Making and Probing
Inflation and Preheating

Non-equilibrium phenomena in the early universe

Making Inflation in QFT and String Theory

Preheating after QFT and String Theory Inflation

Observables from Preheating

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Fig. 1. Left: composition of the expanding universe is changing with time as it cools down. Right: reciprocal universe in terms of physical momenta of particles expands backwards in time to open particle physics at higher and higher temperatures. Icons illustrate physics at different energies.
Early Universe Inflation

Scale factor \( a(t) \)

Equation of State \( t \leq 10^{-35} \text{ sec} \)

\[ p \approx -\epsilon \]

Inflation \( a(t) \approx e^{Ht} \)

Realization of Inflation

Scalar field

\[ p = \frac{1}{2} \dot{\phi}^2 - V \]

\[ \epsilon = \frac{1}{2} \dot{\phi}^2 + V \]

slow roll \( \dot{\phi}^2 \ll V \)
Generation of Cosmological Fluctuations
Light field at inflation

\[ \hat{\Sigma} = \int \mathrm{d}^3k (a_k \hat{\varphi}_k(t) e^{i k \cdot \mathbf{x}} + h.c.) \]

\[ \ddot{\chi}_k + 3H \dot{\chi}_k + \frac{k^2}{a^2} \chi_k = 0 \]

\[ \chi_k \approx \frac{H}{\sqrt{2} k^{3/2}} \]
Hubble radius

wavelength \( \frac{a(t)}{k} \)

during inflation

during recombination
Scalar metric Fluctuations
\[ ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Phi)a^2d\bar{x}^2 \]
Scalar field fluctuations
\[ \phi(t, \bar{x}) = \phi(t) + \delta\phi(t, \bar{x}) \]

\[ \delta R^\mu_\nu = \frac{8\pi}{M_p^2} \delta T^\mu_\nu \]

\[ u = a\delta\phi, \quad z = \frac{a\dot{\phi}}{H}, \quad \eta = \int dt/a \]

\[ u''_k + \left( k^2 - \frac{z''}{z} \right) u_k = 0 \]

spectrum \[ P_s(k) = \frac{k^3}{2\pi^2} \left| \frac{u_k}{z} \right|^2 \sim \frac{V^3}{M_p^6 V^2_{,\phi}} \]

\[ k = aH \]
\[ \Phi_k = \frac{16}{15} \frac{(3\pi V)^{3/2}}{M_p^3 V,\phi} |_{\phi=\phi(t_k)} \cdot \frac{m^2}{2} \phi^2 \]

\[ \phi(t) \approx \phi_0 - \sqrt{\frac{2}{3}} m t, \quad a(t) \approx a_0 \exp \left[ \frac{2\pi}{M_p^2} (\phi_0^2 - \phi(t)^2) \right] \]

\[ \Phi_k \approx 0.4 \frac{m}{M_p} \frac{\log k}{k^{3/2}} \]

\[ V(\phi) \sim \phi^n \]

\[ k^2 |\Phi_k|^2 = A_s \left( \ln \frac{k_0}{k} \right)^{\frac{n+2}{2}} \]
\[
\left( \ln \frac{k_0}{k} \right)^{\frac{n+2}{2}} = \left( \ln \frac{k_0}{k_*} + \ln \frac{k_0}{k_*} \right)^{\frac{n+2}{2}} \approx N^{\frac{n+2}{2}} \left( 1 - \frac{n+2}{2N} \ln \frac{k_*}{k} \right)
\]

number of efoldings \( N = \ln \frac{k_0}{k_*} \)

Often \( P_s(k) = A_s \left( \frac{k}{k_*} \right)^{n_s-1} \)

\[
\left( \frac{k}{k_*} \right)^{(n_s-1)} = e^{(n_s-1) \ln \frac{k}{k_*}} \approx 1 + (n_s - 1) \ln \frac{k}{k_*}
\]

\[
n_s - 1 \approx -\frac{n + 2}{2N}
\]
Logarithmic running

\[ k^3 |\Phi_k|^2 = A_s \left( \ln \frac{k_0}{k} \right)^{\frac{n+2}{2}} \]
Logarithmic running

\[ k^3 |\Phi_k|^2 = A_s \left( \ln \frac{k_0}{k} \right)^{\frac{n+2}{2}}\]
Tensor metric Fluctuations

\[ ds^2 = -dt^2 + (\eta_{ij} + h_{ij})a^2dx^idx^j \]

\[ h^i_i = 0, \quad h^i_{j;i} = 0, \quad i, j = 1, 2, 3. \]

\[ \delta R^\mu_\nu = 0 \]

\[ h''_k + \left( k^2 - \frac{a''}{a} \right) h_k = 0 \]

Spectrum \( P_t(k) \sim \frac{H^2}{M_p^2} \sim \frac{V}{M_p^4} \)

often \( P_t(k) = A_t \left( \frac{k}{k_*} \right)^{n_t} \quad r = A_t/A_s \)
Scalar metric Fluctuations from Inflation
\[ ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Phi)a^2d\vec{x}^2 \]

**Initial conditions from Inflation**

Random Gaussian Field \( \Phi(\vec{x}) \)

\[ \Omega_{tot} = 1 \]
\[ k^3\Phi_k^2 \rightarrow P_s = A_s k^{n_s-1} \]
\[ P_T = \frac{H^2}{M_p^2} k^{n_T} \]
\[ N = 62 - \ln \frac{10^{16} \text{Gev}}{V_{inf}^{1/4}} \]

\[ a = e^{Ht} \]

inflation

Hot FRW

\[ a = \sqrt{t} \]
WMAP3 sees $3^{rd}$ pk, B03 sees $4^{th}$
2007
\[ P_s(k) = A_s \left( \frac{k}{k_\ast} \right)^{n_s-1} \]
ACBAR, CBI see 4th 5th pk

