Viscous Hydrodynamics and Heavy-Ion Collisions

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INT, University of Washington, Seattle

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Outline

1. Motivation
2. Viscous Hydrodynamics Theory
3. $\eta/s$ at RHIC: Status report
Au+Au Collisions at RHIC
Au+Au Collisions at RHIC
Motivation
Viscous Hydrodynamics Theory
$\eta/s$ at RHIC: Status report

Au+Au Collisions at RHIC
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Motivation
Viscous Hydrodynamics Theory
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Viscous Hydrodynamics and Heavy-Ion Collisions
Au+Au Collisions at RHIC
Au+Au Collisions at RHIC
Experimental Observables
Experimental Observables

\[
\frac{dN}{dp/d\phi}
\]
For ultrarelativistic heavy-ion collisions,

\[
\frac{dN}{dp_\perp d\phi dy} = \langle \frac{dN}{dp_\perp d\phi dy} \rangle_\phi (1 + 2v_2(p_\perp) \cos(2\phi) + \ldots)
\]

- Radial flow: \( \langle \frac{dN}{dp_\perp dy} \rangle_\phi \)
- Elliptic flow: \( v_2(p_\perp) \)
A Bit of History
Elliptic Flow: Experiment vs. Ideal Hydro

U.W. Heinz, nucl-th/0412094
“RHIC serves the perfect fluid” – Hydrodynamic flow of the QGP*

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Abstract
The bulk of the hot and dense matter created at RHIC behaves like an almost ideal fluid. I present the evidence for this and also discuss what we can learn

U.W. Heinz, nucl-th/0512051
Transport coefficients characterize deviations from equilibrium

Typical examples: Shear & bulk viscosities, conductivities, diffusion coefficients

In ideal hydrodynamics, all of these are assumed to be negligible
QCD Transport Coefficients
Shear viscosity in weak coupling

- In high-temperature gauge-theories with $N_f$ light fermions

$$\eta_{Weak} = \eta_1 \frac{T^3}{g^4 \ln g^{-1}}$$

with $\eta_1$ a number calculated by Arnold, Moore, Yaffe (hep-ph/0302165)

- For QCD at RHIC scale, typically

$$\left(\frac{\eta}{s}\right)_{Weak} \sim 1$$
In finite-temperature $\mathcal{N} = 4$ SYM in the strongly coupled, large $N$ regime

$$\eta_{\text{Strong}} = \frac{\pi}{8} N^2 T^3$$

(Policastro, Son, Starinets, hep-th/0104066)

This implies the ratio

$$\left( \frac{\eta}{s} \right)_{\text{Strong}} = \frac{1}{4\pi} \ll 1 \simeq \left( \frac{\eta}{s} \right)_{\text{Weak}}$$
Kovtun, Son, Starinets conjecture:

\[ \frac{\eta}{s} \geq \frac{1}{4\pi} \approx 0.08 \]

for all relativistic quantum field theories at finite temperature and zero chemical potential (PRL 94 (2005)).
The KSS Bound and Real Fluids

Kovtun, Son, Starinets, PRL 94 (2005)
Viscosity-entropy ratio of a trapped Fermi gas

\[ \frac{\eta}{s} \sim 4.2 \text{ in units of } \frac{\hbar}{4\pi k_B} \]

T. Schafer, cond-mat/0701251

(based on experimental results by Duke U. group, J.E. Thomas et al., 2005-06)

A. Starinets, talk at Cambridge, 2007
The KSS Bound and RHIC?
Elliptic Flow: Experiment vs. Ideal Hydro

U.W. Heinz, nucl-th/0412094
Questions

- What is the value of $\eta/s$ at RHIC?
- Does $\eta/s$ at RHIC violate the KSS bound?
- What can we expect for $\eta/s$ at the LHC?
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Hydro Energy Momentum Tensor

- Ideal hydro EMT
  \[ T_0^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} \]

- Viscous hydro: departures from equilibrium (here: only shear viscosity)

- Viscous hydro EMT
  \[ T^{\mu\nu} = T_0^{\mu\nu} + \Pi^{\mu\nu} \]

- \( \Pi^{\mu\nu} \) contains first, second, \ldots spatial gradients
Motivation
Viscous Hydrodynamics Theory
\( \eta/s \) at RHIC: Status report

Gradient Expansion Hierarchy

1. Zeroth Order: Ideal Hydrodynamics (“Euler equation”)
2. First-Order: Viscous Hydrodynamics (“Navier-Stokes equation”)
3. Second-Order: Viscous Hydrodynamics (e.g. “Müller-Israel-Stewart theory”)

Remarks:
- 2) contains 1) in the limit of vanishing transport coefficients
- 3) contains 2) in the limit of vanishing second-order transport coefficients
Gradient Expansion Hierarchy

