QCD plasma instability and thermalization

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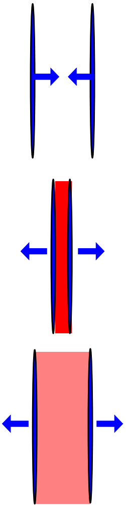
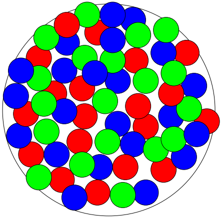
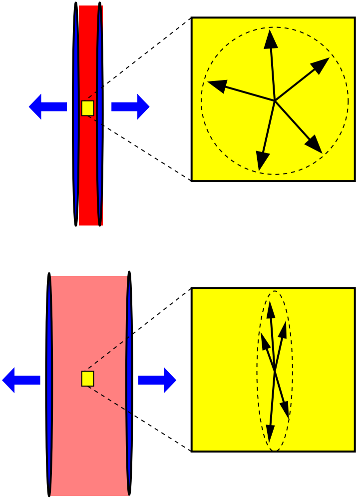
Stages of heavy ion collision:





- **1 TeV/A,** $\gamma \sim 1000$
- $\tau < 0$: initial state:
  Color Glass Condensate (CGC),
  with characteristic momentum scale:
  saturation scale $Q_s \gtrsim$ few GeV
- $\tau \sim 0.1$ fm: "melting" of CGC; exitations with $p \sim Q_s$
  – anisotropic & non-thermal initial distribution!
- $\tau \lesssim 1$ fm: Very rapid isotropization & thermalization
  (observed at RHIC) (topic of this talk!)
- $1 \lesssim \tau \lesssim 10$ fm: Expansion of $\sim$ thermal quark-gluon
  plasma (QGP)
- $\tau \sim 10$ fm: hadronisation

Heavy-Ion collisions & hard modes



- $\tau \lesssim 1/Q_s$: In initial stages of HIC the "plasma" consists of hard ($p_{\rm hard} \sim Q_s$) modes.
- $\tau \gg 1/Q_s$: As the system expands, the hard mode distribution becomes *dilute* (perturbative),

  $$n_{\rm hard} \ll p^3_{\rm hard}/g^2,$$

  and it becomes squeezed along z-direction (free streaming).

  [Baier, Mueller, Schiff, Son]
- Dilute & $Q_s \gg \Lambda_{\rm QCD} \rightarrow$ hard modes behave like on-shell classical particles.

Rapid thermalization

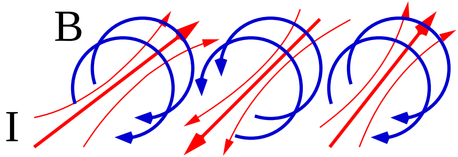
What turns the very non-thermal hard mode distribution to $\sim$ thermal (isotropic) so quickly?

- Bottom-up thermalization: hard-hard collisions [Baier, Mueller, Schiff, Son, ...]
  - Achieve isotropization in $\tau \sim \alpha_s^{-13/5}/Q_s$: 2–4 fm?
- Plasma instabilities:
  - Well-known in electrodynamics (non-trivial current distributions)
  - Can happen in QCD too:
    non-isotropic hard mode distribution
    $\rightarrow$ exponential growth of soft modes ($p \ll Q_s$), *plasma instability*
    $\rightarrow$ strong back reaction to hard modes
    $\rightarrow$ thermalization

    [Mrówczyński; Mrówczyński, Strickland; Arnold, Lenaghan, Moore; Romatschke, Strickland; ...]
  - Parametrically (in g) faster than collisions above

Weibel instability

- In electromagnetic plasmas, anisotropic distribution of current carrier distribution (electrons) which leads to *Weibel (filamentary) instability*:



⇒ Exponential growth of soft magnetic fields; $p_{\rm soft} \ll p_{\rm electron}$. In QED the growth rate can be solved analytically as a function of the anisotropy.

⇒ When magnetic field amplitude is large, $gA_{\rm soft} \sim k_{\rm electron}$, field bends electrons strongly $\rightarrow$ isotropization, thermalization?

- Should play a role in heavy ion collisions too? [Mrówczyński; Arnold, Lenaghan, Moore; Strickland]

Weibel instability in HICs

- QED $\rightarrow$ QCD:
  electrons $\rightarrow$ hard gluons
  soft electromagnetic field $\rightarrow$ soft gluons
- Small-amplitude soft fields ($f_{\rm soft} \ll g^2$): the growth rate can be solved analytically; essentially QED (non-abelian commutators can be neglected)

$\Rightarrow$ exponential growth of soft fields, with characteristic $k_{\rm soft} \sim k^*$

- What happens when magnitude of the soft fields reach the "non-abelian limit" $gA_{\rm soft} \sim k^*$ (or $f_{\rm soft} \sim g^2$)?
  - Continued growth until $gA_{\rm soft} \sim p_{\rm hard}$ (as in QED), leading to efficient isotropization?
  - Just stops? Not so efficient
  - Something else?
- Continued growth may be possible if the fields 'Abelianise', i.e. only one colour component grows. [Arnold, Lenaghan, Moore]
- Special lattice simulations needed.

How to study the system?

- Soft fields: non-perturbative, large occupation numbers ($f_{\rm soft} \gg g^2$): $\sim$ classical evolution
- Hard modes: dilute, weakly coupled $\sim$ classical particles

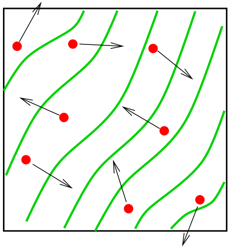
$\Rightarrow$

A) Classical pure gauge field evolution
[Romatschke,Venugopalan; Berges,Scheffler,Sexty]

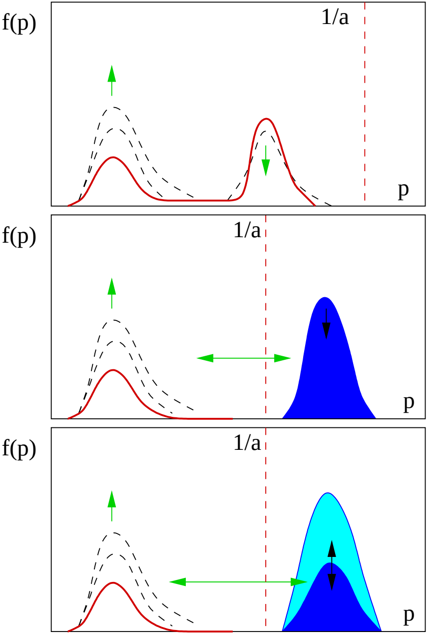
B) System with hard "classical" particles + soft non-perturbative gauge fields ("HTL" theory)

B1) Real particles
[Dumitru,Nara,Strickland]

B2) Particle distribution functions, "W" -fields
[Arnold,Moore,Yaffe; Rebhan,Romatschke,Strickland; Bödeker,KR]
Fixed anisotropic background distribution + fluctuations (W)



- “Classical gauge”:
  - All scales need to fit: large lattices
  - No overcounting
  - Feedback hard↔soft, full isotropization possible
  - Total energy conserved
- “Particles”:
  - Separation of scales
  - Feedback hard↔soft
  - Total energy
  - overcounting?
- “W-fields”:
  - Static anisotropic background + dynamic fluctuations
  - ⇒ Full isotropization not possible
  - Separation of scales
  - Technically “clean”



Hard Thermal Loop effective theory

Hard modes behave as on-shell particles moving in soft background fields, with a distribution function

$$f_{\rm hard}(x,\vec{p}) = \bar{f}(\vec{p}) + \lambda^a f^a(x,\vec{p}) + \ldots$$

where the singlet $\bar{f}(\vec{p})$ is constant in space and time, and is anisotropic.

