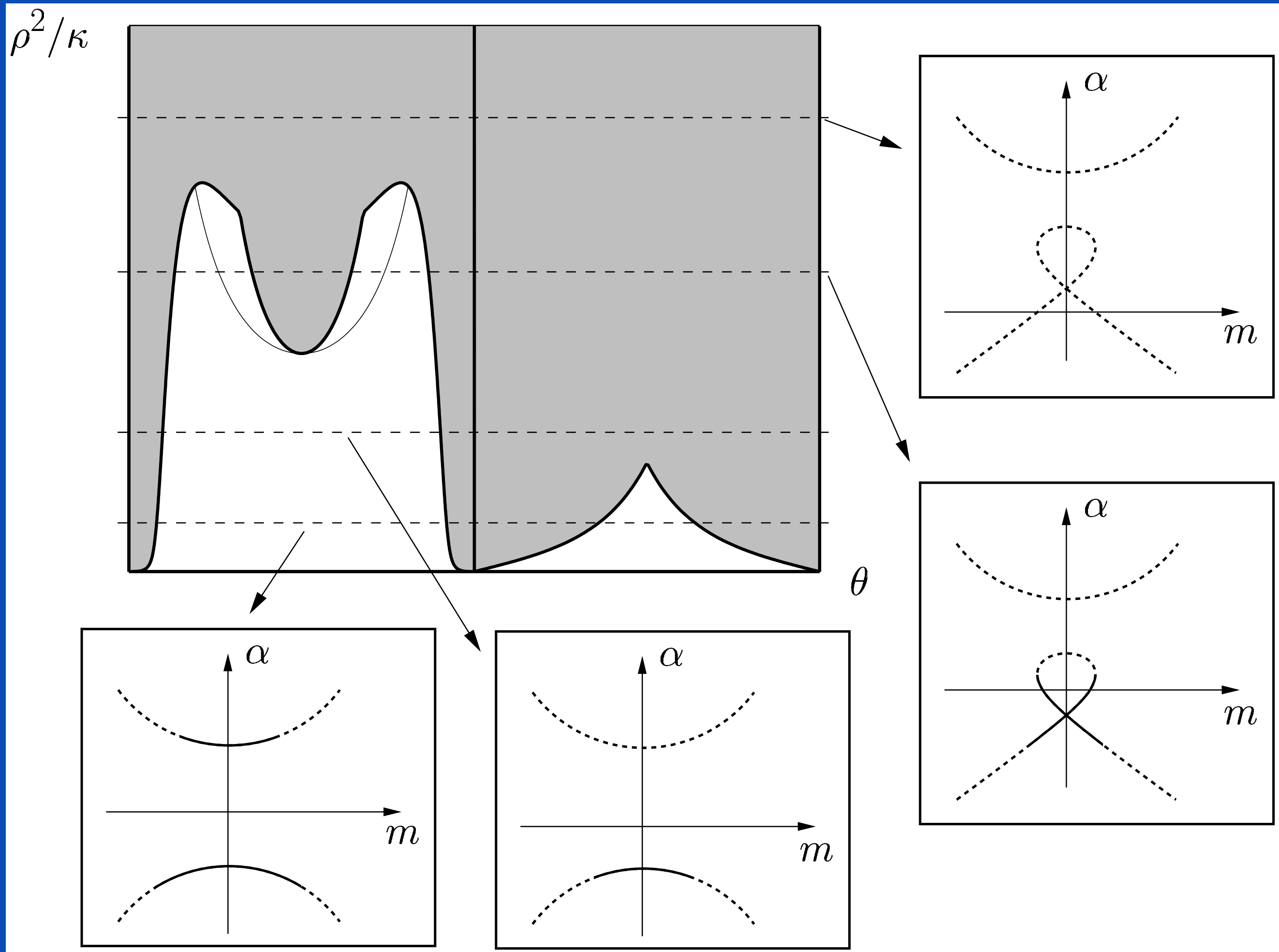
NLCME CW stability ($\varepsilon = 0$)

- ρ_0 uniform ($k = 0$) pert.
- ρ_∞ , $k > k_\star$ pert.
- $k = 1$ pert.
- Sterke JOSA B (1998)

