Heavy quarkonium as a probe of hot QCD

Mikko Laine (University of Bielefeld, Germany)

M. Laine, O. Philipsen, P. Romatschke and M. Tassler, "Real-time static potential in hot QCD," hep-ph/0611300;

M. Laine, "A resummed perturbative estimate for the quarkonium spectral function in hot QCD," arXiv:0704.1720;

M. Laine, O. Philipsen and M. Tassler, "Thermal imaginary part of a real-time static potential from classical lattice gauge theory simulations," arXiv:0707.2458;

Y. Burnier, M. Laine and M. Vepsäläinen, "Heavy quarkonium in any channel in resummed hot QCD", arXiv:0711.1743.

Basic idea

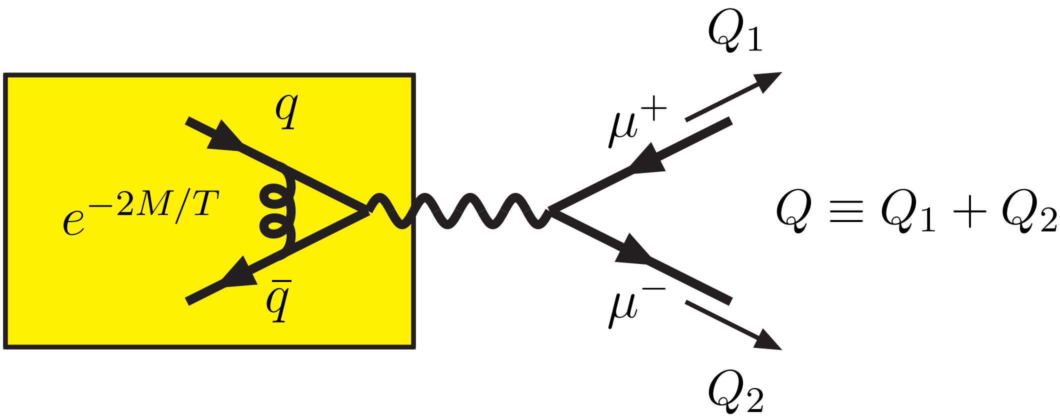
Production, spectroscopy, and decays of heavy quarkonium (charmonium, bottomonium) are precision probes of QCD at zero temperature.

Maybe some related properties could also be useful at finite temperatures?

Matsui, Satz 1986

The relevant physical observable

Heavy quarkonium contribution to the production rate of lepton–antilepton pairs from a thermal plasma:



$\Big($ • Initial state could also be non-thermal, e.g. anisotropic.
• Could also consider light quarks, but here only heavy. $\Big)$

Challenge

Surprisingly, despite asymptotic freedom, the existence of a high temperature does not necessarily make the theoretical determination of the properties of heavy quarkonium more tractable than at $T = 0$.

In other words, most standard approximation methods appear to develop further systematic errors at $T > 0$.

Approximation methods:

1. Potential models
2. Lattice QCD
3. AdS/CFT
4. Perturbation theory

1. Potential models

In the deconfined phase, the quarks are bound together by a Debye-screened Coulomb potential:

$$V(r) \approx -\frac{g^2 C_F}{4\pi} \frac{\exp(-m_{\rm D} r)}{r} ,$$

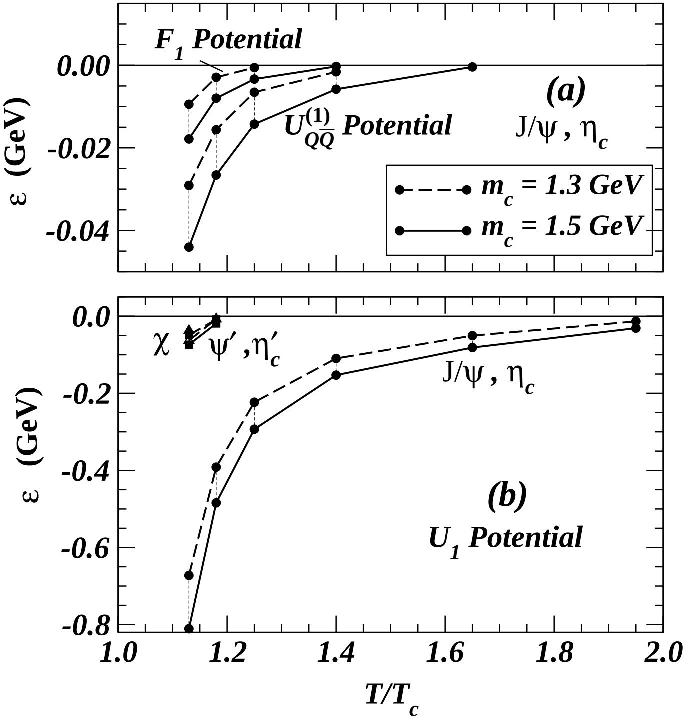
where $m_{\rm D} \sim gT$ is the Debye mass and $C_F \equiv 4/3$.