Yang-Mills-Vlasov equations of motion:

$$(D_\mu F^{\mu\nu})^a = J^{a,\nu}_{\rm hard} = g \int_{\vec{p}} v^\nu f^a$$

$$(v \cdot Df)^a + g v^\mu F^a_{\mu i} \frac{\partial \bar{f}}{\partial p^i} = 0$$

where $v = (1, \vec{p}/p)$. Defining W-function

$$W^a(x,\vec{v}) \equiv 4\pi g \int_0^\infty \frac{dp p^2}{(2\pi)^3} f^a(x,\vec{p})$$

we can integrate EQM over $|p|$, obtaining ...

Hard Thermal Loop effective theory

Yang-Mills-Vlasov EQM:

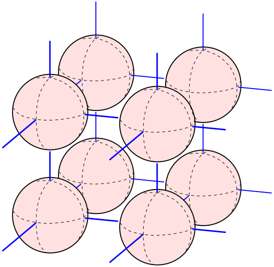
$$
\begin{aligned}
(D_\mu F^{\mu\nu})^a &= \int \frac{d\Omega_{\vec{v}}}{4\pi} v^\nu W^a \\
(v \cdot DW)^a &= m_0^2\, v^\mu F^a_{\mu i} U^i(\vec{v})
\end{aligned}
$$

where $U^i(\vec{v})$ characterises the anisotropic $\bar{f}$:

$$
m_0^2 U^i(\vec{v}) = -4\pi g^2 \int_0^\infty \frac{dpp^2}{(2\pi)^3} \frac{\partial \bar{f}(p\vec{v})}{\partial p^i}
$$

For isotropic $\bar{f}$ we have $\vec{U} = \vec{v}$, and $m_0 = m_{\rm Debye}$. m_0 is the only dimensionful parameter.

Lattice simulations



- The hard mode distribution is modelled with $W^a(x, \vec{v})$ fields. These are expensive: live on $R^3 \times S^2$:
- $\vec{v}$ dependence modelled in 2 ways:
  - expansion in spherical harmonics [Bödeker,Moore,K.R.; Arnold,Moore,Yaffe; Bödeker, K.R.]
  - sample discrete directions [Rebhan,Romatschke,Strickland]

- We use spherical harmonic "W-fields", SU(2) gauge group
- We use similar techniques than Arnold, Moore, Yaffe, but with
  - 5 different values for the anisotropy, both weaker and much stronger than AMY
  - Large lattices (up to 240^3), with a large number of auxiliary W-fields (up to $L_{\max} = 48$, i.e. 14250 auxiliary fields in addition to A^a_μ).

On the lattice:

- We expand W, $\bar{f}$ in spherical harmonics:

$$\begin{aligned} W^a(x,\vec{v}) &= W^a_{\ell m} Y_{\ell m}(\vec{v}), \\ \bar{f}(\vec{p}) &= \bar{f}^a_{\ell m}(p) Y_{\ell m}(\vec{v}), \end{aligned}$$

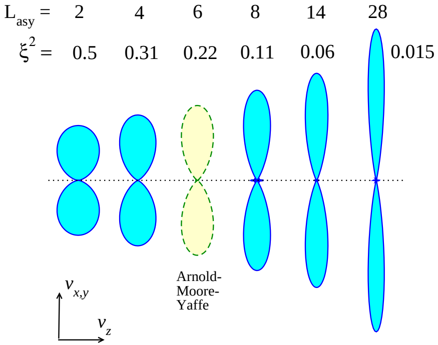
  where $\ell = 0 \ldots L_{\max}$.
- We use $A_0 = 0$ gauge
- The dynamical lattice fields are $U_i \in$SU(2), E^a_i, $W^a_{\ell m}$
- m_0 dimensionful; lattice spacing given by am_0.
- 4 lattice "cutoff" artifacts:
  - finite lattice spacing $a \to 0$
  - finite volume $L^3 \to \infty^3$
  - finite $L_{\max} \to \infty$
  - finite timestep $\delta t \to 0$

Anisotropic hard mode distributions

We parametrise the anisotropic hard mode distributions by expanding in spherical harmonics:

$$\bar{f} = \sum_{\ell=0}^{L_{\rm asym}} f_{\ell 0} Y_{\ell 0},$$

with $L_{\rm asym} = 2 \ldots 28$. For each $L_{\rm asym}$ we try to maximally localise the distribution along xy-plane:

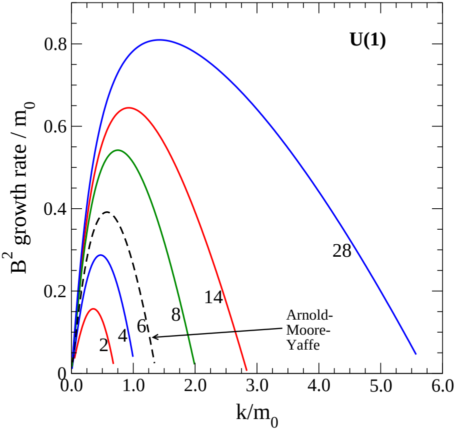


- "Propellor"-shaped distributions
- Asymmetry parameter

$$\xi^2 \equiv \frac{\langle v_z^2 \rangle}{\langle v_\perp^2 \rangle} = 0.5 \ldots 0.015$$

- Naturally $L_{\rm asym} < L_{\rm max}$

Growth rate in U(1) (weak field)



- Growth rate as a function of k
- Much wider range of diverging wave vectors at large asymmetry (large $L_{\max}$)

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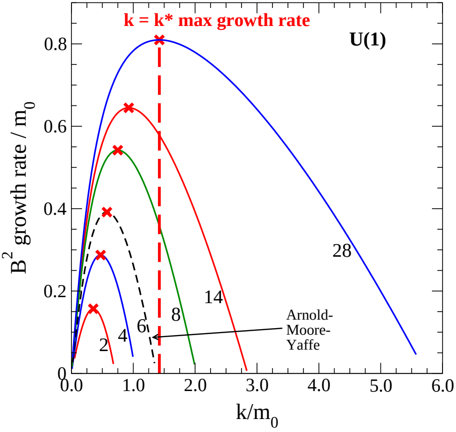
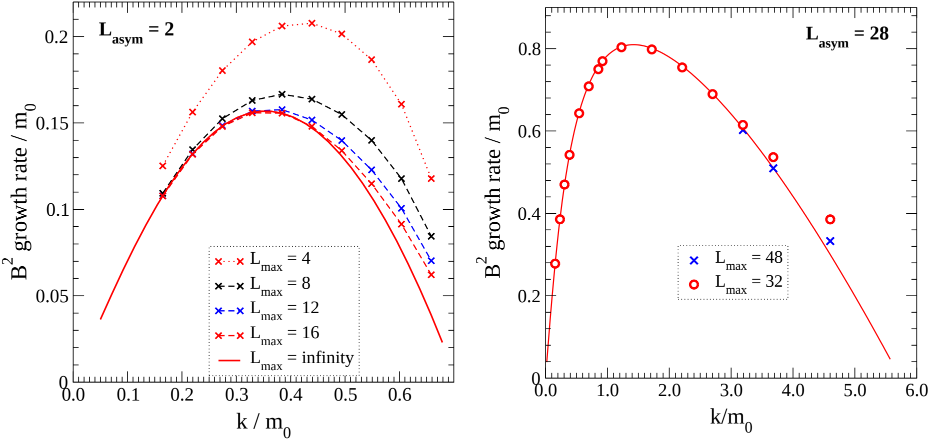


- Growth rate as a function of k
- Much wider range of diverging wave vectors at large asymmetry (large $L_{\max}$)
- Max growth rate varies from $\sim 0.15 \ldots 0.8/m_0$
- Location of maximal growth $k^* \sim m_0$.

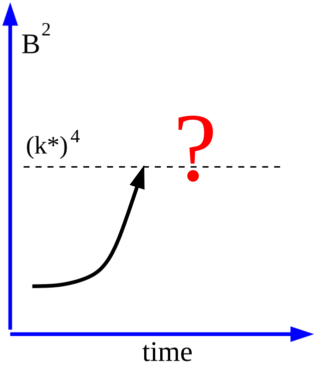
$L_{\max}$ dependence (U(1) or weak field)



$L_{\max}$ cutoff effects small in practice!