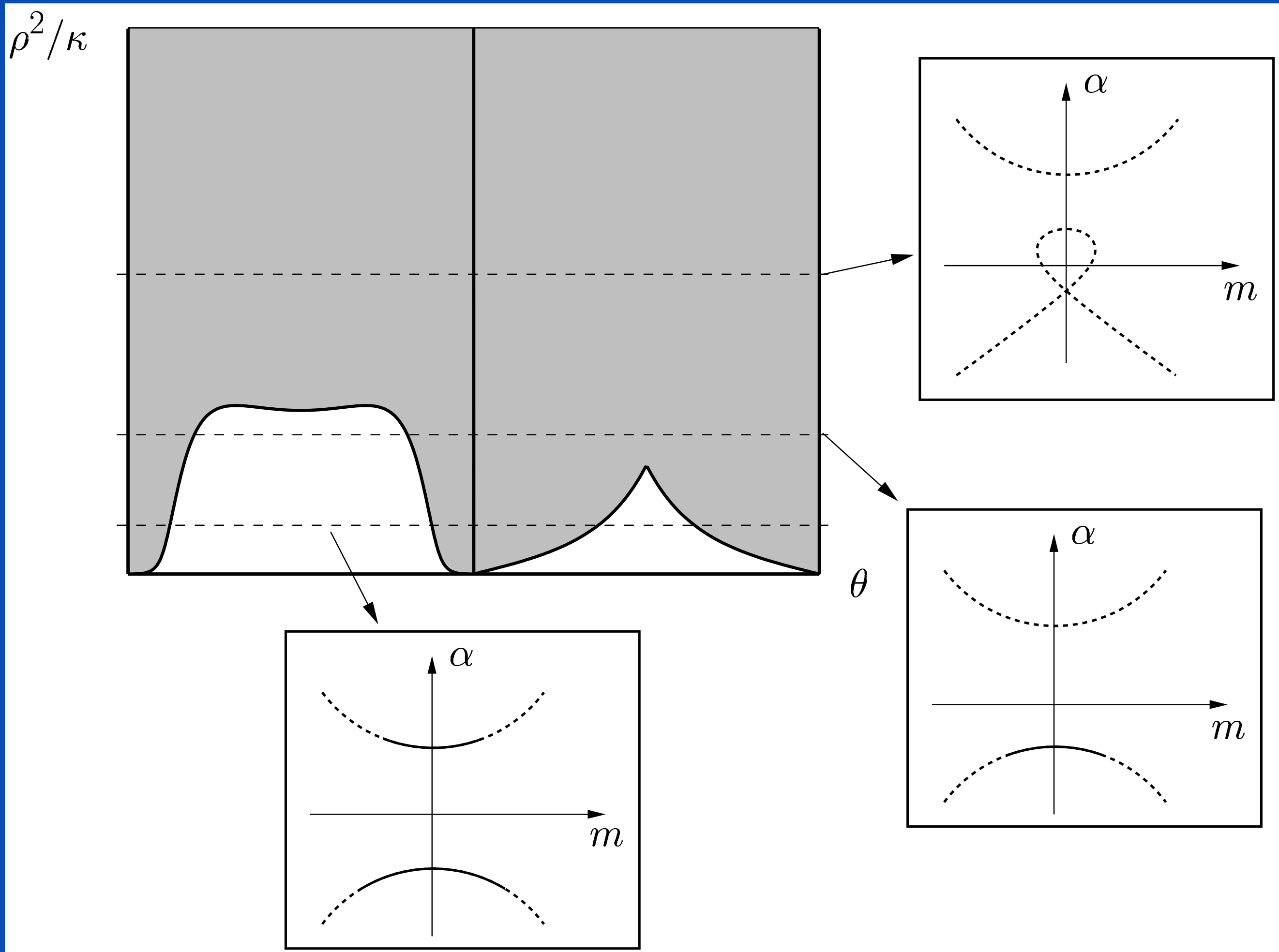




NLCME CW stability ($\kappa < \kappa_c$)



NLCME CW stability ($\kappa > \kappa_c$)



CW dispersive instability

$$A^+ = A^+_{cw}(1 + a^+) \qquad A^- = A^-_{cw}(1 + a^-) \qquad \text{with} \quad |a^\pm| \ll 1$$

$$(a^+, a^-) = \sum_{k=-\infty}^{\infty} (a^+_k(t), a^-_k(t))\, e^{i2\pi kx}$$

$$\begin{aligned}
\frac{da^+_k}{dt} &= i(2\pi k)a^+_k + i\kappa(a^-_k - a^+_k)\tan\theta + i\sigma\rho^2\cos^2\theta(a^+_k + \overline{a^+_{-k}}) \\
&\quad + i\rho^2\sin^2\theta(a^-_k + \overline{a^-_{-k}}) - i\varepsilon(2\pi k)^2 a^+_k \\
\frac{da^-_k}{dt} &= -i(2\pi k)a^-_k + i\kappa(a^+_k - a^-_k)/\tan\theta + i\sigma\rho^2\sin^2\theta(a^-_k + \overline{a^-_{-k}}) \\
&\quad + i\rho^2\cos^2\theta(a^+_k + \overline{a^+_{-k}}) - i\varepsilon(2\pi k)^2 a^-_k
\end{aligned}$$

CW dispersive instability

$$a_K^+ = a_{K0}^+(t,T) + \sqrt{|\varepsilon|}\; a_{K1}^+(t,T) + \cdots\,, \quad a_K^- = a_{K0}^-(t,T) + \sqrt{|\varepsilon|}\; a_{K1}^-(t,T) + \cdots\,,$$

$$T = \mathrm{t}/\sqrt{|\varepsilon|}\,,\ K = (2\pi k)\sqrt{|\varepsilon|} \sim 1$$

$$\begin{aligned} \frac{da_{K0}^+}{dT} - iKa_{K0}^+ &= 0, \\ \frac{da_{K0}^-}{dT} + iKa_{K0}^- &= 0, \end{aligned}$$

