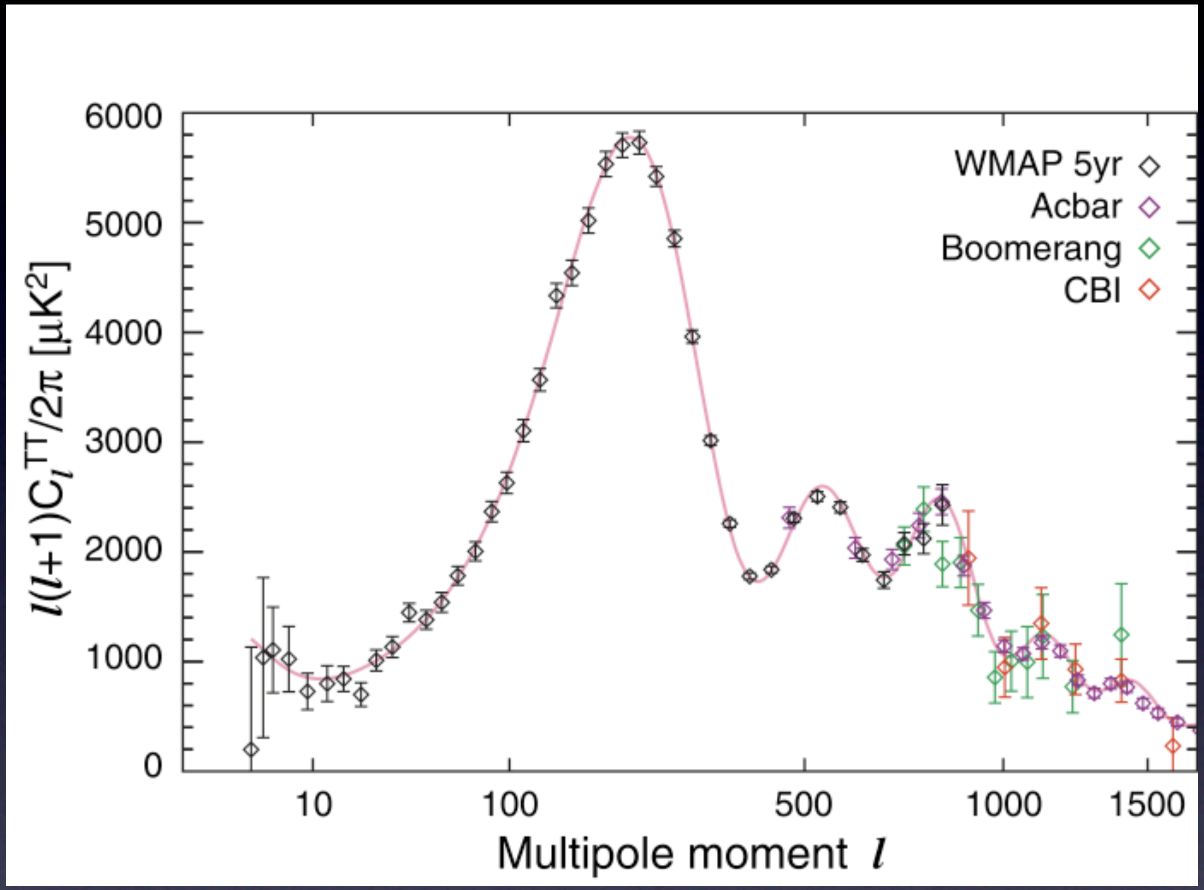
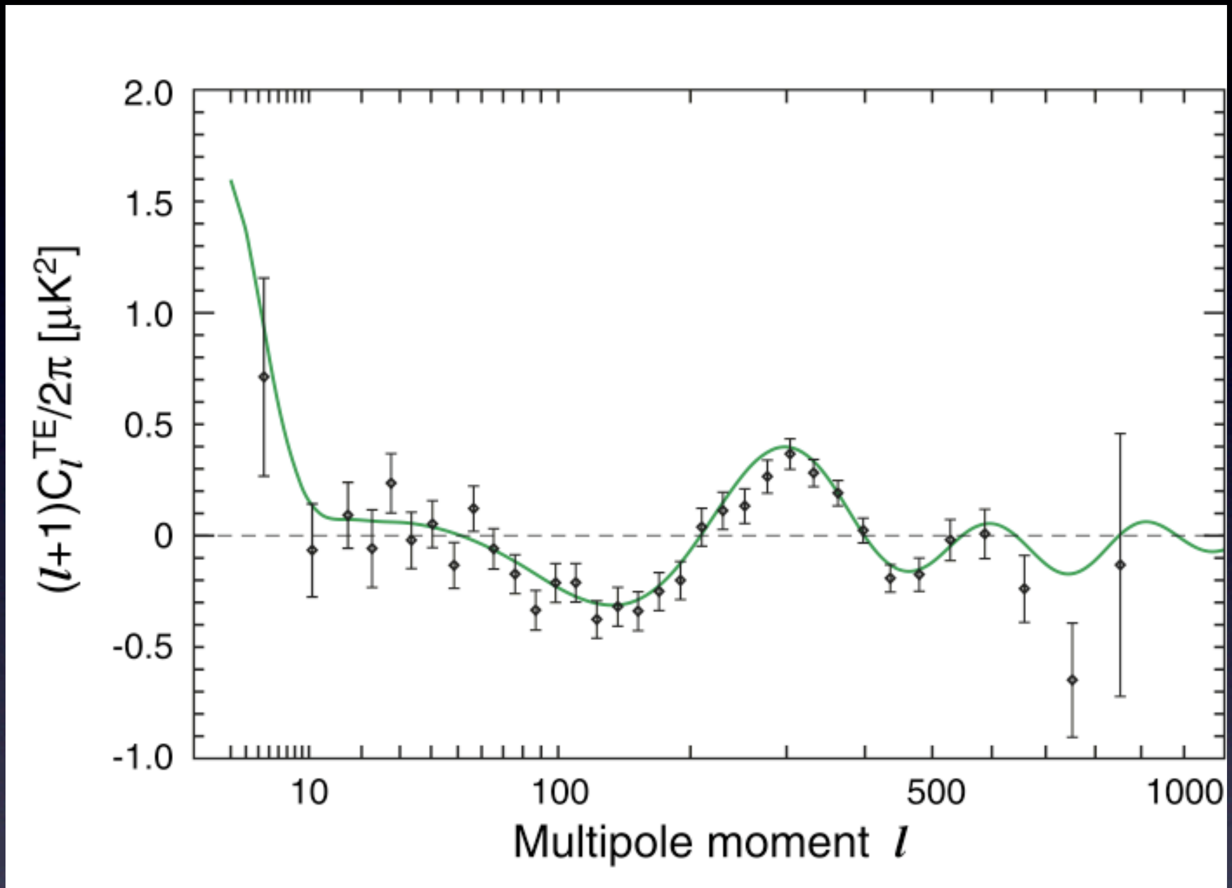
Effective Initial States for Inflation and Trans-Planckian Physics

KITP Non-Equilibrium QFT workshop, March 17 2008
Work done in collaboration with Hael Collins.

