The QCD phase diagram from lattice simulations

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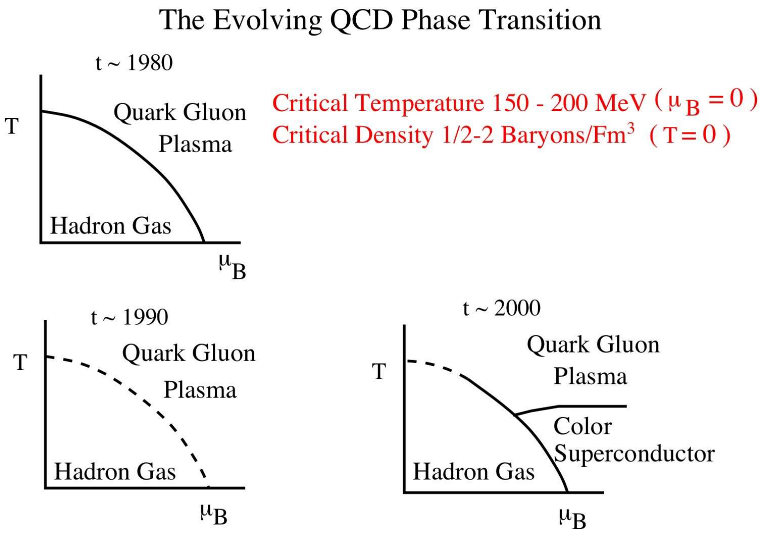
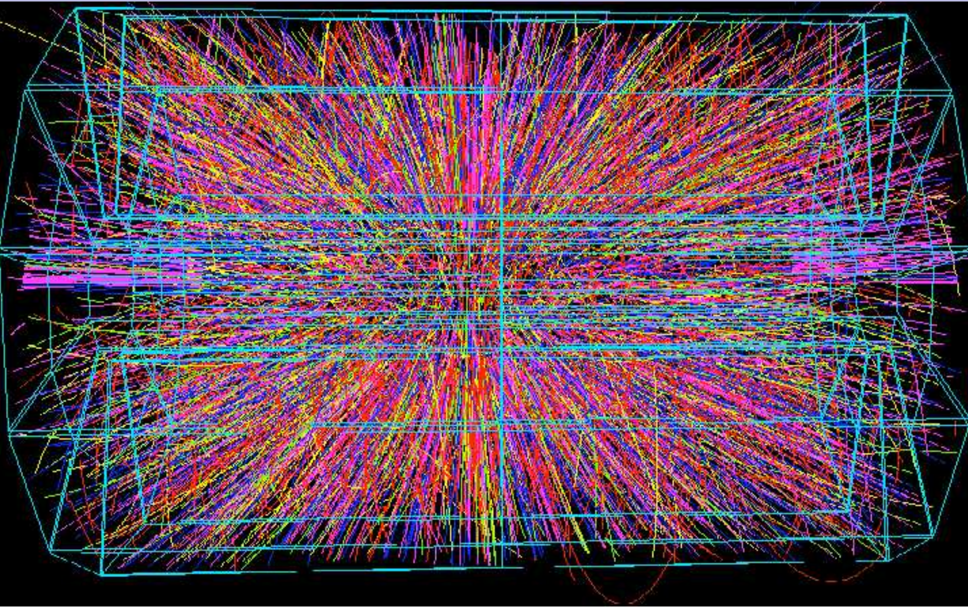
Versions of the QCD phase diagram

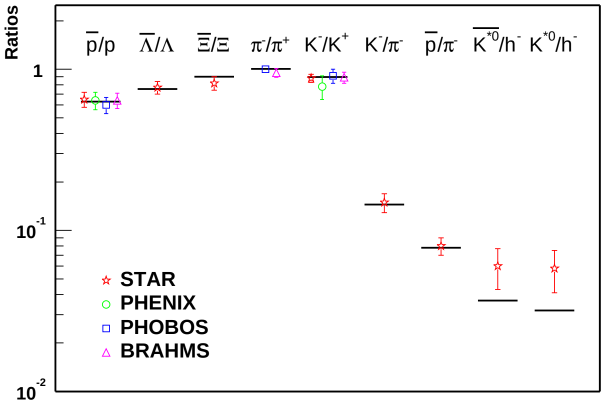


L. McLerran, QM2008

Heavy-ion collisions



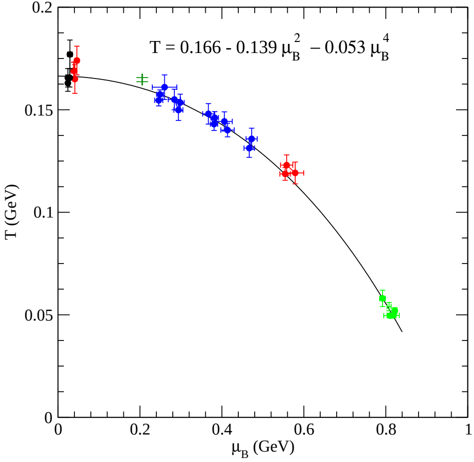
Phase boundary versus freeze-out temperature?



At fixed $\sqrt{s}$, relative abundances fitted well with Boltzmann (T, μ_B)

Phase boundary versus freeze-out temperature?

