Partons in Phase Space

Pawel Danielewicz\textsuperscript{1} \quad David Brown\textsuperscript{2}

\textsuperscript{1}Natl Superconducting Cyclotron Lab, Michigan State U
\textsuperscript{2}Lawrence Livermore Natl Lab

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Introduction

Large systems such as Au+Au at RHIC: final state a result of history where different processes are localized in phase space.

?Can such a perspective be developed for elementary processes?
?Elementary processes embedded in a large system?
– Perturbative processes
– Partons: folding in cross sections
– Generalized Wigner representation:

\[ A(x, p) = \int d^4 q \ A \left( p + \frac{q}{2} \right) \ A^* \left( p - \frac{q}{2} \right) \ e^{-iqx} \]

where \( A(p) \) is an amplitude
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where \( A(p) \) is an amplitude
**γ Absorption**

Transition Amplitude

\[ S_{\gamma B \to B'} = \int d^4 x \langle 0|A^\mu(x)|\bar{q}, \lambda \rangle \langle B'|j_\mu(x)|B \rangle \]

Transition No

\[ |S_{\gamma B \to B'}|^2 = \int d^4 x d^4 k \, \delta^{(4)}(q - k) \, \epsilon_\mu \epsilon^*_\nu \, J^{\mu\nu}_{BB'}(x \, k) \]

where

\[ J^{\mu\nu}_{BB'}(x \, k) \equiv \int d^4 \tilde{k} \, e^{-i\tilde{k}x} \langle B|j^\dagger_\nu(k - \tilde{k}/2)|B' \rangle \langle B'|j_\mu(k + \tilde{k}/2)|B \rangle \]

⇒ Wigner transform of transition current

≡ Phase-Space density for γ absorption/emission
\[ S_{AB \rightarrow A'B'} = \int \, d^4x \, d^4y \, \langle B'|j^{B\nu}(x)|B \rangle \times D_{\nu\mu}(x-y) \langle A'|j^{A\mu}(y)|A \rangle \]

Transition No. = \[ |S_{AB \rightarrow A'B'}|^2 \]

\[ = \int \, d^4y \, d^4x \, d^4q \, J_{AA'}^{\mu\nu}(y\,q) \, D_{\mu\nu\mu'\nu'}(y - x, q) \, J_{BB'}^{\mu'\nu'}(x\,q) \]

where the Wigner transform of a propagator is

\[ D_{\mu\nu\mu'\nu'}(x, q) = \int \, d^4\tilde{x} \, e^{iq\tilde{x}} \, D_{\mu\nu}(x + \tilde{x}/2) \, D_{\mu'\nu'}(x - \tilde{x}/2) \]

In Phase Space:

\[ \rightarrow A \text{ emits } \gamma \text{ of momentum } q \text{ at } y \]
\[ \rightarrow \gamma \text{ propagates from } y \text{ to } x \]
\[ \rightarrow B \text{ absorbs } \gamma \text{ at } x \]
\( \gamma \)-Exchange

Amplitude

\[
S_{AB \rightarrow A'B'} = \int d^4x \ d^4y \ \langle B' | j^{B\nu}(x) | B \rangle \\
\times D_{\nu\mu}(x - y) \ \langle A' | j^{A\mu}(y) | A \rangle
\]

Transition No \( = |S_{AB \rightarrow A'B'}|^2 \)

\[
= \int d^4y \ d^4x \ d^4q \ J^{\mu\nu}_{AA'}(y, q) \ D_{\mu\nu\mu'\nu'}(y - x, q) \ J^{\mu'\nu'}_{BB'}(x, q)
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In Phase Space:

\( \rightarrow A \) emits \( \gamma \) of momentum \( q \) at \( y \)
\( \rightarrow \gamma \) propagates from \( y \) to \( x \)
\( \rightarrow B \) absorbs \( \gamma \) at \( x \)
Towards Weizsäcker-Williams Interpretation

? Vector potential of object A?

\[ A^{\mu}(x) = \int d^{4}y \, D_{\mu\nu}(x - y) \, J_{A}^{\nu}(y) \]

Wigner transform of the vector potential:

\[ A_{\mu\nu}(x, q) = \int d^{4}\tilde{x} \, e^{i\tilde{x}q} \, A_{\mu}(x + \tilde{x}/2) \, A_{\nu}(x - \tilde{x}/2) \]

\[ = \int d^{4}y \, D_{\mu\nu\mu'\nu'}(x - y, q) \, J_{A}^{\mu'\nu'}(y, q) \]

Source-Propagator Form: the current of A creates \( \gamma \) with momentum \( q \) at position \( y \) and the \( \gamma \) propagates to \( x \)

For \( \gamma \)-exchange, the WW interpretation applied in phase space:

Transition No \( = |S_{AB\rightarrow A'B'}|^2 = \int d^{4}x \, d^{4}q \, A_{\mu\nu}(xq) \, J_{B}^{\mu\nu}(xq) \)

\[ \equiv \int d^{4}x \, d^{4}q \, \frac{d \, n_{\gamma}}{d^{3}x \, d^{4}q} \, \mathcal{N}_{\gamma B\rightarrow B'}(xq) \]

In standard WW: \( \gamma \) distribution in energy only
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\[ A^\mu(x) = \int d^4 y \, D_{\mu \nu}(x - y) \, J^\nu_A(y) \]

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\[ = \int d^4 y \, D_{\mu \nu \mu' \nu'}(x - y, q) \, J^{\mu \nu'}_A(y, q) \]

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\[ \text{Transition No} = |S_{AB\rightarrow A'B'}|^2 = \int d^4 x \, d^4 q \, A_{\mu \nu}(xq) \, J^{\mu \nu}_B(xq) \]

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In standard WW: \( \gamma \) distribution in energy *only*
Issue of Current in WW

In standard WW approximation, A-body current classical:

\[ j_\mu(x) = Z e v_\mu \delta^3(\vec{x} - x^0 \vec{v}) \]

Then

\[ J^{\mu\nu}_{\text{classical}}(xq) = \int d^4\tilde{x} e^{i\tilde{x}q} j_\mu(x + \tilde{x}/2) j_\nu^*(x - \tilde{x}/2) = Z^2 e^2 v_\mu v_\nu \delta(q v) \delta^3(\vec{x} - x^0 \vec{v}) \]

\( \delta(q v) \) stems from current conservation, photons purely spacelike in emitter frame; \( q_0 = q_L, v_L \simeq q_L \) in lab frame

Us: good approximation for \( q \ll p \), when one can construct an A-packet such that \( q \ll (\Delta x_A)^{-1} \ll p_A \)

The current of body A in terms of Wigner functions:

\[ J^{\mu\nu}_{AA'}(xq) = \int d^4\tilde{q} e^{-i\tilde{q}x} \langle A|j^*_{\nu}(q - \tilde{q}/2)|A'\rangle \langle A'|j_{\mu}(q + \tilde{q}/2)|A\rangle \]

\[ = \int d^4p_i d^4p_f \delta^4(p_i - p_f - q) \Gamma^{\mu\nu}(q, p_i, p_f) f_A(x p_i) f_{A'}(x p_f) \]

For \( q \ll (\Delta x_A)^{-1} \ll p_A \): \( \Gamma^{\mu\nu}(q, p_i, p_f) \simeq Z^2 e^2 (p_i + p_f)_\mu (p_i + p_f)_\nu \)

⇒ If packet localized on \( q^{-1} \) scale, we could take \( J^{\mu\nu}_{AA'} \sim J^{\mu\nu}_{\text{classical}} \)
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Propagator in $\gamma$-Exchange

$$A^\mu(x) = \int d^4y \ D_{\mu\nu}(x - y) \ J_A^\nu(y)$$

Wigner transform of what propagator?
→ Feynman doable analytically but acausal!
→ Retarded – the same physics results for the same asymptotic states but simpler to interpret

Scalar propagators:
$$G^+(p) = \frac{1}{p^2 - m^2 + i\epsilon p_0} \quad G^c(p) = \frac{1}{p^2 - m^2 + i\epsilon}$$

Propagator in phase space:
$$G(x, q) = \int d^4\tilde{x} \ e^{iq\tilde{x}} \ G(x + \tilde{x}/2) \ G(x - \tilde{x}/2)$$

Massless scalar
$$G^+(x \ p) = \theta(x_0) \ \theta(x^2) \ \theta(\lambda^2) \ \frac{\sin(2\sqrt{\lambda^2})}{\sqrt{\lambda^2}}$$

where $\lambda^2 = (x \ p)^2 - x^2 p^2$. → $D^+_{\mu\nu\mu'\nu'}(x \ q) = g_{\mu\nu} \ g_{\mu'\nu'} \ G^+(x \ q)$
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# Features of Retarded Phase-Space Propagator

- causal, propagation inside light cone only
- ptcles do not propagate beyond $0 \leq \lambda^2 \lesssim 1$

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Off-shell propagation limited, from emission point, by the inverse of energy, momentum or mass

On-shell propagation limited, from the classical trajectory, by the inverse of energy
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Off-shell propagation limited, from emission point, by the inverse of energy, momentum or mass
On-shell propagation limited, from the classical trajectory, by the inverse of energy
\( \gamma \) Phase-Space Distribution for Static Charge

