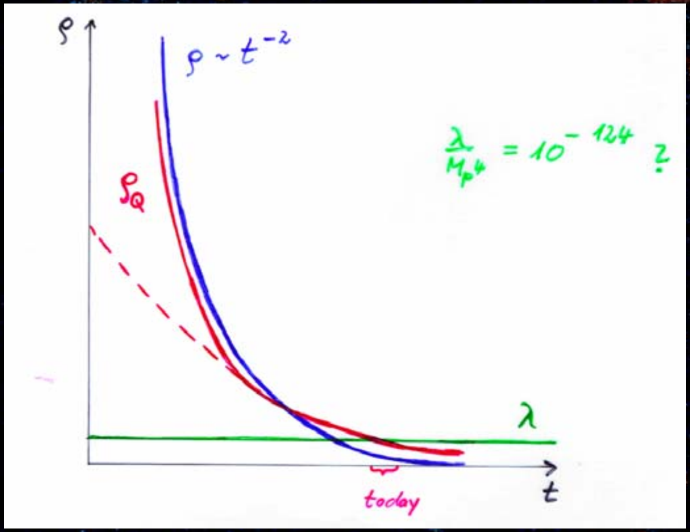
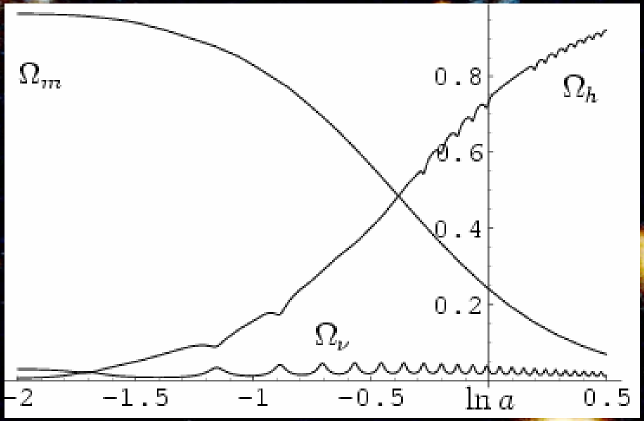
Why the cosmological constant goes to zero, and why we see it now





Quintessence

C.Wetterich

A.Hebecker, M.Doran, M.Lilley, J.Schwindt,
C.Müller, G.Schäfer, E.Thommes,
R.Caldwell, M.Bartelmann, K.Kharwan, G.Robbers,
T.Dent, S.Steffen, L.Amendola, M.Baldi , N.Brouzakis , N.Tetradis,
D.Mota, V.Pettorino, T.Krüger, M.Neubert

Dark Energy dominates the Universe

Energy - density in the Universe

=

Matter + Dark Energy

25 % + 75 %

Cosmological Constant
- Einstein -

- Constant λ compatible with all symmetries
- Constant λ compatible with all observations
- No time variation in contribution to energy density

- Why so small ? $\lambda/M^4 = 10^{-120}$

- Why important just today ?

Cosmological mass scales

- Energy density

$\rho \sim (\, 2.4 \times 10^{-3}\ \mathrm{eV}\,)^{-4}$

- Reduced Planck mass

$M = 2.44 \times 10^{18}\,\mathrm{GeV}$

- Newton's constant

$G_N = (8\pi M^2)$

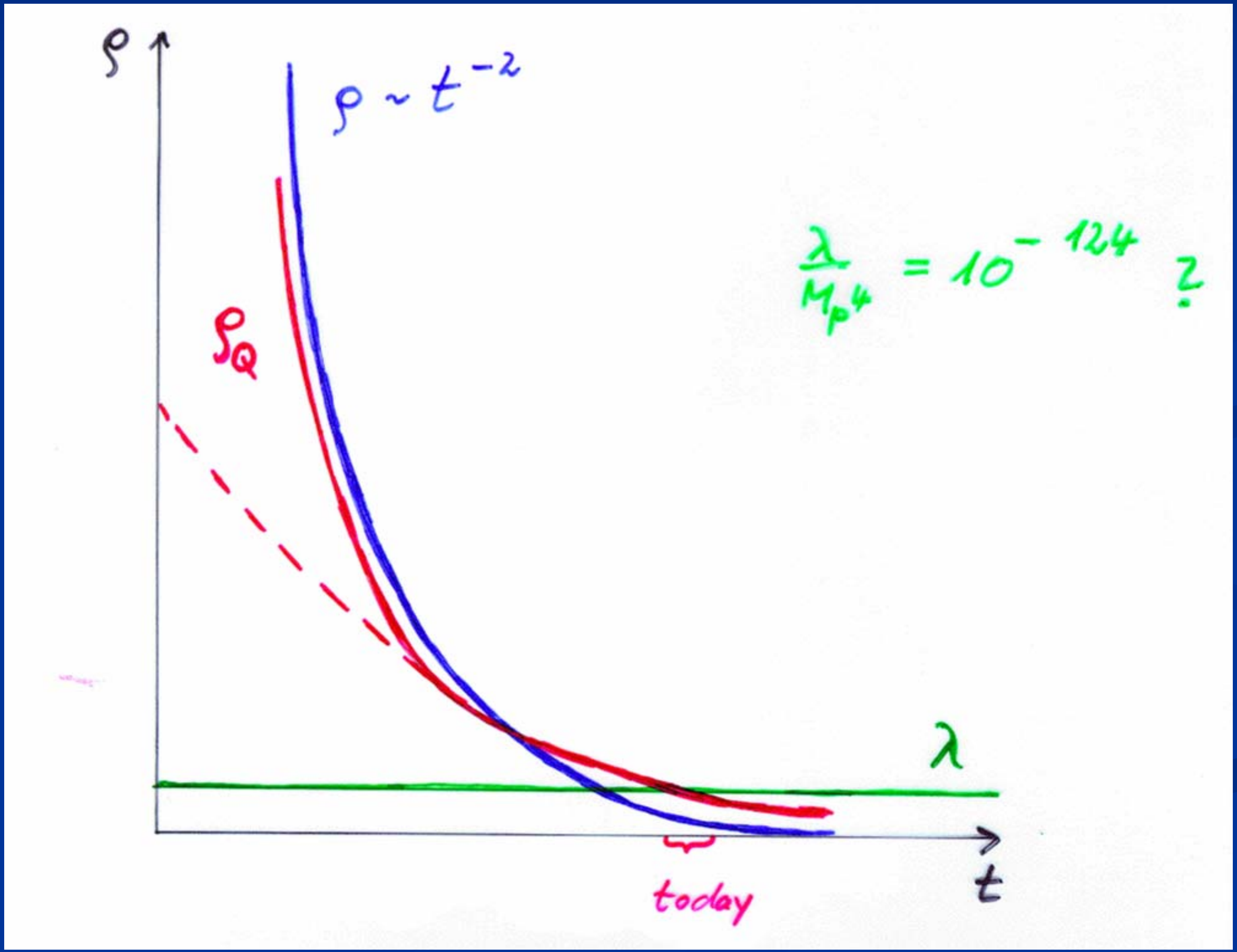
Only ratios of mass scales are observable !

homogeneous dark energy: $\rho_h / M^4 = 7 \cdot 10^{-121}$

matter: $\rho_m / M^4 = 3 \cdot 10^{-121}$

Cosm. Const | Quintessence

static | dynamical



Cosmological Constant
- accident or explanation -

- Why so small ? $\lambda/M^4 = 10^{-120}$

- Why important just today ?

Quintessence

Dynamical dark energy ,
generated by scalar field

(cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations
…. modifications

Cosmon

- *Scalar field changes its value even in the* ***present*** *cosmological epoch*
- *Potential und kinetic energy of cosmon contribute to the energy density of the Universe*

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

- *Time - variable dark energy :*

  $\rho_h(t)$ *decreases with time !*

$V(\varphi) = M^4 \exp(- \alpha\varphi/M)$

two key features

1) Exponential cosmon potential and scaling solution

$$V(\varphi) = M^4 \exp(- \alpha\varphi/M)$$

$$V(\varphi \rightarrow \infty) \rightarrow 0 \;\; !$$

2) Stop of cosmon evolution by cosmological trigger

Evolution of cosmon field

Field equations

$$\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$$

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

Potential V(φ) determines details of the model

$V(\varphi) = M^4 \exp(-\alpha\varphi/M)$

for increasing φ the potential decreases towards zero !