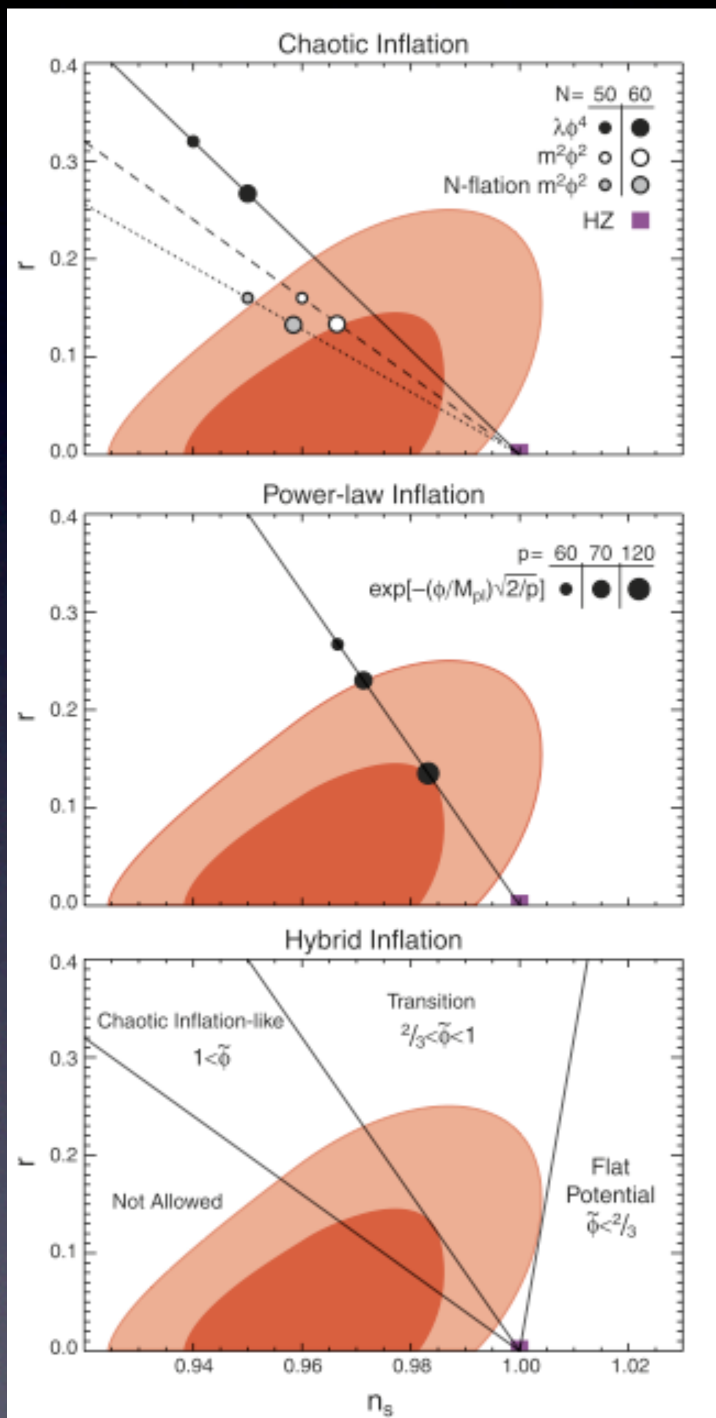
CMB Anisotropies and Inflation

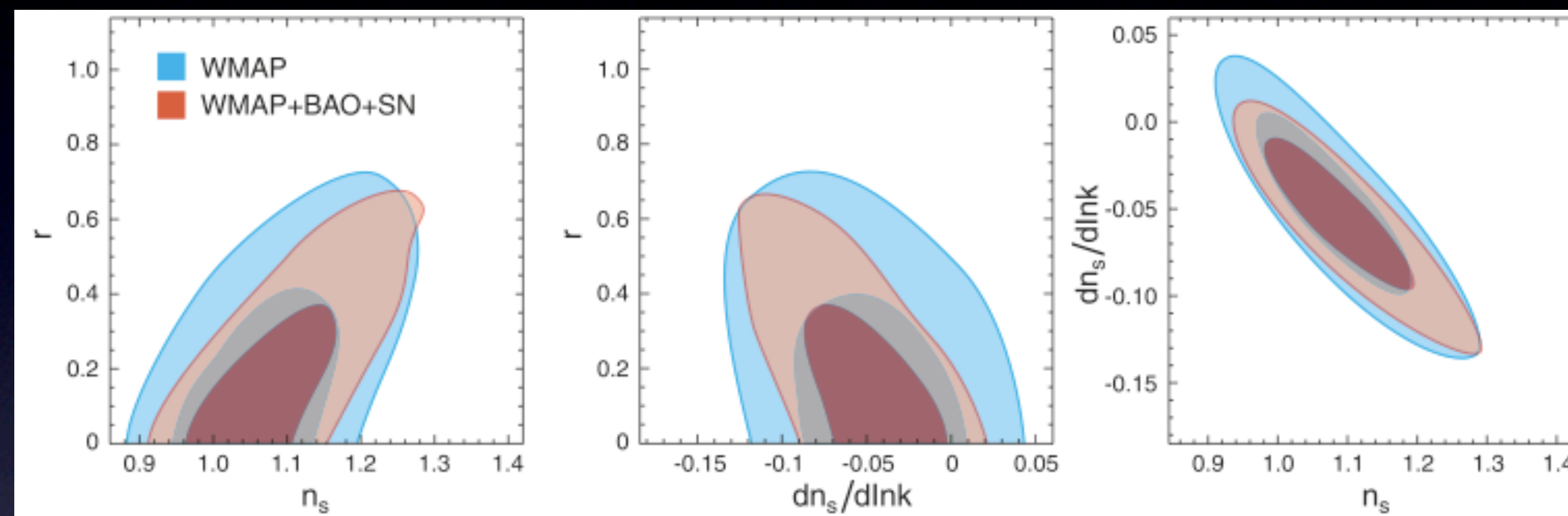




Latest WMAP 5 yr release

WMAP 5-yr data gives even more statistical weight to inflation being the source of metric perturbations that induce CMB temperature anisotropies





We can use the data to put stringent bounds on some of the usual models of inflation.

How reliable are these calculations?

Inflationary Perturbations

- Let's look at how inflationary perturbations evolve.
  - Start as quantum fluctuations in the inflaton field, inside the inflationary horizon.
  - Physical scale is red-shifted outside of horizon and then frozen in amplitude,
  - Once inflation ends, fluctuation can re-enter the matter dominated era horizon, and convert to matter perturbations.
  - CMB photons fall in and out of these wells, giving rise to hot and cold spots.

