Viscous Hydrodynamics and Heavy-Ion Collisions

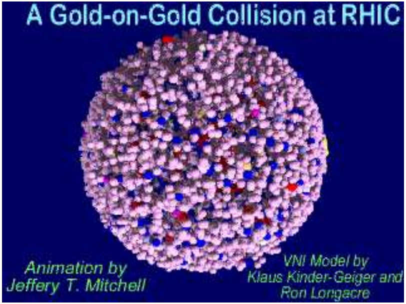
P. Romatschke

INT, University of Washington, Seattle

KITP, January 2008

Outline

Au+Au Collisions at RHIC



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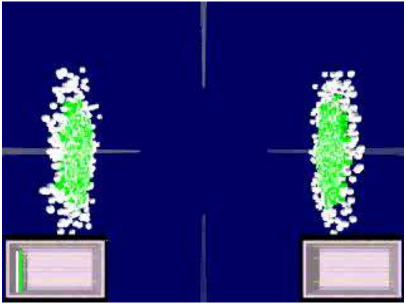

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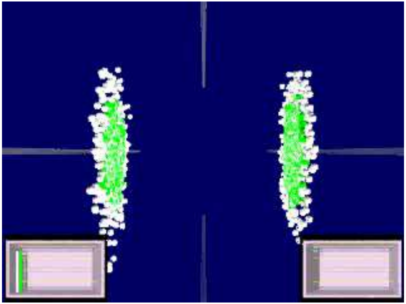

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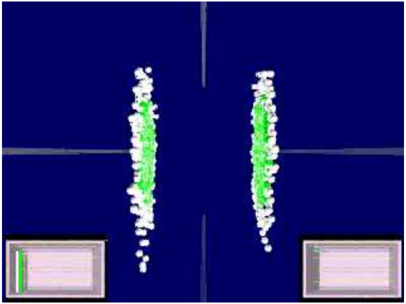

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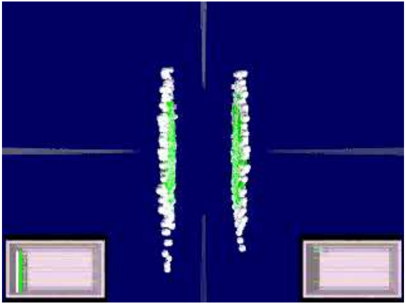

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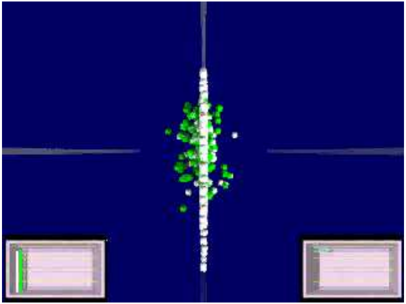

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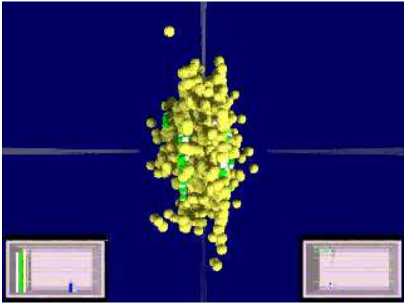

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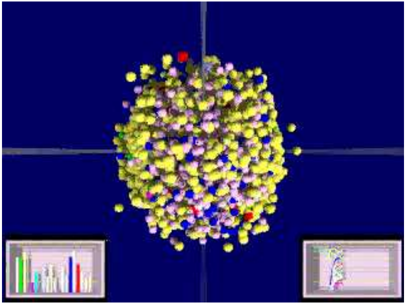
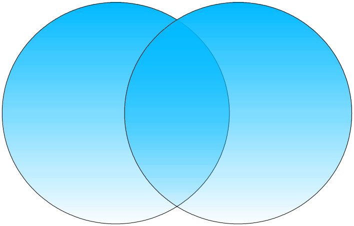


Experimental Observables



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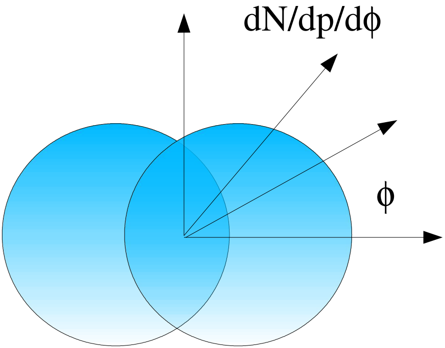