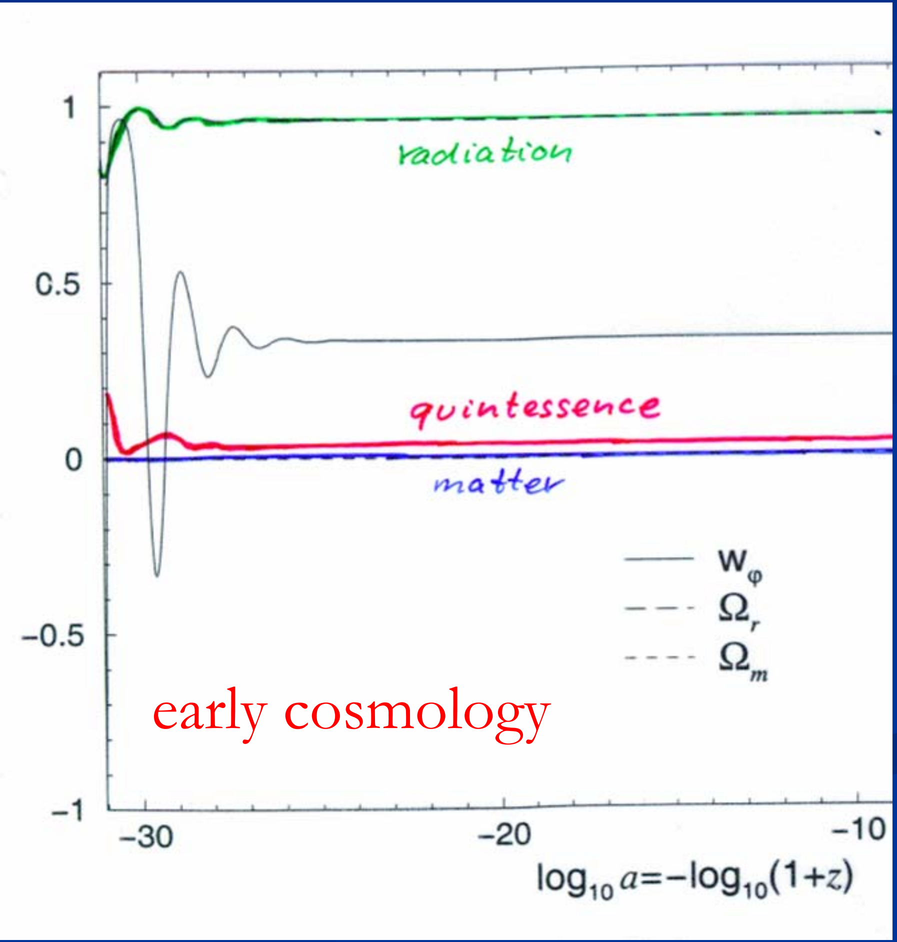
Cosmic Attractor

Solutions independent of initial conditions

$$V \sim t^{-2}$$

$$\varphi \sim \ln(t)$$

$$\Omega_h \sim \text{const.}$$



exponential potential ➡ constant fraction in dark energy

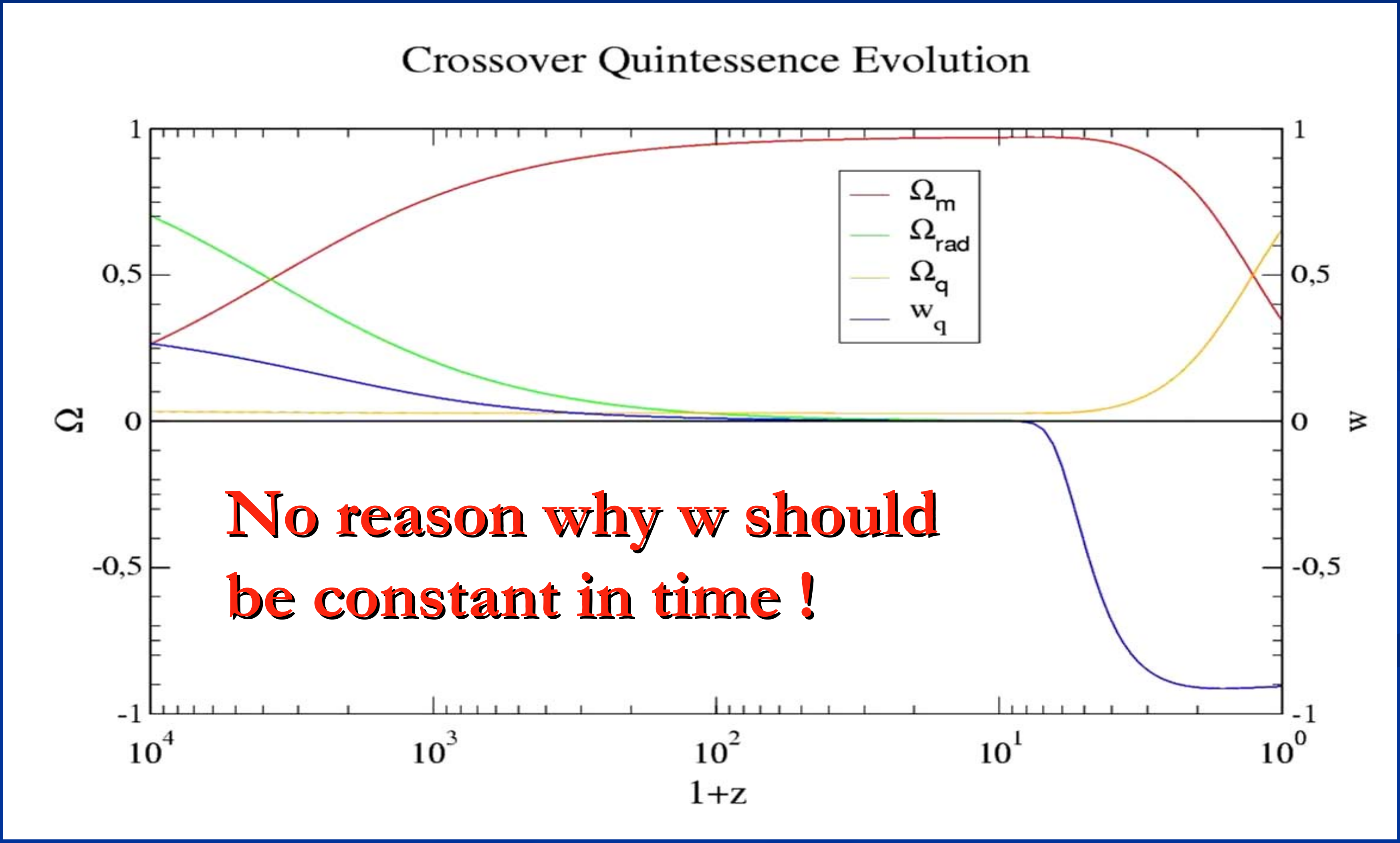
$$\Omega_h = 3(4)/\alpha^2$$

can explain order of magnitude of dark energy !

realistic quintessence

fraction in dark energy has to increase in "recent time" !

Quintessence becomes important "today"

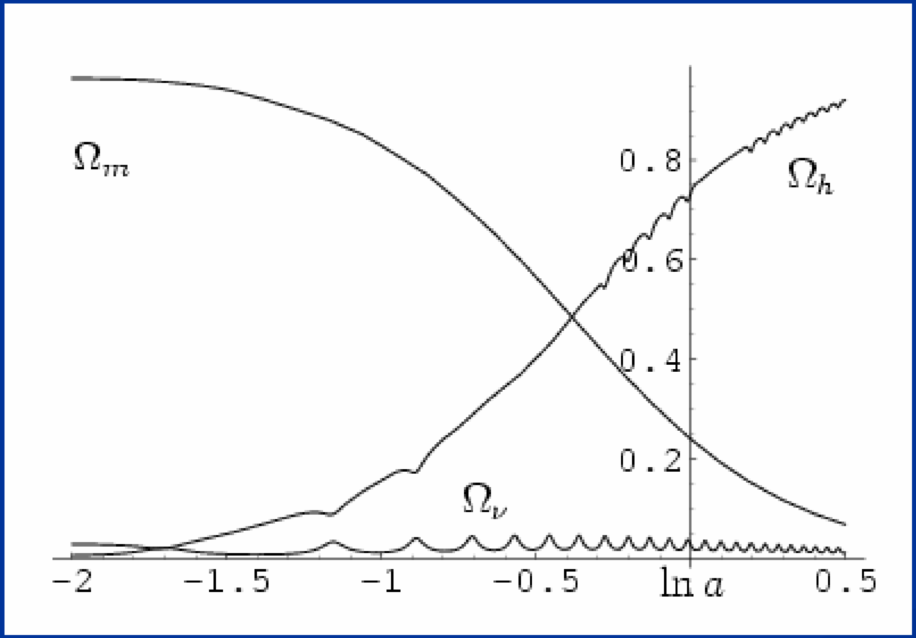


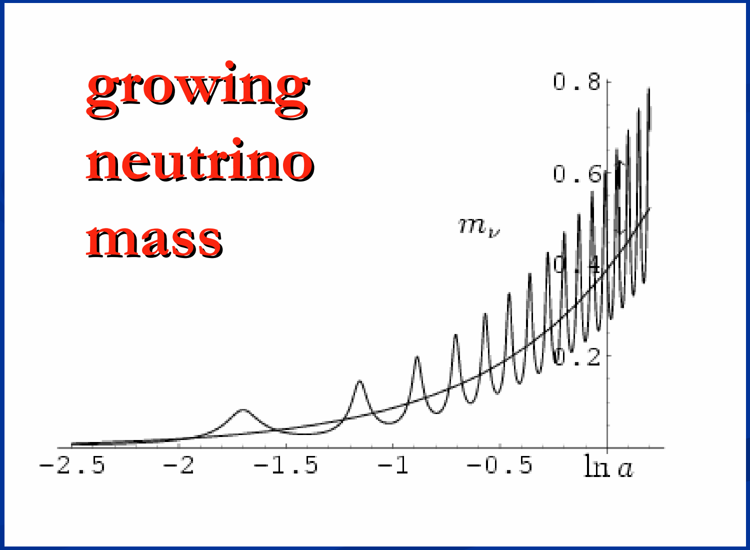
coincidence problem

What is responsible for increase of Ω_h for $z < 6$?

Why now ?

growing neutrino mass triggers transition to almost static dark energy





basic ingredient :

cosmon coupling to neutrinos

Cosmon coupling to neutrinos

- can be large !

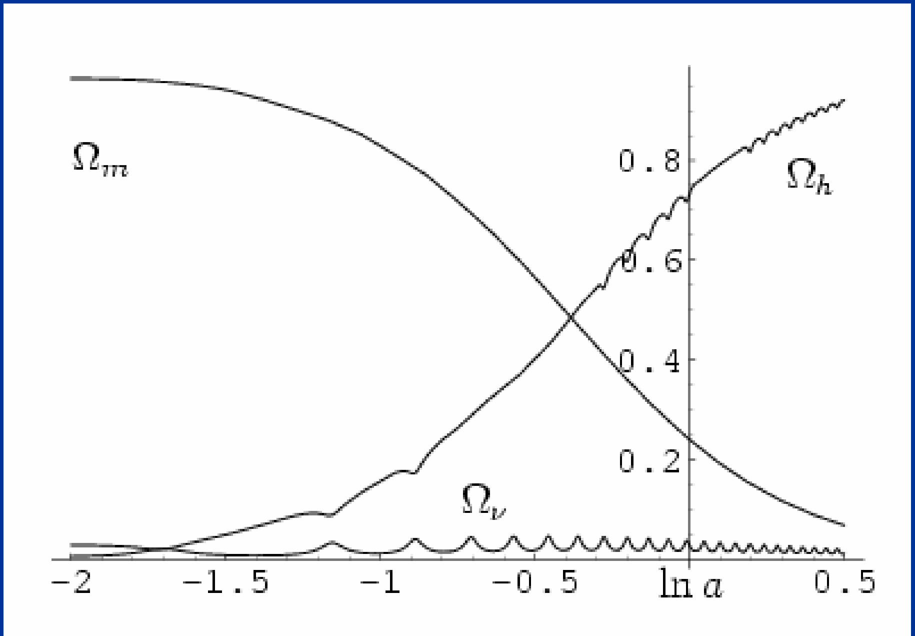
Fardon,Nelson,Weiner

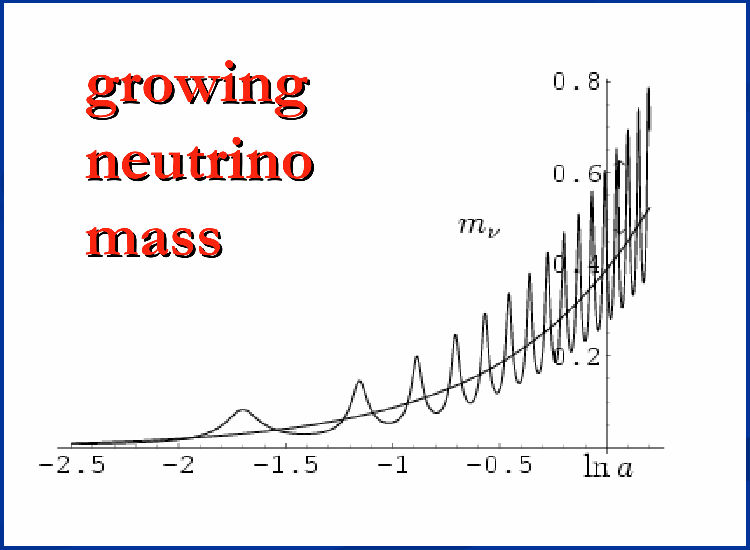
- interesting effects for cosmology if neutrino mass is growing
- growing neutrinos can stop the evolution of the cosmon
- transition from early scaling solution to cosmological constant dominated cosmology

L.Amendola,M.Baldi,…

growing neutrinos

crossover due to non –relativistic neutrinos





end of matter domination

- growing mass of neutrinos

  ➡

- at some moment energy density of neutrinos becomes more important than energy density of dark matter

  ➡

- end of matter dominated period
- similar to transition from radiation domination to matter domination
- this transition happens in the recent past
- cosmon plays crucial role

cosmological selection

- present value of dark energy density set by cosmological event

(neutrinos become non – relativistic)

- not given by ground state properties !

connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.07 \left(\frac{\gamma m_\nu(t_0)}{eV} \right)^{\frac{1}{4}} 10^{-3} eV$$

present dark energy density given by neutrino mass

present equation of state given by neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\text{eV}}$$

dark energy fraction determined by neutrino mass

$$\Omega_h(t_0) \approx \frac{\gamma m_\nu(t_0)}{16eV}$$

$$\gamma = -\frac{\beta}{\alpha}$$

constant neutrino - cosmon coupling β

$$\Omega_h(t_0) \approx -\frac{\epsilon}{\alpha} \frac{m_\nu(t_0)}{\bar{m}_\nu} \frac{m_\nu(t_0)}{16eV}$$

variable neutrino - cosmon coupling

varying neutrino – cosmon coupling

- specific model
- can naturally explain why neutrino – cosmon coupling is much larger than atom – cosmon coupling

neutrino mass

$$M_\nu = M_D M_R^{-1} M_D^T + M_L$$

$$M_L = h_L \gamma \frac{d^2}{M_t^2}$$

seesaw and cascade mechanism

triplet expectation value ~ doublet squared

$$m_\nu = \frac{h_\nu^2 d^2}{m_R} + \frac{h_L \gamma d^2}{M_t^2}$$

omit generation structure

cascade mechanism

$$U = U_0(\varphi) + \frac{\lambda}{2}(d^2 - d_0^2)^2 + \frac{1}{2}M_t^2(\varphi)t^2 - \gamma d^2 t$$

triplet expectation value ~ $\gamma \dfrac{d^2}{M_t^2}$

M.Magg , …
G.Lazarides , Q.Shafi , …

$$M_t^2(\varphi) = \bar{M}_t^2 \left[1 - \exp\left(-\frac{\epsilon}{M}(\varphi - \varphi_t)\right)\right]$$

varying neutrino mass

$$M_t^2 = c_t M_{GUT}^2 \left[1 - \frac{1}{\tau} \exp\left(-\epsilon \frac{\varphi}{M} \right) \right]$$

ε ≈ -0.05

triplet mass depends on cosmon field φ

$$m_\nu(\varphi) = \bar{m}_\nu \left\{ 1 - \exp\left[-\frac{\epsilon}{M} (\varphi - \varphi_t) \right] \right\}^{-1}$$

⟹ neutrino mass depends on φ

“singular” neutrino mass

$$M_t^2 = c_t M_{GUT}^2 \left[1 - \frac{1}{\tau} \exp\left(-\epsilon \frac{\varphi}{M}\right)\right]$$

triplet mass vanishes for $\varphi \rightarrow \varphi_t$

$$\frac{\varphi_t}{M} = -\frac{\ln \tau}{\epsilon}$$

$$m_\nu(\varphi) = \frac{\bar{m}_\nu M}{\epsilon(\varphi - \varphi_t)}$$

➡ neutrino mass diverges for $\varphi \rightarrow \varphi_t$

strong effective neutrino – cosmon coupling for $\varphi \longrightarrow \varphi_t$

$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

crossover from
early scaling solution to
effective cosmological constant

early scaling solution (tracker solution)

$$V(\varphi) = M^4 \exp\left(-\alpha \frac{\varphi}{M}\right)$$

$$\varphi = \varphi_0 + (2M/\alpha)\ln(t/t_0)$$

$$\Omega_{h,e} = \frac{n}{\alpha^2}$$

neutrino mass unimportant in early cosmology

growing neutrinos change cosmon evolution

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} + \frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu),$$

$$\beta(\varphi) = -M\frac{\partial}{\partial \varphi}\ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

modification of conservation equation for neutrinos

$$\dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) = -\frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu)\dot{\varphi} = -\frac{\dot{\varphi}}{\varphi - \varphi_t}(\rho_\nu - 3p_\nu)$$

effective stop of cosmon evolution

cosmon evolution almost stops once

- neutrinos get non –relativistic
- ß gets large

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} + \frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu)$$

$$\beta(\varphi) = -M\frac{\partial}{\partial\varphi}\ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

$$m_\nu(\varphi) = \frac{\beta(\varphi)}{\epsilon}\bar{m}_\nu$$

This always happens for $\varphi \rightarrow \varphi_t$!

effective cosmological trigger for stop of cosmon evolution : neutrinos get non-relativistic

- this has happened recently !
- sets scales for dark energy !

dark energy fraction determined by neutrino mass

$$\Omega_h(t_0) \approx \frac{\gamma m_\nu(t_0)}{16eV}$$

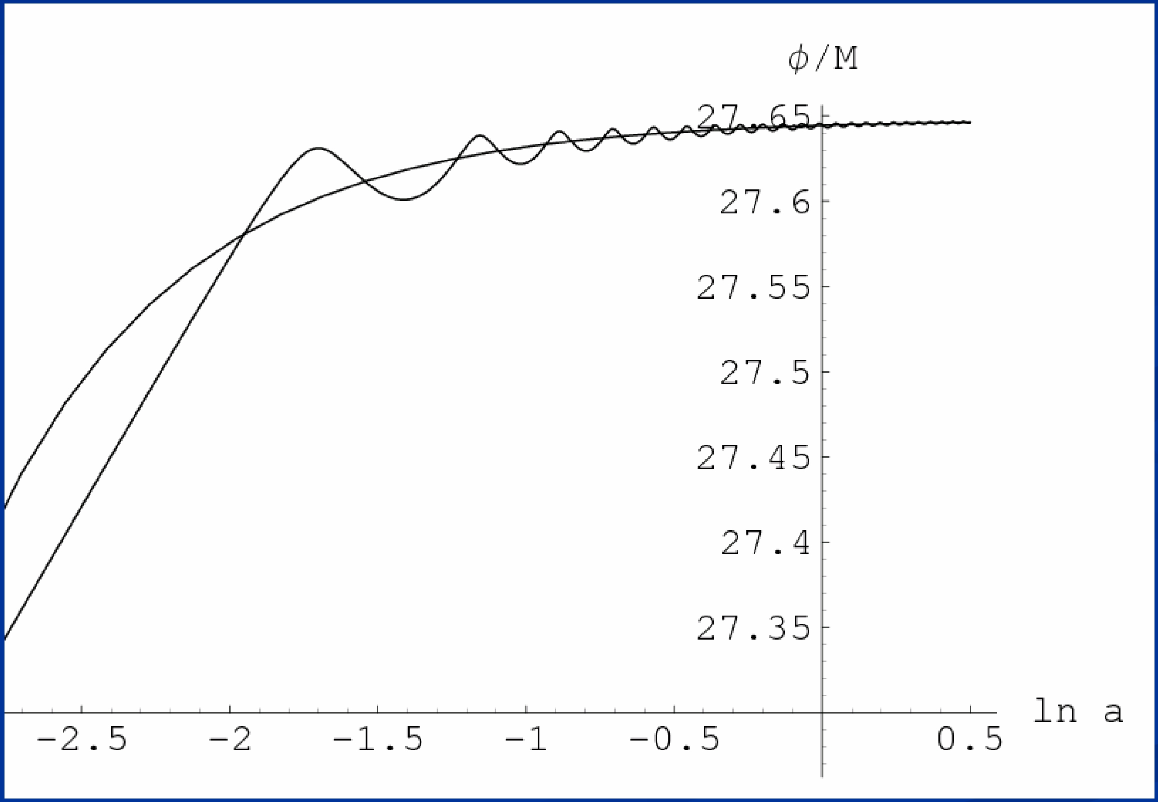
$$\gamma = -\frac{\beta}{\alpha}$$

constant neutrino - cosmon coupling β

$$\Omega_h(t_0) \approx -\frac{\epsilon}{\alpha} \frac{m_\nu(t_0)}{\bar{m}_\nu} \frac{m_\nu(t_0)}{16eV}$$

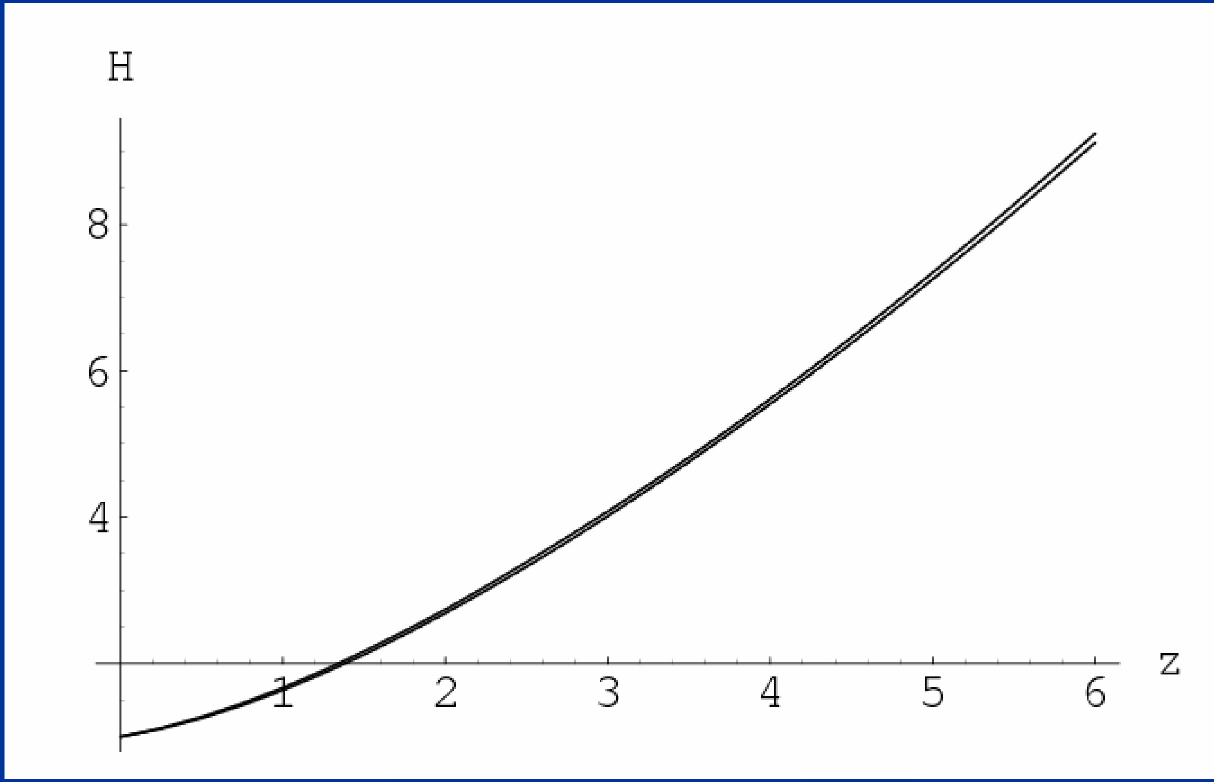
variable neutrino - cosmon coupling

cosmon evolution



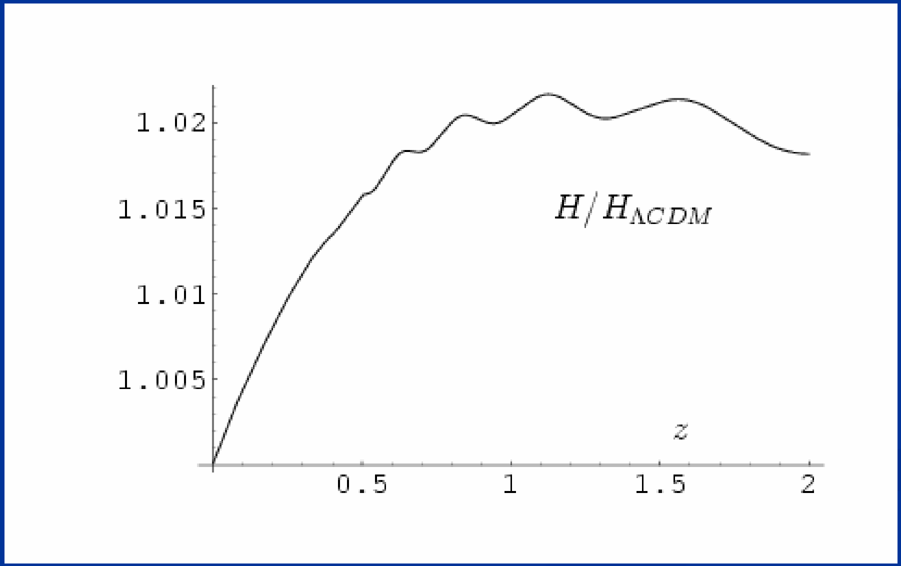
Hubble parameter

as compared to ΛCDM



Hubble parameter (z < z_c)

$$H^2 = \frac{1}{3M^2}\left\{V_t + \rho_{m,0}a^{-3} + 2\tilde{\rho}_{\nu,0}a^{-\frac{3}{2}}\right\}$$