The corresponding Schrödinger equation,

$$\left(-\frac{\nabla_{\mathbf{r}}^2}{M} + V(r) \right) \psi = (E - 2M)\psi ,$$

does not possess any bound state solution for large enough $m_{\rm D}$, $m_{\rm D} > g^2 C_F M/(4\pi \times 1.68...)$.

The same with various non-perturbative potentials:



Wong, hep-ph/0408020

⇒ quarkonium "melts". (Heavy ion experimentalists' Monte Carlo inputs just the melting temperatures.)

Problems:

$T = 0$: using the non-perturbative static potential in a perturbative setup is not theoretically consistent (even though this leads to reasonable spectroscopy) $\Rightarrow$ PNRQCD.

$T > 0$: which non-perturbative potential to use?

Current status: Mócsy, Petreczky, 0705.2559

2. Lattice QCD

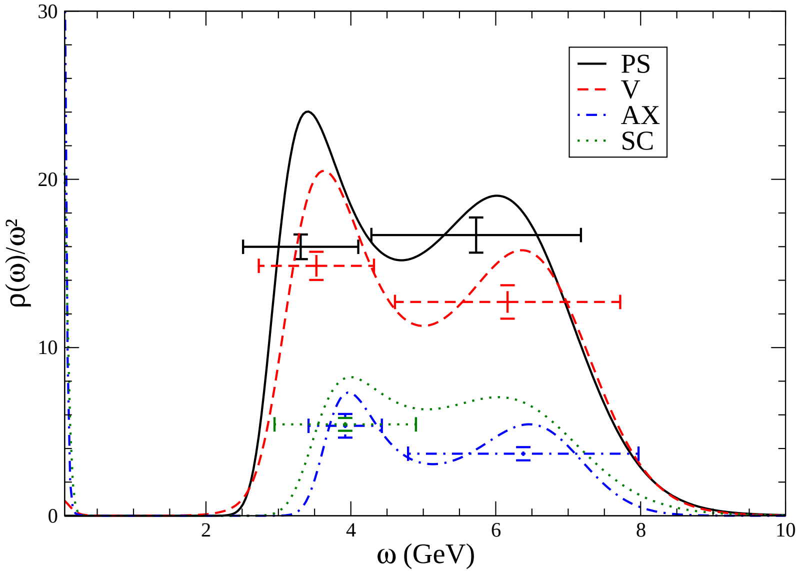
Measure Euclidean correlation function $G_V(\tau)$ between vector currents, for $0 < \tau < \beta \equiv 1/T$.

In the continuum limit this is related to the Minkowskian **spectral function** $\rho_V(\omega)$ through

$$G_V(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_V(\omega) \frac{\cosh\left(\frac{\beta}{2} - \tau\right)\omega}{\sinh\frac{\beta\omega}{2}} .$$

Then simply “invert” this for $\rho_V(\omega)$.

Results in different channels:



Aarts, Allton, Oktay, Peardon, Skullerud, 0705.2198
(see also: Jakovác, Petreczky, Petrov, Velytsky, hep-lat/0611017)

Problems:

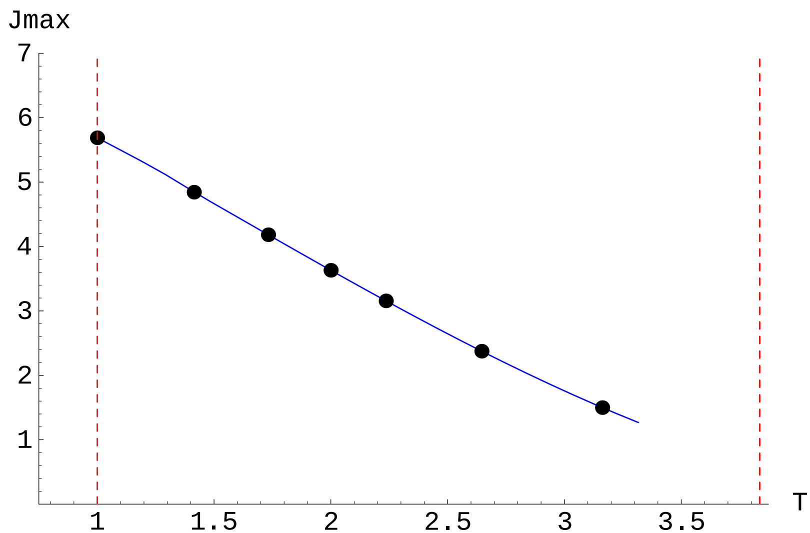
$T = 0$: as usual, $\exists$ finite volume and lattice spacing, and mostly non-chiral quarks with unphysical masses.

$T > 0$: even in the limit of perfect data, it is not a Laplace transform, so strictly speaking it is not clear how to invert for $\rho_V(\omega)$. In practice impose as "prior" the free behaviour at large ω and use e.g. "maximum entropy method" to estimate result at moderate ω.

Recent developments:
Aarts, Allton, Foley, Hands, Kim, hep-lat/0703008;
Meyer, 0704.1801.

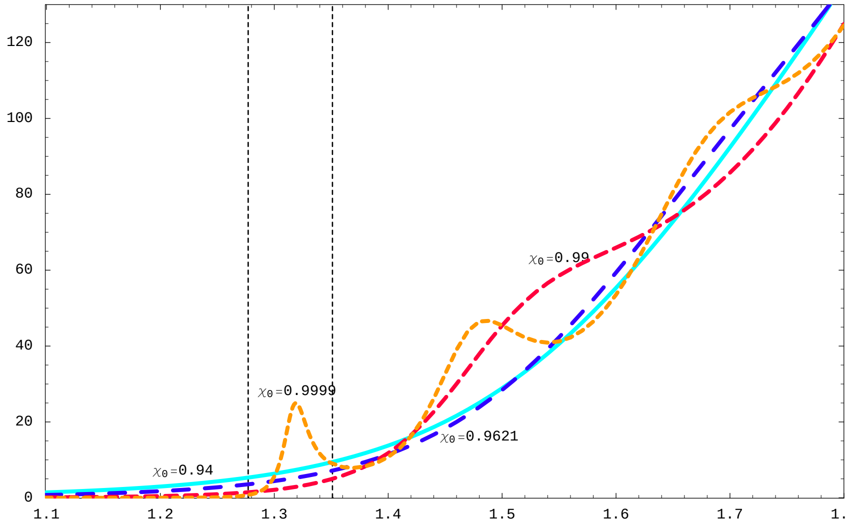
3. AdS/CFT

Melting temperatures:



Peeters, Sonnenschein, Zamaklar, hep-th/0606195, " Holographic melting and related properties of mesons in a quark gluon plasma"

Spectral functions:



Myers, Starinets, Thomson, 0706.0162, " Holographic spectral functions and diffusion constants for fundamental matter"

4. Perturbation theory

At $T > 0$, perturbation theory suffers from infrared divergences, which require complicated resummations (weak-coupling expansion $\neq$ loop expansion). As a result, the weak-coupling series is typically of the form

$$\langle O \rangle \sim 1 + \#_1 \, g^2 + \#_2 \, g^3 + \#_3 \, g^4 \ln \frac{1}{g} + \#_4 \, g^6 + \ldots \, ,$$

where some coefficients can be non-perturbative.

Moreover, even if the coefficients were known, the convergence of the series can be very slow.

Appendix A: momentum scales at $T \neq 0$

QCD $\equiv$ 4d YM + quarks; $\omega_n \sim 2\pi T$

$\Downarrow$ perturbation theory (1)

EQCD $\equiv$ 3d YM + A_0; $m_\mathrm{D} \sim gT$

$\Downarrow$ perturbation theory (2)

