

Making and Probing Inflation and Preheating

Non-equilibrium phenomena in the early universe

Making Inflation in QFT and String Theory

Preheating after QFT and String Theory Inflation

Observables from Preheating

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KITP Santa Barbara February 2008

Concise History of the Early Universe

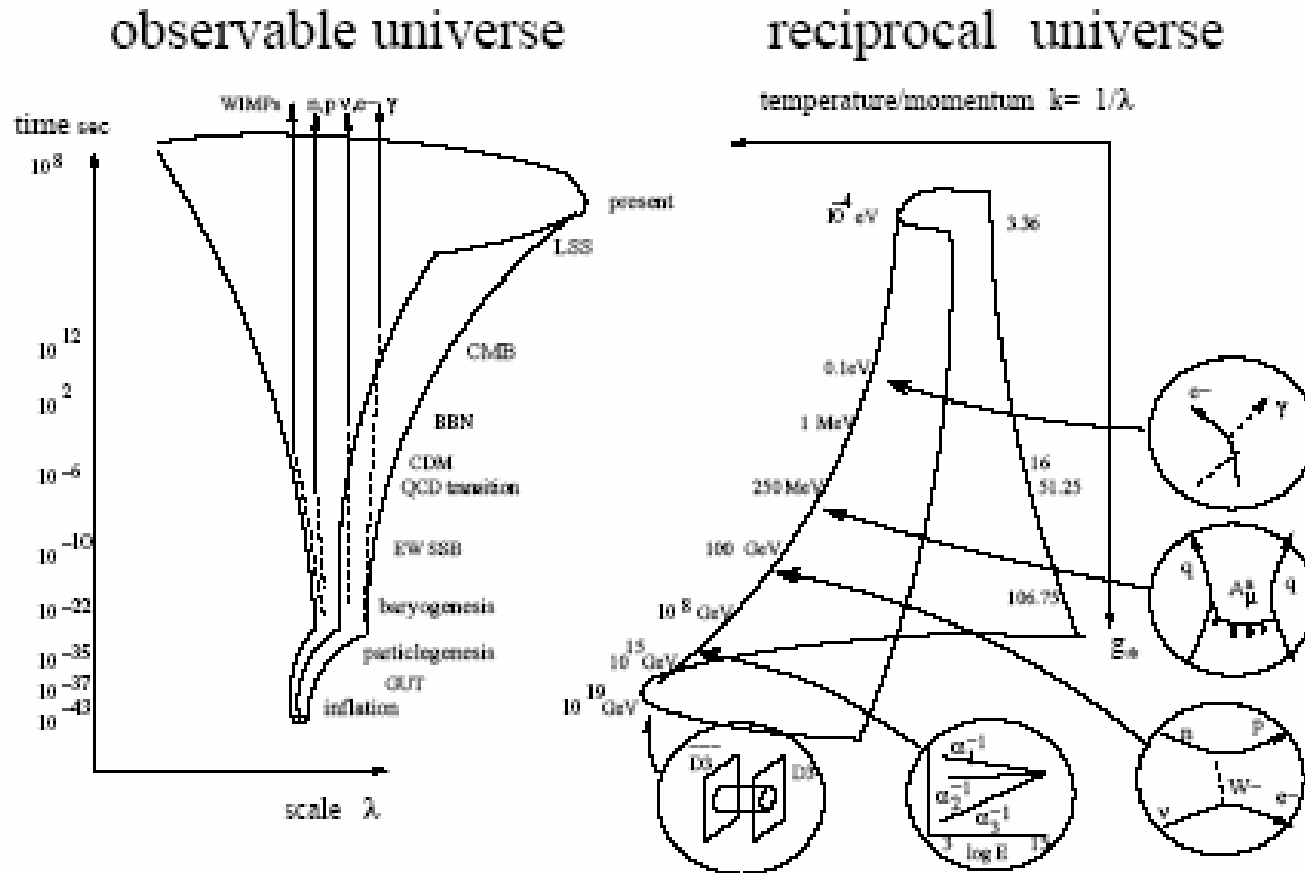
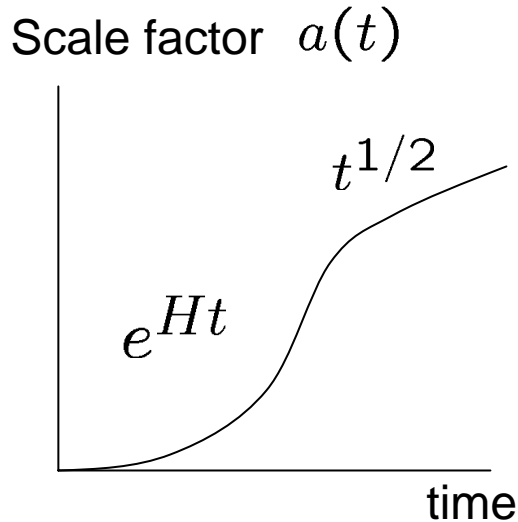


Fig. 1. Left: composition of the expanding universe is changing with time as it cools down. Right: reciprocal universe it terms of physical momenta of particles expands backwards in time to open particle physics at higher and higher temperatures. Icons illustrate physics at different energies.

Early Universe Inflation



Equation of State $t \leq 10^{-35}$ sec

$$p \approx -\epsilon$$

Inflation $a(t) \approx e^{Ht}$

Realization of Inflation

Scalar field

$$p = \frac{1}{2}\dot{\phi}^2 - V$$

$$\epsilon = \frac{1}{2}\dot{\phi}^2 + V$$

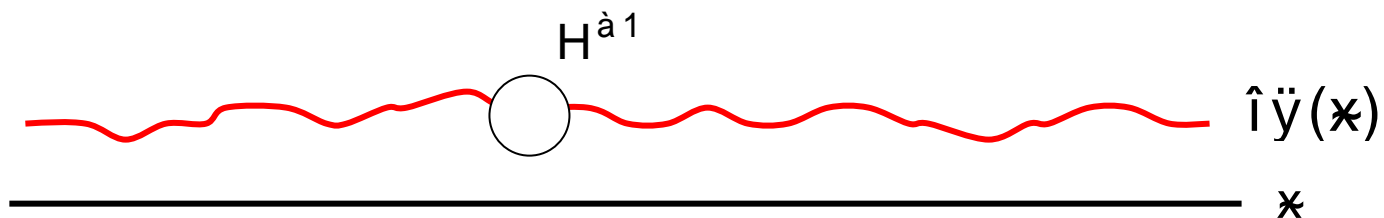
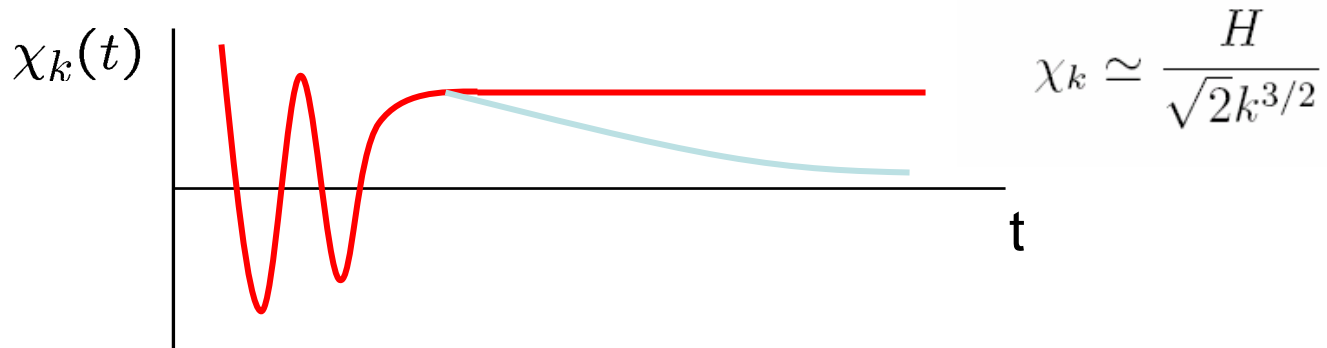
slow roll $\dot{\phi}^2 \ll V$

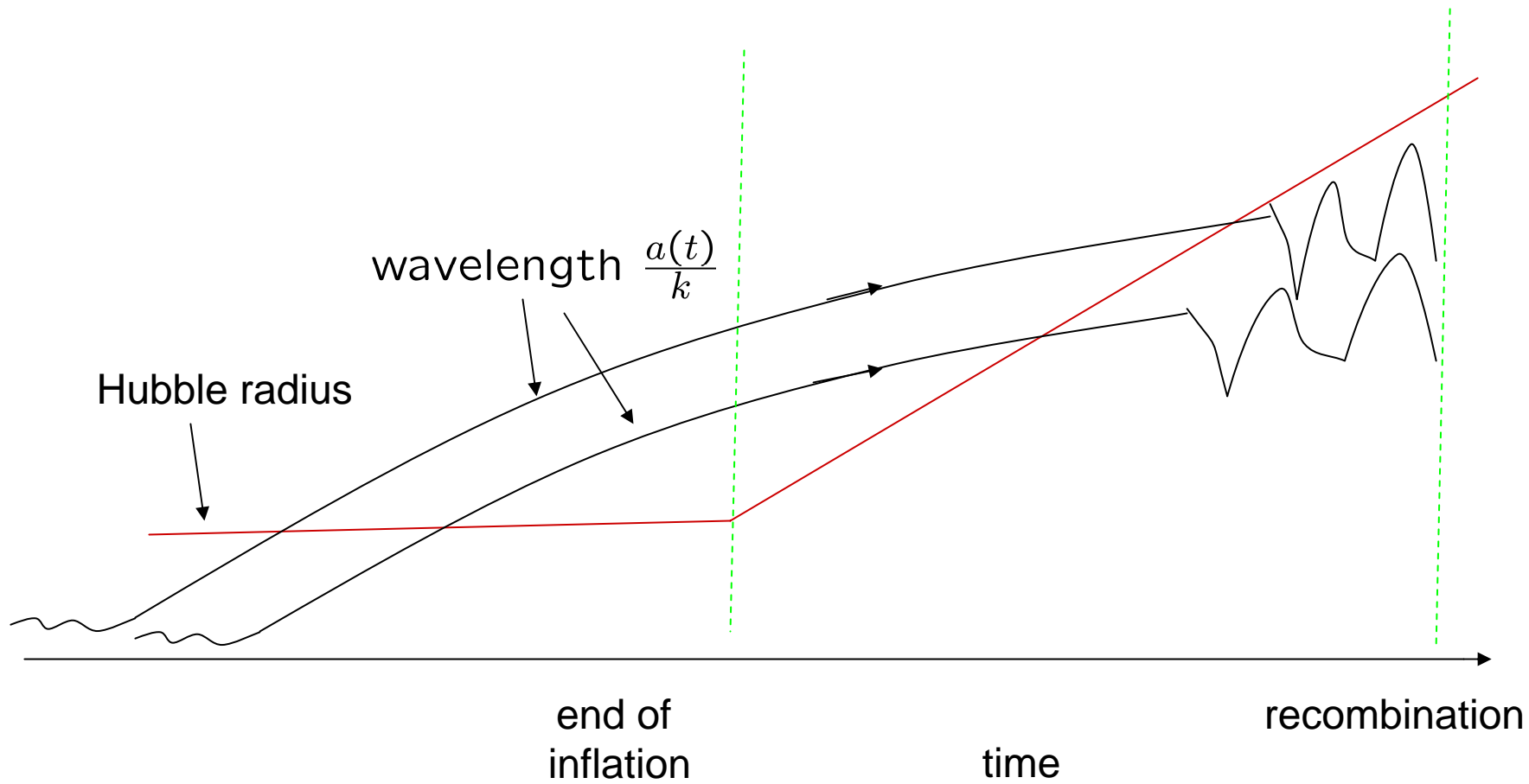
Generation of Cosmological Fluctuations

Light field at inflation

$$\hat{\gamma} = \int d^3k (a_k \ddot{\chi}_k(t) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.})$$

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \frac{k^2}{a^2}\chi_k = 0$$





Scalar metric Fluctuations

$$ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Phi)a^2d\vec{x}^2$$

Scalar field fluctuations

$$\phi(t, \vec{x}) = \phi(t) + \delta\phi(t, \vec{x})$$

$$\delta R_{\nu}^{\mu} = \frac{8\pi}{M_p^2} \delta T_{\nu}^{\mu}$$

$$u = a\delta\phi, \quad z = \frac{a\dot{\phi}}{H}, \quad \eta = \int dt/a$$

$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0$$

$$\text{spectrum } P_s(k) = \frac{k^3}{2\pi^2} \left| \frac{u_k}{z} \right|^2 \sim \frac{V^3}{M_p^6 V_{,\phi}^2}$$

$$k = aH$$

$$\Phi_k = \frac{16}{15} \frac{(3\pi V)^{3/2}}{M_p^3 V_{,\phi}} \Big|_{\phi=\phi(t_k)} .$$

$$\frac{m^2}{2} \phi^2$$

$$\phi(t) \approx \phi_0 - \sqrt{\frac{2}{3}} m t , \quad a(t) \approx a_0 \exp \left[\frac{2\pi}{M_p^2} (\phi_0^2 - \phi(t)^2) \right]$$

$$\Phi_k \approx 0.4 \frac{m}{M_p} \frac{\log k}{k^{3/2}}$$

$$V(\phi) \sim \phi^n$$

$$k^3 |\Phi_k|^2 = A_s \left(\ln \frac{k_0}{k} \right)^{\frac{n+2}{2}} :$$

$$\left(\ln \frac{k_0}{k}\right)^{\frac{n+2}{2}} = \left(\ln \frac{k_*}{k} + \ln \frac{k_0}{k_*}\right)^{\frac{n+2}{2}} \approx N^{\frac{n+2}{2}} \left(1 - \frac{n+2}{2N} \ln \frac{k_*}{k}\right)$$

number of e-folding $N = \ln \frac{k_0}{k_*}$

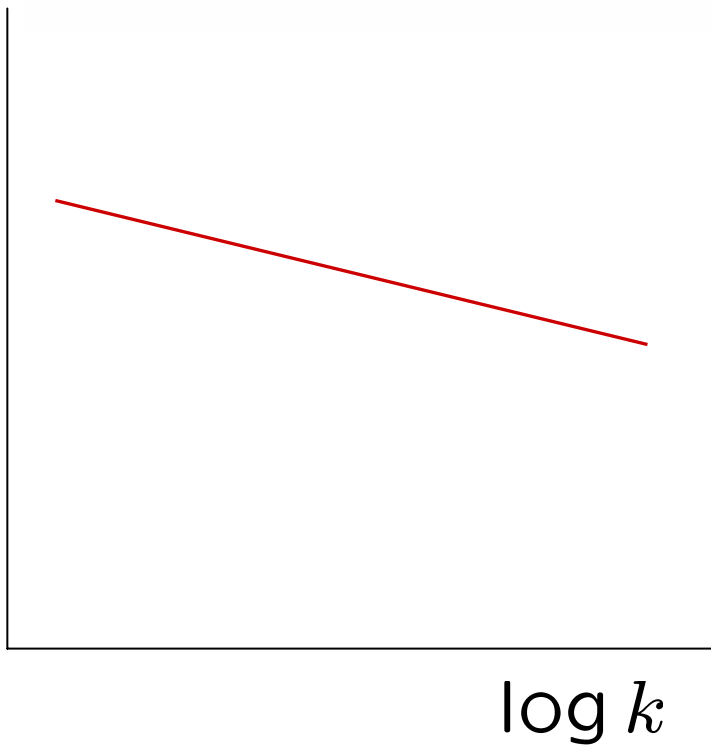
$$\text{often } P_s(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1}$$

$$\left(\frac{k}{k_*}\right)^{(n_s-1)} = e^{(n_s-1) \ln \frac{k}{k_*}} \approx 1 + (n_s - 1) \ln \frac{k}{k_*}$$

$$n_s - 1 \approx -\frac{n+2}{2N}$$

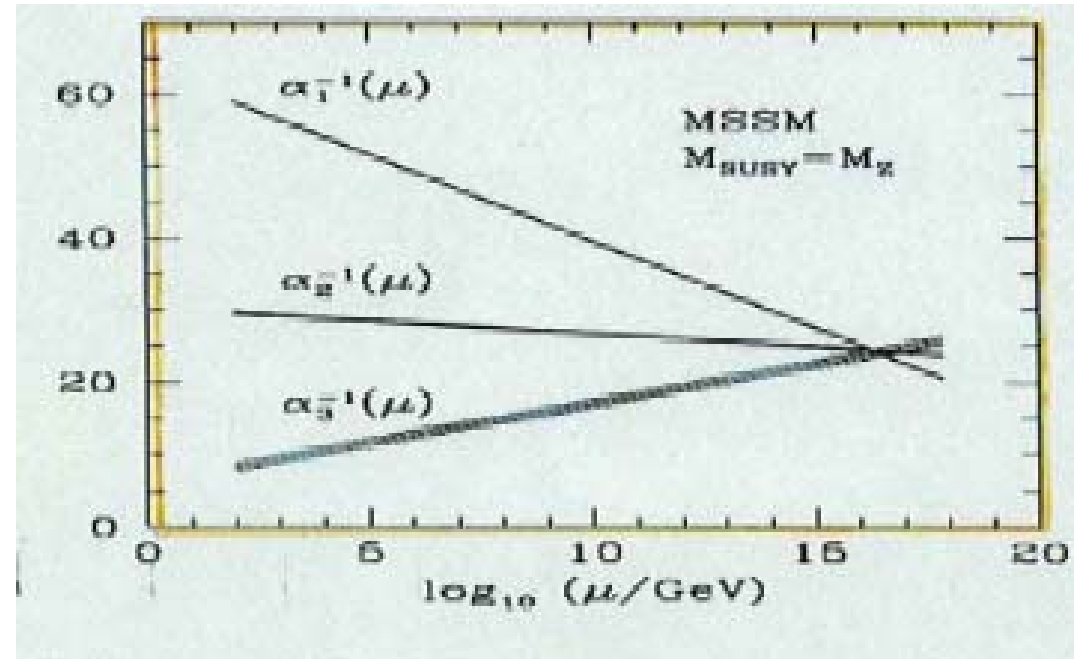
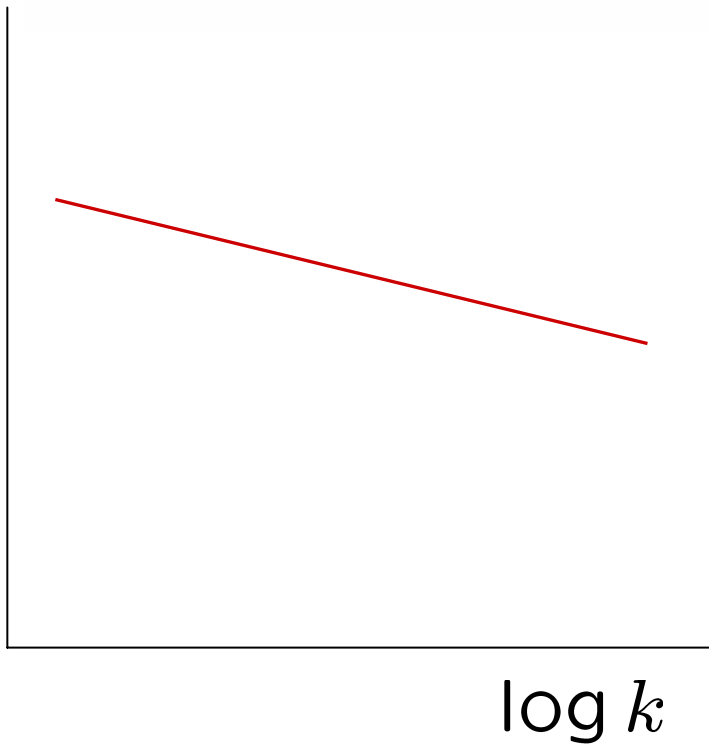
Logarithmic running

$$k^3 |\Phi_k|^2 = A_s \left(\ln \frac{k_0}{k} \right)^{\frac{n+2}{2}} .$$



Logarithmic running

$$k^3 |\Phi_k|^2 = A_s \left(\ln \frac{k_0}{k} \right)^{\frac{n+2}{2}}$$



Tensor metric Fluctuations

$$ds^2 = -dt^2 + (\eta_{ij} + h_{ij})a^2 dx^i dx^j \quad h_{\mathbf{i}}^{\mathbf{i}} = 0, h_{j,i}^i = 0, i, j = 1, 2, 3.$$

$$\delta R_{\nu}^{\mu} = 0$$

$$h_k'' + \left(k^2 - \frac{a''}{a}\right) h_k = 0$$

$$\text{Spectrum } P_t(k) \simeq \frac{H^2}{M_p^2} \sim \frac{V}{M_p^4}$$

$$\text{often } P_t(k) = A_t \left(\frac{k}{k_*}\right)^{n_t} \quad r = A_t/A_s$$

Scalar metric Fluctuations from Inflation

$$ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Phi)a^2d\vec{x}^2$$

Initial conditions from Inflation \rightarrow



Random Gaussian Field $\Phi(\vec{x})$

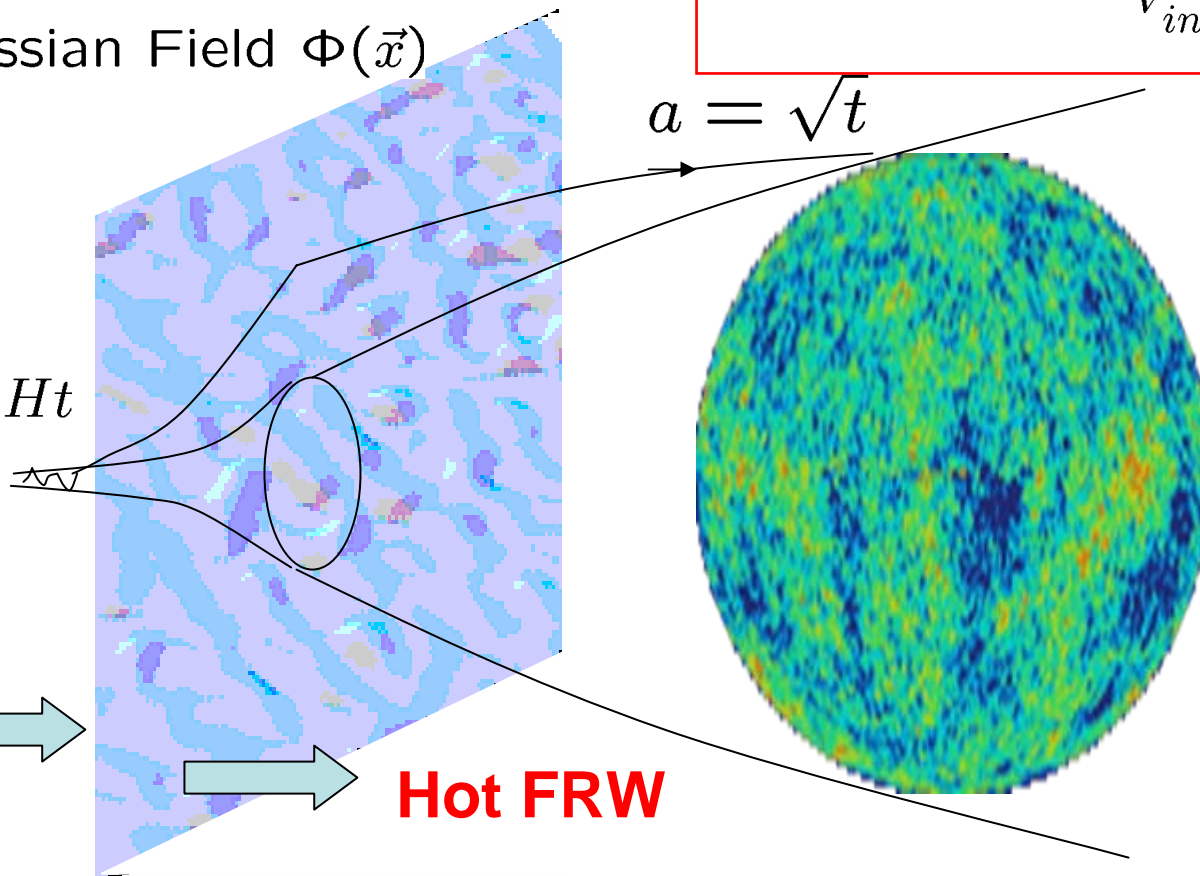
$$a = e^{Ht}$$

$$a = \sqrt{t}$$

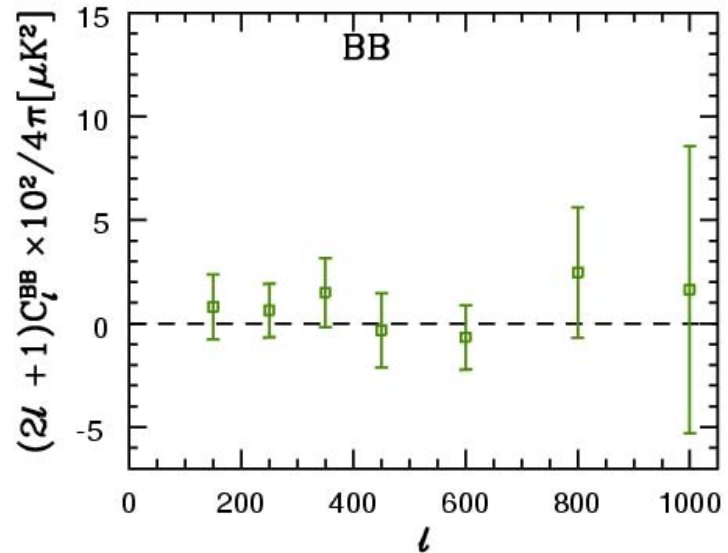
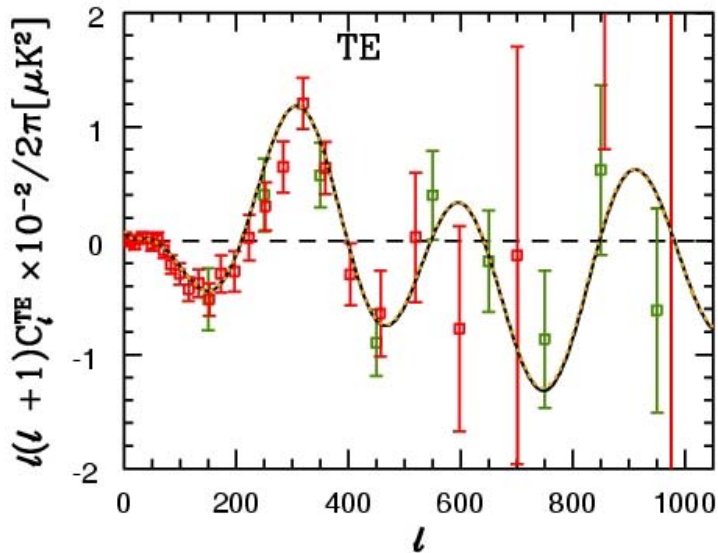
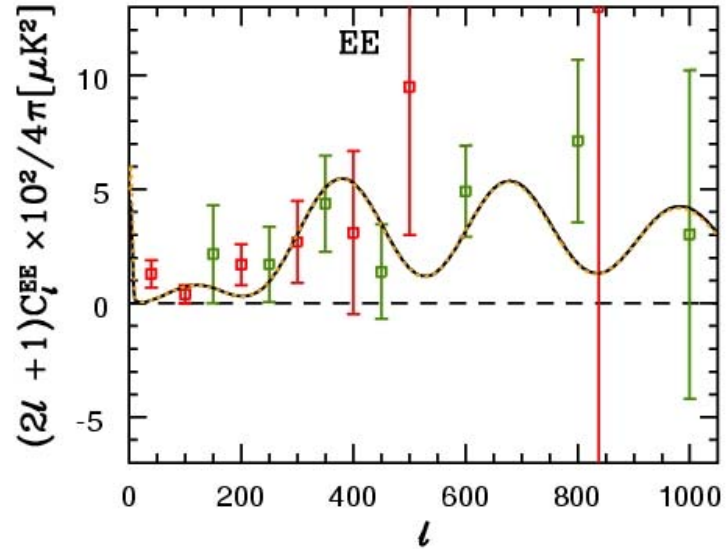
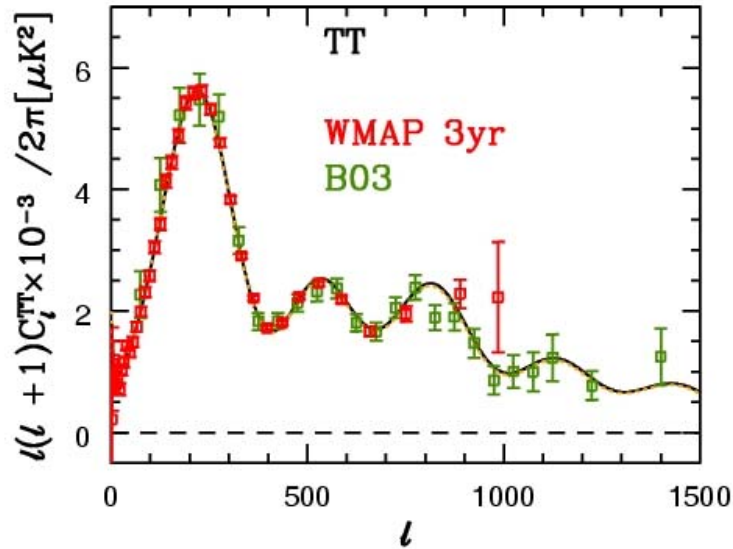
inflation \rightarrow