Repeat for successive $\sqrt{s}$: eg. J. Cleymans et al., hep-ph/0607164



T(freeze-out) $\leq$ T_c but very close

Braun-Munzinger, Stachel & Wetterich, nucl-th/0311005

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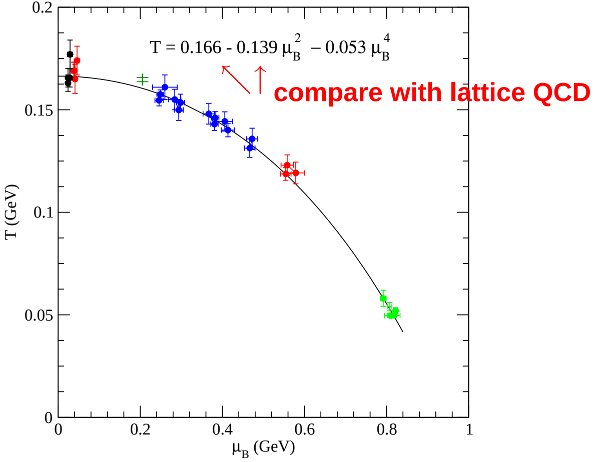

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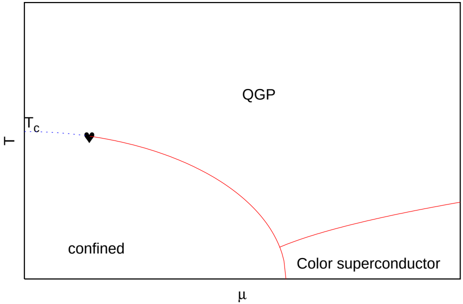
Scope of lattice QCD simulations

What can lattice QCD say about:

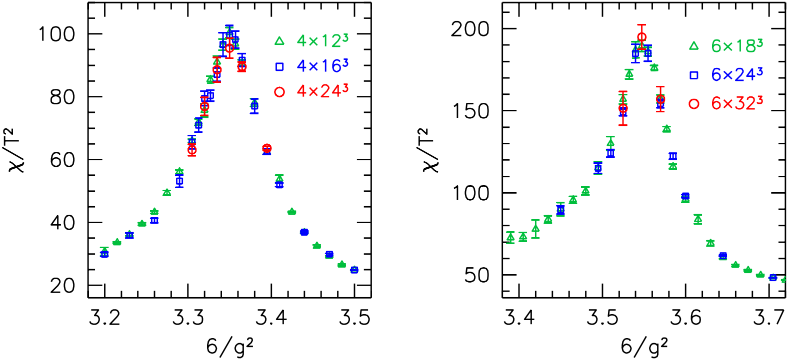
1. The $\mu = 0$ finite-temperature transition/crossover ?
2. The “phase” boundary $T_c(\mu)$?
3. The QCD critical point ?



1. The $\mu = 0$ finite-temperature transition/crossover

The ultimate: Fodor et al. (hep-lat/0611014 → Nature; hep-lat/0609068)
physical quark masses, 4 lattice spacings (but staggered fermions)

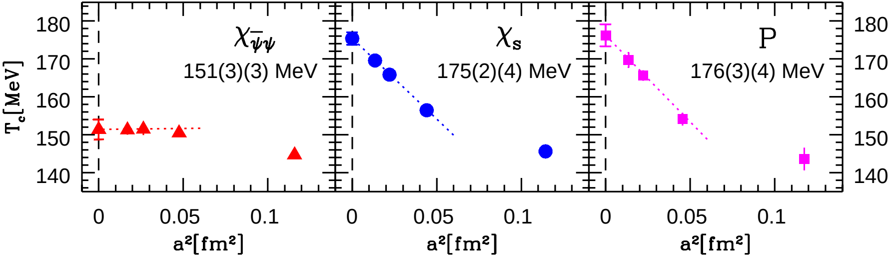
- No phase transition: **crossover**



1. The $\mu = 0$ finite-temperature transition/crossover

The ultimate: Fodor et al. (hep-lat/0611014 $\rightarrow$ Nature; hep-lat/0609068)
physical quark masses, 4 lattice spacings (but staggered fermions)

- "T_c" depends **a lot** on the observable



But: - "$T_c(\bar\psi\psi)$" $<$ $T_{\mathrm{freeze-out}} \approx 166$ MeV ?
- $T_c = 192(7)(4)$ MeV (Karsch et al.), from $N_t = 4$ and 6

The dust should settle soon.. ($N_t = 8$, two actions from HotQCD)

Comparing finite *a* data: Karsch vs Fodor (1)

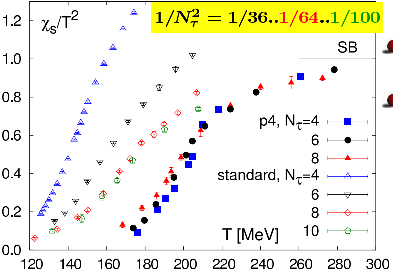
DECONFINEMENT:

Light and Strange Susceptibilities

1-link, stout, physical quark masses; T-scale from f_K, but $f_K r_0$ consistent with asqtad value for r_0 in the continuum limit

p4fat3, $m_q = 0.1 m_s$, i.e. $m_\pi \simeq 220$ MeV; T-scale from r_0 using the asqtad r_0 deduced from 'gold plated observables'

expect still a shift of T-scale ~ 5 MeV for physical quark masses



p4fat3 versus 1-link, stout

strange quark number susceptibility:

- $T \rightarrow \infty$, ideal gas limit: $\chi_l/T^2, \chi_s/T^2 \quad \rightarrow \quad 1$
- similar cut-off dependence as pressure $\Rightarrow$ strong cut-off dependence in simulations with not $\mathcal{O}(a^2)$ improved actions

1-link, stout
Y. Aoki et al., PLB643, 46 (2006)

p4fat3, $\mathcal{O}(a^2)$ improved
RBC-Bielefeld, preliminary
$N_\tau = 8$: hotQCD, preliminary

F. Karsch, xQCD, August 2007 – p. 16/28

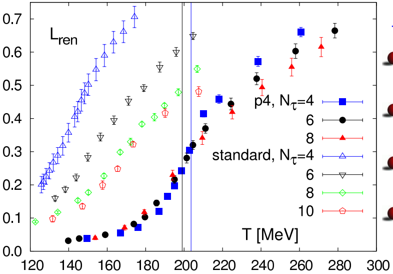
Comparing finite *a* data: Karsch vs Fodor (2)