What we observe:

Small initial fields, exponential (analytically solvable) growth at a wave vector $k^* \ll p_{\rm hard}$. What happens when $gA_{k^*} \sim k^*$, or $B^2 \sim k^{*4}/g^2$?



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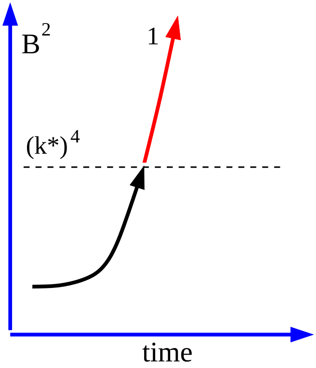

1. Abelianisation and continued exponential growth at $k \sim k^*$ [Arnold,Lenaghan,Moore]
   - **Not seen in QCD**; QED OK

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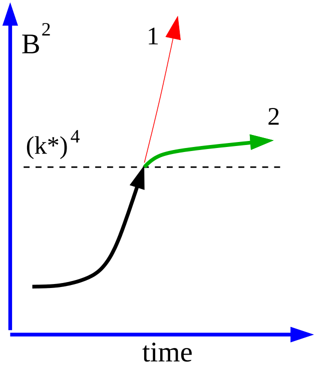

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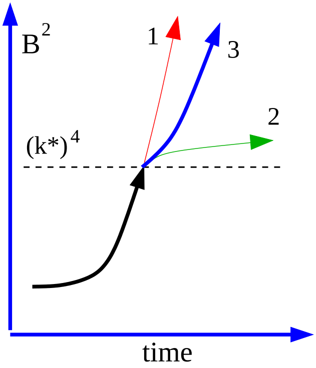

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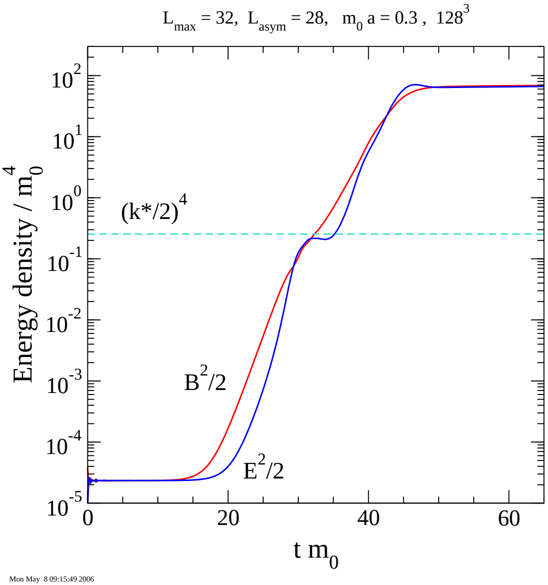
   - Weak to moderate anisotropy [Arnold, Moore, Yaffe]
3. Growth of A_{k^*} stops, rapid avalanche to UV with $\sim$ exponential growth of energy
   - We observe this at strong anisotropy
   - almost full saturation of lattice modes $\Rightarrow$ direct thermalization?

Generic growth of energy:



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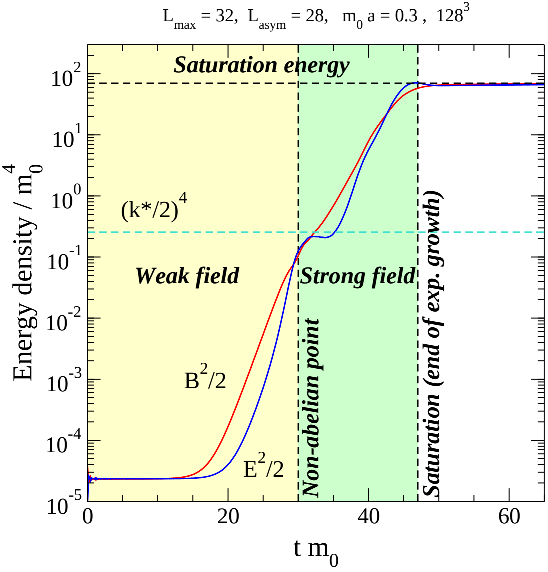

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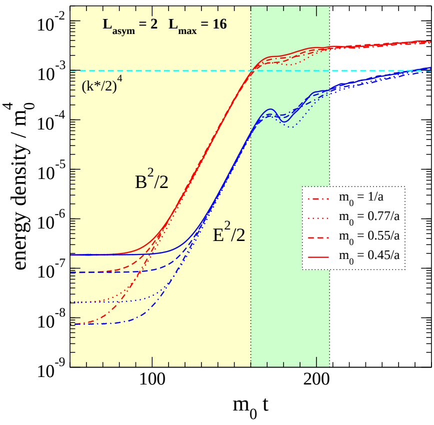
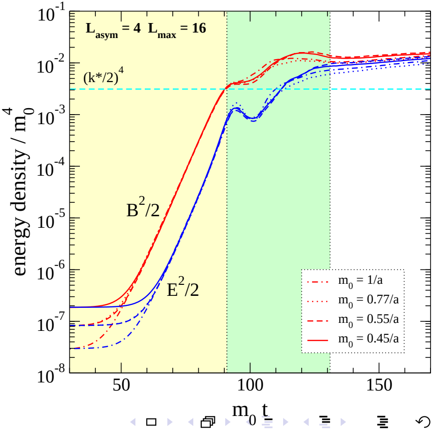
Results: growth of energy with small anisotropy

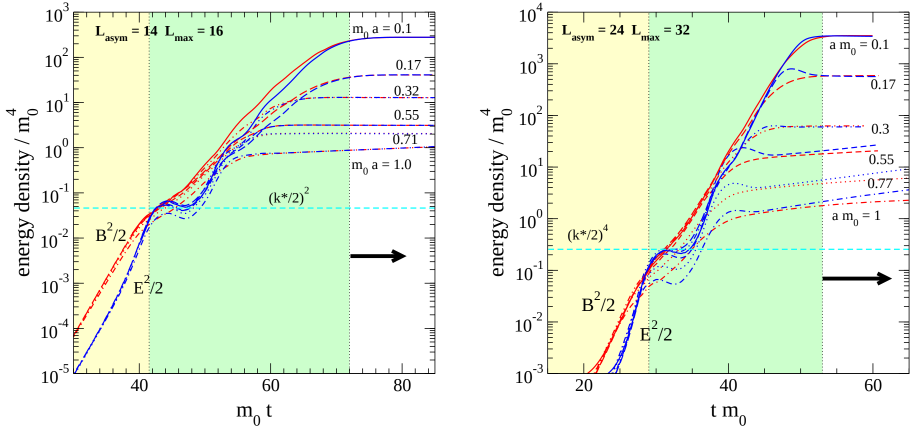
- Little growth seen beyond the weak field region at $L_{\max} = 2, 4$
- lattice UV modes far from saturated
- very small lattice spacing dependence
- agrees with Arnold, Moore, Yaffe ($L_{\max} = 6$)





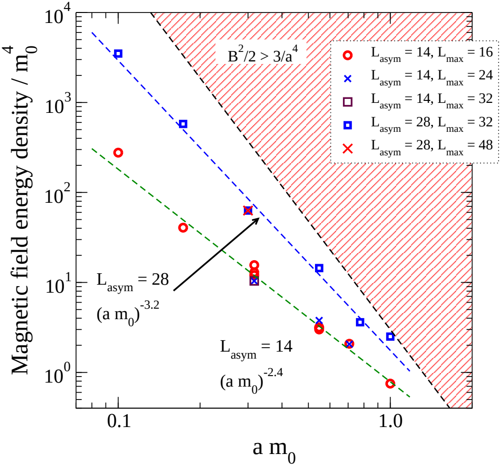
Results: growth of energy with large anisotropy

- Continued exponential growth in strong field region at $L_{\max} = 14, 28$
- stops when lattice UV modes saturate: a dependence
- How far does it continue when $a \to 0$?



