

# Complex periodic spiral waves

(in a BZ system & in vitro cardiac system)

**Kyoung J. Lee**

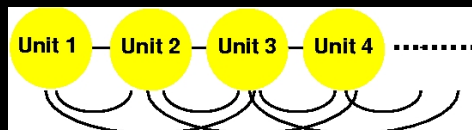
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# Coupled network of nonlinear oscillators in biology

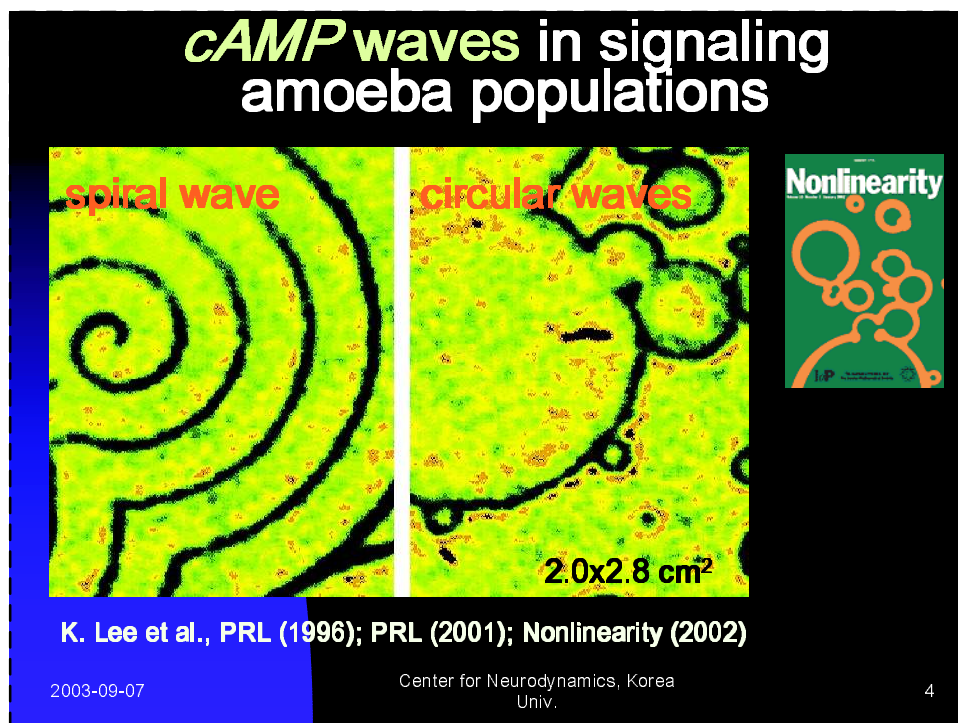
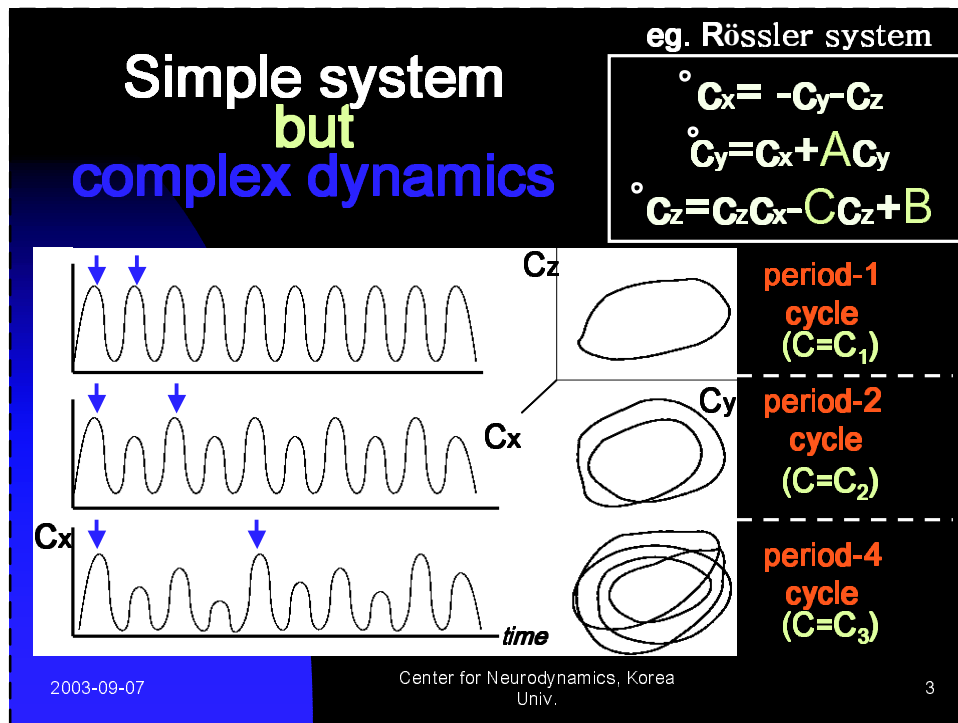