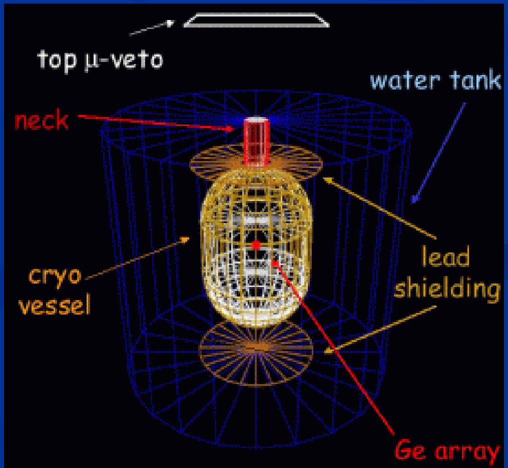


only small difference from ΛCDM !

Can time evolution of neutrino mass be observed ?

- Experimental determination of neutrino mass may turn out higher than upper bound in model for cosmological constant

(KATRIN, neutrino-less double beta decay)



GERDA

neutrino fluctuations

- time when neutrinos become non – relativistic
- sets free streaming scale

$$a_R = \left(\frac{\tilde{m}_\nu(t_0)}{3T_{\nu,0}} \right)^{-\frac{2}{5}} = 0.05 \left(\frac{\tilde{m}_\nu(t_0)}{eV} \right)^{-2/5}$$

- neutrino structures become nonlinear at z~1 for supercluster scales

D.Mota , G.Robbers , V.Pettorino , …

- stable neutrino-cosmon lumps exist

N.Brouzakis , N.Tetradis ,…

Conclusions

- Cosmic event triggers qualitative change in evolution of cosmon
- Cosmon stops changing after neutrinos become non-relativistic
- Explains why now
- Cosmological selection
- Model can be distinguished from cosmological constant

two key features

1) Exponential cosmon potential and scaling solution

$$V(\varphi) = M^4 \exp(- \alpha\varphi/M)$$

$$V(\varphi \rightarrow \infty) \rightarrow 0 \;\; !$$

2) Stop of cosmon evolution by cosmological trigger

Why goes the cosmological constant to zero ?

Time dependent Dark Energy : Quintessence

- What changes in time ?
- **Only dimensionless ratios of mass scales are observable !**
- V : potential energy of scalar field or cosmological constant
- V/M^4 is observable
- **Imagine the Planck mass M increases …**

Cosmon and fundamental mass scale

- Assume all mass parameters are proportional to scalar field χ (GUTs, superstrings,…)
- $M_p \sim \chi$, $m_{proton} \sim \chi$, $\Lambda_{QCD} \sim \chi$, $M_W \sim \chi$,…

- χ may evolve with time : **cosmon**
- m_n/M : (almost) constant - ***observation*** !

Only ratios of mass scales are observable

Example :

Field χ is connected to mass scale of transition from higher dimensional physics to effective four dimensional description

theory without explicit mass scale

- Lagrange density:

$$L = \sqrt{g}\left(-\frac{1}{2}\chi^2 R + \frac{1}{2}(\delta - 6)\partial^\mu\chi\partial_\mu\chi + V(\chi) + h\chi\overline{\psi}\psi\right)$$

realistic theory

- χ has no gauge interactions
- χ is effective scalar field after "integrating out" all other scalar fields

Dilatation symmetry

- Lagrange density:

$$L = \sqrt{g}\left(-\frac{1}{2}\chi^2 R + \frac{1}{2}(\delta - 6)\partial^\mu \chi \partial_\mu \chi + V(\chi) + h\chi\overline{\psi}\psi\right)$$

- Dilatation symmetry for

$$V = \lambda\chi^4,\ \lambda = const.,\ \delta = const.,\ h = const.$$

- Conformal symmetry for $\delta=0$

Asymptotically vanishing effective "cosmological constant"

- Effective cosmological constant ~ V/M^4
- $\lambda \sim (\chi/\mu)^{-A}$
- $V \sim (\chi/\mu)^{-A} \chi^4$ → $V/M^4 \sim (\chi/\mu)^{-A}$
- $M = \chi$

It is sufficient that V increases less fast than χ^4 !

Cosmology

Cosmology : χ increases with time !

(due to coupling of χ to curvature scalar)

for large χ the ratio V/M^4 decreases to zero

→

Effective cosmological constant vanishes asymptotically for large t !

Weyl scaling

Weyl scaling : $g_{\mu\nu} \to (M/\chi)^2 g_{\mu\nu}$,

$\varphi/M = \ln(\chi^4/V(\chi))$

$$L = \sqrt{g}\left(-\frac{1}{2}M^2R + \frac{1}{2}k^2(\phi)\partial^\mu\phi\partial_\mu\phi + V(\phi) + m(\phi)\overline{\psi}\psi\right)$$

Exponential potential : $V = M^4 \exp(-\varphi/M)$

No additional constant !

Quintessence from higher dimensions

geometrical runaway and the problem of time varying constants

- It is not difficult to obtain quintessence potentials from higher dimensional (or string ?) theories
- Exponential form rather generic

  (after Weyl scaling)
- Potential goes to zero for $\varphi \to \infty$
- But most models show too strong time dependence of constants !

runaway solutions

- geometrical runaway
- anomalous runaway
- geometrical adjustment

Quintessence from higher dimensions

An instructive example:

with J. Schwindt
hep-th/0501049

Einstein – Maxwell theory in six dimensions

$$S = \int d^6x \sqrt{-g} \left\{ -\frac{M_6^4}{2} R + \lambda_6 + \frac{1}{4} F^{AB} F_{AB} \right\}$$

Metric

Ansatz with particular metric (not most general !)

which is consistent with

d=4 homogeneous and isotropic Universe

and internal U(1) x Z_2 isometry

$$ds^2 = \exp\left(-\frac{\phi(t)}{\bar{M}}\right)\{-dt^2 + a^2(t)d\vec{x}d\vec{x}\}$$

$$+\exp\left(\frac{\phi(t)}{\bar{M}}\right) r_0^2\{d\rho^2 + B^2\sin^2\rho\, d\theta^2\}$$

$$r_0^2 = \frac{\bar{M}^2}{4\pi B M_6^4}$$

B ≠ 1 : football shaped internal geometry

Conical singularities

deficit angle

$$\Delta = 2\pi(1 - B)$$

singularities can be included with
energy momentum tensor on brane

$$(T^{(B)})_\mu^\nu = \frac{B-1}{Br_0^2 e^{\phi/\bar{M}}} M_6^4 \left(\frac{\delta(\rho)}{\rho} + \frac{\delta(\rho - \pi)}{\pi - \rho} \right) \delta_\mu^\nu$$

bulk point of view :
describe everything in terms of bulk geometry
(not possible for modes on brane without tail in bulk)

Exact solution

$$A_\theta = \frac{m}{2e_6}(1 - \cos\rho)$$

m : monopole number (integer)

$$H^2 = \frac{1}{3\bar{M}^2}(\frac{1}{2}\dot{\phi}^2 + V(\phi))$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

cosmology with scalar

and potential V :

$$V(\phi) = \bar{M}^4 \left\{ \frac{\lambda_6}{M_6^4 \bar{M}^2}\, e^{-\frac{\phi}{\bar{M}}} - 4\pi B \frac{M_6^4}{\bar{M}^4}\, e^{-\frac{2\phi}{\bar{M}}} + 2\pi^2 m^2 \frac{M_6^4}{e_6^2 \bar{M}^6}\, e^{-\frac{3\phi}{\bar{M}}} \right\}$$

Asymptotic solution for large t

$$H = 2t^{-1}, \quad \phi = 2\bar{M} \ln \frac{t}{\sqrt{10} M_6^2 \lambda_6^{-1/2}}$$

$$\Omega_h = \frac{V + \frac{1}{2}\dot{\phi}^2}{3\bar{M}^2 H^2} \longrightarrow 1$$

$$V + \frac{1}{2}\dot{\phi}^2 \propto t^{-2}$$

Naturalness

- No tuning of parameters or integration constants
- Radiation and matter can be implemented
- Asymptotic solution depends on details of model, e.g. solutions with constant $\Omega_h \neq 1$

geometrical runaway

$$V \sim L^{D}$$

$$M_p^{2} \sim L^{D}$$

$$V/M_p^{4} \sim L^{-D}$$

problem :

time variation of fundamental constants

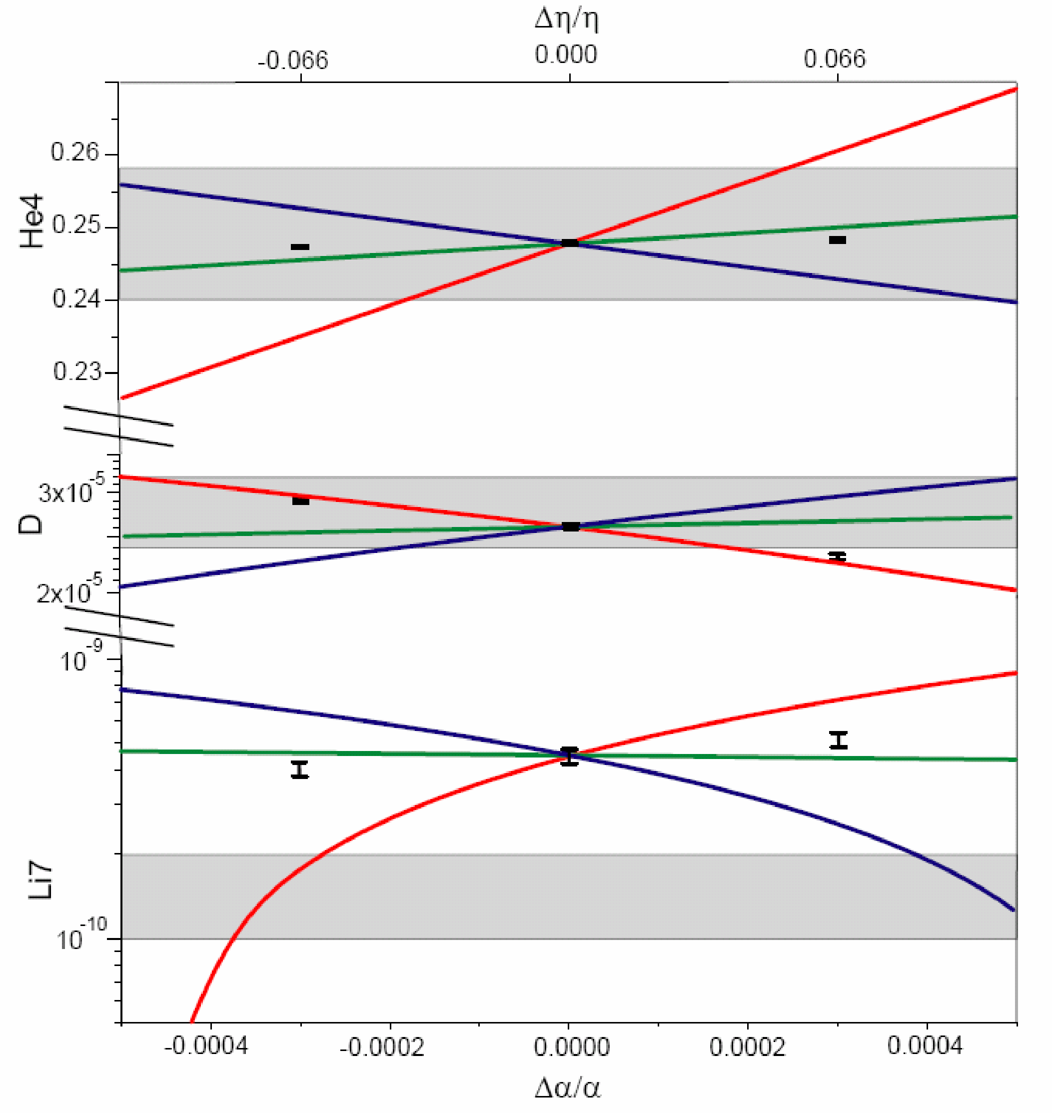
relative change order one for z around one