Results: growth of the saturation scale

Magnetic field energy density ($\frac{1}{2}B^2$) when the exponential growth stops:



- Both for $L_{\mathrm{asym}} = 14, 28$ the scale grows with a power of lattice spacing a

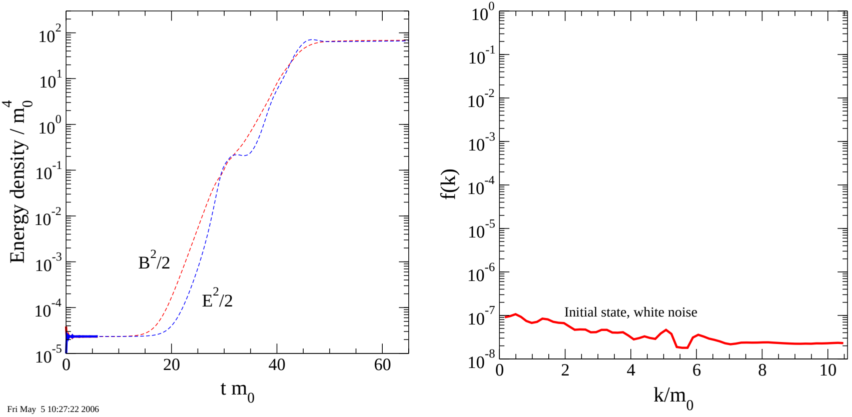
$\Rightarrow$ Growth regulated by a

$\Rightarrow$ Exponential avalanche to far UV in the continuum limit

$\Rightarrow$ Thermalization?

Results: gauge field spectrum

Fixing to Coulomb gauge, $f_k \propto k\langle A_k^2 \rangle$



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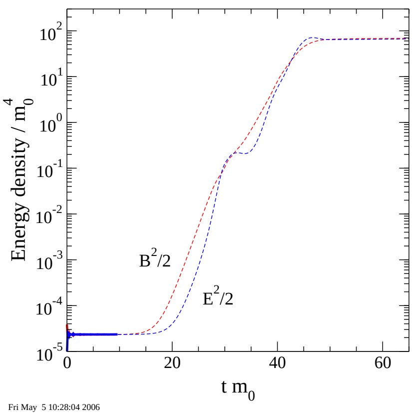

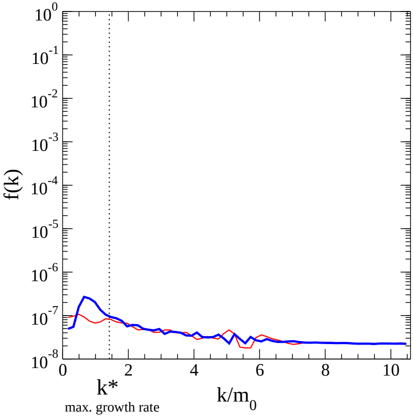

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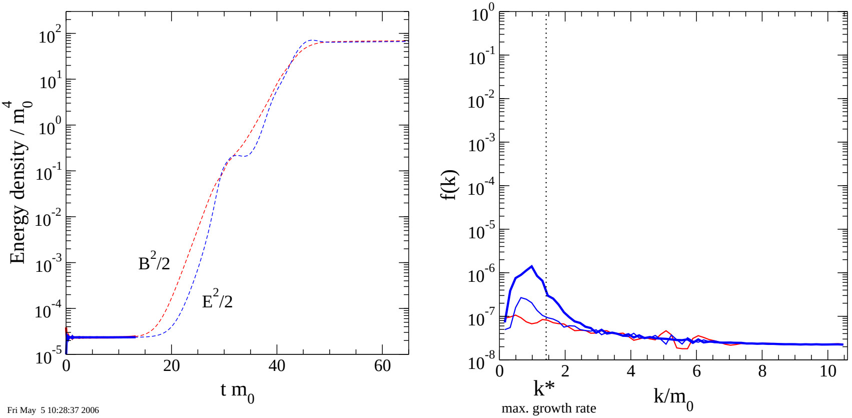

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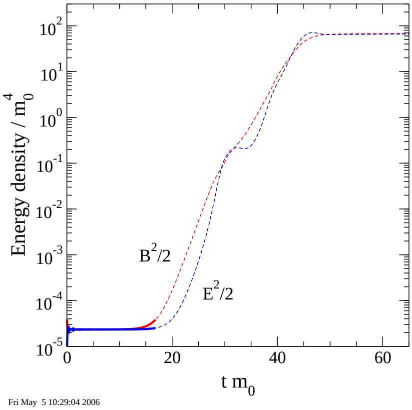

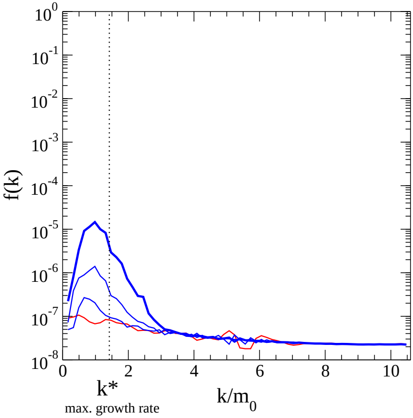

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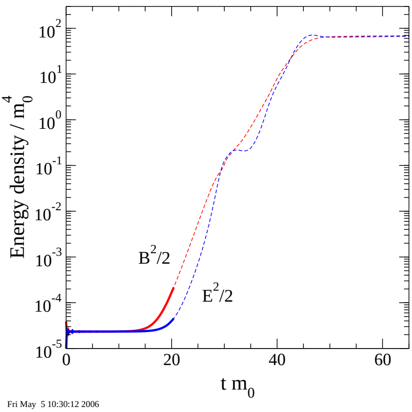

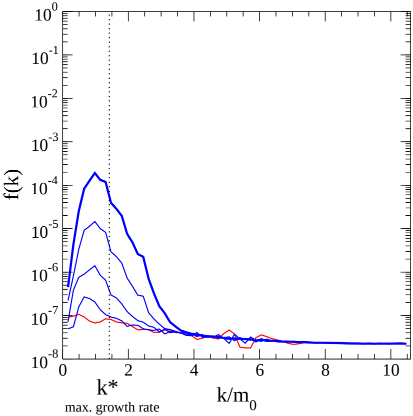

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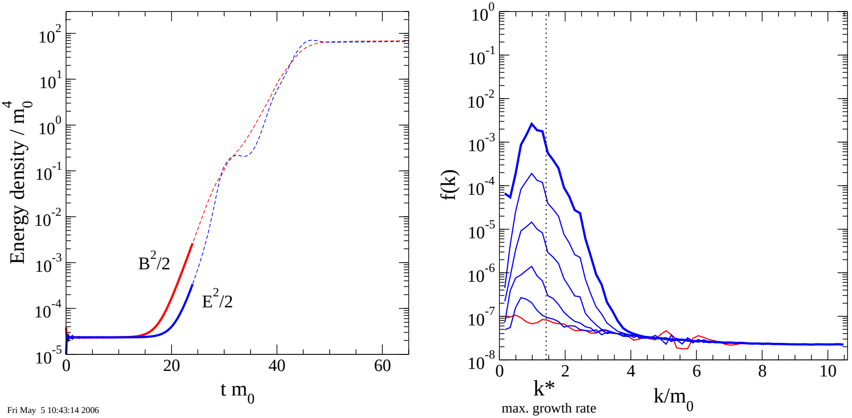

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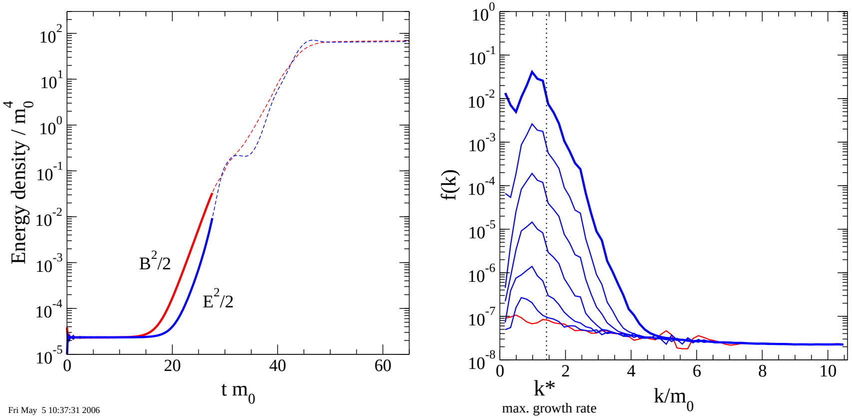