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Relativistic Navier Stokes:

\[ \Pi_{\mu\nu} = \eta \langle \nabla^\mu u^\nu \rangle \]

where

\[ \langle \nabla^\mu u^\nu \rangle = \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3} \nabla^\alpha u^\alpha \]
Consider small perturbations $\delta \epsilon, \delta u^i$

In Fourier-Space, split $\delta u^i$ into

$$\delta u^i = \delta u_L \frac{k^i}{|k^i|} + \delta u_T^i$$

From hydrodynamic equations $\partial_\mu \delta T^{\mu\nu} = 0$,

$$\partial_t \delta u_T^i + \frac{\eta}{\epsilon_0 + p_0} k^2 \delta u_T^i = 0$$
Why do we need 2nd order? (3/4)

- Equation for $\delta u_T$ is diffusion equation
- Diffusion equation has “dispersion relation”

$$\omega = \frac{\eta}{\epsilon_0 + p_0} k^2$$

such that $v_T \equiv d\omega/dk \to \infty$ for $k \to \infty$.

- Perturbations propagate at superluminal speed, theory is not causal!
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Why do we need 2nd order? (3/4)

- Problematic modes are $k \gg 1$: outside of hydrodynamic regime
- Nevertheless problematic in numeric problems (almost always for hydro!)
- Can regulate theory “by hand” (see e.g. Dusling, Teaney 07)
- Look for regulator from microscopic physics: 2nd order gradients!
History: Müller-Israel-Stewart theory

- Instead of the Navier-Stokes relation
  \[ \Pi^{\mu\nu} = \eta \langle \nabla^\mu u^\nu \rangle, \]
  in second-order theories
  \[ \tau_\Pi \Delta_\alpha^\mu \Delta_\beta^\nu D \Pi^{\alpha\beta} + \Pi^{\mu\nu} = \eta \langle \nabla^\mu u^\nu \rangle + \ldots \]

- Differential equation for \( \Pi^{\mu\nu} \) with new parameter \( \tau_\Pi \)
  (=second order transport coefficient)
Motivation
Viscous Hydrodynamics Theory
\( \eta/s \) at RHIC: Status report

History: Müller-Israel-Stewart theory

- For weak-coupling QCD, \( \tau_\Pi = 6\eta/(\epsilon + p) \)
- Consequence: transverse perturbations move with

\[
\lim_{k \to \infty} v_\perp^2 = \frac{\eta}{(\epsilon + p)\tau_\Pi} = \frac{\eta}{s} \frac{1}{T\tau_\Pi}
\]

so that \( v_\perp^2 = 1/6 < 1 \)!
History: Müller-Israel-Stewart theory

Problems of Müller-Israel-Stewart theory:

- $\tau_\Pi \Delta^\mu_\alpha \Delta^\nu_\beta D^\alpha_\beta$ is not the only possible second-order term
- Get different terms/coefficients if matching MIS-theory to Boltzmann (Muronga; Baier, Romatschke, Wiedemann) or coarse-grained Heisenberg equation (Koide)
- Possible to match MIS theory to strongly coupled field theories? Would $\tau_\Pi$ be such that e.g. for $\mathcal{N} = 4$ SYM $\nu_\perp < 1$? Is $\tau_\Pi$ quantitatively important (see e.g. Lublinsky, Shuryak 07)?

Recently clarified:
History: Müller-Israel-Stewart theory

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Recently clarified:
Consider hydrodynamics of conformal fluids, e.g. fluids with EMT

\[ T_{\mu \nu} \rightarrow e^{6\omega} T_{\mu \nu} \]

when \( g_{\mu \nu} \rightarrow e^{-2\omega} g_{\mu \nu} \).

- OK for ideal hydro
- OK for Navier-Stokes, since \( \eta \langle \nabla^\mu u^\nu \rangle \rightarrow e^{6\omega} \eta \langle \nabla^\mu u^\nu \rangle \)
- NOT OK for \( \tau_\Pi \Delta^\mu_\alpha \Delta^\nu_\beta D\Pi^\alpha\beta \)!!!
Not theories are conformal, but the correct viscous hydrodynamic theory should be able to describe conformal fluids.

