# Phases of Holographic QCD

Andrei Parnachev

C.N.Yang ITP, Stony Brook, USA

# Outline

- Introduction and motivation
- $D4 - D8 - \bar{D}8$ (Sakai-Sugimoto) model a.k.a. Holographic QCD
- Remarks on the spectrum; energy scales
- Review: phase diagram of real QCD
- Finite temperature
- Finite baryon chemical potential
- Finite isospin chemical potential
- Conclusions/Open problems

# Introduction and motivation

We don't know the holographic dual of QCD.

Next best thing: holographic dual of large $N$ QCD. Strings can be solved in the regime where there is no asymptotic freedom (coupling doesn't have a chance to run to small values)

Interesting qualitative physics at strong coupling. Confinement/deconfinement and chiral phase transitions. Mesons and baryons.

# Introduction and motivation

Many questions in QCD are hard to answer because the theory is strongly coupled in the IR. Sometimes lattice is hard to do (transport properties, chemical potential). String theory provides an independent technology!

On the other hand, gravity/string physics in the bulk is related to gauge theory physics. (e.g. open string tachyon is related to quark condensate) Can understand gravity/strings better?

Perhaps can get universal quantities?

# Holographic QCD

Brane construction: $N$ $D4$-branes spanning $x^0 \ldots x^4$
$N_f \ll N_c$ $D8 - \bar{D}8$ pairs spanning every coordinate but $\tau = x^4 \in [0, 2\pi R_4)$. The $x^4$ circle has antiperiodic boundary conditions for fermions.

Theory at energies much smaller then $1/l_s$: large $N$ 5d Yang-Mills with $N_f$ 4d quarks. Theory at energies much smaller then $1/R_4$: large $N$ QCD with quarks. No adjoint fields, conformal symmetry or supersymmetry!

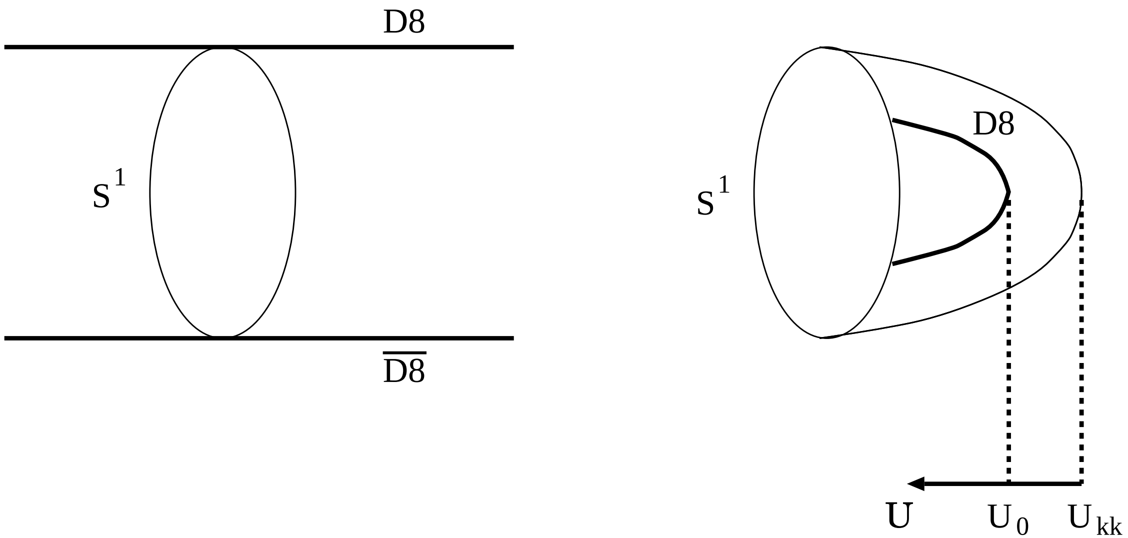
# Holographic QCD

Parameters: $\lambda = g_{YM}^2 N = g_s l_s$; 4-dimensional t'Hooft coupling at scale $1/R_4$ $\lambda_4 = \lambda/R_4$; brane-antibrane asymptotic separation $L$.

$$\Lambda_{QCD} = \frac{1}{R_4} e^{-\frac{1}{\lambda_4}}$$

so we have good approximation to QCD in the regime $\lambda_4 \ll 1$. Holographic (stringy) dual is solvable in the opposite regime $\lambda_4 \gg 1$. Yang-Mills=gravity; quarks=Dirac-Born-Infield (DBI) action for D8 branes

# Holographic QCD




Gravity dual has a form $R^{3,1} \times S^1_{x^4} \times R_{+(U)} \times S^4$. In the $U - x^4$ plane geometry has a cigar-like form. $D8 - \bar{D}8$ branes connect in the IR (small $U$), hence chiral symmetry is broken.