- 1) Bacterial colony
- 2) Firefly population
- 3) amoeba population
- 4) cardiac tissue
- 5) neural networks: eg., suprachiasmatic nucleus (the master biological clock)

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## Period-1 spiral wave in a diffusively coupled Rössler system

$$\partial \vec{c}(\vec{r}, t) / \partial t = \vec{R}[\vec{c}(\vec{r}, t)] + D \nabla^2 \vec{c}(\vec{r}, t)$$

$$R_x = -c_y - c_z$$

$$R_y = c_x + A c_y$$

$$R_z = c_x c_z - C c_z + B$$



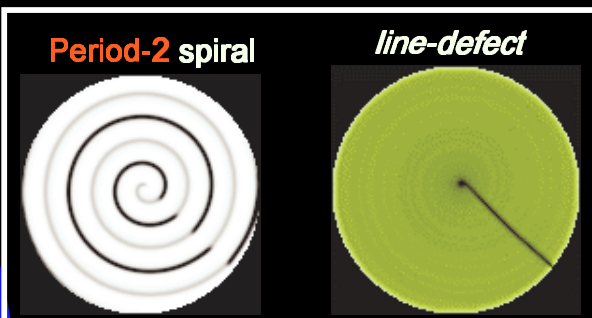
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## Period-2 spiral wave with one line-defect in a Rössler system

Parameters:  
 $A=0.2,$   
 $B=0.2,$   
 $C=3.84,$   
 $D\Delta t/(\Delta r)^2=1.6 \times 10^{-2}$

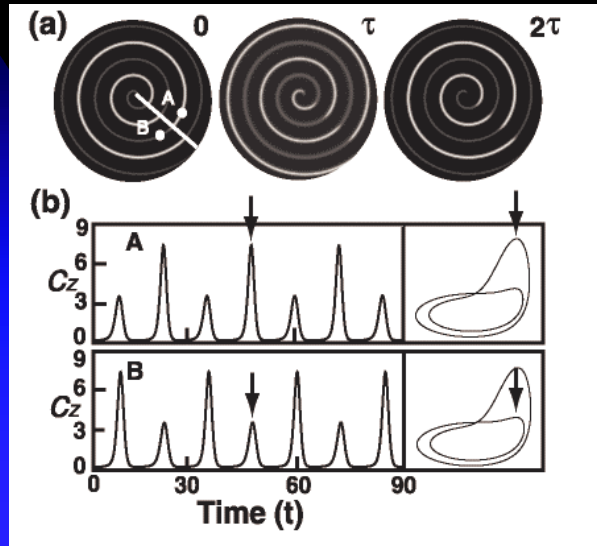


$$V(x, t) = \frac{1}{\tau_0} \int_0^{\tau_0} [G(x, t+t') - G(x, t+t'-\tau)] dt'$$

### [References]

Goryachev et al., *PRL* 76 (1996), *PRE* 54 (1996), *PRL* 80 (1998); *PRL* 83 (1999); *Chaos* 10 (2000).

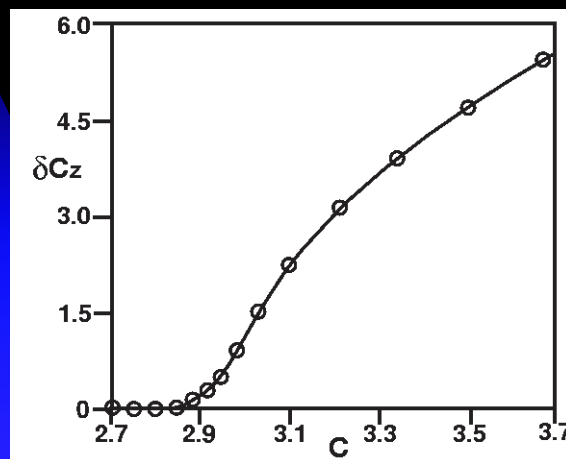
## Period-2 spiral wave in the coupled Rössler system