CBI excess 07
ACBAR excess 08

WMAP3 sees 3rd pk, B03 sees 4th

ACBAR, CBI see 4th 5th pk

CBI excess 07
ACBAR excess 08
Early Universe Inflation

Scale factor $a(t)$

$$e^{Ht}$$

$t^{1/2}$

time

Equation of State $t \leq 10^{-35}$ sec

$p \approx -\epsilon$

Inflation $a(t) \approx e^{Ht}$

Realization of Inflation

Scalar field

$p = \frac{1}{2} \dot{\phi}^2 - V$

$\epsilon = \frac{1}{2} \dot{\phi}^2 + V$

slow roll $\dot{\phi}^2 \ll V$

choice of $V(\phi)$
Inflation in the context of ever changing fundamental theory

1980
- $R^2$-inflation
- Old Inflation
- New Inflation
- Chaotic inflation
- Double Inflation
- Power-law inflation
- SUGRA inflation
- Extended inflation

1990
- Hybrid inflation
- SUSY F-term inflation
- SUSY D-term inflation
- Assisted inflation
- Brane inflation

2000
- SUSY P-term inflation
- Super-natural Inflation
- K-flation
- Roulette
- DBI inflation
- Warped Brane inflation
- $D3 - D7$ inflation
- Tachyon inflation
- Racetrack inflation
Family phase portrait of inflation

\[ \ddot{\phi} + 3H\dot{\phi} + V,\phi = 0 \]

\[ 3H^2 = \frac{8\pi}{M_p^2} \left( \frac{\dot{\phi}^2}{2} + V \right) \]
Fig. 8. Phase portrait for the theory $V(\phi) = \frac{1}{2} m^2 \phi^2 + V_0$ for $V_0 < 0$. The branches describing stages of expansion and contraction (upper and lower parts of the hyperboloid) are connected by a throat.
Inflation in String Theory: Cosmology with Compactification
Inflation in String Theory: Cosmology with Compactification
String Theory inflation models

**Modular Inflation.** They use Kahler moduli/axion like the fields that are a present in the KKLT stabilization.

**Brane inflation.** The inflaton field corresponds to the distance between branes in Calabi-Yau space. Historically, this was the first class of string inflation models.
Inflation with branes in String Theory

Prototype of hybrid inflation

4-dim picture
Inflation with branes in String Theory

Prototype of hybrid inflation

\[ V = V_0 \left[ 1 - \left( \frac{\Delta}{\kappa \phi} \right)^4 \right] \]
Kahler moduli Inflation (Conlon&Quevedo hep-th/050912) 
Roulette Inflation-Kahler moduli/axion 
(Bond, LK, Prokushkin&Vandrevange hep-th/0612197)

Figure 1: Schematic illustration of the ingredients in Kähler moduli inflation. The four-cycles of the CY are the Kähler moduli $T_i$ which govern the sizes of different holes in the manifold. We assume $T_3$ and the overall scale $T_1$ are already stabilized, while the last modulus to stabilize, $T_2$, drives inflation while settling down to its minimum. The imaginary parts of $T_i$ have to be left to the imagination. The outer $3 + 1$ observable dimensions are also not shown.
\[ V(\phi, \bar{\phi}) = e^{\mathcal{K}/M_P^2} \left( \mathcal{K}^{ij} D_i \hat{W} D_j \bar{\hat{W}} - \frac{3}{M_P^2} \hat{W} \bar{\hat{W}} \right) + \text{D-terms} \]

\[ \frac{\mathcal{K}}{M_P^2} = -2 \ln \left( \mathcal{V} + \frac{\xi}{2} \right) + \ln g_s + \mathcal{K}_{cs} \]

\[ \hat{W} = \frac{g_s^3 M_P^3}{\sqrt{4\pi}} \left( W_0 + \sum_{i=1}^{h^{1,1}} A_i e^{-a_i T_i} \right), \quad W_0 = \frac{1}{l_s^2} \int_M G_3 \wedge \Omega \]

\[ T_i = \tau_i + i\theta_i \]

\[ V(T_1, ..., T_n) = \frac{12 W_0^2 \xi}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} + \sum_{i=2}^{n} \frac{12 e^{-2a_i \tau_i} \xi A_i^2}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} + \frac{16(a_i A_i)^2 \sqrt{\tau_i} e^{-2a_i \tau_i}}{3a_2 (2\mathcal{V} + \xi)} \]

\[ + \frac{32 e^{-2a_i \tau_i} A_i^2 \tau_i (1 + a_i \tau_i)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)} + \frac{8 W_0 A_i e^{-a_i \tau_i} \cos(a_i \theta_i)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)} \left( \frac{3\xi}{2\mathcal{V} + \xi} + 4a_i \tau_i \right) \]

\[ + \sum_{i,j=2}^{n} \frac{A_i A_j \cos(a_i \theta_i - a_j \theta_j)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} e^{-(a_i \tau_i + a_j \tau_j)} \left[ 32(2\mathcal{V} + \xi)(a_i \tau_i + a_j \tau_j) + 2a_i a_j \tau_i \tau_j \right] + 24 \xi \] + \text{V}_{\text{uplift}}. \]
String theory landscape of the Kahler moduli/axion Inflation

\[ T_i = \tau_i + i\theta_i \]
Ensemble of Inflationary trajectories
Lessons:

Multiple fields Inflation

Ensemble of acceleration histories (trajectories) for the same underlying theory

Prior probabilities of trajectories $P(H(t))$

Small amplitude of gravity waves $r$ from inflation $r \sim 10^{-10}$
for the brane inflation \( \frac{\Delta \phi}{M_p} < \frac{2}{\sqrt{n}} \)

\[
r \leq 0.01 \left( \frac{4}{n} \right) \left( \frac{60}{N} \right)^2
\]

\[n = MK\]
Measurement of GW from CMB anisotropy polarization

- **Current limit**: 0.1
- **Planck Satellite 2008.6**
- **CMBPol 2017**
- **Ultimate limit**: 0.001