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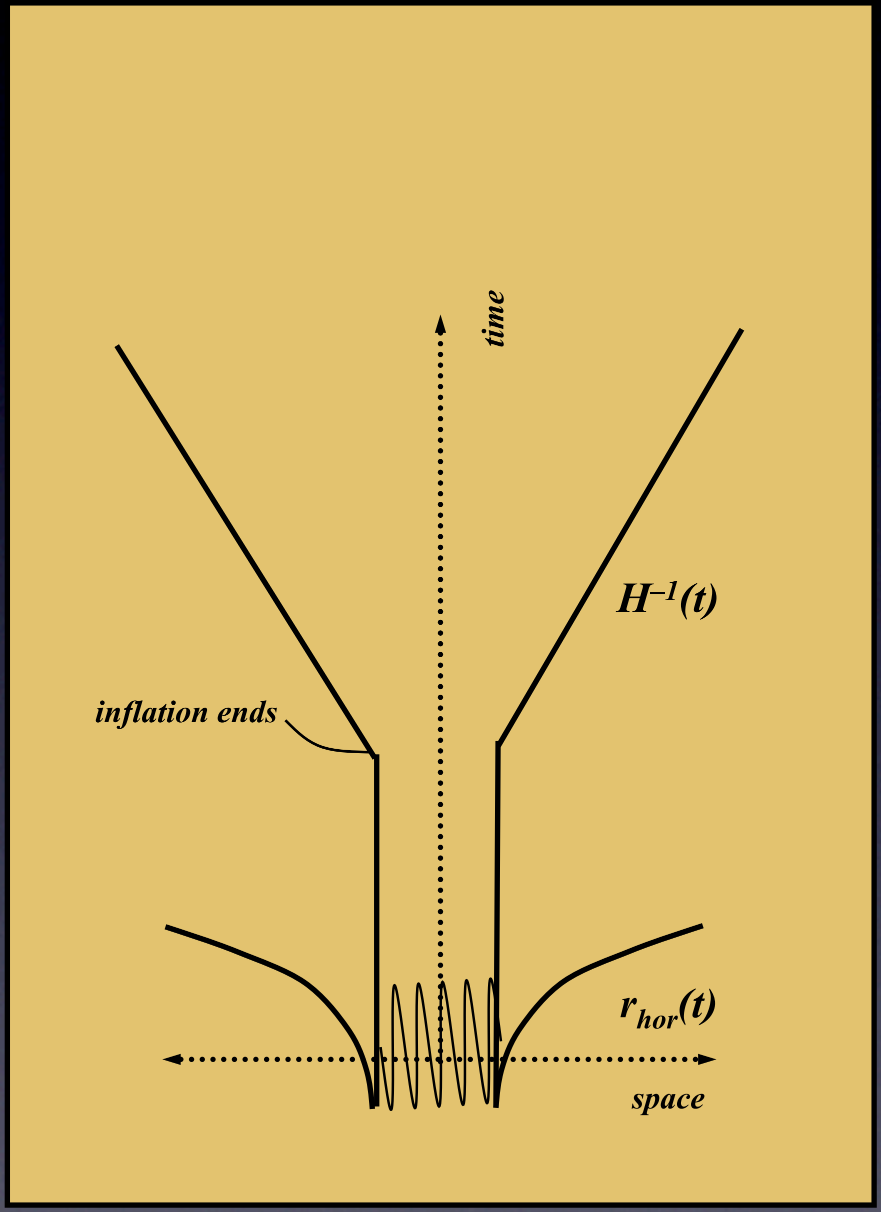

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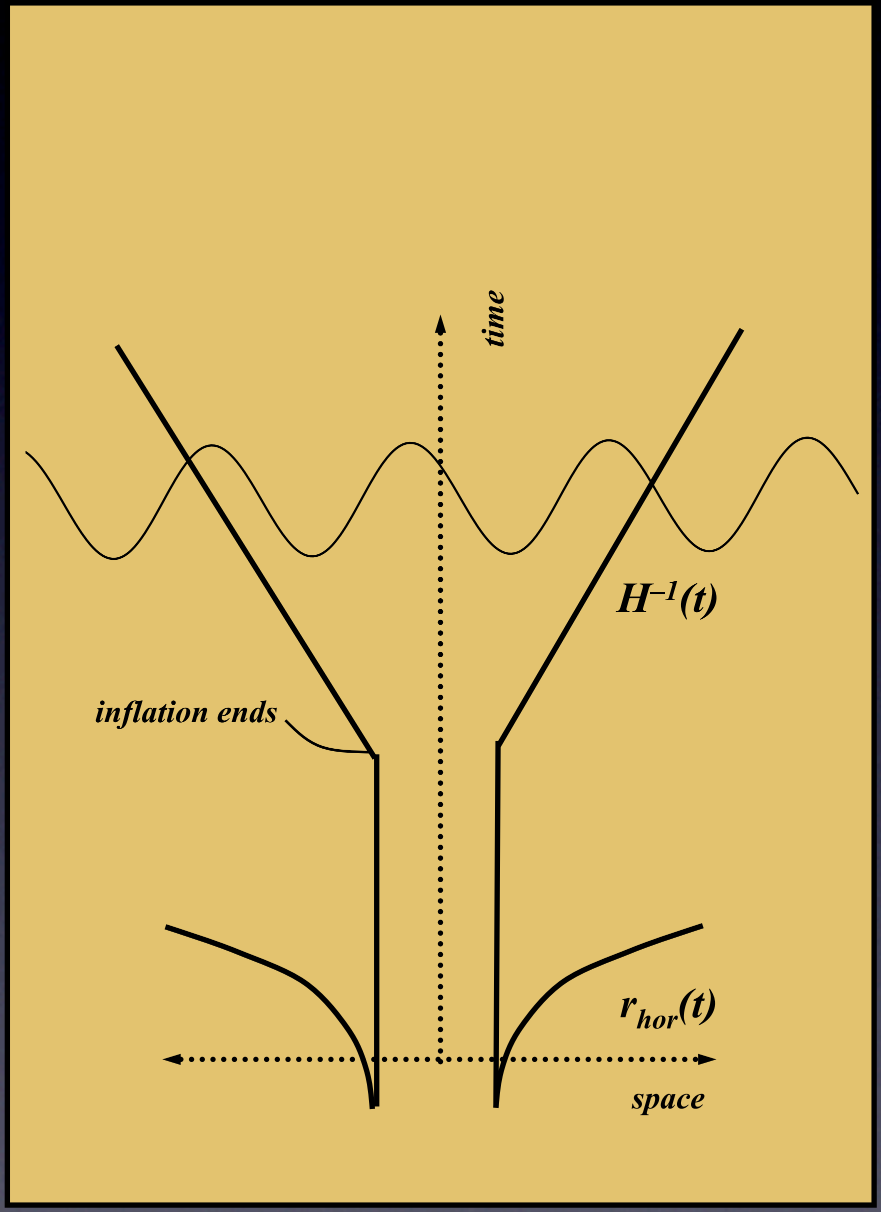