Classify all allowed terms to second order in gradients that are conformal (there are five, see Baier, Romatschke, Son, Starinets, Stephanov 07).
The most general viscous hydrodynamic theory to second order in $d$ dimensions is then

\[\Pi^{\mu\nu} = -\eta \sigma^{\mu\nu} - \tau \Pi \left[ <D\Pi^{\mu\nu}> + \frac{d}{d-1} \Pi^{\mu\nu} (\nabla \cdot u) \right] + \kappa \left[ R^{\mu\nu} - (d-2) u_\alpha R^{\alpha<\mu\nu>\beta} u_\beta \right] + \frac{\lambda_1}{\eta^2} \Pi^{<\mu\gamma\nu><\lambda>\lambda} - \frac{\lambda_2}{\eta} \Pi^{<\mu\lambda\Omega><\nu>\lambda} + \lambda_3 \Omega^{<\mu\lambda\Omega><\nu>\lambda} .\]
Five allowed second-order transport coefficients: \( \tau_\Pi, \kappa, \lambda_1, \lambda_2, \lambda_3 \)

- MIS does not allow for non-vanishing \( \kappa \)

- Explicit calculation shows: weak coupling (Boltzmann equation) implies \( \kappa = 0 \)

- Explicit calculation shows: strongly coupled \( N = 4 \) SYM requires \( \kappa = \frac{\eta}{\pi T} \)

- MIS cannot be the correct theory!
Hydrodynamics, conformal invariance, and holography

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MIS cannot be the correct theory!
Hydrodynamics, conformal invariance, and holography

Coefficients for $\mathcal{N} = 4$ SYM

- Matching the more general viscous hydro theory to the scalar/sound correlators of strongly coupled $\mathcal{N} = 4$ SYM one finds

$$\tau_\Pi = \frac{2 - \ln 2}{2\pi T}$$

Together with $\eta/s = \frac{1}{4\pi}$ this implies $\nu_\perp < 1$!

- Using result by Heller & Janik 07 one finds

$$\lambda_1 = \frac{\eta}{2\pi T}$$

- Bhattacharyya, Hubeny, Minwalla, Rangamani 07 confirm values for $\tau_\Pi, \lambda_1$ and furthermore extract

$$\lambda_2 = \frac{2\eta \ln 2}{\pi T}, \quad \lambda_3 = 0$$
There has been a longstanding dispute over the relativistic viscous hydrodynamic equations.

For the case of shear viscosity, a recently developed framework gives the most general relativistic viscous hydrodynamic equation to second order.

That theory can be matched to both weak-coupling approaches (Boltzmann equation) as well as strong-coupling approaches (AdS/CFT).
There are five second-order transport coefficients, one of them the MIS "relaxation time"

This theory is causal ($v_\perp < 1$) for both weakly coupled theories and strongly-coupled $N = 4$ SYM

The regime of validity is still the hydrodynamic regime, e.g. sufficiently far from equilibrium the general second-order theory breaks down.
Let’s use viscous hydro and try extract $\eta/s$ from data!
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For any hydrodynamic model of a heavy-ion collision

- Hydrodynamics = differential equations. Need to fix initial/boundary conditions!
  - the time when to start the hydrodynamic evolution
  - the initial distribution of energy density (Glauber? CGC?)
  - the equation of state for QCD (lattice!)
  - the freeze-out procedure (Cooper-Frye?)

There is much more to RHIC hydro than just fluid dynamics!
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- There is much more to RHIC hydro than just fluid dynamics!
Are we allowed to use hydrodynamics at RHIC?

\[ \tau_{\Pi} = C \frac{\eta}{(\varepsilon + p)} \]

Spatial eccentricity

Momentum eccentricity (~v_2)

Dependence on second-order transport coefficients is small!
Are we allowed to use hydrodynamics at RHIC?

- Dependence on second-order transport coefficients is small!
- For $\eta/s = 0.08$, second-order viscous hydrodynamics gives results close to first-order approximation
- Dusling+Teaney 07: First order should be reliable up to $\eta/s \simeq 0.3$
- Heinz+Song 07: Do not agree, but miss terms in hydro equations

Even though there is no consensus yet, there is a good chance that one can apply hydrodynamics at RHIC!
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Use (some) RHIC data to fix freedom:

- use centrality dependence of multiplicity to fix the initial distribution of energy density
- use centrality dependence of $<p_T>$ to fix hydro starting/stopping time/temperature
- Once this is done, $v_2$ in the hydro model is fixed and can be compared to data (“prediction”)
Motivation
Viscous Hydrodynamics Theory
$\eta/s$ at RHIC: Status report

Multiplicity (Glauber)

PR+UR, arxiv:0706.1522v1

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Viscous Hydrodynamics and Heavy-Ion Collisions
Mean transverse momentum (Glauber)

PR+UR, arxiv:0706.1522v1
Integrated v2 (Glauber)

PR+UR, arxiv:0706.1522v1
Min. Bias $v_2$ (Glauber)

$\eta/s$ at RHIC: Status report

$0 \ 1 \ 2 \ 3 \ 4$
$p_T [GeV]$  

$0 \ 5 \ 10 \ 15 \ 20 \ 25$
$v_2$ (percent)  

ideal  
$\eta/s=0.03$  
$\eta/s=0.08$  
$\eta/s=0.16$  
STAR

PR+UR, arxiv:0706.1522v1
b=7 fm, Cu+Cu (Glauber)