# Holographic QCD

Metric:

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} \left((dx_\mu)^2 + f(U)(dx^4)^2\right) + \left(\frac{U}{R}\right)^{-\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

where $f(U) = 1 - U_K^3/U^3$.
$U = U_K$ bounds $U$ from below; determined via
$2\pi R_4 = 4\pi R^{3/2}/3U_K^{1/2}$

Dilaton is $e^\Phi = \left(\frac{U}{R}\right)^{\frac{3}{4}}$
Similarly to $AdS_5$, $R^3 = \pi\lambda_5$

# Holographic QCD; matter

Fundamental matter comes from $N_f$ $D8 - \bar{D}8$ pairs with asymptotic separation $L$. Their shape is determined by solving DBI equations of motion.

Results: branes are connected at $U = U_0$; chiral symmetry is broken; pion lagrangian can be derived. $U_0$ and $L$ are related: $L = R^{3/2}/U_0^{1/2}$

Fluctuations of the flavor branes give rise to scalar and vector mesons. Baryons are described by the $D4$ branes wrapping the $S^4$. Strings are streched to the flavor branes and source the electric field dual to the baryon current.

# Spectrum; energy scales

Spectrum of mesons is studied using DBI action. At high energy, WKB can be used to show

$$m_n = \frac{\mathcal{K} n}{L}$$

Glueball masses are set by $1/R_4$. Anticipate: $1/R_4$ is the scale of confinement/deconfinement and $1/L$ of chiral phase transition.

There is no asymptotic restoration of chiral symmetry.

Scalars and pseudoscalars correspond to even and odd $n$.

# Phase diagram of QCD

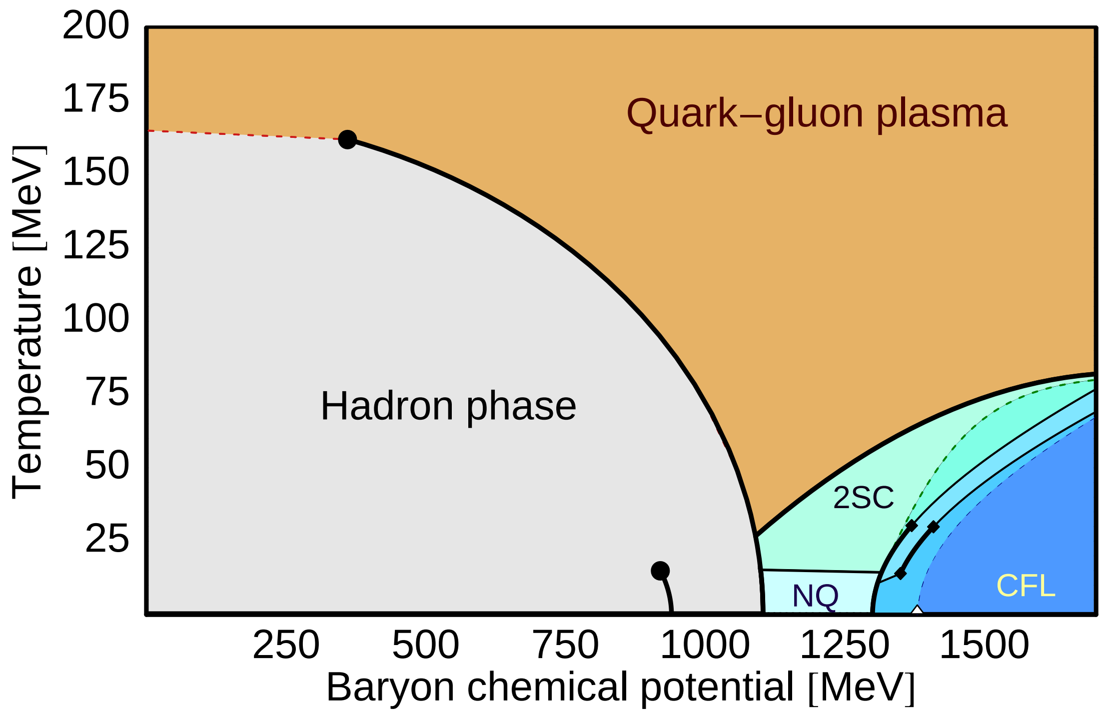
Consider QCD at finite temperature $T$. Matter part of QCD lagrangian