Experimental Observables

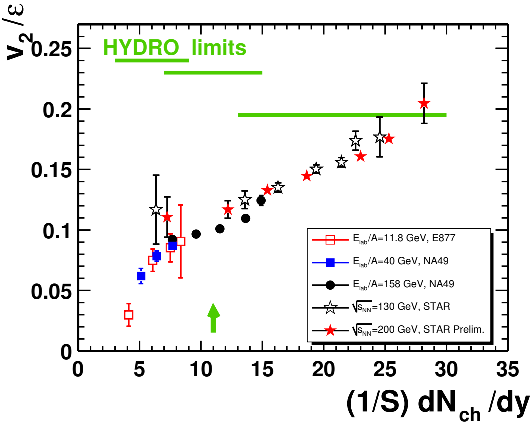
- For ultrarelativistic heavy-ion collisions,

$$\frac{dN}{dp_\perp d\phi dy} = \langle \frac{dN}{dp_\perp d\phi dy} \rangle_\phi (1 + 2v_2(p_\perp)\cos(2\phi) + \ldots)$$

- Radial flow: $\langle \frac{dN}{dp_\perp dy} \rangle_\phi$
- Elliptic flow: $v_2(p_\perp)$

A Bit of History

Elliptic Flow: Experiment vs. Ideal Hydro



U.W. Heinz, nucl-th/0412094

A Bit of History

"RHIC serves the perfect fluid" – Hydrodynamic flow of the QGP*

Ulrich Heinz
Department of Physics, The Ohio State University, Columbus, OH 43210, USA

Abstract
The bulk of the hot and dense matter created at RHIC behaves like an almost ideal fluid. I present the evidence for this and also discuss what we can learn

U.W. Heinz, nucl-th/0512051

QCD Transport Coefficients

- Transport coefficients characterize deviations from equilibrium
- Typical examples: Shear & bulk viscosities, conductivities, diffusion coefficients
- In ideal hydrodynamics, all of these are assumed to be negligible

QCD Transport Coefficients

Shear viscosity in weak coupling

- In high-temperature gauge-theories with N_f light fermions

$$\eta_{Weak} = \eta_1 \frac{T^3}{g^4 \ln g^{-1}}$$

with η_1 a number calculated by Arnold, Moore, Yaffe (hep-ph/0302165)

- For QCD at RHIC scale, typically

$$(\eta/s)_{Weak} \sim 1$$

QCD Transport Coefficients

Shear viscosity in strong coupling

- In finite-temperature $\mathcal{N}=4$ SYM in the strongly coupled, large N regime

$$\eta_{Strong} = \frac{\pi}{8} N^2 T^3$$

(Policastro, Son, Starinets, hep-th/0104066)

- This implies the ratio

$$(\eta/s)_{Strong} = \frac{1}{4\pi} \ll 1 \simeq (\eta/s)_{Weak}$$

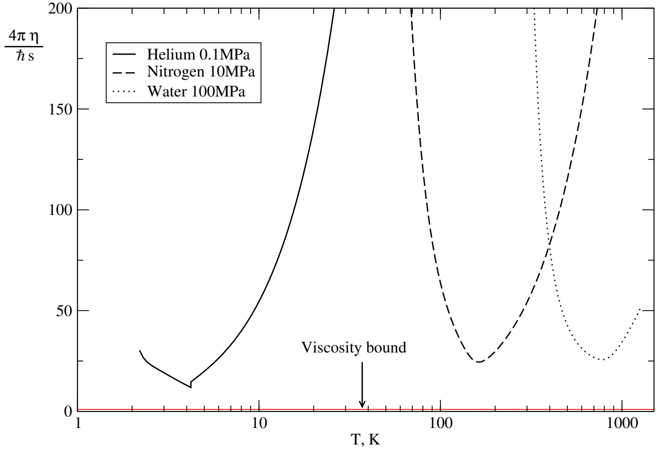
"The KSS Bound"

Kovtun, Son, Starinets conjecture:

$$\eta/s \geq \frac{1}{4\pi} \simeq 0.08$$

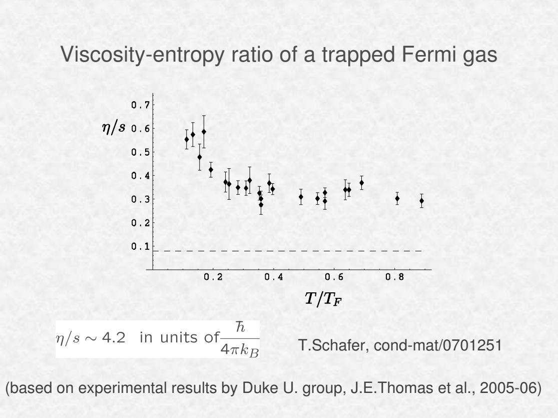
for all relativistic quantum field theories at finite temperature and zero chemical potential (PRL 94 (2005)).