$$(a_{K0}^+, a_{K0}^-) = (A_{K0}^+(t)e^{iKT}, A_{K0}^-(t)e^{-iKT}).$$

CW dispersive instability

$$\frac{da_{K1}^{+}}{dT} - iKa_{K1}^{+} = [-\frac{dA_{K0}^{+}}{dt} - i(\kappa \tan\theta + \frac{\varepsilon}{|\varepsilon|}K^2)A_{K0}^{+} + i\sigma\rho^2\cos^2\theta(A_{K0}^{+} + \overline{A_{-K0}^{+}})]e^{iKT}$$
$$+[i\kappa\tan\theta A_{K0}^{-} + i\rho^2\sin^2\theta(A_{K0}^{-} + \overline{A_{-K0}^{-}})]e^{-iKT},$$

$$\frac{da_{K1}^{-}}{dT} + iKa_{K1}^{-} = [-\frac{dA_{K0}^{-}}{dt} - i(\kappa/\tan\theta + \frac{\varepsilon}{|\varepsilon|}K^2)A_{K0}^{-} + i\sigma\rho^2\sin^2\theta(A_{K0}^{-} + \overline{A_{-K0}^{-}})]e^{-iKT}$$
$$+[i\kappa/\tan\theta A_{K0}^{+} + i\rho^2\cos^2\theta(A_{K0}^{+} + \overline{A_{-K0}^{+}})]e^{iKT}.$$

$$\frac{dA_{K0}^{+}}{dt} = -i(\kappa\tan\theta + \frac{\varepsilon}{|\varepsilon|}K^2)A_{K0}^{+} + i\sigma\rho^2\cos^2\theta(A_{K0}^{+} + \overline{A_{-K0}^{+}}),$$

$$\frac{dA_{K0}^{-}}{dt} = -i(\kappa/\tan\theta + \frac{\varepsilon}{|\varepsilon|}K^2)A_{K0}^{-} + i\sigma\rho^2\sin^2\theta(A_{K0}^{-} + \overline{A_{-K0}^{-}}).$$

CW dispersive instability

$$\Omega^+ = \pm\sqrt{(\kappa \tan\theta + \frac{\varepsilon}{|\varepsilon|}K^2)(2\sigma\rho^2\cos^2\theta - (\kappa\tan\theta + \frac{\varepsilon}{|\varepsilon|}K^2))}$$

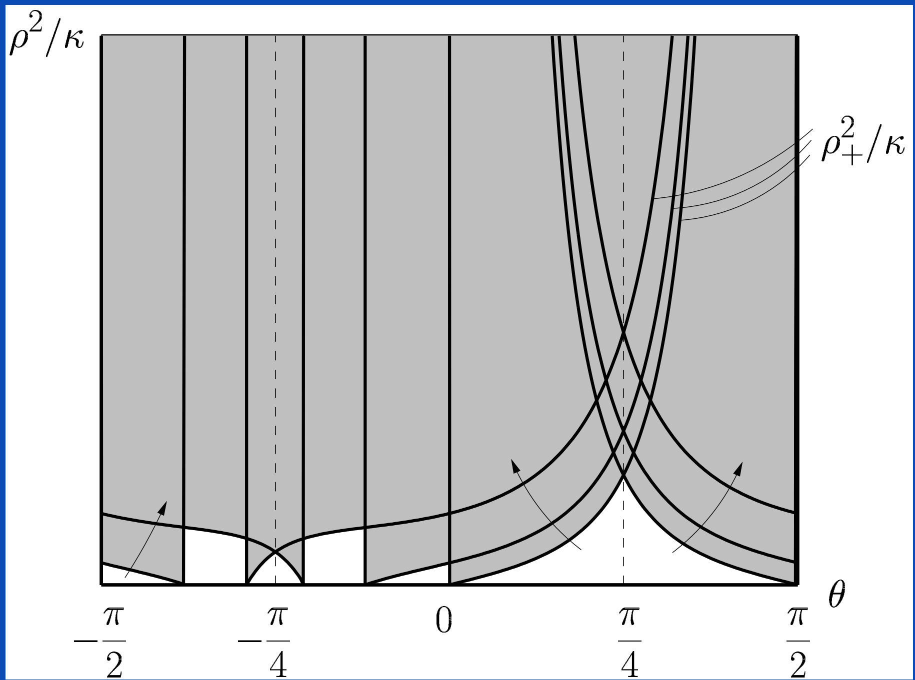
$$\Omega^- = \pm\sqrt{(\kappa/\tan\theta + \frac{\varepsilon}{|\varepsilon|}K^2)(2\sigma\rho^2\sin^2\theta - (\kappa/\tan\theta + \frac{\varepsilon}{|\varepsilon|}K^2))}$$

$$\rho^2 \geq \rho_+^2 = \frac{\tan\theta}{\sigma\sin(2\theta)}(\kappa\tan\theta + \frac{\varepsilon}{|\varepsilon|}K^2) \quad \text{with} \quad \tan\theta \geq \frac{\varepsilon}{|\varepsilon|}\frac{K^2}{\kappa}$$