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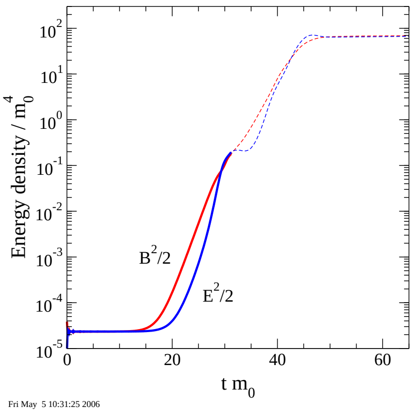

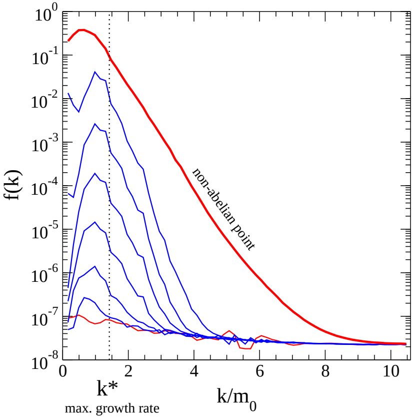

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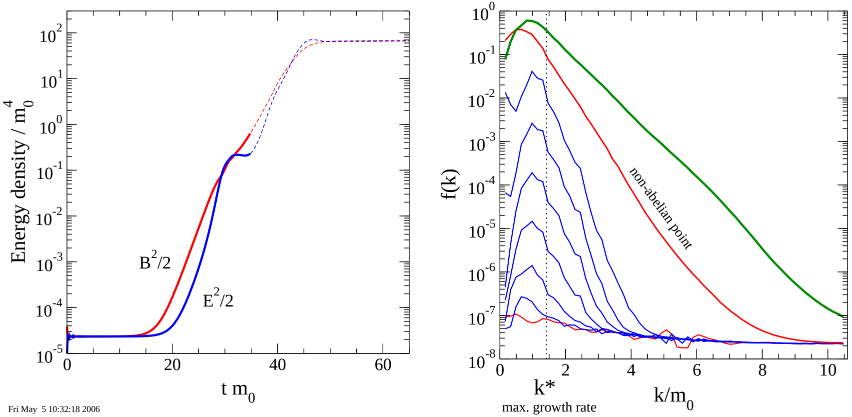

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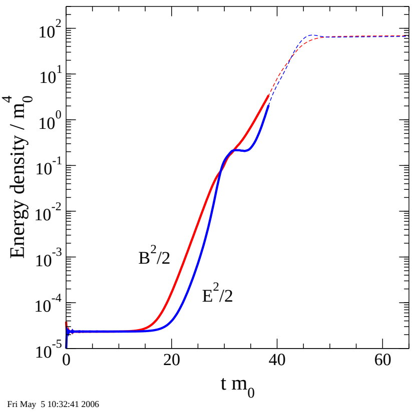

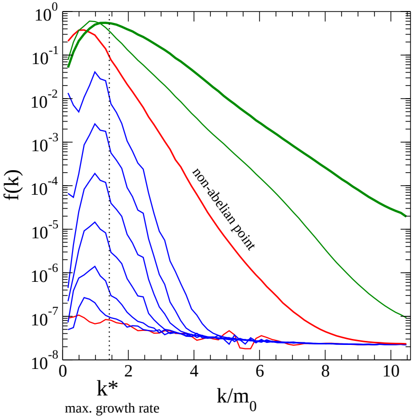

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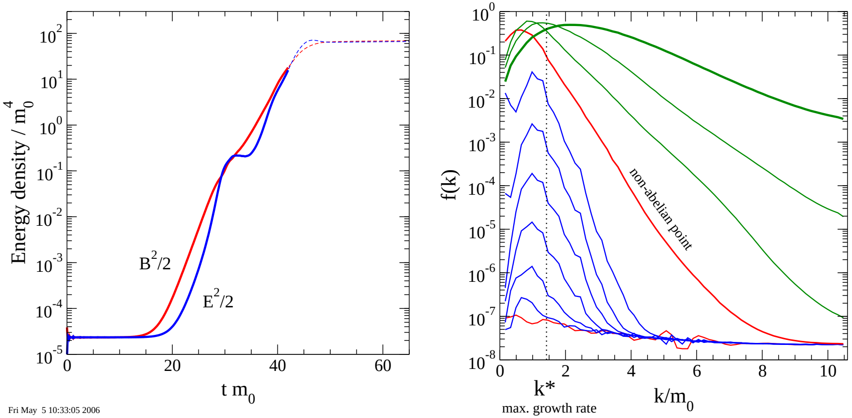

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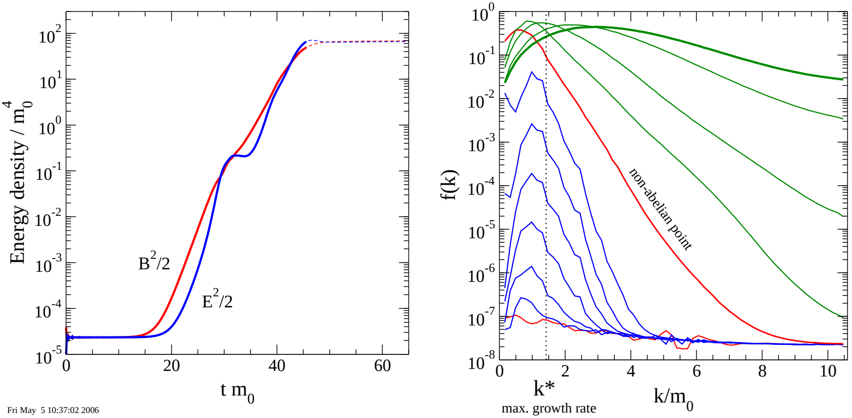

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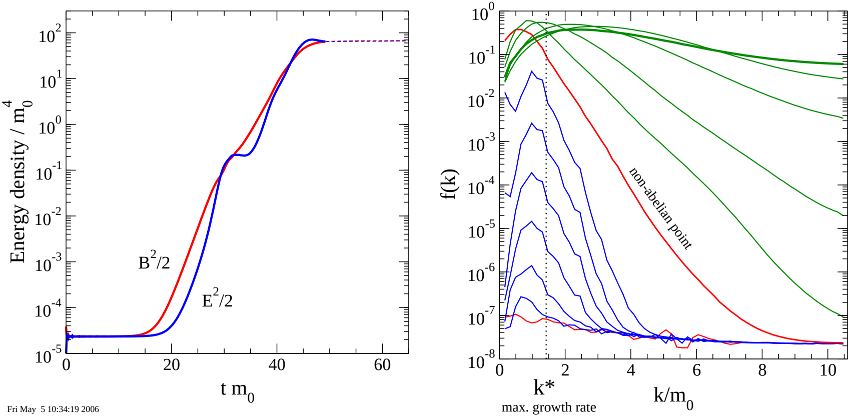

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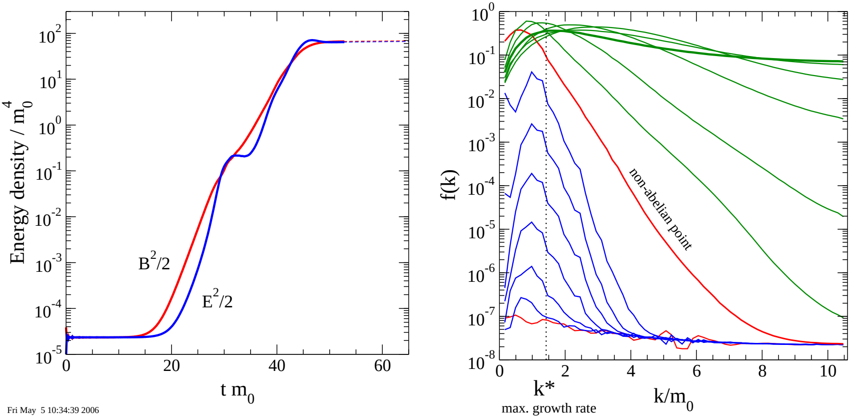