$$\mathcal{L}_m = \sum_{i=1}^{N_f} \bar{\psi}_i [\gamma^\mu (\partial_\mu + i A_\mu) + m_i + \gamma^0 \mu_i] \psi_i$$

Simplest case $\mu_i = \mu_B$ (baryon chemical potential). Relevant for RHIC physics, neutron stars...

In the bulk of the $\mu_B - T$ plane lattice is hard to do. Can't say much rigorously– models like NJL, random matrix are used.

# Phase diagram of QCD



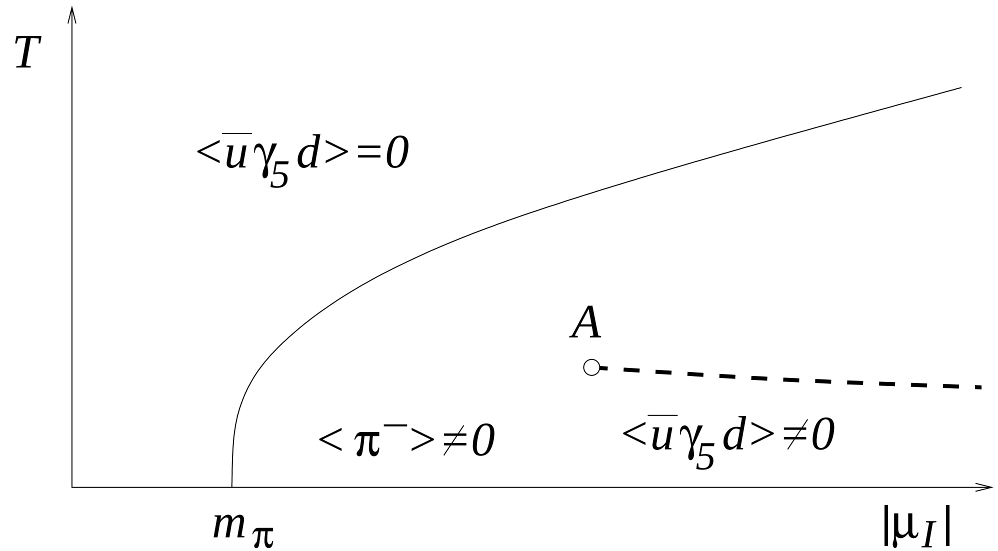

One version of phase diagram [hep-ph/0503184]. $\mu_B = 0$ line is accessible on the lattice. large $\mu_B$ regime is accessible in perturbation theory. The rest is less well established. What does string theory have to say?

# QCD with isospin chemical potential

Consider $\mu_B = 0$ but $\mu_I = (\mu_u - \mu_d)/2 \neq 0$. At small $\mu_I$ chiral perturbation theory can be used to show