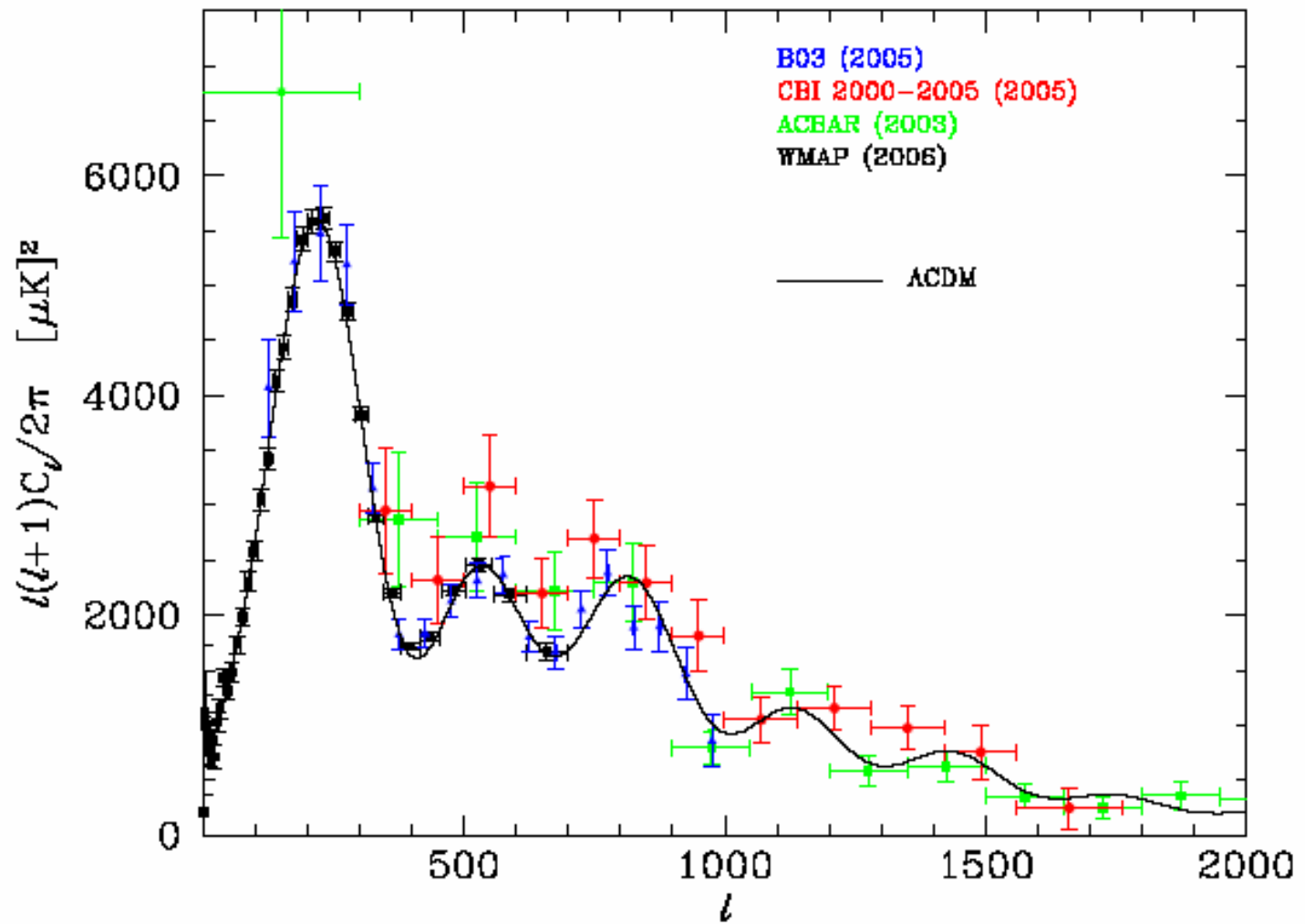
Hot FRW

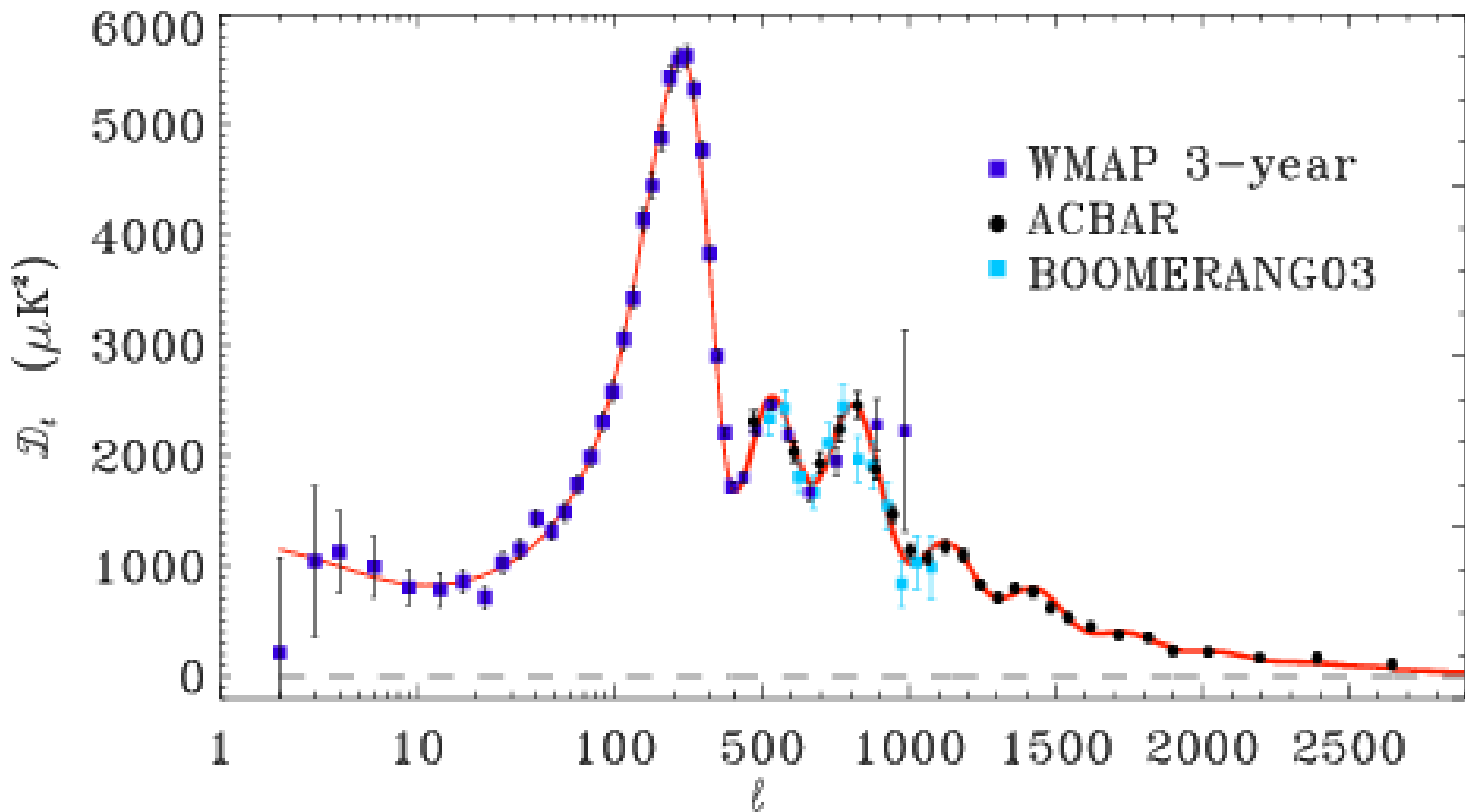
$$\begin{aligned}\Omega_{tot} &= 1 \\ k^3 \Phi_k^2 &\rightarrow P_s = A_s k^{n_s - 1} \\ P_T &= \frac{H^2}{M_p^2} k^{n_T} \\ N &= 62 - \ln \frac{10^{16} \text{Gev}}{V_{inf}^{1/4}}\end{aligned}$$



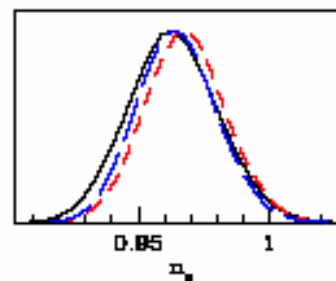
WMAP3 sees 3rd pk, B03 sees 4th

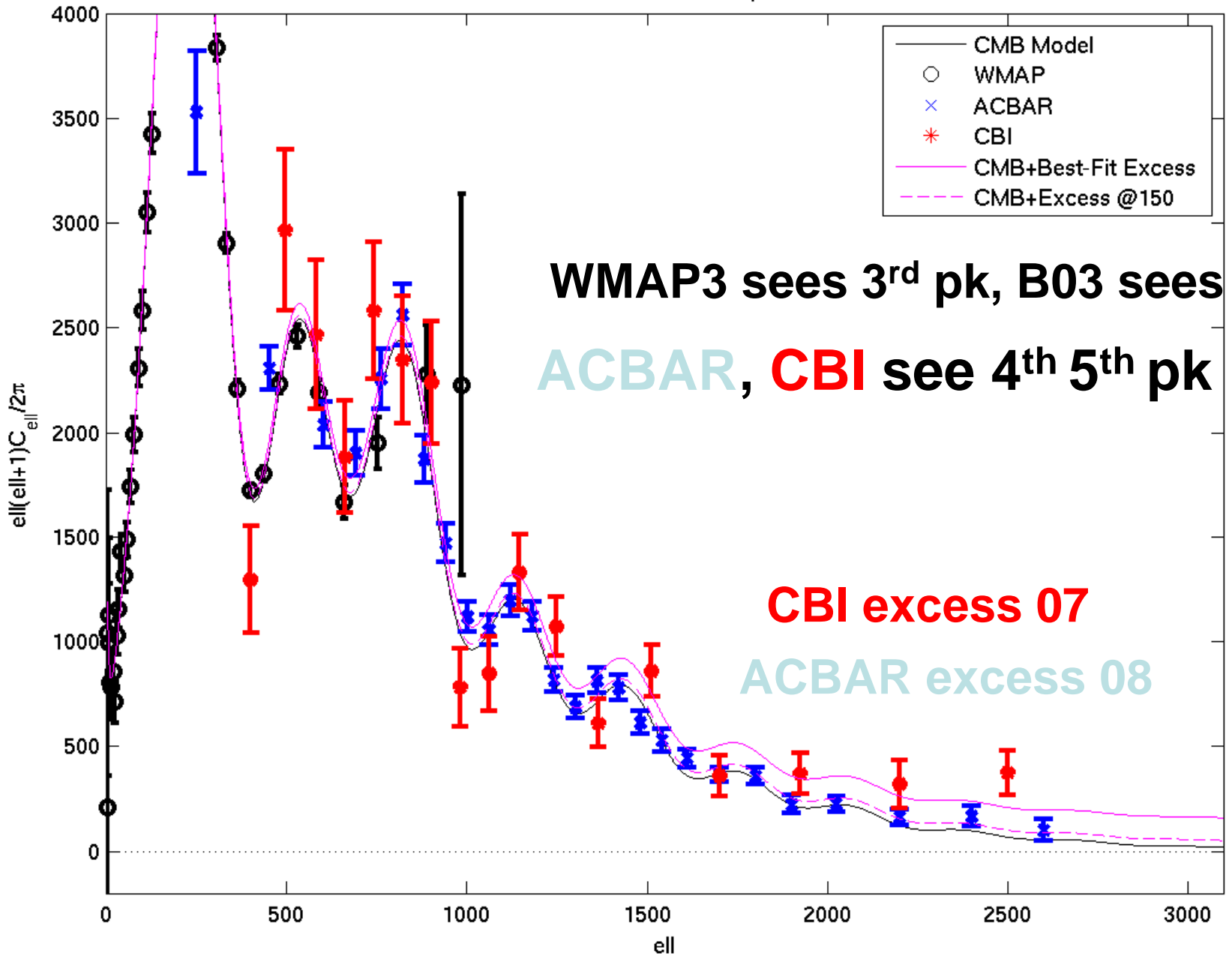




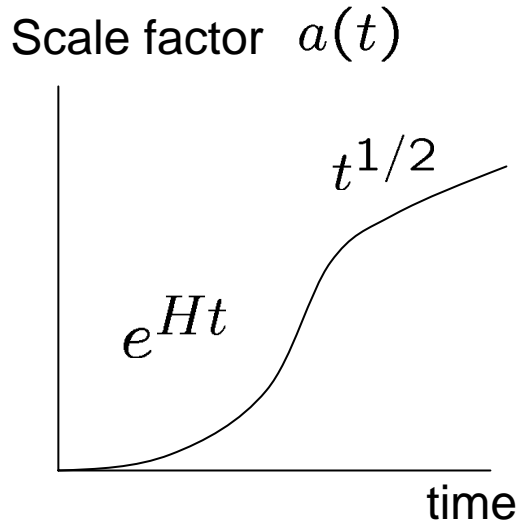


$$P_s(k) = A_s \left(\frac{k}{k_*} \right)^{n_s - 1}$$





Early Universe Inflation



Equation of State $t \leq 10^{-35}$ sec

$$p \approx -\epsilon$$

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Realization of Inflation

Scalar field

$$p = \frac{1}{2}\dot{\phi}^2 - V$$

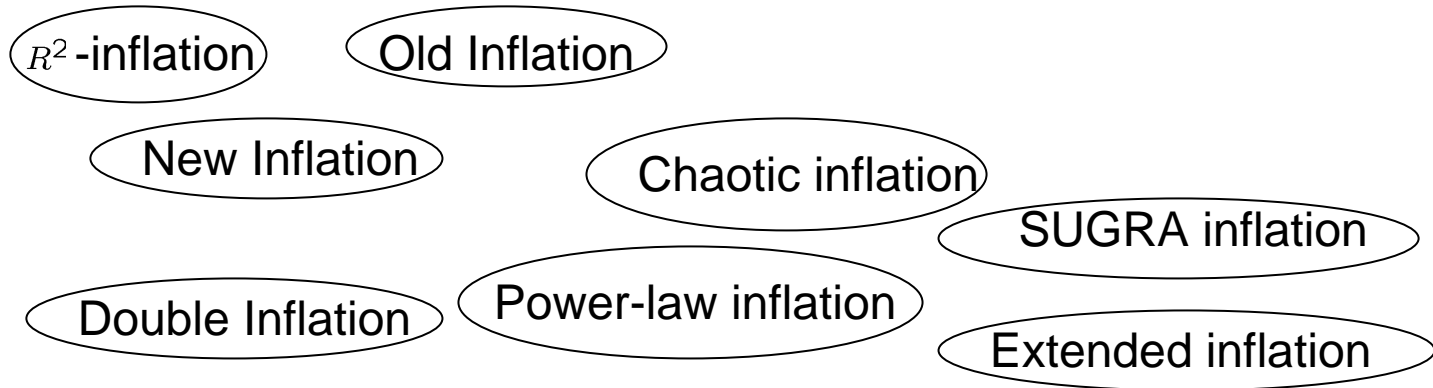
$$\epsilon = \frac{1}{2}\dot{\phi}^2 + V$$

slow roll $\dot{\phi}^2 \ll V$

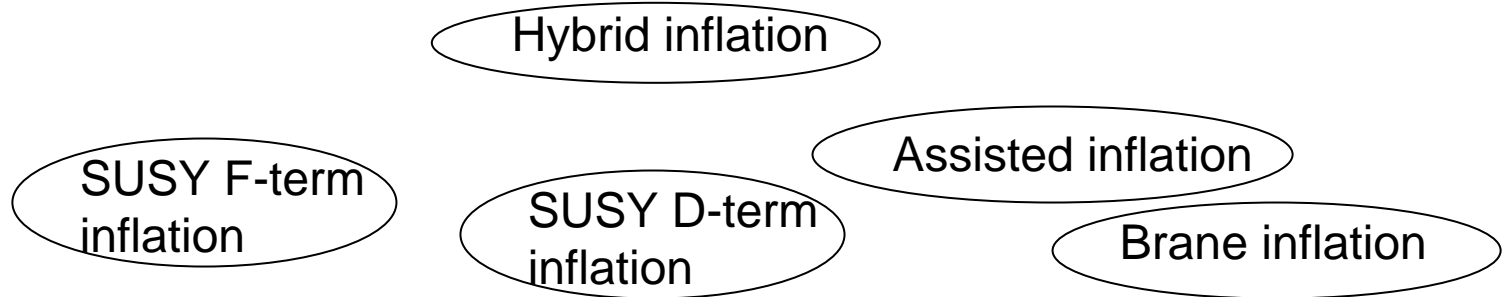
choise of $V(\phi)$

Inflation in the context of ever changing fundamental theory

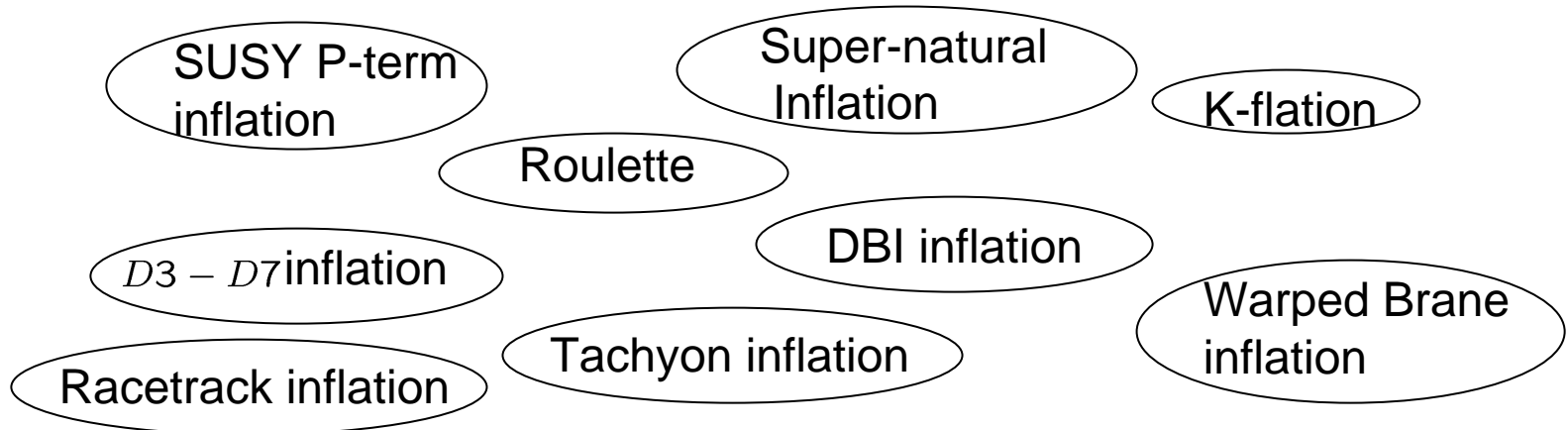
1980



1990

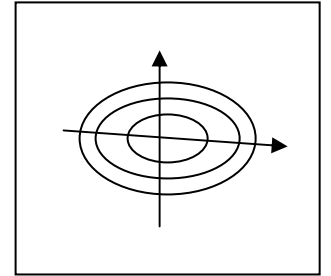
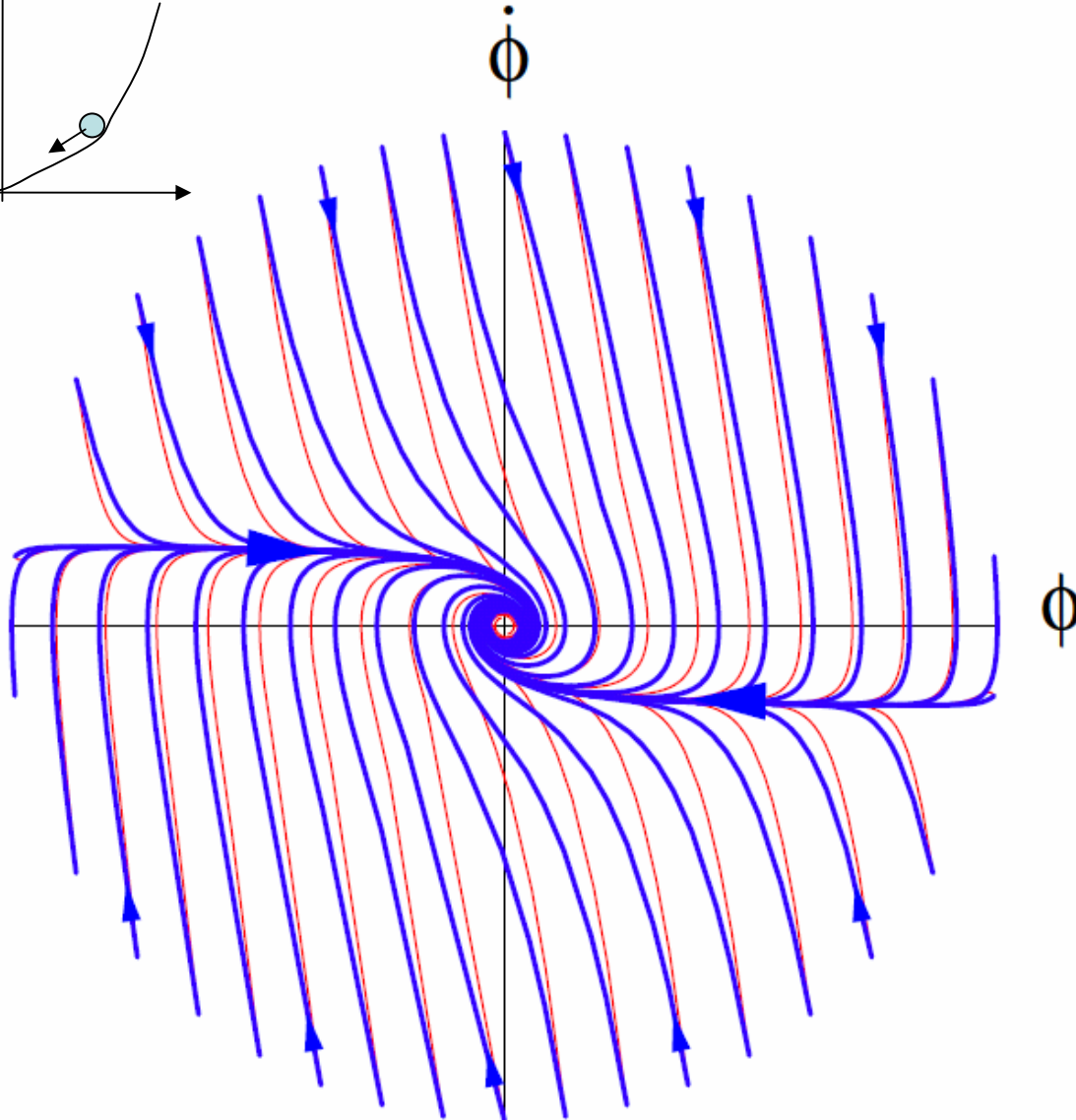
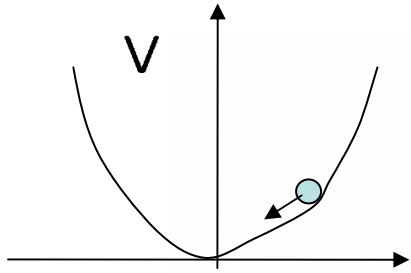


2000



Family phase portrait of inflation

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$
$$3H^2 = \frac{8\pi}{M_p^2} \left(\dot{\phi}^2/2 + V \right)$$



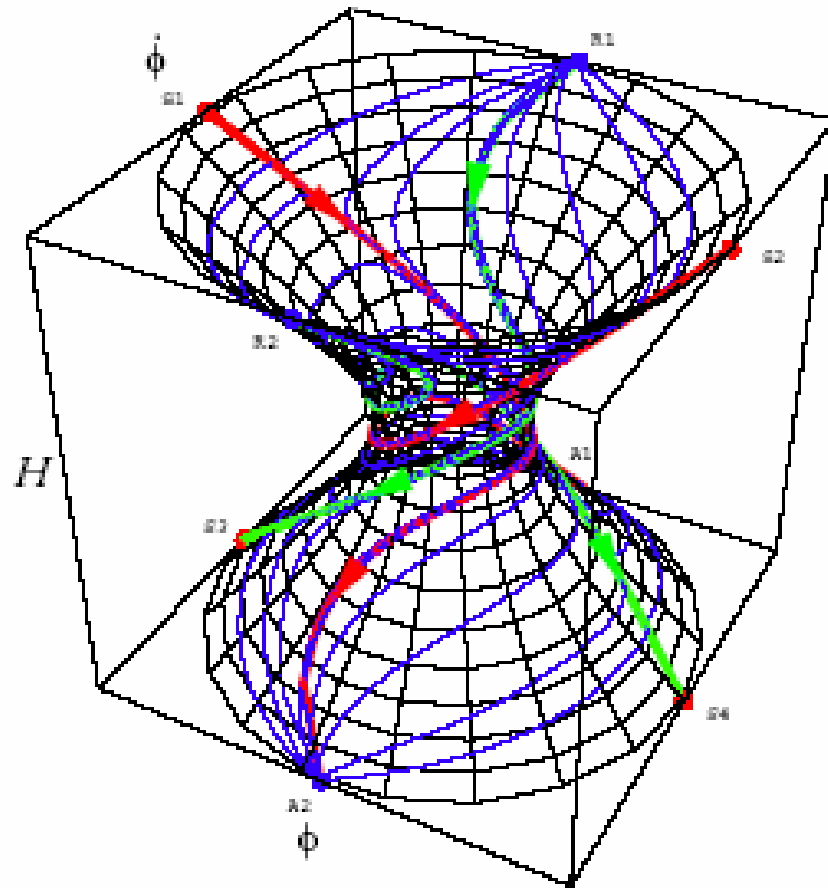
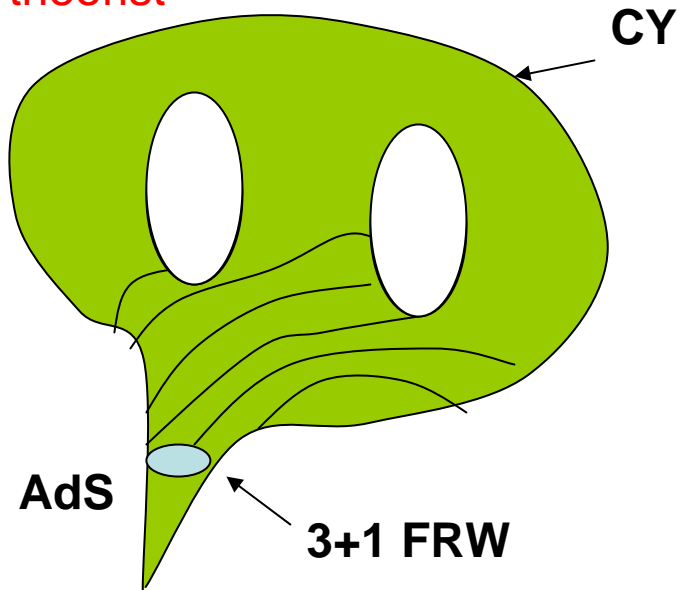


Fig. 8. Phase portrait for the theory $V(\phi) = \frac{1}{2}m^2\phi^2 + V_0$ for $V_0 < 0$. The branches describing stages of expansion and contraction (upper and lower parts of the hyperboloid) are connected by a throat

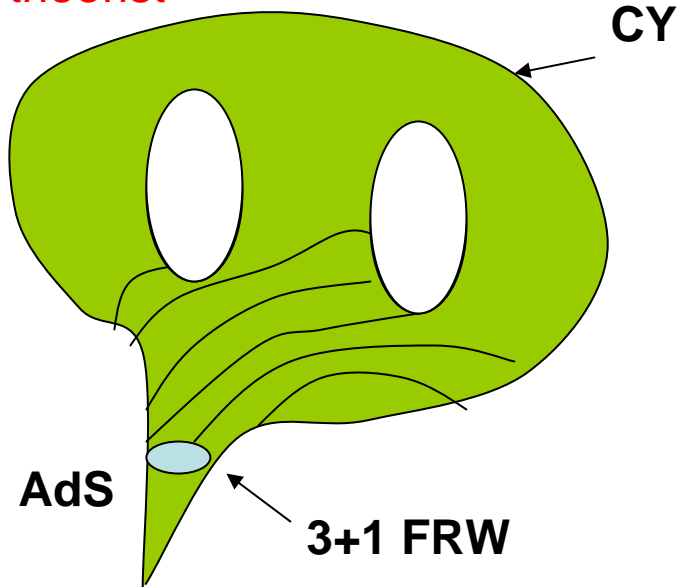
Inflation in String Theory: Cosmology with Compactification

string theorist

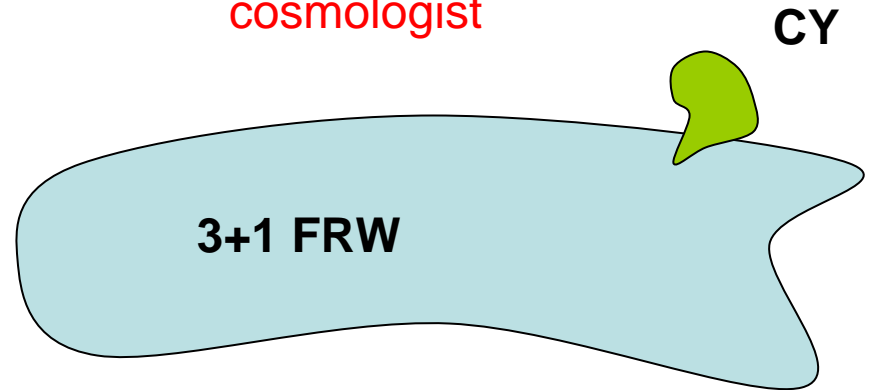


Inflation in String Theory: Cosmology with Compactification

string theorist



cosmologist

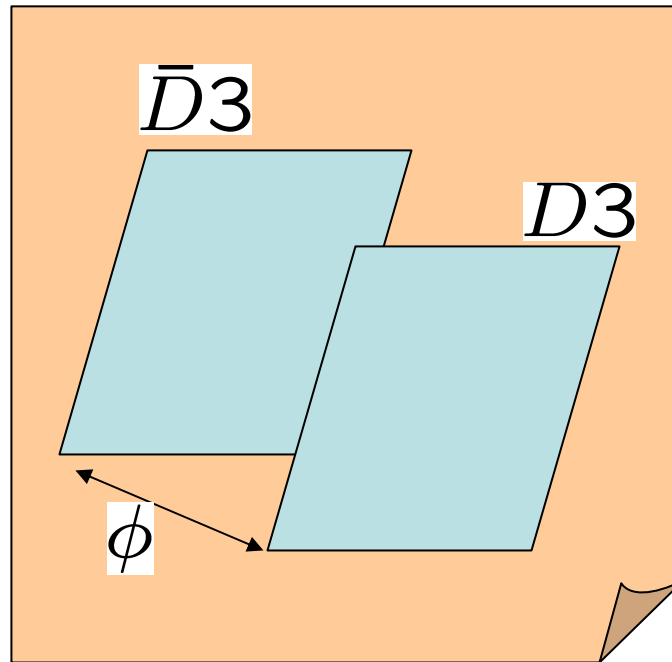


String Theory inflation models

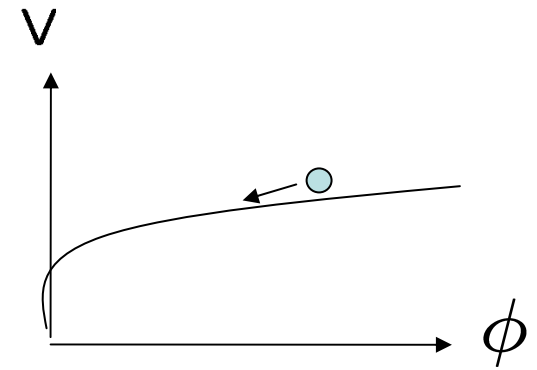
Modular Inflation. They use Kahler moduli/axion like the fields that are present in the KKLT stabilization.

Brane inflation. The inflaton field corresponds to the distance between branes in Calabi-Yau space. Historically, this was the first class of string inflation models.