Results: gauge field spectrum

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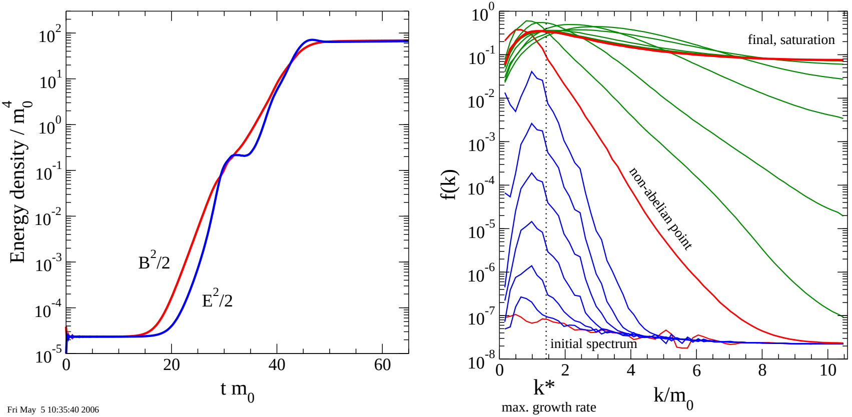
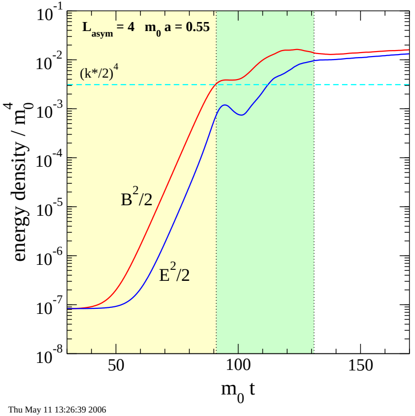
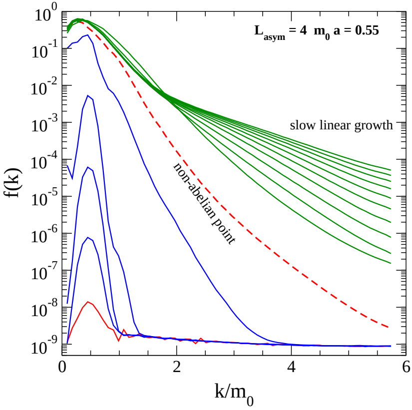


The final spectrum is $\sim$ thermal ($f_k \propto 1/k$)

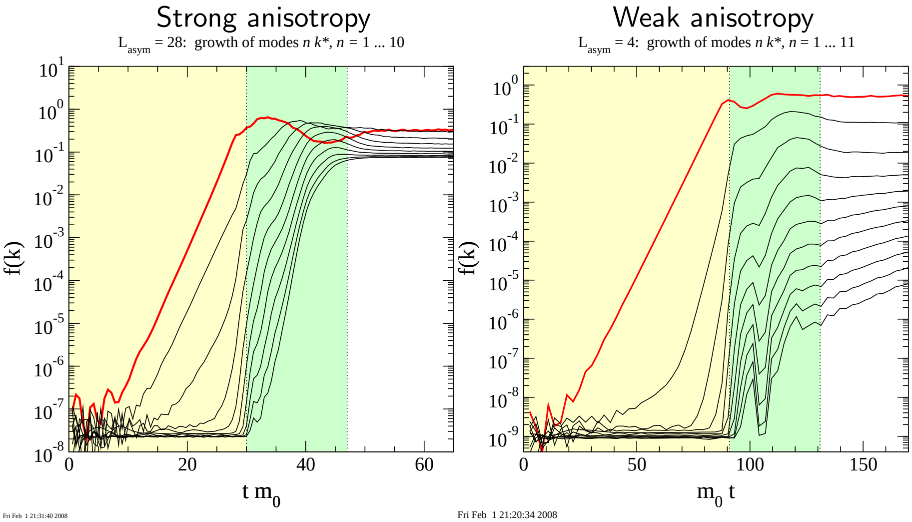
Small anisotropy remains IR dominated

- Exponential growth stops without full UV saturation.
- Slow ∼ linear growth





Growth of individual modes



(See also [Berges,Scheffler,Sexty])

Why UV modes grow so rapidly?

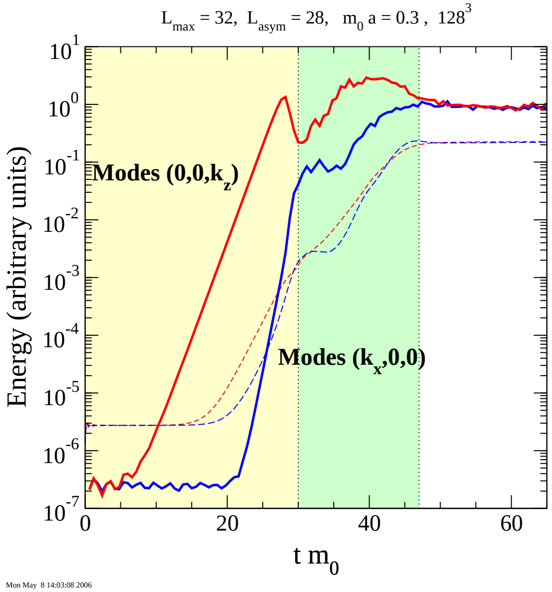
Shape of the spectrum:

- Spectrum looks like $A_k \sim e^{-\alpha k}$ in the "Strong field" domain. At $k \gg k^*$, growth caused by non-linear (commutator) terms in EQM
  $\Rightarrow \partial_i A \sim \partial_0 A \sim g A^2$
  $\Rightarrow k A_k \sim \partial_0 A_k \sim g \int_{k'} A_{k'} A_{k-k'} \approx g (A_{k/2})^2$
  $\Rightarrow A_k \sim e^{-\alpha k (t_f - t)}$,
  where $t < t_f$ and $\alpha = O(1)$.
- Exponential shape, growth rate $\propto k$. $\sim$ OK.

What powers the non-linear exponential growth?

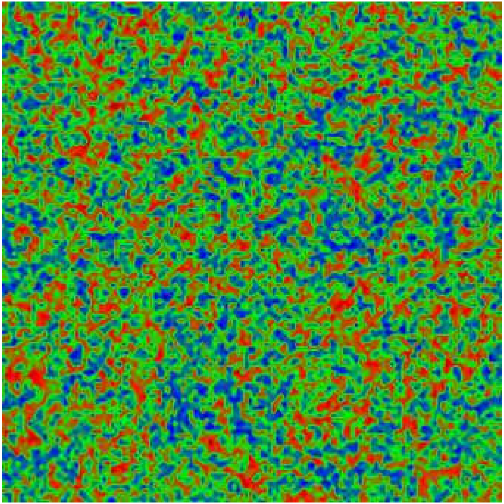
- Exponential flow of energy from hard modes to soft fields $\Rightarrow$ some kind of instability must still be active.
- Not like the linear (Weibel) instability! Different characteristics, mechanism unknown.
- Gauge fixing artifacts? Checked with gauge-invariant measurements (e.g. cooling).