The KSS Bound and Real Fluids



Kovtun, Son, Starinets, PRL 94 (2005)

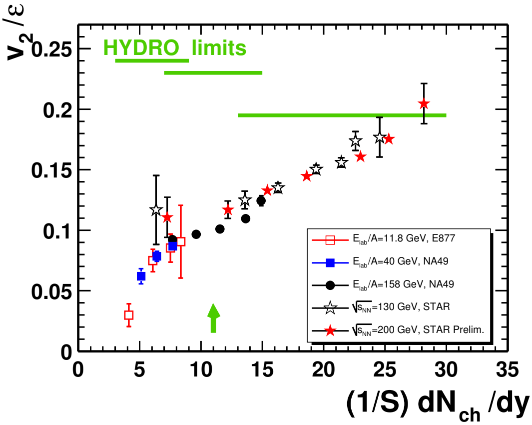
The KSS Bound and Cold Atoms



A. Starinets, talk at Cambridge, 2007

The KSS Bound and RHIC?

Elliptic Flow: Experiment vs. Ideal Hydro



U.W. Heinz, nucl-th/0412094

Questions

- What is the value of η/s at RHIC?
- Does η/s at RHIC violate the KSS bound ?
- What can we expect for η/s at the LHC?

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Outline

Hydro Energy Momentum Tensor

- Ideal hydro EMT

$$T_0^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p g^{\mu\nu}$$

- Viscous hydro: departures from equilibrium (here: only shear viscosity)
- Viscous hydro EMT

$$T^{\mu\nu} = T_0^{\mu\nu} + \Pi^{\mu\nu}$$

- $\Pi^{\mu\nu}$ contains first, second, ...spatial gradients

Gradient Expansion Hierachy

1. Zeroth Order: Ideal Hydrodynamics (“Euler equation”)
2. First-Order: Viscous Hydrodynamics (“Navier-Stokes equation”)
3. Second-Order: Viscous Hydrodynamics (e.g. “Müller-Israel-Stewart theory”)

Remarks:

- 2) contains 1) in the limit of vanishing transport coefficients
- 3) contains 2) in the limit of vanishing second-order transport coefficients

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Why do we need 2nd order? (1/4)

Relativistic Navier Stokes:

$$\Pi^{\mu\nu} = \eta\langle\nabla^\mu u^\nu\rangle$$

where

$$\langle\nabla^\mu u^\nu\rangle = \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\nabla_\alpha u^\alpha$$

Why do we need 2nd order? (2/4)

- Consider small perturbations $\delta\epsilon$, δu^i
- In Fourier-Space, split δu^i into

$$\delta u^i = \delta u_L \frac{k^i}{|k^i|} + \delta u^i_T$$

- From hydrodynamic equations $\partial_\mu \delta T^{\mu\nu} = 0$,

$$\partial_t \delta u^i_T + \frac{\eta}{\epsilon_0 + p_0} k^2 \delta u^i_T = 0$$

Why do we need 2nd order? (3/4)

- Equation for δu_T is diffusion equation
- Diffusion equation has “dispersion relation”

$$\omega = \frac{\eta}{\epsilon_0 + p_0} k^2$$

such that $v_T \equiv d\omega/dk \to \infty$ for $k \to \infty$.

- Perturbations propagate at superluminal speed, theory is not causal!

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Why do we need 2nd order? (3/4)

- Problematic modes are $k \gg 1$: outside of hydrodynamic regime
- Nevertheless problematic in numeric problems (almost always for hydro!)
- Can regulate theory "by hand" (see e.g. Dusling, Teaney 07)
- Look for regulator from microscopic physics: 2nd order gradients!

History: Müller-Israel-Stewart theory

- Instead of the Navier-Stokes relation

$$\Pi^{\mu\nu} = \eta\langle\nabla^\mu u^\nu\rangle,$$

in second-order theories

$$\underbrace{\tau_\Pi \Delta^\mu_\alpha \Delta^\nu_\beta D\Pi^{\alpha\beta}}_{\sim\tau_\Pi\partial_t\Pi^{\mu\nu}} + \Pi^{\mu\nu} = \eta\langle\nabla^\mu u^\nu\rangle + \ldots$$

- Differential equation for $\Pi^{\mu\nu}$ with new parameter τ_Π (=second order transport coefficient)

History: Müller-Israel-Stewart theory

- For weak-coupling QCD, $\tau_\Pi = 6\eta/(\epsilon + p)$
- Consequence: transverse perturbations move with

$$\lim_{k\to\infty} v_\perp^2 = \frac{\eta}{(\epsilon + p)\tau_\Pi} = \frac{\eta}{s}\frac{1}{T\tau_\Pi}$$

so that $v_\perp^2 = 1/6 < 1$!