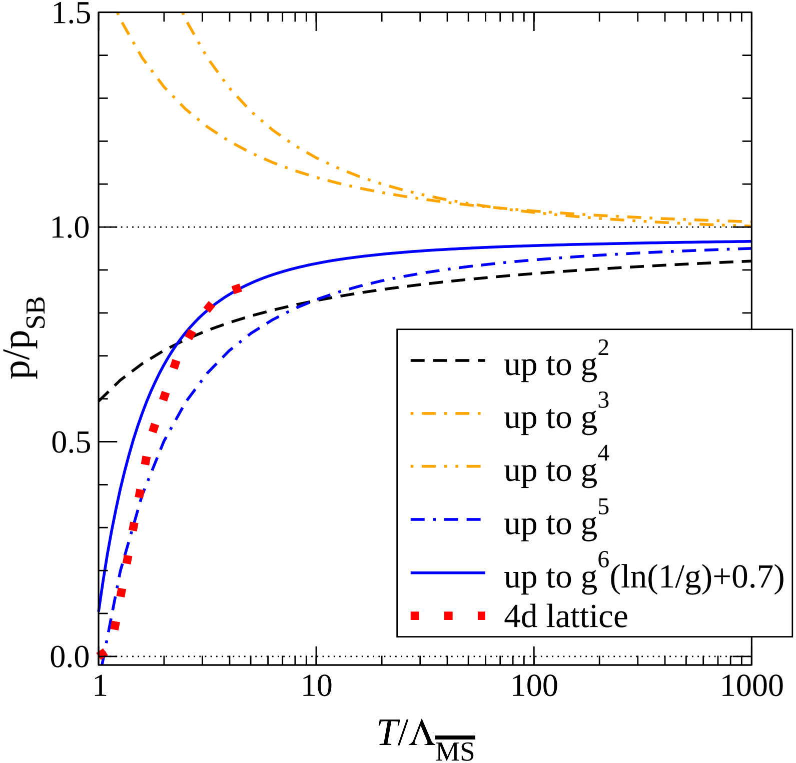
MQCD $\equiv$ 3d YM; $g_3^2 \sim g^2 T$

$\Downarrow$ non-perturbative computation (3)

PHYSICS

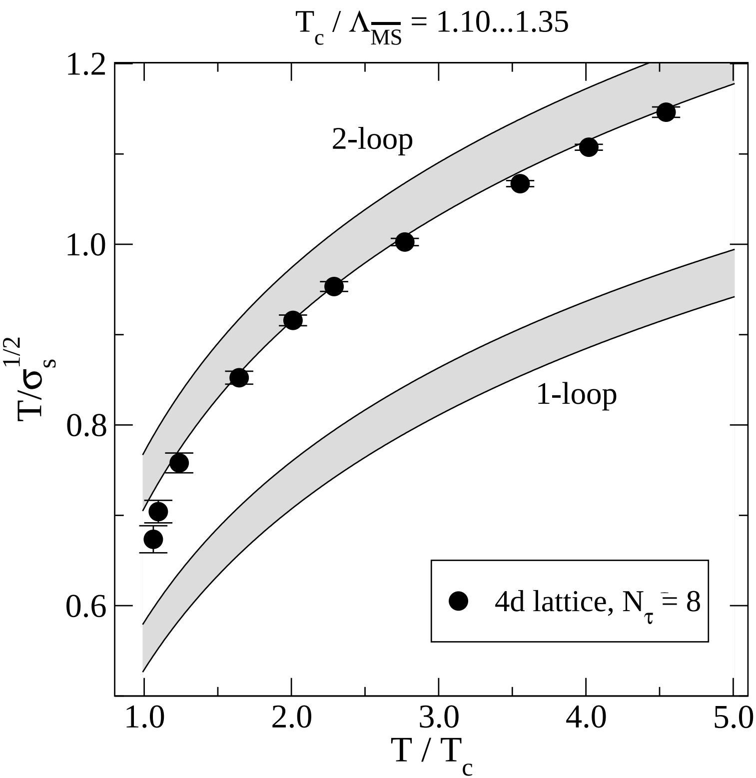
Expansion parameter: $\epsilon_{(i)} \sim g^2 T / 4\pi |\mathbf{k}|_{(i)}$.

