Understanding equilibration in strongly coupled quark-gluon plasma (sQGP)

(KITP program
Santa Barbara, Jan. 2008)

Edward Shuryak
Stony Brook
The emerging theory of sQGP

- Manybody theory
  - Strongly coupled cold trapped Quantum gases
- Bose-Einstein Condensation -> confinement
- Flux tubes ->
- Molecular dynamics
- Plasma physics
  - Energy loss, Collective modes Mach cones
- Transport properties
- Hydrodynamics
- Monopoles E/M duality
- AdS/CFT duality
- Quantum mechanics
- Quasiparticles
  - Bound states of EQP and MQP
  - J/ψ, mesons, baryons, calorons
- Lattice simulations
- Quasi particles
  - Potentials
  - Correlators
- EoS
- RHIC data
- Gauge theories, SUSY models
- String theory
• **Part I (in equilibrium)**
• **RHIC findings: collective flows and jet quenching**

Why is quark-gluon plasma (sQGP) at RHIC such a good liquid? Is it related to deconfinement?
What is the role of e/m duality and magnetic objects in sQGP?


- transport summary; RHIC vs both dualities - AdS/CFT and sQGP with monopoles
- - seem to work. Are they related??? LHC will tell

- Few aps of AdS/CFT in equilibrium, T
Part II: equilibraiton in AdS/CFT

- Objects falling into black hole
- Colliding heavy quarks: AdS/CFT “Lund model”
- Scaling and nonscaling open string solutions
- **Hologramm** of a falling string as seen on the boundary, nonthermal explosion
- Many strings: beyond probe approximation: 2 dynamical membranes -- matter and horizon ones -- produce entropy
- Instead of summary - list of homework problems
Thermo and hydrodynamics: can they be used at sub-fm scale?

• Here are three people who asked this question first:
  • Fermi (1951) proposed strong interaction leading to equilibration: (predicted \( <n> \) scales as \( s^{1/4} \))
  • Pomeranchuck (1952): interaction strong till freezeout
  • Landau (1953): used hydro in between, saving Fermi’s prediction via entropy conservation {he also suggested it should work because coupling runs to strong at small distance! No asymptotic freedom in 1950’s yet, but Landau pole}
My hydro before RHIC

- Hydro for e+e- as a spherical explosion PLB 34 (1971) 509
  ➞ killed by as.freedom (1973) and (1976) discovery of jets in e+e- at SLAC
- Looking for it in pp: radial flow at ISR? ES+Zhirov, PLB (1979) 253:
  ➞ Killed by apparent absence of transverse flow in pp
  ➞ ES+Hung, 1996 (PRC57 1891), radial flow at PbPb at CERN SPS with correct freezeout surface worked!
  ➞ So we made predictions for RHIC based on that…
RHIC findings (2000-2007)

• Strong radial and elliptic flows are very well described by ideal hydro => small eta/s => "the most perfect liquid known"

• Strong jet quenching, well beyond pQCD gluon radiation rate, same for heavy charm quarks (b quark data coming…)

• Jets destroyed and their energy goes into hydrodynamical "conical flow"
Contrary to expectations of most, hydrodynamics does work at RHIC!

Elliptic flow

How does the system respond to initial spatial anisotropy?

is it macro or microscopic?

\[
\frac{dN}{p_T dp_T dy d\phi} = \frac{1}{2\pi p_T dp_T dy} \left( 1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi) + \cdots \right)
\]

\[
v_2(p_T, y) = \frac{\int d\phi \cos(2\phi) \frac{dN}{p_T dp_T dy d\phi}}{\int d\phi \frac{dN}{p_T dp_T dy d\phi}} = \langle \cos(2\phi) \rangle
\]
So it is even less than presumed
Lower bound (Son et al) $>1/4\pi$!
Why it may be possible, read Lublinsky, ES hep-ph0704.1647
One more surprise from RHIC: strong jet quenching and flow of heavy quarks

Heavy quark quenching as strong as for light gluon-q jets!

Radiative energy loss only fails to reproduce $v_2^{HF}$.

Heavy quark elliptic flow: $v_2^{HF}(p_T<2\text{GeV})$ is about the same as for all hadrons!