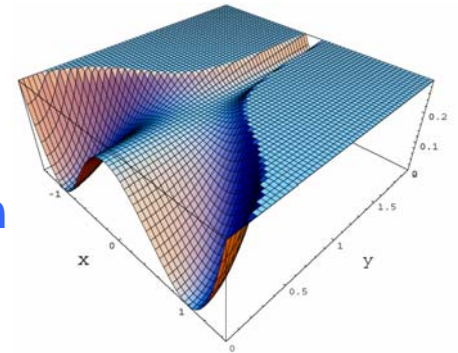
Inflation with branes in String Theory



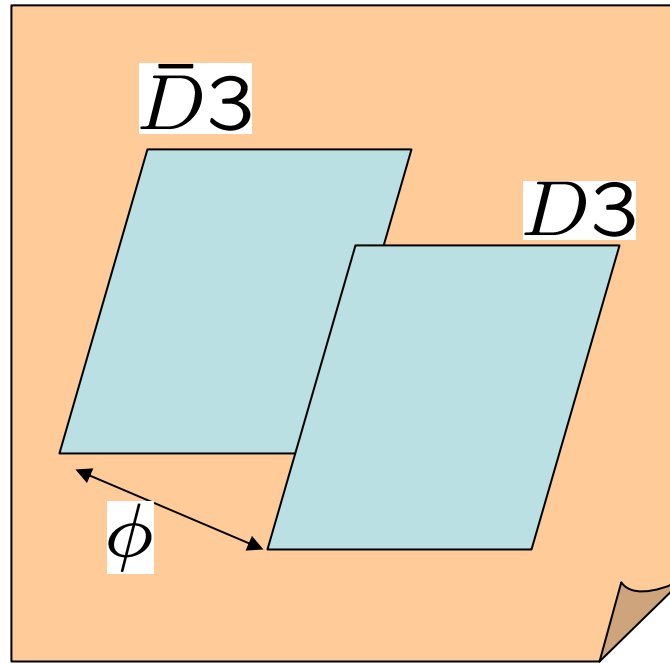
4-dim picture



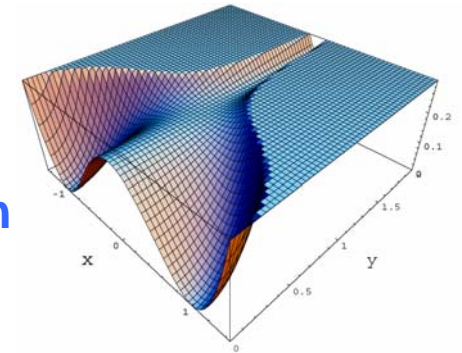
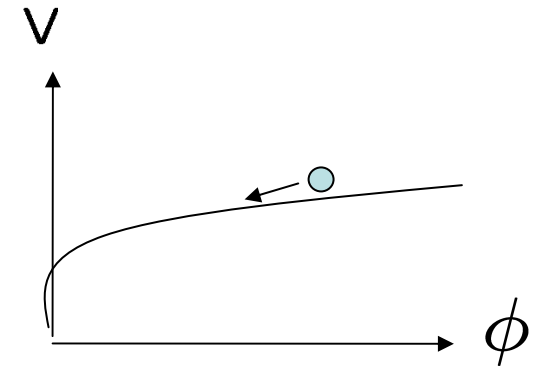
Prototype of hybrid inflation



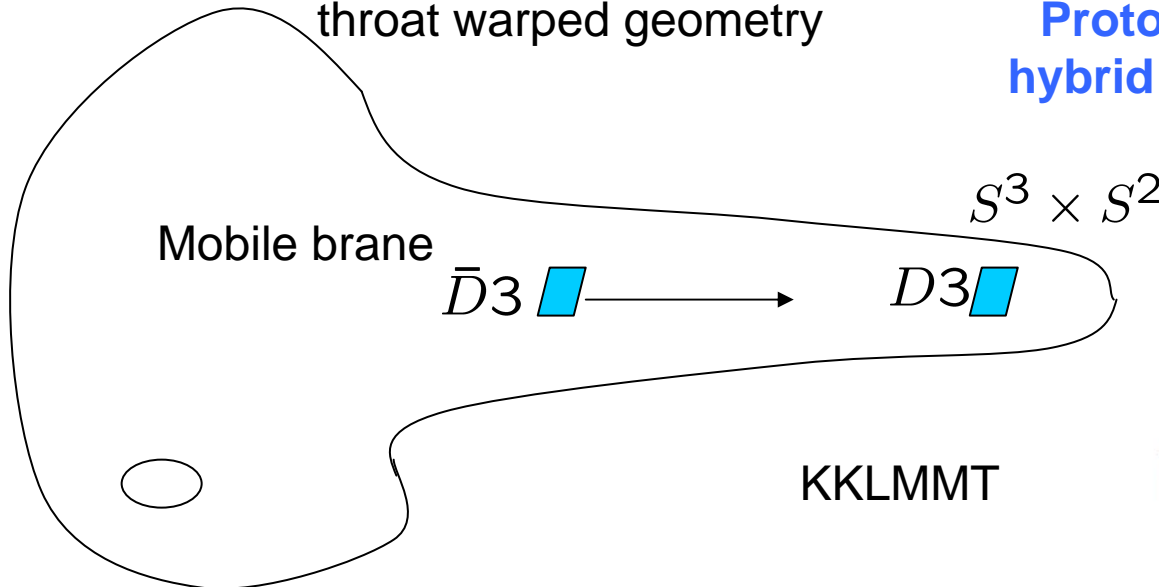
Inflation with branes in String Theory



4-dim picture



throat warped geometry



Prototype of hybrid inflation

$$V = V_0 \left[1 - \left(\frac{\Delta}{\kappa\phi} \right)^4 \right]$$

Kahler moduli Inflation (Conlon&Quevedo hep-th/050912)

Roulette Inflation-Kahler moduli/axion (Bond, LK, Prokushkin&Vandrevange hep-th/0612197)

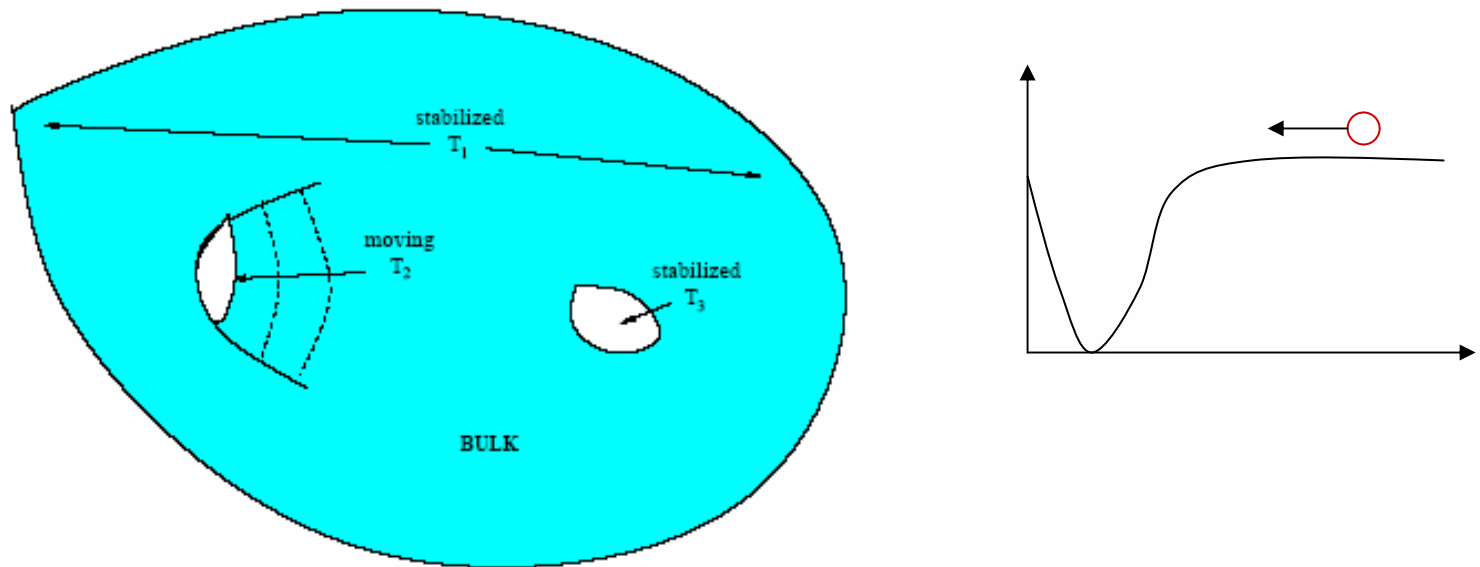


Figure 1: Schematic illustration of the ingredients in Kähler moduli inflation. The four-cycles of the CY are the Kähler moduli T_i which govern the sizes of different holes in the manifold. We assume T_3 and the overall scale T_1 are already stabilized, while the last modulus to stabilize, T_2 , drives inflation while settling down to its minimum. The imaginary parts of T_i have to be left to the imagination. The outer $3 + 1$ observable dimensions are also not shown.

$$V(\phi, \bar{\phi}) = e^{\mathcal{K}/M_P^2} \left(\mathcal{K}^{i\bar{j}} D_i \hat{W} D_{\bar{j}} \bar{\hat{W}} - \frac{3}{M_P^2} \hat{W} \bar{\hat{W}} \right) + \text{D-terms}$$

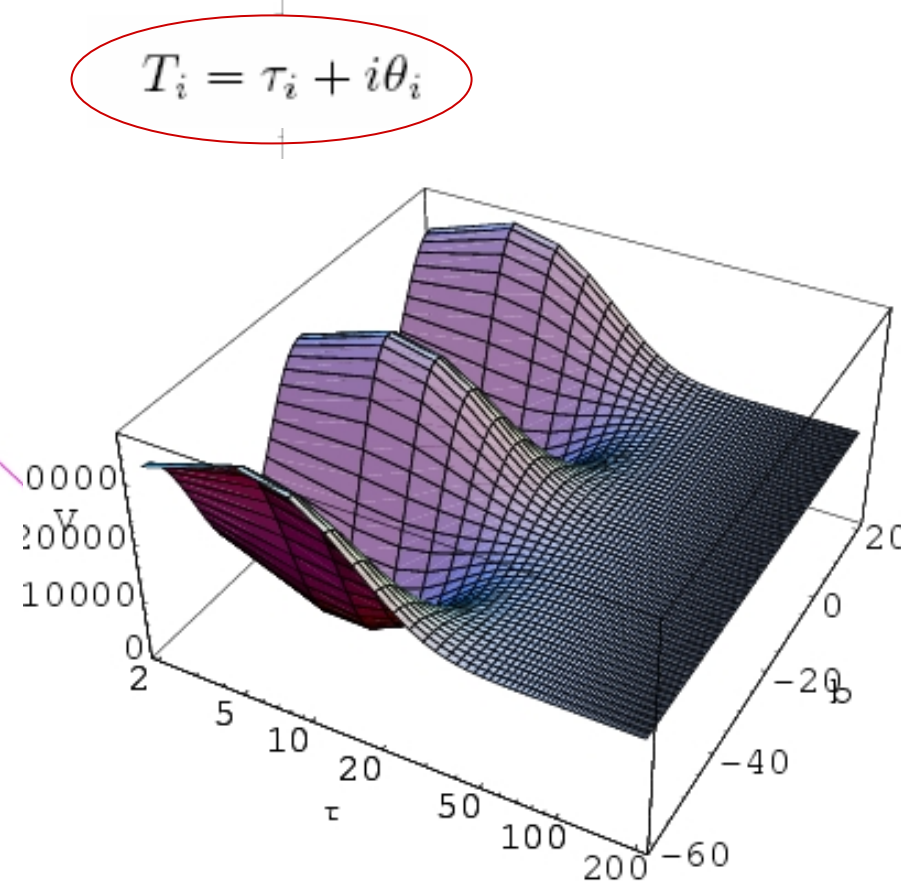
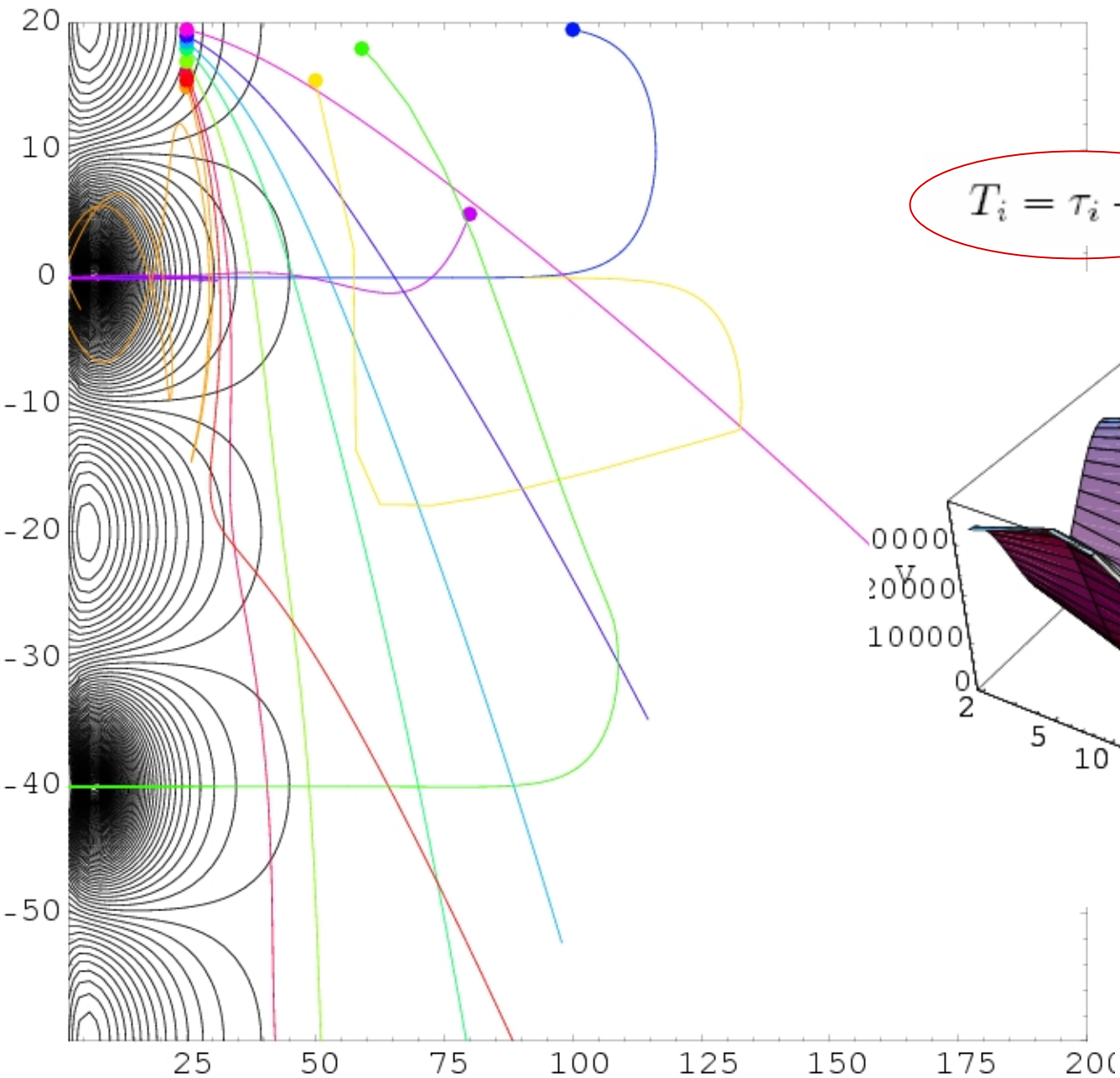
$$\frac{\mathcal{K}}{M_P^2} = -2 \ln \left(\mathcal{V} + \frac{\xi}{2} \right) + \ln g_s + \mathcal{K}_{cs}$$

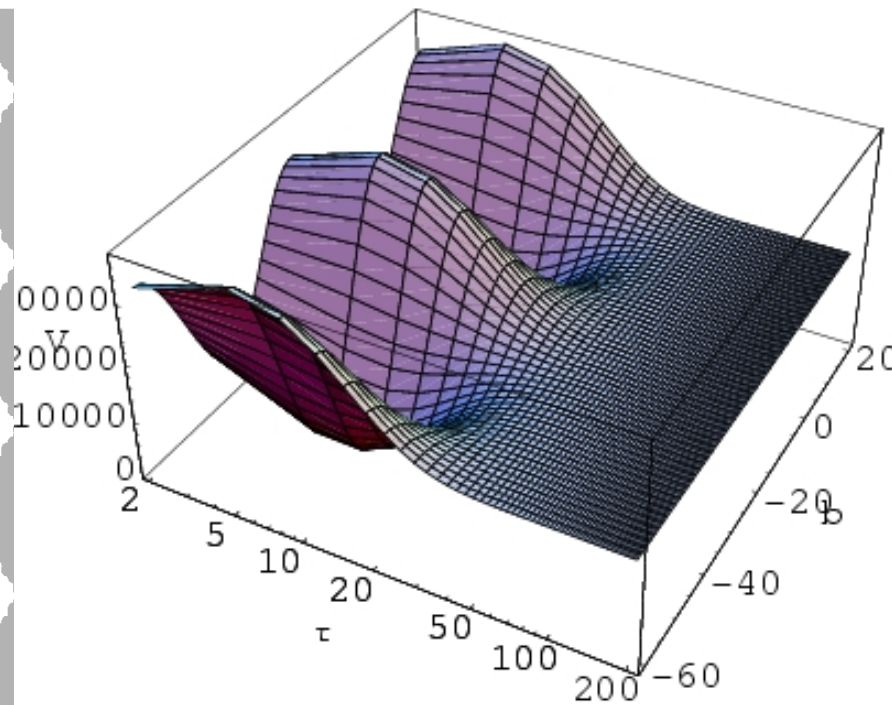
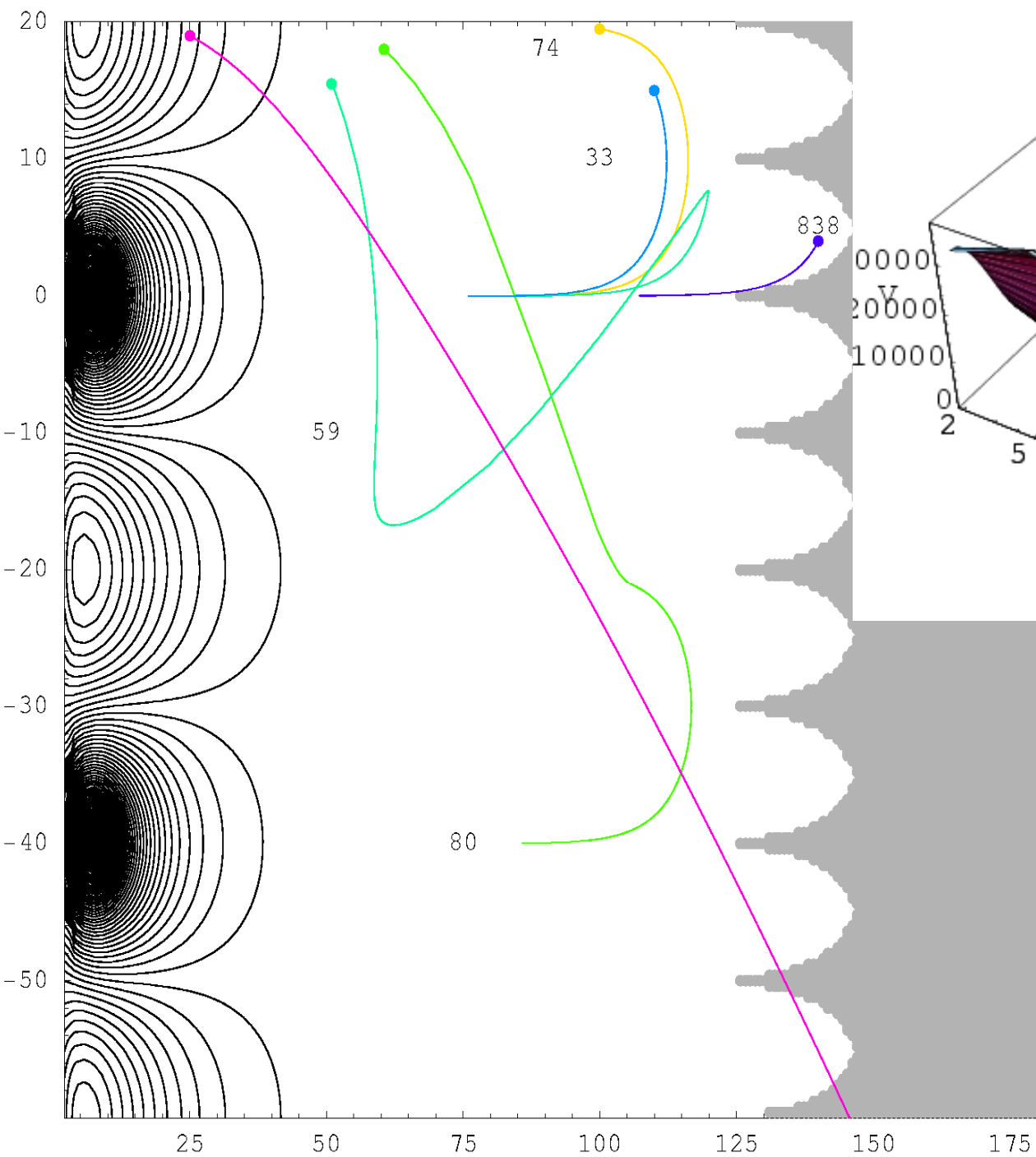
$$\hat{W} = \frac{g_s^{\frac{3}{2}} M_P^3}{\sqrt{4\pi}} \left(W_0 + \sum_{i=1}^{h^{1,1}} A_i e^{-a_i T_i} \right), \quad W_0 = \frac{1}{l_s^2} \int_M G_3 \wedge \Omega$$

$$T_i = \tau_i + i\theta_i$$

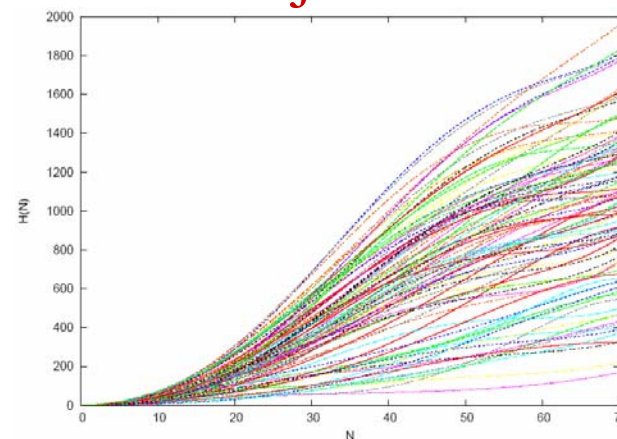
$$\begin{aligned} V(T_1, \dots, T_n) = & \frac{12W_0^2 \xi}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} + \sum_{i=2}^n \frac{12e^{-2a_i \tau_i} \xi A_i^2}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} + \frac{16(a_i A_i)^2 \sqrt{\tau_i} e^{-2a_i \tau_i}}{3\alpha \lambda_2 (2\mathcal{V} + \xi)} \quad (18) \\ & + \frac{32e^{-2a_i \tau_i} a_i A_i^2 \tau_i (1 + a_i \tau_i)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)} + \frac{8W_0 A_i e^{-a_i \tau_i} \cos(a_i \theta_i)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)} \left(\frac{3\xi}{(2\mathcal{V} + \xi)} + 4a_i \tau_i \right) \\ & + \sum_{\substack{i,j=2 \\ i < j}}^n \frac{A_i A_j \cos(a_i \theta_i - a_j \theta_j)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} e^{-(a_i \tau_i + a_j \tau_j)} [32(2\mathcal{V} + \xi)(a_i \tau_i + a_j \tau_j \\ & + 2a_i a_j \tau_i \tau_j) + 24\xi] + V_{\text{uplift}} . \end{aligned}$$

String theory landscape of the Kahler moduli/axion Inflation





**Ensemble of
Inflationary
trajectories**



Lessons:

Multiple fields Inflation

Ensemble of acceleration histories (trajectories)
for the same underlying theory

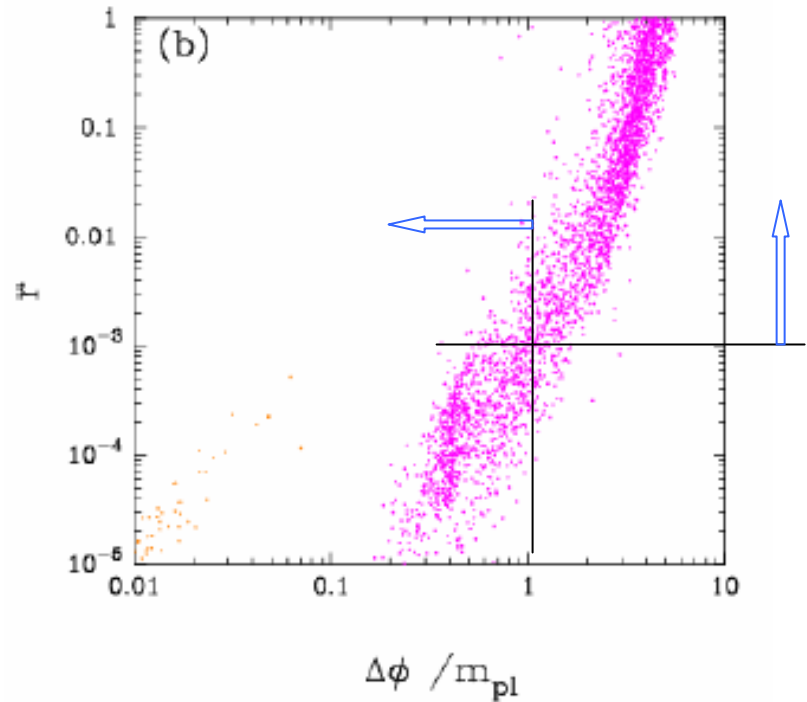
Prior probabilities of trajectories $P(H(t))$

Small amplitude of gravity waves r from inflation $r \sim 10^{-10}$

Lyth hep-ph/9606387

$$\Delta\phi \sim \frac{m_{\text{pl}}}{8} \left(\frac{r}{\pi}\right)^{1/2} \Delta N$$

Efstathiou & Mack
astro-ph/0503360

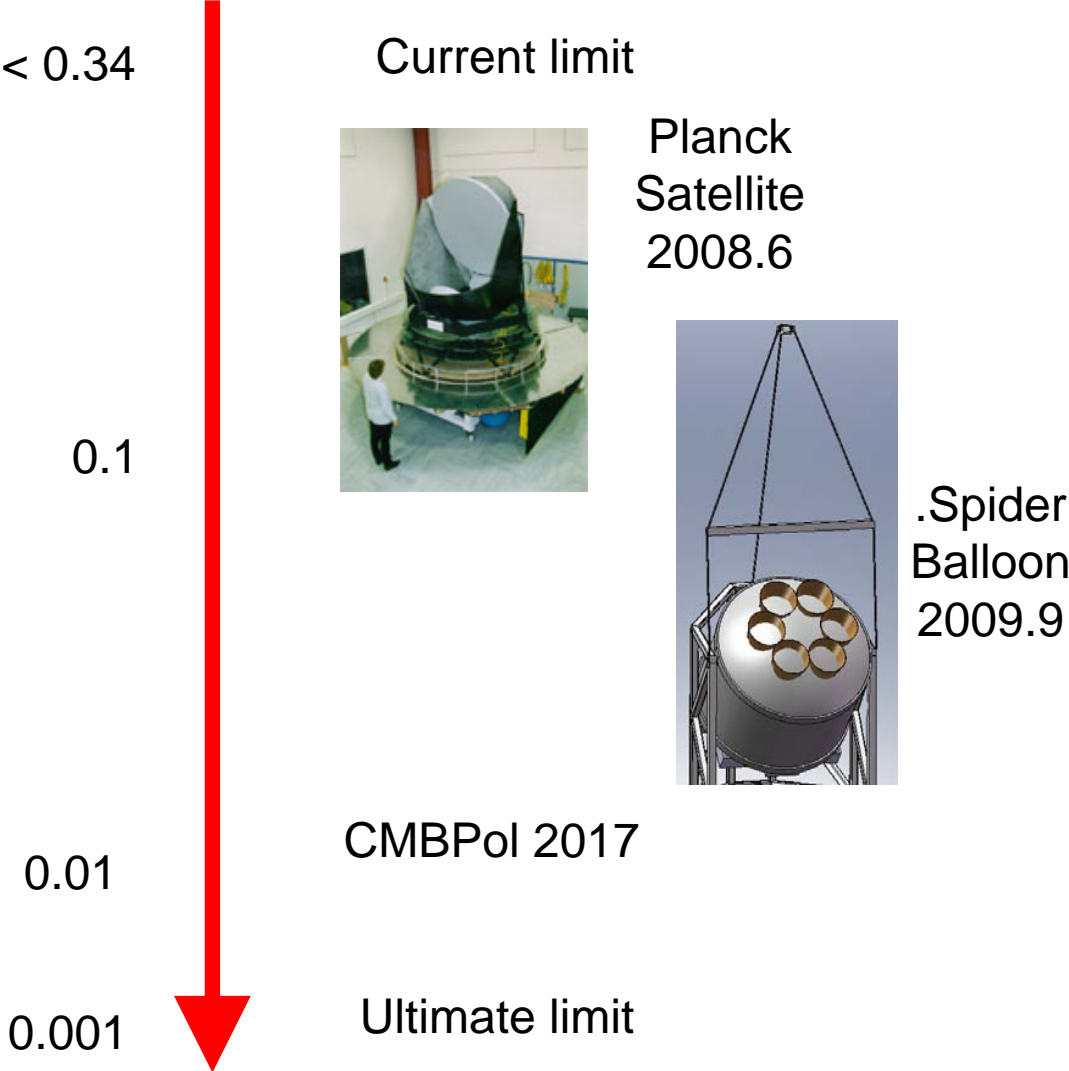


Baumann & McAllister
hep-th/0610285

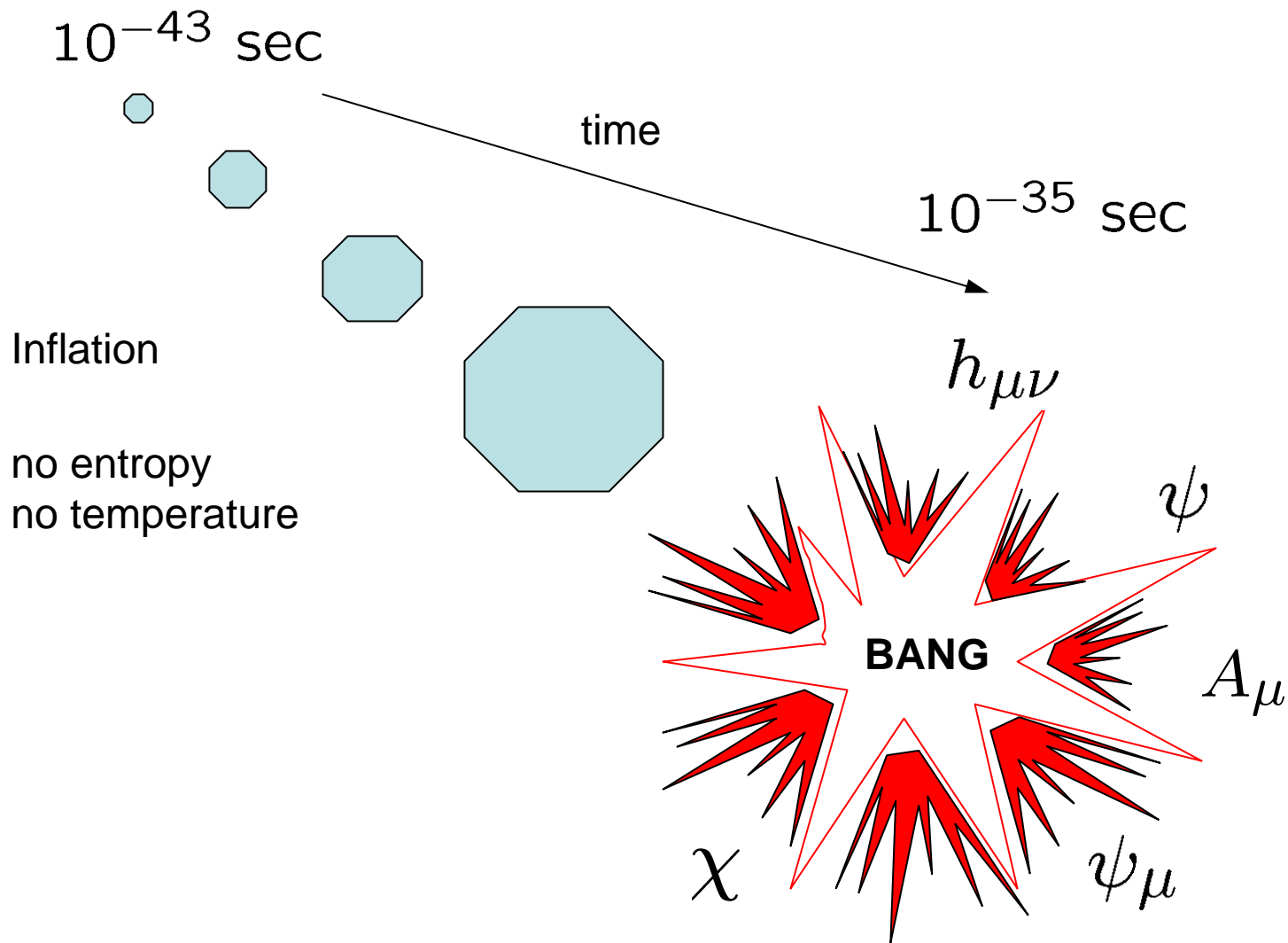
for the brane inflation $\frac{\Delta\phi}{M_p} < \frac{2}{\sqrt{n}}$

$$r \leq 0.01 \left(\frac{4}{n}\right) \left(\frac{60}{N}\right)^2 \quad n = MK$$