Example of slow convergence: pressure



Kajantie et al, hep-ph/0211321

Example of faster convergence: spatial string tension

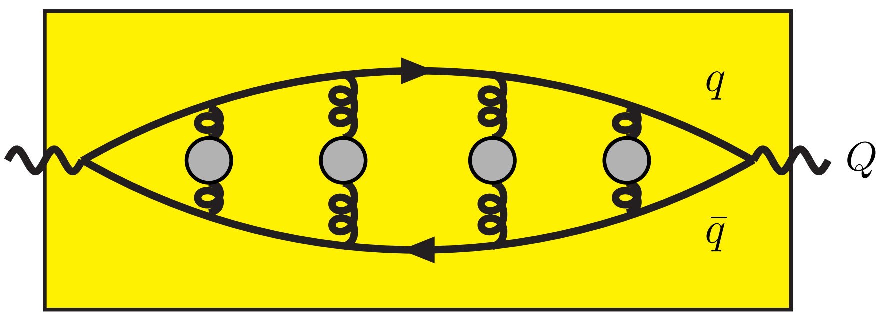


Laine, Schröder, hep-ph/0503061

$\Rightarrow$ **Practical mindset these days:**

Compute the observable with all available methods, possessing complementary systematic errors, and hope to find a consistent picture!

Here: weak-coupling expansion, i.e. "just" graphs:



Momentum/energy scales

Vacuum: $M, g^2M, g^4M, \ldots$

Finite temperature: $T, gT, g^2T, \ldots$

The procedure now depends on the ratio of M and T.

$T \sim g^2M \Rightarrow$ width $\sim g^6M \ll$ binding energy $\sim g^4M$
$\Rightarrow$ bound state exists.

$T \sim gM \Rightarrow$ width $\sim g^3M \gg$ binding energy $\sim g^4M$
$\Rightarrow$ bound state has melted.

In the following assume, formally, $g^2M < T < gM$.
Then the computation proceeds in the following steps:

1. Relation of production rate to Green's function

$$\frac{\mathrm{d}N_{\mu^+\mu^-}}{\mathrm{d}^4x\mathrm{d}^4Q} = -\frac{e^2}{3(2\pi)^5Q^2}\Big(1+\frac{2m_\mu^2}{Q^2}\Big)\Big(1-\frac{4m_\mu^2}{Q^2}\Big)^{\frac{1}{2}}e^{-\frac{q^0}{T}}\tilde{C}_>(Q) \; ,$$

$$\tilde{C}_>(Q) \equiv \int_{-\infty}^{\infty}\mathrm{d}t \int \mathrm{d}^3\mathbf{x}\, e^{iQ\cdot x}\langle \hat{\mathcal{J}}^\mu(x)\hat{\mathcal{J}}_\mu(0)\rangle \; ,$$

$$\hat{\mathcal{J}}^\mu(x) = \ldots + \frac{2}{3}e\,\hat{\bar{c}}\,(x)\gamma^\mu\hat{c}(x) - \frac{1}{3}e\,\hat{\bar{b}}\,(x)\gamma^\mu\hat{b}(x) \; ,$$

$$\langle\ldots\rangle \equiv \mathcal{Z}^{-1}\,\mathrm{Tr}[\exp(-\hat{H}/T)(\ldots)] \; .$$

[Rather than $\tilde{C}_>$ one often considers the spectral function:

$$\rho(Q) = \frac{1}{2}(1-e^{-\frac{q^0}{T}})\tilde{C}_>(Q) \; .$$]

Appendix B: Time orderings at finite temperature

Consider 2-point functions; $x \equiv (t, \mathbf{x})$; $\tilde{x} \equiv (\tau, \mathbf{x})$;

$\hat{A}(t) = e^{i\hat{H}t}\hat{A}(0)e^{-i\hat{H}t}$; $\hat{A}(\tau) = e^{\hat{H}\tau}\hat{A}(0)e^{-\hat{H}\tau}$.

$$\tilde{C}_>(Q) \equiv \int \mathrm{d}t\, \mathrm{d}^3\mathbf{x}\, e^{iQ\cdot x} \langle \hat{A}(x)\hat{B}(0)\rangle \ ,$$

$$\tilde{C}_<(Q) \equiv \int \mathrm{d}t\, \mathrm{d}^3\mathbf{x}\, e^{iQ\cdot x} \langle \hat{B}(0)\hat{A}(x)\rangle \ ,$$

$$\tilde{C}_R(Q) \equiv i \int \mathrm{d}t\, \mathrm{d}^3\mathbf{x}\, e^{iQ\cdot x} \langle [\hat{A}(x), \hat{B}(0)]\theta(t)\rangle \ ,$$

$$\tilde{C}_T(Q) \equiv \int \mathrm{d}t\, \mathrm{d}^3\mathbf{x}\, e^{iQ\cdot x} \langle \hat{A}(x)\hat{B}(0)\theta(t) + \hat{B}(0)\hat{A}(x)\theta(-t)\rangle ,$$

$$\tilde{C}_E(\tilde{Q}) \equiv \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3\mathbf{x}\, e^{i\tilde{Q}\cdot\tilde{x}} \langle \hat{A}(\tilde{x})\hat{B}(0)\rangle \ .$$