=>

Small relaxation time $\tau$ or diffusion coefficient $D_{HQ}$ inferred for charm.

nucl-ex/0611018

$R_{AA}$

$V_2^{HF}$

$Au+Au \@ S_{NN} = 200 \text{ GeV}$

$\pi^0 R_{AA}, p_T > 4 \text{ GeV/c}$

$\pi^0 v_2, p_T > 2 \text{ GeV/c}$

$e^\pm R_{AA}, e^\pm v_2^{HF}$

$PT [\text{GeV/c}]$
Sonic boom from quenched jets  
Casalderrey, ES, Teaney, hep-ph/0410067; H. Stocker…

- the energy deposited by jets into liquid-like strongly coupled QGP must go into conical shock waves
- We solved relativistic hydrodynamics and got the flow picture
- If there are start and end points, there are two spheres and a cone tangent to both

Wake effect or “sonic boom”
Two hydro modes can be excited (from our linearized hydro solution):

\[ \epsilon_{dt_0}(t = t_0, \vec{x}) = e_0(z, r) , \quad \gamma_{dt_0}(t = t_0, \vec{x}) = g_0(z, r) \delta^{ir} + \partial g_1(z, r) . \]

A "diffusion" and a sound.
Note: it is only projection of a cone on phi

Note 2: there is also a minimum in \( <p_t(\phi)\) at 180 degr., with a value consistent with background.

The most peripheral bin, here there is no QGP.
Electric/magnetic duality and transport in sQGP

magnetically charged Quasiparticles in sQGP (monopoles and dyons)
⇒ Completely new plasmas
⇒ Unexpected properties
New Insight: MQPs Are Important in sQGP

- 't Hooft-Mandelstam: QCD Vacuum as Magnetic Superconductor
  - Condensate of magnetic D.o.F occupies vacuum
  - Original electric D.o.F in $L_{QCD}$ gets confined.
  - Best explored on lattice (Bali; Chernodub; Suzuki; Bornyakov; ...)
- **Bottom-up**: heating up condensate $\rightarrow$ vaporization $\rightarrow$ MQPs in plasma
- **Top-down**: cooling down to $T_c$ $\rightarrow$ monopoles become light and weakly-coupled $\rightarrow$ electric coupling becomes strong.

Lesson from Seiberg-Witten Theory ($\mathcal{N} = 2$ SYM)

![Figure 3: Quantum moduli space of Seiberg-Witten Theory.](image)
New (compactified) phase diagram describing an electric-vs-magnetic competition

Dirac condition (old QED-type units $e^2 = \alpha$, deliberately no $N_c$ yet)

\[
\frac{eg}{\hbar c} = \frac{n}{2}
\]

Thus at the $e=g$ line

\[
e^2/\hbar c = g^2/\hbar c = 1
\]

Near deconfinement line $g \rightarrow 0$ in IR

$\Rightarrow e$-strong

There are $e$-flux tubes in all blue region, not only in the confined phase! In fact, they are maximally enhanced \textit{at} $T_c$
Note that the relative monopole density grows as $T \rightarrow T_c$

\[ \rho(T)/T^2 \] as a function of $T/T_c$. Data have been obtained on a $45^3 \times L_4$ lattice, with variable $L_4$ and at $\beta = 2.75$ (first 9 points), and variable $\beta$ at $L_4 = 4$ (last 10 points).

\[ g(r) = \exp(-V(r)/T) \] with $V(r)$ a Yukawa potential (see Eqs. (2.9) and (2.10)).

C.Ratti, ES: monopole masses explain this trend, they get 3-4 times lighter than Electric ones $(q,g)$. Cristoforetti, ES: mass from Bose condensation condition.
Electric and magnetic screening

Masses, Nakamura et al, 2004

My arrow shows the "self-dual" E=M point

Me\(<\)Mm
Magnetic
Dominated

At T=0 magnetic
Screening mass
Is about 2 GeV
(de Forcrand et al)
(a glueball mass)

Other data
(Karsch et al)
better show how
Me
Vanishes at Tc

Me\(\geq\)Mm
Electric
Dominated

\[
\frac{M_E}{T} = O(g) \\
ES 78 \\
\frac{M_M}{T} = O(g^2) \\
Polyakov 79
\]
So why is such plasma a good liquid? Because of magnetic-bottle trapping:

**static eDipole+MPS**

Note that Lorentz force is $O(v)$!

\[ M \frac{d^2 \vec{r}}{dt^2} = \frac{q}{c} \vec{E} \times \frac{d\vec{r}}{dt} \]

\[ \vec{E} = e \left[ \frac{\vec{r} - az}{|\vec{r} - az|^3} - \frac{\vec{r} + az}{|\vec{r} + az|^3} \right] \]

Monopole rotates around the electric field line, bouncing off both charges (whatever the sign)
We found that two charges play ping-pong by a monopole without even moving!