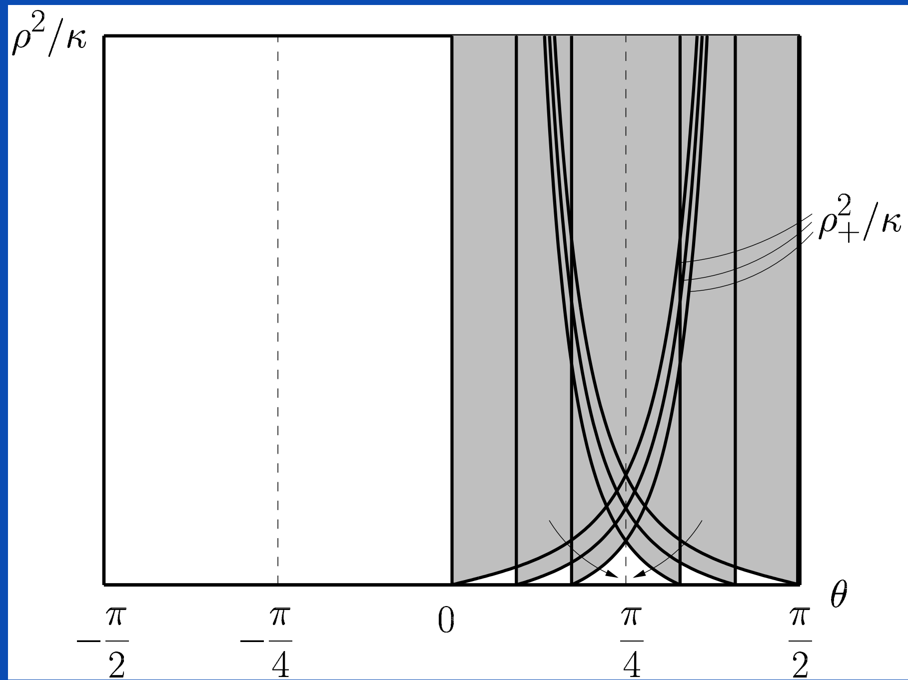
$$\rho^2 \geq \rho_-^2 = \frac{\tan^{-1}\theta}{\sigma\sin(2\theta)}(\kappa\tan^{-1}\theta + \frac{\varepsilon}{|\varepsilon|}K^2) \quad \text{with} \quad \tan^{-1}\theta \geq \frac{\varepsilon}{|\varepsilon|}\frac{K^2}{\kappa}$$

CW dispersive instability

- $\varepsilon > 0$



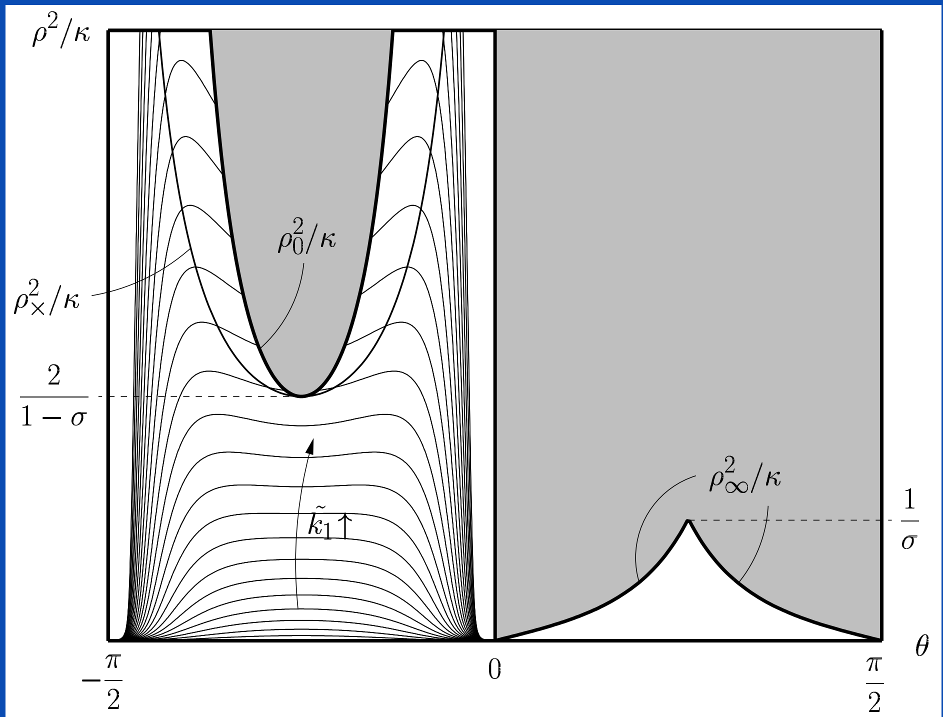
- $\varepsilon < 0$

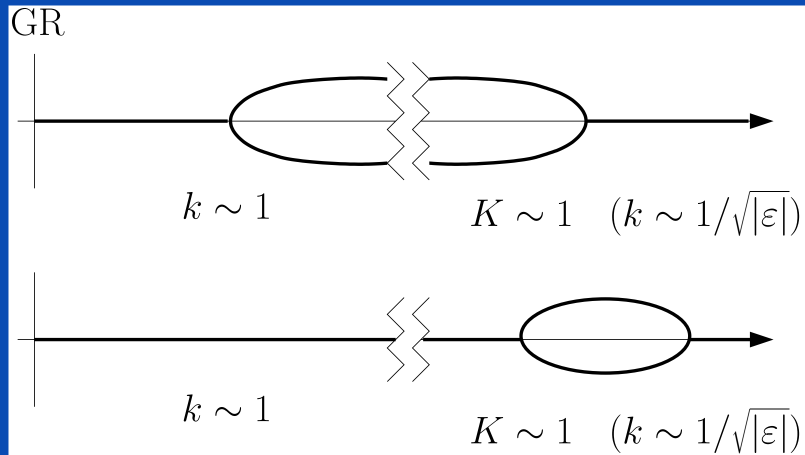


For $\varepsilon > 0$ ($\varepsilon < 0$), all CW with $\theta < 0$ ($\theta > 0$) are unstable