2. Properties of the Green's function at large M

Restrict to $\mathbf{q} = \mathbf{0}$ and introduce point-splitting:

$$C_>(t;\mathbf{r},\mathbf{r}') \equiv \int \mathrm{d}^3\mathbf{x} \left\langle \hat{\bar{\psi}}\left(t,\mathbf{x}+\frac{\mathbf{r}}{2}\right)\gamma^\mu\, W\, \hat{\psi}\left(t,\mathbf{x}-\frac{\mathbf{r}}{2}\right)\ \hat{\bar{\psi}}\,(0,-\frac{\mathbf{r}'}{2})\gamma_\mu W'\hat{\psi}(0,\frac{\mathbf{r}'}{2})\right\rangle .$$

The $\mathbf{r}$-dependence is not physical ...

$$\tilde{C}_>(Q) = \int_{-\infty}^{\infty} \mathrm{d}t\, e^{iq^0 t} C_>(t;\mathbf{0},\mathbf{0}) \ ,$$

... but it facilitates perturbative solution: to $\mathcal{O}(g^0)$,

$$\left\{ i\partial_t - \left[2M - \frac{\nabla_\mathbf{r}^2}{M} + \mathcal{O}\left(\frac{1}{M^3}\right)\right]\right\} C_>(t;\mathbf{r},\mathbf{r}') = 0 \ ,$$

$$C_>(0;\mathbf{r},\mathbf{r}') = -6N_\mathrm{c}\,\delta^{(3)}(\mathbf{r}-\mathbf{r}') + \mathcal{O}\left(\frac{1}{M^2}\right) .$$

3. Definition of a real-time static potential

The static potential $V_>(t, r)$ is defined to be the term independent of M in the "exact" Schrödinger equation:

$$\Big\{ i\partial_t - \Big[2M + V_>(t,r) - \frac{\nabla_{\mathbf{r}}^2}{M} + \mathcal{O}\big(\frac{1}{M^2}\big) \Big] \Big\} C_> = 0 \ .$$

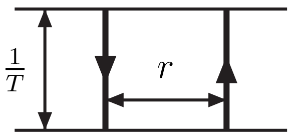
It can thus be obtained in the limit $M \to \infty$, whereby the heavy quarks can be replaced by Wilson lines.

Noting that $C_>(t; \mathbf{r}, \mathbf{r}) \propto C_E(it, r)$, where $C_E(\tau, r)$ is the Euclidean Wilson loop, we are lead to

$$i\partial_t C_E(it, r) \equiv V_>(t, r) C_E(it, r) \ .$$

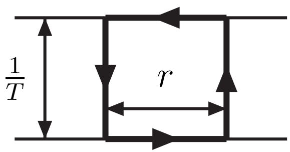
Appendix C: A few different static potentials

From Polyakov loops:



$$\langle \mathrm{Tr}[P]\,\mathrm{Tr}[P^\dagger]\rangle \equiv e^{-\frac{V_\mathsf{a}(r,T)}{T}} \ .$$

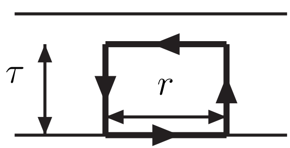
From a Wilson loop:



$$\langle \mathrm{Tr}[W_E(\frac{1}{T},r)]\rangle \equiv e^{-\frac{V_\mathsf{b}(r,T)}{T}} \ .$$

Or may also Legendre transform from "free energy" to "internal energy": $U_\mathsf{i} = V_\mathsf{i} + TS_\mathsf{i} = V_\mathsf{i} - T\partial_T V_\mathsf{i}$.

From an analytic continuation:



$$\langle \mathrm{Tr}[W_E(\tau,r)]\rangle \equiv C_E(\tau,r) \ .$$

4. Result for real-time static potential

In Hard Thermal Loop perturbation theory, to $\mathcal{O}(g^2)$:

$$
\begin{aligned}
\mathrm{Re}\, V_{>}^{(2)}(\infty, r) &= -\frac{g^2 C_F}{4\pi}\left[m_{\mathrm{D}} + \frac{\exp(-m_{\mathrm{D}} r)}{r}\right] , \\
\mathrm{Im}\, V_{>}^{(2)}(\infty, r) &= -\frac{g^2 T C_F}{4\pi}\, \phi(m_{\mathrm{D}} r) ,
\end{aligned}
$$

where

$$
\phi(x) = 2\int_0^\infty \frac{\mathrm{d}z\, z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right] ,
$$

is finite and strictly increasing, with the limiting values $\phi(0) = 0$, $\phi(\infty) = 1$.