Current limit values:
- ≤ 0.34
- 0.1
- 0.01
- 0.001

Ultimate limit values:
- 0.001
Particlegenesis

$10^{-43}$ sec

Inflation
no entropy
no temperature

$10^{-35}$ sec

\[ \mathcal{L}(\phi, \chi, \psi, A_\mu, \psi_\mu, h_{\mu\nu}) \]

BANG
Output of Preheating

- Reheat temperature $T_R$

- Out-of-equilibrium state

- Evolution of EoS

- Number of e-folds
  \[ N = 62 - \ln \frac{10^{16} \text{GeV}}{V_h^{1/4}} + \frac{1}{4} \ln \frac{V_h}{V_{end}} - \frac{1}{12} \ln \frac{V_{end}}{\rho_{rad}} \]

- Potential observables
Large field models

\[ V = \phi^\alpha \]

\[ n - 1 = -\frac{2 + \alpha}{2N} \quad r = \frac{4\alpha}{N} \]
Resonant Preheating in Chaotic Inflation

\[ \frac{1}{2} \phi_{\mu} \phi^{\mu} + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \chi_{\mu} \chi^{\mu} + g^2 \phi^2 \chi^2 \]

**Quantum**

\[ \hat{\chi}(t, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3 k \left( \hat{a}_k \chi_k(t) e^{-i k \mathbf{x}} + \hat{a}^+_k \chi \right) \]

\[ \ddot{\chi}_k + 3H \dot{\chi}_k + \left( \frac{k^2}{a^2} + g^2 \phi^2 \right) \chi_k = 0 \]

\[ \tau = mt \]

\[ \ddot{x}_k + \left( \frac{k^2}{m^2} + q \sin^2 \tau \right) x_k = 0 \]

Parameter \( q = \frac{g^2 \phi^2}{m^2} \sim g^2 10^{10} \)

Occupation number

\[ n_k = \frac{\omega_k}{2} \left( \frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2 \right) - \frac{1}{2} \]

\[ n_k \sim e^{\mu t} \]
Resonant Preheating in Chaotic Inflation

\[
\frac{1}{2} \phi_\mu \phi^\mu + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \chi_\mu \chi^\mu + g^2 \phi^2 \chi^2
\]

Quantum

\[
\hat{\chi}(t, x) = \frac{1}{(2\pi)^{3/2}} \int d^3 k \left( \hat{a}_k \chi_k(t) e^{-i k x} + \hat{a}_k^+ \chi \right)
\]

\[
\ddot{\chi}_k + 3H \dot{\chi}_k + \left( \frac{k^2}{a^2} + g^2 \phi^2 \right) \chi_k = 0
\]

\[
\tau = mt
\]

\[
\ddot{X}_k + \left( \frac{k^2}{m^2} + q \sin^2 \tau \right) X_k = 0
\]

Parameter \( q = \frac{g^2 \phi^2}{m^2} \sim g^2 \times 10^{10} \)

Occupation number

\[
n_k = \frac{\omega_k}{2} \left( \frac{\dot{\chi}_k^2}{\omega_k^2} + |\chi_k|^2 \right) - \frac{1}{2}
\]

\( n_k \sim e^{\mu t} \)
\[ X_k^j(t) = \frac{\alpha_k^j}{\sqrt{2\omega}} e^{-i \int_0^t \omega dt} + \frac{\beta_k^j}{\sqrt{2\omega}} e^{+i \int_0^t \omega dt} \]

\[ X_k^{j+1}(t) = \frac{\alpha_k^{j+1}}{\sqrt{2\omega}} e^{-i \int_0^t \omega dt} + \frac{\beta_k^{j+1}}{\sqrt{2\omega}} e^{+i \int_0^t \omega dt} \]

\[
\begin{pmatrix}
\alpha_k^{j+1} e^{-i \theta_k^j} \\
\beta_k^{j+1} e^{+i \theta_k^j}
\end{pmatrix} = \begin{pmatrix}
\frac{1}{D_k} & R_k^* \\
\frac{R_k}{D_k} & \frac{1}{D_k^*}
\end{pmatrix} \begin{pmatrix}
\alpha_k^j e^{-i \theta_k^j} \\
\beta_k^j e^{+i \theta_k^j}
\end{pmatrix}
\]

\[ \frac{d^2 X_k}{d\tau^2} + (\kappa^2 + \tau^2) X_k = 0. \]
\[ \kappa^2 = \frac{k^2}{gm\phi_0} \]

**Method of successive scatterings**

\[
n_k^{j+1} = e^{-\pi \kappa^2} + \left(1 + 2e^{-\pi \kappa^2}\right)n_k^j - 2e^{-\frac{\pi}{2} \kappa^2} \sqrt{1 + e^{-\pi \kappa^2}} \sqrt{n_k^j (1 + n_k^j)} \sin \theta_{tot}^j
\]

\[
n_k^{j+1} \approx \left(1 + 2e^{-\pi \kappa^2} - 2 \sin \theta_{tot}^j e^{-\frac{\pi}{2} \kappa^2} \sqrt{1 + e^{-\pi \kappa^2}}\right)n_k^j
\]

\[
n_{j+1} \approx e^{4\pi j \mu_k} = e^{2\mu_k m t}
\]
Resonant Preheating in Chaotic Inflation

\[ g^2 \phi^2 \chi^2 \]

\[ \text{parameter } q = \frac{g^2 \phi_0^2}{m^2} \sim g^2 \times 10^{10} \gg 1 \]

\[ \delta \chi \]
FIG. 1: Evolution of spectra in the combination $k^2 \omega_k n_k$ of the $\phi$ and $\chi$ fields during and immediately after preheating. Bluer plots show later spectra. Horizontal axis $k$ is in units of $m$.

FIG. 2: Evolution of comoving number density of $\phi$ (red, lower plot) and $\chi$ (blue, upper plot) in units of $mass^3$. Time is in units of $1/m$.