CW dispersive instability

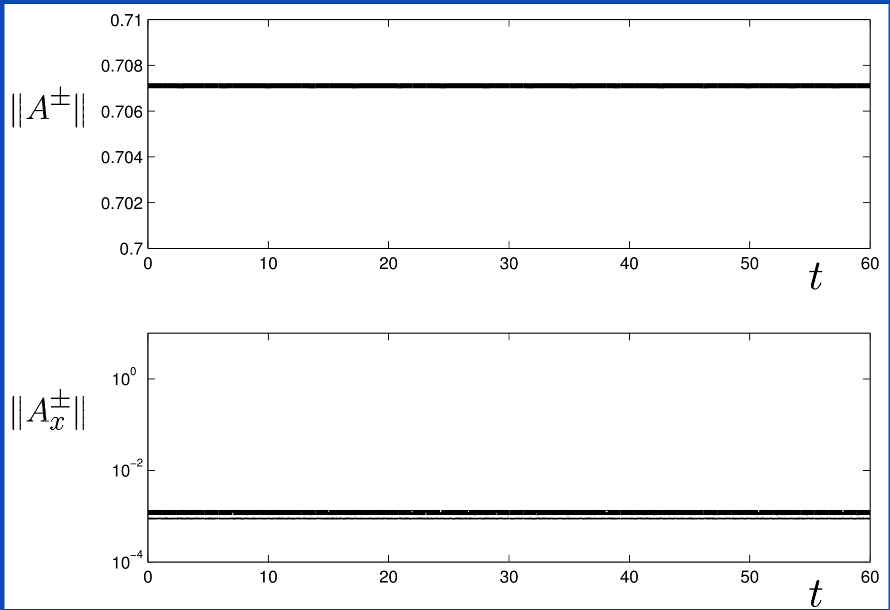
For $\varepsilon > 0$ ($\varepsilon < 0$), all CW with $\theta < 0$ ($\theta > 0$) are unstable



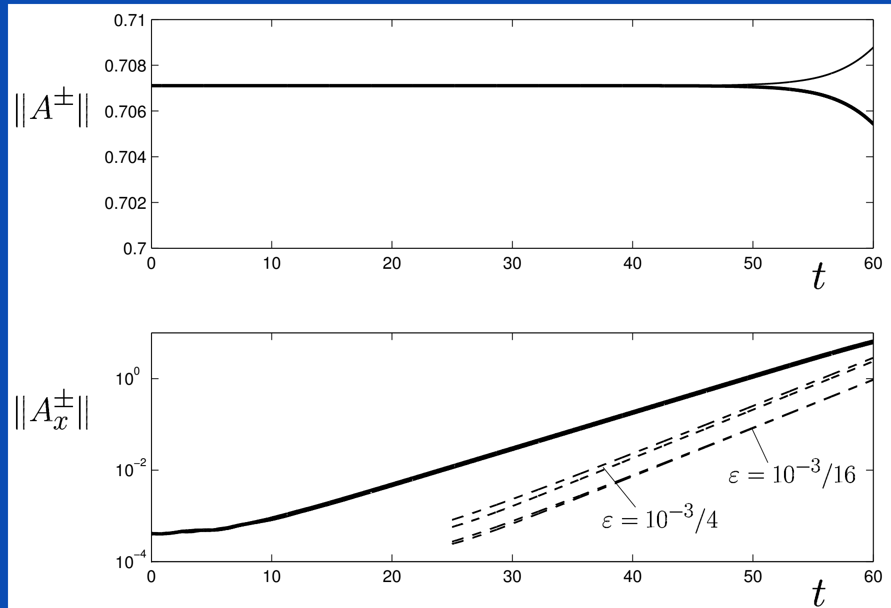


Numerical Simulations

- CW $\kappa = 1$, $\rho = 1$, $\theta = -\frac{\pi}{4}$, $\varepsilon = -10^{-3}$