Nothing REALLY matters: Choosing the inflationary vacuum

- Let's go over the standard procedure for computing the power spectrum of fluctuations
  - Decompose the fluctuations into modes and solve the mode equations
  - Now we need to pick the initial state, i.e. which linear combination will be used to compute the power spectrum.
  - Equal time commutation relations can give a partial solution and fix overall normalization

$$
\begin{aligned}
\Phi(\vec{x},\eta) &= \phi(\eta) + \psi(\vec{x},\eta) \\
\psi(\vec{x}.\eta) &= \int \frac{d^3k}{(2\pi)^3}\ \left[\mathcal{U}_k\ a_k\ e^{i\mathbf{k}\cdot\mathbf{x}} + \mathrm{h.c}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{d^2\mathcal{U}_k}{d\eta^2} &- \frac{2}{\eta}\frac{d\mathcal{U}_k}{d\eta} + \left(k^2 + \frac{1}{\eta^2}\frac{m^2}{H^2}\right)\mathcal{U}_k = 0 \\
\mathcal{U}_k &= A_k\ \eta^{\frac{3}{2}} H_\nu^{(2)}(k\eta) + B_k\ \eta^{\frac{3}{2}} H_\nu^{(1)}(k\eta) \\
\nu &= \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}
\end{aligned}
$$

$$
[\psi(\eta,\vec{x}),\pi(\eta,\vec{y})] = i\delta^{(3)}\left(\vec{x}-\vec{y}\right) \Rightarrow
$$

$$
\mathcal{U}_k(\eta) = N_k\left(\frac{\sqrt{\pi}}{2} H\eta^{\frac{3}{2}} H_\nu^{(2)}(k\eta) + f_k \frac{\sqrt{\pi}}{2} H\eta^{\frac{3}{2}} H_\nu^{(1)}(k\eta)\right)
$$

But what fixes relative strength of the solutions?

The usual statement is that at short distances or high energy, spacetime looks like flat space so fields should match to flat space vacua

Mathematically

$$\text{As } k\eta \to -\infty, \quad \mathcal{U}_k(\eta) \to -\frac{H\eta}{\sqrt{2k}} e^{-ik\eta}$$

This fixes the modes as

$$\mathcal{U}_k(\eta) = \frac{\sqrt{\pi}}{2} H \eta^{\frac{3}{2}} H_\nu(k\eta)$$

This is the Bunch-Davies state

BUT: Is this a reasonable requirement? What if, as is most likely, there is some scale M at which new physics relating to the inflaton occurs?

Maybe the inflaton is a composite at energies larger than M!

The Trans-Planckian Problem
(Brandenberger & Martin)

- We need at least 60-65 e-folds of inflation to solve the horizon, flatness and monopole problems.
- Most models give far more e-folds, unless the dynamics is fine tuned.
- Length scales in the CMB sky, would correspond to distance scales SMALLER than the Planck length!
- DO WE NEED TO UNDERSTAND QG TO DO ANY CALCULATIONS AT ALL? HOW COULD THIS BE DONE RELIABLY?

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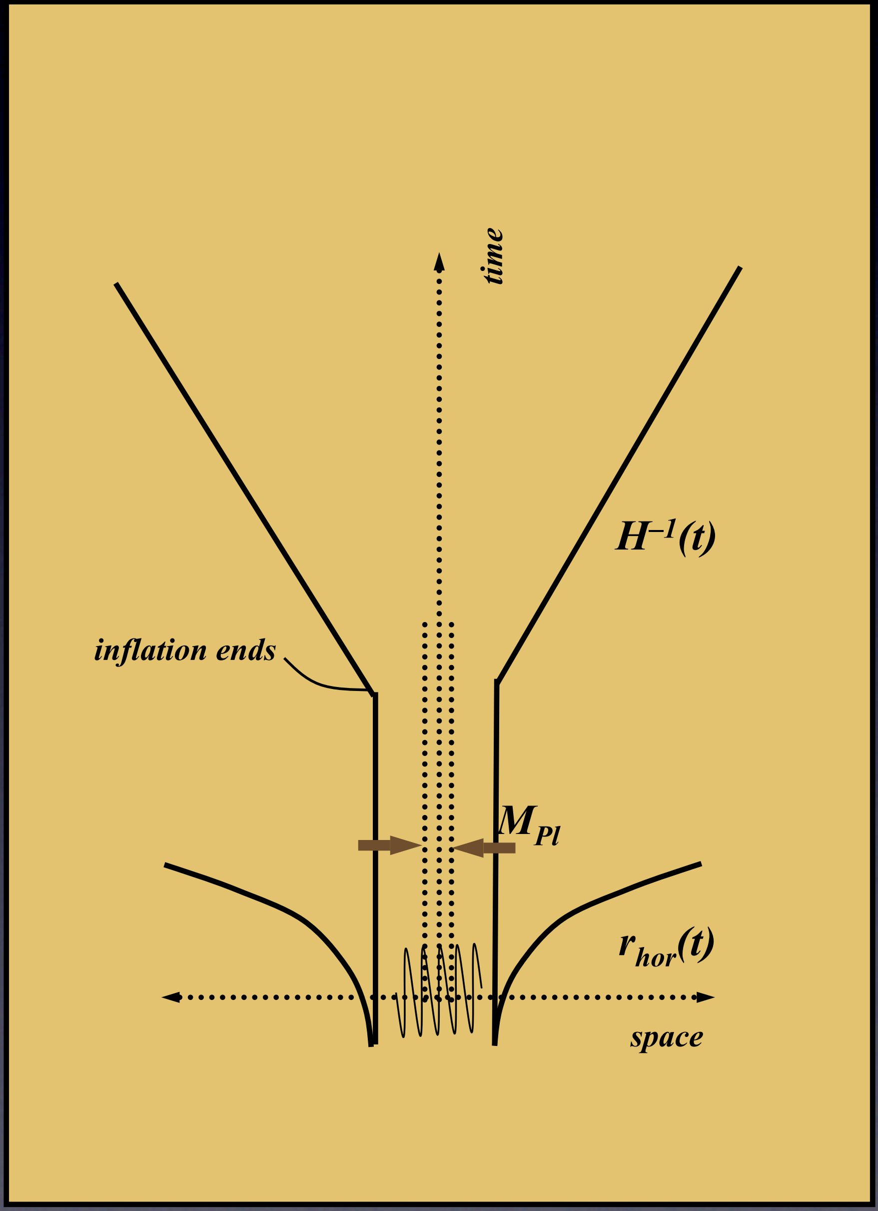