History: Müller-Israel-Stewart theory

Problems of Müller-Israel-Stewart theory:

- $\tau_\Pi \Delta^\mu_\alpha \Delta^\nu_\beta D \Pi^{\alpha\beta}$ is not the only possible second-order term
- Get different terms/coefficients if matching MIS-theory to Boltzmann (Muronga; Baier,Romatschke,Wiedemann) or coarse-grained Heisenberg equation (Koide)
- Possible to match MIS theory to strongly coupled field theories? Would τ_Π be such that e.g. for $\mathcal{N}=4$ SYM $v_\perp < 1$? Is τ_Π quantitatively important (see e.g. Lublinsky, Shuryak 07)?

Recently clarified:

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Recently clarified:

Hydrodynamics, conformal invariance, and holography

- Consider hydrodynamics of conformal fluids, e.g. fluids with EMT
$$T^{\mu\nu} \to e^{6\omega} T^{\mu\nu}$$
when $g_{\mu\nu} \to e^{-2\omega} g_{\mu\nu}$.
- OK for ideal hydro
- OK for Navier-Stokes, since $\eta\langle\nabla^\mu u^\nu\rangle \to e^{6\omega}\eta\langle\nabla^\mu u^\nu\rangle$
- NOT OK for $\tau_\Pi \Delta^\mu_\alpha \Delta^\nu_\beta D\Pi^{\alpha\beta}$!!!

Hydrodynamics, conformal invariance, and holography

- Not theories are conformal, but the correct viscous hydrodynamic theory should be able to describe conformal fluids
- Classify all allowed terms to second order in gradients that are conformal (there are five, see Baier,Romatschke,Son,Starinets,Stephanov 07)

Hydrodynamics, conformal invariance, and holography

The most general viscous hydrodynamic theory to second order in d dimensions is then

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\Pi \left[{}^{<}D\Pi^{\mu\nu>} + \frac{d}{d-1}\Pi^{\mu\nu}(\nabla\cdot u) \right] + \kappa \left[R^{<\mu\nu>} - (d-2) u_\alpha R^{\alpha<\mu\nu>\beta} u_\beta \right] + \frac{\lambda_1}{\eta^2}\Pi^{<\mu}{}_\lambda \Pi^{\nu>\lambda} - \frac{\lambda_2}{\eta}\Pi^{<\mu}{}_\lambda \Omega^{\nu>\lambda} + \lambda_3 \Omega^{<\mu}{}_\lambda \Omega^{\nu>\lambda} .$$

Hydrodynamics, conformal invariance, and holography

- Five allowed second-order transport coefficients: $\tau_\Pi, \kappa, \lambda_1, \lambda_2, \lambda_3$
- MIS does not allow for non-vanishing κ
- Explicit calculation shows: weak coupling (Boltzmann equation) implies $\kappa = 0$
- Explicit calculation shows: strongly coupled $N = 4$ SYM requires $\kappa = \frac{\eta}{\pi T}$
- MIS cannot be the correct theory!

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Hydrodynamics, conformal invariance, and holography

Coefficients for $\mathcal{N}=4$ SYM

- Matching the more general viscous hydro theory to the scalar/sound correlators of strongly coupled $\mathcal{N}=4$ SYM one finds
$$\tau_\Pi = \frac{2-\ln 2}{2\pi T}$$
Together with $\eta/s = \frac{1}{4\pi}$ this implies $v_\perp < 1$!
- Using result by Heller & Janik 07 one finds
$$\lambda_1 = \frac{\eta}{2\pi T}$$
- Bhattacharyya, Hubeny, Minwalla, Rangamani 07 confirm values for τ_Π, λ_1 and furthermore extract
$$\lambda_2 = \frac{2\eta \ln 2}{\pi T}, \qquad \lambda_3 = 0$$