$$\rho = \langle \bar{\psi} \gamma_5 \tau^2 \psi \rangle \neq 0$$

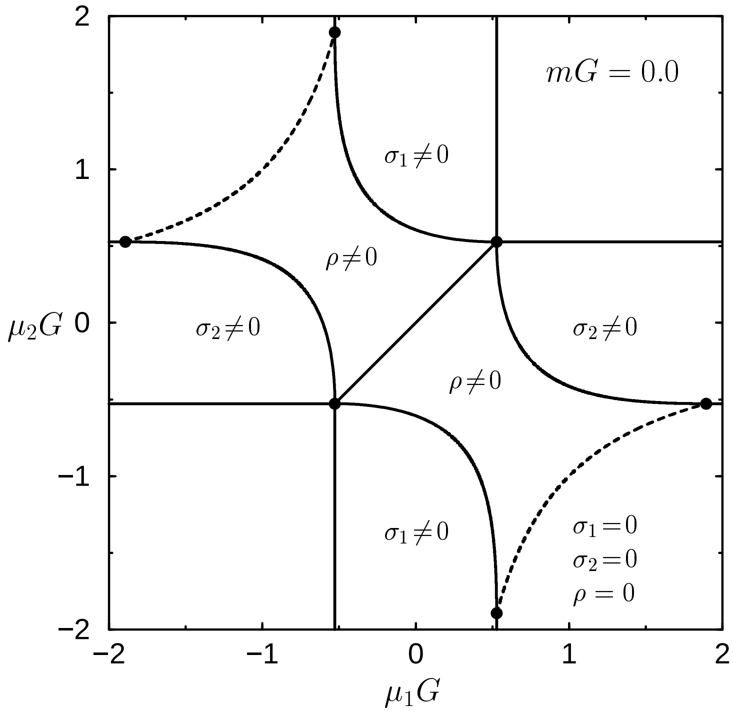
at large $\mu_I$ perturbation theory leads to $\rho \neq 0$ again!




[Son, Stephanov]

# QCD with isospin chemical potential

When $\mu_B$ is allowed to be nonzero, the phase diagram should be drawn in the 3-dimensional space $(T, \mu_B, \mu_I)$. Slice at $T = const$, from RMM:




[Klein, Toublan, Verbaarschot] What does string theory say?

# Finite temperature; Yang-Mills

Compactify euclidean time $t_E$ with asymptotic circumference $\beta = 1/T$. There are 2 geometries which asymptote to this:

- (1) $(x^4, U > U_K)_{cigar} \times t_E$
- (2) $(t_E, U > U_T)_{cigar} \times x^4$

The Lorenzian version of $(2)$ is a black hole (BH); it has a horizon. The geometries really are the same; smaller cigar dominates partition function.

For $T < 1/2\pi R_4$ $(1)$ dominates; gluon confining state
For $T > 1/2\pi R_4$ $(2)$ (BH) dominates; gluon deconfinement

# Finite temperature; matter

In the gluon confining phase $D8$ and $\bar{D}8$ branes connect and chiral symmetry is broken. In the following we consider gluon-deconfining phase. There are two possibilities:

- They connect as before (curved)
- The go straight into the BH horizon (straight)

curved branes= chiral symmetry is broken; straight branes=chiral symmetry is restored.

Analyse DBI action to find thermodynamically preferred configuration. 1st order phase transition occurs at $T = 1/L$.

# Some formulas

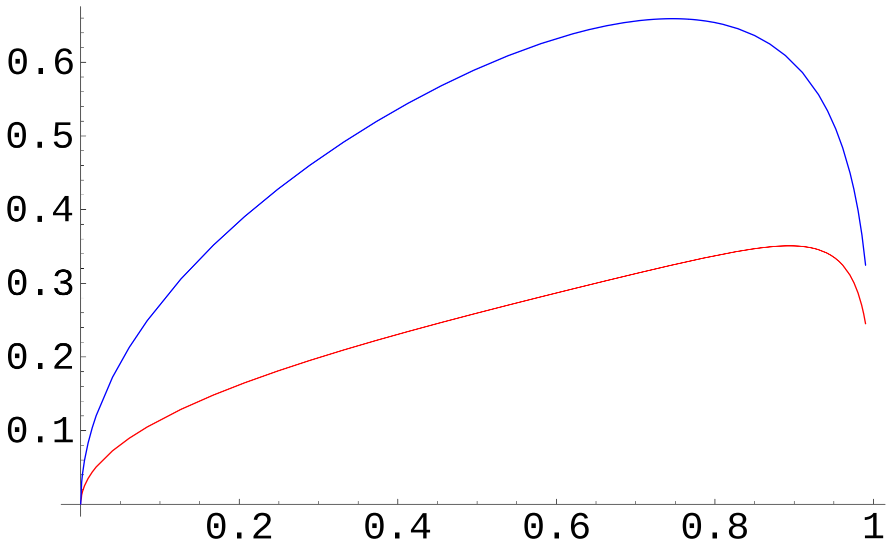
$$S = 2\int dU U^{\frac{5}{2}} \sqrt{1 + f_T(U)\left(\frac{U}{R}\right)^3 (\partial_U X^4)^2}$$

where $f_T(U) = 1 - U_T^3/U^3$, $U_T \sim \lambda T^2$
Equation of motion:

$$(\partial_U X^4)^2 = \frac{U_0^8 R^3 f_T(U_0)}{U^{11} f_T^2(U) - U_0^8 U^3 f_T(U_0) f_T(U)}$$