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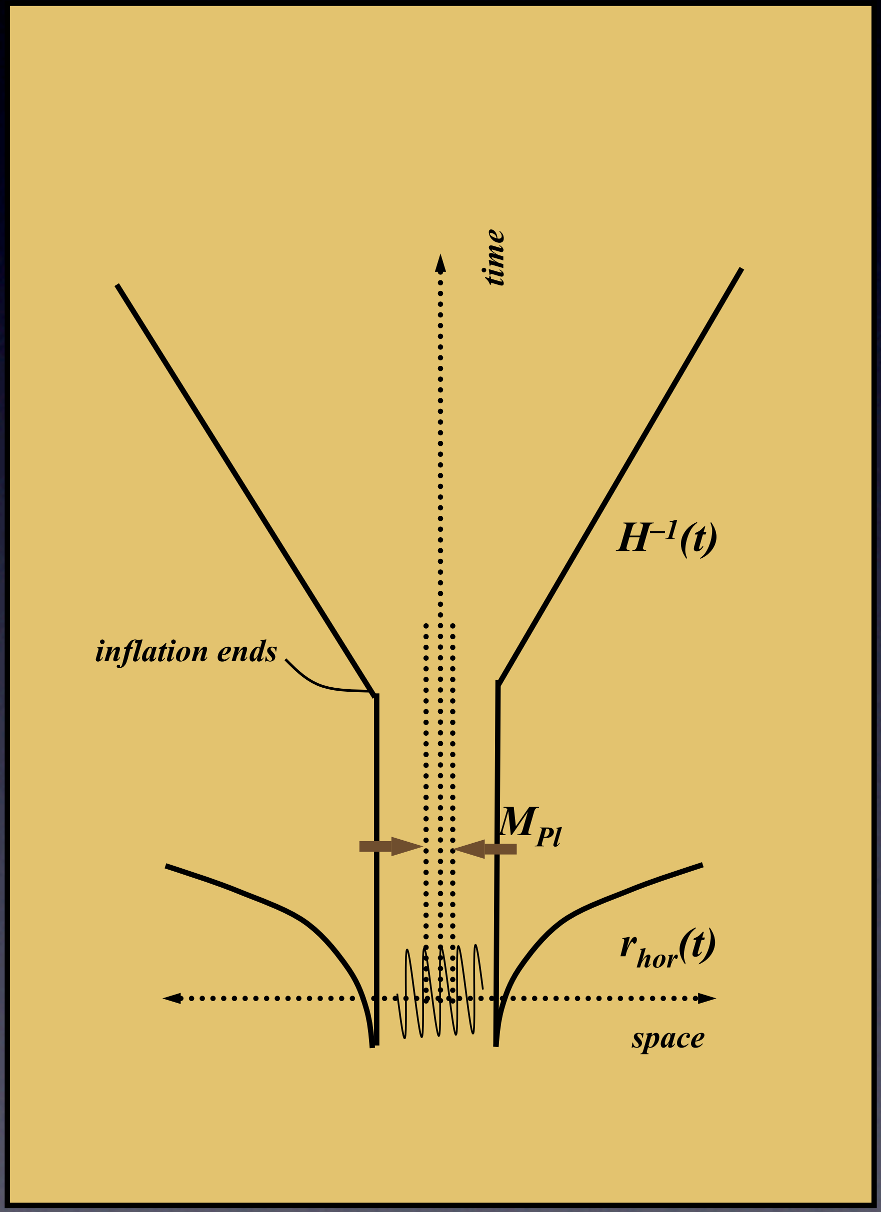

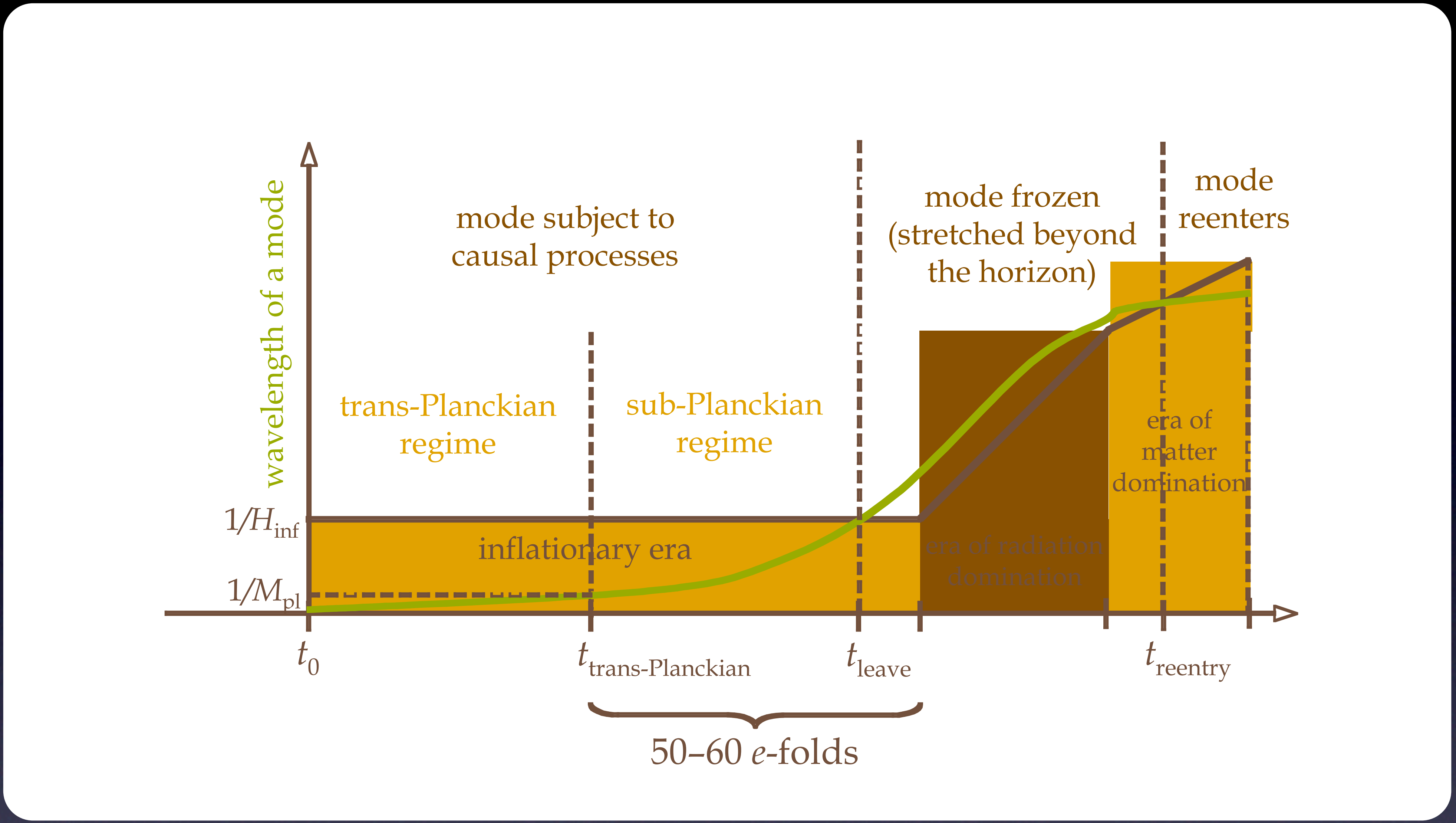

The take-home lesson from the Trans-Planckian discussion:

There's no escaping new physics thresholds when defining the inflaton modes!

Approaches to the Trans-Planckian Problem

- How shall we deal with the trans-Planckian modes?

- One way is to construct models of what that physics may be and try to infer general trends from those models.