FIG. 3: Evolution of the ratio $(\langle f^2 \rangle^2 / \langle f^4 \rangle)$, where $f$ represents the $\phi$ field (red, solid) or the $\chi$ field (blue, dashed) and angle brackets represent a spatial average, is a measure of gaussianity. This ratio is one for a random gaussian field. Time is in units $1/m$. 
Decay of inflaton and preheating after inflation

\[ \frac{1}{2} \phi_{\mu} \phi^{\mu} + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \chi_{\mu} \chi^{\mu} + g^2 \phi^2 \chi \]

Classical

\[ \phi_0 + \phi \]

Quantum

\[ \chi \]

\[ \ddot{\phi} - \nabla^2 \phi + g^2 \chi^2 \phi = 0 \]

\[ \ddot{\chi} - \nabla^2 \chi + g^2 \phi^2 \chi = 0 \]

Felder, LK

hep-ph/0606256
Classical

\[ \phi_0 + \phi \]

Quantum

\[ \chi \]

\[
\frac{1}{2} \phi_\mu \phi^\mu + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \chi_\mu \chi^\mu + g^2 \phi^2 \chi^2
\]

Decay of inflaton and preheating after inflation

movie
Classical Quantum

Decay of inflaton and preheating after inflation

Classical

\[ \phi_0 + \phi \]

Quantum

\[ \chi \]
Evolution of energy density

Evolution of gravitational potential
Tachyonic Preheating in Hybrid Inflation

\[ V(\phi, \sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{g^2}{2} \phi^2 \sigma^2 \]

bifurcation point \( \phi_c = \frac{\sqrt{\lambda}v}{g} \)
Tachyonic Preheating

\[ V_F = V_0 + \frac{\lambda}{4} \sigma^4 - \frac{\lambda^3}{4} \sigma^3 + \lambda \sigma^2 \]

\[ \sigma(t, \vec{x}) \]
FIG. 10. Deviations from Gaussianity for the field \( \phi \) as a function of time. The solid, red line shows \( 3 \langle \delta \phi^2 \rangle^2 / \langle \delta \phi^4 \rangle \) and the dashed, blue line shows \( 3 \langle \delta \phi^4 \rangle^2 / \langle \delta \phi^4 \rangle \).
preheating exp
excitation

Kolmogorov
turbulence
of bosons

Non-linear
“bubbly” stage

thermalization
1. In many, if not all viable models of inflation there exists a mechanism for exponentially amplifying fluctuations of at least one field $\chi$. These mechanisms tend to excite long-wavelength excitations, giving rise to a highly infrared spectrum.

2. Exciting one field $\chi$ is sufficient to rapidly drag all other light fields with which $\chi$ interacts into a similarly excited state.

3. The excited fields will be grouped into subsets with identical characteristics (spectra, occupation numbers, effective temperatures) depending on the coupling strengths.

4. Once the fields are amplified, they will approach thermal equilibrium by scattering energy into higher momentum modes.

5. There is a stage of turbulence before thermalization. EoS very rapidly evolves towards radiation domination before thermalization.
\[ D(t)^2 \equiv \sum_A (|f'_A - f_A|)^2 + (|\dot{f}'_A - \dot{f}_A|)^2. \]

FIG. 8. The Lyapunov exponent $\lambda'$ for the fields $\phi$ and $\chi$ using the normalized distance function $\Delta$. 
Three-linear interaction

In expanding universe complete inflaton decay requires 3-legs interactions

\[ V = \frac{m^2}{2} \phi^2 + \frac{\sigma}{2} \phi \chi^2 + \frac{g^2}{2} \phi^2 \chi^2 + \frac{\lambda}{4} \chi^4 \rightleftharpoons W = \frac{m}{2\sqrt{2}} \phi^2 + \frac{g}{2\sqrt{2}} \phi \chi^2 \]

\[ \lambda = \frac{g^2}{2} \text{ and } \sigma = gm \]

\[ \chi''_k + (A_k - 2q \cos 2z)\chi_k = 0 \]

\[ mt = 2z - \frac{\pi}{2}, \ A_k = \frac{4k^2}{m^2} \text{ and } q = \frac{2\sigma \Phi}{m^2} \]

\[ A_k \geq 2q \]

\[ 0 < A_k < 2q \]

Broad Parametric Resonance

Tachyonic Resonance
Above the barrier

\[ \chi_k^j(t) = \frac{\alpha_k^j}{\sqrt{2\omega_k(t)}} \exp \left( -i \int_{t_0}^{t} \omega_k(t') dt' \right) + \frac{\beta_k^j}{\sqrt{2\omega_k(t)}} \exp \left( i \int_{t_0}^{t} \omega_k(t') dt' \right) \]

Below the barrier

\[ \chi_k(t) \sim \frac{\alpha_k^j}{\sqrt{2\Omega_k(t)}} \exp \left( - \int_{t_{kj}}^{t} \Omega_k(t') dt' \right) + \frac{b_k^j}{\sqrt{2\Omega_k(t)}} \exp \left( \int_{t_{kj}}^{t} \Omega_k(t') dt' \right) \]

\[
\begin{pmatrix}
\alpha_k^{j+1} \\
\beta_k^{j+1}
\end{pmatrix} = e^{X_k^j} \begin{pmatrix} 1 & i e^{2i\theta_k^j} \\ -i e^{-2i\theta_k^j} & 1 \end{pmatrix} \begin{pmatrix}
\alpha_k^j \\
\beta_k^j
\end{pmatrix}
\]

\[ n_k^j = |\beta_k^j|^2 = \exp(2jX_k) (2\cos\Theta_k)^{2(j-1)} \]

\[ X_k \simeq -\frac{x}{\sqrt{q}} A_k + 2x \sqrt{q} \quad \text{for} \quad x \simeq 0.85 \]

\[ j = 4, \quad q = 20 \]
Three- vs four-legs

\[ q_3 = \frac{\sigma \Phi}{m^2} \quad \text{and} \quad q_4 = \frac{g^2 \Phi^2}{m^2} \]