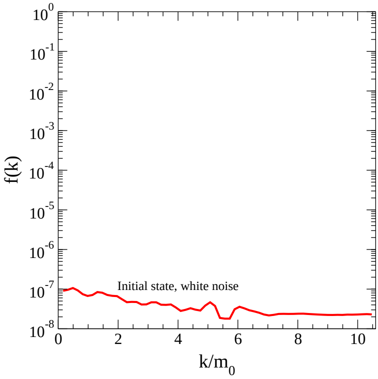
Results: isotropization



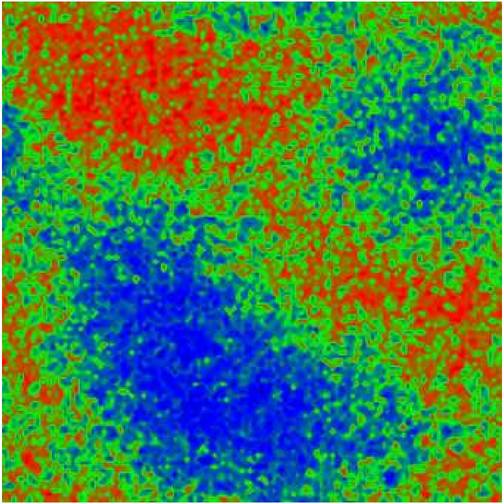
Soft fields become nearly isotropic when entering the "Strong field" domain; fully isotropic after UV saturation.

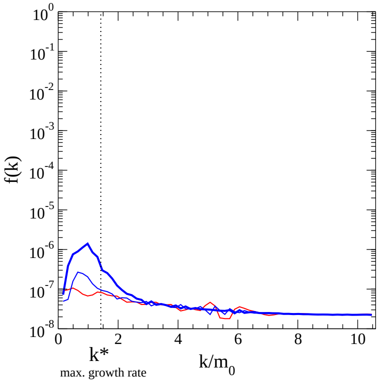
B^a (1 color component) along $\perp$-plane





B^a (1 color component) along ⊥-plane





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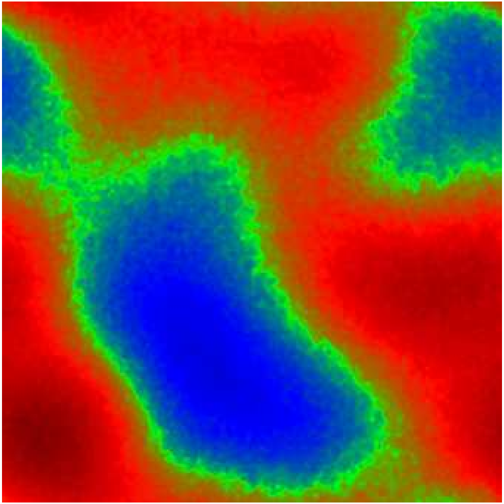

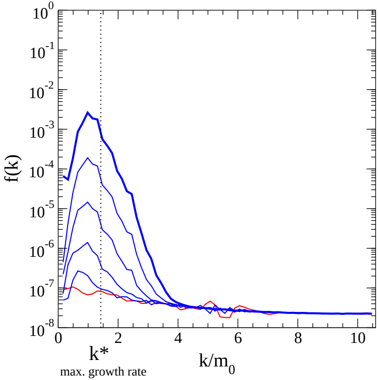

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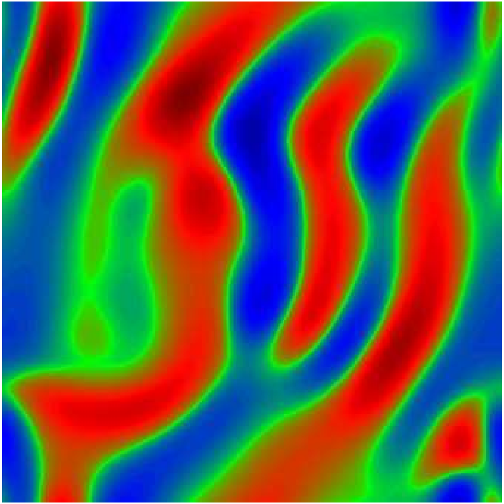

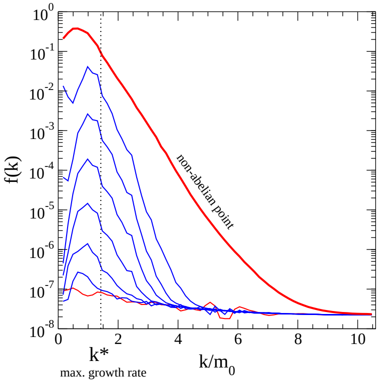

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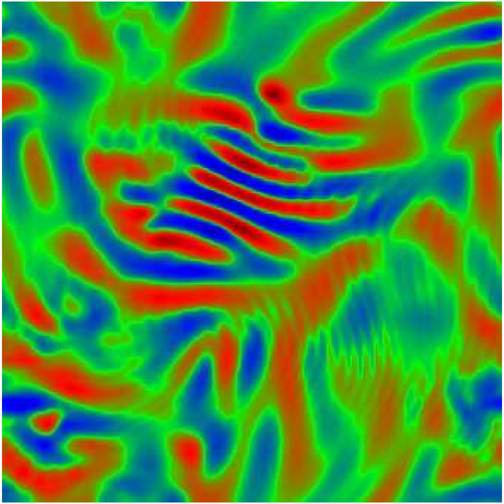

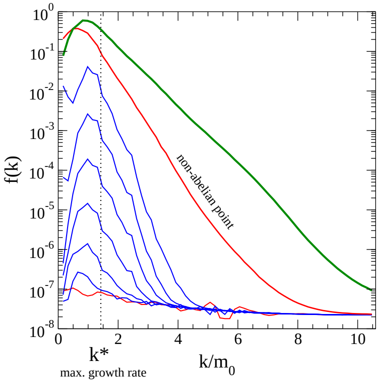

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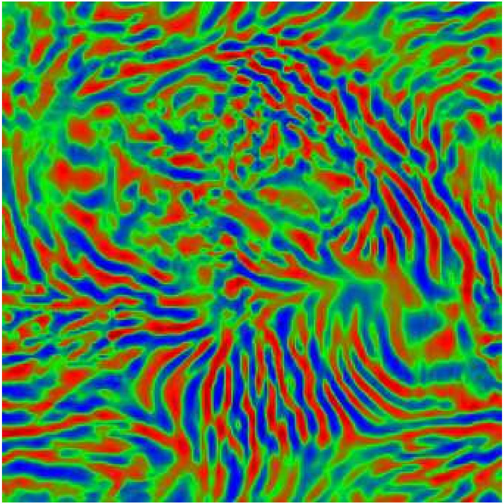
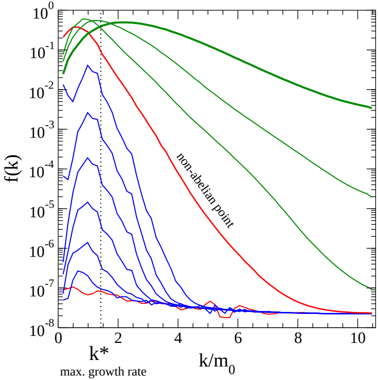


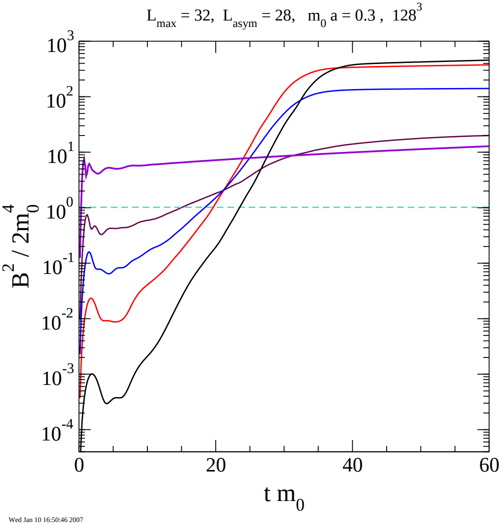


Large initial fields

- The growth is suppressed if the initial amplitude of soft fields is too large!
- Initial condition: random $E_i(k)$ with amplitude

$$E_i(k) \sim Ce^{-k^2/(2m_0)^2}$$

- Vary C $\Longrightarrow$
- Linear growth with very weak initial fields generate favourable conditions for further (non-linear) growth!
- Energy slowly "cascades" to UV [Arnold, Moore]
- Needs further study



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  - large at large anisotropy (UV avalanche)