  - Modified dispersion relations (Brandenberger and Martin)

  - Alpha vacua in de Sitter space (Daniellson; Collins, RH, Martin)

  - Couplings to excited fields (Burgess, Cline, RH, Lemieux)

<u>Some ideas</u>

Consider a dispersion relation that is modified at the Planck scale:

$$k_{\rm eff}^2(k,\eta) = k^2 - k^2\frac{|b_m|}{a(\eta)}\frac{k}{M_{\rm pl}}$$

A modified uncertainty relation at short distances,

$$[\mathrm{x},\mathrm{p}] = i\hbar(1+\beta\,\mathrm{p}\cdot\mathrm{p}+\cdots)$$

Long-distance given by an α-state,

$$U_k^\alpha = N_k\left[U_k^{BD} + e^\alpha U_k^{BD*}\right]$$

An inflaton (φ) coupled to to a heavy, excited field (χ),

$$L = \sqrt{g}\Big\{\tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \tfrac{1}{2}\partial_\mu\chi\partial^\mu\chi + \tfrac{1}{2}m^2\varphi^2 + \lambda(\chi^2-v^2)^2 + \tfrac{1}{2}g\chi^2\varphi^2 + \gamma\varphi^4\Big\}$$

Possible Effects of TP Physics on the CMB?

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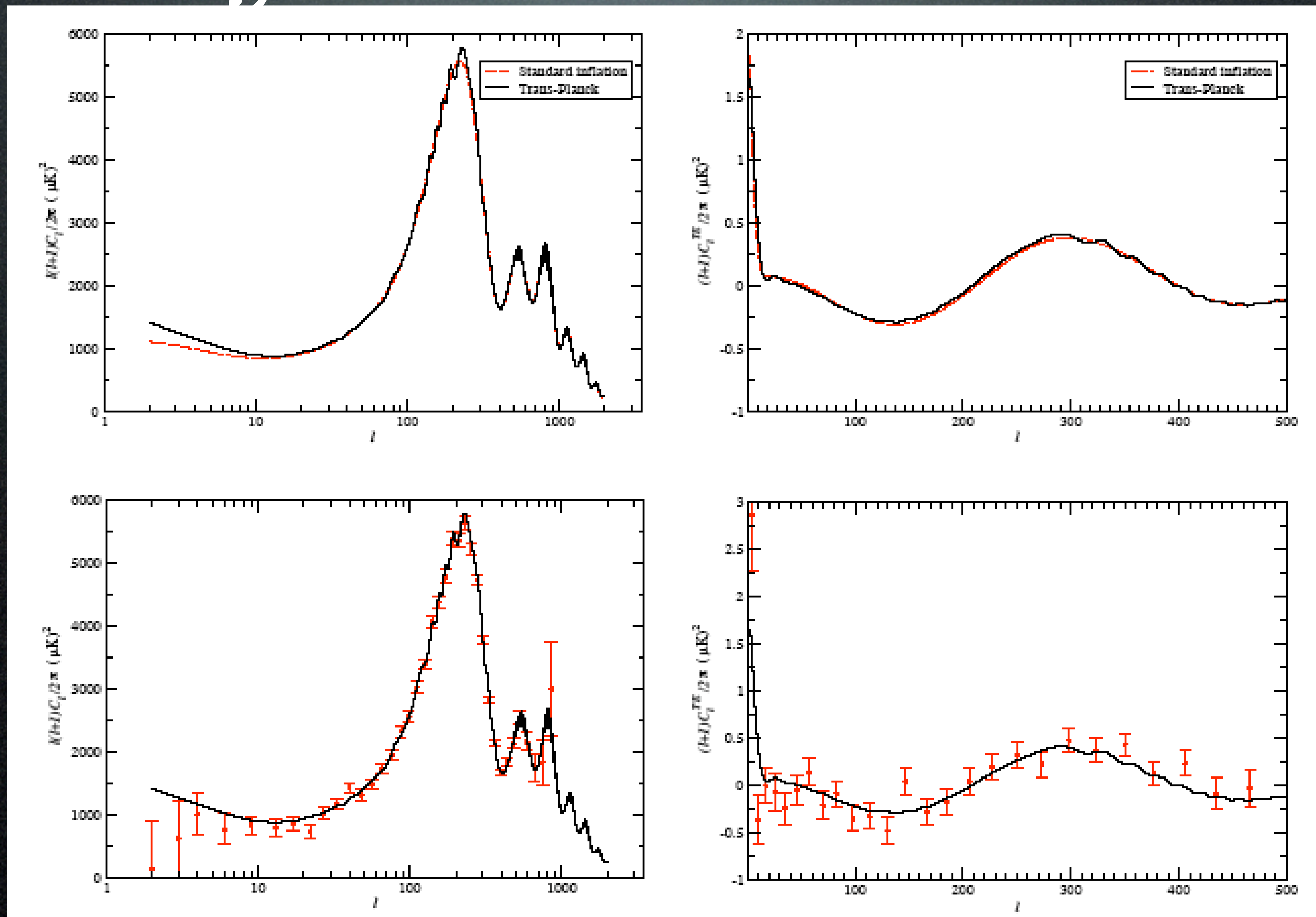


From Martin and Ringeval: arXiv:astro-ph/0310382

What do we learn from these models?

There are indeed corrections to the power spectrum. The scale of these corrections tends to be of order H/M

As an example, Daniellson finds:

$$P_\phi = \frac{H^2}{4\pi^2}\left(1 - \frac{H}{M_{\rm Pl}}\sin\frac{2M_{\rm Pl}}{H}\right)$$

Some questions remain:

- To what extent are the results universal?
- Can different models be distinguished?
- Calculations use a crude cutoff, but in QFT we are used to integrating beyond the regime of validity of the theory.

Effective Field Theory in an Expanding Universe

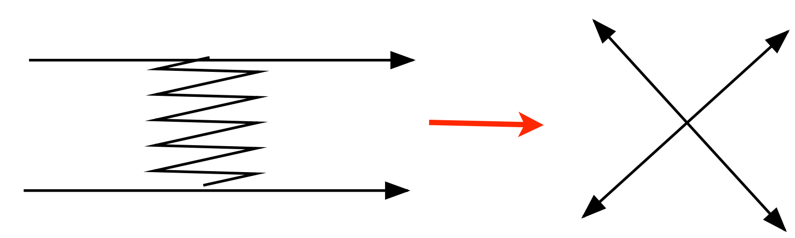
In EFT, we divide phenomena according to whether or not they occur at energies larger than some fixed scale M.

- The fields and symmetries of the low-energy theory fix the renormalizable operators.
- High energy physics appears as higher dimension operators, suppressed by powers of M

- The physics is consistent since for experiments at a scale E, all high energy physics will be suppressed by powers of E/M.
- In principle, renormalizability of the low-energy theory would require an infinite number of operators, but in practice, how well we can measure determines the dimension of the operators we should keep.

Integrate out the W boson in the standard model to go the Fermi theory. This will be valid for

$$E << M_W$$



EFT in an Expanding Universe

- Separation of scales non-trivial due to redshifting of UV modes into the IR.
- The questions asked are different; not S-matrix elements, but in-in ones.
- Problem to be solved is an IVP; specified initial state is also only defined up to modifications suppressed by powers of M.