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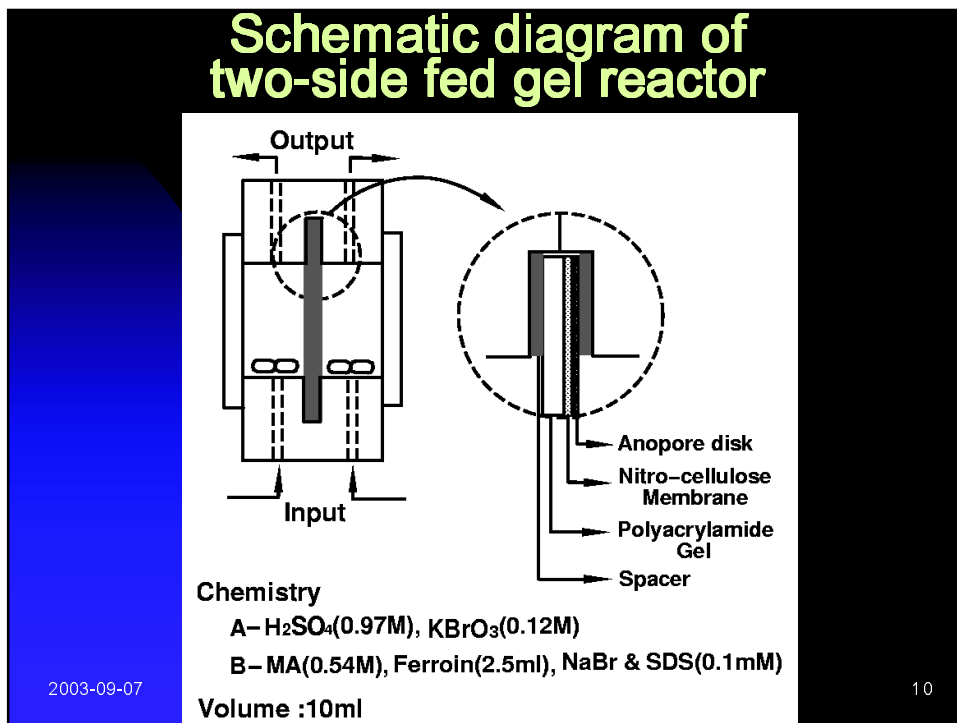
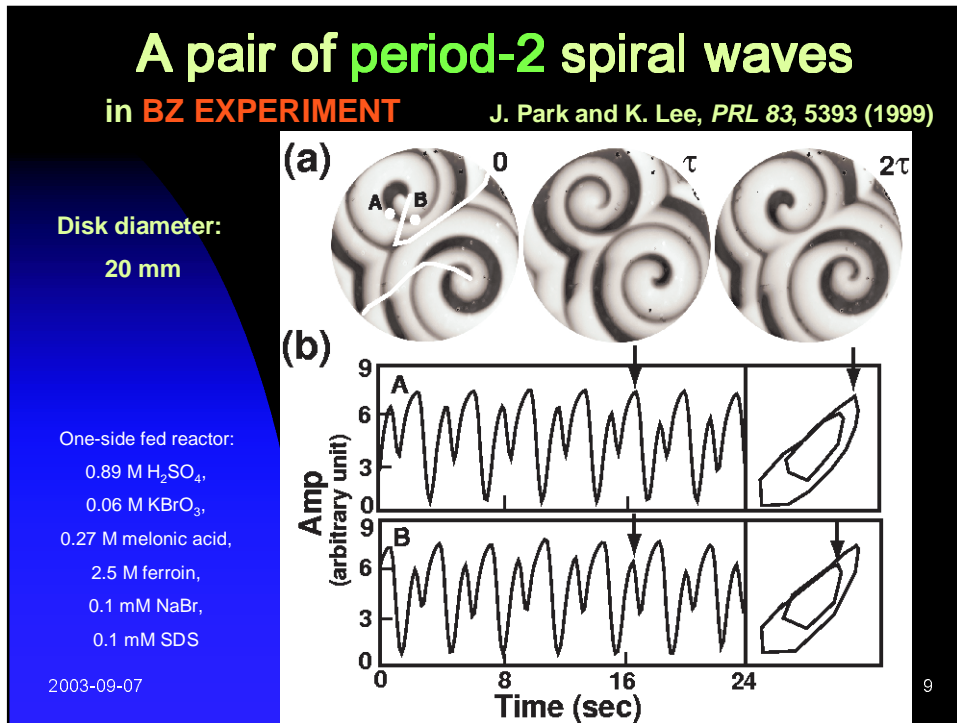
## P1-P2 spiral wave transition in the coupled Rössler system