Four-legs dominates at preheating

\[ q_3 < q_4^{3/4} \quad \leftrightarrow \quad \text{SUSY} \quad q_3 = \sqrt{q_4} \]

\[ \Phi \sim \frac{1}{a^{3/2}} \]

Three-legs dominates after preheating
Light field at inflation

\[ \hat{\chi}(t) = \int d^3k (a_k \chi_k(t) e^{i \mathbf{k} \cdot \mathbf{x}} + \text{h.c.}) \]

\[ \ddot{\chi}_k + 3H \dot{\chi}_k + \frac{k^2}{a^2} \chi_k = 0 \]

\[ \chi_k \simeq \frac{H}{\sqrt{2}k^{3/2}} \]
Modulated Fluctuations

\[ g^2 \phi^2 \sigma^2 \]

\[ \phi \quad g^2 \quad \sigma \]

coupling depends on moduli \( g^2 = g^2(\chi) \)

Light field at inflation develops fluctuations

\[ \chi_k \approx \frac{H}{\sqrt{2k^{3/2}}} \]

spacial variations \( \delta g^2 = \frac{\partial g^2}{\partial \chi} \delta \chi \)

\[ H^{\hat{a}1} \]

\[ \hat{y}(x) \]

varying \( g^2 = 10^{-7} \) by 5%

Generation of metric fluctuations

\[ \delta \chi_k \rightarrow \delta g^2 \rightarrow \Phi_k \]
Generation of gravitational waves from random media

\[ ds^2 = -dt^2 + a(t)^2 (\delta_{ij} + h_{ij}) \, dx^i \, dx^j \]

\[ h_i^i = 0, \quad h_{j;i} = 0, \quad i, j = 1, 2, 3. \]

\[ \Box h_{ij} = \frac{8\pi}{M_p^2} T^{TT}_{ij} \]
Hubble radius

end of inflation

wavelength \( \frac{a(t)}{k} \)

recombination

\[ h_{ij} = \frac{8\pi}{M_p^2} T^{TT}_{ij} \]
Stochastic background of gravitational waves emitted from preheating after inflation

Khlebnikov, Tkachev, PRD56(1997)653
Easther and Lim, astro-ph/0601617
Felder and LK, hep-ph/0606256
Easther, Giblin and Lim, astro-ph/0612294
Garcia-Bellido and Figueroa, astro-ph/0701014
Dufaux, LK et al astro-ph/0707:0875
Garcia-Bellido, Figueroa, astro-ph/0707:0839
For isolated sources

\[ \square h_{ij} = \frac{8\pi}{M_p^2} T^{TT}_{ij} \]

\[ \frac{dE}{d\Omega} = 2G\Lambda_{ij,lm} \omega^2 T^{ij*}(\vec{k}, \omega) T^{lm}(\vec{k}, \omega) d\omega \]

\[ T_{ij}(k, \omega) = \int \frac{d\tau}{2\pi} e^{i\omega \tau} \int d^3x e^{-i k x} T_{ij}(\tau, x) \]

\[ \Lambda_{ij,lm}(\hat{k}) = \delta_{ij} \delta_{lm} - 2 \hat{k}_j \hat{k}_m \delta_{il} + \frac{1}{2} \hat{k}_i \hat{k}_j \hat{k}_l \hat{k}_m \]

\[ - \frac{1}{2} \delta_{ij} \delta_{lm} + \frac{1}{2} \delta_{ij} \hat{k}_l \hat{k}_m + \frac{1}{2} \delta_{jl} \hat{k}_i \hat{k}_m \]
Emission of stochastic GW by random media

Theory and Numerics of Gravitational Waves from Preheating after Inflation.

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astro-ph:0707.0875
Emission of stochastic GW by random scalar fields

\[
\dddot{h}_{ij}(\tau, k) + k^2 \ddot{h}_{ij}(\tau, k) = 16\pi G a(\tau) T_{ij}^{TT}(\tau, k)
\]

\[
\dot{h}_{ij}(\tau, k) = \frac{16\pi G}{k} \int_{\tau_i}^{\tau} d\tau' \sin[k(\tau - \tau')] a(\tau') T_{ij}^{TT}(\tau', k)
\]

\[
T_{ij}^{TT}(k) = O_{ij,lm}(\vec{k}) \left\{ \partial_l \phi_a \partial_m \phi_a \right\}(k) = O_{ij,lm}(\vec{k}) \int \frac{d^3p}{(2\pi)^{3/2}} p_l p_m \phi_a(p) \phi_a(k-p)
\]

First order phase transitions
Second order phase transitions
Topological defects formation
Thermal bath of scalars
Tachyonic preheating
Resonant preheating
No-go Theorem: No Gravity Waves from Scalar Field Waves

\[ \bar{h}_{ij}(\tau, k) = \frac{16\pi G}{k} \mathcal{O}_{ij,lm}(\hat{k}) \int_{\tau_i}^{\tau} d\tau' \sin[k(\tau - \tau')] a(\tau') \int \frac{d^3 p}{(2\pi)^{3/2}} p_l p_m \phi_a(\tau', p) \phi_a(\tau', k - p) \]

\[ \phi(p, \tau)e^{i p x} = b(p) e^{\pm i \omega_p \tau + i px} \]

\[ \omega_p^2 = p^2 + m^2 + g^4 \psi^2 \]

\[ h_{ij}(\tau, k) \propto e^{\pm i k \tau} \mathcal{O}_{ij,lm}(\hat{k}) \int d^3 p p_l p_m b(p)b(k - p) \int_{\tau_i}^{\tau} d\tau' e^{i (\pm \omega_p \pm \omega_{|k-p|} \pm k) \tau'} \]

k parallel to p

\[ \mathcal{O}_{ij,lm}(\hat{k}) p_l p_m = 0 \]

