Aspects of thermodynamics of the $\mathcal{N} = 4$ theory on S^3

S. Prem Kumar

Swansea University

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(KITP, Santa Barbara)

Introduction

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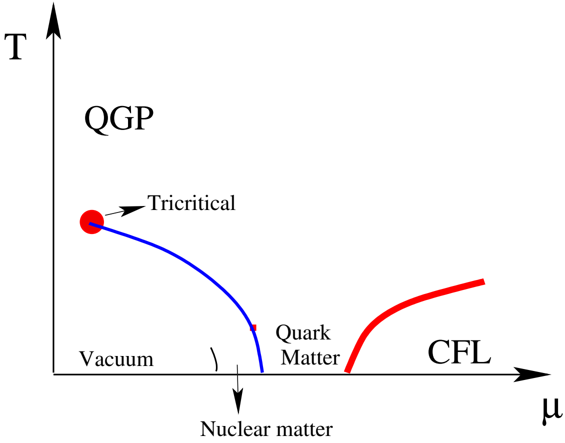
E.g. QCD phase diagram



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Polyakov loop

- Low T: $\langle u_1 \rangle = 0 \implies$ Confined Phase

First Order Transition (Svetitsky, Yaffe)

- High T: $\langle u_1 \rangle \neq 0 \implies$ Deconfined Phase $\rightarrow$ $\mathbb{Z}_N$ breaking

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- Field theory on $S^3 \times R_t \simeq$ conformal boundary of global AdS_5.

- $T \neq 0$: $\mathcal{N} = 4$ SYM on $S^3 \times S^1 \simeq$ boundary of Euclidean AdS_5 space with thermal S^1.

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- SUGRA on AdS_5 yields $\lambda \to \infty$ field theory dynamics

- Can gauge theory at $\lambda \ll 1$ provide a window into AdS gravity ?

Free theory ($\lambda = 0$) on $S^3 \times S^1$

(Sundborg; Aharony, Marsano, Minwalla, Papadodimas, van Raamsdonk)

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$$\text{E.g. } \mathrm{Tr}\underbrace{\left[\phi_1\phi_2\ldots\phi_2\phi_2\ldots\right]}_{L} \qquad \text{Energy} \sim L$$

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- $\mathcal{Z} = \mathrm{Tr}\, e^{-\beta\Delta}$ can be computed in a Wilsonian approach:

- A_0 has a zero mode on $S^3 \times S^1$

$$\alpha = \int_0^\beta A_0 d\tau \qquad U \equiv e^{i\alpha}$$

• Integrating out all KK harmonics on $S^1 \times S^3$, obtain an effective action for the zero mode of $U = e^{i\alpha}$

$$\mathcal{Z} = \int [dU] \;\; \exp[\sum_{m=1}^{\infty} a_m(TR) \;\mathrm{Tr}U^m \;\mathrm{Tr}U^{\dagger m}]$$

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• Eigenvalues $(\alpha_1, \alpha_2, \ldots \alpha_N)$ experience

Vandermonde repulsion $\sim$ $\log|\sin(\frac{\alpha_i - \alpha_j}{2})|$ $+$

T-dependent attraction

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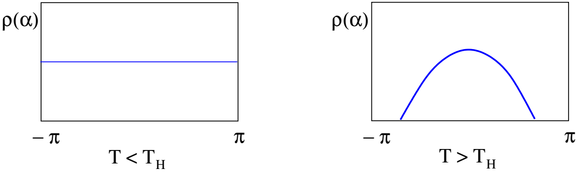

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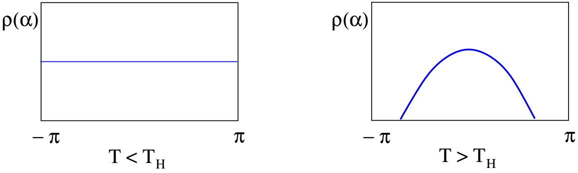

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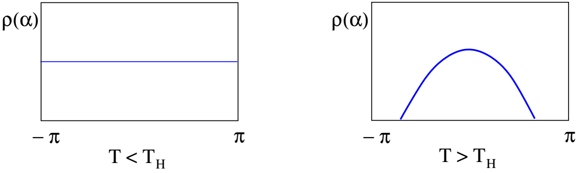