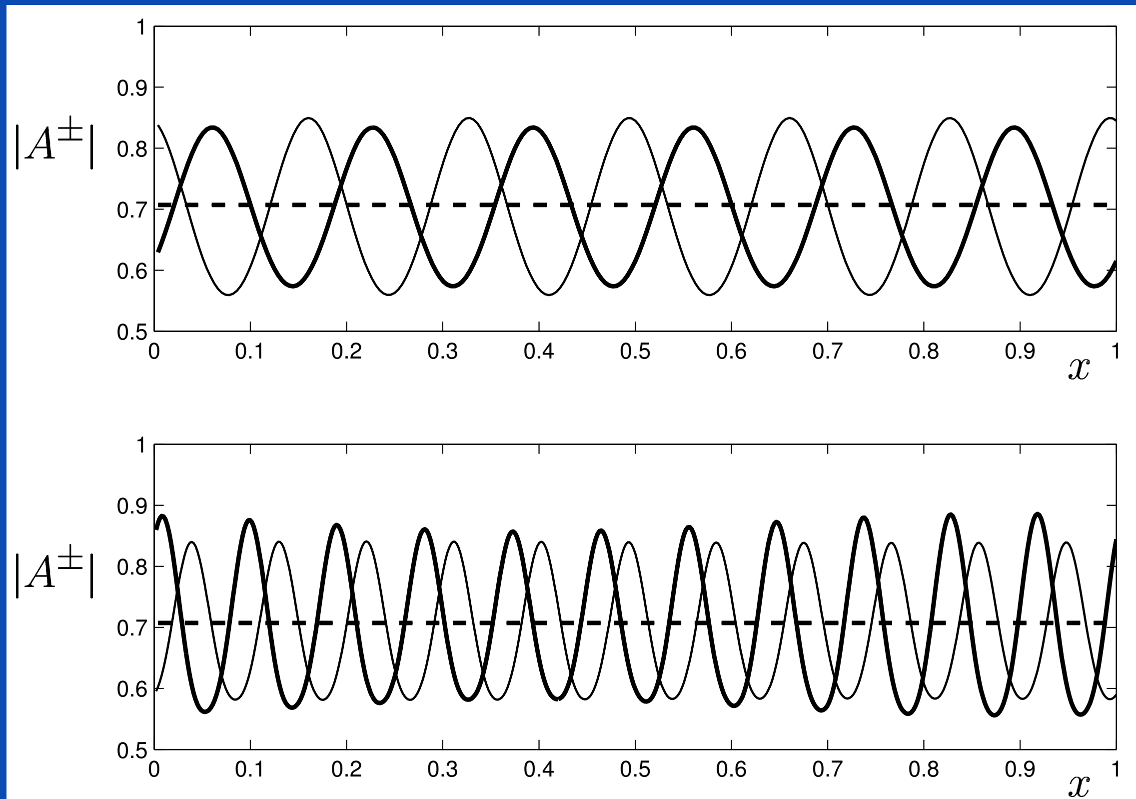


- CW $\kappa = 1$, $\rho = 1$, $\theta = -\frac{\pi}{4}$, $\varepsilon = 10^{-3}$



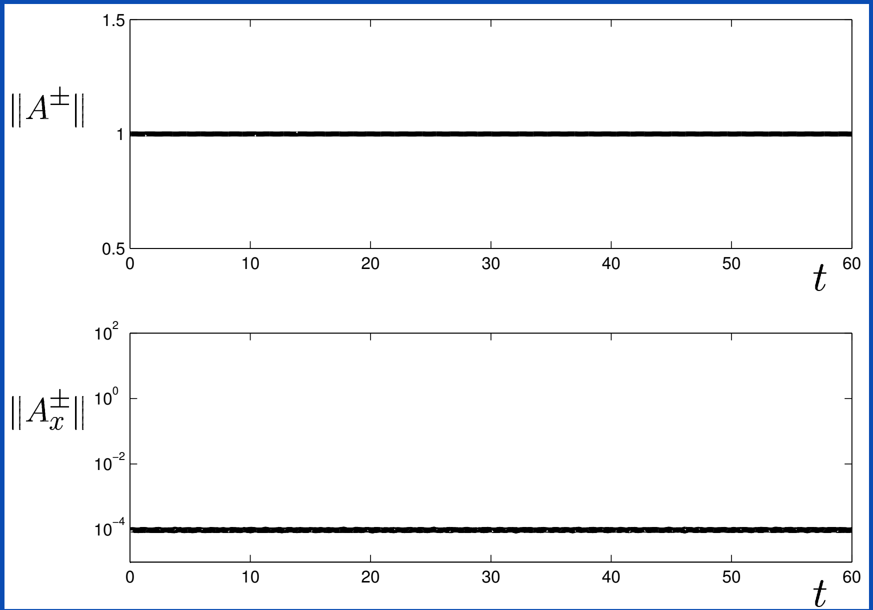
Numerical Simulations

- CW $\kappa = 1$, $\rho = 1$, $\theta = -\frac{\pi}{4}$, $\varepsilon = 10^{-3}$ and $\varepsilon = 10^{-3}/4$