$U = U_0 \sim \lambda_5/L^2$ is the “turning point” of the brane,
$U > U_0 > U_T$

# Calculation of $LT$



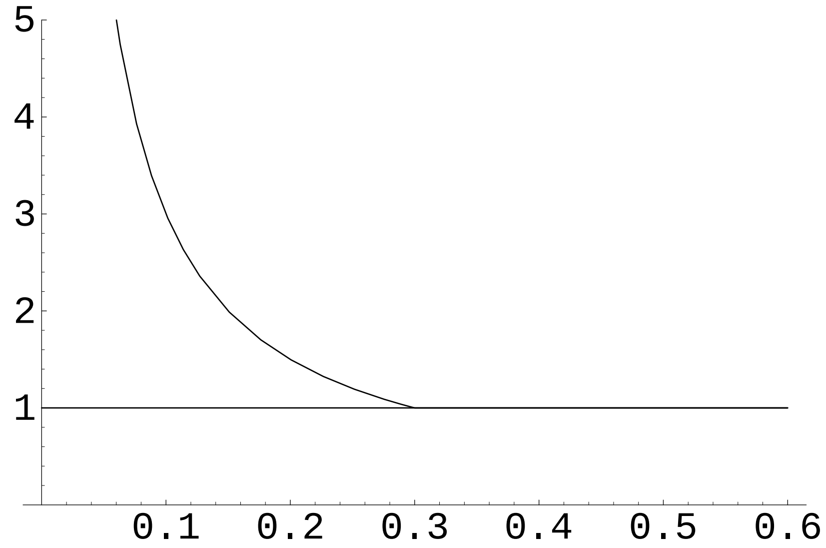

This is $2\pi LT/3$ as a function of (roughly) the position of the brane tip, $U_0$. Red line $\mu_I = 0$, blue line $\mu_I \to \infty$.

There're two curved solutions, but the one which connects to the vacuum always dominates. (But the straigt brane dominates for large enough $LT$)

# Results at finite $T$

First order chiral phase transition happens at

$$T_c = \frac{0.15}{L} = \frac{0.3}{2\pi R_4}\frac{1}{L/\pi R_4}$$




$T_c(x)/2\pi R_4$ where $x = L/\pi R_4$

# Finite T and $\mu_B$

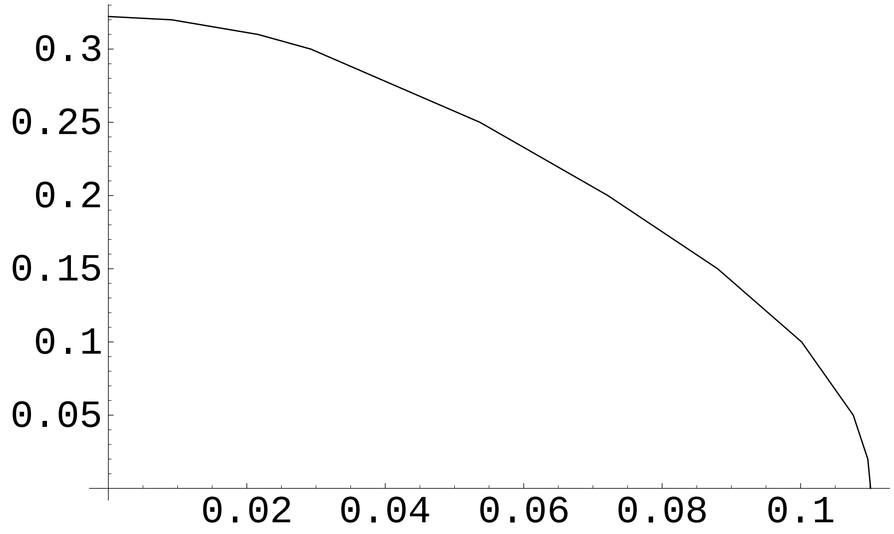
Baryon chemical potential corresponds to turning on gauge field $A_0$ on the D8-brane. Curved solution cannot have flux. Staight solution can have nonzero flux, and its action depends on it.