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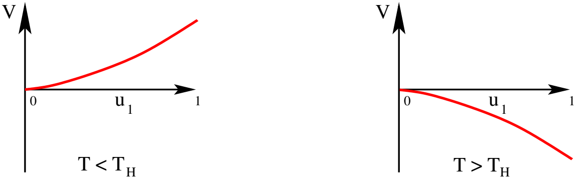
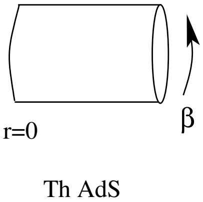
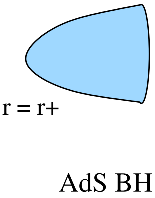


$\langle u_1 \rangle = 0$ $\langle u_1 \rangle \neq 0$ $\mathbb{Z}_N$ breaking

• $\lambda = 0$ picture consistent with $\lambda = \infty$

At $\lambda = \infty$: first order Hawking-Page transition between Thermal AdS and the Big AdS-Schwarzschild Black Hole





$W = \langle \frac{1}{N} \mathrm{Tr} e^{i \int_\beta (A_0 + \ldots)} \rangle = 0$

$W = e^{-S_{F1}} \neq 0.$

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$$V = N^2(m^2(T)|u_1|^2 + b|u_1|^4); \qquad b \sim \lambda^2$$

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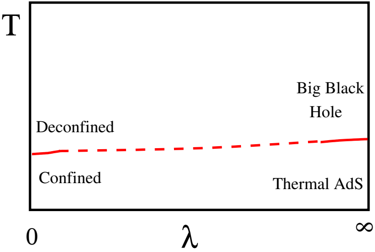
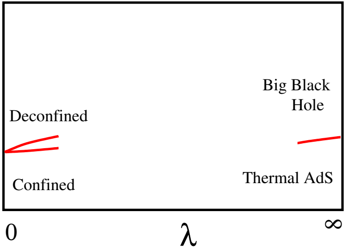
$b < 0$ $\qquad$ $b > 0$



Possibility 1



Possibility 2

Introducing Chemical Potentials

• Chemical potentials (μ_1, μ_2, μ_3) for $U(1)^3 \subset SU(4)_R$ global symmetry.

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- On S^3, all scalars have a conformal mass $\frac{1}{R^2}$

$$V_0 = \frac{N}{\lambda}\mathrm{Tr}\left(\frac{1}{2}(R^{-2} - \mu_p^2)(\phi_{2p}^2 + \phi_{2p-1}^2) - [\phi_a, \phi_b]^2\right)$$

Phase diagram at $\lambda = 0$

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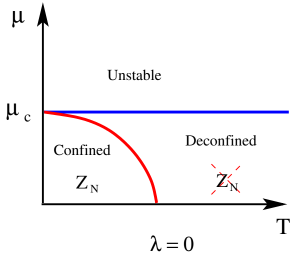

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• Along the classically flat directions

$$\begin{pmatrix} \phi_{a1} & . & . & . \\ . & \phi_{a2} & . & . \\ . & . & \phi_{a3} & . \\ . & . & . & . \end{pmatrix}$$

integrate out all heavy off-diagonal modes, $m^2 \sim |\phi_i - \phi_j|^2 + \ell^2$.

- Background field gauge on S^3

$$\mathcal{L}^{(\mathrm{gf})} = \frac{1}{2g^2}\mathrm{Tr}\left[\left(\nabla_i A^i + \tilde{D}_0 A^0 - i[\phi, \delta\phi]\right)^2 \right.$$
$$\left. + \bar{c}(-\tilde{D}_0^2 - \Delta^{(s)} + [\phi, .]^2)c \ \right].$$

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- Fluctuation determinants yield Casimir sums at $T = 0$:

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- With critical μ_p fermions can have integer moding on S^3:
$\varepsilon_F = \sqrt{(\ell + \frac{1}{2})^2 + \phi_{ij}^2} \to \sqrt{(\ell + \frac{1}{2} \pm \frac{\mu_1}{2})^2 + \phi_{ij}^2} \pm \frac{\mu_2}{2} \pm \frac{\mu_3}{2}.$