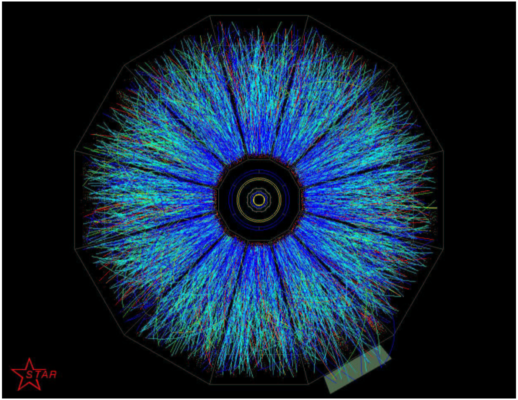
Viscous Hydro Theory – Summary 1/2

- There has been a longstanding dispute over the relativistic viscous hydrodynamic equations
- For the case of shear viscosity, a recently developed framework gives the most general relativistic viscous hydrodynamic equation to second order
- That theory can be matched to both weak-coupling approaches (Boltzmann equation) as well as strong-coupling approaches (AdS/CFT)

Viscous Hydro Theory – Summary 2/2

- There are five second-order transport coefficients, one of them the MIS “relaxation time”
- This theory is causal ($v_\perp < 1$) for both weakly coupled theories and strongly-coupled $N = 4$ SYM
- The regime of validity is still the hydrodynamic regime, e.g. sufficiently far from equilibrium the general second-order theory breaks down.

Let's use viscous hydro and try extract η/s from data!



Outline

Things to know about Hydro @ RHIC

For any hydrodynamic model of a heavy-ion collision

- Hydrodynamics = differential equations. Need to fix initial/boundary conditions!
- the time when to start the hydrodynamic evolution
- the initial distribution of energy density (Glauber? CGC?)
- the equation of state for QCD (lattice!)
- the freeze-out procedure (Cooper-Frye?)
- There is much more to RHIC hydro than just fluid dynamics!

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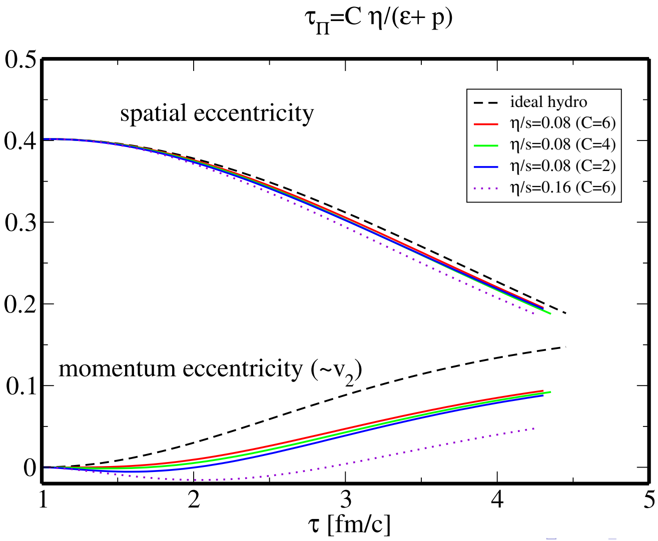
Are we allowed to use hydrodynamics at RHIC?



Are we allowed to use hydrodynamics at RHIC?

- Dependence on second-order transport coefficients is small!
- For $\eta/s = 0.08$, second-order viscous hydrodynamics gives results close to first-order approximation
- Dusling+Teaney 07: First order should be reliable up to $\eta/s \simeq 0.3$
- Heinz+Song 07: Do not agree, but miss terms in hydro equations
- Even though there is no consensus yet, there is a good chance that one can apply hydrodynamics at RHIC!

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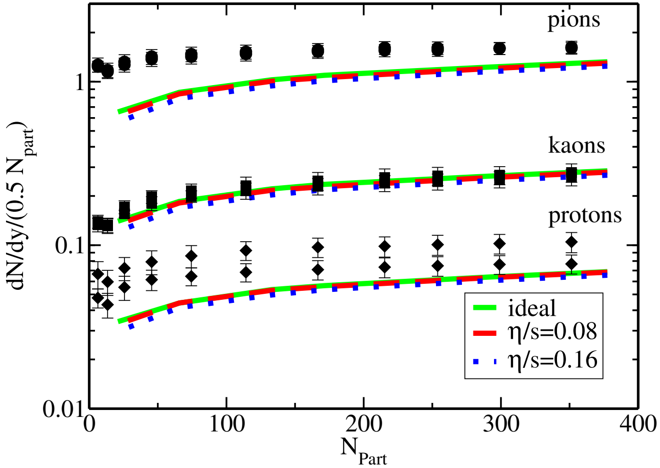
Hydro @ RHIC: mode d'emploi

Use (some) RHIC data to fix freedom:

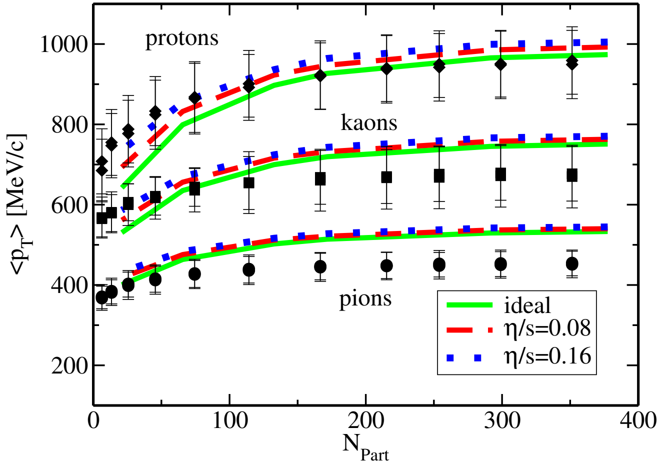
- use centrality dependence of multiplicity to fix the initial distribution of energy density
- use centrality dependence of $< p_T >$ to fix hydro starting/stopping time/temperature
- Once this is done, v_2 in the hydro model is fixed and can be compared to data ("prediction")

Multiplicity (Glauber)



PR+UR, arxiv:0706.1522v1

Mean transverse momentum (Glauber)



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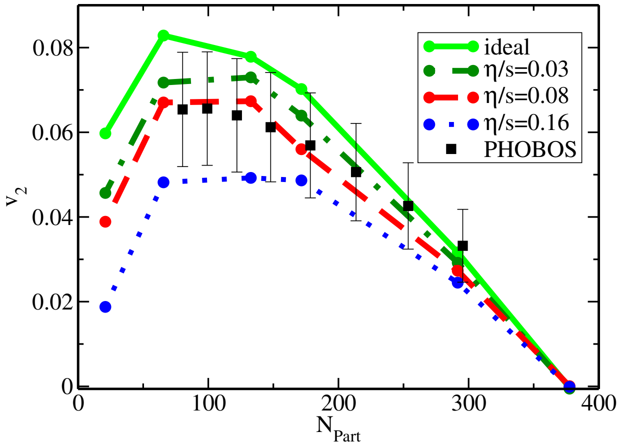
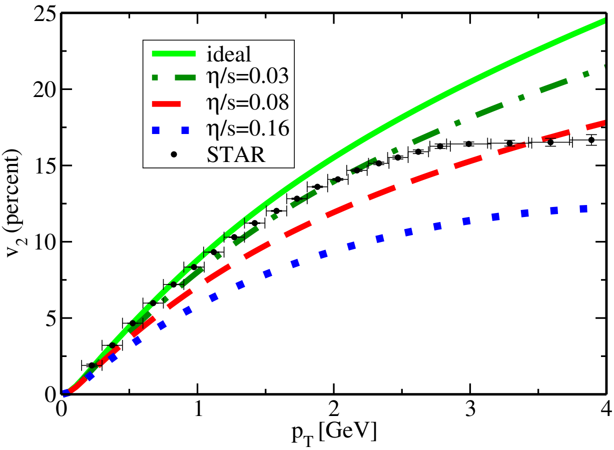
Integrated v2 (Glauber)



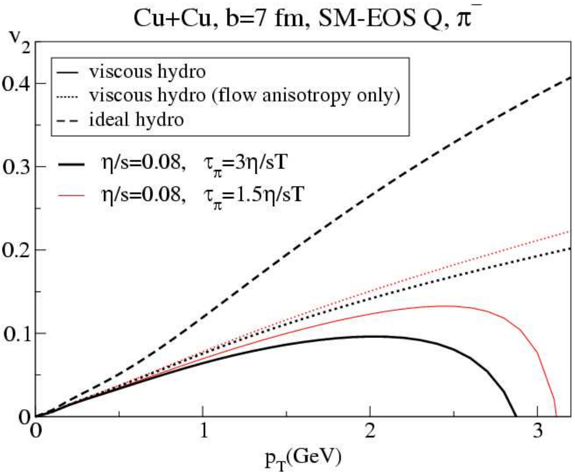
PR+UR, arxiv:0706.1522v1

Min. Bias v2 (Glauber)