primordial abundances for three GUT models

He

D

Li



present observations : 1σ

T.Dent, S.Stern,…

three GUT models

- unification scale ~ Planck scale
- 1) All particle physics scales $\sim\Lambda_{QCD}$
- 2) Fermi scale and fermion masses ~ unification scale
- 3) Fermi scale varies more rapidly than Λ_{QCD}

$\Delta\alpha/\alpha \approx 4\ 10^{-4}$ allowed for GUT 1 and 3 , larger for GUT 2

$\Delta\ln(M_n/M_P) \approx 40\ \Delta\alpha/\alpha \approx 0.015$ allowed

stabilizing the couplings…

gauge couplings go to zero as volume of internal space increases

ways to solve this problem:

- volume or curvature of internal space is irrelevant for modes on brane
- possible stabilization by fixed points in scale free models

Warped branes

- model is similar to first co-dimension two

  warped brane model : C.W. Nucl.Phys.B255,480(1985);

  see also B253,366(1985)
- first realistic warped model
- see Rubakov and Shaposhnikov for earlier work (no stable solutions, infinitely many chiral fermions)
- see Randjbar-Daemi, C.W. for arbitrary dimensions

Brane stabilization

idea :

- all masses and couplings of standard model depend only on characteristic scale and geometry of brane
- generalized curvature invariant , which is relevant for V, scales with inverse power of characteristic length scale L for volume of internal space
- $L \to \infty$ while brane scale remains constant
- analogy with black hole in cosmological background

scales in gravity

- gravity admits solutions with very different length or mass scales
- example : black hole in expanding universe

quantum fluctuations and dilatation anomaly

Dilatation symmetry

- Lagrange density:

$$L = \sqrt{g}\left(-\frac{1}{2}\chi^2 R + \frac{1}{2}(\delta - 6)\partial^\mu \chi \partial_\mu \chi + V(\chi) + h\chi\overline{\psi}\psi\right)$$

- Dilatation symmetry for

$$V = \lambda\chi^4, \lambda = const., \delta = const., h = const.$$

- Conformal symmetry for $\delta=0$

Dilatation anomaly

- Quantum fluctuations responsible for dilatation anomaly
- Running couplings: **hypothesis**

$$\partial\lambda/\partial \ln\chi = -A\lambda$$

- Renormalization scale μ : (momentum scale)
- $\lambda \sim (\chi/\mu)^{-A}$

Asymptotic behavior of effective potential

- $\lambda \sim (\chi/\mu)^{-A}$
- $V \sim (\chi/\mu)^{-A} \chi^4$

$$V \sim \chi^{4-A}$$

crucial : behavior for large χ !

Without dilatation – anomaly :

V= const.

Massless Goldstone boson = dilaton

Dilatation – anomaly :

V (φ)

Scalar with tiny time dependent mass :

cosmon

Dilatation anomaly and quantum fluctuations

- Computation of running couplings (beta functions) needs unified theory !
- Dominant contribution from modes with momenta $\sim\chi$!
- No prejudice on "natural value " of anomalous dimension should be inferred from tiny contributions at QCD- momentum scale !

quantum fluctuations and naturalness

- Jordan- and Einstein frame completely equivalent on level of effective action and field equations (**after** computation of quantum fluctuations !)
- Treatment of quantum fluctuations depends on frame : Jacobian for variable transformation in functional integral
- What is natural in one frame may look unnatural in another frame

quantum fluctuations and frames

- Einstein frame : quantum fluctuations make zero cosmological constant look unnatural
- Jordan frame : quantum fluctuations are at the origin of dilatation anomaly;
- may be key ingredient for solution of cosmological constant problem !

fixed points and fluctuation contributions of individual components

If running couplings influenced by fixed points:

individual fluctuation contribution can be huge overestimate !

here : fixed point at vanishing quartic coupling and anomalous dimension ⟹ $V \sim \chi^{4-A}$

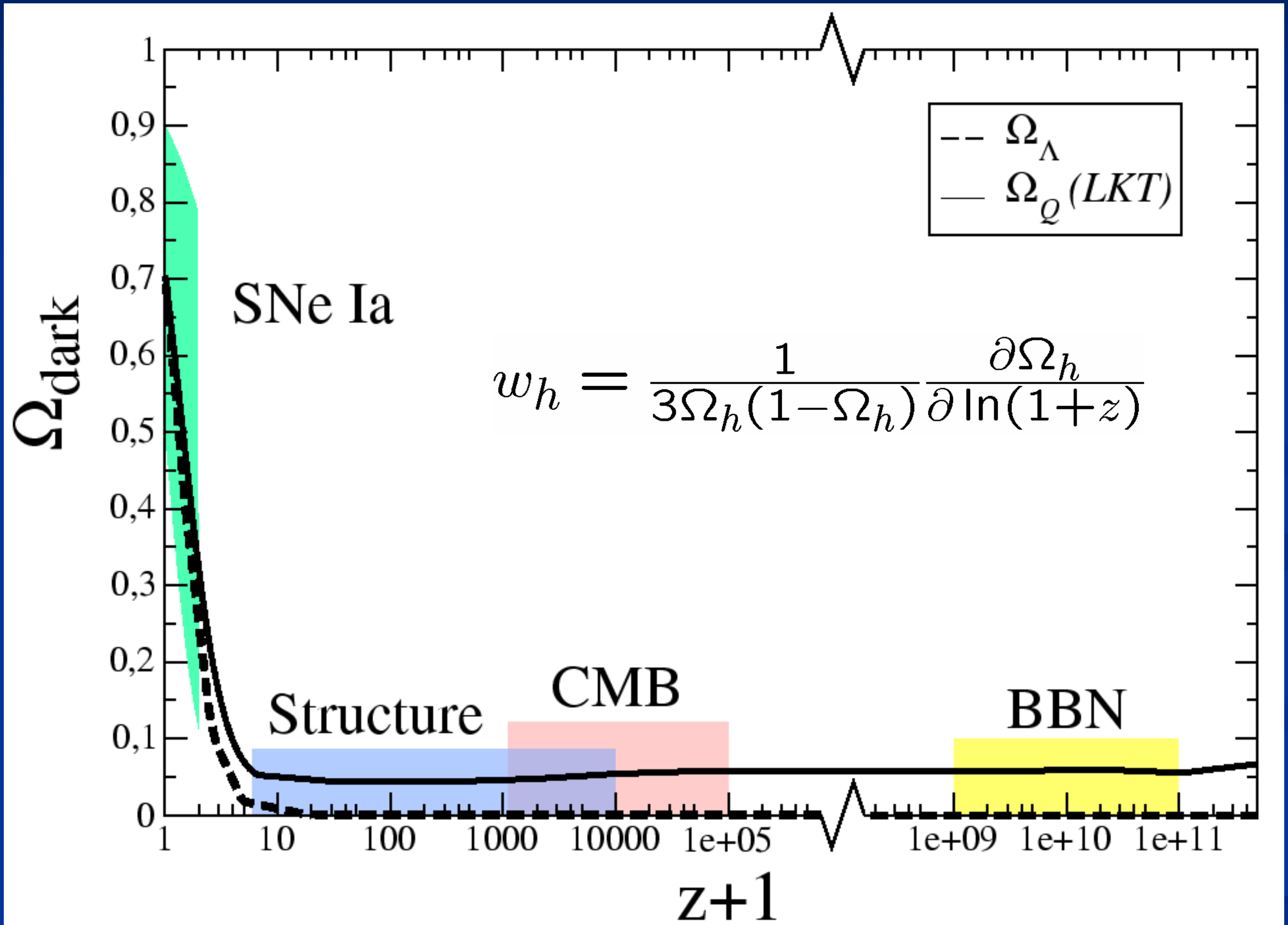
it makes no sense to use naïve scaling argument to infer individual contribution $V \sim h\,\chi^{4}$

conclusions

- naturalness of cosmological constant and cosmon potential should be discussed in the light of dilatation symmetry and its anomalies
- Jordan frame
- higher dimensional setting
- four dimensional Einstein frame and naïve estimate of individual contributions can be very misleading !

How can quintessence be distinguished from a cosmological constant ?