FIG. 1: Would be emission of a graviton \( h_{ij} \) with momentum \( k \) from the annihilation of two scalar waves \( \phi(p) \) and \( \phi(k - p) \) with momenta \( p \) and \( p - k \). Helicity 2 of the emitted graviton cannot match the helicity zero of the incoming scalar waves.
No-go Theorem: No Gravity Waves from Scalar Field Waves

\[ \bar{h}_{ij}(\tau, \mathbf{k}) = \frac{16\pi G}{k} \mathcal{O}_{ij,lm}(\hat{k}) \int_{\tau_i}^{\tau} d\tau' \sin[k(\tau - \tau')] \ a(\tau') \int \frac{d^3 \mathbf{p}}{(2\pi)^{3/2}} \ p_i \ p_m \ \phi_a(\tau', \mathbf{p}) \ \phi_a(\tau', \mathbf{k} - \mathbf{p}) \]

\[ \phi(\mathbf{p}, \tau) e^{i\mathbf{p} \cdot \mathbf{x}} = b(\mathbf{p}) \ e^{\pm i\omega_p \tau + i\mathbf{p} \cdot \mathbf{x}} \]

\[ \omega_p^2 = p^2 + m^2 + g^2 \psi^2 \]

\[ h_{ij}(\tau, \mathbf{k}) \propto e^{\pm ik\tau} \mathcal{O}_{ij,lm}(\hat{k}) \int d^3 \mathbf{p} \ p_i \ p_m \ b(\mathbf{p}) b(\mathbf{k} - \mathbf{p}) \int_{\tau_i}^{\tau} d\tau' \ e^{i(\pm \omega_p \pm \omega|\mathbf{k} - \mathbf{p}| \pm k) \tau'} \]

No GW emission

FIG. 1: Would be emission of a graviton \( h_{ij} \) with momentum \( \mathbf{k} \) from the annihilation of two scalar waves \( \phi(\mathbf{p}) \) and \( \phi(\mathbf{k} - \mathbf{p}) \) with momenta \( \mathbf{p} \) and \( \mathbf{p} - \mathbf{k} \). Helicity 2 of the emitted graviton cannot match the helicity zero of the incoming scalar waves.
Modulated fluctuations in Hybrid Inflation

\[
V(\phi, \sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{g^2}{2}\phi^2\sigma^2
\]

bifurcation point \( \phi_c = \frac{\sqrt{\lambda}v}{g} \)

\[
\phi_c(\chi_0 + \delta\chi)
\]

inhomogeneous waterfall

Bernardeau, LK, Uzan 03
No-go Theorem: No Gravity Waves from Scalar Field Waves

\[ \tilde{h}_{ij}(\tau, k) = \frac{16\pi G}{k} O_{ij,lm}(\hat{k}) \int_{\tau_i}^{\tau} \sin[k(\tau - \tau')] a(\tau') \int \frac{d^3 p}{(2\pi)^{3/2}} p_l p_m \phi_a(\tau', p) \phi_a(\tau', k - p) \]

\[ \phi(p, \tau) e^{ipx} = b(p) e^{\pm i\omega_p \tau + ipx} \]

\[ \omega_p^2 = p^2 + m^2 + g^2 \psi^2 \]

\[ h_{ij}(\tau, k) \propto e^{\pm ik\tau} O_{ij,lm}(\hat{k}) \int d^3 p p_l p_m b(p)b(k - p) \int_{\tau_i}^{\tau} d\tau' e^{i(\pm \omega_p \pm \omega_{|k-p|} \pm k)\tau'} \]

GW generated for non-trivial dispersion relation

\[ \omega^2 = \vec{p}^2 + g^2 \phi(t)^2 - \frac{a''}{a} \]

FIG. 1: Would be emission of a graviton \( h_{ij} \) with momentum \( k \) from the annihilation of two scalar waves \( \phi(p) \) and \( \phi(k - p) \) with momenta \( p \) and \( p - k \). Helicity 2 of the emitted graviton cannot match the helicity zero of the incoming scalar waves.
\[
\rho_{gw} = \frac{1}{32\pi G} \langle \dot{h}_{ij}(t, x) \dot{h}_{ij}(t, x) \rangle
\]

\[
\frac{(16\pi G)^2}{2} \sum_{i,j} \left\{ \left| \int_{\tau_i}^{\tau_f} d\tau' \cos[k(\tau_f - \tau')] \ a(\tau') \ T_{ij}^{TT}(\tau', k) \right|^2 + \left| \int_{\tau_i}^{\tau_f} d\tau' \sin[k(\tau_f - \tau')] \ a(\tau') \ T_{ij}^{TT}(\tau', k) \right|^2 \right\}
\]

Random gaussian fields

\[
\langle T_{ij}^{TT}(\tau', k) T_{ij}^{TT*}(\tau'', k') \rangle = \mathcal{O}_{ij,lm}(\hat{k}) \mathcal{O}_{ij,rs}(\hat{k}') \int \frac{d^3p}{(2\pi)^{3/2}} \int \frac{d^3p'}{(2\pi)^{3/2}} \ p_l p_m p'_r p'_s \langle \phi_a(p, \tau') \phi_a(k - p, \tau') \phi_b^*(p', \tau'') \phi_b^*(k' - p', \tau'') \rangle
\]

\[
\left( \frac{d\rho_{gw}}{d \ln k} \right)_{\tau > \tau_f} = \frac{S_k(\tau_f)}{a^4(\tau)}
\]

\[
S_k(\tau_f) = \frac{2}{\pi} G k^3 \int \frac{d^3p}{(2\pi)^3} p^4 \sin^4(\hat{k} \cdot \hat{p}) \int_{\tau_i}^{\tau_f} d\tau' \int_{\tau_i}^{\tau_f} d\tau'' \cos[k(\tau' - \tau'')] \ a(\tau') \ a(\tau'') \ F_{ab}(p, \tau', \tau'') \ F_{ab}(|k - p|, \tau', \tau'')
\]

\[
\langle \phi_a(p, \tau') \phi_b^*(p', \tau'') \rangle = F_{ab}(p, \tau', \tau'') \delta(p - p')
\]
Emission of GW from preheating

$$\Omega_{gw} h^2 = 7.8 \times 10^{-5} \ S_k(\tau_f) \ \frac{a^{-4}}{M_{Pl}^2 H_j^2}$$

$$\frac{2}{\pi} G k^3 \int \frac{dP}{(2\pi)^3} p^4 \sin(\hat{k}, \hat{p})$$

$$\left\{ \left| \int_{\tau_i}^{\tau_f} d\tau \cos(k \cdot \tau) a(\tau) \chi_p(\tau) \chi_{|k-p|}(\tau) \right|^2 + \left| \int_{\tau_i}^{\tau_f} d\tau \sin(k \cdot \tau) a(\tau) \chi_p(\tau) \chi_{|k-p|}(\tau) \right|^2 \right\}$$
Numerical calculations of GW emission from Preheating