Appendix D: Hard Thermal Loop propagators

Introducing the projection operators

$$P^T_{00}(\tilde{Q}) \;=\; P^T_{0i}(\tilde{Q}) = P^T_{i0}(\tilde{Q}) \equiv 0 \;, \quad P^T_{ij}(\tilde{Q}) \equiv \delta_{ij} - \frac{\tilde{q}_i \tilde{q}_j}{\tilde{\mathbf{q}}^2} \;,$$

$$P^E_{\mu\nu}(\tilde{Q}) \;\equiv\; \delta_{\mu\nu} - \frac{\tilde{q}_\mu \tilde{q}_\nu}{\tilde{Q}^2} - P^T_{\mu\nu}(\tilde{Q}) \;,$$

the Euclidean gluon propagator reads

$$\langle A^a_\mu A^b_\nu \rangle = \delta^{ab} \Big[\frac{P^T_{\mu\nu}(\tilde{Q})}{\tilde{Q}^2 + \Pi_T(\tilde{Q})} + \frac{P^E_{\mu\nu}(\tilde{Q})}{\tilde{Q}^2 + \Pi_E(\tilde{Q})} + \xi \frac{\tilde{q}_\mu \tilde{q}_\nu}{(\tilde{Q}^2)^2} \Big] \;,$$

where ξ is the gauge parameter.

The Hard Thermal Loop self-energies read

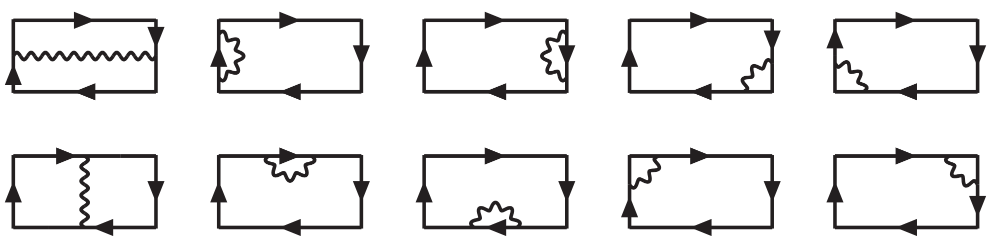
$$\Pi_T(\tilde{Q}) \quad = \quad \frac{m_{\rm D}^2}{2}\Big\{\frac{(i\tilde{q}_0)^2}{\tilde{\mathbf{q}}^2} + \frac{i\tilde{q}_0}{2|\tilde{\mathbf{q}}|}\Big[1 - \frac{(i\tilde{q}_0)^2}{\tilde{\mathbf{q}}^2}\Big] \ln\frac{i\tilde{q}_0 + |\tilde{\mathbf{q}}|}{i\tilde{q}_0 - |\tilde{\mathbf{q}}|}\Big\} \,,$$

$$\Pi_E(\tilde{Q}) \quad = \quad m_{\rm D}^2\Big[1 - \frac{(i\tilde{q}_0)^2}{\tilde{\mathbf{q}}^2}\Big]\Big[1 - \frac{i\tilde{q}_0}{2|\tilde{\mathbf{q}}|} \ln\frac{i\tilde{q}_0 + |\tilde{\mathbf{q}}|}{i\tilde{q}_0 - |\tilde{\mathbf{q}}|}\Big] \,,$$

where $\tilde{q}_0$ denotes bosonic Matsubara frequencies, and

$$m_{\rm D}^2 = g^2T^2\Big(\frac{N_{\rm c}}{3} + \frac{N_{\rm f}}{6}\Big) \,.$$

Graphs:

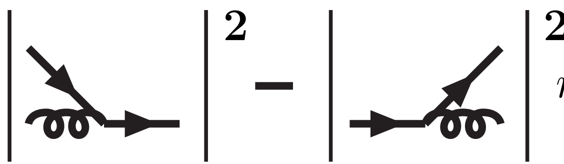


Physics of real part:

2 $\times$ thermal mass correction for a heavy quark $+$
r-dependent Debye-screened potential.