Measurement of GW from CMB anisotropy polarization



Particlegenesis



$$\mathcal{L}(\phi, \chi, \psi, A_\mu, \psi_\mu, h_{\mu\nu})$$

Output of Preheating

- Reheat temperature T_R
- Out-of-equilibrium state
- Evolution of EoS

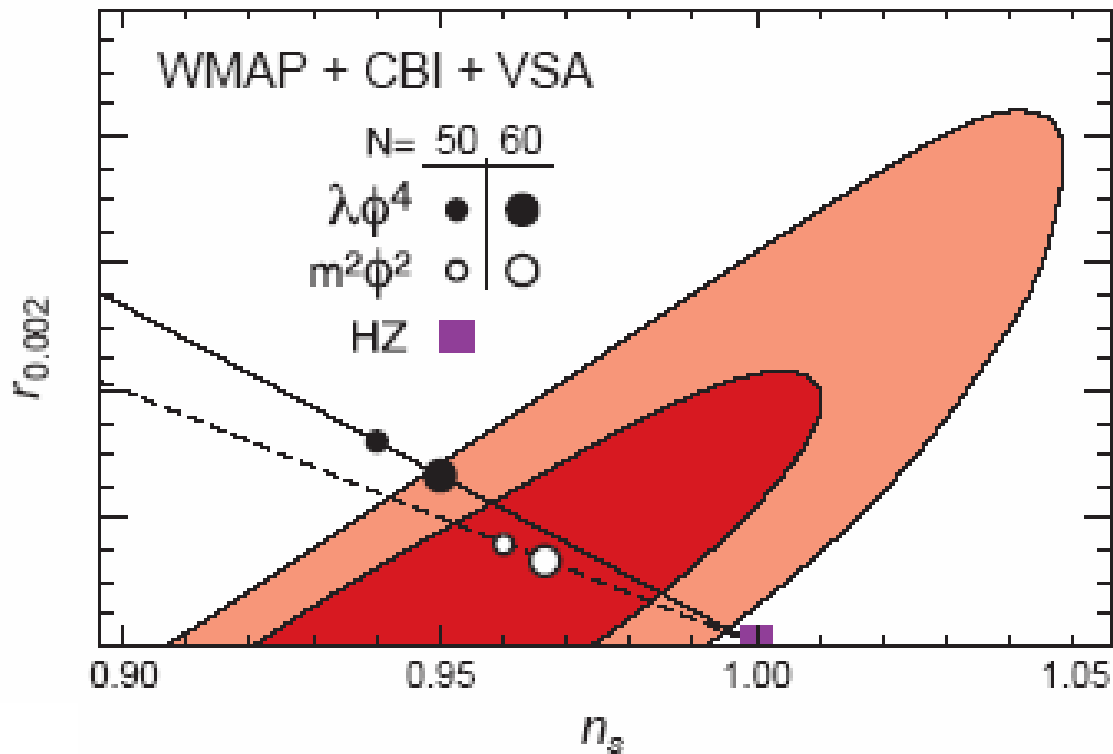
- Number of efolds

$$N = 62 - \ln \frac{10^{16} \text{Gev}}{V_h^{1/4}} + \frac{1}{4} \ln \frac{V_h}{V_{end}} - \frac{1}{12} \ln \frac{V_{end}}{\rho_{rad}}$$

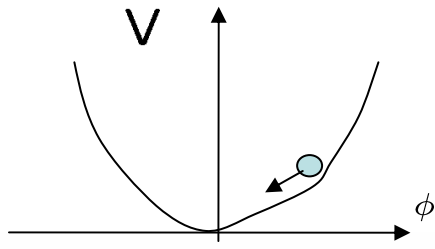
- Potential observables

Large field models

$$V = \phi^\alpha \quad n_s - 1 = -\frac{2 + \alpha}{2N} \quad r = \frac{4\alpha}{N}$$



Resonant Preheating in Chaotic Inflation



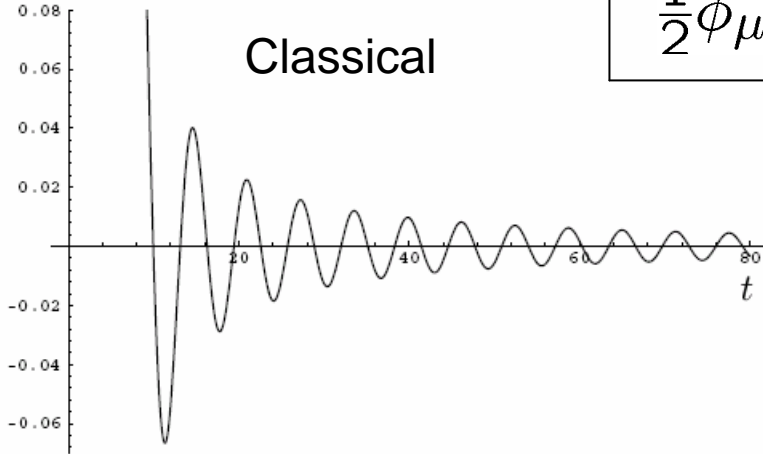
Classical

$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Quantum

$$\hat{\chi}(t, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left(\hat{a}_k \chi_k(t) e^{-i\mathbf{k}\mathbf{x}} + \hat{a}_k^\dagger \chi_k^\dagger(t) e^{i\mathbf{k}\mathbf{x}} \right)$$

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2 \right) \chi_k = 0$$



$$\tau = mt$$

$$\ddot{X}_k + \left(\frac{k^2}{m^2} + q \sin^2 \tau \right) X_k = 0$$

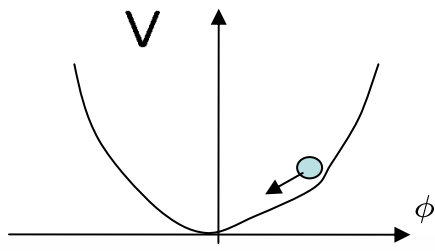
parameter $q = \frac{g^2\phi_0^2}{m^2} \sim g^2 10^{10}$

Occupation number

$$n_k = \frac{\omega_k}{2} \left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2 \right) - \frac{1}{2}$$

$$n_k \sim e^{\mu t}$$

Resonant Preheating in Chaotic Inflation



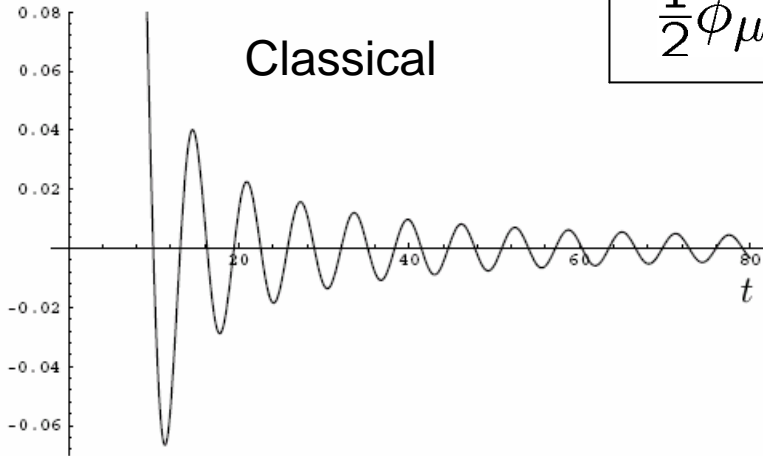
Classical

$$\frac{1}{2}\phi_{,\mu}\phi^{,\mu} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_{,\mu}\chi^{,\mu} + g^2\phi^2\chi^2$$

Quantum

$$\hat{\chi}(t, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left(\hat{a}_k \chi_k(t) e^{-i\mathbf{k}\cdot\mathbf{x}} + \hat{a}_k^\dagger \chi_k^\dagger(t) e^{i\mathbf{k}\cdot\mathbf{x}} \right)$$

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2 \right) \chi_k = 0$$



$$\tau = mt$$

$$\ddot{X}_k + \left(\frac{k^2}{m^2} + q \sin^2 \tau \right) X_k = 0$$

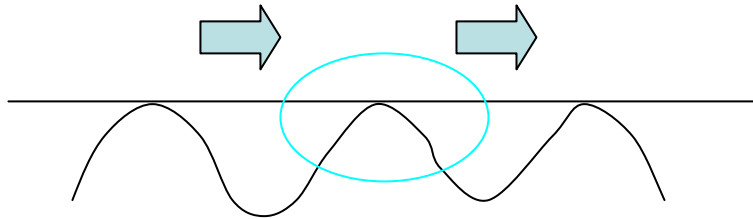
parameter $q = \frac{g^2\phi_0^2}{m^2} \sim g^2 10^{10}$

Occupation number

$$n_k = \frac{\omega_k}{2} \left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2 \right) - \frac{1}{2}$$

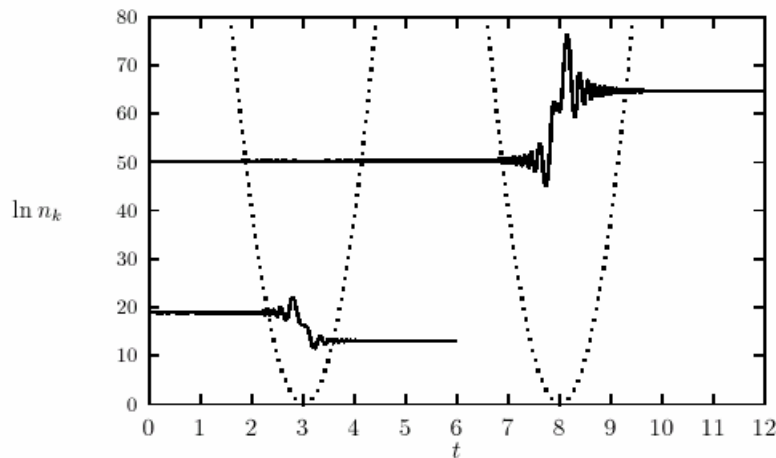
$$n_k \sim e^{\mu t}$$

$$X_k^j(t) = \frac{\alpha_k^j}{\sqrt{2\omega}} e^{-i \int_0^t \omega dt} + \frac{\beta_k^j}{\sqrt{2\omega}} e^{+i \int_0^t \omega dt} \quad X_k^{j+1}(t) = \frac{\alpha_k^{j+1}}{\sqrt{2\omega}} e^{-i \int_0^t \omega dt} + \frac{\beta_k^{j+1}}{\sqrt{2\omega}} e^{+i \int_0^t \omega dt}$$



$$\begin{pmatrix} \alpha_k^{j+1} e^{-i\theta_k^j} \\ \beta_k^{j+1} e^{+i\theta_k^j} \end{pmatrix} = \begin{pmatrix} \frac{1}{D_k} & \frac{R_k^*}{D_k^*} \\ \frac{R_k}{D_k} & \frac{1}{D_k^*} \end{pmatrix} \begin{pmatrix} \alpha_k^j e^{-i\theta_k^j} \\ \beta_k^j e^{+i\theta_k^j} \end{pmatrix}$$

+



$$\frac{d^2 X_k}{d\tau^2} + (\kappa^2 + \tau^2) X_k = 0 .$$

$$\kappa^2 = \frac{k^2}{gm\phi_0}$$

Method of successive scatterings

$$n_k^{j+1} = e^{-\pi\kappa^2} + \left(1 + 2e^{-\pi\kappa^2}\right) n_k^j - 2e^{-\frac{\pi}{2}\kappa^2} \sqrt{1 + e^{-\pi\kappa^2}} \sqrt{n_k^j(1 + n_k^j)} \sin \theta_{tot}^j$$

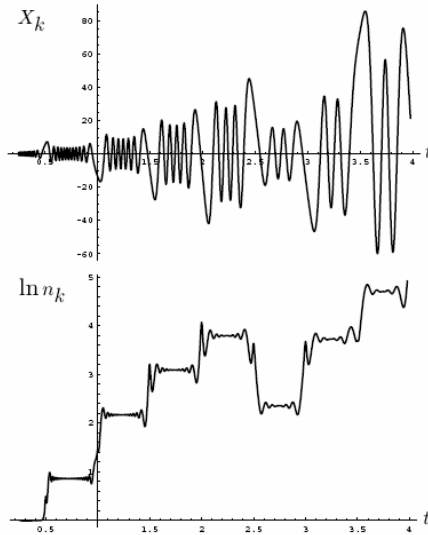
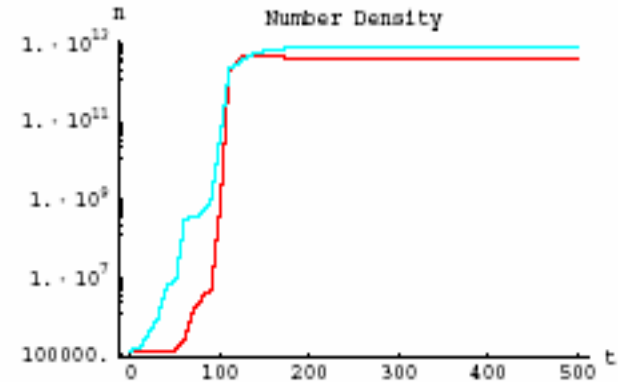
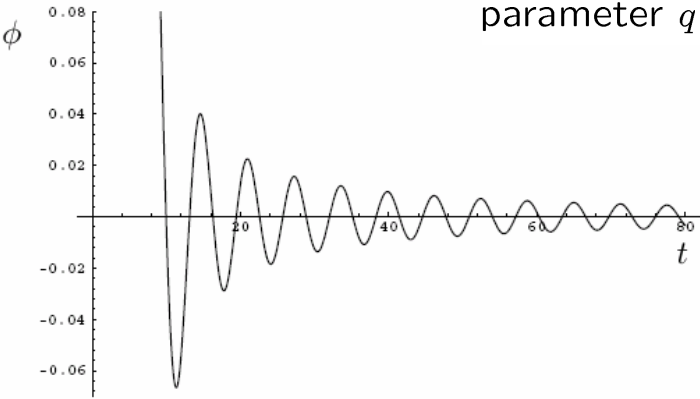
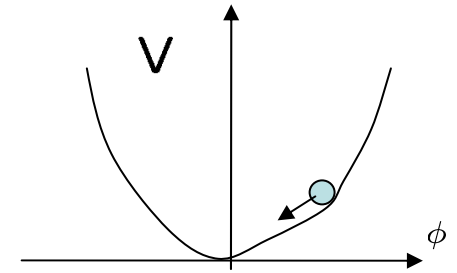
$$n_k^{j+1} \approx \left(1 + 2e^{-\pi\kappa^2} - 2 \sin \theta_{tot}^j e^{-\frac{\pi}{2}\kappa^2} \sqrt{1 + e^{-\pi\kappa^2}}\right) n_k^j$$

$$n_{j+1} \approx e^{4\pi j \mu_k} = e^{2\mu_k m t}$$

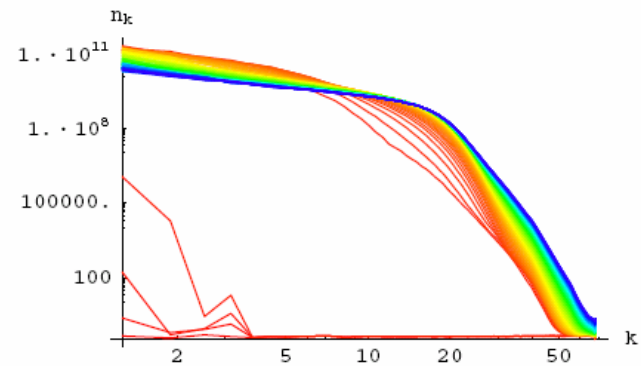
Resonant Preheating in Chaotic Inflation

$$g^2 \phi^2 \chi^2$$

parameter $q = \frac{g^2 \phi_0^2}{m^2} \sim g^2 10^{10} \gg 1$



$\delta\chi$



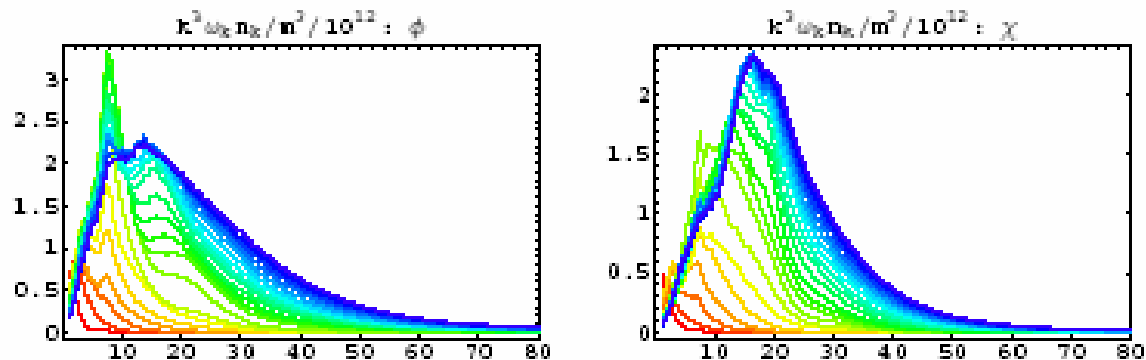


FIG. 1: Evolution of spectra in the combination $k^2 \omega_k n_k$ of the ϕ and χ fields during and immediately after preheating. Blue plots show later spectra. Horizontal axis k is in units of m .

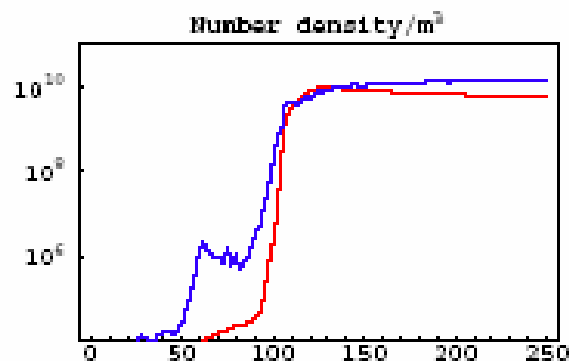


FIG. 2: Evolution of comoving number density of ϕ (red, lower plot) and χ (blue, upper plot) in units of $mass^2$. Time is in units of $1/m$.

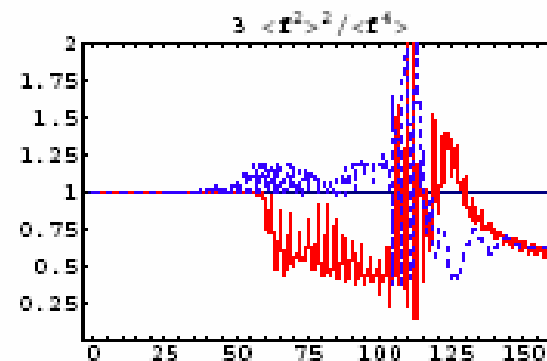
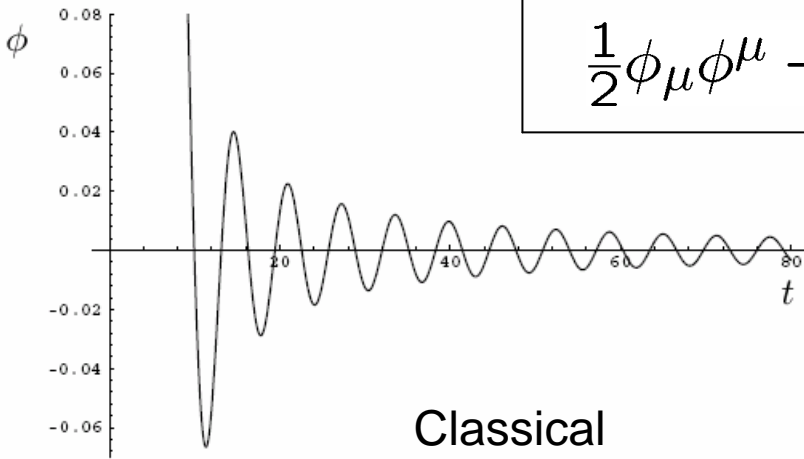


FIG. 3: Evolution of the ratio $(\langle f^2 \rangle^2 / \langle f^4 \rangle)$, where f represents the ϕ field (red, solid) or the χ field (blue, dashed) and angle brackets represent a spatial average, is a measure of gaussianity. This ratio is one for a random gaussian field. Time is in units $1/m$.

$$\frac{1}{2}\phi_{,\mu}\phi^{,\mu} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_{,\mu}\chi^{,\mu} + g^2\phi^2\chi^2$$



Classical

$$\phi_0 + \phi$$

**Decay of inflaton
and preheating after inflation**

Quantum

$$\chi$$

$$\ddot{\phi} - \nabla^2\phi + g^2\chi^2\phi = 0$$

$$\ddot{\chi} - \nabla^2\chi + g^2\phi^2\chi = 0$$

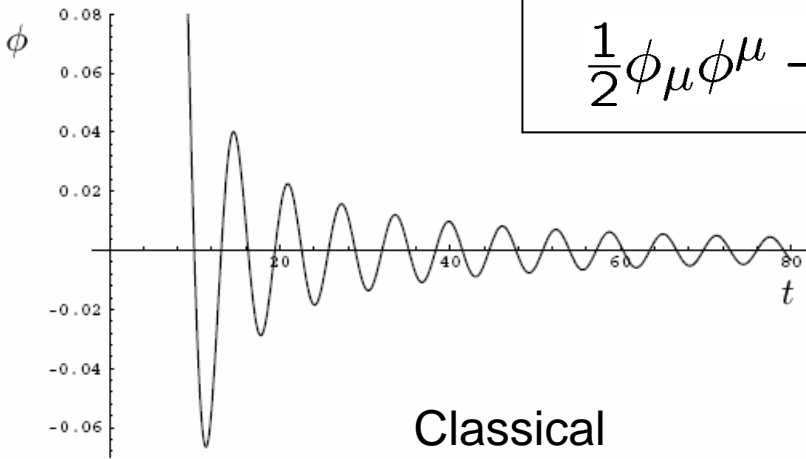
Felder, LK
hep-ph/0606256

movie



$$\frac{1}{2}\phi_{,\mu}\phi^{,\mu} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_{,\mu}\chi^{,\mu} + g^2\phi^2\chi^2$$

Decay of inflaton and preheating after inflation



Classical

$$\phi_0 + \phi$$

Quantum

$$\chi$$

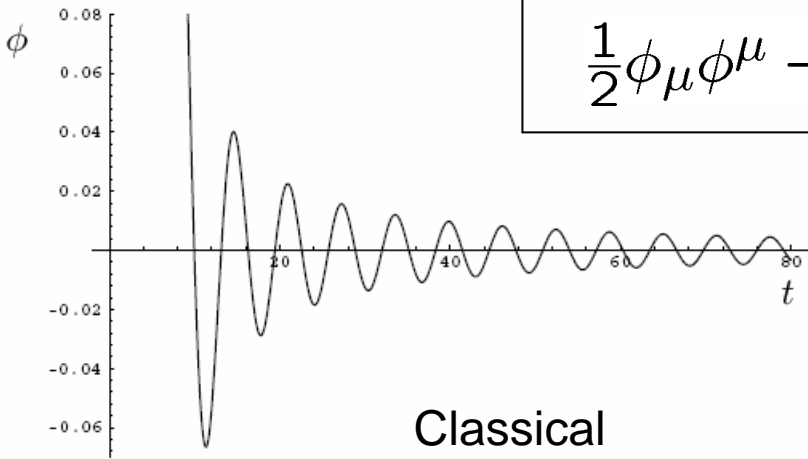


movie



$$\frac{1}{2}\phi_{\mu}\phi^{\mu} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_{\mu}\chi^{\mu} + g^2\phi^2\chi^2$$

**Decay of inflaton
and preheating after inflation**



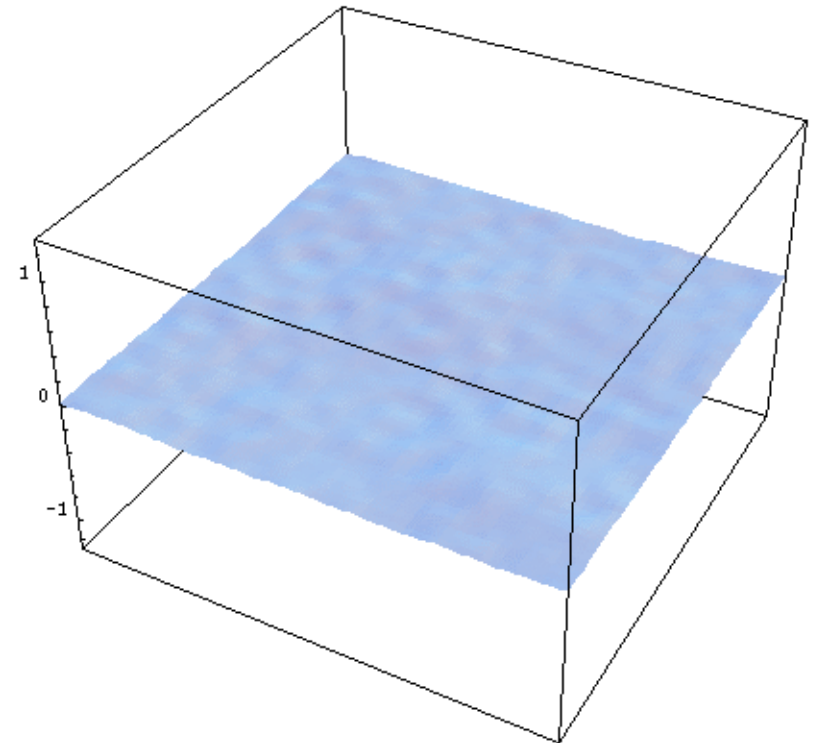
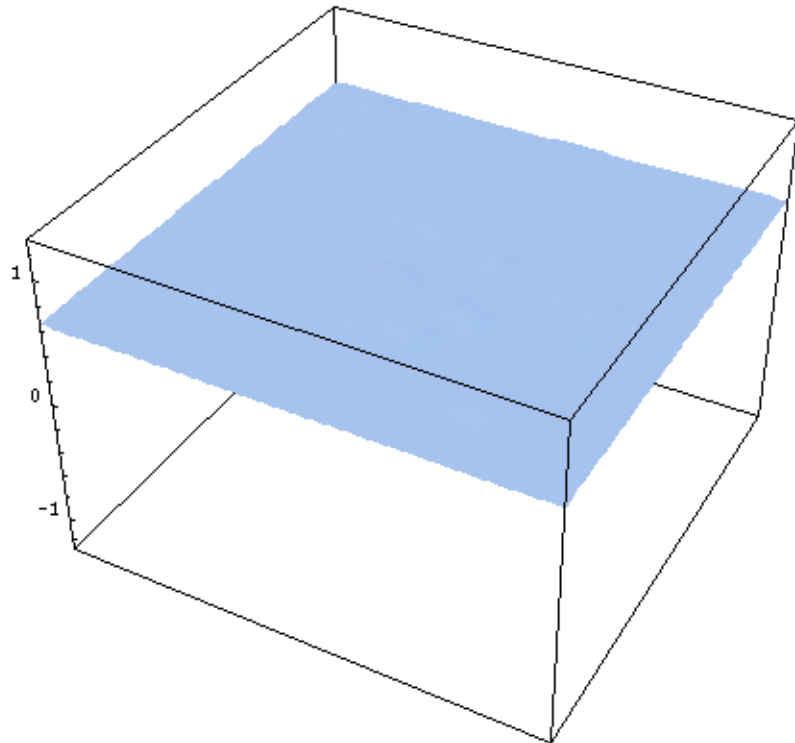
Classical

Quantum

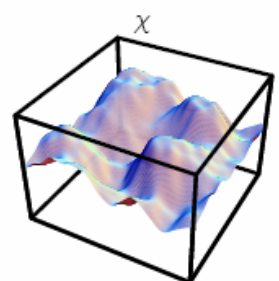
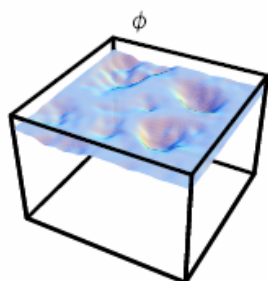
$\phi_0 + \phi$

χ

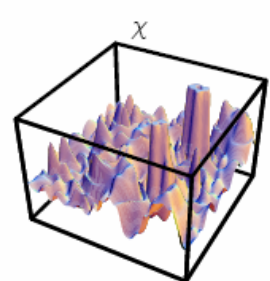
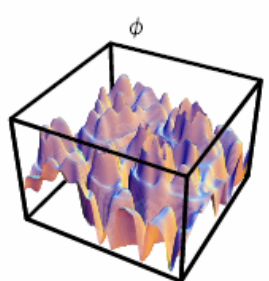
Slices for t=95.2019



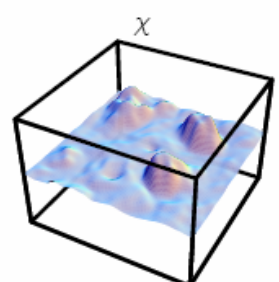
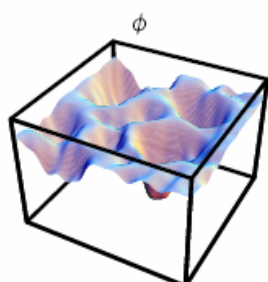
t=106



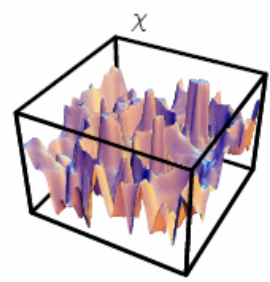
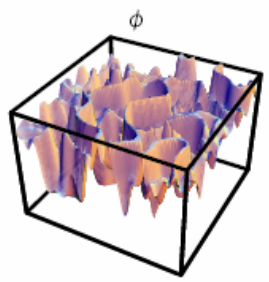
t=116



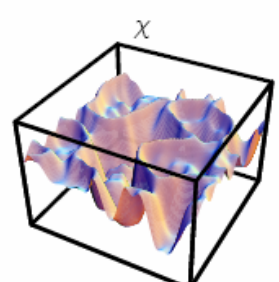
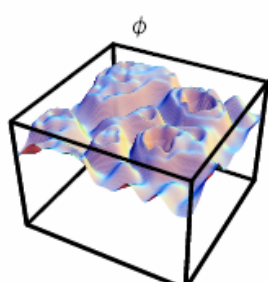
t=107