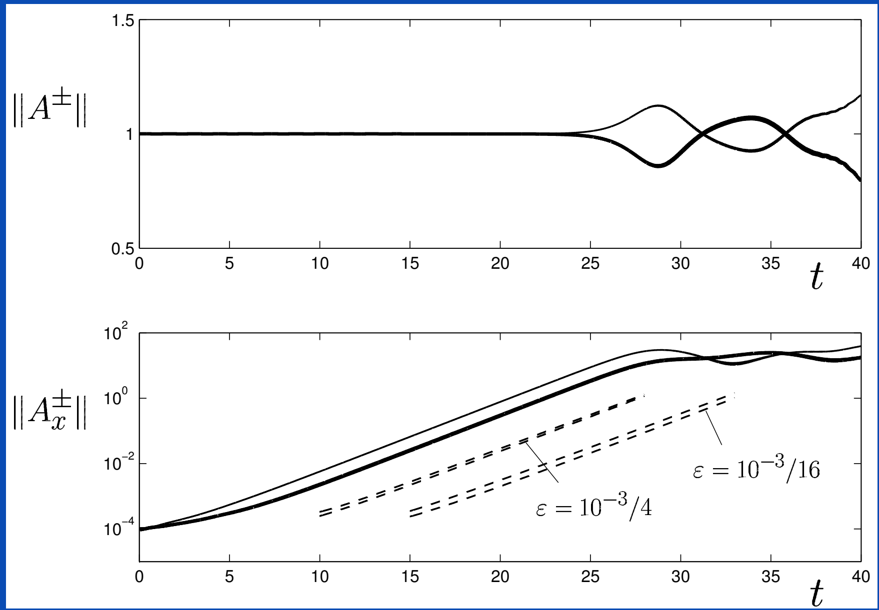


Numerical Simulations

- CW $\kappa = 2$, $\rho = \sqrt{2}$, $\theta = \frac{\pi}{4}$, $\varepsilon = 10^{-3}$

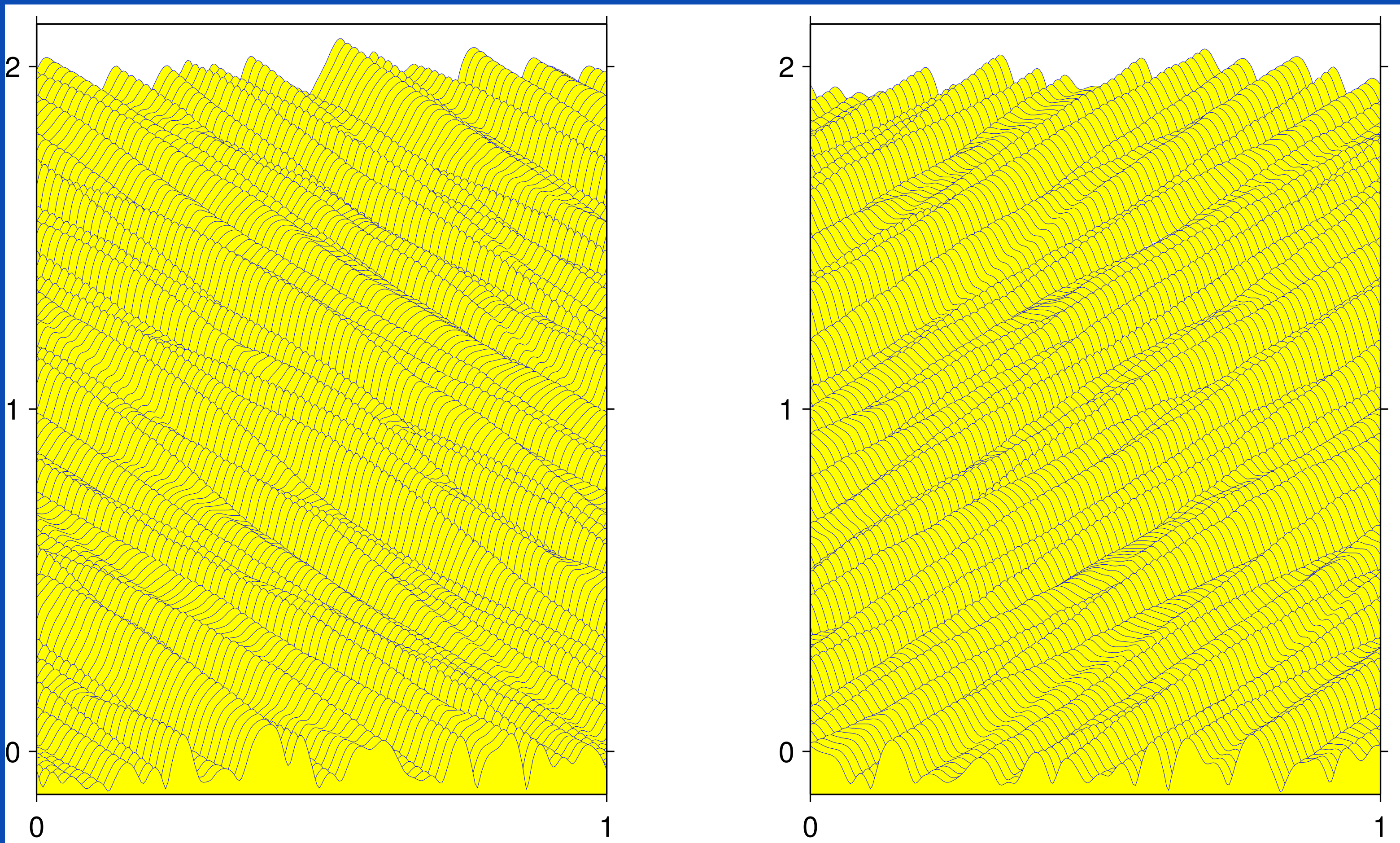


- CW $\kappa = 2$, $\rho = \sqrt{2}$, $\theta = \frac{\pi}{4}$, $\varepsilon = -10^{-3}$



