Nonabelian Plasma Instabilities

Michael Strickland
Frankfurt Institute for Advanced Studies

Nonequilibrium Dynamics in Particle Physics and Cosmology
31 January 2008
Heavy-ion collision timescales and “epochs” @ LHC

- Semi-hard particle production: $0 < \tau < 0.1 \text{ fm/c}$
- Non-equilibrium QGP: $0.1 < \tau < 2 \text{ fm/c}$
- Equilibrium QGP: $2 < \tau < 15 \text{ fm/c}$
- Hot Hadron Gas: $15 < \tau < 18 \text{ fm/c}$
- Chemical "Freezeout": $\tau > 18 \text{ fm/c}$

$*1 \text{ fm/c} \simeq 3 \times 10^{-23} \text{ seconds}$
Motivation – Isotropization/Thermalization

• Need to understand mechanisms and time scales necessary for the isotropization and equilibration of a QGP at weak coupling.

• Consider pure glue. Processes include:
  ◦ $2 \leftrightarrow 2$ elastic scattering (super slow)
  ◦ Inelastic processes, e.g. $2 \rightarrow 3$ and $3 \rightarrow 2$ processes
  ◦ Effect of *soft background fields*: expansion is in $gA$ not $g$; CGC $A \sim 1/g$

• Equilibrium: background fields screen the interaction (Debye)

• Non-equilibrium: background fields can have non-trivial dynamics and can have a large effect on the particles’ motion
Improving upon Bottom-Up Thermalization

• Previous leading order perturbative results included $2 \leftrightarrow 2$, $2 \rightarrow 3$, and $3 \rightarrow 2$ processes [R. Baier, A. Mueller, D. Son, and D. Schiff, hep-ph/0009237]

• "Bottom-up" thermalization: soft modes isotropize and equilibrate first, then the hard modes $\rightarrow \tau_{\text{therm}} \sim \alpha_s^{-13/5} Q_s^{-1}$

• At RHIC $Q_s \sim 1.5 - 2$ GeV and $\alpha_s \sim 0.3 \rightarrow \tau_{\text{therm}} \sim 2 - 3$ fm/c

• Bottom-up calculation ignored effect of local anisotropy in momentum space on soft physics (field dynamics)

In anisotropic systems plasma instabilities are present which will accelerate isotropization and thermalization.
Momentum Space Anisotropy Time Dependence

Expansion rate and isotropization via interactions balance

\[ \tau_{\text{iso}} \sim Q_s^{-1} \]

Expansion rate is much faster than the interaction time scale
\[ 1/\tau \gg 1/\tau_{\text{int}} \]

System is momentarily isotropic

Viscous Hydro

Boltzmann-Vlasov Transport

CGC/Glasma

\[ \langle p_L \rangle \]
\[ \langle p_T \rangle \]

0.1-0.2 fm/c

1-3 fm/c

\[ \tau \]
- Analytic Results from Linearized Transport Theory -
Gluon Polarization – The Chromo-Weibel Instability

The high-energy medium gluon polarization tensor can be obtained by linearizing collisionless transport theory: \( f(p, x) \rightarrow f(p) + \delta f(p, x) \)

\[
[v \cdot D_x, \delta f(p, x)] + g v_\mu F^{\mu\nu} \partial_\nu^{(p)} f(p) = 0
\]

\[
D_\mu F^{\mu\nu} = J^\nu = g \int_p \nu^\nu \delta f(p, x)
\]

or diagrammatically using “hard-loop” perturbation theory

\[
\Pi_{ij}^{ab}(\omega, k) = -g^2 \delta_{ij} \int_p v^i \frac{\partial f(p)}{\partial p^l} \left( \delta^{jl} - \frac{v^j k^l}{\omega - \mathbf{v} \cdot \mathbf{k} + i\epsilon} \right)
\]

S. Mrówczyński (1994); P. Romatschke and MS, hep-ph/0304092
The nature of the anisotropy

For simplicity assume that the anisotropic distribution function can be obtained from an arbitrary isotropic distribution function by a change of its argument.

\[ f(p^2) \rightarrow f(p^2 + \xi(p \cdot n)^2) \]

The polarization tensor can then be written as

\[ \Pi_{ab}^{ij}(\omega, k) = m_D^2 \delta_{ab} \int \frac{d\Omega}{4\pi} v^i v^l + \xi(v \cdot n)n^l \left( \delta^{jl} - \frac{v^j k^l}{\omega - v \cdot k + i\epsilon} \right) \]

where \( m_D \) is the isotropic Debye mass

\[ m_D^2 = -\frac{g^2}{2\pi^2} \int_0^\infty dp p^2 \frac{df(p^2)}{dp} \sim g^2 p_{\text{hard}}^2 \]
Anisotropic Gluonic Collective Modes ($\xi > 0$)

Unstable Modes

Magnetic

"Electric"