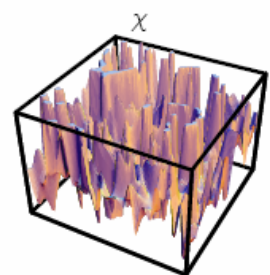
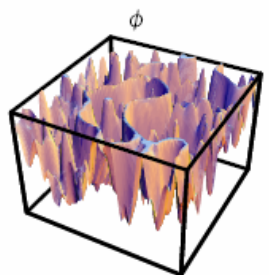
t=119



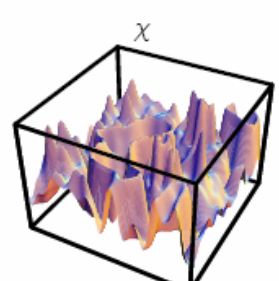
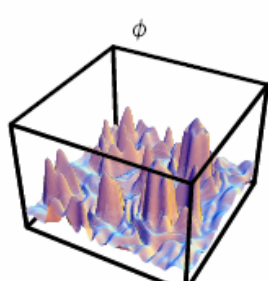
t=112



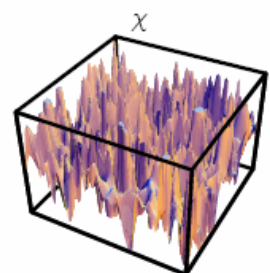
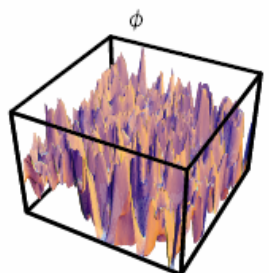
t=124



t=115



t=128



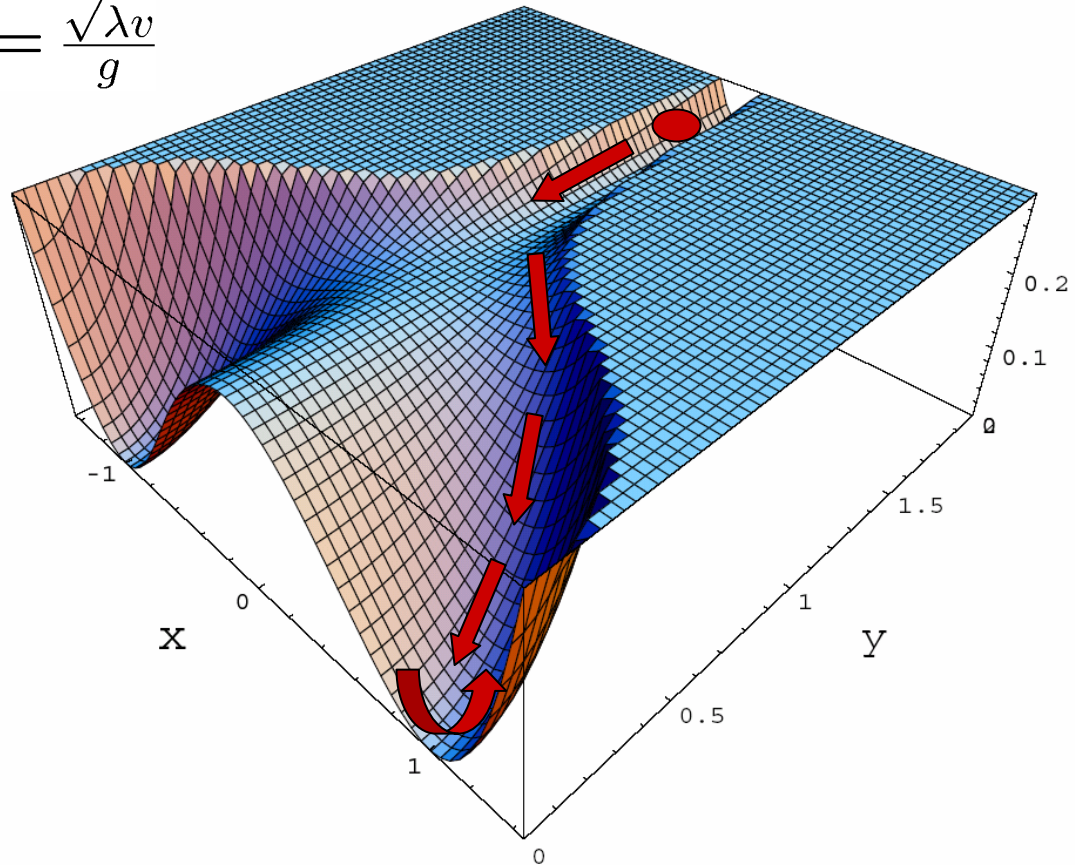
Evolution of energy density

Evolution of gravitational potential

Tachyonic Preheating in Hybrid Inflation

$$V(\phi, \sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{g^2}{2}\phi^2\sigma^2$$

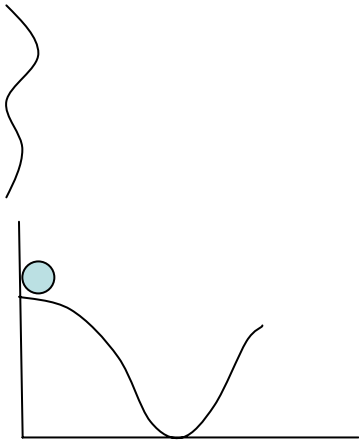
bifurcation point $\phi_c = \frac{\sqrt{\lambda}v}{g}$



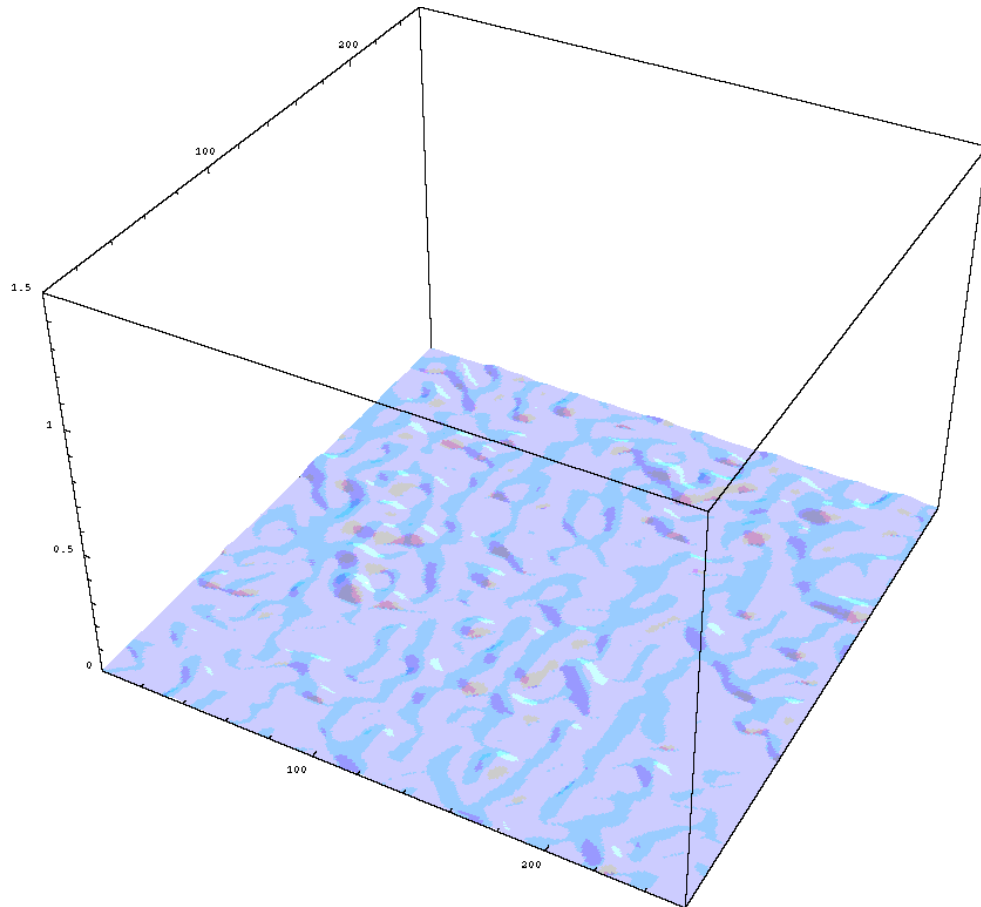
movie 

Tachyonic Preheating

$$V_F = V_0 + \frac{\lambda}{4}\sigma^4 - \frac{\lambda^3}{4}\sigma^3 + \lambda\sigma^2$$



$\sigma(t, \vec{x})$



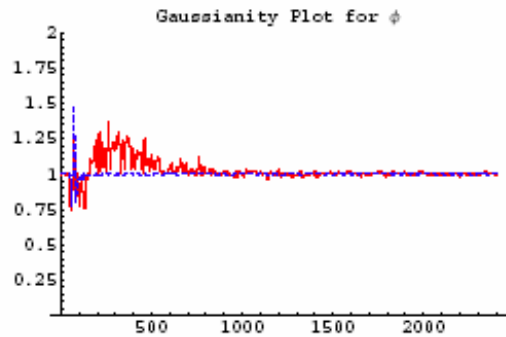
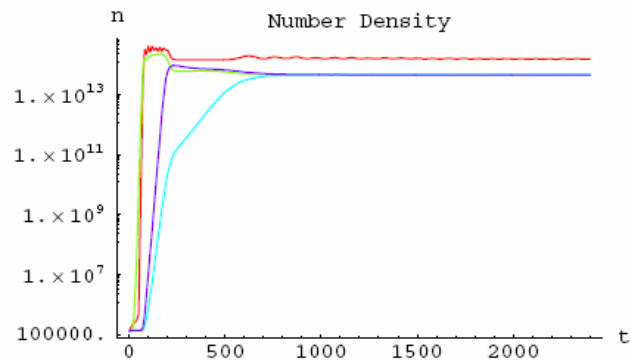
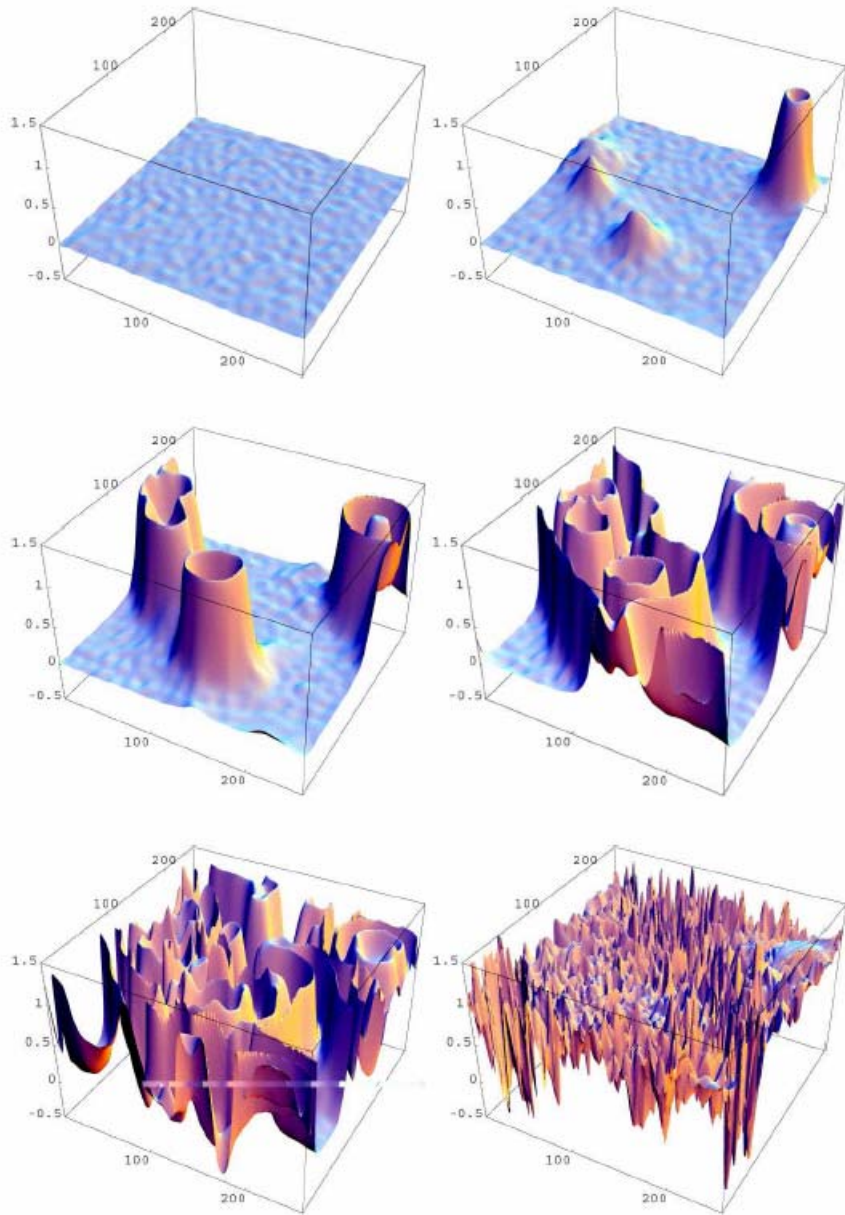
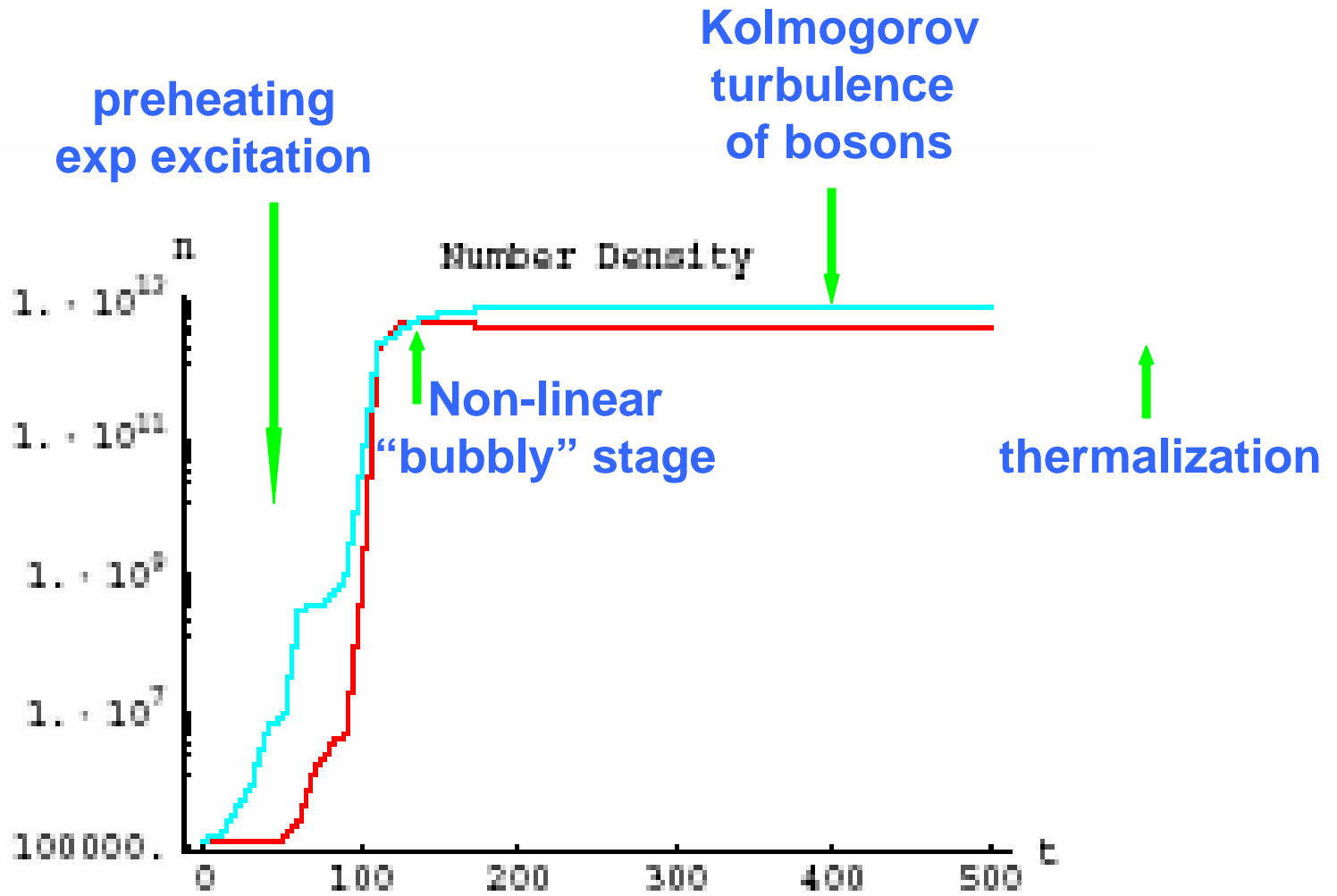


FIG. 10. Deviations from Gaussianity for the field ϕ as a function of time. The solid, red line shows $3\langle\delta\phi^2\rangle^2/\langle\delta\phi^4\rangle$ and the dashed, blue line shows $3\langle\delta\phi^2\rangle^2/\langle\delta\phi^4\rangle$.



1. In many, if not all viable models of inflation there exists a mechanism for exponentially amplifying fluctuations of at least one field χ . These mechanisms tend to excite long-wavelength excitations, giving rise to a highly infrared spectrum.

2. Exciting one field χ is sufficient to rapidly drag all other light fields with which χ interacts into a similarly excited state.

3. The excited fields will be grouped into subsets with identical characteristics (spectra, occupation numbers, effective temperatures) depending on the coupling strengths.

4. Once the fields are amplified, they will approach thermal equilibrium by scattering energy into higher momentum modes.

5. There is a stage of turbulence before thermalization. EoS very rapidly evolves towards radiation domination before thermalization.

$$D(t)^2 \equiv \sum_A (|f'_A - f_A|)^2 + (|\dot{f}'_A - \dot{f}_A|)^2$$

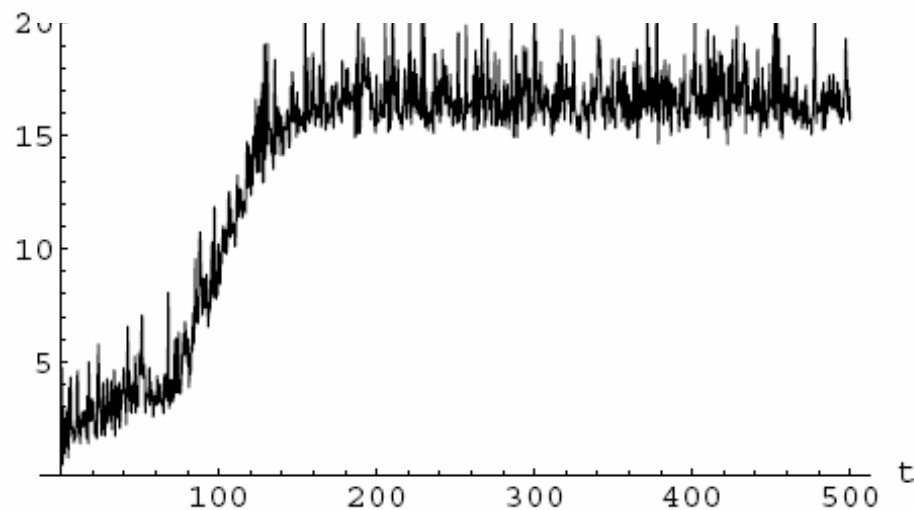


FIG. 8. The Lyapunov exponent λ' for the fields ϕ and χ using the normalized distance function Δ .

Three-linear interaction

In expanding universe complete inflaton decay requires 3-legs interactions

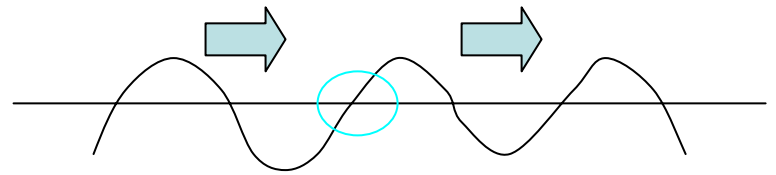
$$V = \frac{m^2}{2} \phi^2 + \frac{\sigma}{2} \phi \chi^2 + \frac{g^2}{2} \phi^2 \chi^2 + \frac{\lambda}{4} \chi^4 \quad \Longleftrightarrow \quad W = \frac{m}{2\sqrt{2}} \phi^2 + \frac{g}{2\sqrt{2}} \phi \chi^2$$
$$\lambda = g^2/2 \text{ and } \sigma = gm$$

$$\chi_k'' + (A_k - 2q \cos 2z) \chi_k = 0$$

$$mt = 2z - \frac{\pi}{2}, \quad A_k = \frac{4k^2}{m^2} \text{ and } q = \frac{2\sigma\Phi}{m^2}$$

$$A_k \geq 2q \quad \text{Broad Parametric Resonance}$$

$$0 < A_k < 2q \quad \text{Tachyonic Resonance}$$



Above the barrier

$$\chi_k^j(t) = \frac{\alpha_k^j}{\sqrt{2\omega_k(t)}} \exp\left(-i \int_{t_0}^t \omega_k(t') dt'\right) + \frac{\beta_k^j}{\sqrt{2\omega_k(t)}} \exp\left(i \int_{t_0}^t \omega_k(t') dt'\right)$$

Below the barrier

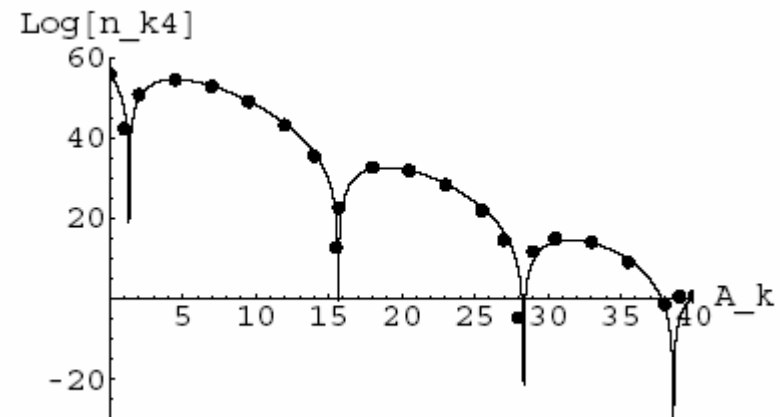
$$\chi_k(t) \simeq \frac{a_k^j}{\sqrt{2\Omega_k(t)}} \exp\left(-\int_{t_{kj}^-}^t \Omega_k(t') dt'\right) + \frac{b_k^j}{\sqrt{2\Omega_k(t)}} \exp\left(\int_{t_{kj}^-}^t \Omega_k(t') dt'\right)$$

$$\begin{pmatrix} \alpha_k^{j+1} \\ \beta_k^{j+1} \end{pmatrix} = e^{X_k^j} \begin{pmatrix} 1 & i e^{2i\theta_k^j} \\ -i e^{-2i\theta_k^j} & 1 \end{pmatrix} \begin{pmatrix} \alpha_k^j \\ \beta_k^j \end{pmatrix}$$

$$X_k^j = \int_{t_{kj}^-}^{t_{kj}^+} \Omega_k(t') dt'$$

$$n_k^j = |\beta_k^j|^2 = \exp(2jX_k) (2 \cos \Theta_k)^{2(j-1)}$$

$$X_k \simeq -\frac{x}{\sqrt{q}} A_k + 2x \sqrt{q} \quad x \simeq 0.85.$$



$$j = 4, q = 20$$

Three- vs four-legs

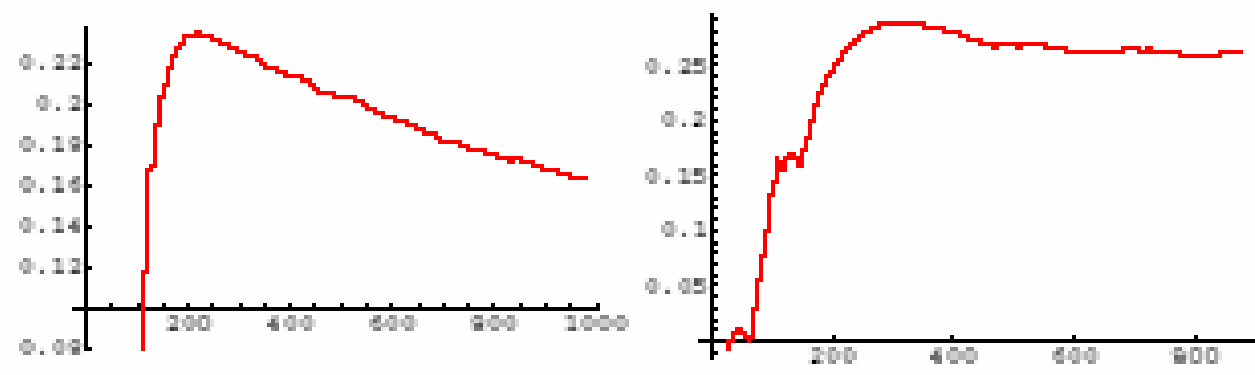
$$q_3 = \frac{\sigma \Phi}{m^2} \quad \text{and} \quad q_4 = \frac{g^2 \Phi^2}{m^2}$$

Four-legs dominates at preheating

$$q_3 < q_4^{3/4} \quad \leftarrow \quad \text{SUSY} \quad q_3 = \sqrt{q_4}$$

$$\Phi \sim \frac{1}{a^{3/2}}$$

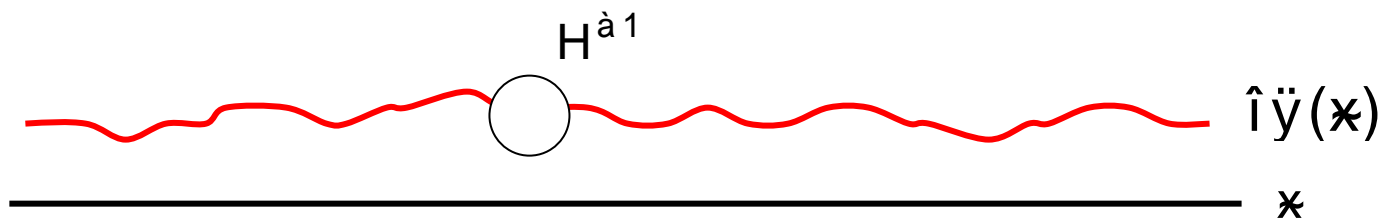
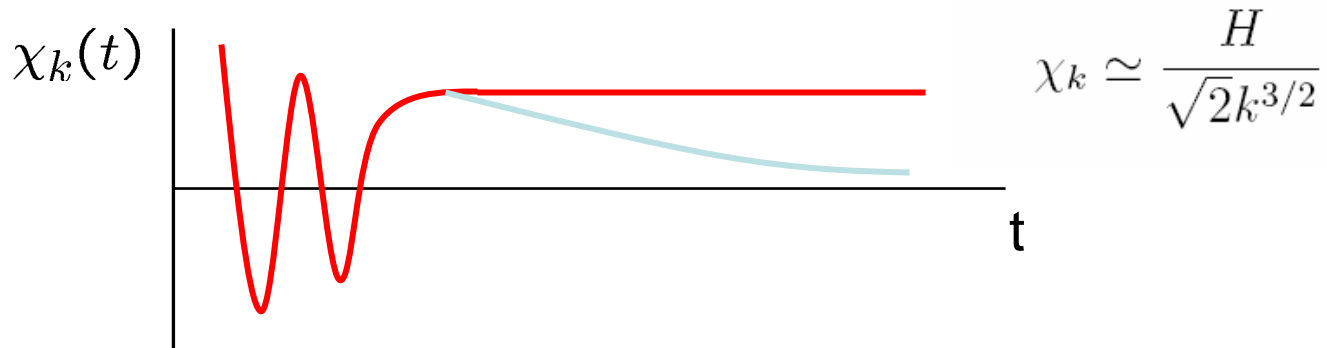
Three-legs dominates after preheating



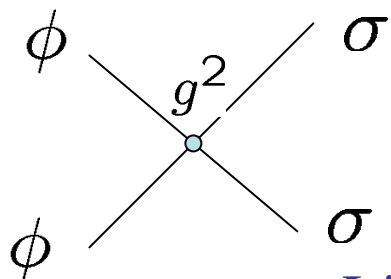
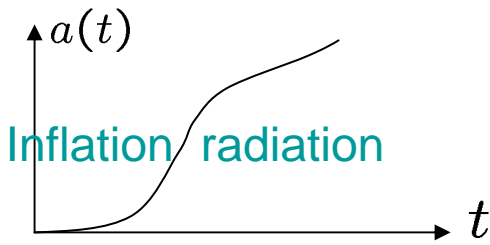
Light field at inflation

$$\hat{\gamma} = \int d^3k (a_k \ddot{\gamma}_k(t) e^{ikx} + \text{h.c.})$$

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \frac{k^2}{a^2}\chi_k = 0$$



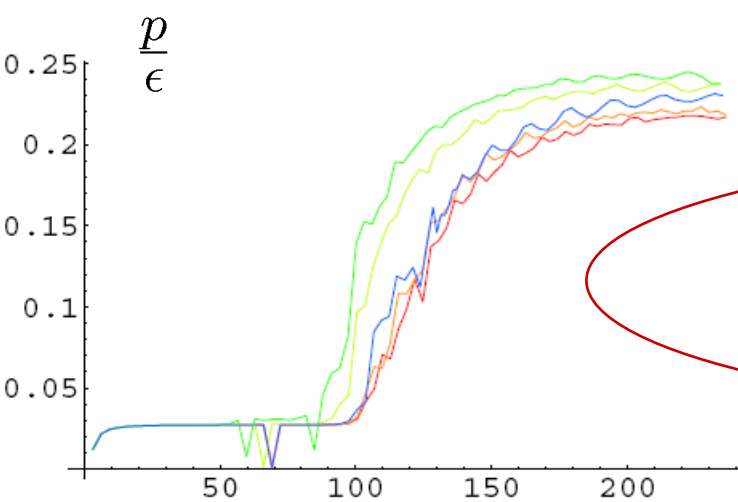
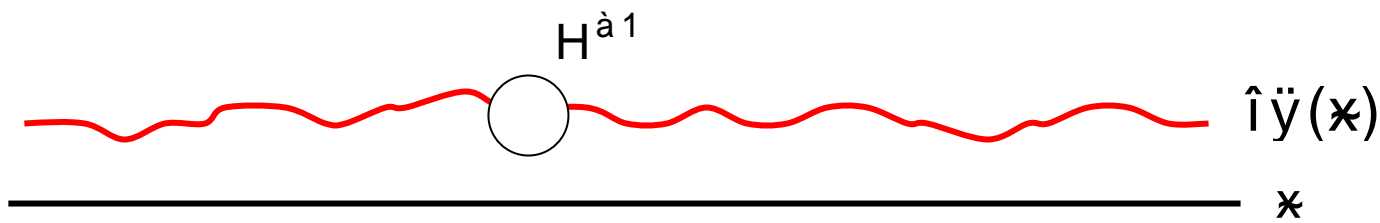
$g^2 \phi^2 \sigma^2$ **Modulated Fluctuations**



coupling depends on moduli $g^2 = g^2(\chi)$

Light field at inflation develops fluctuations $\chi_k \simeq \frac{H}{\sqrt{2}k^{3/2}}$

spacial variations $\delta g^2 = \frac{\partial g^2}{\partial \chi} \delta \chi$



varying $g^2 = 10^{-7}$ by 5%

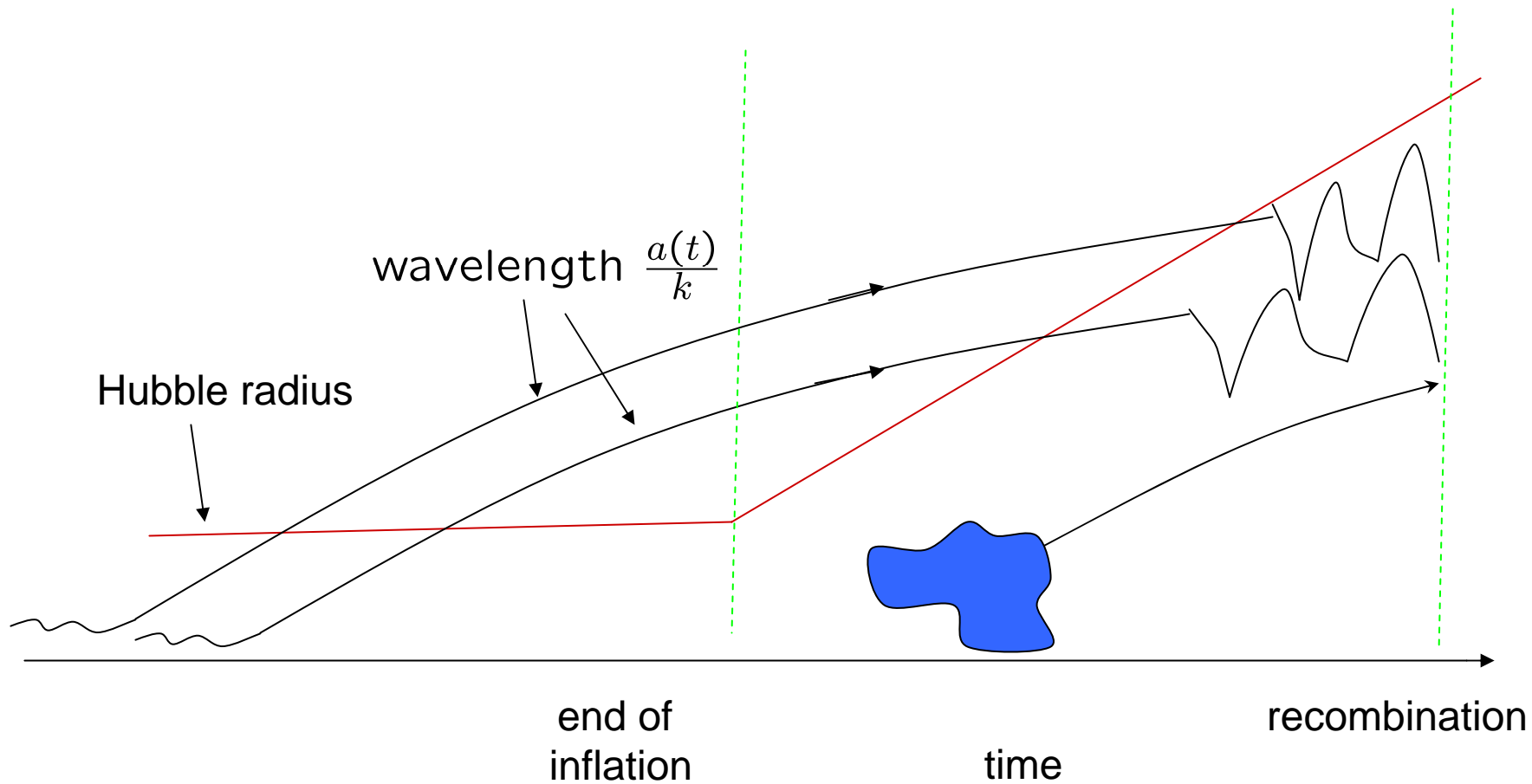
Generation of metric fluctuations
 $\delta \chi_k \rightarrow \delta g^2 \rightarrow \Phi_k$