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$$V_1^{\mathrm{b}} = (2\pi^2 R^3)^{-1} \sum_{ij=1}^{N} \Lambda^4 R^3 - \frac{1}{2} R\Lambda^2 - R^3 \phi_{ij}^2 \Lambda^2 + \frac{1}{12R} - \frac{1}{4}\phi_{ij}^2 R$$

$$+\frac{1}{2}\phi_{ij}^4 R^3 \log\left(\frac{|\phi_{ij}| e^{1/4}}{2\Lambda}\right) + 8\int_{R\phi_{ij}}^{\infty} \frac{x^2\sqrt{x^2 R^{-2} - \phi_{ij}^2}}{e^{2\pi x} - 1}.$$

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$$V_1^{\mathrm{f}} = (2\pi^2 R^3)^{-1} \sum_{ij=1}^{N} -\Lambda^4 R^3 + \tfrac{1}{2} R\Lambda^2 + R^3 \phi_{ij}^2 \Lambda^2 + \tfrac{5}{48R}$$

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- For two and three critical μ_p, the ground states are the $\frac{1}{4}$ and $\frac{1}{8}$ BPS states.

$TR << 1$ and $|\mu_1 - \mu_c| \lesssim \mathcal{O}(\lambda)$

- At $\mu_1 = \mu_c$, switch on a small non-zero T ($TR \ll 1$)
- Joint potential for α_i and scalars:

$$V_1 = \sum_{ij=1}^{N} \left(\frac{1}{\mathrm{Vol}(S^3)} \left[\frac{3}{16R} - 8\,T e^{-\frac{1}{TR}\sqrt{1+R^2\phi_{ij}^2}} \times \cos\left(\frac{\alpha_i - \alpha_j}{T}\right) + \mathcal{O}(e^{-2/TR}) \right] \right).$$

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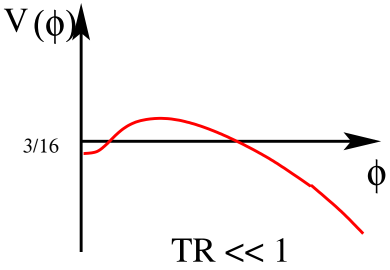
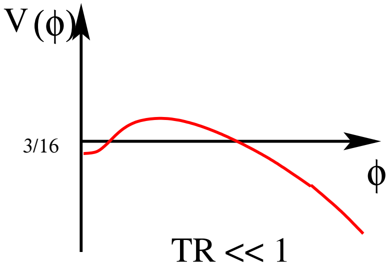
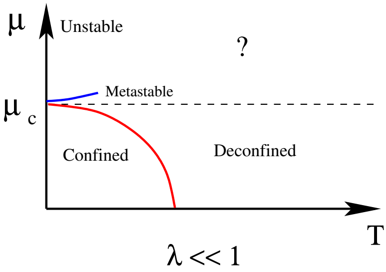
- All $\alpha_i = 0$ – deconfined phase: $u_1 = 1$.
- 1-loop term vanishes at large ϕ_{ij}, and has positive curvature near $\phi_i = 0$.
- For some values of $\mu \gtrsim \mu_c$, V_1 can overcome tree level instability near $\phi_i = 0$.


V (ϕ)
3/16
ϕ
TR << 1



- For $TR \ll 1$ metastable state with $(\mu_1 - \mu_c) \leq \frac{1}{R} \lambda \exp(\frac{-1}{TR})$

- Thermal activation and tunnelling rates $\propto \exp(-N e^{-\frac{1}{TR}})$



- Width of metastable band $\sim \lambda \, e^{\frac{-1}{TR}}$

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- Scalars have a thermal mass λT^2 near the origin $\phi_i = 0$.
- At large $|\phi_{ij}|$, quantum corrections vanish, effective potential has classical behaviour.