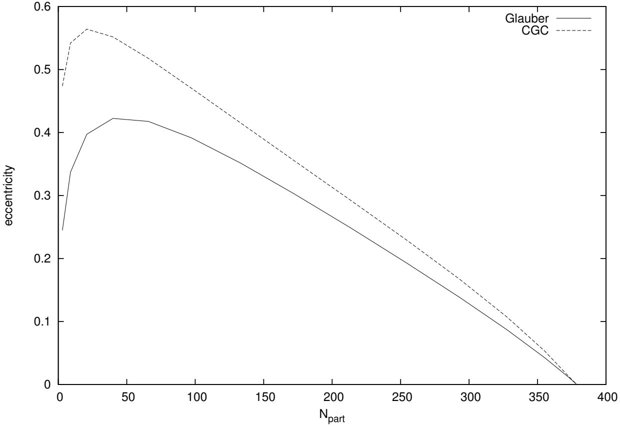
PR+UR, arxiv:0706.1522v1

b=7 fm, Cu+Cu (Glauber)

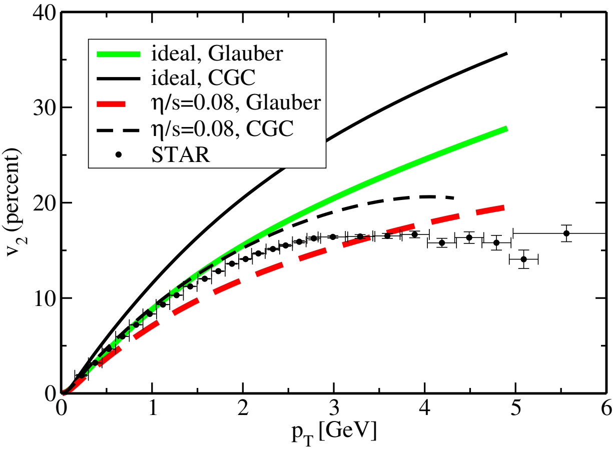


H.Song + U. Heinz, arxiv:0712.3715v1

Eccentricity: Glauber vs CGC



Min. Bias v2 (CGC), Preliminary



Matt Luzum + PR, in preparation

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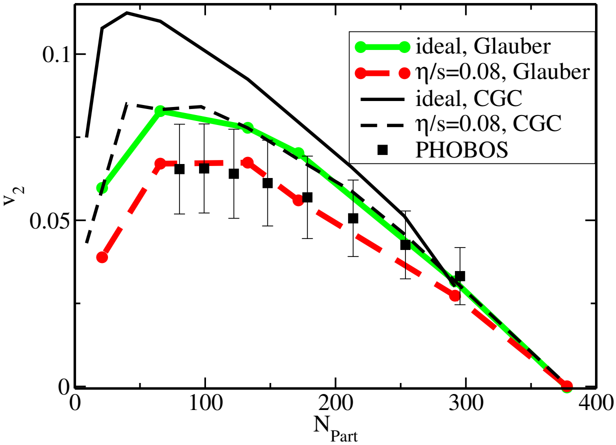

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Does RHIC violate the KSS bound?

Maybe.

- If Glauber is the right IC, saving the bound seems hard
- CGC implies somewhat larger values of η/s
- Experimental systematic error bars could prove vital to decide!
- Mean η/s at RHIC seems to be in the range $\eta/s \sim 0 - 0.2$

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More results indicating $\eta/s < 0.2$ at RHIC

- Gavin e.a., nucl-th/0606061, estimate $\eta/s \simeq 0.19 \pm 0.11$ from transverse momentum correlators
- Lacey e.a., nucl-ex/0609025, estimate $\eta/s \simeq 0.09 \pm 0.015$ from $\eta/s \simeq T\lambda_f c_s$ with $\lambda_f \sim 0.3$ fm (λ_f obtained from pQCD simulation by Xu/Greiner 05)
- Adare e.a, nucl-ex/0611018v3, estimate $\eta/s \simeq 0.13 \pm 0.03$ from heavy-quark energy loss (based on $D = 6\eta/(sT)$ from Moore, Teaney 05)
- Drescher e.a., arXiv:0704.3553, estimate $\eta/s \simeq 0.15 \pm 0.04$ from $\lambda_f \sim 0.34 - 0.6$ (obtained from two-parameter fit of v_2/ϵ from Au+Au and Cu+Cu data)
- Meyer, arXiv:0704.1801, estimates $\eta/s = 0.134 \pm 0.033$ from LGT

Summary

Things we know:

- We now have a relativistic theory of fluid dynamics with shear viscosity for weakly/strongly coupled plasmas
- The RHIC plasma seems to be close enough to equilibrium such that a hydrodynamic description can be attempted
- η/s for RHIC seems to be in the range $0 - 0.2$

Things we know that we don't know:

- Currently we cannot decide whether RHIC violates the KSS bound

There's lots of work to do, join the fun!