Cu+Cu, b=7 fm, SM-EOS Q, $\pi^-$

- viscous hydro
- viscous hydro (flow anisotropy only)
- ideal hydro

$\eta/s=0.08$, $\tau_{\pi}=3\eta/sT$
$\eta/s=0.08$, $\tau_{\pi}=1.5\eta/sT$

H. Song + U. Heinz, arxiv:0712.3715v1
Eccentricity: Glauber vs CGC

The graph shows the comparison between Glauber and CGC models for eccentricity as a function of $N_{part}$. The solid line represents the Glauber model, and the dashed line represents the CGC model. The eccentricity reaches a peak and then decreases as $N_{part}$ increases.
Min. Bias $v_2$ (CGC), Preliminary

$\eta/s$ at RHIC: Status report

$0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6$
$p_T$ [GeV]

$v_2$ (percent)

ideal, Glauber
ideal, CGC
$\eta/s=0.08$, Glauber
$\eta/s=0.08$, CGC
STAR

Matt Luzum + PR, in preparation
Integrated $v_2$ (CGC), Preliminary

Matt Luzum + PR, in preparation
Does RHIC violate the KSS bound?

Maybe.

- If Glauber is the right IC, saving the bound seems hard
- CGC implies somewhat larger values of $\eta/s$
- Experimental systematic error bars could prove vital to decide!
- Mean $\eta/s$ at RHIC seems to be in the range $\eta/s \sim 0 - 0.2$
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More results indicating $\eta/s < 0.2$ at RHIC

- Gavin e.a., nucl-th/0606061, estimate $\eta/s \simeq 0.19 \pm 0.11$ from transverse momentum correlators.
- Lacey e.a., nucl-ex/0609025, estimate $\eta/s \simeq 0.09 \pm 0.015$ from $\eta/s \simeq T\lambda_f c_s$ with $\lambda_f \sim 0.3$ fm ($\lambda_f$ obtained from pQCD simulation by Xu/Greiner 05).
- Adare e.a, nucl-ex/0611018v3, estimate $\eta/s \simeq 0.13 \pm 0.03$ from heavy-quark energy loss (based on $D = 6\eta/(sT)$ from Moore, Teaney 05).
- Drescher e.a., arXiv:0704.3553, estimate $\eta/s \simeq 0.15 \pm 0.04$ from $\lambda_f \sim 0.34 - 0.6$ (obtained from two-parameter fit of $v_2/\epsilon$ from Au+Au and Cu+Cu data).
- Meyer, arXiv:0704.1801, estimates $\eta/s = 0.134 \pm 0.033$ from LGT.
Things we know:

- We now have a relativistic theory of fluid dynamics with shear viscosity for weakly/strongly coupled plasmas.
- The RHIC plasma seems to be close enough to equilibrium such that a hydrodynamic description can be attempted.
- $\eta/s$ for RHIC seems to be in the range $0 - 0.2$.

Things we know that we don’t know:

- Currently we cannot decide whether RHIC violates the KSS bound.

There’s lots of work to do, join the fun!
Summary

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Backup slides
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\( \eta/s \) at RHIC: Status report

Speed of Sound from Laine and Schröder, PRD73

\[ \begin{align*}
0 & \quad 0.1 & \quad 0.2 & \quad 0.3 & \quad 0.4 & \quad 0.5 \\
T \ [\text{GeV}] & \quad & \quad & \quad & \quad & \quad \\
0 & \quad 0.1 & \quad 0.2 & \quad 0.3 & \quad 0.4 & \quad 0.5 \\
\beta^2 & \quad & \quad & \quad & \quad & \quad \\
\text{pQCD} & \quad \text{resonance gas} & \quad \text{crossover transition (interpolated)} & \quad \\
\end{align*} \]
Dependence on $\tau_0$

![Graph showing $v_2$ vs $p_T$ for different $\eta/s$ and $\tau_0$ values.]

- Ideal hydro ($\tau_0 = 1.0$ fm/$c$)
- $\eta/s = 0.08$ ($\tau_0 = 1.0$ fm/$c$)
- $\eta/s = 0.08$ ($\tau_0 = 0.5$ fm/$c$)
- $\eta/s = 0.08$ ($\tau_0 = 2.0$ fm/$c$)
- $\eta/s = 0.16$ ($\tau_0 = 1.0$ fm/$c$)
Viscous Hydro creates $\sim 0.75 \eta/s$ more final multiplicity!
Motivation
Viscous Hydrodynamics Theory
$\eta/s$ at RHIC: Status report

Multiplicity (CGC), Preliminary

\[ \frac{dN}{dy}(0.5 N_{\text{part}}) \]

- Ideal
- $\eta/s = 0.08$
- $\eta/s = 0.16$

pions, kaons, protons

Matt Luzum + PR, in preparation
Mean $p_T$ (CGC), Preliminary

Matt Luzum + PR, in preparation

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