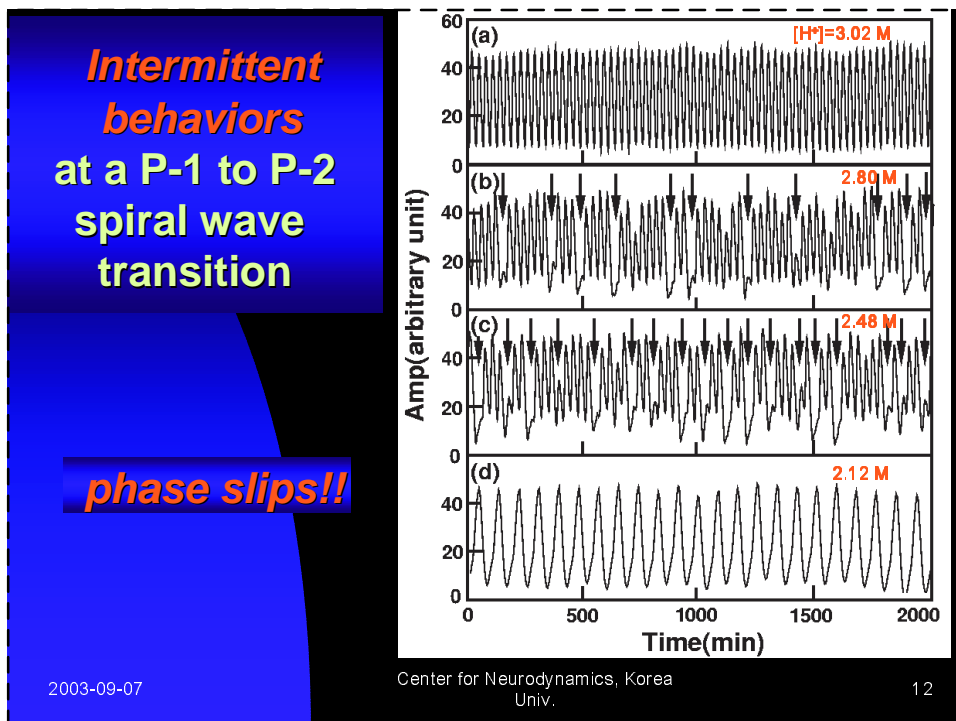
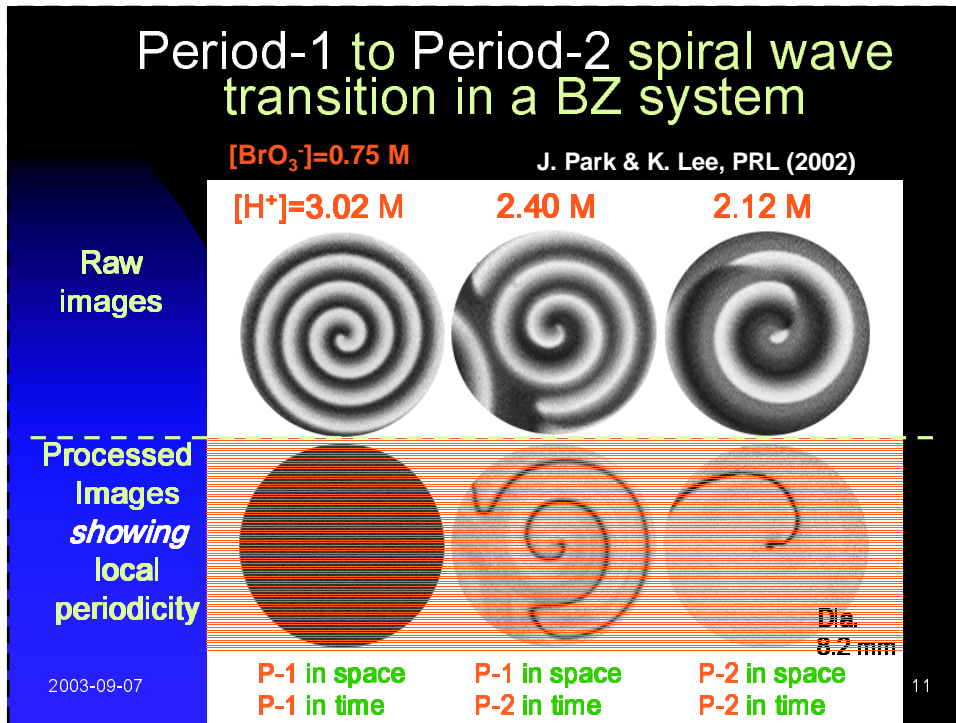