Renormalized Polyakov loop

- Polyakov loop expectation value $\langle L\rangle = \exp(-F_q(T)/T)$; needs renormalization of divergent quark self-energies:

$$L_{ren}(T) = Z(\beta)^{N_\tau}\langle L\rangle(T)$$

expect still a shift of T-scale ~ 5 MeV for physical quark masses



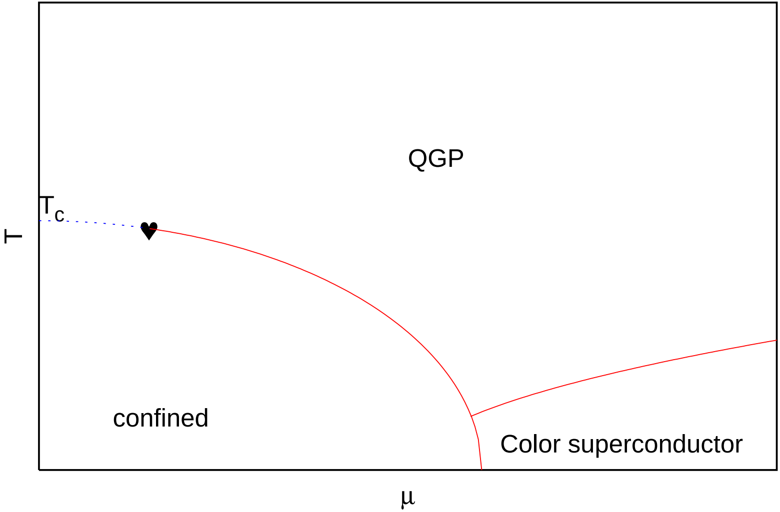
p4 versus standard

used $T=0$ potential to determine $Z(\beta)$ for each $T>0$ parameter set

- good scaling behavior of the renormalized Polyakov loop
- no significant cut-off dependence; confirms $SU(3)$ experience
- standard action L_{ren} rescaled with 1/5 (different renormalization scheme)
- quite different from findings with non $\mathcal{O}(a^2)$ improved actions; Y. Aoki et al., PLB643, 46 (2006)

F. Karsch, xQCD, August 2007 – p. 17/28

2. The “phase” boundary $T_c(\mu)$



Phase diagram: to be checked by lattice QCD simulations

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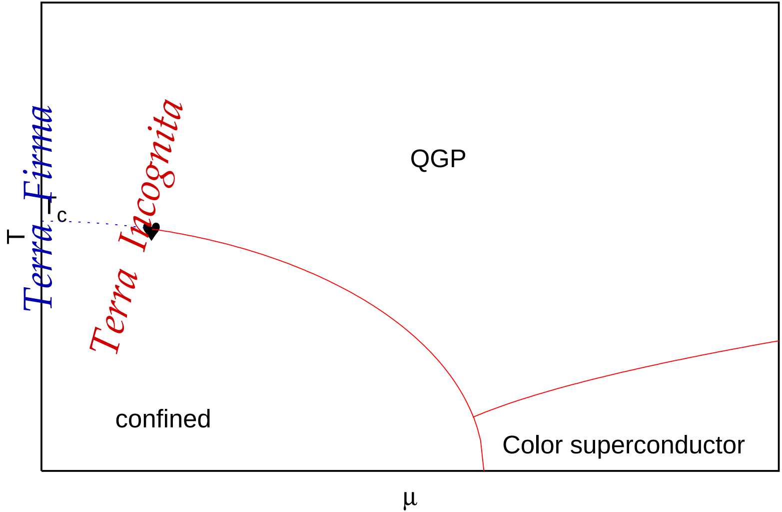

Phase diagram: to be checked by lattice QCD simulations

The difficulty: "sign" problem

- quarks anti-commute $\longrightarrow$ integrate analytically: $\det(\not{D}(U) + m + \mu\gamma_0)$
  $\gamma_5(i\not{p} + m + \mu\gamma_0)\gamma_5 = (-i\not{p} + m - \mu\gamma_0) = (i\not{p} + m - \mu^*\gamma_0)^\dagger$

$$\boxed{\det \not{D}(\mu) = \det{}^* \not{D}(-\mu^*)}$$

det complex unless $\mu = 0$ (or $i\mu_I$)

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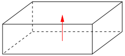
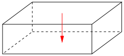
- Corollary: measure ϖ must be complex when $\mu \neq 0$



$$\langle \mathrm{Tr\ Polyakov}\ \rangle = \exp(-\tfrac{1}{T}F_q) = \langle \mathrm{Re\ Pol} \times \mathrm{Re}\varpi - \mathrm{Im\ Pol} \times \mathrm{Im}\varpi \rangle$$



$$\langle \mathrm{Tr\ Polyakov}^\dagger \rangle = \exp(-\tfrac{1}{T}F_{\bar{q}}) = \langle \mathrm{Re\ Pol} \times \mathrm{Re}\varpi + \mathrm{Im\ Pol} \times \mathrm{Im}\varpi \rangle$$

$$F_q \neq F_{\bar{q}} \;\Rightarrow\; \mathrm{Im}\varpi \neq 0$$

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$$\boxed{\det \not{D}(\mu) = \det{}^* \not{D}(-\mu^*)}$$

det complex unless $\mu = 0$ (or $i\mu_I$)

- Need auxiliary partition function for Monte Carlo sampling

$$Z(\mu) = \int \mathcal{D}U\, e^{-S_g}\, \det \not{D}(\mu) \quad \longrightarrow \text{no Monte Carlo}$$

$$Z_{MC} = \int \mathcal{D}U\, e^{-S_g}\, |\det \not{D}(\mu)| \qquad \text{(or } \det(\mu=0) \text{ or ...)}$$

$$Z(\mu)/Z_{MC} = \langle \frac{\det \not{D}(\mu)}{|\det \not{D}(\mu)|} \rangle = \langle e^{i\phi} \rangle, \quad \textbf{average "sign"}$$

$$\langle e^{i\phi} \rangle \sim \exp(-V \frac{\delta F(\mu)}{T}), \text{ but each measurement } \mathcal{O}(1)$$

$\Longrightarrow$ Need statistics $\propto$ $\exp(+V)$ for constant accuracy

Numerical approaches: I. Conservative

I. Conservative: evaluate **coefficients of Taylor series about** $\mu = 0$

No sign problem $\Longrightarrow$ can control thermodynamic/continuum limits

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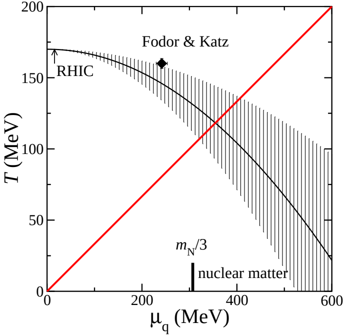
- Simple-minded: simulate at $\mu = 0$, measure susceptibilities