Time dependence of dark energy



cosmological constant : $\Omega_h \sim t^2 \sim (1+z)^{-3}$

M.Doran,…

small early and large present dark energy

fraction in dark energy has substantially increased since end of structure formation

→

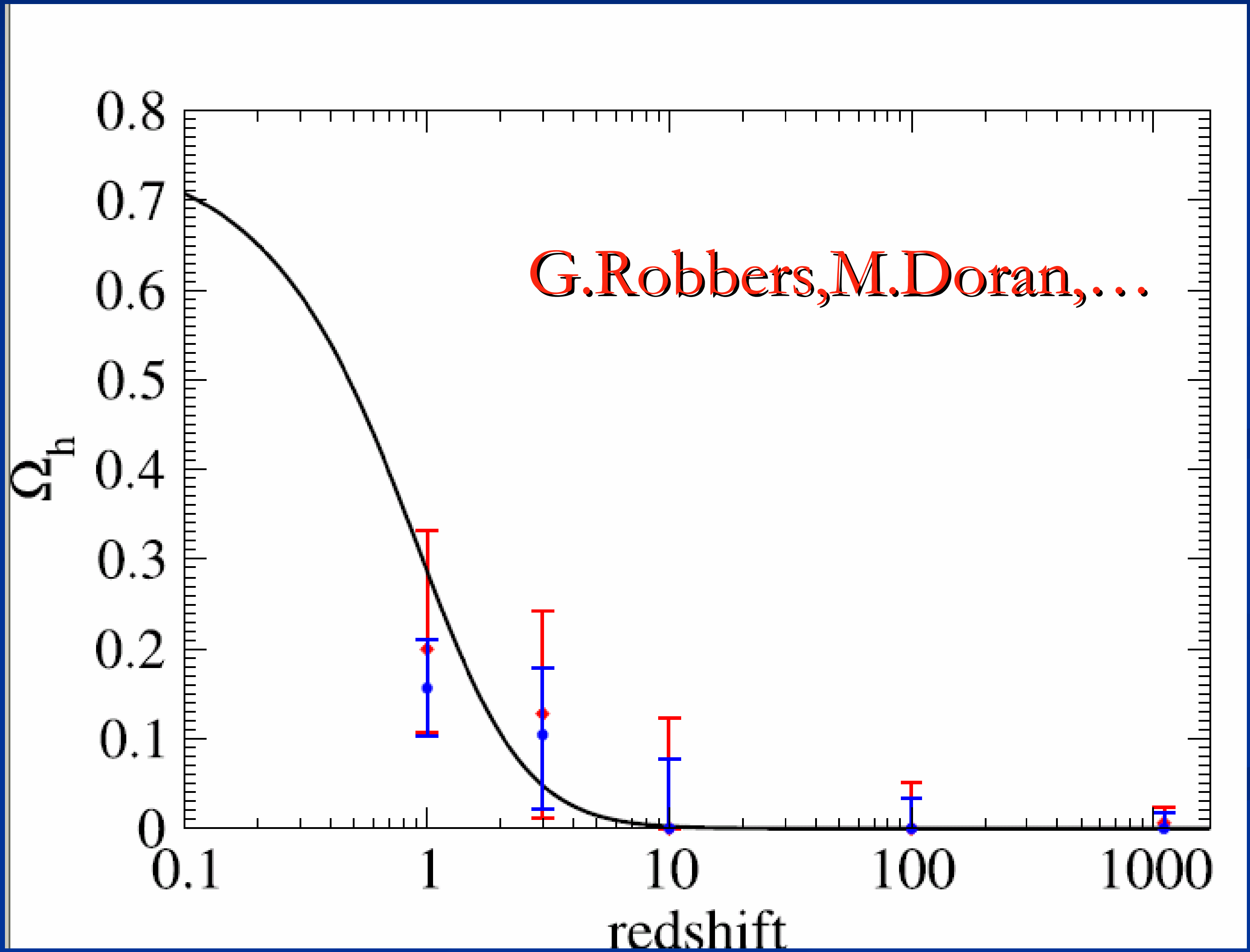
expansion of universe accelerates in present epoch

$$w_h = \frac{1}{3\Omega_h(1-\Omega_h)} \frac{\partial \Omega_h}{\partial \ln(1+z)}$$

effects of early dark energy

- modifies cosmological evolution (CMB)
- slows down the growth of structure

interpolation of Ω_h



Summary

- $\Omega_h = 0.75$
- Q/Λ : dynamical und static dark energy will be distinguishable
- growing neutrino mass can explain why now problem
- Q : time varying fundamental coupling “constants”
  violation of equivalence principle

End

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Cosmon coupling to atoms

- Tiny !!!
- Substantially weaker than gravity.
- Non-universal couplings bounded by tests of equivalence principle.
- Universal coupling bounded by tests of Brans-Dicke parameter ω in solar system.
- Only very small influence on cosmology.

effective cosmological constant linked to neutrino mass

realistic value $\alpha\, \varphi_t / M \approx 276$:
needed for neutrinos to become non-relativistic in recent past -
as required for observed mass range of neutrino masses

φ_t / M : essentially determined by present neutrino mass

adjustment of one dimensionless parameter
in order to obtain for the present time the
correct ratio between dark energy and neutrino
energy density

no fine tuning !

effective cosmological constant

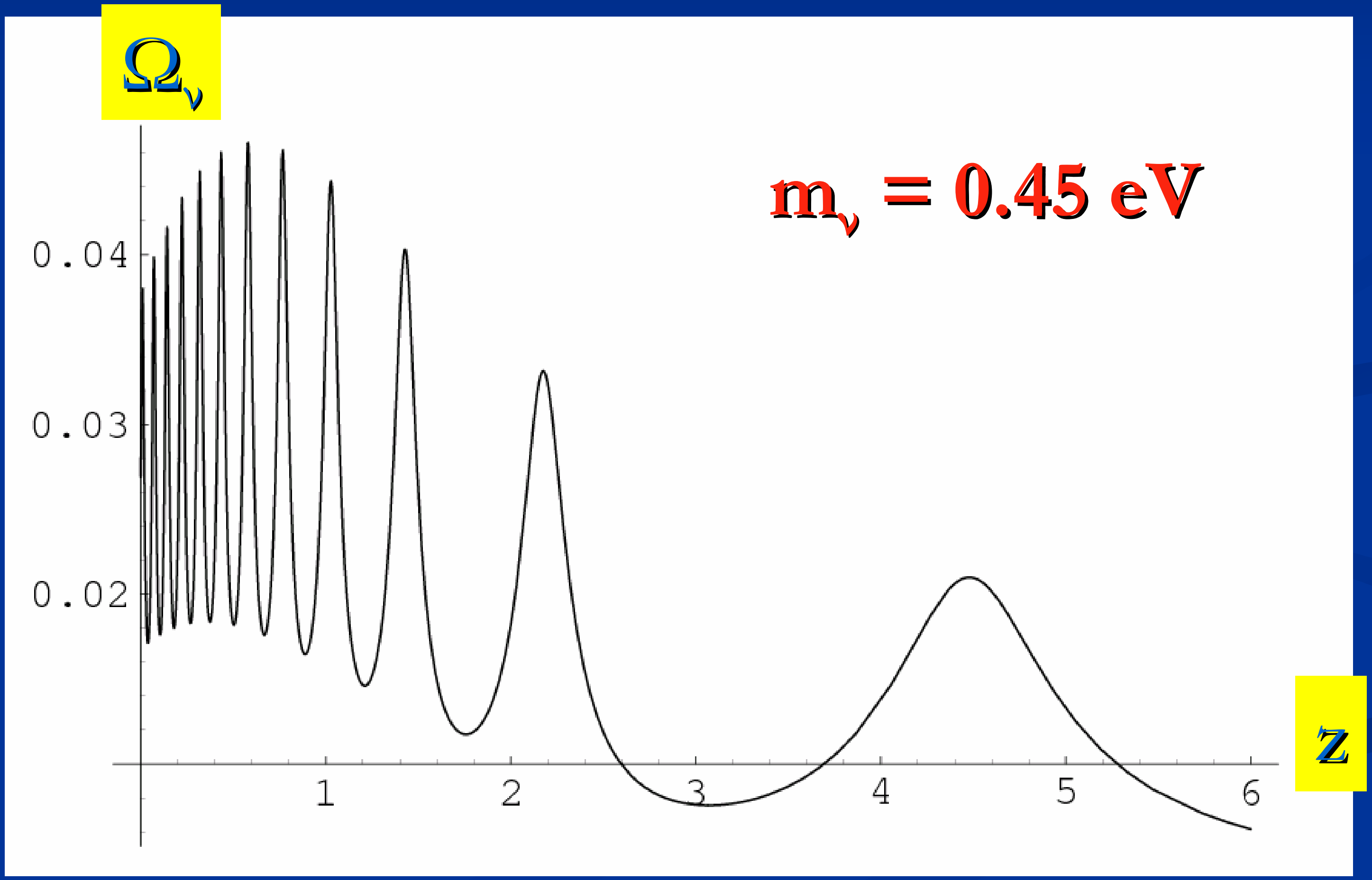
$$V_t = M^4 \exp\left(-\alpha \frac{\varphi_t}{M}\right)$$

realistic value
for
$\alpha \varphi_t / M \approx 276$

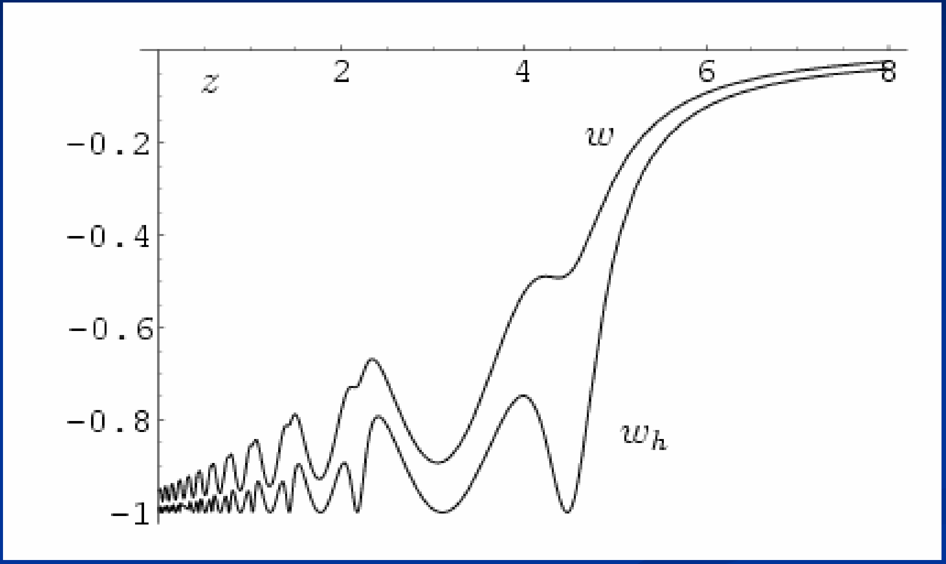
⟺

$$\epsilon = -\frac{\alpha \ln \tau}{276}$$

neutrino fraction remains small



equation of state

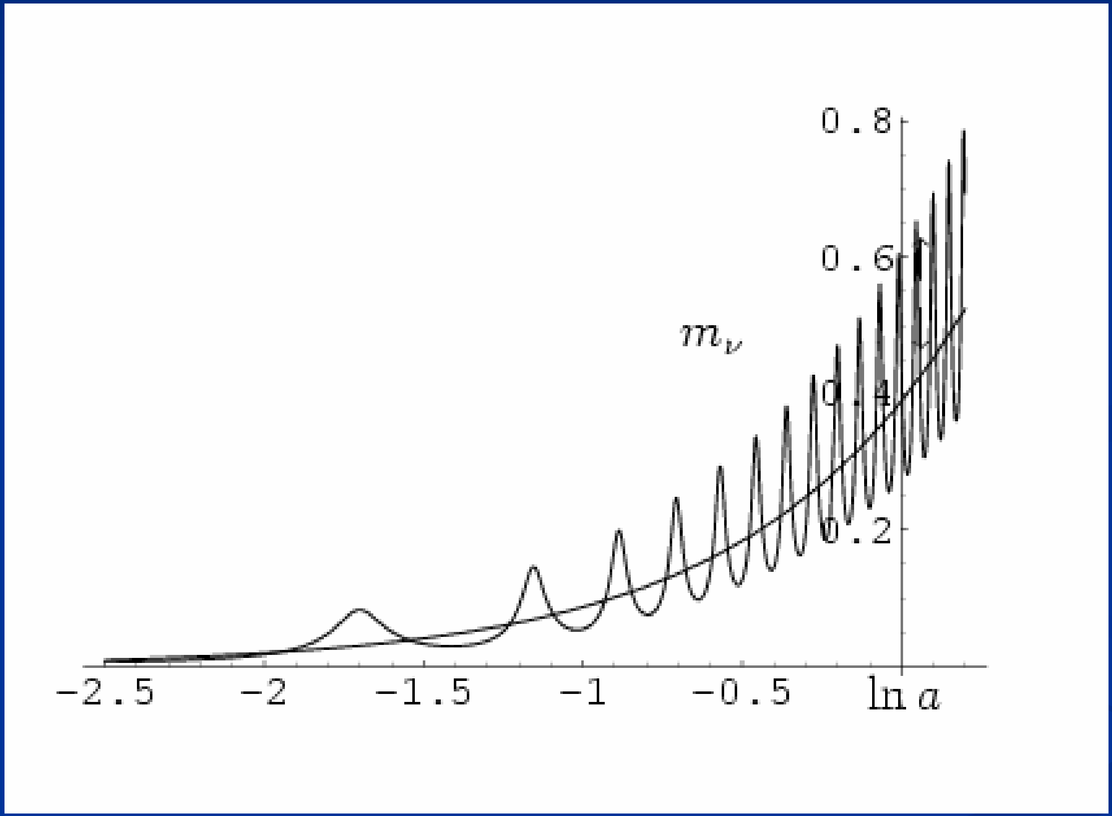


$$w = \frac{T - V + w_\nu \rho_\nu}{T + V + \rho_\nu} \approx -1 + \frac{\rho_\nu}{V} \approx -1 + \frac{\Omega_\nu}{\Omega_h},$$