Physics of imaginary part:

almost static (off-shell) gluons may disappear due to interactions with hard particles in the plasma.

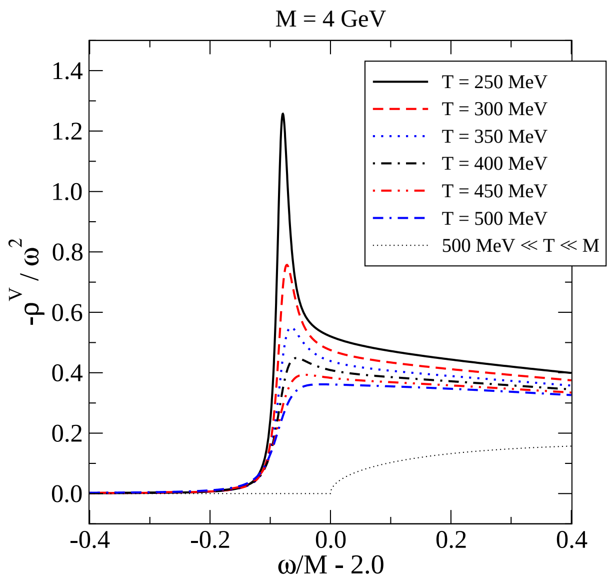
$n_{\mathbf{F}} n_{\mathbf{B}}(1-n_{\mathbf{F}})$  $n_{\mathbf{F}}(1+n_{\mathbf{B}})(1-n_{\mathbf{F}})$

This is the phenomenon of Landau-damping.

Consequently, there is no stationary wave function:
the bound state is a short-lived transient!

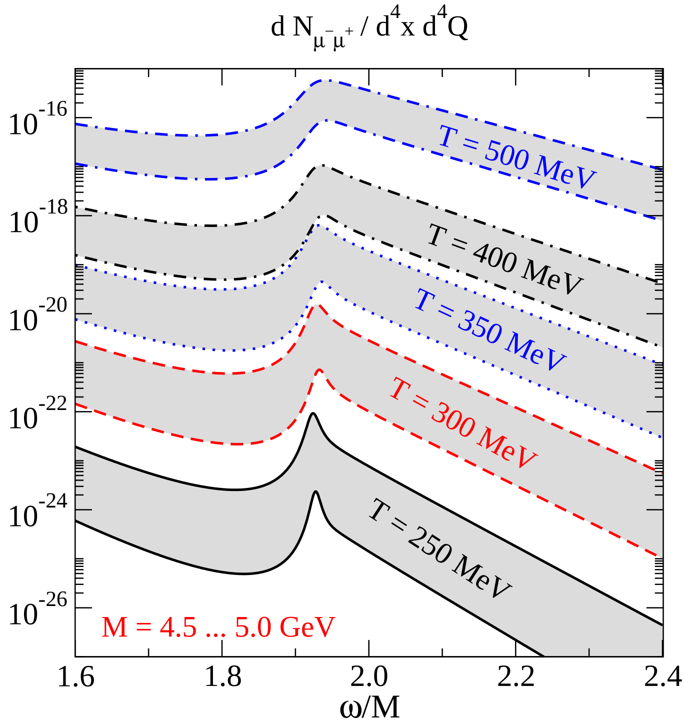
5. Result for the spectral function

Insert $V_>(\infty, r)$ to the time-dependent Schrödinger equation, solve, and Fourier transform to $\omega \equiv q^0$.



Basic structure as suggested by Matsui and Satz (1986) from phenomenological arguments. Melting temperature $\sim$ consistent with potential models and lattice QCD within ± 50 MeV.

Dilepton production rate: $\frac{\mathrm{d}N_{\mu^+\mu^-}}{\mathrm{d}^4x\mathrm{d}^4Q} \propto \frac{\rho_V(\omega)}{\omega^2}e^{-\frac{\omega}{T}}$



The peak is boosted because of the Boltzmann factor.

Conclusions

Useful definition of a finite-temperature real-time static potential is non-trivial. $V_>$ originates from a physical observable; it has both a real and an imaginary part.

Using this potential, the existence and disappearance of a peak in the quarkonium spectral function is qualitatively a **weak-coupling** phenomenon.

At high temperatures, there is no stationary wave function. The bound state is a short-lived transient!

In the end, for quantitative understanding, need to go to higher orders / compare with other methods.