Chaotic, regular and escape trajectories for a monopole, all different in initial condition by 1/1000 only!

Dual to Budker’s magnetic bottle
MQP in the field of a cube with alternating charges at corners.

Another example: a monopole in a “grain of salt”
Liao and ES, in progress
escape time $\Gamma^{(.5-.6)}$
short transport summary

log(inverse viscosity $s/\eta$) vs. log(inverse heavy $q$ diffusion const $D^*2\pi T$) (avoids messy discussion of couplings)

$\rightarrow$ Stronger coupled $\rightarrow$

• RHIC data: very small viscosity and diffusion
• vs theory - AdS/CFT and our MD

50-50% E/M is the most ideal liquid

Weak coupling end $\Rightarrow$
(Perturbative results shown here)
Both related to mean free path
From RHIC to LHC:
(no answers, only 1bn$ questions)
(I don’t mean the price of LHC but ALICE and the rest of heavy ion program)

- Will "perfect liquid" be still there?
- Is jet quenching as strong, especially for c, b quark jets and much larger pt?
- Is matter response (conical flow at Mach angle) similar? (This is most sensitive to viscosity...
From SPS to LHC

- lifetime of QGP phase nearly doubles, but $v2$ grows only a little, to a universal value corresponding to EoS $p=(1/3) \epsilon$
- radial flow grows by about 20% => less mixed / hadronic phase (only 33% increase in collision numbers of hadronic phase in spite of larger multiplicity)

(hydro above from S.Bass)
AdS/CFT duality
from gravity in AdS$_5$ to strongly coupled CFT (N=4 SYM) plasma

what LHC people dream about
-- a black hole formation --

does happen, in each and every RHIC AuAu event!

What we see at RHIC is a hologramm of this process...
The first gauge-string duality found in 1997

- AdS/CFT correspondence known as "Maldacena duality"
- Along the long path illuminated by Witten, Polyakov, Klebanov…
The duality setting

- CFT (conformal gauge theory) N=4 SYM a cousin of QCD (chromodynamics=theory of strong interaction) in which the coupling $\lambda = g^2 N_c$ does not run.
- It lives on flat 4-dim boundary of 5-d curved AdS (anti-de-Sitter) space where (super)gravity is a description of (super) string theory.
- Correspondence dictionary: everything in the “bulk” reflects on the boundary.
- Hint; think of extra dimension as a complex variable trick: instead of functions on the real axes one may think of poles in a complex plane …
The 5th coordinate

- $z$ is the 5th coordinate, $\text{dim}=\text{length}=1/\text{momentum}$
- its physical meaning is "scale" as in renorm.group
- $z \to 0$ is "high scale" UV or very high energies,
  $z \to \infty$ is low scale or IR
- $ds^2 = (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dz^2)/z^2$ so distances in $z$
  are logarithmic. Light speed is still 1 in all directions
- $z = L^2/r$ where $r$ is distance from b.h. =>
  Gravity force is acting toward large $z$, "stones" fall there
- (unless they are BPS states which levitate --Newton cancels Coulomb)
Maldacena’s dipole
strong coupled Coulomb law

• Maldacena, Rey, Yee -98 one of the first apps:
  • The pending string (=flux tube) has minimal action
  • **Modified Coulomb law** at strong coupling, sqrt of the coupling << coupling
  • Can it be just a factor, like dielectric constant?

\[ E = -\frac{4\pi^2 (g_{YM}^2 N)^{\frac{1}{2}}}{\Gamma (\frac{1}{4})^4 L} \]
A hologramm of a dipole in a strongly coupled vacuum: not just electric E!


Here is large r behavior:

- \( T_{00} \rightarrow (g^2N)^{1/2}d^3/r^7 \)

Times function of the Angle which is plotted by a solid line.

(to be compared to zero coupling)

\( \rightarrow (g^2N)d^2/r^6 \)

Times another function (dashed).