Generation of gravitational waves from random media

$$ds^2 = -dt^2 + a(t)^2 (\delta_{ij} + h_{ij}) dx^i dx^j$$

$$h_{\mathbf{i}}^{\mathbf{i}} = 0, h_{\mathbf{j}\mathbf{i}}^{\mathbf{i}} = 0, \mathbf{i}, \mathbf{j} = 1, 2, 3.$$

$$\square h_{ij} = \frac{8\pi}{M_p^2} T_{ij}^{TT}$$



$$\square h_{ij} = \frac{8\pi}{M_p^2} T_{ij}^{TT}$$

Stochastic background of gravitational waves emitted from preheating after inflation

Khlebnikov, Tkachev, PRD56(1997)653

Easther and Lim, astro-ph/0601617

Felder and LK, hep-ph/0606256

Easther, Giblin and Lim, astro-ph/0612294

Garcia-Bellido and Figueroa, astro-ph/0701014

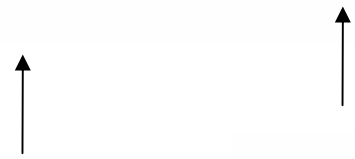
Dufaux, LK et al astro-ph/0707:0875

Garcia-Bellido, Figueroa, astro-ph/0707:0839

For isolated sources

$$\square h_{ij} = \frac{8\pi}{M_p^2} T_{ij}^{TT}$$

$$\frac{dE}{d\Omega} = 2G\Lambda_{ij,lm}\omega^2 T^{ij*}(\vec{k}, \omega) T^{lm}(\vec{k}, \omega) d\omega$$



$$T_{ij}(\mathbf{k}, \omega) = \int \frac{d\tau}{2\pi} e^{i\omega\tau} \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} T_{ij}(\tau, \mathbf{x})$$

$$\Lambda_{ij,lm}(\hat{k}) = \delta_{ij}\delta_{lm} - 2\hat{k}_j\hat{k}_m\delta_{il} + \frac{1}{2}\hat{k}_i\hat{k}_j\hat{k}_l\hat{k}_m - \frac{1}{2}\delta_{ij}\delta_{lm} + \frac{1}{2}\delta_{ij}\hat{k}_l\hat{k}_m + \frac{1}{2}\delta_{jl}\hat{k}_i\hat{k}_m$$

Emission of stochastic GW by random media

Theory and Numerics of Gravitational Waves from Preheating after Inflation.

Jean-François Dufaux¹, Amanda Bergman², Gary Felder², Lev Kofman¹ and Jean-Philippe Uzan³

astro-ph:0707.0875

$$h''_{ij} + 2 \frac{a'}{a} h'_{ij} - \nabla^2 h_{ij} = 16\pi G a^2 \Pi_{ij}^{\text{TT}}$$

$$\bar{h}_{ij} = a h_{ij}$$

$$\bar{h}''_{ij}(\mathbf{k}) + \left(k^2 - \frac{a''}{a} \right) \bar{h}_{ij}(\mathbf{k}) = 16\pi G a^3 \Pi_{ij}^{\text{TT}}(\mathbf{k})$$

$$\Pi_{ij}^{\text{TT}}(\mathbf{k}) = \mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \Pi_{lm}(\mathbf{k}) = \left[P_{il}(\hat{\mathbf{k}}) P_{jm}(\hat{\mathbf{k}}) - \frac{1}{2} P_{ij}(\hat{\mathbf{k}}) P_{lm}(\hat{\mathbf{k}}) \right] \Pi_{lm}(\mathbf{k})$$

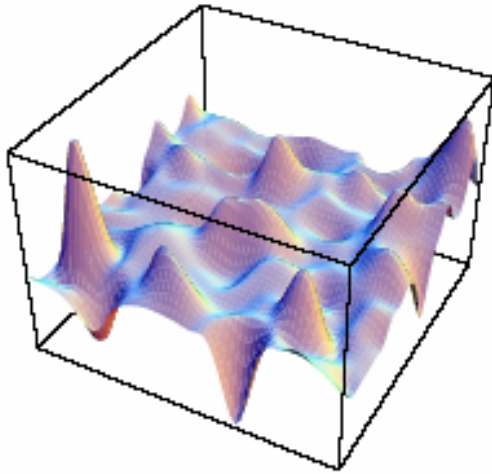
$$P_{ij}(\hat{\mathbf{k}}) = \delta_{ij} - \hat{k}_i \hat{k}_j$$

Emission of stochastic GW by random scalar fields

$$\bar{h}_{ij}''(\tau, \mathbf{k}) + k^2 \bar{h}_{ij}(\tau, \mathbf{k}) = 16\pi G a(\tau) T_{ij}^{\text{TT}}(\tau, \mathbf{k})$$

$$\bar{h}_{ij}(\tau, \mathbf{k}) = \frac{16\pi G}{k} \int_{\tau_i}^{\tau} d\tau' \sin[k(\tau - \tau')] a(\tau') T_{ij}^{\text{TT}}(\tau', \mathbf{k})$$

$$T_{ij}^{\text{TT}}(\mathbf{k}) = O_{ij,lm}(\hat{\mathbf{k}}) \{ \partial_l \phi_a \partial_m \phi_a \}(\mathbf{k}) = O_{ij,lm}(\hat{\mathbf{k}}) \int \frac{d^3 \mathbf{p}}{(2\pi)^{3/2}} p_l p_m \phi_a(\mathbf{p}) \phi_a(\mathbf{k} - \mathbf{p})$$



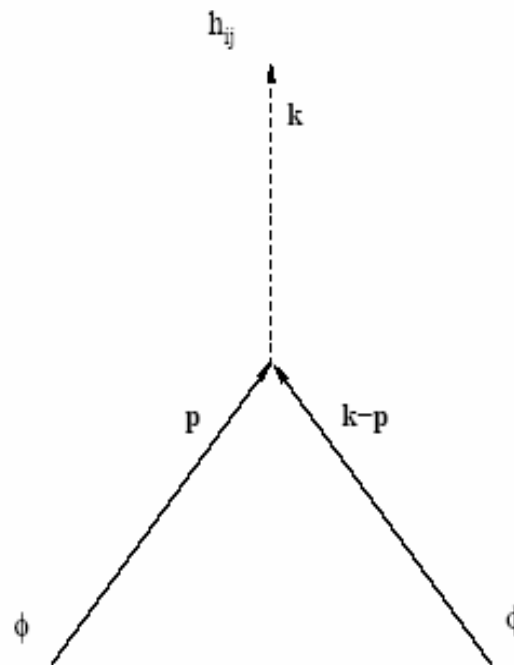
First order phase transitions
Second order phase transitions
Topological defects formation
Thermal bath of scalars
Tachyonic preheating
Resonant preheating

No-go Theorem: No Gravity Waves from Scalar Field Waves

$$\bar{h}_{ij}(\tau, \mathbf{k}) = \frac{16\pi G}{k} \mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int_{\tau_i}^{\tau} d\tau' \sin[k(\tau - \tau')] a(\tau') \int \frac{d^3 \mathbf{p}}{(2\pi)^{3/2}} p_l p_m \phi_a(\tau', \mathbf{p}) \phi_a(\tau', \mathbf{k} - \mathbf{p})$$

$$\phi(\mathbf{p}, \tau) e^{i\mathbf{p}\mathbf{x}} = b(\mathbf{p}) e^{\pm i\omega_p \tau + i\mathbf{p}\mathbf{x}} \quad \omega_p^2 = p^2 + m^2 + g^2 \psi^2$$

$$h_{ij}(\tau, \mathbf{k}) \propto e^{\pm ik\tau} \mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int d^3 \mathbf{p} p_l p_m b(\mathbf{p}) b(\mathbf{k} - \mathbf{p}) \int_{\tau_i}^{\tau} d\tau' e^{i(\pm \omega_p \pm \omega_{|\mathbf{k}-\mathbf{p}|} \pm k)\tau'}$$



\mathbf{k} parallel to \mathbf{p}

$$\mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) p_l p_m = 0$$

FIG. 1: Would be emission of a graviton h_{ij} with momentum \mathbf{k} from the annihilation of two scalar waves $\phi(\mathbf{p})$ and $\phi(\mathbf{k} - \mathbf{p})$ with momenta \mathbf{p} and $\mathbf{p} - \mathbf{k}$. Helicity 2 of the emitted graviton cannot match the helicity zero of the incoming scalar waves.

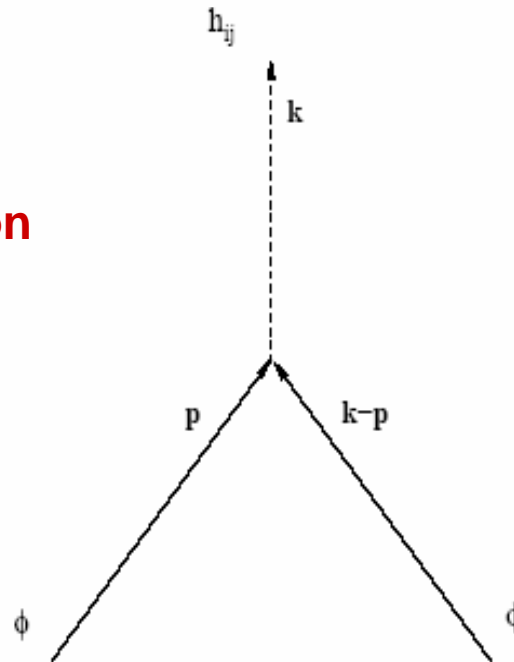
No-go Theorem: No Gravity Waves from Scalar Field Waves

$$\bar{h}_{ij}(\tau, \mathbf{k}) = \frac{16\pi G}{k} \mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int_{\tau_i}^{\tau} d\tau' \sin[k(\tau - \tau')] a(\tau') \int \frac{d^3 \mathbf{p}}{(2\pi)^{3/2}} p_l p_m \phi_a(\tau', \mathbf{p}) \phi_a(\tau', \mathbf{k} - \mathbf{p})$$

$$\phi(\mathbf{p}, \tau) e^{i\mathbf{p}\mathbf{x}} = b(\mathbf{p}) e^{\pm i\omega_p \tau + i\mathbf{p}\mathbf{x}} \quad \omega_p^2 = p^2 + m^2 + g^2 \psi^2$$

$$h_{ij}(\tau, \mathbf{k}) \propto e^{\pm ik\tau} \mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int d^3 \mathbf{p} p_l p_m b(\mathbf{p}) b(\mathbf{k} - \mathbf{p}) \int_{\tau_i}^{\tau} d\tau' e^{i(\pm \omega_p \pm \omega_{|\mathbf{k}-\mathbf{p}|} \pm k) \tau'}$$

No GW emission



k parallel to p

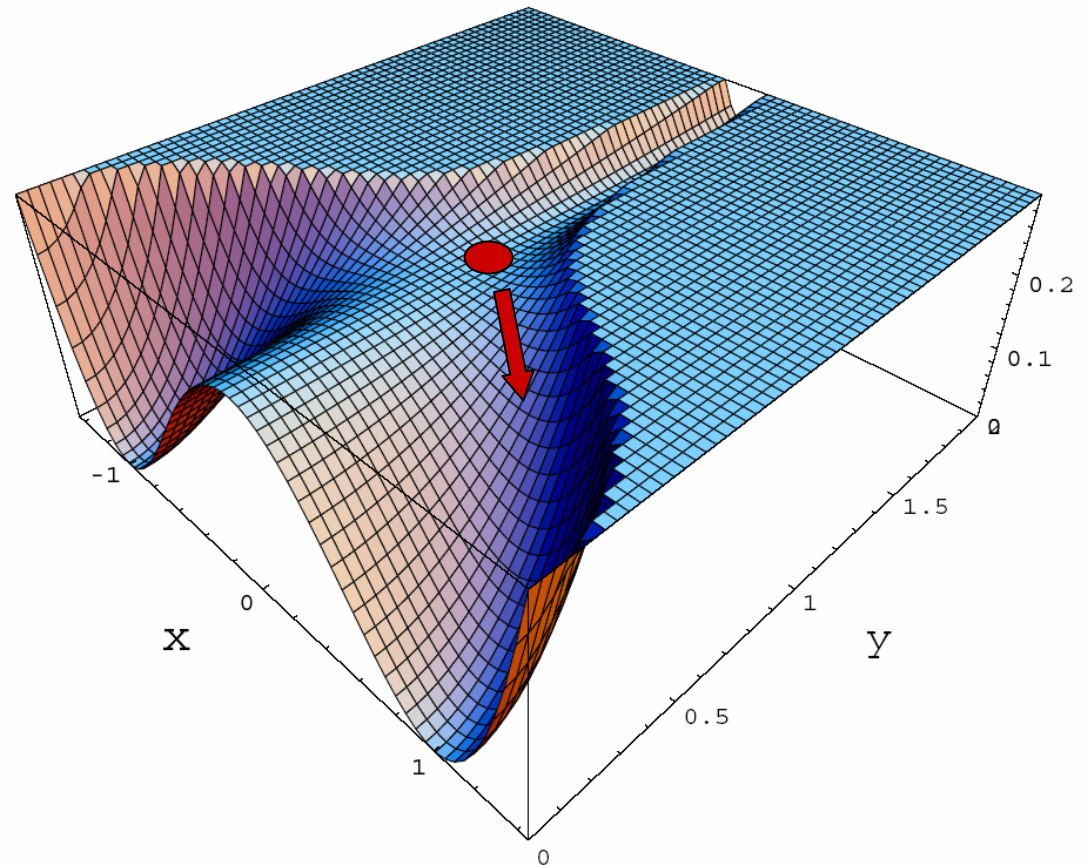
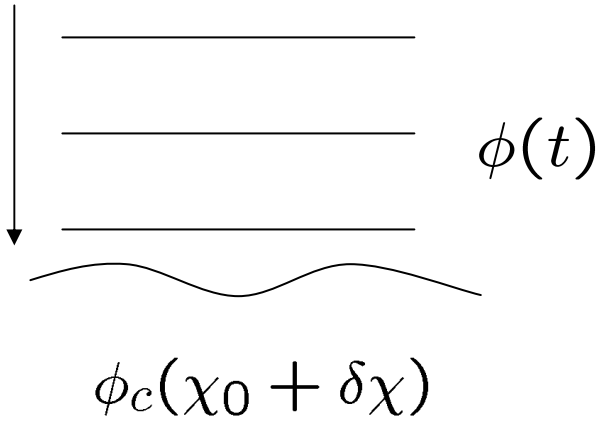
$$\mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) p_l p_m = 0$$

FIG. 1: Would be emission of a graviton h_{ij} with momentum \mathbf{k} from the annihilation of two scalar waves $\phi(\mathbf{p})$ and $\phi(\mathbf{k} - \mathbf{p})$ with momenta \mathbf{p} and $\mathbf{p} - \mathbf{k}$. Helicity 2 of the emitted graviton cannot match the helicity zero of the incoming scalar waves.

Modulated fluctuations in Hybrid Inflation

$$V(\phi, \sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{g^2}{2}\phi^2\sigma^2$$

bifurcation point $\phi_c = \frac{\sqrt{\lambda}v}{g}$



inhomogeneous waterfall

No-go Theorem: No Gravity Waves from Scalar Field Waves

$$\bar{h}_{ij}(\tau, \mathbf{k}) = \frac{16\pi G}{k} \mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int_{\tau_i}^{\tau} d\tau' \sin[k(\tau - \tau')] a(\tau') \int \frac{d^3 \mathbf{p}}{(2\pi)^{3/2}} p_l p_m \phi_a(\tau', \mathbf{p}) \phi_a(\tau', \mathbf{k} - \mathbf{p})$$

$$\phi(\mathbf{p}, \tau) e^{i\mathbf{p}\mathbf{x}} = b(\mathbf{p}) e^{\pm i\omega_p \tau + i\mathbf{p}\mathbf{x}} \quad \omega_p^2 = p^2 + m^2 + g^2 \psi^2$$

$$h_{ij}(\tau, \mathbf{k}) \propto e^{\pm i k \tau} \mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \int d^3 \mathbf{p} p_l p_m b(\mathbf{p}) b(\mathbf{k} - \mathbf{p}) \int_{\tau_i}^{\tau} d\tau' e^{i(\pm \omega_p \pm \omega_{|\mathbf{k}-\mathbf{p}|} \pm k) \tau'}$$

GW generated for non-trivial dispersion relation

$$\omega^2 = \vec{p}^2 + g^2 \phi(t)^2 - \frac{a''}{a}$$

k parallel to p

$$\mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) p_l p_m = 0$$

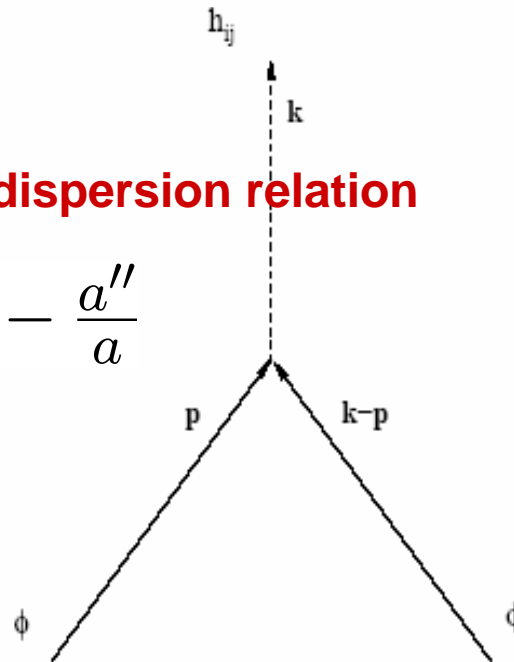


FIG. 1: Would be emission of a graviton h_{ij} with momentum \mathbf{k} from the annihilation of two scalar waves $\phi(\mathbf{p})$ and $\phi(\mathbf{k} - \mathbf{p})$ with momenta \mathbf{p} and $\mathbf{p} - \mathbf{k}$. Helicity 2 of the emitted graviton cannot match the helicity zero of the incoming scalar waves.

$$\rho_{\text{gw}} = \frac{1}{32\pi G} \langle \dot{h}_{ij}(t, \mathbf{x}) \dot{h}_{ij}(t, \mathbf{x}) \rangle$$

$$\frac{(16\pi G)^2}{2} \sum_{i,j} \left\{ \left| \int_{\tau_i}^{\tau_f} d\tau' \cos[k(\tau_f - \tau')] a(\tau') T_{ij}^{\text{TT}}(\tau', \mathbf{k}) \right|^2 + \left| \int_{\tau_i}^{\tau_f} d\tau' \sin[k(\tau_f - \tau')] a(\tau') T_{ij}^{\text{TT}}(\tau', \mathbf{k}) \right|^2 \right\}$$

Random gaussian fields

$$\langle T_{ij}^{\text{TT}}(\tau', \mathbf{k}) T_{ij}^{\text{TT}*}(\tau'', \mathbf{k}') \rangle =$$

$$\mathcal{O}_{ij,lm}(\hat{\mathbf{k}}) \mathcal{O}_{ij,rs}(\hat{\mathbf{k}}') \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} \int \frac{d^3\mathbf{p}'}{(2\pi)^{3/2}} p_l p_m p'_r p'_s \langle \phi_a(\mathbf{p}, \tau') \phi_a(\mathbf{k} - \mathbf{p}, \tau') \phi_b^*(\mathbf{p}', \tau'') \phi_b^*(\mathbf{k}' - \mathbf{p}', \tau'') \rangle$$

$$\left(\frac{d\rho_{\text{gw}}}{d \ln k} \right)_{\tau > \tau_f} = \frac{S_k(\tau_f)}{a^4(\tau)}$$

$$S_k(\tau_f) = \frac{2}{\pi} G k^3 \int \frac{d^3\mathbf{p}}{(2\pi)^3} p^4 \sin^4(\hat{\mathbf{k}}, \hat{\mathbf{p}}) \int_{\tau_i}^{\tau_f} d\tau' \int_{\tau_i}^{\tau_f} d\tau'' \cos[k(\tau' - \tau'')] a(\tau') a(\tau'') F_{ab}(p, \tau', \tau'') F_{ab}(|\mathbf{k} - \mathbf{p}|, \tau', \tau'')$$

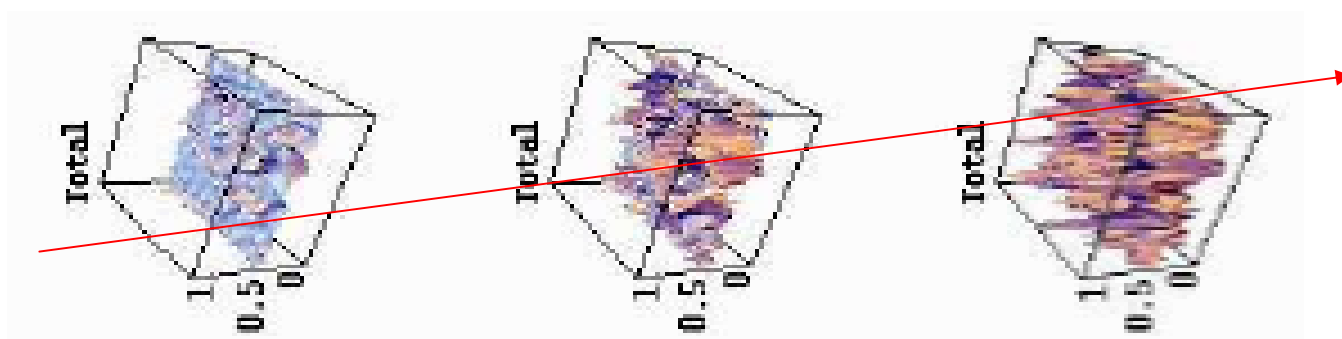
$$\langle \phi_a(\mathbf{p}, \tau') \phi_b^*(\mathbf{p}', \tau'') \rangle = F_{ab}(p, \tau', \tau'') \delta(\mathbf{p} - \mathbf{p}')$$

Emission of GW from preheating

$$\Omega_{\text{gw}} h^2 = 7.8 \times 10^{-5} S_k(\tau_f) \frac{a_j^{-4}}{M_{\text{Pl}}^2 H_j^2}$$

$$\frac{2}{\pi} G k^3 \int \frac{d\mathbf{p}}{(2\pi)^3} p^4 \sin^4(\hat{\mathbf{k}}, \hat{\mathbf{p}})$$

$$\left\{ \left| \int_{\tau_i}^{\tau_f} d\tau \cos(k\tau) a(\tau) \chi_{\mathbf{p}}(\tau) \chi_{|\mathbf{k}-\mathbf{p}|}(\tau) \right|^2 + \left| \int_{\tau_i}^{\tau_f} d\tau \sin(k\tau) a(\tau) \chi_{\mathbf{p}}(\tau) \chi_{|\mathbf{k}-\mathbf{p}|}(\tau) \right|^2 \right\}$$



Numerical calculations of GW emission from Preheating

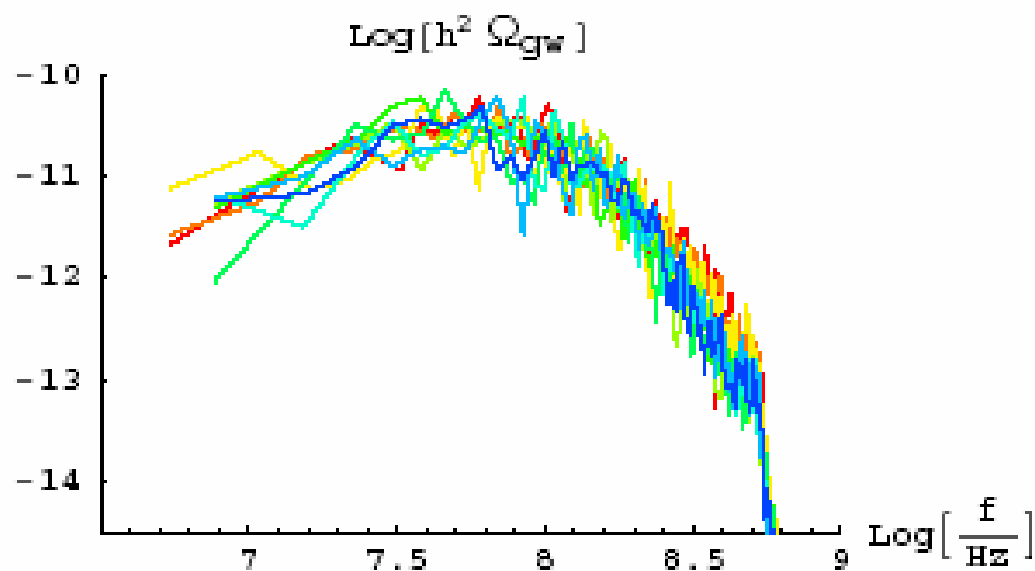


FIG. 3: Spectrum of energy density in gravity waves calculated along nine different directions in k-space. The

$$V = \frac{\lambda}{4} \phi^4 + \frac{g^2}{2} \phi^2 \chi^2 \quad q = \frac{g^2}{\lambda}$$

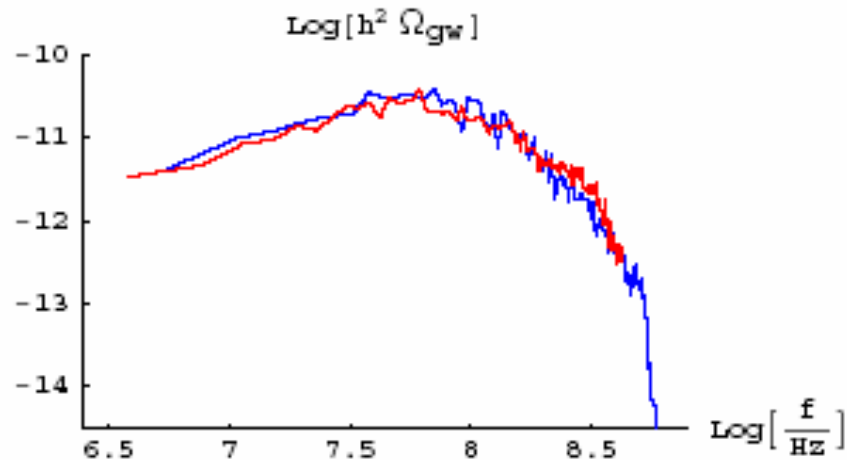


FIG. 1: Spectrum of gravity waves energy density in physical variables today, accumulated up to the time $x_f = 240$, for the model (48) with $q = 120$. The 2 spectra were obtained from simulations with different box sizes, and averaged over different directions in k -space.

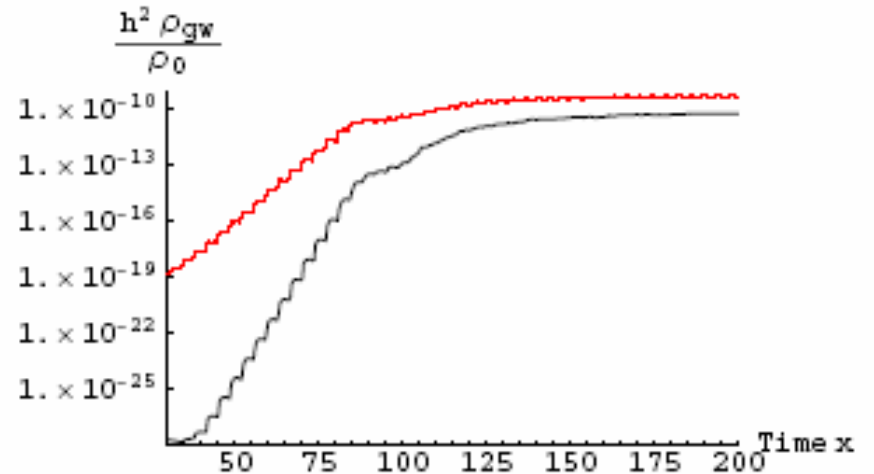


FIG. 2: The thick curve shows the total energy density in gravity waves (53) accumulated up to the time x_f , as a function of x_f . The thin curve shows the evolution with time of the total number density, $n_{\text{tot}} = n_\chi + n_\phi$, rescaled to fit on the same figure.