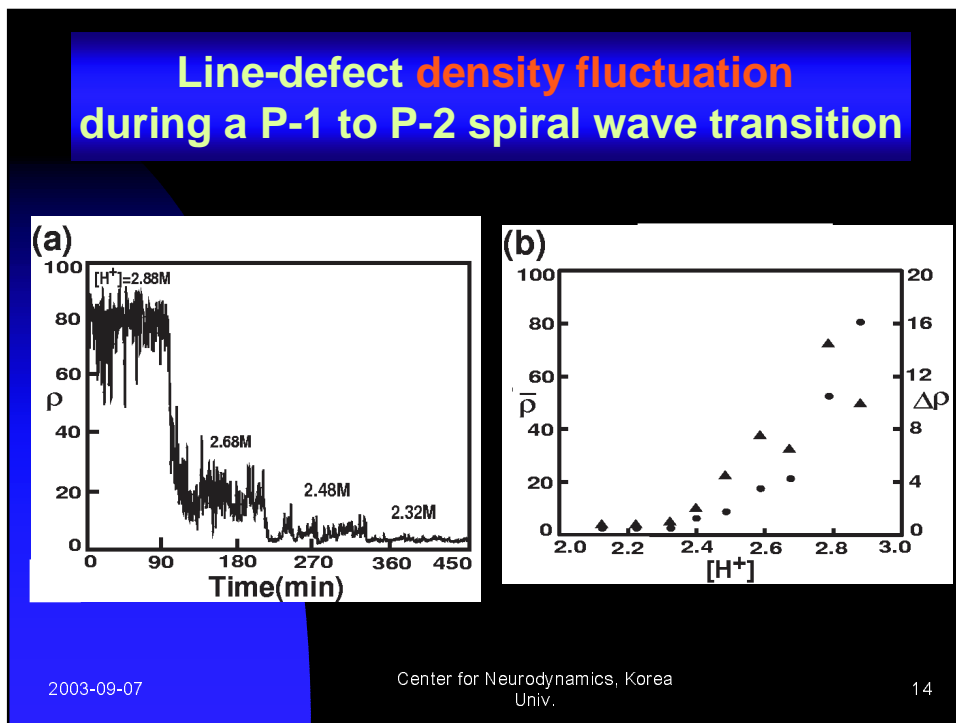
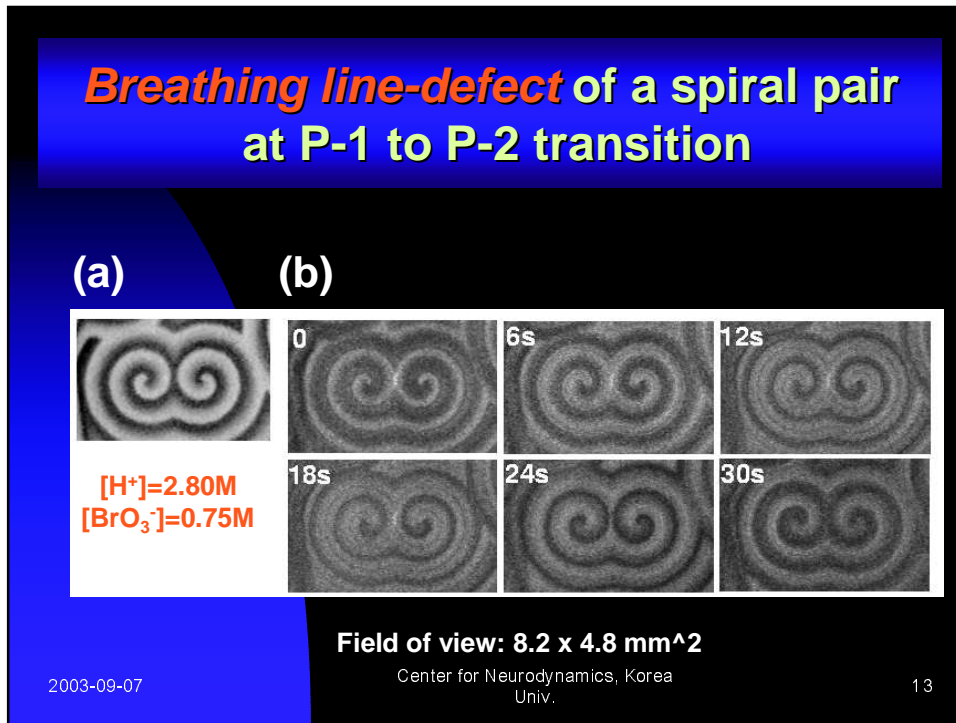
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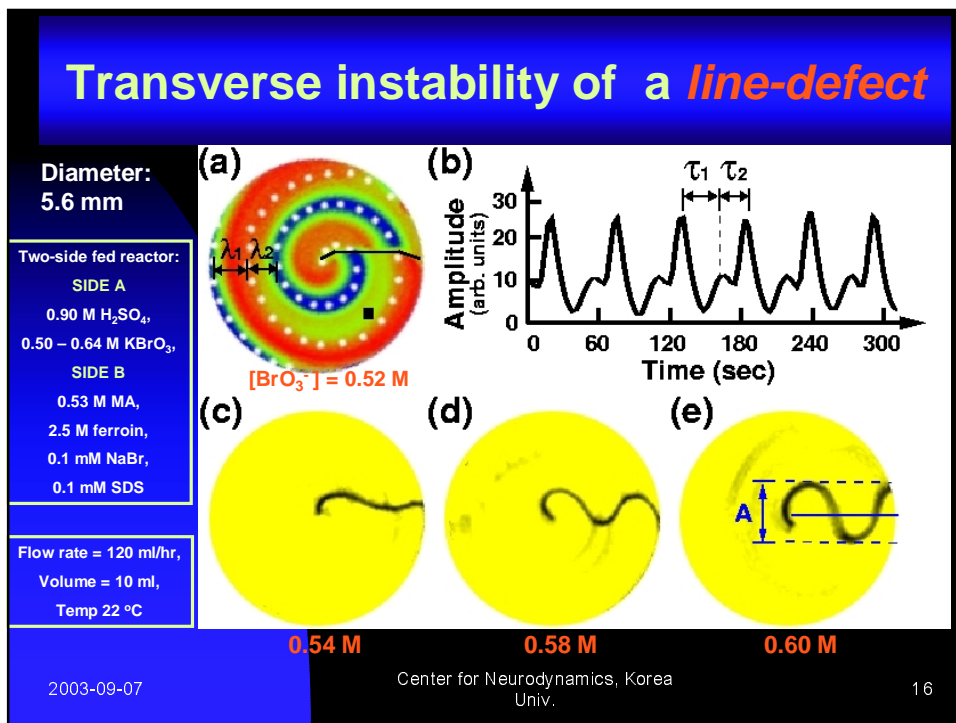