Figure 1: (Color online) The far field energy distribution in polar angle \( \theta(\cos(\theta) = y_1/y_1) \), normalized at zero angle. Solid (black) line is our result, compared to the perturbative result \((3\cos^2 + 1)/4\) given by the dashed (blue) line.
Finite T AdS/CFT  
(Witten 98)

viscosity from Kubo formula $<T_{ij}(x)T_{ij}(y)>$
(Polykastro, Son, Starinets 03)

Horisontal line is our 4d Universe, (x,y are on it)
- Temperature is given by position of a horizon $z_h$ of non-extreme BH
- $T = T_{\text{Hawking}}$
- **Correlator is just the graviton propagator**
  - Blue graviton path does not contribute to $\text{Im } G$, but red graviton can be lost

The answer is so simple because of boundary condition (universal “black membrane”) at the horizon

Sound is a hologram of a wave on the bottom

$$\frac{\eta}{s} = \frac{\hbar \alpha}{4\pi}$$
Heavy quark diffusion


\[ \kappa_T = \sqrt{\gamma \lambda T^3 \pi} \]

string solution spans the full Kruskal plane and gives access to contour correlations. The diffusion coefficient is \( D = 2/\sqrt{\lambda T} \) and is therefore parametrically smaller than momentum diffusion, \( \eta/(e + p) = 1/4\pi T \). The quark mass must be much greater than \( T\sqrt{\lambda} \) in order to treat the quark as a heavy quasi-particle. The result is discussed in the context of the RHIC experiments.

One quark (fisherman) is
In our world,
The other (fish) in
Antiworld (=conj.amplitude)
String connects them and conduct waves in one direction through the black hole
Heavy quark in CFT plasma has a string deformed by "hot wind"

Herzog, Yaffe, Gubser... May 06

calculated the
drag force = momentum
Flow down the string
Einstein's relation between drag and diffusion works
But how gravity knows?

\[ \eta_D = \frac{T}{M_c D_c} \]

Figure 1: The AdS5-Schwarzschild background is part of the near-extremal D3-brane, which encodes a thermal state of \( \mathcal{N} = 4 \) supersymmetric gauge theory [25]. The external quark trails a string into the five-dimensional bulk, representing color fields sourced by the quark's fundamental charge and interacting with the thermal medium.
Hologramm from P. Chesler, L. Yaffe (also Gubser et al have detailed papers On this)

Both groups made amazingly detailed Description of the conical flow from AdS/CFT.>

Note that it is not hydro but a full solution: the shape of the wave is correct Even at micro scales
FIG. 1: Plot of $|x|E(x)/(T^3 \sqrt{\lambda})$ for $v = 1/4$, with the zero temperature and near zone (20) contributions removed. Note the absence of structure in the region $|x| \gg 1/\pi T$.

FIG. 2: Plot of $|x|E(x)/(T^3 \sqrt{\lambda})$ for $v = 3/4$, with the $T=0$ and near zone (20) contributions removed. A Mach cone is clearly visible, with an opening half-angle $\theta \approx 50^\circ$.

Left: P.Chesler, L.Yaffe
Up- from Gubser et al

Both groups made
Amasingly detailed
Description of the conical flow from
AdS/ CFT => not much is diffused
Part II

Non-equilibrium physics in AdS/CFT setting
Gravity dual for the heavy ion collisions

- **T=0 AdS metric** corresponds to extreme BH (mass is minimal for its charge => no horizon)
- As collision creates “debris”, they fall, add extra mass and form a **non-extreme BH with a horizon** => **T**
- **Advantages: naturally dissipative+ classical**
  - Expanding/cooling fireball= departing b.h. horizon,
  - Different geometries: 1+d, d=1,2,3 collapses
    Sin,ES and Zahed 04,
    - BH is longitudinally stretching 1+1 - rapidity independent example Janik-Peschanski 05 proposed **late-time** solution
  - 1+3 approaching/departing BH without entropy change Gubser et al, 06
Gravity dual to the (heavy quark) collision: “Lund model” in AdS/CFT
(Shu Lin, ES, I+II papers)

If colliding objects are made of heavy quarks

- Stretching strings -- are falling under the AdS gravity
- Strings are flux tubes, they don’t break

AdS$_5$ center = Extremal b.h.
Toward the AdS/CFT gravity dual for High Energy Collisions:
I. Falling into the AdS

Shu Lin and Edward Shuryak

Department of Physics and Astronomy, Stony Brook University, Stony Brook NY 11794-3800, USA

- EOM and solutions for various objects falling in AdS: a "stone", closed circular string, 3d membrane
- Falling open string, with ends fixed $x=(+/-)vt$
- Analytic scaling solution $z=\tau f(y)$, only exist till $v<1/2$ and remains stable till $v<.27$ or so
- Numerical solutions: near free fall in the middle
Scaling solution is analytic, but we found it gets unstable at endpoint rapidity $Y > .27$!