FIG. 3: Spectrum of energy density in gravity waves calculated along nine different directions in k-space. The
\[ V = \frac{\lambda}{4} \phi^4 + \frac{g^2}{2} \phi^2 \chi^2 \]

\[ q = \frac{g^2}{\lambda} \]

**FIG. 1:** Spectrum of gravity waves energy density in physical variables today, accumulated up to the time \( x_f = 240 \), for the model (48) with \( q = 120 \). The 2 spectra were obtained from simulations with different box sizes, and averaged over different directions in \( k \)-space.

**FIG. 2:** The thick curve shows the total energy density in gravity waves (53) accumulated up to the time \( x_f \), as a function of \( x_f \). The thin curve shows the evolution with time of the total number density, \( n_{\text{tot}} = n_\chi + n_{\phi} \), rescaled to fit on the same figure.
FIG. 3: Spectrum (55) of the gravity waves energy density, accumulated up to different times $x_f$, as a function of the comoving momentum $k$ (in units of $\lambda \phi_0$). The spectra grow from $x_f = 90$ to $x_f = 240$ with spacing $\Delta x_f = 10$.

FIG. 4: Measure of the (unnormalised) total energy density in the two scalar fields per logarithmic momentum interval at different moments of time. The same times as in Fig. (3) are shown, the spectra moving towards UV from $x = 90$ to $x = 240$ with spacing $\Delta x = 10$. 
Numerical calculations of GW emission from Preheating

\[
\hat{\chi}(\tau, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} \left( \hat{a}_k \chi_k(\tau) e^{i \mathbf{k} \cdot \mathbf{x}} + \hat{a}_k^+ \chi_k^*(\tau) e^{-i \mathbf{k} \cdot \mathbf{x}} \right)
\]

\[
S_k(\tau_f) = \frac{2}{\pi} G k^3 \int \frac{d^3 p}{(2\pi)^3} p^4 \sin^4(\mathbf{k} \cdot \mathbf{p}) \left\{ \left| \int_{\tau_i}^{\tau_f} d\tau \cos(k \tau) a(\tau) \chi_p(\tau) \chi_{|\mathbf{k} - \mathbf{p}|}(\tau) \right|^2 + \left| \int_{\tau_i}^{\tau_f} d\tau \sin(k \tau) a(\tau) \chi_p(\tau) \chi_{|\mathbf{k} - \mathbf{p}|}(\tau) \right|^2 \right\}
\]
Analytic check
Shortcut to the answer

estimation

\[ \Omega_{GW} \sim 10^{-6}(RH)^2 \]

size of structures \( R \) vs Hubble radius \( 1/H \)

\[ f \sim \frac{1}{RH} \frac{M}{10^{15} \text{Gev}} \times 10^8 \text{ Hz} \]
topological effects after hybrid inflation (unstable)
formation of defects results in GW emission

\[ \frac{\rho_{gw}}{\rho_r} \sim (R_H)^2 \]
The story of stochastic gravitational waves is CMB anisotropies of 21 century

GW from high energy inflation are targeted by CMB B-mode polarization experiments

GW from low-energy inflation are targeted by GW astronomy
Reheating after String Theory Inflation

Barnaby, Burgess, Cline, hep-th/0412095

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Chialva, Shiu, Underwood, hep-th/0508229

Chen, Tye, hep-th/0512000; 0602136

Dufaux, LK, Peloso 08
Realization of String Theory Hybrid Inflation

throat warped geometry

Mobile brane $\tilde{D}3 \rightarrow D3$

Warped brane inflation

Prototype of hybrid inflation

4-dim picture

$V$ vs. $\phi$
Open strings between branes are unstable

End point of inflation

$\bar{D3}$ $D3$

Closed strings

$\mathcal{h}_{\mu\nu}$

Unstable KK modes

SM particles

BANG

Long-living KK modes related to inner isometries

LK, Yi 05
Cascading Energy from Inflaton to Radiation

Figure 2: Identifying the channels of D-brane decay
KK story

\[ R^4 \times \mathcal{M} \]

\[ h_{AB}(x, y) = \sum_m h^{(m)}(x)f_m(y) \]

\[ m = 0: \text{ usual } 4 \text{ dim gravitons } \Omega_{GW} \simeq e^{-2\Lambda} \]

other \( m \): modes \( m_{KK} \simeq e^{-\Lambda}/R \)

KK particles are thermalized first
SM particles are thermalized much later

KK from M with isometries are stable

No complete decay

KK particles freeze out \( \Omega_{KK} \gg 1 \)
Fluctuations in Cosmology with Compactification

string theorist

Practical cosmologist

cosmologist

\[ \Phi_m(y^c) = \sum_{L,M} Y_L^M(\Omega) \psi_{m,L}(y) \]
metric

\[ G = H^{-1/2}(y) g_{\mu\nu} dx^\mu dx^{\nu} + h_{IJ} dy^I dy^J \]

\[ h \equiv H^{1/2} g \]

wave equation for KK mode

\[ \left[ \frac{H(y)}{\sqrt{h}} \partial_I h^{IJ} \frac{\sqrt{h}}{H(y)} \partial_J + H^{1/2}(y) \nabla_\mu \nabla^\mu \right] \Phi = 0 \]

\[ m_{KK}^2 \]