Putting current + propagation together yields \( \gamma \) distribution:

\[
A^{\mu\nu}(x, q) = Z^2 e^2 v_\mu v_\nu \frac{\delta(qv)}{\sqrt{-q^2}} A
\]

where \( A \) is dimensionless, semi-analytic and gives shape:

- Photon 4-momentum
  \( q_\mu = (0, 0.788, \vec{0}_T) \) MeV/c

- Lorentz contraction in the direction of photon momentum
\( \gamma \) Phase-Space Distribution for Moving Charge

Charge moving at \( \vec{v} = (0.9c, \vec{0}_T) \)

Photon \( q^\mu = (m_e, m_e/v_L, \vec{0}_T) \) & \( q^\mu = (m_e, m_e/v_L, 0.56 \text{ MeV/c}, 0) \)

Again contraction in the photon direction
Positron Production

\( \gamma + B \rightarrow \bar{e} + B' \)

Process amplitude

\[
S_{\gamma B \rightarrow \bar{e}B'} = \int d^4x \, d^4y \, A_\mu(x) \, \psi_{\bar{e}}(x, s) \times e^{-\gamma \mu} \, S(x - y) \, \mathcal{V}_{Be \rightarrow B'}(y)
\]

Transition No. = \(|S_{\gamma B \rightarrow \bar{e}B'}|^2 = e^2 \int d^4y \, d^4p \, d^4x \, d^4q \, \frac{d^3k}{|k^0|} \, A_{\mu\nu}(x, q) \times \mathcal{G}(y - x, p) \, \delta^{(4)}(k + q - p) \, \text{Tr}\{\mathcal{V}_{Be \rightarrow B'}(y, p) (\not{p} - \ldots) \ldots\}\]

Distribution of virtual electrons for a photon:

\[
\frac{dn_{\bar{e}}}{d^3y \, d^3p \, dp^2} \propto e^2 \int d^4x \, \frac{d^3k}{|k^0|} \, \mathcal{G}(y - x, p) \, A_{\mu\nu}(x, p - k)
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Positron Production

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\[ \times e^{\gamma \mu} \, S(x - y) \, \mathcal{V}_{Be \rightarrow B'}(y) \]

Transition No \( = |S_{\gamma B \rightarrow \bar{e} B'}|^2 \)

\[ = e^2 \int d^4 y \, d^4 \rho \, d^4 x \, d^4 q \, \frac{d^3 k}{|k^0|} \, A_{\mu\nu}(x, q) \]
\[ \times G(y - x, \rho) \, \delta^{(4)}(k + q - \rho) \, \text{Tr}\{\mathcal{V}_{Be \rightarrow B'}(y, \rho) (\rho - \ldots) \ldots\} \]

Distribution of virtual electrons for a photon:

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**Positron Production**

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**Distribution of virtual electrons for a photon:**

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\]
Virtual Electron Distribution

Point charge, moving at 0.9c to the right, is the source of photons that are the source of virtual electrons

Electron momentum

\[ p^\mu = (2.0, 2.05, 0) \text{ MeV/c} \]

Forward-backward asymmetry due to \( \bar{e} \) recoil
Interference in $2\gamma$ Interaction

Transition No $= |S_{12\rightarrow 1'2'e\bar{e}}|^2 = \int d^4 R d^4 r \ldots$

\[
\times G(r \cdot p) \delta^{(4)} (q_1 + q_2 - k_1 + k_2) \\
\times \{ A_{1}^{\mu\mu'} (R - r/2, q_1) A_{2}^{\nu\nu'} (R + r/2, q_2) \\
\times \delta^{(4)} (q_1 + p - k_1) + A_{1}^{\nu\nu'} (R + r/2, q_1) \\
\times A_{2}^{\mu\mu'} (R - r/2, q_2) \delta^{(4)} (q_1 + k_2 - p) \\
+ \int d^4 \tilde{r} 2 \cos [\tilde{r} \cdot (-p + \frac{k_1 + k_2}{2}) - r \cdot (q_1 - q_2)] \\
\times A_{1}^{\nu\nu'} (R - \tilde{r}/4, q_1) A_{2}^{\mu\mu'} (R + \tilde{r}/4, q_2) \} 
\]

2 direct terms + HBT-like interference term

The direct terms factorize into e-density $\times$ absorption.
The interference term does not.