MILC, .., TARO, Bielefeld-Swansea, Gavai & Gupta

ie. Taylor expansion $\frac{P(\mu)-P(\mu=0)}{T^4} = \sum_{n=1} c_{2n}(T)(\frac{\mu}{\pi T})^{2n}$

A few Taylor coeffs (max. 4); expect convergence for $|\frac{\mu}{\pi T}| \lesssim 1$



Bielefeld-Swansea, $N_f = 2, m_q/T = 0.4$

Numerical approaches: I. Conservative

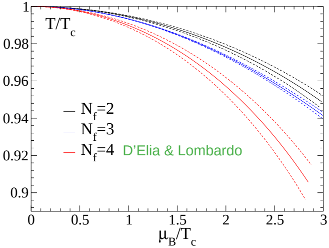
I. Conservative: evaluate **coefficients of Taylor series about $\mu = 0$**

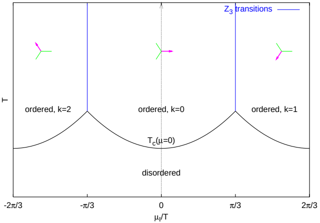
No sign problem $\Longrightarrow$ can control thermodynamic/continuum limits

- [Much] better: simulate at $\mu = i\mu_I$ imaginary

PdF & Philipsen, D'Elia & Lombardo, Chen & Luo, Azcoiti et al.,..

- two control parameters: β and μ_I
- fit with truncated Taylor expansion, then analytically continue $\mu_I^2 \rightarrow \mu^2$
- systematics: can check significance of higher-order terms
- works also for critical line $T_c(\mu)$
- limited by singularity $\mu_I = \frac{\pi}{3}T$





$\frac{T_c(\mu)}{T_c(\mu=0)} = 1 - c(N_f, m_q)(\frac{\mu}{\pi T})^2 + ..$ $\quad c \approx 0.500(34), 0.602(9), 0.93(10)$ for $N_f = 2, 3, 4$, $\frac{m_q}{T} \ll 1$ $\quad (c \propto N_f/N_c)$

Numerical approaches: II. Adventurous

II. Adventurous: evaluate **full result at finite μ**

Sign problem $\Longrightarrow$ small, coarse lattices $\rightarrow$ crosscheck essential

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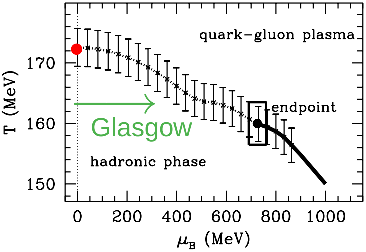
- Double reweighting in (μ, β) from $(\mu = 0, \beta_c)$

Fodor & Katz

$$Z(\mu,\beta) = \langle \frac{\exp(-\beta S_g)\det M(\mu)}{\exp(-\beta_c S_g)\det M(\mu=0)} \rangle Z_{MC}(\mu=0,\beta_c)$$



Errors under control ? Sign problem ?, Overlap problem ?

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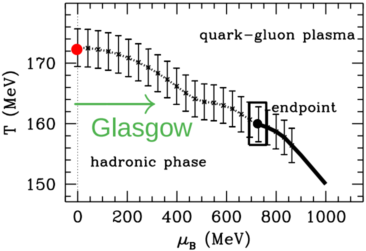

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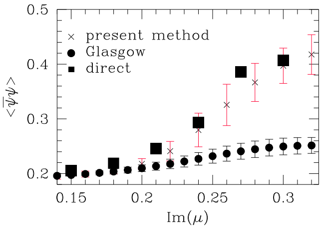

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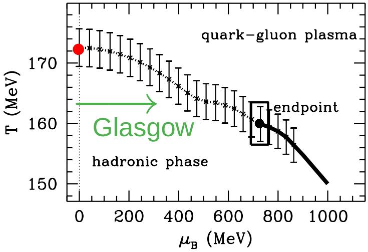

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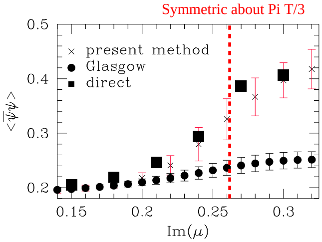
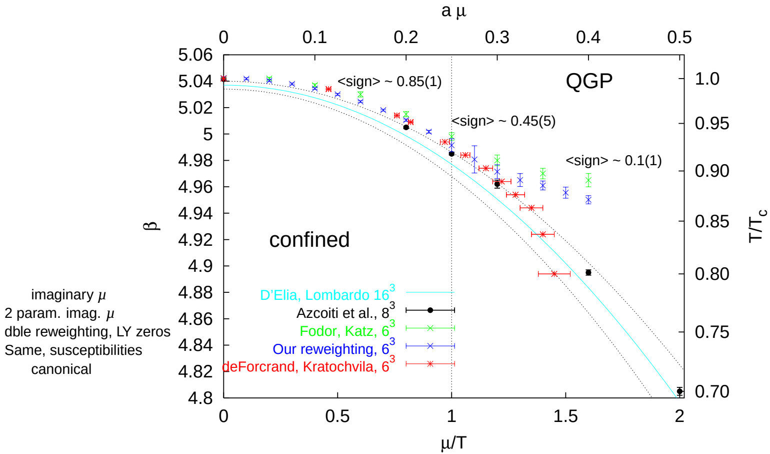


Phase Diagram $T-\mu$: comparing apples with apples

All with $N_f = 4$ staggered fermions, $am_q = 0.05, N_t = 4$ $(a \sim 0.3$ fm)