Comparing the actions, we find a line of 1st order phase transitions in the $\mu/E_0 - TL$ plane where

$$E_0 = \frac{\lambda}{L^2} \gg \frac{1}{L}$$

Note that $E_0$ is the energy of constituent quark.

# Finite T and $\mu_B$




The line of first order phase transitions between the phase whith broken chiral symmetry (small $TL$, $\mu/E_0$) and the restored phase

# Finite T, $\mu_B$ and $\mu_I$

Consider $N_f = 2$. When $\mu_I$ is turned on, we can have curved solutions with nonzero flux.

These are the solutions where “u”-brane connects to “d” antibrane. They correspond to

$$\rho = \langle \bar{\psi}\gamma_5\tau^2\psi \rangle \neq 0$$

For small $\mu_{u,d}$ the phase with $\rho \neq 0$ is preferred to the phase with nonzero $\sigma_{u,d} = \bar{\psi}_{u,d}\psi_{u,d}$

# Formulas

Action

$$S = 2\int dU U^{\frac{5}{2}}\sqrt{1-(\partial_U A_0)^2 + f_T(U)\left(\frac{U}{R}\right)^3(\partial_U X^4)^2}$$

Define $y = \left(\frac{U_T}{U_0}\right)^3$, $\tilde{t} = \frac{2\pi LT}{3}$, $\tilde{\mu} = \mu/E_0$.

Then

$$\tilde{t} = y^{\frac{1}{2}} G(\tilde{c}, y)$$

$$\frac{1}{2}|\tilde{\mu}_1 - \tilde{\mu}_2| = \frac{\tilde{t}^2}{y} F(\tilde{c}, y)$$

where

$$G(\tilde{c}, y) = \int_1^\infty dx \sqrt{\frac{(1+\tilde{c}^2)(1-y^2)}{(x^5+\tilde{c}^2)(x^3-y^3)^2-(1+\tilde{c}^2)(x^3-y^3)(1-y^3)}}$$

$$F(\tilde{c}, y) = \int_1^\infty dx \sqrt{\frac{\tilde{c}^2(x^3-y^3)}{(x^5+\tilde{c}^2)(x^3-y^3)-(1+\tilde{c}^2)(1-y^3)}}$$

# Formulas

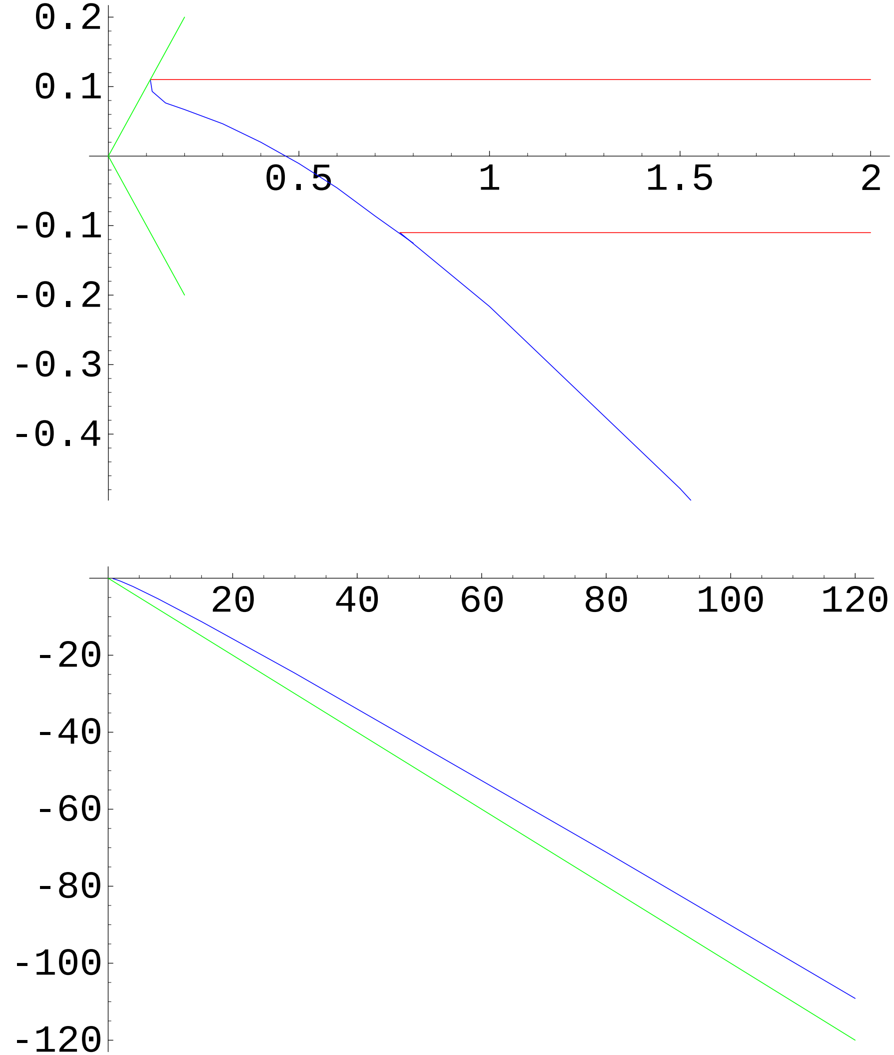
Hence, $LT$ and $\mu/E_0 = \mu/(\lambda_4/L)$ completely determine $y$ and $\tilde{c}$. Then the value of (subtracted) action is computed via