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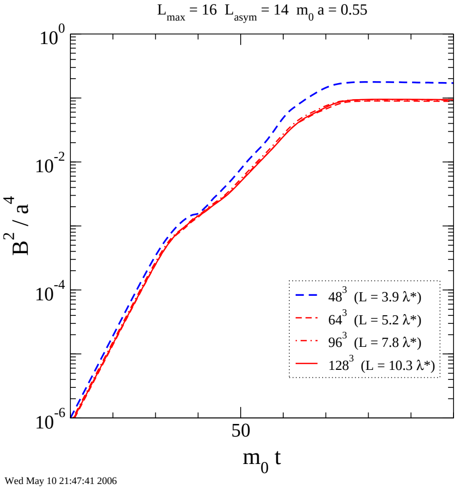

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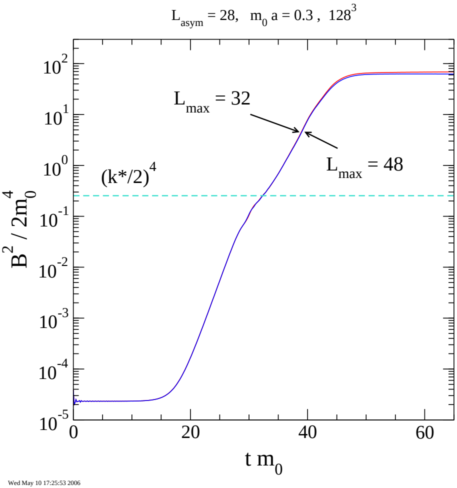

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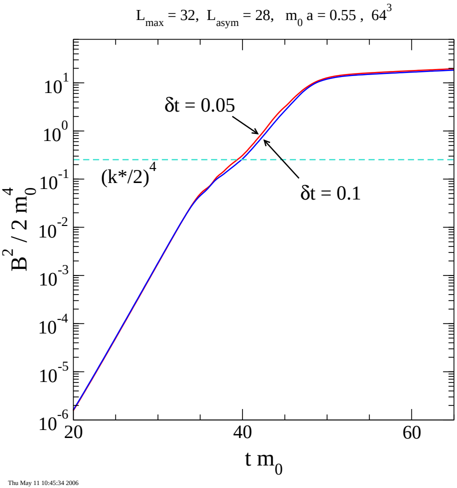

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- Statistics of one:
  - only 1 or 2 runs for each parameter set
  - OK, because statistical variation $\ll$ physical variation

Conclusions

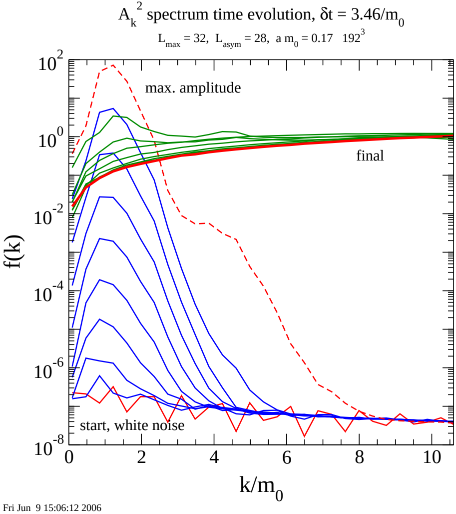
- We observe a fast growth in UV part of the soft fields if the asymmetry of the hard mode distribution is large enough.
- Growth fastest to $\hat{z}$-direction: "soft" modes fill up the $\hat{z}$ deficit in hard modes?
- Rate itself is sufficient for rapid thermalization.
  For large anisotropy

$$\text{rate} \sim m_0 \to m_{Debye} \Rightarrow \quad \text{growth rate less than} \;\; 1/fm.$$

- Warrants further study!
- Open problems:
  - Right initial field configuration?
  - Expanding system tends to slow down the onset of growth further [Romatschke,Venugopalan; Strickland, Nara, Rebhan]

UV runoff in compact U(1)

- compact lattice U(1) becomes non-linear when we hit the lattice limit $A_k \sim a^{-4}k^{-2}$. Causes runoff to UV too!
- Check signature by directly simulating compact U(1):
- Fourier spectrum: $f_{k,\max} \gg 1$



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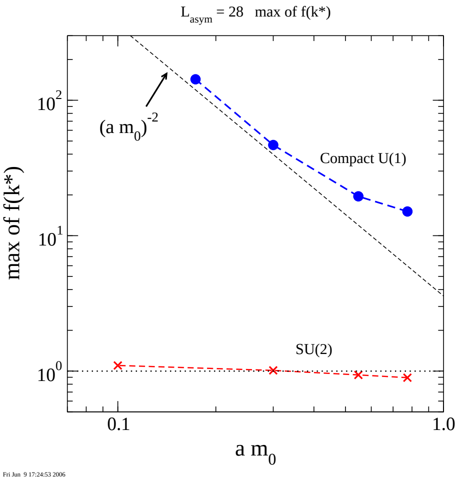
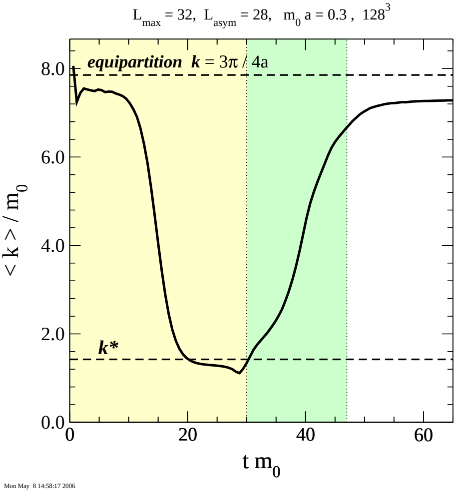
- $f_{k,\max}$ diverges when $a \to 0$. Very different behaviour wrt. non-Abelian theory!



Results: where is the energy?



- Initial equipartition due to white noise initial state; i.e. each lattice mode equally populated
- weak field growth: energy in modes with $k \sim k^*$
- strong field growth: energy runs to UV
- apporoaches lattice equipartition

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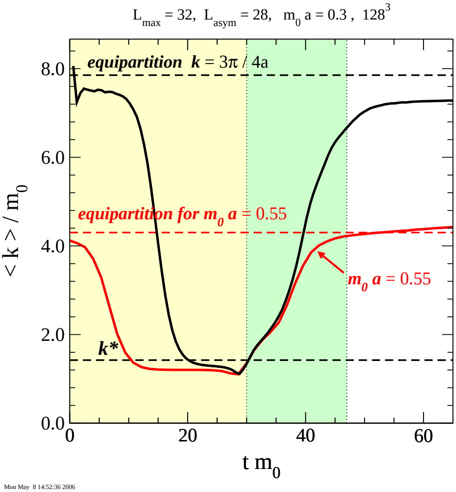
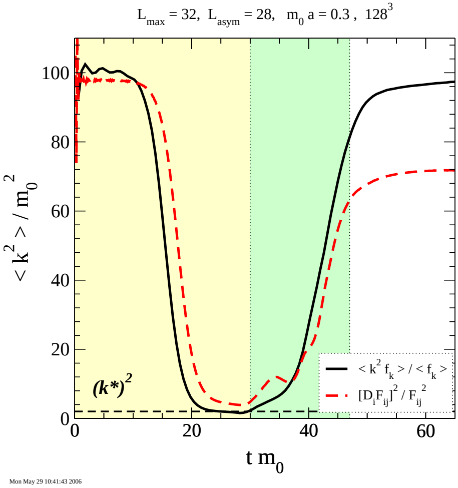


- Initial equipartition due to white noise initial state; i.e. each lattice mode equally populated
- weak field growth: energy in modes with $k \sim k^*$
- strong field growth: energy runs to UV
- apporoaches lattice equipartition
- UV divergent, depends on lattice spacing

Results: checking the gauge fixing



- Gauge fixing always suspect with large fields and/or IR modes due to Gribov copies.
- Compare gauge fixed

  $\langle k^2 \rangle = \int dk\, k^2 f_k$

  with gauge invariant

  $\langle k^2 \rangle = \langle [D_i F_{ij}]^2 \rangle / \langle F_{ij}^2 \rangle$
- works well!