PdF & Kratochvila



Agreement for $\mu/T \lesssim 1$

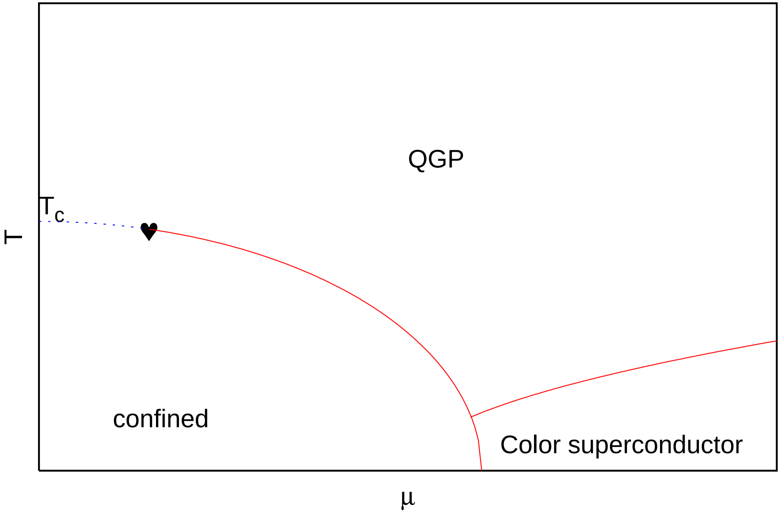
Summary for phase boundary

- Under control for $\mu/T \lesssim 1$
- Well described by parabola $\longrightarrow$ curvature $\frac{d(T/T_c)}{d(\mu/T_c)^2}|_{\mu=0}$
- Curvature about 1/3 freeze-out parabola (using pert. scaling)
- **Can study $a \to 0$ continuum limit** ($\sim$ susceptibility)

Preliminary:

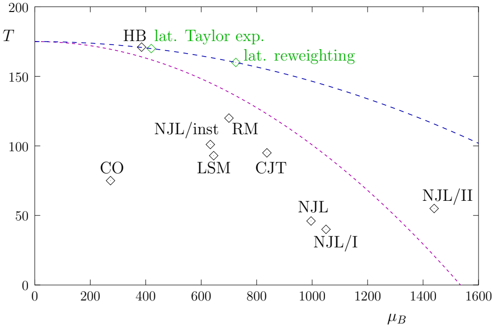
- curvature increases towards freeze-out value ($m_q = m_q^{\text{phys}}$) Fodor
- curvature decreases for $N_f = 3, m_q = m_q^{\text{crit}}$ PdF & OP

3. The QCD critical point



Can one locate the critical point (μ_E, T_E) ?

Locating the critical point



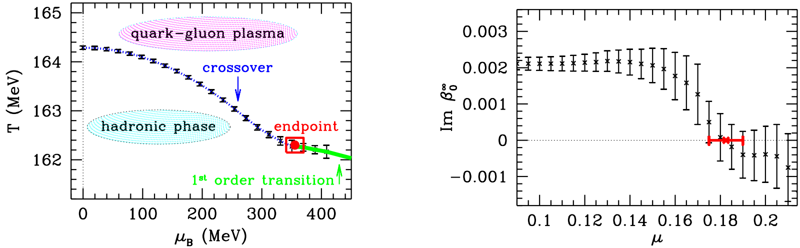
M. Stephanov, hep-ph/0402115

- Much harder task:

  detect divergence of correlation length on small lattice (??)

Already determined, but...

Fodor & Katz: hep-lat/0402006 ($\sim$ physical quark masses)



$$(\mu_E^q, T_E) = (120(13), 162(2))\ \text{MeV}$$

Legitimate **concerns**:

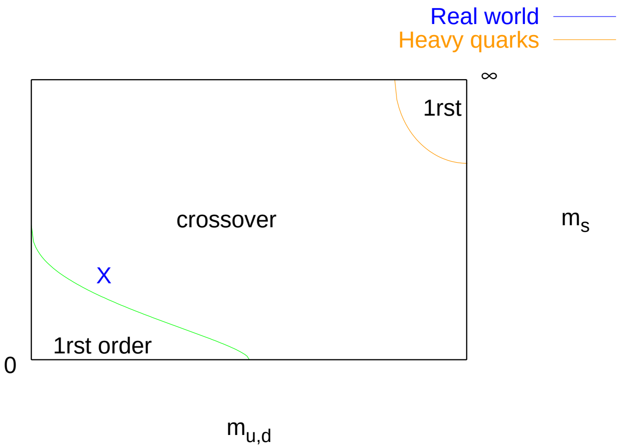
- Discretization error? $N_t = 4 \implies a \sim 0.3$ fm
- Abrupt qualitative change near μ_E:

  abrupt change of physics or breakdown of algorithm (Splittorff)?

$\longrightarrow$ repeat with conservative approach (derivative), with $N_t = 4$ first

Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram

$\mu = 0$



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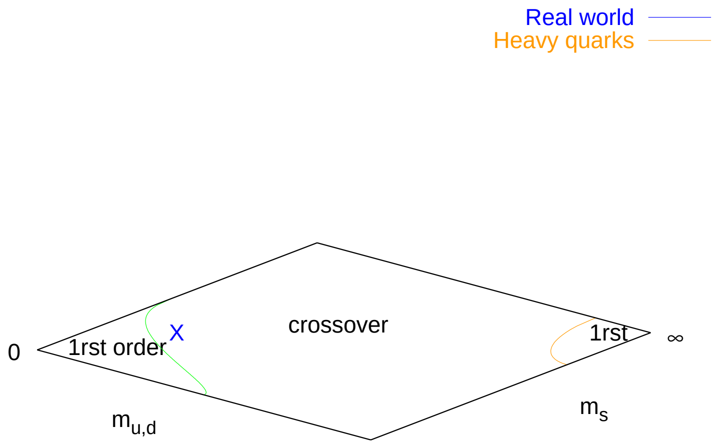
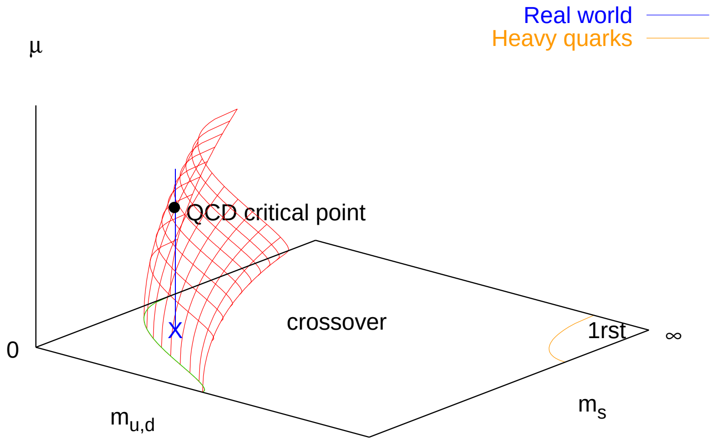