$$\delta S = 2E_0^{\frac{7}{2}} \frac{\tilde{t}^7}{y^{\frac{7}{2}}} H(\tilde{c}, y)$$

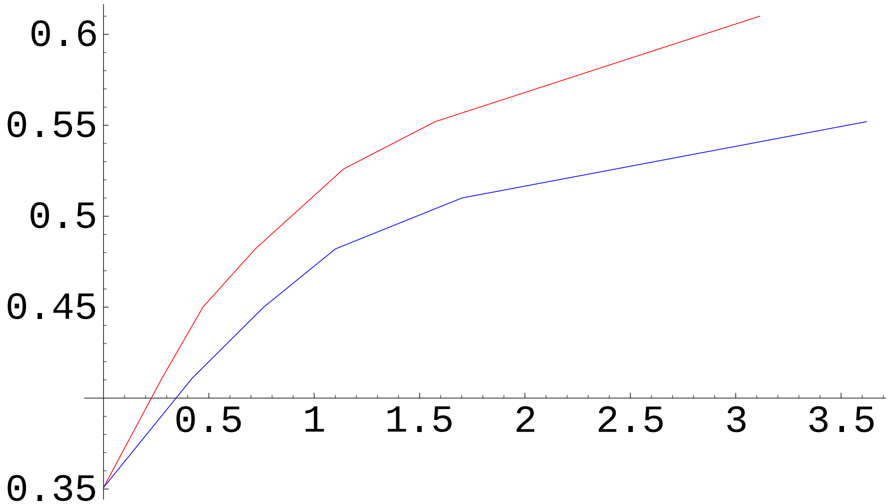
where

$$H(\tilde{c}, y) = \frac{2}{7}(1 - y^{\frac{7}{2}}) +$$
$$\int_1^\infty dx x^{\frac{5}{2}} \left(1 - \sqrt{\frac{x^5(x^3-y^3)}{(x^5+\tilde{c}^2)(x^3-y^3)-(1+\tilde{c}^2)(1-y^3)}}\right)$$

# Phases in the $\mu_B - \mu_I$ plane



# $\mu_I - T$ plane at $\mu_B = 0$




Blue line– chiral phase transition;
Red line– boundary of $\rho \neq 0$ phase

Note that there is no phase transition along the $T = 0,\ \mu_B = 0$ axis

# Conclusions:

- Holographic QCD describes chiral symmetry breaking/restoration
- Spectrum is not linear in $n$
- Phase diagram resembles QCD, differs from that of other models in some ways

# Open problems

- Nuclear matter
- Massive pions
- Non-homogeneous phases (CDW?)
- Universal quantities?