⇒ Scale separation needed to achieve a factorization.
Interference in 2-\(\gamma\) Interaction

Transition No = \( |S_{12 \rightarrow 1'2'\bar{e}e}|^2 = \int d^4R d^4r \ldots \)

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\(\Rightarrow\) Scale separation needed to achieve a factorization.
Propagation with Mass and in Medium

?Massive propagator?

\[ G^+(x \ p) = G_0^+(x \ p) - \frac{2m}{\pi} \frac{\theta(x^0)}{\theta(x^2)} \]

\[ \times \int_0^{\sqrt{x^2}} d\xi \ J_1(2m\xi) \frac{\sin \left(2\sqrt{\lambda^2 + \xi^2 p^2}\right)}{\sqrt{\lambda^2 + \xi^2 p^2}} \]

\( G_0^+ \) – massless propagator; the correction removes pieces close to the light cone.

For an on-shell particle, as before \( |x_0 - \vec{v} \cdot \vec{x}| \lesssim \frac{1}{|p_0|} \)

In medium, fluctuation-dissipation theorem: \( G^\gtrless = G^+ \Sigma^\gtrless G^- \)

In Wigner representation:

\[ G^\gtrless(x \ p) = \int d^4y \ d^4p' \ G^+(x \ p; y \ p') \Sigma^\gtrless(y \ p') \]

\[ \gtrsim \int d^4y \ G^+(x - y, \ p) \Sigma^\gtrless(y \ p') \]

\( G^\gtrless \) – density, \( \Sigma^\gtrless \) – source
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?Massive propagator?
\[ G^+_0(x,p) = G^0_0(x,p) - \frac{2m}{\pi} \theta(x^0) \theta(x^2) \]
\[ \times \int_0^{\sqrt{x^2}} d\xi \, J_1(2m\xi) \frac{\sin \left(2\sqrt{\lambda^2 + \xi^2 p^2}\right)}{\sqrt{\lambda^2 + \xi^2 p^2}} \]

\[ G^+_0 \] – massless propagator; the correction removes pieces close to the light cone.

For an on-shell particle, as before
\[ |x_0 - \vec{v} \cdot \vec{x}| \lesssim \frac{1}{|p_0|} \]

In medium, fluctuation-dissipation theorem: \( G^\Xi = G^+ \Sigma^\Xi G^- \)

In Wigner representation:
\[ G^\Xi(x,p) = \int d^4 y \, d^4 p' \, G^+(x,p;y,p') \Sigma^\Xi(y,p') \]
\[ \simeq \int d^4 y \, G^+(x-y,p) \Sigma^\Xi(y,p') \]

\( G^\Xi \) – density, \( \Sigma^\Xi \) – source
QCD DGLAP Cloud

Large $Q^2$ & $-q_1^2 \ll -q_2^2 \ldots \ll -q_n^2$

increasing virtuality

decreasing portion of beam momentum

$1 \leq x_1 \leq \ldots \leq x_{n-1} \leq x_n$

Each parton a source for the subsequent one. Propagation determines size of virtual cloud.

First source localized to $(\frac{R_{\text{bag}}}{\gamma}, R_{\text{bag}}^\perp)$.

Propagation increases $R_{\perp}$ by $\sim \frac{1}{\sqrt{-q_n^2}}$.

Because of large virtuality $R_{\text{bag}}^\perp \sim R_{\text{bag}}^\perp$.

In $\parallel$ direction size increases by $\frac{1}{|q_{n0}|}$, i.e. $R_n^\parallel \sim \frac{R_{\text{bag}}}{\gamma} + \frac{1}{x_n P}$.

If $\frac{1}{x_n P} > \frac{R_{\text{nuc}}}{\gamma}$ partons delocalized beyond the nucleus...
QCD DGLAP Cloud

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If $\frac{1}{x_n P} > \frac{R_{nuc}}{\gamma}$ partons delocalized beyond the nucleus...
QCD BFKL Cloud

\[ 1 \gg x_1 \gg \ldots \gg x_{n-1} \gg x_n \]

strongly ordered low-\(x\)

No constraints on \(q^2_\perp\); in practice \(q^2_{\perp 1}, \ldots, q^2_{\perp n} \gg 1/R_{\text{bag}}\).

If \(\frac{1}{x_n P} > \frac{R_{\text{nuc}}}{\gamma}\) partons delocalized beyond the nucleus.

Small-\(x\) partons can travel beyond the longitudinal nuclear size and then see color charge of any other nucleon in a longitudinal tube centered on the parent nucleon.

⇒ Support for McLerran-Venugopalan
Without interference, probabilities from tree-type diagrams can be expressed in terms of transparent phase-space convolutions of source-propagator form.

Massless phase-space propagators in analytic form; massive involve 1D integral.

Interferences produce HBT-type terms in probability.

Phase-space propagation is causal for the retarded propagators. Deviations from classical motion are limited by energy, momentum or mass.

Interference terms will get suppressed when there is space-momentum scale separation and that is the case in different limits where the partonic model is employed.

Wee partons extend farthest from a nucleus and should be first to interact.
Summary

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