Stable Modes

Damped Modes

Unstable modes are generic if system is not isotropic

$\omega/k$

$\text{Im } \omega/k$

$\text{Re } \omega/k$

$\times$:
- isotropic
- oblate
Using $\alpha_s = 0.3$ and $Q_s \sim 1.5 - 2$ GeV

$$m_D = g Q_s$$

$$\rightarrow 3 - 4 \text{ GeV}$$

$$m_D \sim \sqrt{\frac{g^2 N_c n_g}{Q_s}}$$

$$\rightarrow 2 - 3 \text{ GeV}$$

$$\Gamma \sim 0.5 - 2.4 \text{ GeV}$$

**e-Folding time**

0.1 - 0.4 fm/c

Instability growth rates as a function of momentum for $\langle p_T^2 \rangle / \langle p_L^2 \rangle \simeq 10$ and $\theta_{\text{glue}} = \pi/8$ with respect to the beamline.
Time scales

- This picture strictly only holds at leading order in $\alpha_s = g^2/4\pi$.
- Instability time scale: $t_{\text{instability}} \sim m_{D,\text{iso}}^{-1} \sim (\sqrt{\alpha_s Q_s})^{-1}$
- Collisional time scale: $t_{\text{hard collisions}} \sim (\alpha_s^2 Q_s)^{-1}$

<table>
<thead>
<tr>
<th>$\alpha_s$</th>
<th>$t_{\text{collisions}}/t_{\text{instability}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>1000</td>
</tr>
<tr>
<td>0.1</td>
<td>30</td>
</tr>
<tr>
<td>0.3</td>
<td>6</td>
</tr>
</tbody>
</table>

Can include collisions in the Boltzmann-Vlasov equation and it has been shown that for $\xi \gtrsim 1$ instabilities persist even for $\alpha_s = 0.3$.

[ B. Schenke, MS, C. Greiner, and M. Thoma, hep-ph/0603029 ]
Current Filamentation in Abelian (QED) Plasmas

Induced Current

Magnetic Fluctuation

E. Weibel, PRL 2, 83 (1959)
Anisotropic Abelian Plasma – Weibel Instability

Non-abelian effects would kick in at these energy densities.
Anisotropic QCD Hard-loop Effective Action

Require gauge invariance

\[ S_{\text{soft}} = S_{\text{QCD}} + S_{\text{HL}} \]

\[
S_{\text{HL}} = \frac{g^2}{2} \int_{x,p} \left[ f(p) F_{\mu \nu}(x) \frac{p^\nu p^\rho}{(p \cdot D)^2} F^\rho_{\mu}(x) + i \frac{C_F}{2} \tilde{f}(p) \bar{\Psi}(x) \frac{p \cdot \gamma}{p \cdot D} \Psi(x) \right]
\]
3+1 Real-Time Lattice Simulation (Pure Glue)

Numerically solve the equations of motion resulting from the hard-loop effective action on a space + velocity lattice in temporal gauge.

\[ j^\mu[A] = -g^2 \int_p \frac{1}{2|p|} p^\mu \frac{\partial f(p)}{\partial p^\beta} W^\beta(x; v) \]

with

\[ [p \cdot D(A)] W_\beta(x; v) = F_{\beta\gamma}(A) p^\gamma \]

This has to be solved with the Yang-Mills equation

\[ D_\mu(A) F^{\mu\nu} = j^\nu \]

where \( \nu = 0 \) is the Gauss law constraint.

Rebhan, Romatschke, and MS, 2005; Arnold, Moore, and Yaffe, 2005
3D SU(2) Hard-Loop Results

\[ \langle p_z^2 \rangle / \langle p_T^2 \rangle = 0.1 \]

No expansion!
Late growth is linear and approx isotropic!

Kolmogorov cascade $\rightarrow$ Turbulent Fields?

Non-Perturbative Soft Occupation Number: $f \sim 1/g^2$

Increasing simulation time $\sim 1/k^2$
- Come to Kari Rummukainen’s discussion for more details and systematics -
Hard-loop approximation strictly only applies when we ignore the back-reaction of the particles on their self-generated fields. How can we go beyond hard-loops?

Include back-reaction by solving collision-less transport equation without linearization

\[
p^\mu [\partial_\mu - g q^a F^a_{\mu \nu} \partial_\nu - g f_{abc} A^b_\mu q^c \partial q^a] f(t, x, p, q) = 0
\]

Coupled to the Yang-Mills equation for the soft gluon fields

\[
D_\mu F^{\mu \nu} = J^\nu = g \int \frac{d^3 p}{(2\pi)^3} dq q^\nu f(t, x, p, q)
\]

A. Dumitru, Y. Nara, and MS, hep-ph/0604149
CPIC Results – Ultraviolet Avalanche

Energy density \( \langle m_\infty \rangle^4 / g^2 \) vs. \( m_\infty t \)

- \( B^2/2 \)
- \( E^2/2 \)

Fields isotropic throughout the simulation

Saturated amplitude increases as lattice spacing decreases!