This implies the existence of an earliest time. Take scales of interest today and blueshift them until they reach M

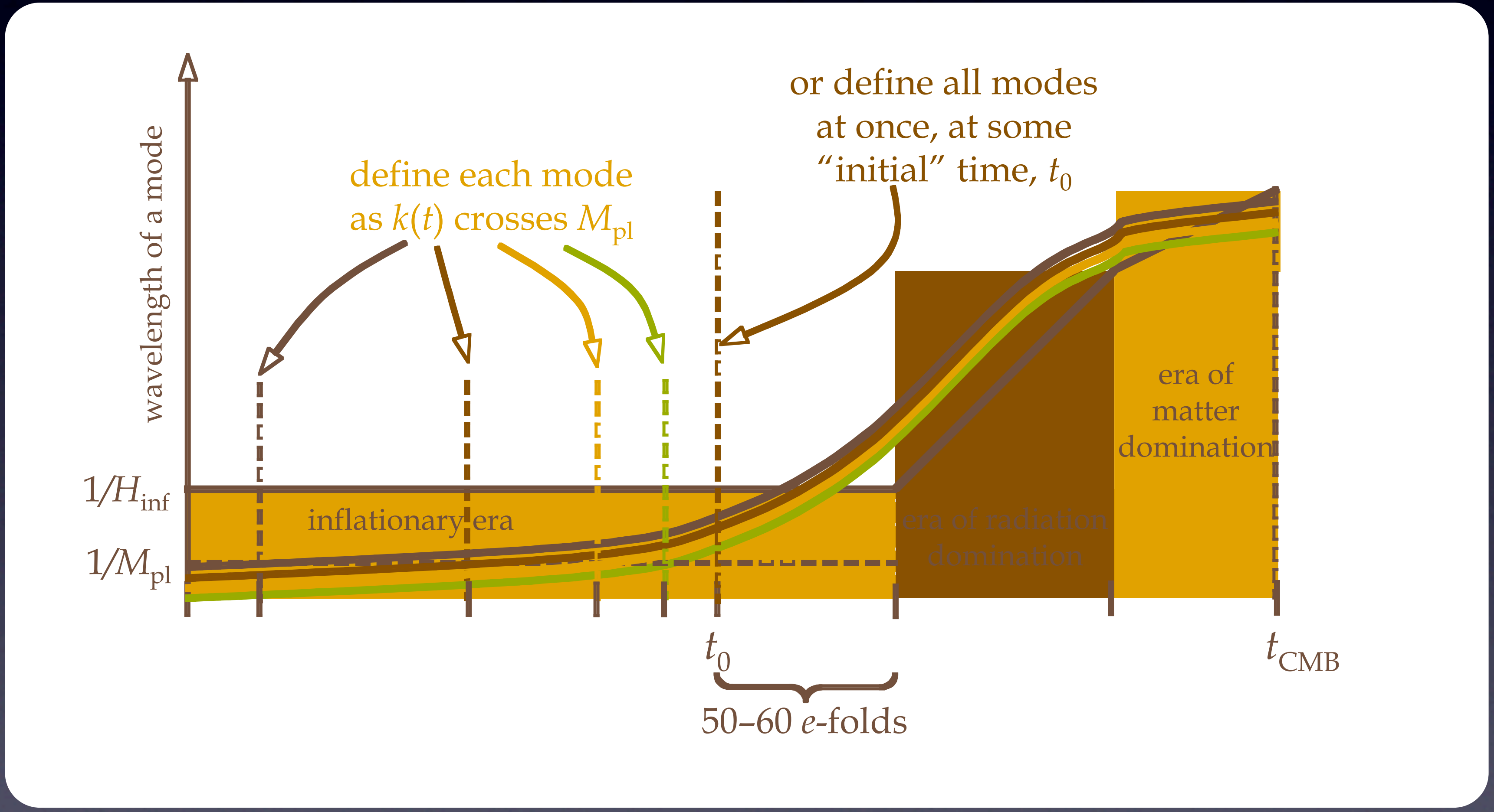
$$\frac{k_{\mathrm{CMB}}}{a(\eta_0)} = M$$

Want a formalism that takes this time into account and allows for perturbatively controlled calculations of cosmologically interesting quantities

Effective State Formalism

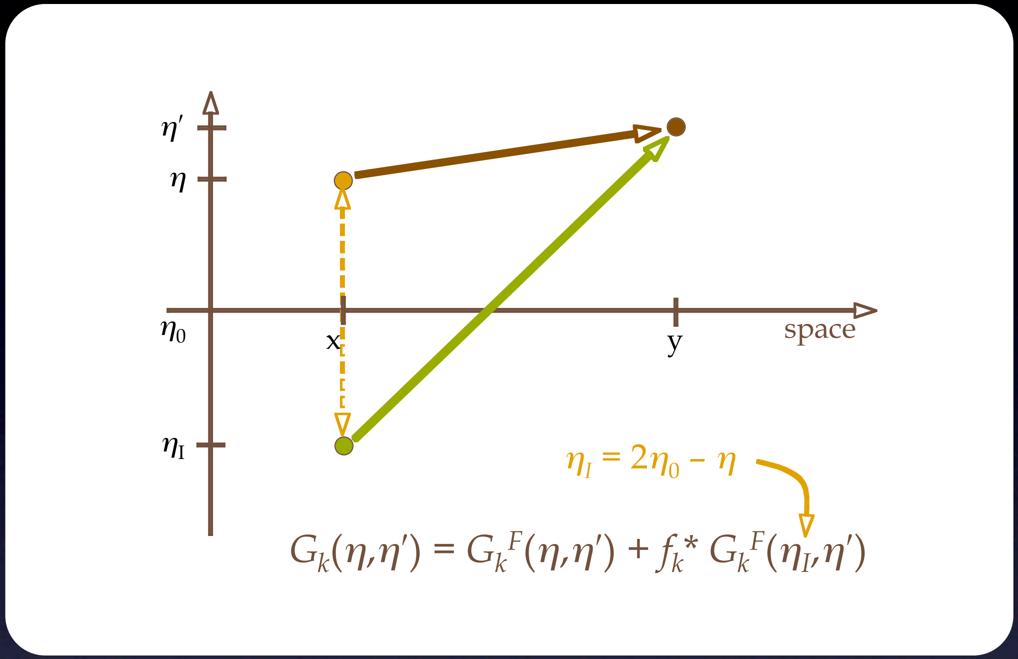
An important point about our formalism: we set up an initial value problem for ALL our modes at the earliest time η_0 i.e. on a space-like surface.

This is as opposed to approaches that define the modes as each physical wavenumber crosses M, which corresponds to a IVP using a **timelike** surface.



Loops

Part of the exercise is to find a way to control higher order corrections to things like the power spectrum.



With new initial conditions, propagators have to be modified to incorporate them. The propagators now have two pieces; point source and boundary influence.

$$-iG_k^{++}(\eta,\eta') = \Theta(\eta-\eta')U_k(\eta)U_k^*(\eta') + \Theta(\eta'-\eta)U_k^*(\eta)U_k(\eta') + f_k^* U_k(\eta)U_k(\eta)$$

$$-iG_k^{+-}(\eta,\eta') = U_k^*(\eta)U_k(\eta') + f_k^* U_k(\eta)U_k(\eta)$$

$$-iG_k^{-+}(\eta,\eta') = U_k(\eta)U_k^*(\eta') + f_k^* U_k(\eta)U_k(\eta)$$

$$-iG_k^{--}(\eta,\eta') = \Theta(\eta'-\eta)U_k(\eta)U_k^*(\eta') + \Theta(\eta-\eta')U_k^*(\eta)U_k(\eta') + f_k^* U_k(\eta)U_k(\eta)$$

In flat space, the modifications can be found via a method of images construction.