Looking for classical stability: Lyapunov exponents

Action at small $v$ gives "AdS/CFT Ampere law"

Which instability in the hologramm does it correspond to? Is it generic or present for this solution only?
Non-scaling solution at large $y$ studied numerically: it develops cusps

- There is a well seen fragmentation and nearly-free falling central parts
- Stress self-focuses at the ``corners'' => cusps

*FIG. 8: The dynamics of the string(half) $g(\tau, y)$ with $y = 0.6$. The profiles from the innermost to the outermost correspond to $\tau = 1$(solid red), $\tau = 2$(dotted blue), $\tau = 4$(dashed green), $\tau = 8$(dot-dashed black).*
• What observer on the boundary sees is a "holographic image" of this process,
• => can be calculated using time-dependent Green function for linearized Einstein eqns, as in examples above

How does it look for a falling string?
Is it hydro-like explosion or not?
Holographic image of a falling string shows **an explosion**

(as far as we know the first time-dependent hologram)

Which however cannot be represented as hydro fluid => anisotropic pressure in the "comoving frame"
Many strings falling together

- Imagine 2 walls of heavy quarks ⇒ multiple strings falling ⇒ no dependence on transverse coordinates $x_2, x_3$
- The falling object is thus not a string but 3d membrane-like, to be called $M_m$ (matter or string membrane)
- (Are there instabilities or other dissipative phenomena in it, creating entropy?)
Including gravity of debries:

=> another (more famous) membrane $M_h$, (of the "membrane paradigm") is hovering just above the horizon

- Horizon not only has Hawking T and Bekenstein S, but many other universal properties
- T.Damour (1978..1982) introduced electric conductivity, shear and bulk viscosity
- K.Thorne et al (1980s) put it in the form in which many astrophysical problems were solved
  - (e.g. planets rotating around and plunging into B.H., accretion discs with magnetic and electric fields, thermal atmospheres etc)
How falling strings got equilibrated?

• Solid line is a falling "matter membrane" $M_m$, its ends have $+/\pm v$
• Dashed lines are horizon membrane $M_h$ which bulge upward due to gravity of $M_m$
• One possible simplification => flat falling membranes

Before equilibration

Curvature jumps => Israel junction condition

After part of $M_m$ gets
Substituted by $M_h$, observer at $z=0$
sees hydro and near-thermal $T_{\mu\nu}$
The membrane which is longitudinally stretched is being contracted, **as soap film**

- $M_h$ is 3-dimensional in 5d, if stretched longitudinally a la Hubble $x=vt$, it moves in $z=O(t^{1/3})=1/r$ so $A=4S=O(x r^3)=\text{const}$ (Janik et al)
- In next order in time there is dissipation induced by the $M_h$ viscosity
Before conclusions, homework problems

• Are there (analogs of filamentation etc?) instabilities for falling strings? Unruh T?
• If so, what part of Bekenstein entropy do they create?
• Calculate the other part, created by viscosity of the horizon membrane
• Using AdS/CFT, answer the dilemma of top-down vs down-up cascades (collisional vs bremmstrahlung equilibration)
Conclusions

- Strongly coupled QGP is produced at RHIC T=(1-2)Tc
- This is the region where transition from magnetic to electric dominance happen
- at T<1.4 Tc of magnetic dominance => E-flux tubes
- Good liquid because of magnetic-bottle trapping the lowest viscosity for 50-50% electric/magnetic plasma

- AdS/CFT => natural applications to finite-T nonconfining and Strongly coupled, sQGP
- RHIC data on transport (eta,D), ADS/CFT and classical MD all qualitatively agree!
- Are these two pictures related? LHC

Non-equilibrium
AdS/CFT has
Advantages:
(i) classical
(ii) t-odd dissipative boundary cond. At horizon
(iii) 30 years or work on grav. collapse

Holograms of 2 membranes...