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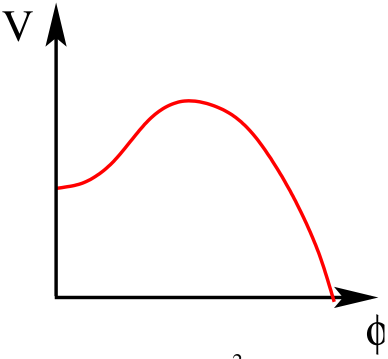
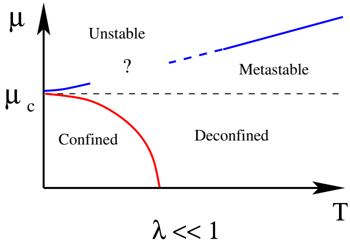
- At high temperatures $\frac{1}{\sqrt{\lambda}} \gtrsim TR \gg 1$, theory is deconfined $(\alpha_i = 0)$.
- Scalars have a thermal mass λT^2 near the origin $\phi_i = 0$.
- At large $|\phi_{ij}|$, quantum corrections vanish, effective potential has classical behaviour.
- Thus for $\mu_c < \mu < \sqrt{\lambda T^2 + \mu_c^2}$, there is a metastable phase near the origin, with decay rate $\sim e^{-N/\lambda^{\frac{3}{2}}}$.

High T metastable potential

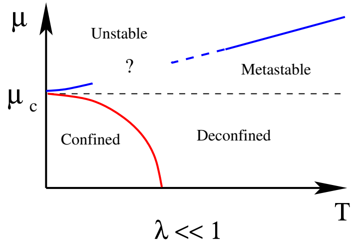


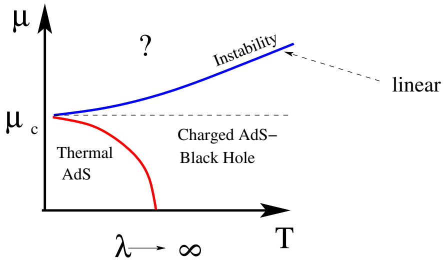
$1/\lambda > (TR)^2 >> 1$

Weak-strong comparison



Weak-strong comparison





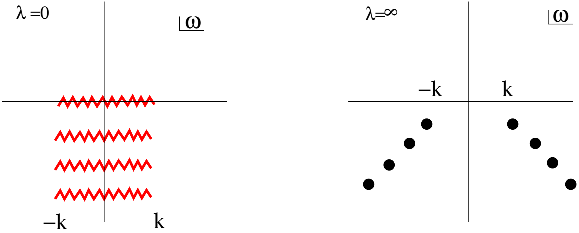
(Cvetic,Gubser;Behrndt,Cvetic,Sabra;Yamada)

Further directions

- Unitary matrix model for on S^3, truncted to the 'b' term, as a model for extracting small black holes;blackhole -string phase transition. (Alvarez-Gaume,Gomez,Liu,Wadia; Basu,Wadia; Dutta,Gopakumar)
- An effective potential for the Polyakov loop from gravity. (Headrick)
- Eigenvalue distributions for the Polyakov-Maldacena both at weak and strong coupling. (Hartnoll,SPK)

• Real time correlators at high temperature, $TR \to \infty$- Poles vs. Cuts.

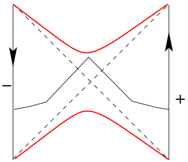
E.g. $\langle \mathrm{Tr} F^2(t,\vec{x}) \mathrm{Tr} F^2(0) \rangle^{\mathrm{ret}}_{\omega,\vec{k}}$

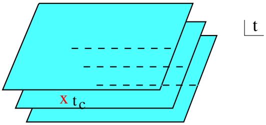


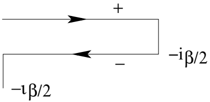
(Hartnoll,SPK)

• More generally, branch cuts from graphs at $\lambda \ll 1$ should turn into poles corresponding to BH quasinormal frequencies at $\lambda \to \infty$.

• Real time correlators as probes of black hole singularities. (Fidkowski,Hubeny,Kleban,Shenker)







$$\langle \mathcal{O}^{+}(t)\mathcal{O}^{-}(-t)\rangle \sim \frac{1}{(t-t_c)^{2\Delta}}; \quad \Delta \gg 1.$$

• Exponential falloff of correlator at large imaginary frequency. (Festuccia,Liu)

• Remnants of such signals in weakly coupled field theory?