Now turn on μ

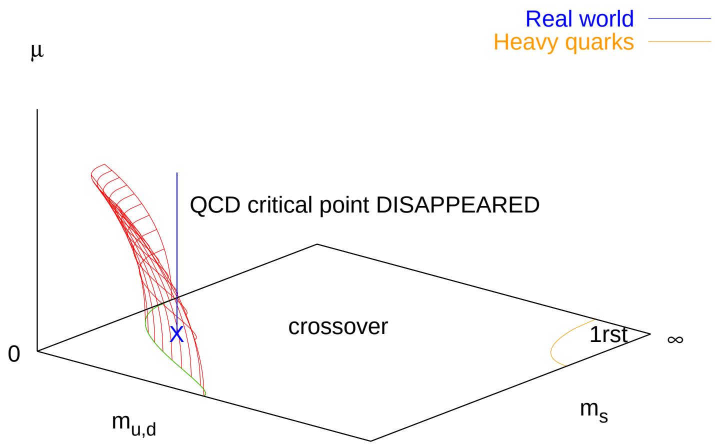
Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram

$\mu \neq 0$



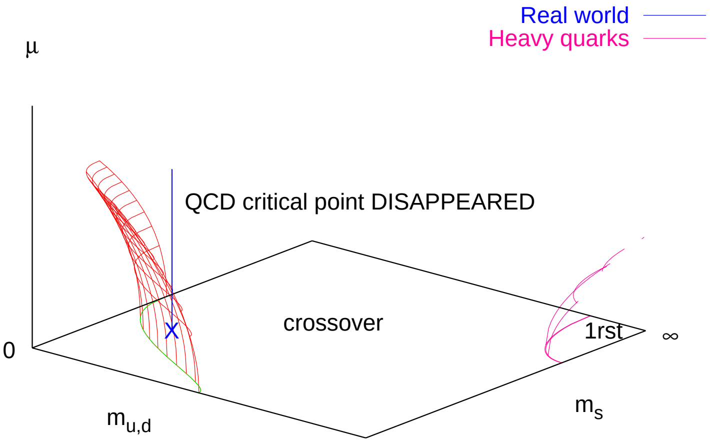
Conventional wisdom: first-order region expands with real $|\mu|$

Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram



Exotic scenario: first-order region shrinks with real $|\mu|$ $\frac{d\ m_c}{d\mu^2}|_{\mu=0} < 0$

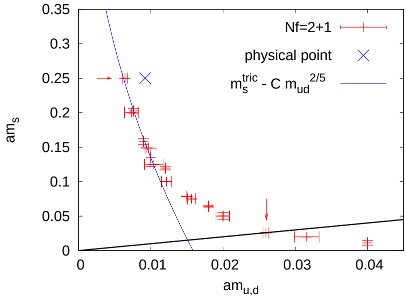
Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram

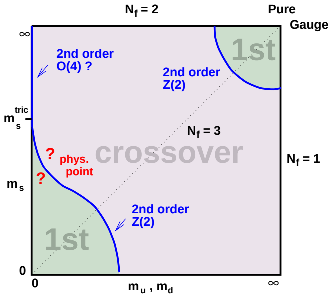


For heavy quarks, first-order region shrinks (PdF, Kim, Takaishi, hep-lat/0510069)

Lattice study with Owe Philipsen (hep-lat/0607017)

1. Line of second-order phase transitions in the quark mass plane $(m_{u,d}, m_s)$ via Binder cumulant $B_4 = \langle(\delta\bar{\psi}\psi)^4\rangle / \langle(\delta\bar{\psi}\psi)^2\rangle^2$



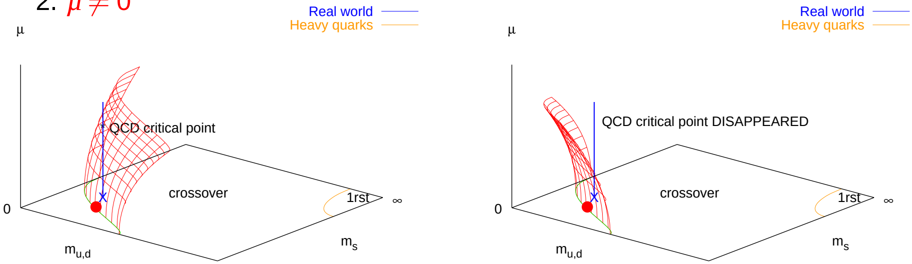


$\mu = 0$:

- data consistent with tricritical point at $m_{u,d} = 0,\ m_s \sim 2.8T_c$
- physical point in crossover region cf. Fodor & Katz

Lattice study with Owe Philipsen (hep-lat/0607017)

2. $\mu \neq 0$



Strategy: tune m_q for 2nd-order P.T. at $\mu = 0$, then turn on [imaginary] μ

Does the transition become 1rst-order (left) or crossover (right)?