\( L = 5 \text{ fm}, \ p_{\text{hard}} = 16 \text{ GeV}, \ g^2 n_g = 10/\text{fm}^3, \ m_\infty = 0.12 \text{ GeV} \)

Coulomb gauge-fixed color-magnetic field spectrum at four different times.

A. Dumitru, Y. Nara, and MS, hep-ph/0604149
Instabilities in classical YM – The unstable glasma

Instabilities also seen in expanding classical Yang-Mills solutions which include rapidity fluctuations.

Growth $\sim e^{\sqrt{Q_s}\tau}$ agrees with HL calculation!

[P. Romatschke and A. Rebhan, hep-ph/0605064]

Initial spectrum of rapidity fluctuations from CGC camp

Instabilities in classical YM – Non-expanding

Recently there have also been measurements of the instability growth rate, induced spectrum, etc within classical SU(2) Yang-Mills by Berges, Scheffler, and Sexty (arXiv:0712.3514v2).
- Including expansion + field-particle coupling -
Instabilities induced by expansion - Free Streaming Bkg

Assuming a color neutral background distribution function $f_0(p, x, t)$ which satisfies

$$v \cdot \partial f_0(p, x, t) = 0, \quad v^\mu = p^\mu / p^0,$$

the gauge covariant Boltzmann-Vlasov equations for colored perturbations $\delta f_a$ of an approximately collisionless plasma have the form

$$v \cdot D \delta f_a(p, x, t) = g v_\mu F^{\mu \nu}_a \partial_\nu f_0(p, x, t),$$

which have to be solved self-consistently with the non-Abelian Maxwell equations

$$D_\mu F^{\mu \nu}_a = j_\nu^a = g \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu}{2p^0} \delta f_a(p, x, t).$$

Instabilities induced by expansion - Free Streaming Bkg

Go to comoving coordinates $\tilde{x}^\alpha = (\tau, x^1, x^2, \eta)$ with metric
\[
 ds^2 = d\tau^2 - d\mathbf{x}_\perp^2 - \tau^2 d\eta^2 \quad \text{and} \quad \tilde{V}^\alpha = (\cosh(y - \eta), \cos \phi, \sin \phi, \frac{1}{\tau} \sinh(y - \eta))
\]

\[
 \frac{1}{\tau} \tilde{D}_\alpha (\tau \tilde{F}^{\alpha\beta}) = \tilde{j}^\beta
\]

\[
 \tilde{V} \cdot \tilde{D} \mathcal{W} = \left( \tilde{V}^i \tilde{F}_{i\tau} + \frac{\tau^2}{\tau_{iso}^2} \tilde{V}^\eta \tilde{F}_{\eta\tau} \right) \tilde{V}^\tau + \tilde{V}^i \tilde{V}^\eta \tilde{F}_{i\eta} \left( 1 - \frac{\tau^2}{\tau_{iso}^2} \right).
\]

\[
 \tilde{j}^\alpha = -m_D^2 \frac{1}{2} \int_0^{2\pi} \frac{d\phi}{2\pi} \int_{-\infty}^{\infty} dy \tilde{V}^\alpha \left( 1 + \frac{\tau^2}{\tau_{iso}^2} \sinh^2(y - \eta) \right)^{-2} \mathcal{W}(\tilde{x}; \phi, y)
\]

1s×3v Numerical Results - Energies and Pressures

Perform simulation assuming that induced fields only depend on rapidity and are constant in the transverse directions → 1s × 3v simulations. Captures the physics of the most unstable modes and provides a reference point for future 3s × 3v simulations.

A. Rebhan, MS, and M. Attems, forthcoming.
**1s $\times$ 3v Numerical Results - Spectra**

Spectra from (left) an abelian run and (right) a non-abelian run showing Fourier decomposition of modes at different times.

Non-abelian run shows “quasi-thermal” spectra at intermediate times; qualitatively different than abelian case.

---

A. Rebhan, MS, and M. Attems, forthcoming.
Conclusions and Outlook

• Anisotropic plasmas are qualitatively different than isotropic ones.
• Hard-Loop: Fields show isotropic linear growth and UV cascade.
• CPIC: Rapid isotropic field growth followed by UV “avalanche”.
• Classical YM: rapidity fluctuations → the “glasma” is unstable to becoming a QGP! Instabilities also seen in a static box.
• The same instability exists in weakly-coupled supersymmetric gauge theories; just need to rescale the Debye mass. QUESTION: Do these instabilities also exist in the strong coupling limit????
• Need to pin down the possible phenomenological effect of plasma instabilities: Systematic calculations of $p_T-p_L$ anisotropy observables such as jet effects and E&M signatures.
• CPICv2: code now includes stochastic collisions between particles. Can now be used to measure transport properties, jet energy deposition, medium response to energy deposition, etc.