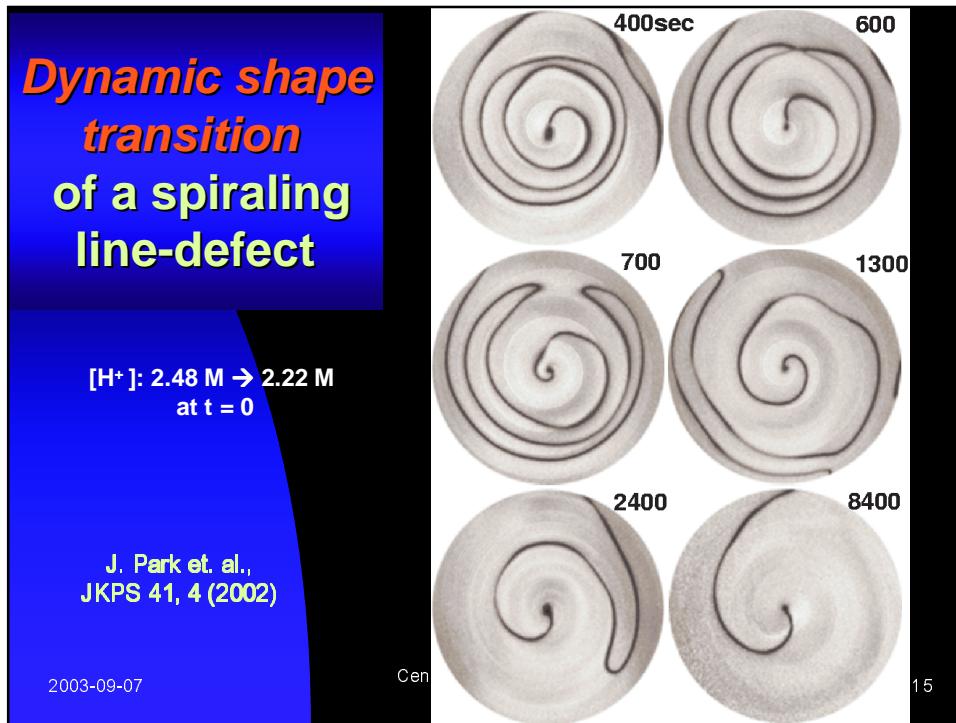
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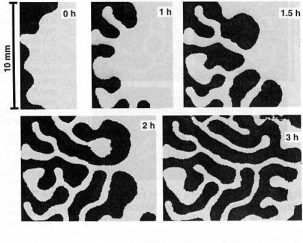
Line-defects of complex oscillatory spiral waves