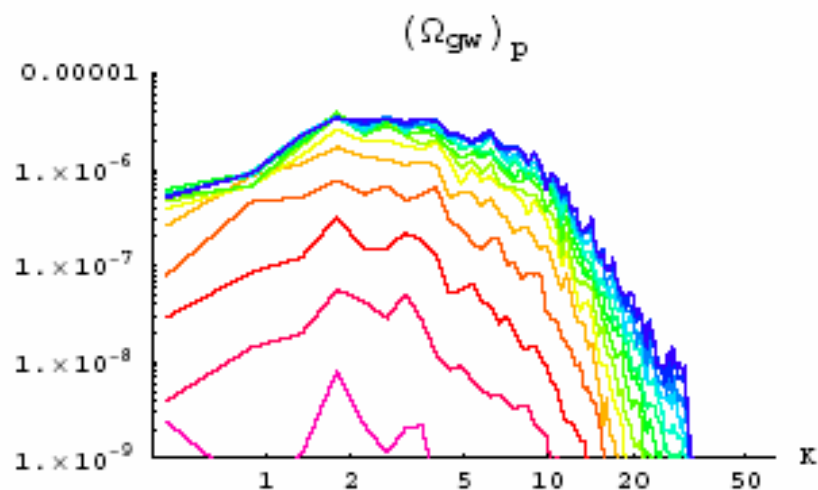


FIG. 3: Spectrum (55) of the gravity waves energy density, accumulated up to different times x_f , as a function of the comoving momentum k (in units of $\lambda\phi_0$). The spectra grow from $x_f = 90$ to $x_f = 240$ with spacing $\Delta x_f = 10$.

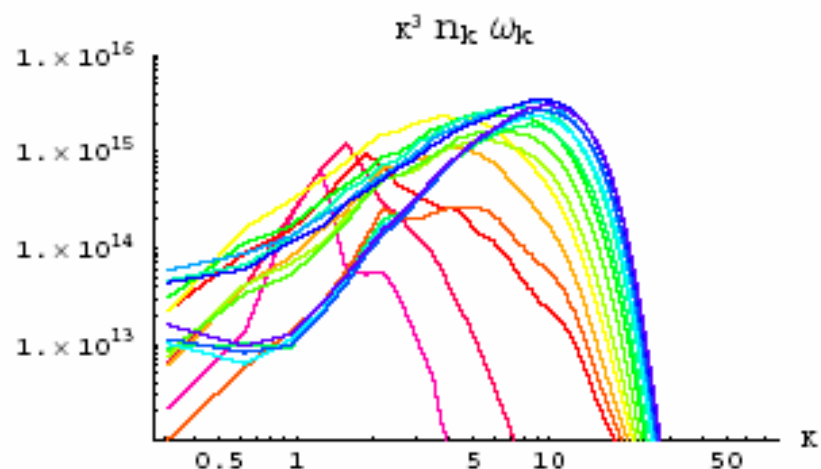
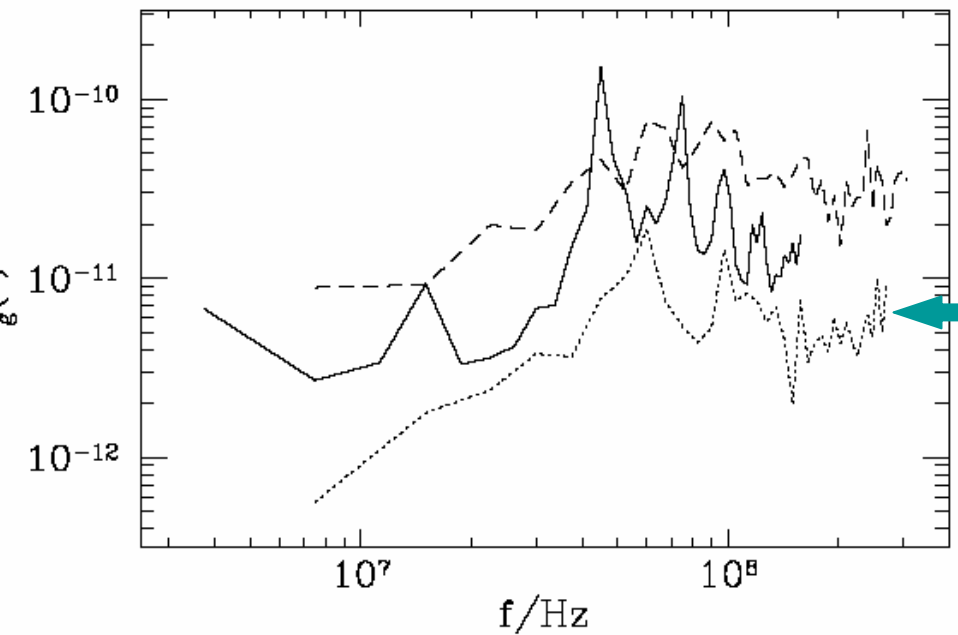
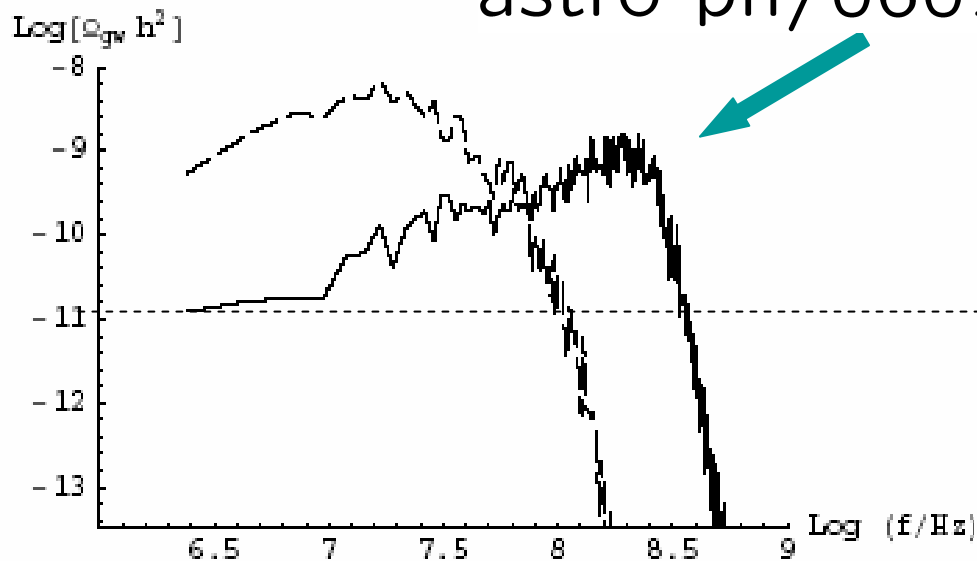


FIG. 4: Measure of the (unnormalised) total energy density in the two scalar fields per logarithmic momentum interval at different moments of time. The same times as in Fig. (3) are shown, the spectra moving towards UV from $x = 90$ to $x = 240$ with spacing $\Delta x = 10$.



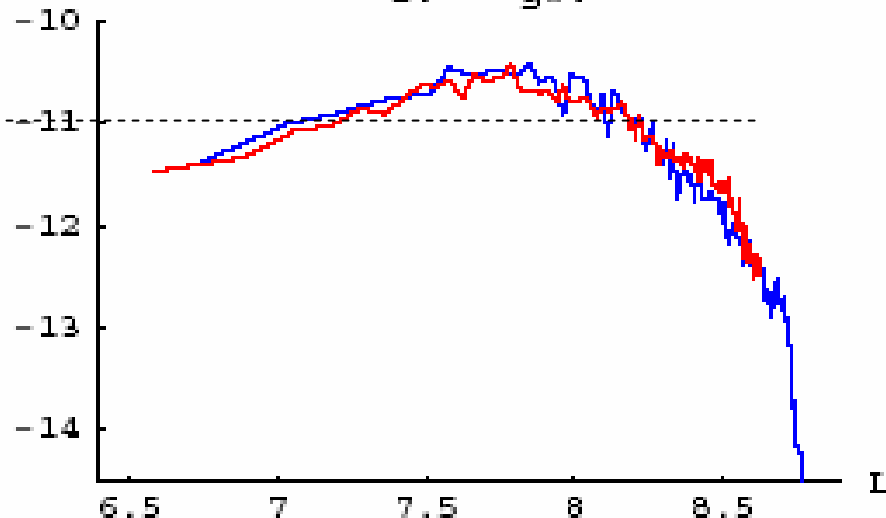
hep-ph/9701423

astro-ph/0601617



astro-ph/0707:0875

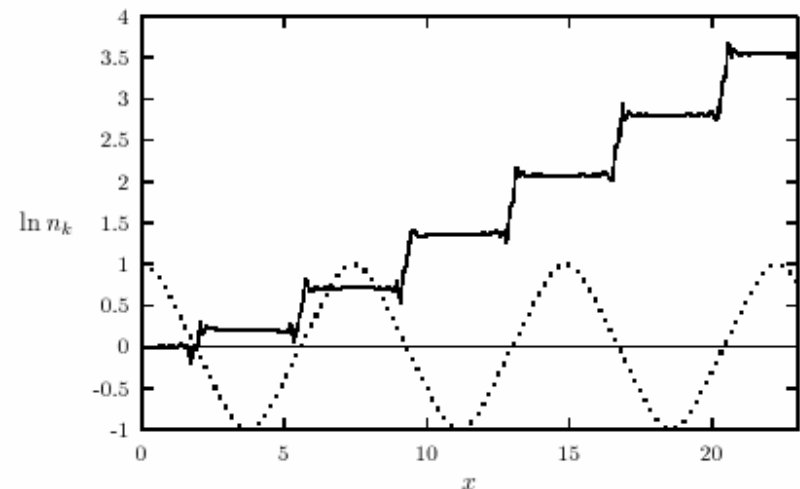
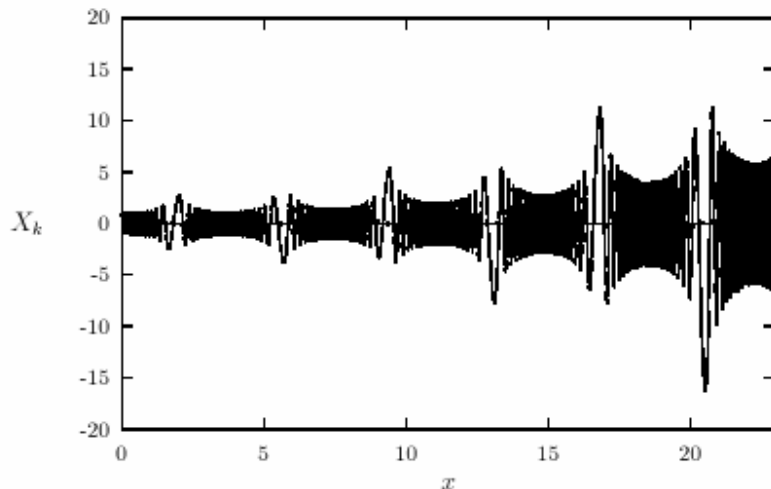
$\text{Log}[h^2 \Omega_{\text{gw}}]$

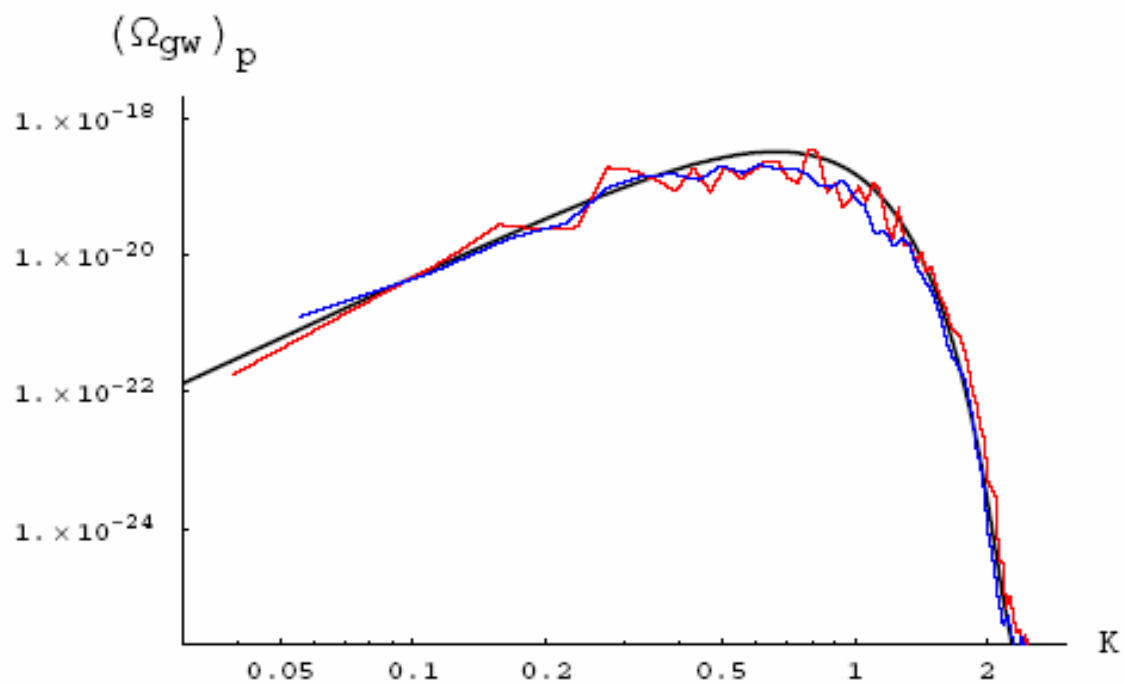
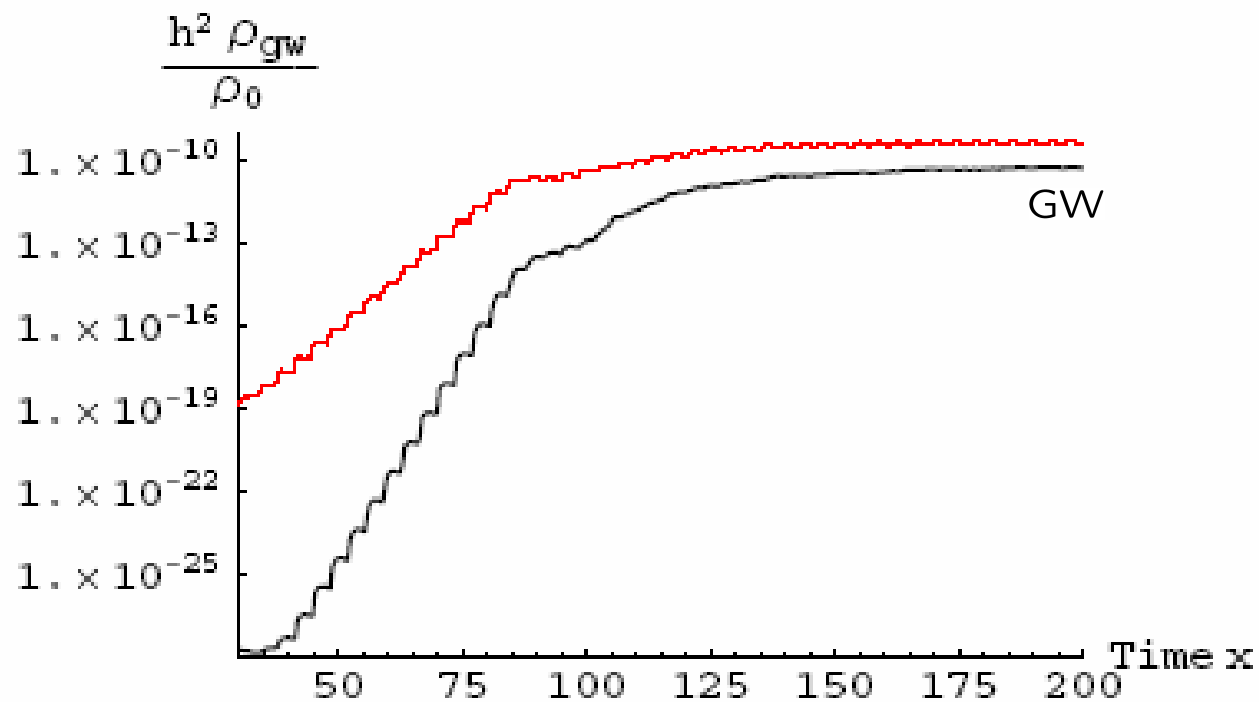


Numerical calculations of GW emission from Preheating

$$\hat{\chi}(\tau, \mathbf{x}) = \int \frac{d^3 \mathbf{k}}{(2\pi)^{3/2}} \left(\hat{a}_{\mathbf{k}} \chi_{\mathbf{k}}(\tau) e^{i\mathbf{k}\mathbf{x}} + \hat{a}_{\mathbf{k}}^\dagger \chi_{\mathbf{k}}^*(\tau) e^{-i\mathbf{k}\mathbf{x}} \right)$$

$$S_k(\tau_f) = \frac{2}{\pi} G k^3 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} p^4 \sin^4(\hat{\mathbf{k}}, \hat{\mathbf{p}}) \left\{ \left| \int_{\tau_i}^{\tau_f} d\tau \cos(k\tau) a(\tau) \chi_{\mathbf{p}}(\tau) \chi_{|\mathbf{k}-\mathbf{p}|}(\tau) \right|^2 + \left| \int_{\tau_i}^{\tau_f} d\tau \sin(k\tau) a(\tau) \chi_{\mathbf{p}}(\tau) \chi_{|\mathbf{k}-\mathbf{p}|}(\tau) \right|^2 \right\}$$





Analytic check

Shortcut to the answer

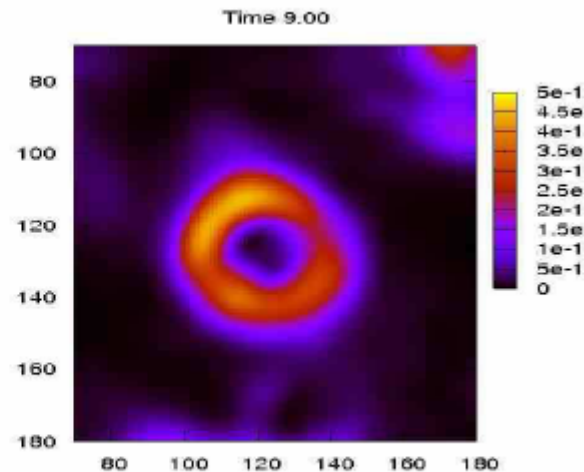
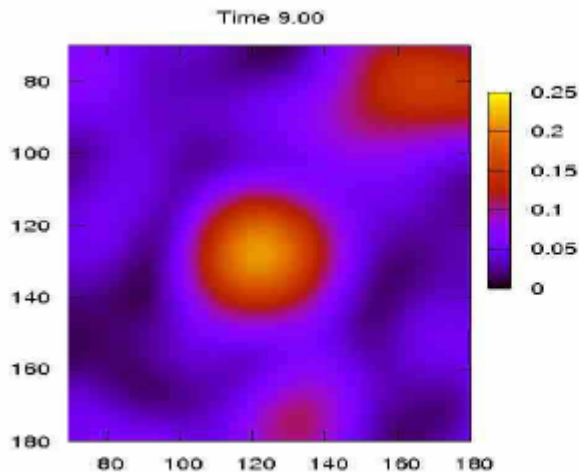
Felder, LK
hep-ph/0606256

estimation

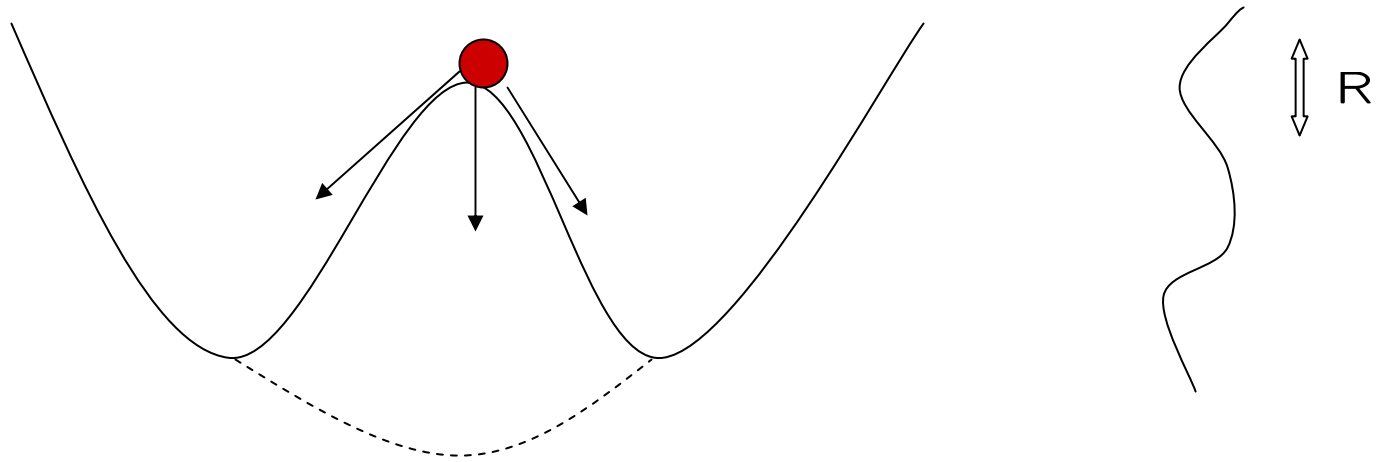
$$\Omega_{GW} \sim 10^{-6} (RH)^2$$

size of structures R vs Hubble radius $1/H$

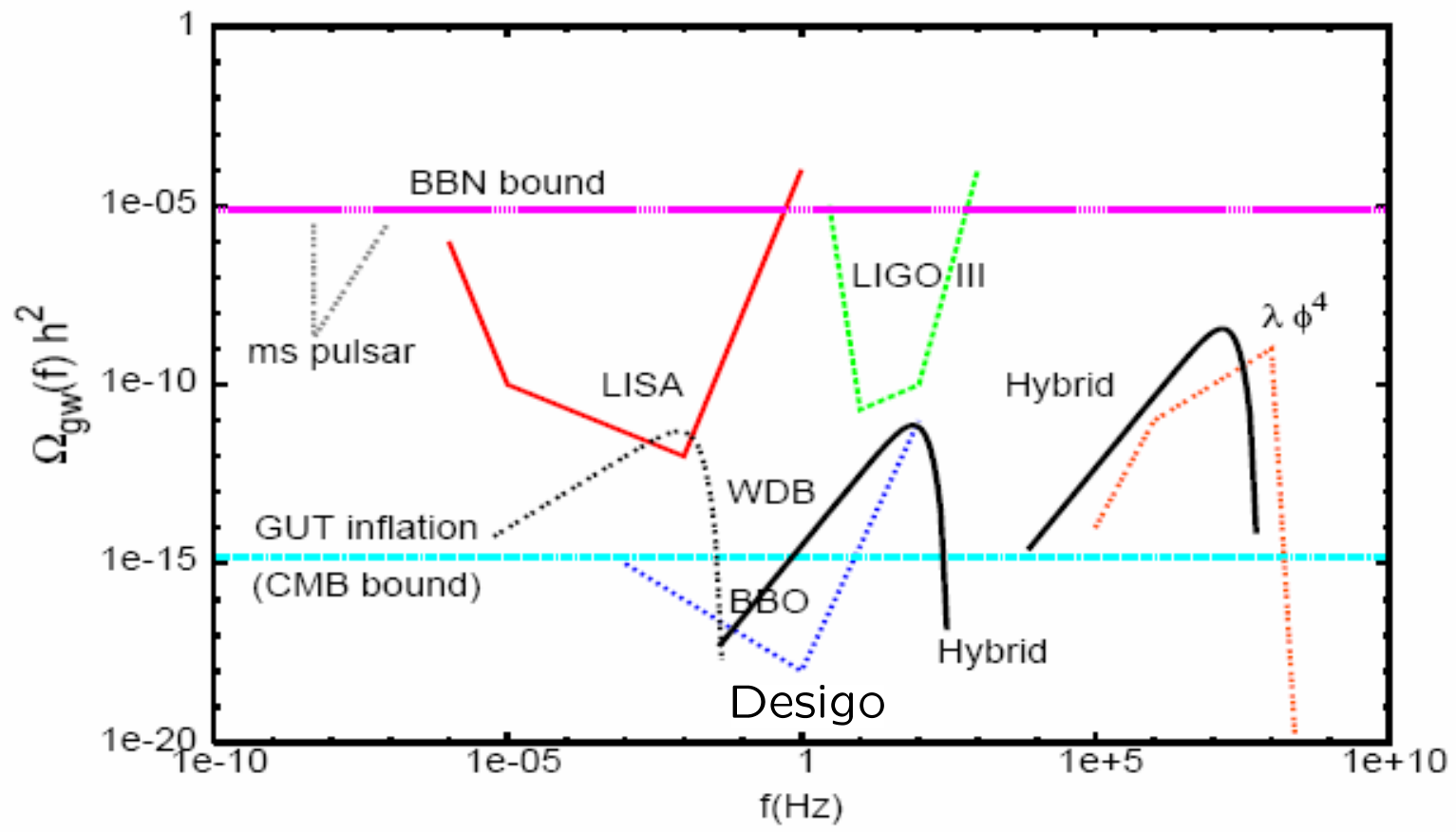
$$f \sim \frac{1}{RH} \frac{M}{10^{15} \text{Gev}} 10^8 \text{ Hz}$$

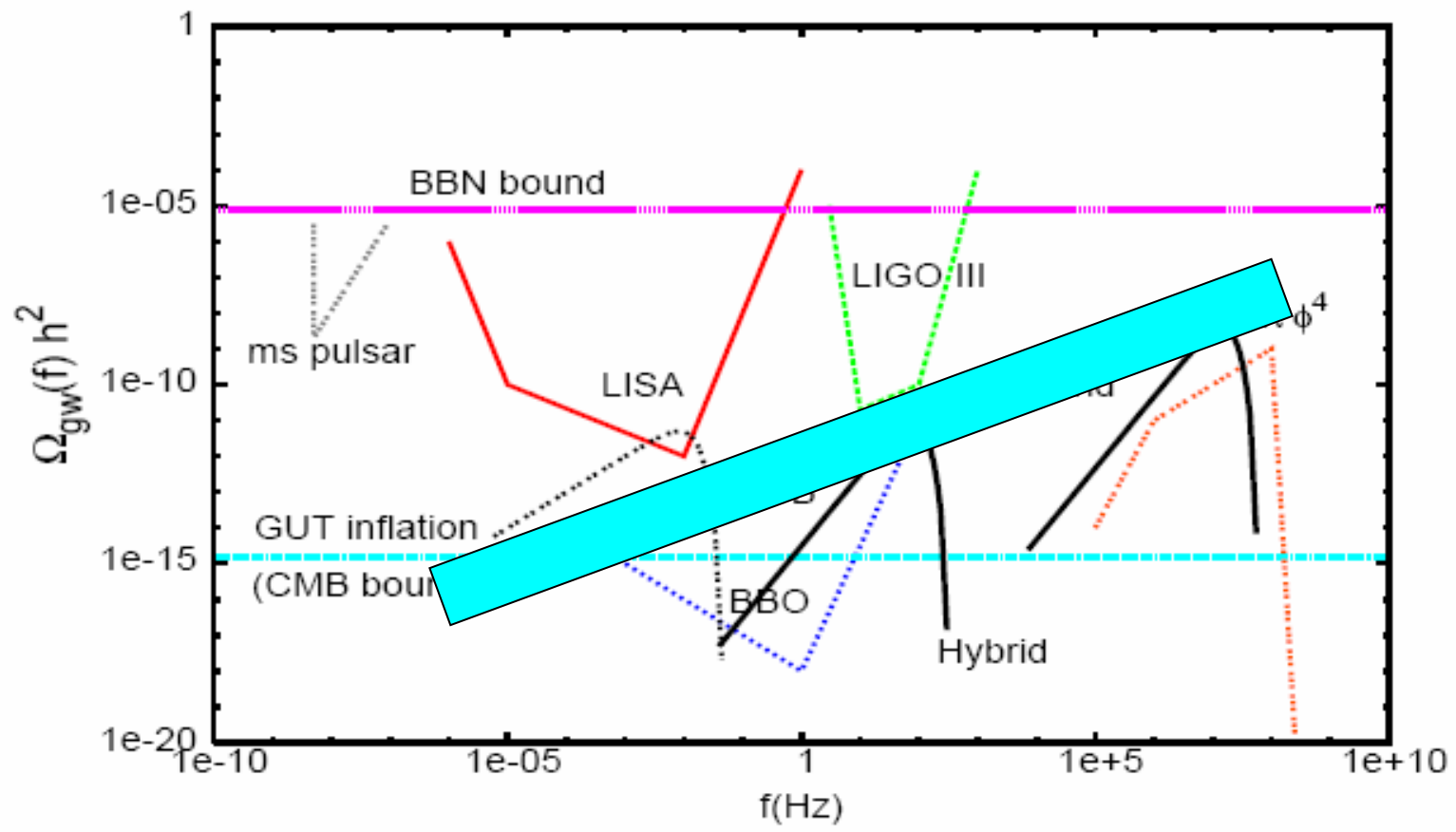


**topological effects after hybrid inflation (unstable)
formation of defects results in GW emission**



$$\frac{\rho_{gw}}{\rho_r} \sim (RH)^2$$





**The story of stochastic gravitational waves is
CMB anisotropies of 21 century**

**GW from high energy inflation are targeted by
CMB B-mode polarization experiments**

**GW from low-energy inflation are targeted by
GW astronomy**

Reheating after String Theory Inflation

Barnaby, Burgess, Cline, hep-th/0412095

LK, Yi, hep-th/0507257

Frey, Mazumdar, Myers, hep-th/0508139

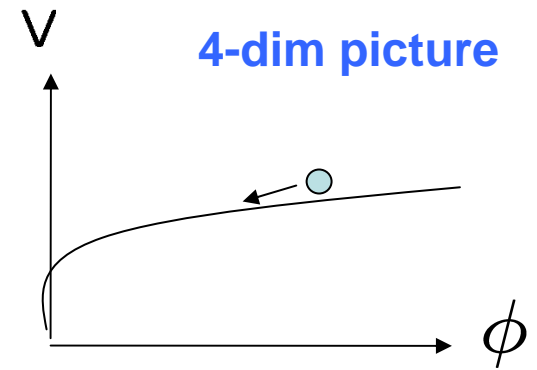
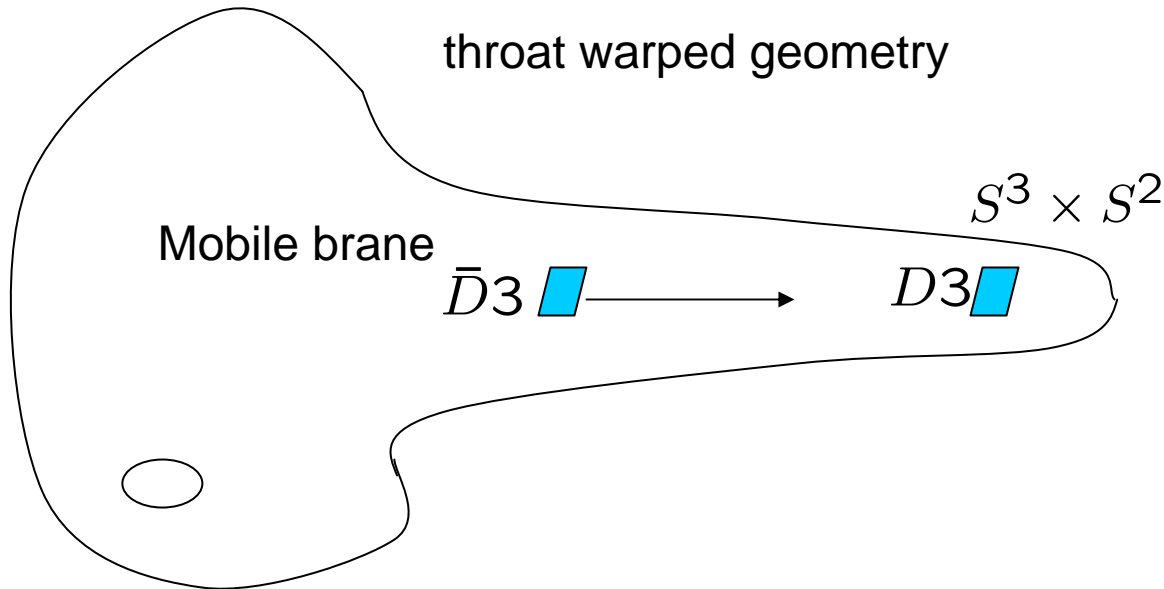
Chialva, Shiu, Underwood, hep-th/0508229