Boundary Renormalization

Consider the following simple description of an effective state

$$f_k = \sum_{n=1}^{\infty} d_n \frac{k^n}{(a(\eta_0)M)^n}$$

This is a short-distance modification, where we expect Trans-Planckian signals to be.

Correspondence

UV structures in initial state $\longleftrightarrow$ Irrelevant boundary counterterms

Loops create ***NEW*** UV divergences, when we sum over all modes and ***ONLY*** on the initial time hypersurface. Counter-terms also live only on this hypersurface.

$$\Omega_k(\eta) \simeq \frac{\sqrt{k^2 + m_{\mathrm{eff}}^2}}{a(\eta)}$$

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UV piece: Need non-renormalizable boundary counterterms

$$\begin{aligned} \frac{\partial}{\partial \eta} \mathcal{U}_k(\eta_0) &\simeq -ik\mathcal{U}_k(\eta_0) \\ &+ i \left[\frac{2d_0}{1+d_0} k + 2 \frac{H(\eta_0) d_1}{(1+d_0)^2} \right] \mathcal{U}_k(\eta_0) \\ &+ \mathcal{O}\left(\frac{H^2(\eta_0)}{k} \right) \mathcal{U}_k(\eta_0) \end{aligned}$$

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Example: $\lambda\phi^4$ theory

IR: Marginal or relevant operators

$$\begin{aligned} d_0 &\rightarrow \phi\partial_\eta\phi \\ d_1 &\rightarrow K\phi^2 = 3H\phi^2 \end{aligned}$$

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UV: irrelevant operators

$$\begin{aligned} c_1 \rightarrow \quad & \phi\ddot{\phi} \quad , \dot{\phi}^2, K\phi\dot{\phi}, \\ & \dot{K}\phi^2 \quad , K^2\phi^2 \end{aligned}$$

Application: Primordial Spectrum Correction

Use $$f_k = d_1 \frac{k}{(a(\eta_0)M)} + \cdots$$

The power spectrum is given by $$P_k = \frac{H^2}{4\pi^2}\left[1 + d_1 \frac{k}{k_*} \sin\left(2 \frac{k}{k_*} \frac{M}{H}\right)\right] \text{ where } k_*/a(\eta_0) = M.$$

It's worth noting that time-like defined states will give something like $$P_k = \frac{H^2}{4\pi^2}\left[1 + \mathcal{O}(1) \frac{H}{M} \sin\left(2 \frac{M}{H} + \phi\right)\right]$$

Stress Energy Tensor Renormalization

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$$\rho = \frac{1}{2}\frac{1}{a^2}\int \frac{d^3\vec{k}}{(2\pi)^3}\left\{U_k' U_k'^* + (k^2 + a^2 m^2) U_k U_k^* \right.$$
$$\left. + \underline{f_k^*}\left[U_k' U_k' + (k^2 + a^2 m^2) U_k U_k\right]\right\},$$

$$p = -\rho + \frac{1}{a^2}\int \frac{d^3\vec{k}}{(2\pi)^3}\left\{U_k' U_k'^* + \tfrac{1}{3}k^2 U_k U_k^* \right.$$
$$\left. + \underline{f_k^*}\left[U_k' U_k' + \tfrac{1}{3}k^2 U_k U_k\right]\right\},$$

Stress Energy Tensor Renormalization (Cont'd)

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1. Expand metric about FRW,

$$g_{\mu\nu} = a^2(\eta)\left[\eta_{\mu\nu} + h_{\mu\nu}(\eta, \vec{x})\right]$$

Stress Energy Tensor Renormalization (Cont'd)

The Procedure:

1. Expand metric about FRW,
2. Construct interaction Hamiltonian linear in fluctuations,

$$g_{\mu\nu} = a^2(\eta)\left[\eta_{\mu\nu} + h_{\mu\nu}(\eta, \vec{x})\right]$$

$$H_I(\eta) = \frac{1}{2}a^2(\eta)\int d^3\vec{x}\, h^{\mu\nu}\{-2\tilde{G}_{\mu\nu} + T^{\rm cl}_{\mu\nu} + \hat{T}_{\mu\nu}\}$$

Stress Energy Tensor Renormalization (Cont'd)

The Procedure:

1. Expand metric about FRW,
2. Construct interaction Hamiltonian linear in fluctuations,
3. Compute tadpole using S-K formalism.

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$$\begin{aligned}
&\langle 0_{\rm eff}(\eta)|h^{+}_{\mu\nu}(\eta,\vec{x})|0_{\rm eff}(\eta)\rangle \\
&= \langle 0_{\rm eff}|T\Big(h^{+}_{\mu\nu}(\eta,\vec{x})e^{-i\int_{\eta_0}^{0} d\eta'[H_I^{+}(\eta')-H_I^{-}(\eta')]}\Big)|0_{\rm eff}\rangle \\
&= \frac{1}{2}\int_{\eta_0}^{\eta} d\eta' a^2(\eta')\Big\{\big[\Pi^{>}_{\mu\nu,}{}^{\lambda\rho}(\eta,\eta';\vec{0}) - \Pi^{<}_{\mu\nu,}{}^{\lambda\rho}(\eta,\eta';\vec{0})\big] \\
&\qquad \times\big[2\tilde{G}_{\lambda\rho}(\eta') - T^{\rm cl}_{\lambda\rho}(\eta') - T_{\lambda\rho}(\eta')\big] + \cdots\Big\}.
\end{aligned}$$

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For TP corrections

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$$\rho_{\rm surf}(\eta_0) \;\propto\; \frac{H^4}{16\pi^2}\frac{H^n}{M^n}\frac{d_n^*}{\epsilon}\left[1+\mathcal{O}\big(m^2/H^2\big)+\mathcal{O}\big(H'/H^2\big)\right]$$
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$$\frac{\rho^R_{\rm surf}}{\rho_{\rm vac}} \sim \frac{1}{16\pi^2}\frac{H^2}{M_{\rm pl}^2}\frac{H^n}{M^n}$$

Backreaction is under control!

Greene et al vs. Porrati et al.

Observability

Signals that scale as H/M could be seen in future surveys!

Far from being an academic exercise, we need to understand how to control the possible infiltration of short-distance physics into the CMB just to have predictive power!

Observability

Power Spectrum prospect summary

- Today: 10^{-2}
- Soon (WMAP/Planck) : 10^{-3}
- Planned Galaxy Surveys (KAOS, LSST, Pan-Starr): 10^{-4}
- Future Galaxy Surveys (21 cm survey up to z~30) : 10^{-5}
- Theoretical Bound: 10^{-6}

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Conclusions

- To extract maximum information early Universe from the CMB we need to know how to reliably calculate all relevant effects.
- There is a real possibility of using the CMB power spectra to get information about possible trans-Planckian physics effects.
- We now have an effective initial state that allows for reliable, controllable calculations. We've shown that as expected, back-reaction effects are small after renormalization of the effective theory.