present equation of state given by neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\mathrm{eV}}$$

oscillating neutrino mass



crossing time

from matching between

early solution and late solution

$$V_t \approx V(t_c) \approx \frac{3}{2}\Omega_{h,e} M^2 H^2(t_c)$$
$$= \frac{9}{2\alpha^2} M^2 H^2(t_c) = \frac{2M^2}{\alpha^2 t_c^2}$$

$$t_c^2 H_0^2 = \frac{2}{3\Omega_{h,0}\alpha^2} \approx \frac{8}{9\alpha^2}$$

approximate late solution

variables :

$$s = -\alpha(\varphi - \varphi_t)/M,$$
$$x = \ln a$$

$$\partial_x \ln \rho_\nu + \partial_x \ln s = -3 \ , \ \partial_x \ln \rho_m = -3$$

$$\rho_\nu = \frac{c_\nu}{sa^3} \ , \ \rho_m = \frac{\rho_{m,0}}{a^3}$$

approximate smooth solution
(averaged over oscillations)

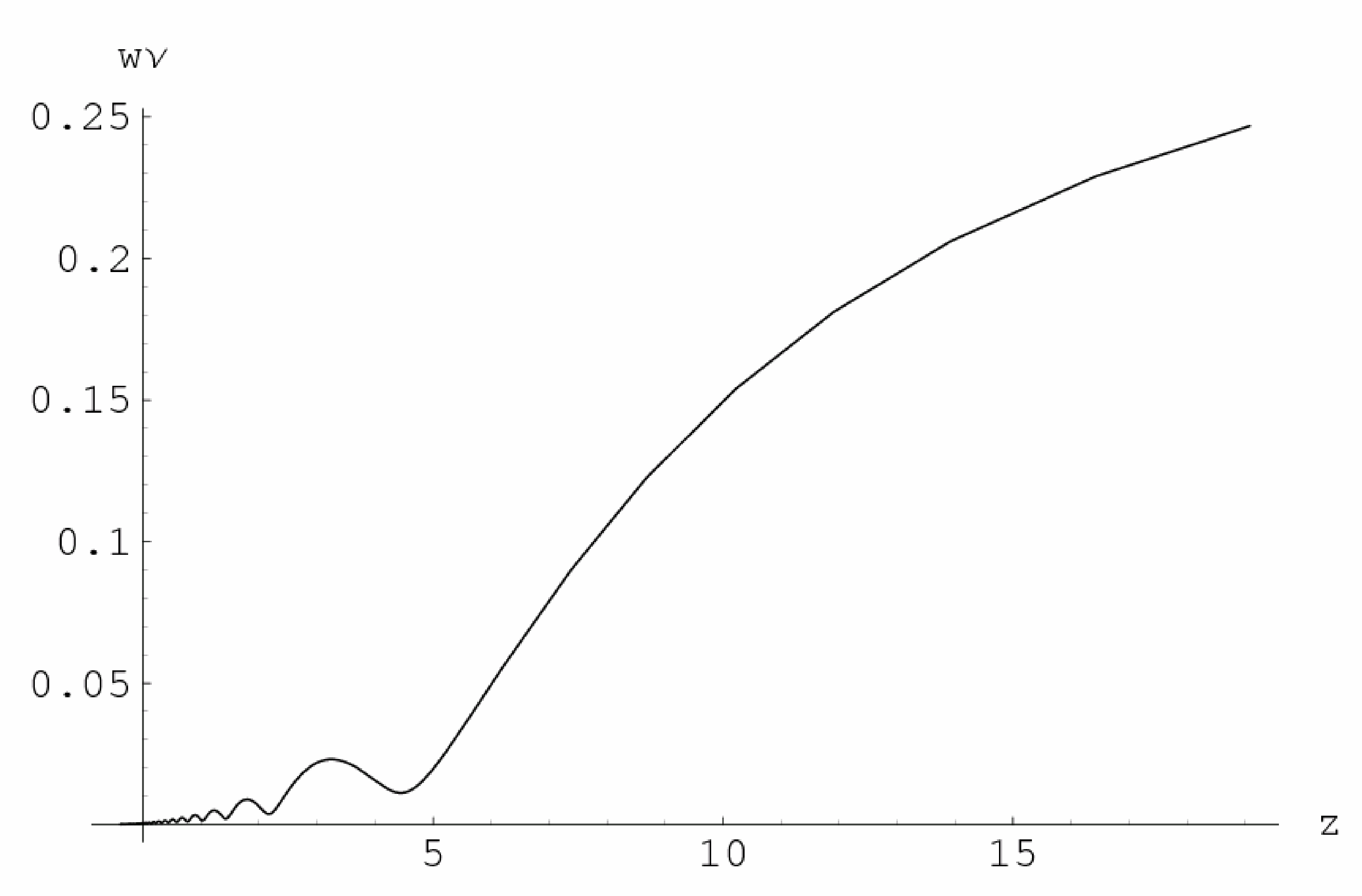
$$s^{(0)}(x) = \left(\frac{c_\nu}{V_t}\right)^{1/2} e^{-\frac{3x}{2}} = \frac{\tilde{\rho}_\nu(x)}{V_t}$$

$$s_0^{(0)} = \left(\frac{c_\nu}{V_t}\right)^{1/2} = \frac{\tilde{\rho}_{\nu,0}}{V_t} \approx \frac{\Omega_\nu(t_0)}{\Omega_h(t_0)}$$

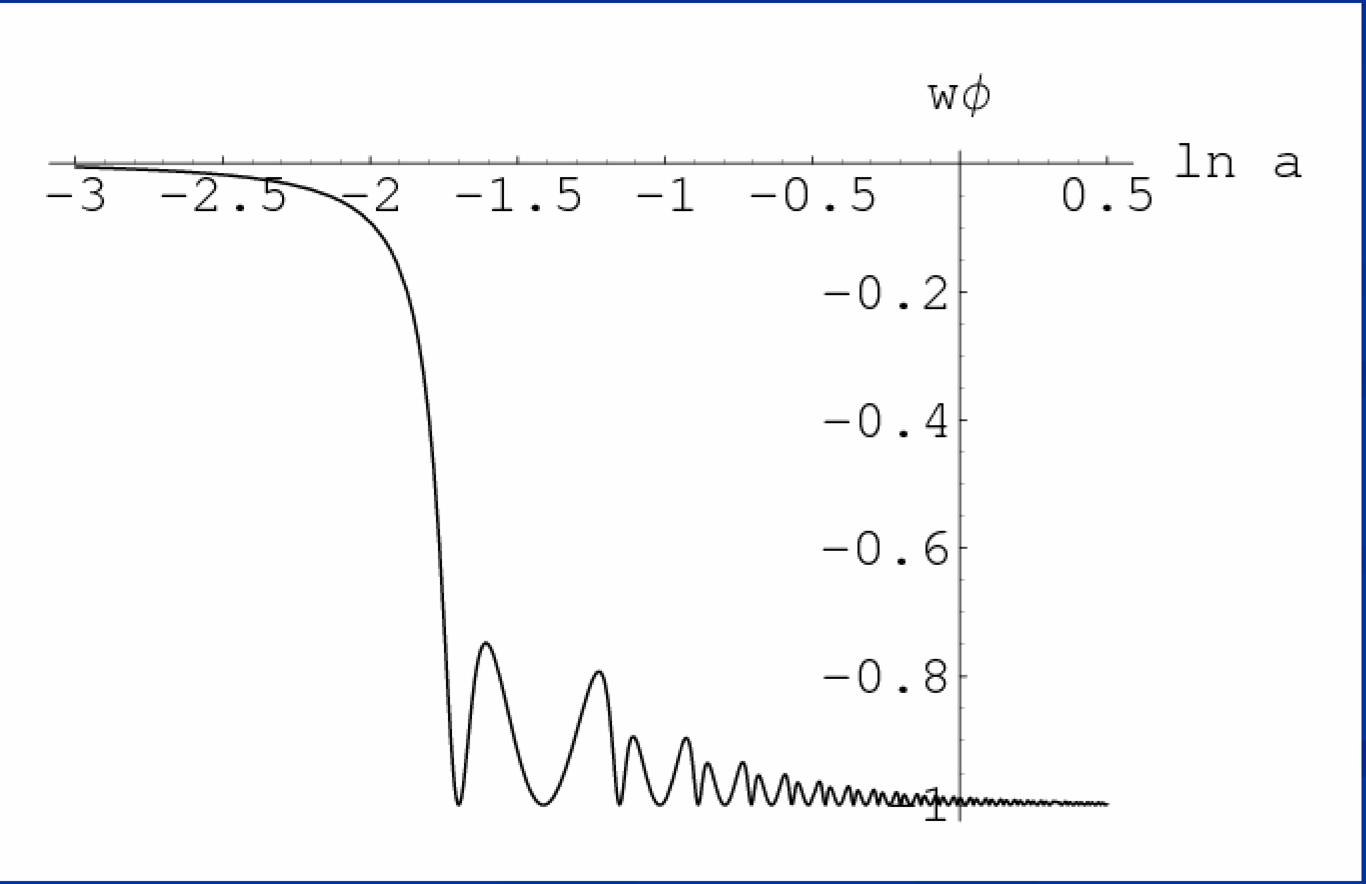
dark energy fraction

$$\tilde{\Omega}_h(a) = \begin{cases} \dfrac{\tilde{\Omega}_{h,0}a^3+2\Omega_{\nu,0}(a^{3/2}-a^3)}{1-\tilde{\Omega}_{h,0}(1-a^3)+2\Omega_{\nu,0}(a^{3/2}-a^3)} & \text{for } a > a_c \\ \dfrac{3}{\alpha^2} & \text{for } a < a_c \end{cases}$$

neutrino equation of state



cosmon equation of state



fixed point behaviour : apparent tuning

$$V(\varphi) = U_0(\varphi) - \frac{\lambda d_0^4 \gamma^2}{2(\lambda M_t^2(\varphi) - \gamma^2)}$$

$$V(\varphi) = U_0(\varphi) - \frac{m_\nu(\varphi) d^2 \gamma}{2 h_L}$$

Growth of density fluctuations

- Matter dominated universe with constant Ω_h :