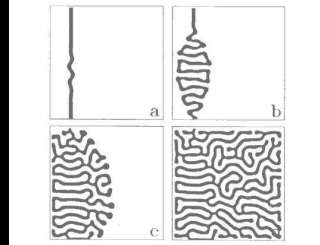
## Analogy with bistable systems

Iodate-ferrocyanide-sulfite reaction-diffusion system, SCIENCE 1993, K. Lee et al.



**Fig. 1.** Time evolution of a pattern initiated by a perturbation with intense ultraviolet light at the left boundary. The pattern achieved 3 hours (3 h) after the localized perturbation is removed was essentially stationary. (A subsequent slow evolution, perhaps due to a slow drift in feed conditions, occurs on the time scale of days.) The pattern was visualized with bromothymol blue pH indicator; white regions correspond to low pH and black regions to high pH. Chemical concentrations are given in the caption for Fig. 3; reservoir flow rate, 130 ml/hour.


FitzHugh-Nagumo type model reaction-diffusion system, PRL 1994, A. Hagberg and E. Meron

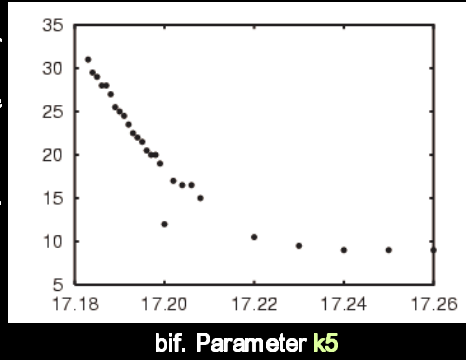


**FIG. 3.** The development of a labyrinthine pattern from a single stripe in the Ising front regime. The light and dark regions correspond to the down and up states, respectively. (a), (b), (c), and (d) pertain to times  $t=100, 950, 1900,$  and  $5000$ . Parameters used:  $\alpha_0=-0.1, \alpha_1=2, \epsilon=0.05, \delta=4$ .

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## Dynamics of LINE-DEFECT in a Williamskii- Rössler system





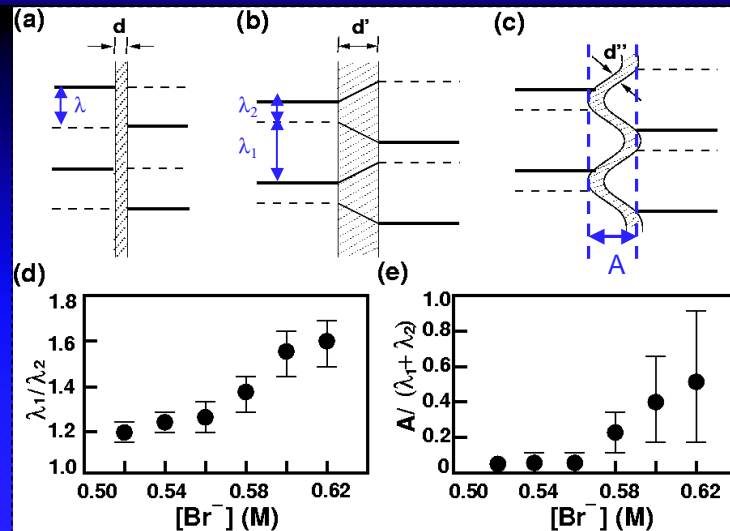
bif. Parameter $k_5$	min. Separation (pixels)
17.18	30
17.19	25
17.20	20
17.21	15
17.22	10
17.23	10
17.24	10
17.25	10
17.26	10

**Simulation by S. Woo (2002)**

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## Heuristic picture & quantifications