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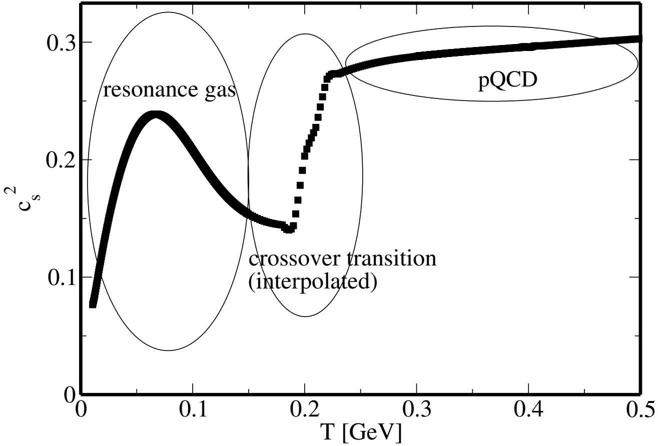
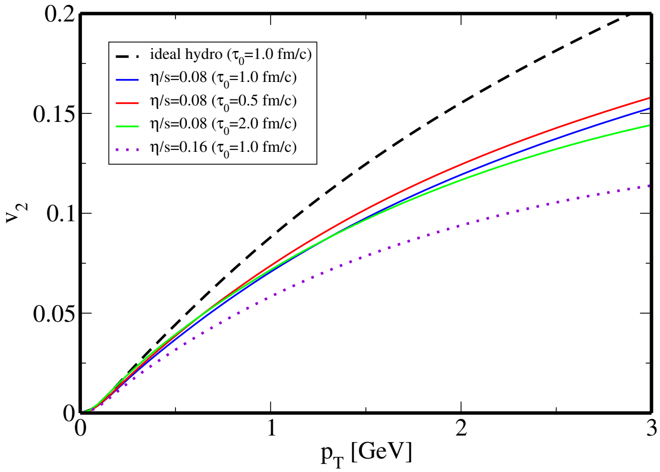
Backup slides

Speed of Sound from Laine and Schröder, PRD73



Dependence on τ_0

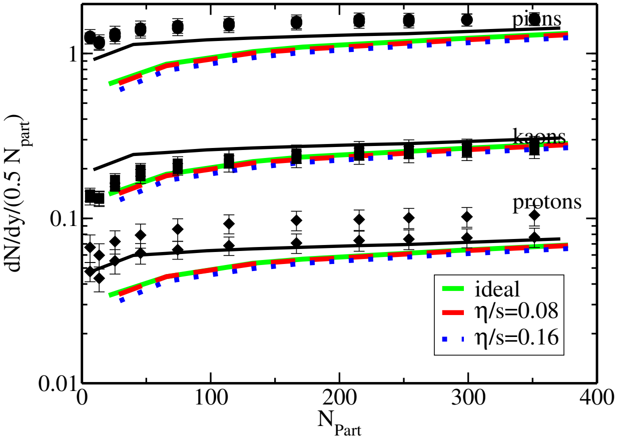


Backup: Multiplicity in Viscous Hydro

	$\frac{dN_{\pi,\mathrm{visc}}}{dy} / \frac{dN_{\pi,\mathrm{ideal}}}{dy}$	$\frac{dN_{K,\mathrm{visc}}}{dy} / \frac{dN_{K,\mathrm{ideal}}}{dy}$
$\eta/s = 0.08$	1.06	1.06
$\eta/s = 0.16$	1.12	1.12
$\eta/s = 0.24$	1.18	1.19
$\eta/s = 0.32$	1.23	1.23
$\eta/s = 0.40$	1.28	1.28

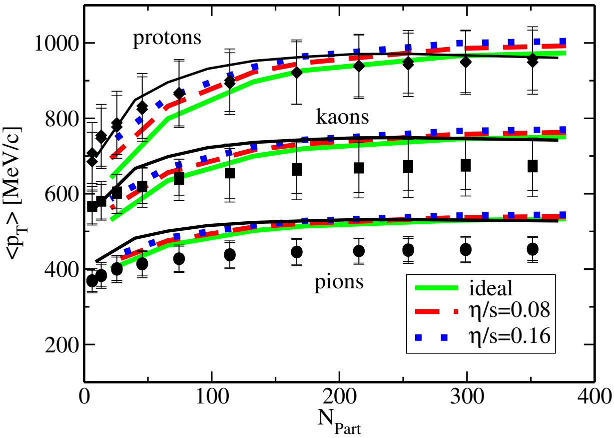
Viscous Hydro creates $\sim 0.75\,\eta/s$ more final multiplicity!

Multiplicity (CGC), Preliminary



Matt Luzum + PR, in preparation

Mean p_T (CGC), Preliminary



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