Chen, Tye, hep-th/05120000; 0602136

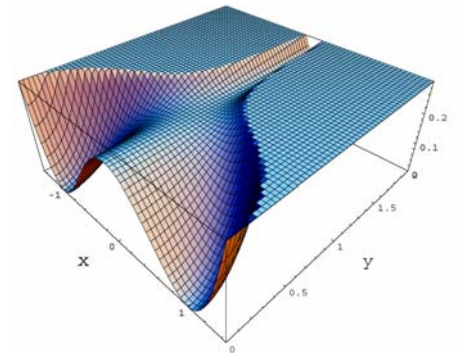
Dufaux, LK, Peloso 08

Realization of String Theory Hybrid Inflation

Warped brane inflation

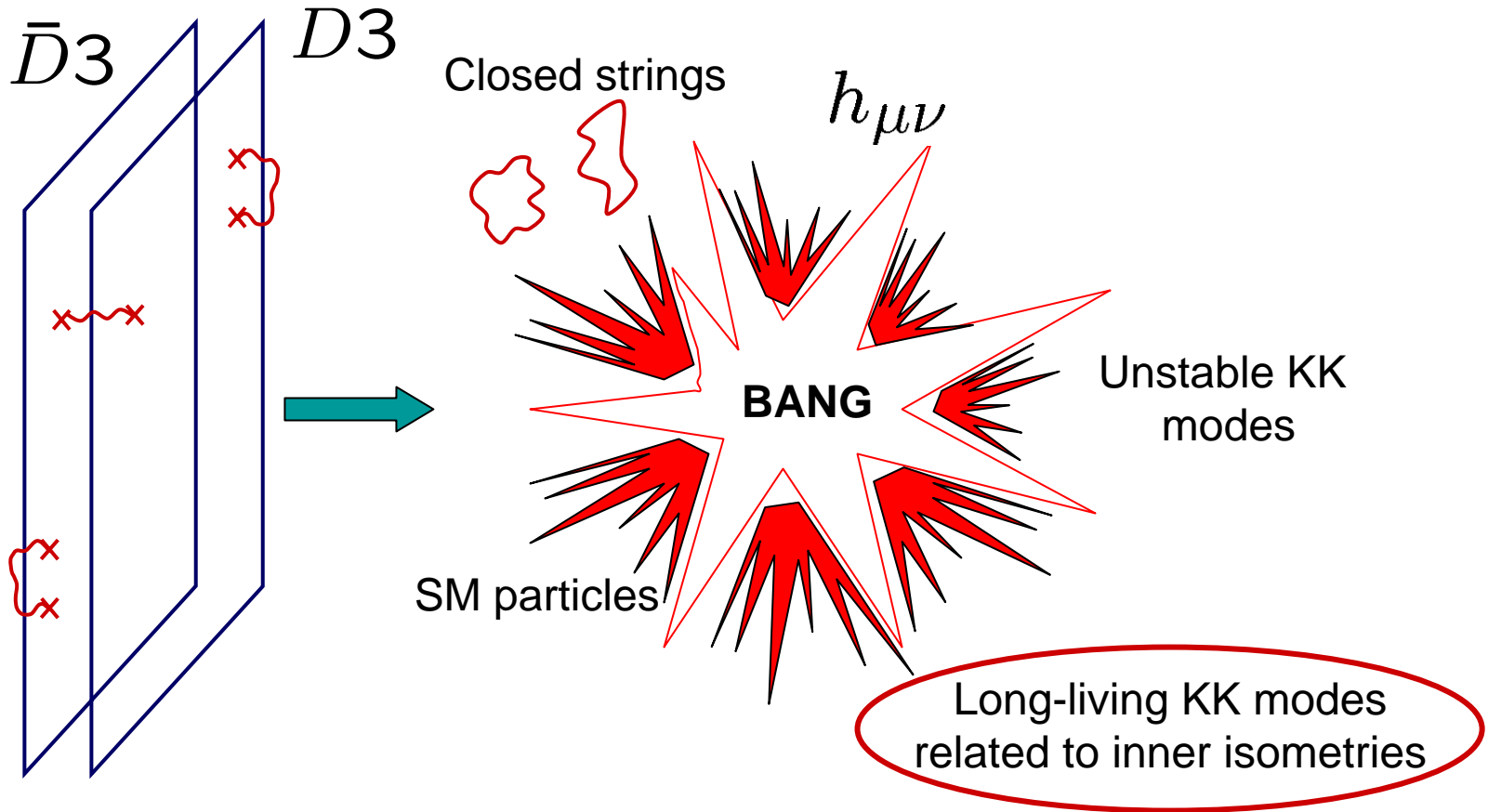


Prototype of hybrid inflation



End point of inflation

LK, Yi 05



Open strings $x \sim x$

between branes are unstable

Cascading Energy from Inflaton to Radiation

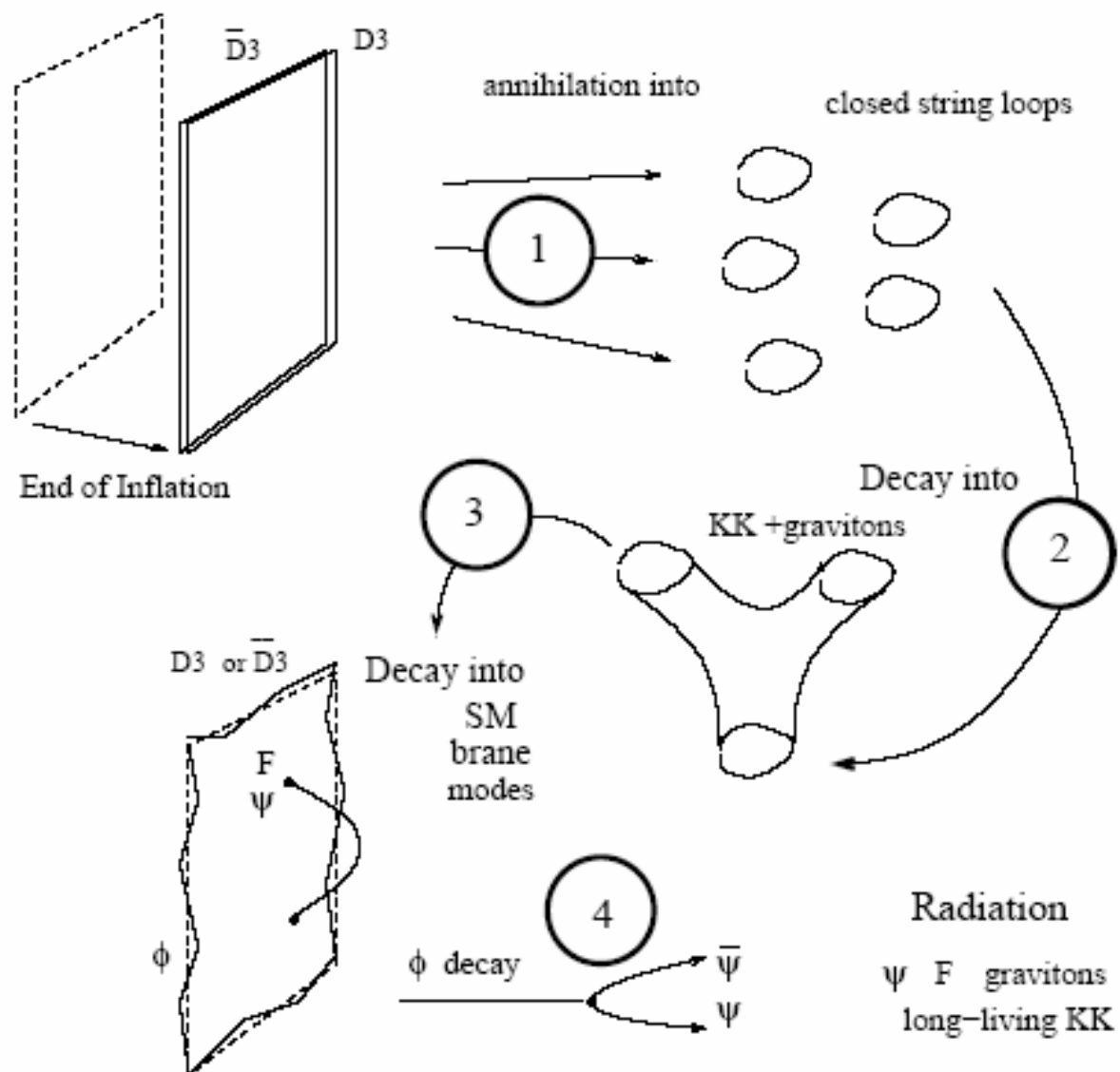
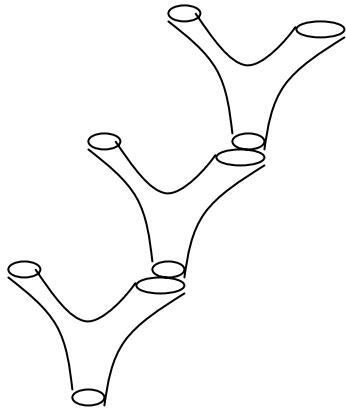


Figure 2: Identifying the channels of D-brane decay

KK story



$$R^4 \times \mathcal{M}$$

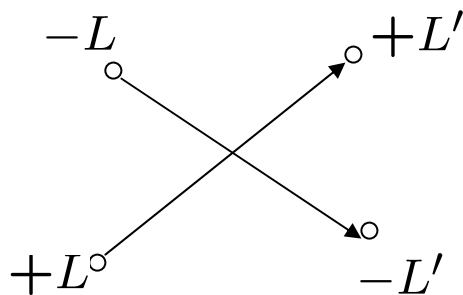


$$h_{AB}(x, y) = \sum_m h^{(m)}(x) f_m(y)$$

$m = 0$: usual 4 dim gravitons $\Omega_{GW} \simeq e^{-2A}$

other m : modes $m_{KK} \simeq e^{-A}/R$

KK particles are thermalized first
SM particles are thermalized much later



KK from M with isometries
are stable

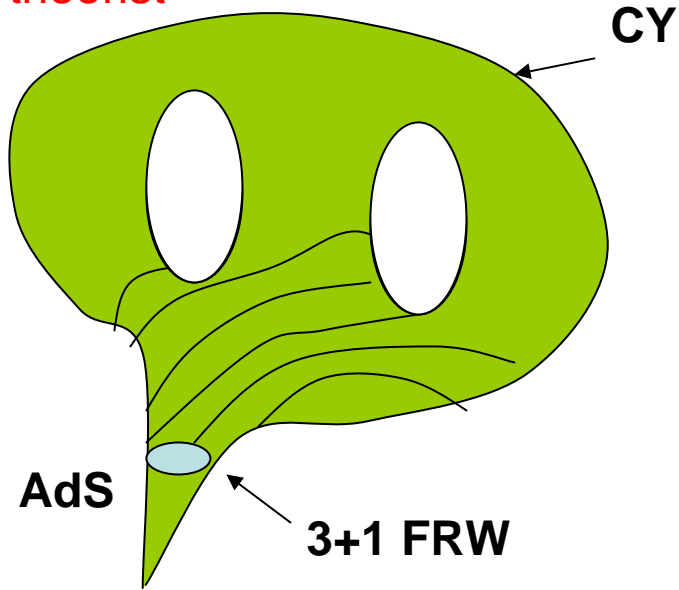
No complete decay

KK particles freeze out

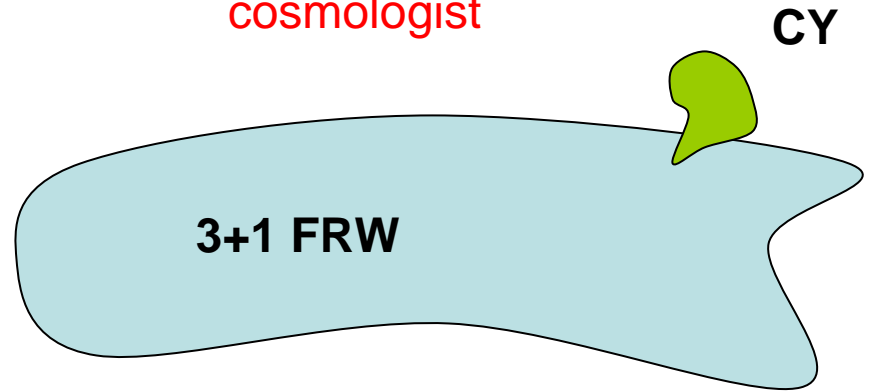
$$\Omega_{KK} \gg 1$$

Fluctuations in Cosmology with Compactification

string theorist

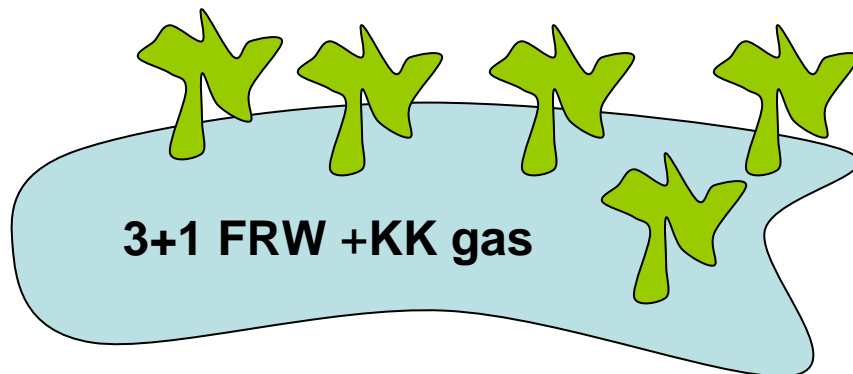


cosmologist



Practical cosmologist

CY +fluctuations



KK atom



$$\Phi_m(y^c) = \sum_{L,M} Y_L^M(\Omega) \psi_{m,L}(y)$$

metric

$$G = H^{-1/2}(y) g_{\mu\nu} dx^\mu dx^\nu + h_{IJ} dy^I dy^J$$

$$h \equiv H^{1/2} \mathcal{G}$$

wave equation for KK mode

$$\left[\frac{H(y)}{\sqrt{h}} \partial_I h^{IJ} \frac{\sqrt{h}}{H(y)} \partial_J + H^{1/2}(y) \nabla_\mu \nabla^\mu \right] \Phi = 0$$

m_{KK}^2

Throat geometry

$$H \simeq e^{4y}, \quad h = R^2 (dy^2 + ds_{T_{1,1}}^2)$$

$$\Phi = \sum_L \Phi_{m_{KK};L}(y) Y_L(\theta)$$

$$\left[e^{4y} \partial_y e^{-4y} \partial_y + m_{KK}^2 R^2 e^{2y} - L^2 \right] \Phi_{m_{KK};L} = 0 \quad \nu^2 = 4 + L^2$$

“big” CY

$$H(y) \sim H_0,$$

$$\left[\partial_y^2 + m_{KK}^2 R^2 H_0^{1/2} - L^2 \right] \Phi_{m_{KK};L} = 0 \quad \Phi_{m_{KK};L} \sim e^{\pm Ly}$$

KK modes interactions

$$S_1 = \int d^D x \sqrt{\hat{g}} \sqrt{-g} e^{2A} R^{(4)}[g]$$

$$S_2 = \int d^D x \sqrt{\hat{g}} \sqrt{-g} e^{4A} \left[\frac{1}{4} g^{\mu\nu} g^{\lambda\rho} (\partial_c g_{\mu\rho} \partial^c g_{\nu\lambda} - \partial_c g_{\lambda\rho} \partial^c g_{\mu\nu}) - \frac{1}{2} \partial^c g^{\mu\nu} \partial_c g_{\mu\nu} - g^{\mu\nu} \hat{\nabla}_c \hat{\nabla}^c g_{\mu\nu} \right. \\ \left. - \partial^c A (6 g^{\mu\nu} \partial_c g_{\mu\nu} + g_{\mu\nu} \partial_c g^{\mu\nu}) - 2 \hat{\nabla}_c \hat{\nabla}^c A - 8 \partial_c A \partial^c A \right]$$

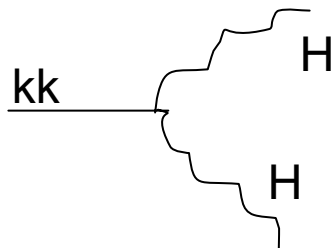
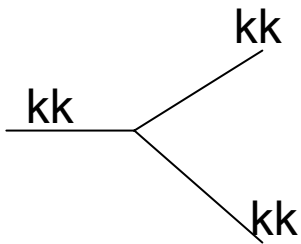
the spin 2 perturbations $h_{\mu\nu}$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x^a, y^c) \quad \text{with} \quad \partial^\mu h_{\mu\nu} = \eta^{\mu\nu} h_{\mu\nu} = 0$$

$$h_{\mu\nu}(x^\lambda, y^a) = \sum_m \Phi_m(y^a) \gamma_{\mu\nu}^{(m)}(x^\lambda)$$

$$\int d^{(D-4)} y \sqrt{\hat{g}} e^{2A} \Phi_m \Phi_{m'} \Phi_{m''} \int d^4 x \gamma_\mu^{(m)\nu} \partial_\sigma \gamma_{\nu\rho}^{(m)} \partial^\sigma \gamma^{(m)\mu\rho}$$

$$\int d^4 x e^{2A(y_b^c)} g^{\mu\nu}(y_b^c) \partial_\mu H \partial_\nu H$$



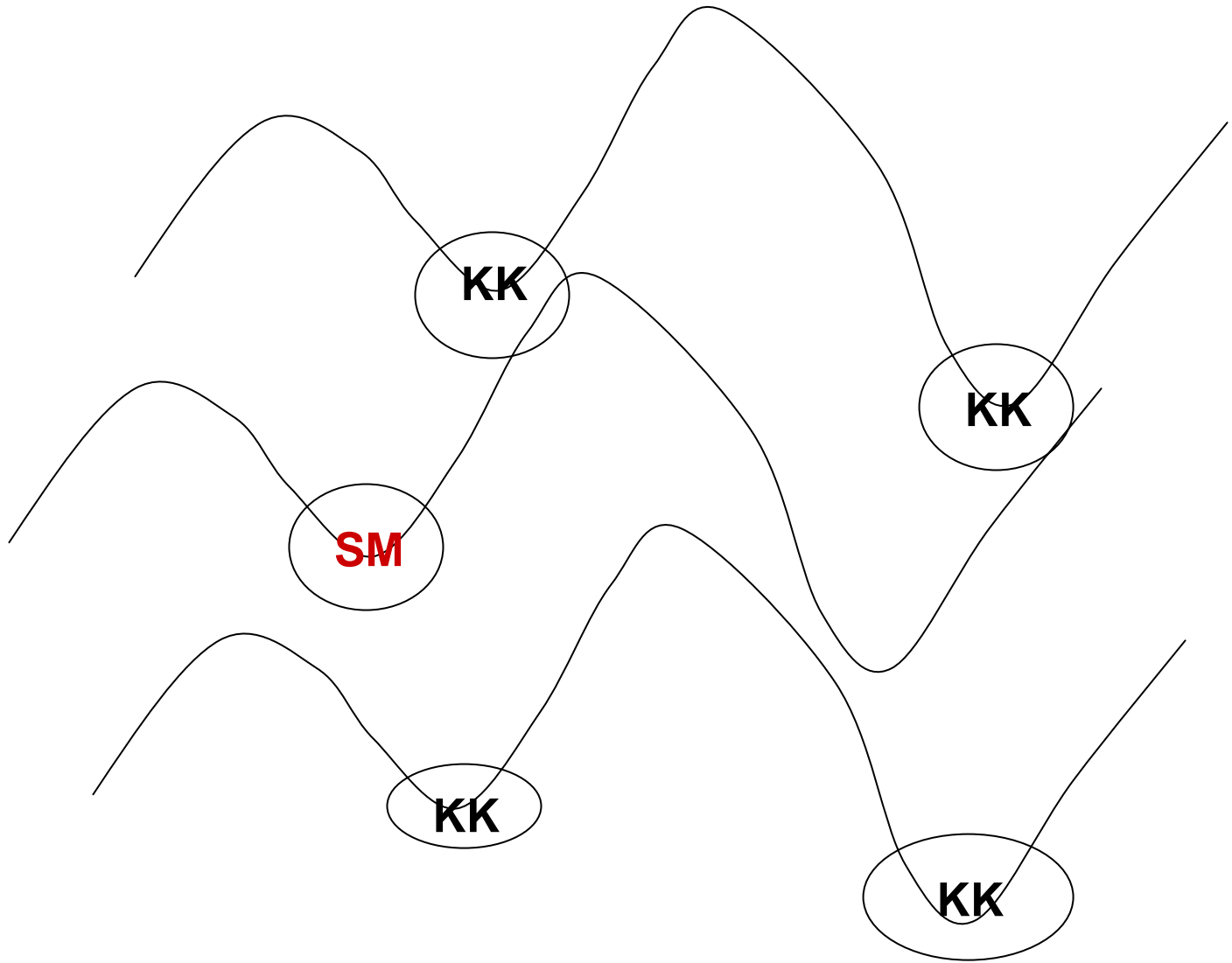
$$\Omega_{KKst} h^2 \sim 10^{17} \frac{m}{T_F} \left(\frac{V_6^{1/6}}{\sqrt{\alpha'}} \right)^{-6} e^{-2y_t/R} \left(\frac{R}{\sqrt{\alpha'}} \right)^{14}$$

$$\frac{m}{T_F} - \frac{1}{2} \text{Log} \left(\frac{m}{T_F} \right) \approx \text{Log} \left[10^{10} \left(\frac{V_6^{1/6}}{\sqrt{\alpha'}} \right)^3 e^{y_t/R} \left(\frac{R}{\sqrt{\alpha'}} \right)^{-15} \right]$$

Inflationary throat $e^{-A} \sim 10^{-4}$

$$\Omega_{KKst} h^2 \gg 1$$

Standard Model throat $e^{-A} \sim 10^{-16}$ $\Omega_{KKst} h^2 \sim 10^{-5}, 0.1$ and 10^3
for $R/\sqrt{\alpha'} = 5, 10$ and 20 .



Resolution?

- Attachment of KS throat to a compact CY
Induces symmetry breaking perturbations.
- Tip of KS throat is a particular case of Sasaki-Einstein manifolds.
There are asymmetric SE manifolds, but no examples of asymmetric throats

Impact of isometry breaking perturbation on KK modes decay

Dufaux, LK, Peloso 08

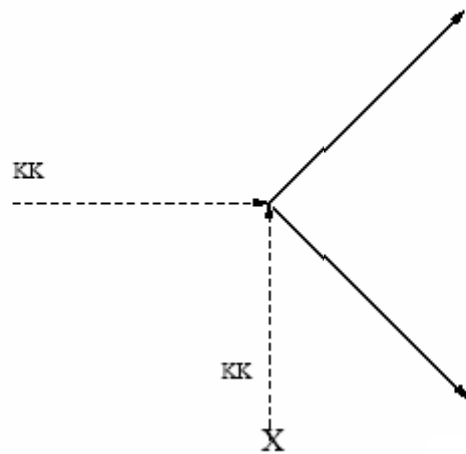
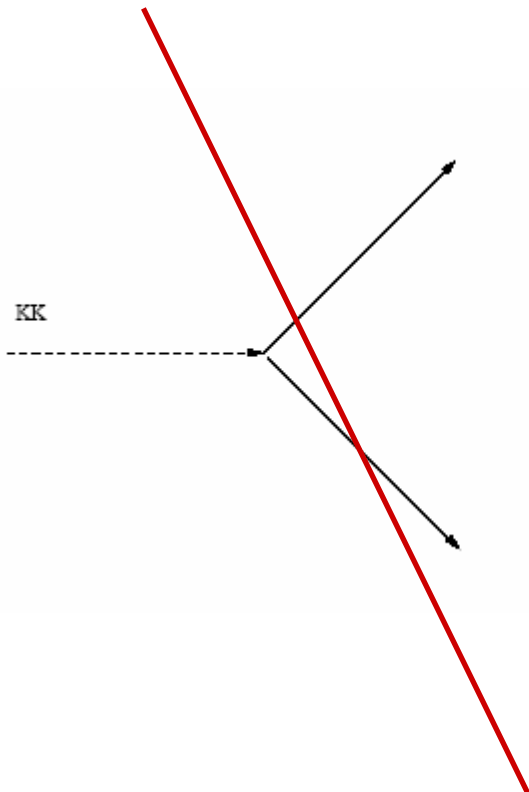
$$AdS_5 \times S^5$$

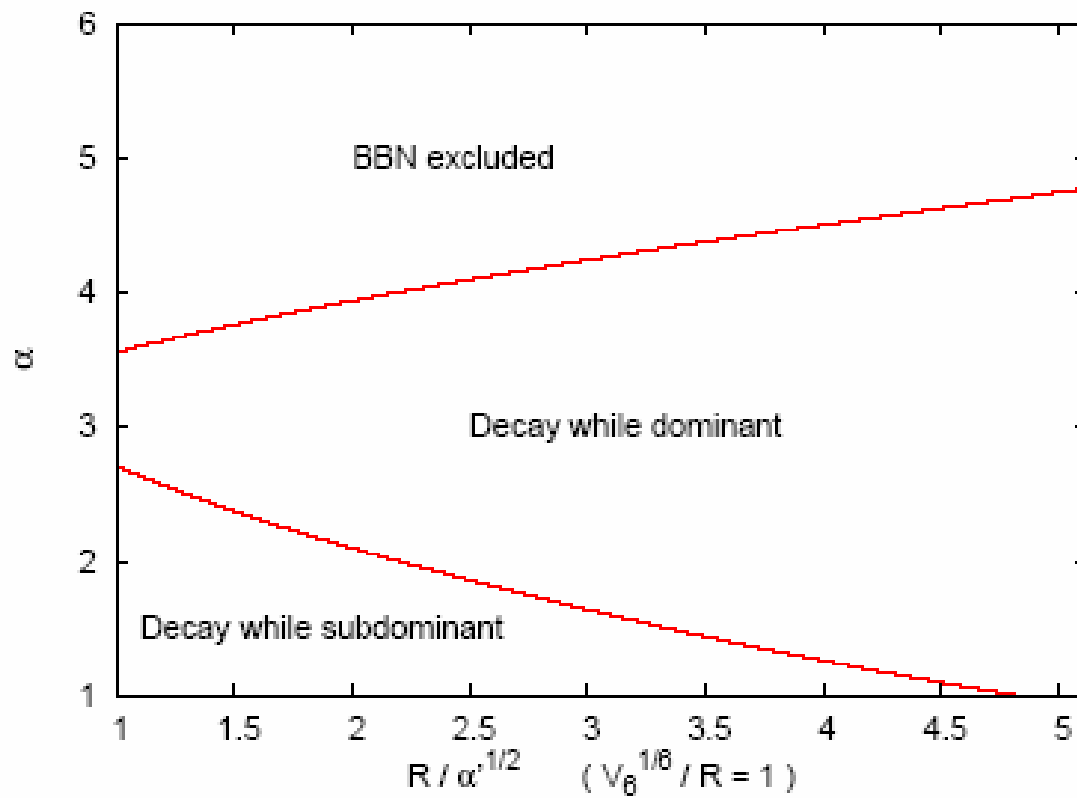
$$ds^2 = e^{-2y/R} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 + R^2 f_{ij}(\Omega) d\theta^i d\theta^j$$

$$ds^2 = e^{-2y/R} [1 + \epsilon(y) w(\Omega)] \eta_{\mu\nu} dx^\mu dx^\nu + \\ + dy^2 + R^2 [f_{ij}(\Omega) + \epsilon(y) \delta f_{ij}(\Omega)] d\theta^i d\theta^j$$

O. Aharony, Y. E. Antebi and M. Berkooz, "Open string moduli in KKLT compactifications," (2005) [arXiv:hep-th/0508080].

$$\epsilon(y) = e^{-\alpha y/R} \quad \alpha = \sqrt{28} - 4 = 1.29$$





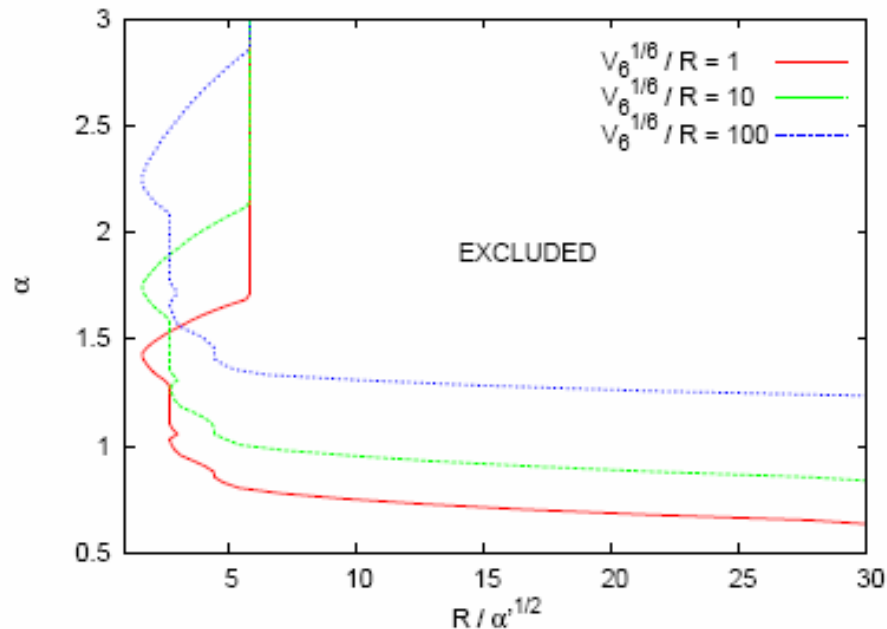


FIG. 10: Final exclusion region for the parameter in the long throat. The three lines correspond to three reference value of $V_6^{1/6}/R$. For each case, values of the parameters on the right of the corresponding curve conflict with the phenomenological limits shown in fig. 9. The highest values of α shown result in KK_{\pm} particles with a much longer lifetime than the age of the universe. In this case, the only relevant bound is that the energy density of the KK_{\pm} particles does not exceed the one of dark matter in our universe.

Non-equilibrium early universe

EW Phase Transition GW generation from the bubble collisions and turbulence

Dark Matter freeze-out Close to the 250 MeV phase transition, QGP is involved

Inflation Scalar field condensate+fluctuation are out-of-equilibrium

Preheating after Inflation Creation of particles, Inflaton fragmentation and thermalization