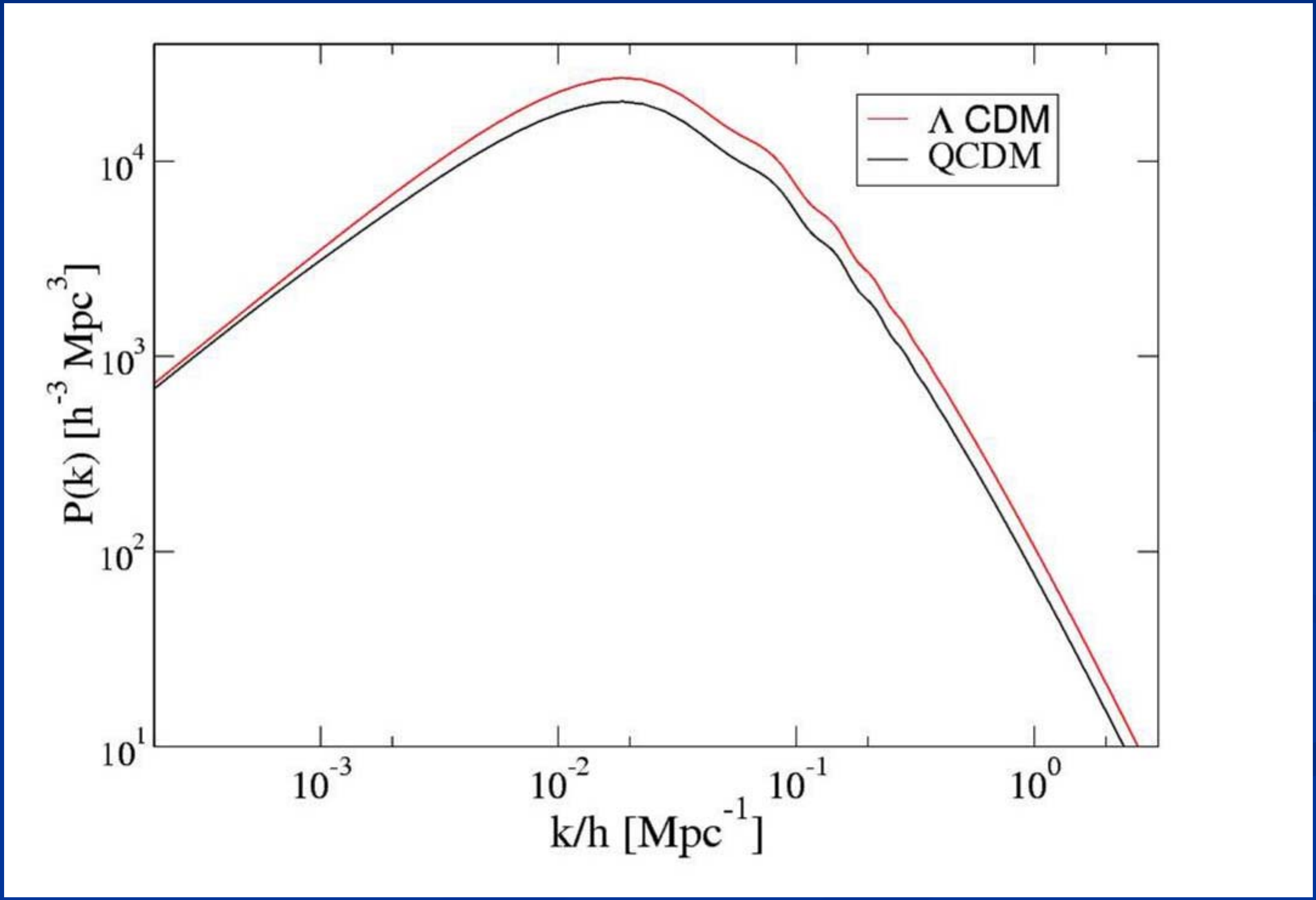
$$\Delta\rho \sim a^{1-\frac{\epsilon}{2}} \ , \ \epsilon = \frac{5}{2}\left(1 - \sqrt{1 - \frac{24}{25}\Omega_h}\right)$$

P.Ferreira,M.Joyce

- Dark energy slows down structure formation

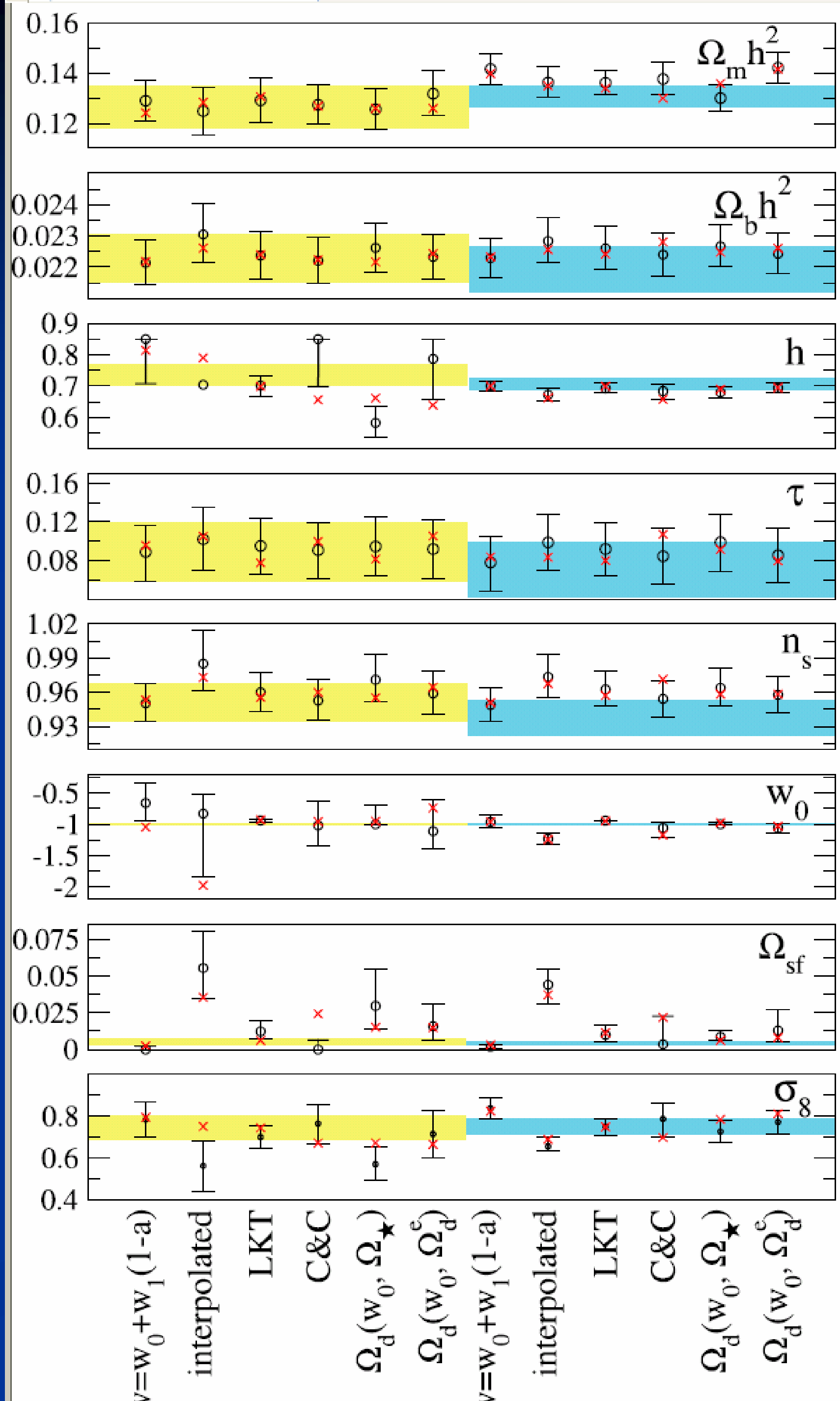
  $\Longrightarrow$ $\Omega_h < 10\%$ during structure formation

Early quintessence slows down the growth of structure

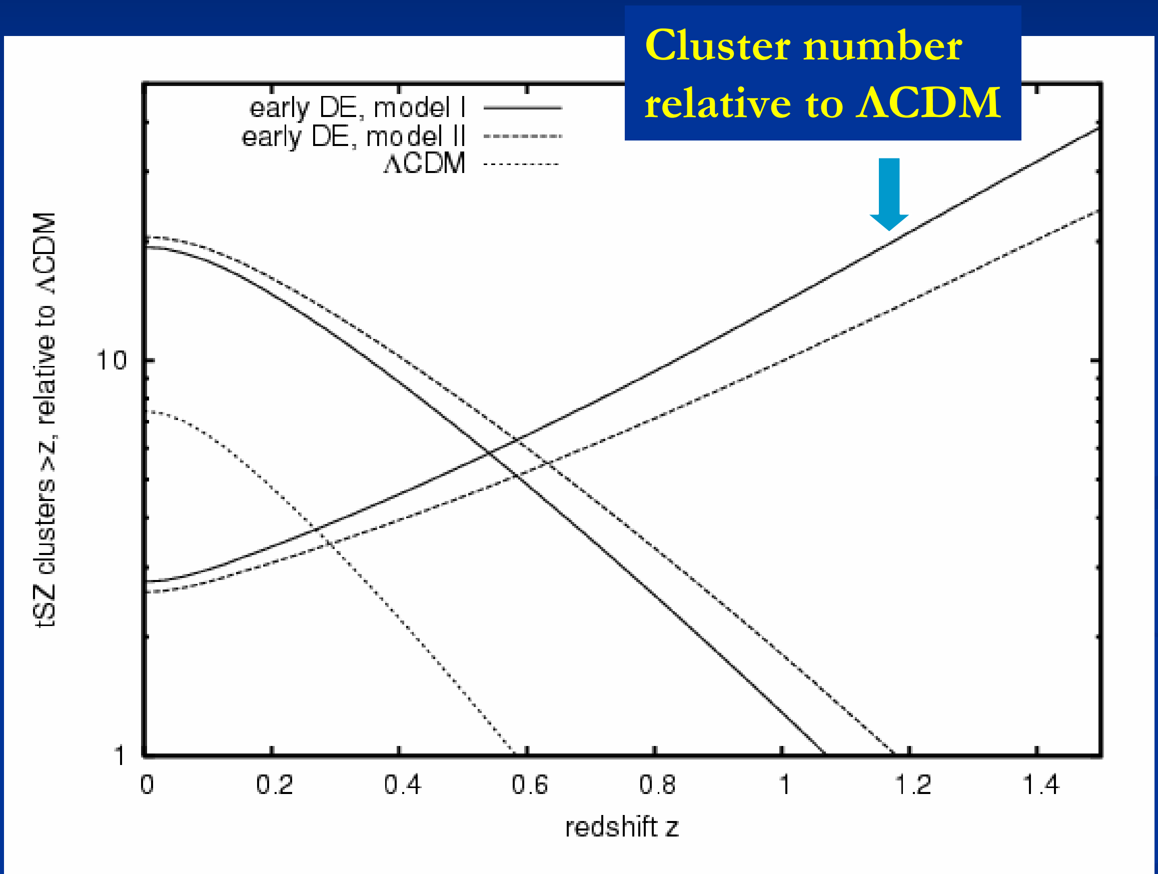


bounds on
Early Dark Energy
after WMAP'06

G.Robbers,M.Doran,…



Little Early Dark Energy can make large effect !
Non – linear enhancement



Two models with 4% Dark Energy during structure formation

Fixed σ_8
(normalization dependence !)

More clusters at high redshift !

Bartelmann,Doran,…