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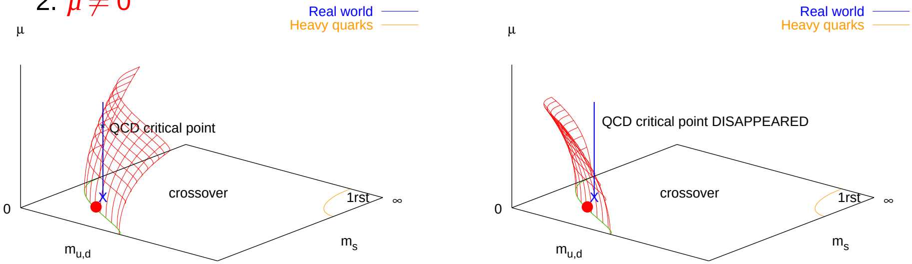

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Answer: very little change ($\longrightarrow$ surface almost vertical)

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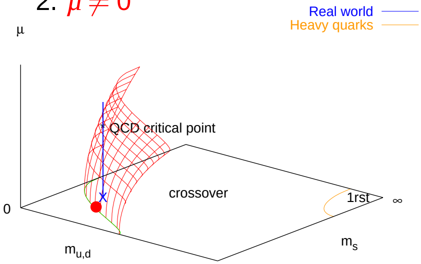

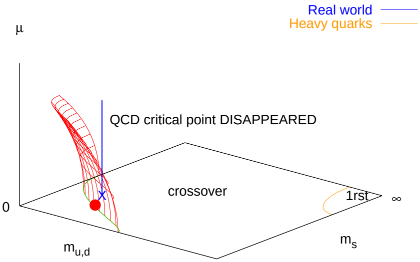

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0711.0262: measure δB_4 under $\delta\mu^2 \longrightarrow$ **crossover**: $\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(5)\left(\frac{\mu}{\pi T}\right)^2$

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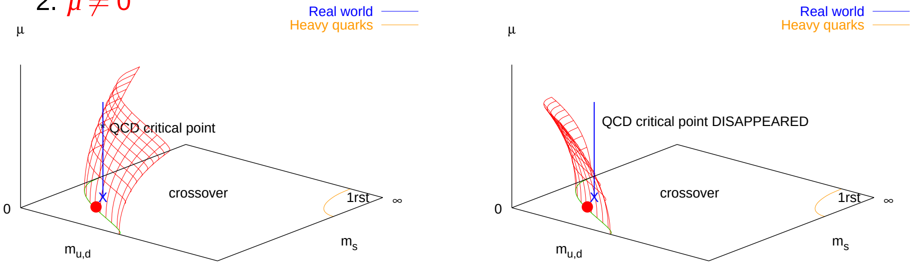
2. $\mu \neq 0$



Strategy: tune m_q for 2nd-order P.T. at $\mu = 0$, then turn on [imaginary] μ
Does the transition become 1rst-order (left) or crossover (right)?

Answer: very little change ($\longrightarrow$ surface almost vertical)

0711.0262: measure δB_4 under $\delta\mu^2$ $\longrightarrow$ **crossover**: $\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(5)\left(\frac{\mu}{\pi T}\right)^2$
(preliminary) $-12(6)\left(\frac{\mu}{\pi T}\right)^4$

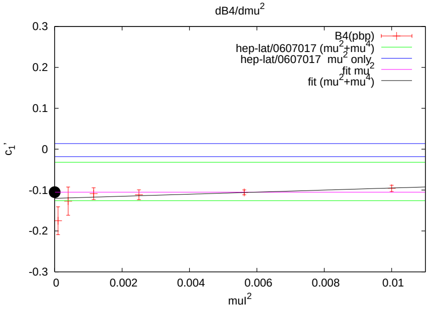
Status of numerical results

Measure variation of $B_4(\bar{\psi}\psi)$ and apply chain rule:

$$c_1' = \frac{d(am_c)}{d(a\mu)^2}\Big|_{\mu=0} = \frac{\partial B_4}{\partial(a\mu)^2} \times \left(\frac{\partial B_4}{\partial(am_c)}\right)^{-1}$$

- Consistency under increase of volume:

$8^3 \times 4$



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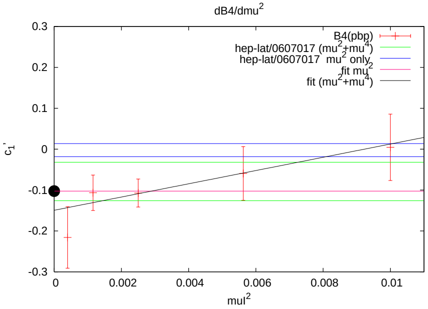

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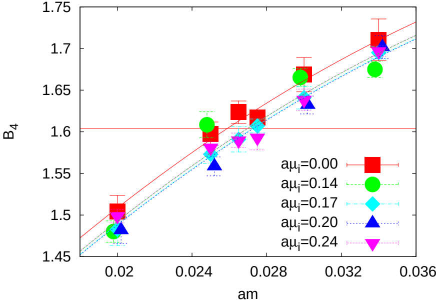
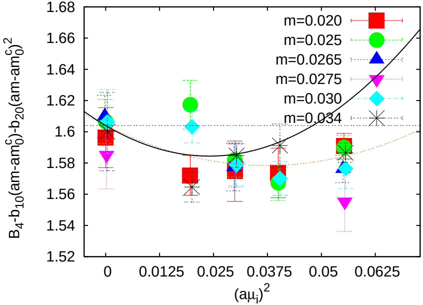
- NLO fits of B_4 consistent with direct meas. of derivative c_1':

Fitted meas. of $B_4(am, a\mu_I)$

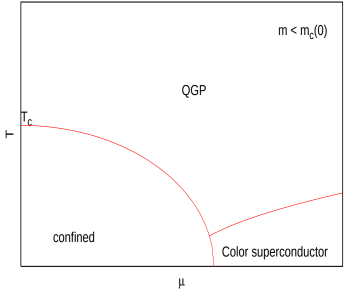


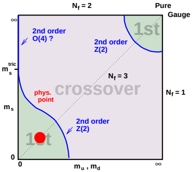
with mass dependence subtracted



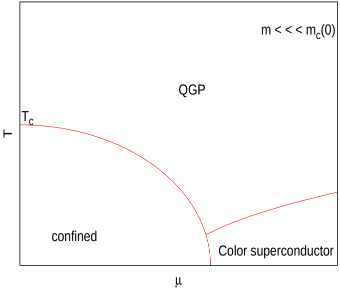
Resulting phase diagram (simplest possibility)

Standard scenario





Exotic scenario



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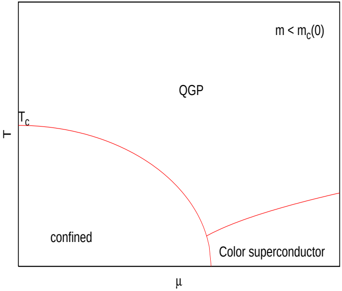