Throat geometry

\[ H \approx e^{4y}, \quad h = R^2 \left( dy^2 + ds^2_{T_{1,1}} \right) \]

\[ \left[ e^{4y} \partial_\mu e^{-4y} \partial_\mu + m_{KK}^2 R^2 e^{2y} - L^2 \right] \Phi_{m_{KK};L} = 0 \]

\[ J_{\pm\nu}(m_{KK} R e^{y}) \]

\[ \nu^2 = 4 + L^2 \]

“big” CY

\[ H(y) \sim H_0, \]

\[ \left[ \partial_\mu^2 + m_{KK}^2 R^2 H_0^{1/2} - L^2 \right] \Phi_{m_{KK};L} = 0 \]

\[ \Phi_{m_{KK};L} \sim e^{\pm Ly} \]
KK modes interactions

\[ S_1 = \int d^D x \sqrt{\hat{g}} \sqrt{-g} e^{2A} R^{(4)}[g] \]

\[ S_2 = \int d^D x \sqrt{\hat{g}} \sqrt{-g} e^{4A} \left[ \frac{1}{4} g^\mu\nu g^{\lambda\rho} \left( \partial_c g_{\mu\rho} \partial_c g_{\nu\lambda} - \partial_c g_{\lambda\rho} \partial_c g_{\mu\nu} \right) - \frac{1}{2} \partial^\nu g^{\mu\nu} \partial_c g_{\mu\nu} - g^{\mu\nu} \hat{\nabla}_c \hat{\nabla}_c g_{\mu\nu} \right.

\left. \partial^\nu A \left( 6 g^{\mu\nu} \partial_c g_{\mu\nu} + g_{\mu\nu} \partial_c g^{\mu\nu} \right) - 2 \hat{\nabla}_c \hat{\nabla}_c A - 8 \partial_c A \partial^\nu A \right] \]

the spin 2 perturbations \( h_{\mu\nu} \)

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x^\mu, y^i) \quad \text{with} \quad \partial^\mu h_{\mu\nu} = \eta^{\mu\nu} h_{\mu\nu} = 0 \]

\[ h_{\mu\nu}(x^\lambda, y^a) = \sum_m \Phi_m(y^a) \gamma^{(m)}_{\mu\nu}(x^\lambda) \]

\[ \int d^{(D-4)} y \sqrt{\hat{g}} e^{2A} \Phi_m \Phi_{m'} \Phi_{m''} \int d^4 x \gamma^{(m)}_{\mu\nu} \partial_c \gamma^{(m)}_{\nu\rho} \partial^\sigma \gamma^{(m)}_{\mu\rho} \]

\[ \int d^4 x e^{2A(y^i)} g^{\mu\nu} (y^i) \partial_\mu H \partial_\nu H \]
\[ \Omega_{KKst} h^2 \sim 10^{17} \frac{m}{T_F} \left( \frac{V_6^{1/6}}{\sqrt{\alpha'}} \right)^{-6} e^{-2y/\alpha'} \left( \frac{R}{\sqrt{\alpha'}} \right)^{14} \]

\[ \frac{m}{T_F} - \frac{1}{2} \log \left( \frac{m}{T_F} \right) \approx \log \left[ 10^{10} \left( \frac{V_6^{1/6}}{\sqrt{\alpha'}} \right)^{3} e^{y/R} \left( \frac{R}{\sqrt{\alpha'}} \right)^{-15} \right] \]

Inflationary throat \( e^{-A} \sim 10^{-4} \)

\[ \Omega_{KKst} h^2 \gg 1 \]

Standard Model throat \( e^{-A} \sim 10^{-16} \)

\[ \Omega_{KKst} h^2 \sim 10^{-5}, 0.1 \text{ and } 10^{3} \]

for \( R/\sqrt{\alpha'} = 5, 10 \text{ and } 20 \).
Resolution?

- Attachment of KS throat to a compact CY
  Induces symmetry breaking perturbations.

- Tip of KS throat is a particular case of Sasaki-Einstein manifolds.
  There are asymmetric SE manifolds, but no examples of asymmetric throats
Impact of isometry breaking perturbation on KK modes decay

\[ AdS_5 \times S^5 \]

\[ ds^2 = e^{-2y/R} \eta_{\mu\nu} \, dx^\mu \, dx^\nu + dy^2 + R^2 \, f_{ij}(\Omega) \, d\theta^i \, d\theta^j \]

\[ ds^2 = e^{-2y/R} \left[ 1 + \epsilon(y) \, w(\Omega) \right] \eta_{\mu\nu} \, dx^\mu \, dx^\nu + \]

\[ + \, dy^2 + R^2 \left[ f_{ij}(\Omega) + \epsilon(y) \, \delta f_{ij}(\Omega) \right] \, d\theta^i \, d\theta^j \]


\[ \epsilon(y) = e^{-\alpha y/R} \]

\[ \alpha = \sqrt{28} - 4 = 1.29 \]
FIG. 10: Final exclusion region for the parameter in the long throat. The three lines correspond to three reference value of $V^{1/6}/R$. For each case, values of the parameters on the right of the corresponding curve conflict with the phenomenological limits shown in fig. 9. The highest values of $\alpha$ shown result in KK$_\pm$ particles with a much longer lifetime than the age of the universe. In this case, the only relevant bound is that the energy density of the KK$_\pm$ particles does not exceed the one of dark matter in our universe.
Non-equilibrium early universe

**EW Phase Transition**  GW generation from the bubble collisions and turbulence

**Dark Matter freeze-out**  Close to the 250 MeV phase transition, QGP is involved

**Inflation**  Scalar field condensate+fluctuation are out-of-equilibrium

**Preheating after Inflation**  Creation of particles, Inflaton fragmentation and thermalization