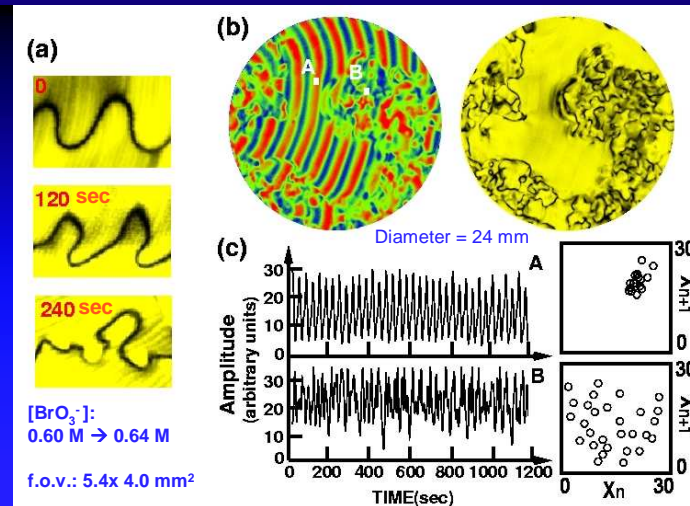


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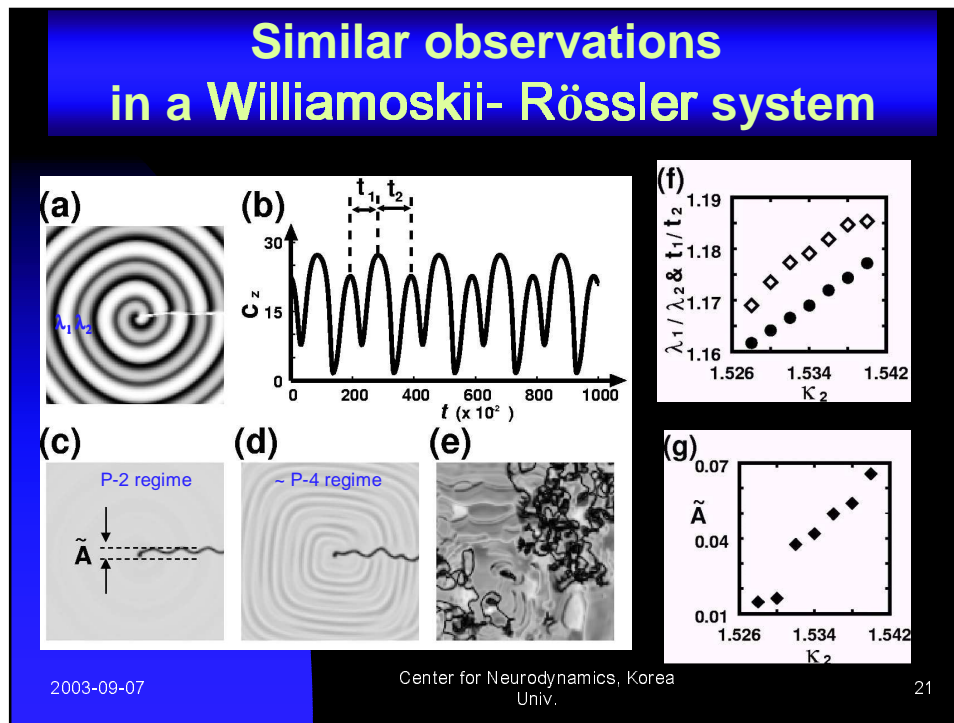
## Line-defect proliferation & chaotic domains



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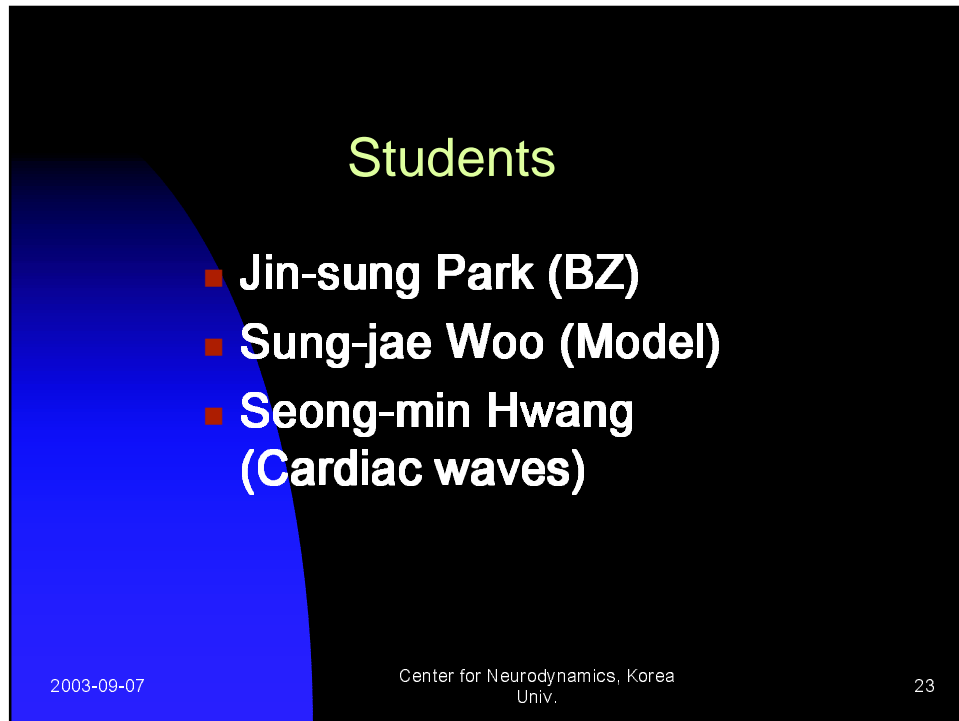
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### Complex periodic spiral waves: summary

- So far, we discussed several phenomena associated with diffusively (i.e., linearly) coupled nonlinear oscillators, mostly, in Period-1 & Period-1/Period-2 (mixed) regimes.
- Obviously, the system will become more complex, if it 1) moves into more complex periodic regime, 2) involves a more complex coupling mechanism, or 2) becomes three-dimensional in space.
- Any complex periodic spiral wave in biology?

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**Students**

- **Jin-sung Park (BZ)**
- **Sung-jae Woo (Model)**
- **Seong-min Hwang (Cardiac waves)**

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