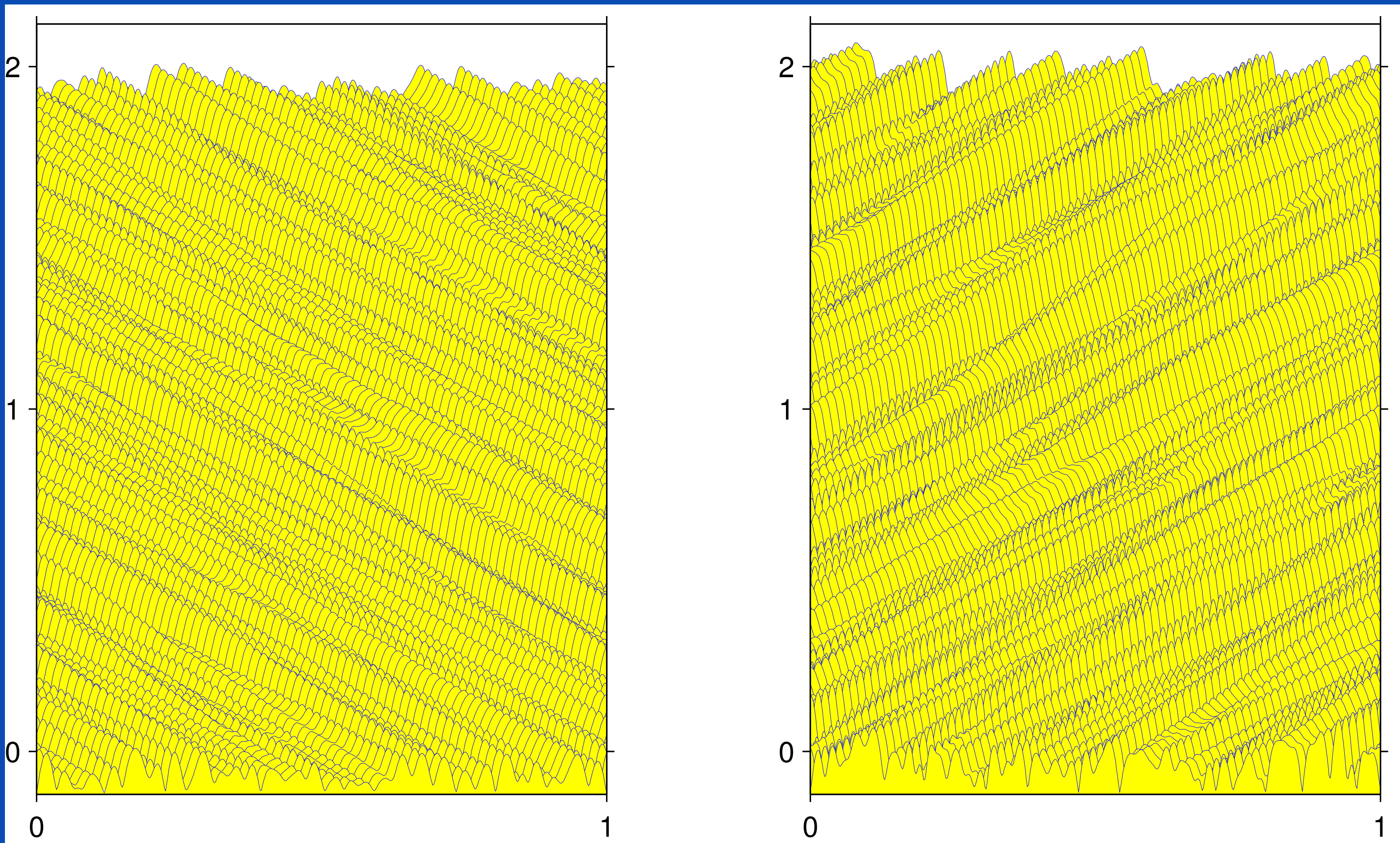
Numerical Simulations

- x-t diagram CW $\kappa = 2$, $\rho = \sqrt{2}$, $\theta = \frac{\pi}{4}$, $\varepsilon = -10^{-3}$



Numerical Simulations

- x-t diagram CW $\kappa = 2$, $\rho = \sqrt{2}$, $\theta = \frac{\pi}{4}$, $\varepsilon = -10^{-3}/4$



Summary

- NLCME (nonlinearity+transport, $\varepsilon = 0$), **not complete**.
- Dispersion terms $|\varepsilon| \ll 1$ must be retained (intermediate scales).
- Dispersive scales destabilization ($t \sim 1$), complicated spatiotemporal dynamics.
- NLCME rigorous estimates: Schneider & Uecker 2001, Goodman et al. 2001 (Contradiction?).
- **Generic situation** (OI,TC,Water waves,...):
    - Spatial reflection symmetry: $x \longrightarrow -x$.
    - Instability, k_0 and ω_0.
    - Group Velocity $v_g \sim 1$ + Dispersion (diffusion).