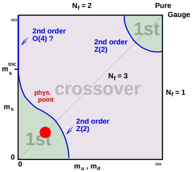

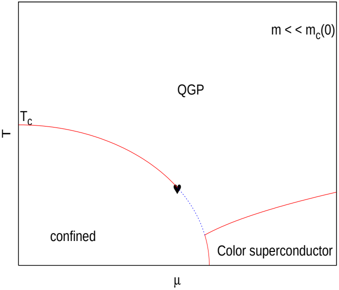

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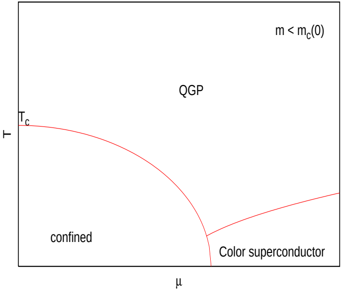

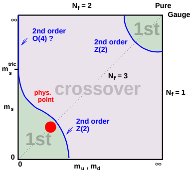

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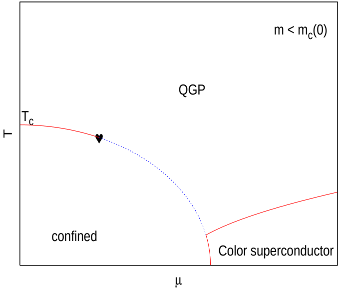

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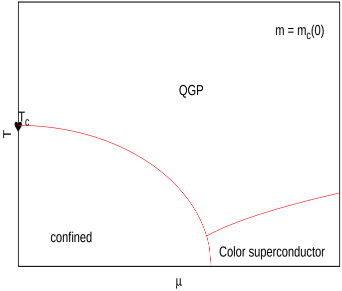

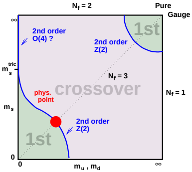

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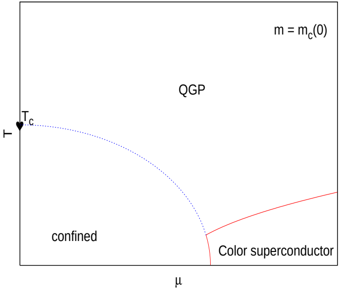

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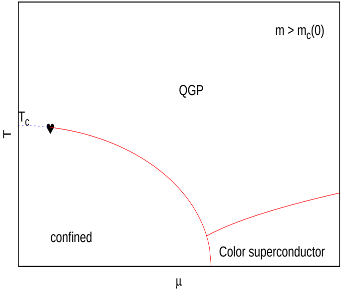

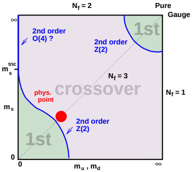

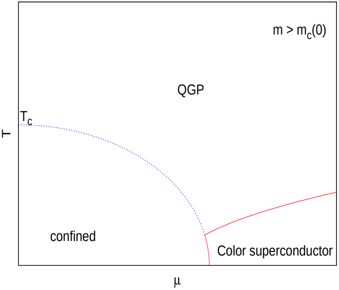

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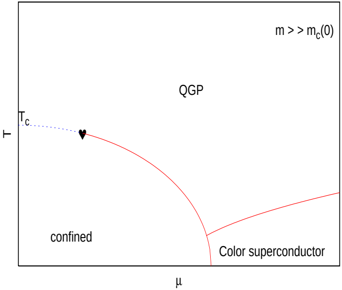

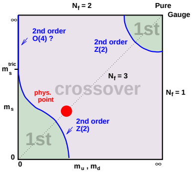

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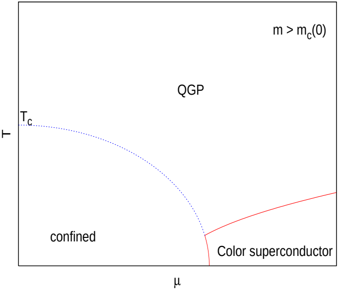

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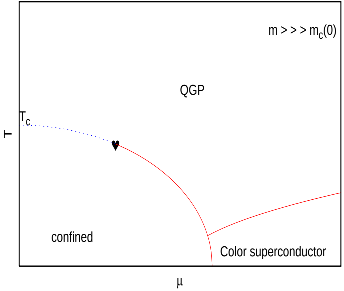

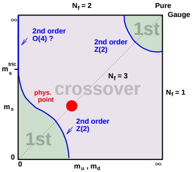

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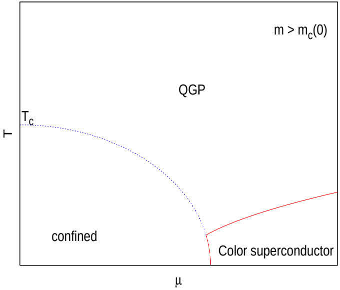

Discretization errors? Recall that $N_t = 4 \Rightarrow a \sim 0.3$ fm

Location of critical point depends on:

1) curvature of critical surface

2) distance physical point $\longleftrightarrow$ critical surface

Discretization errors on **(1)** and **(2)** ?

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Discretization errors on **(1)** and **(2)** ?

(2) increases by $\mathcal{O}(100\%)$ as $a \to 0$

As $a \to 0$, it takes much lighter quarks to have first-order transition

0711.0262, PdF & Philipsen; also 0710.0998, Fodor & Katz; Bielefeld, MILC

Pion mass (measured at $T = 0$) decreases: $\frac{m_\pi}{T_c} \approx 1.6\ (N_t = 4)\ \longrightarrow\ 0.95\ (N_t = 6)$

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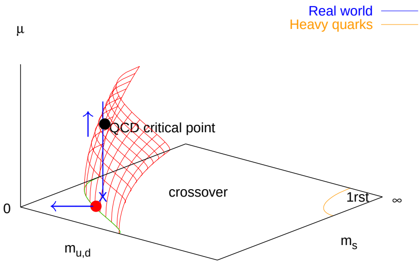
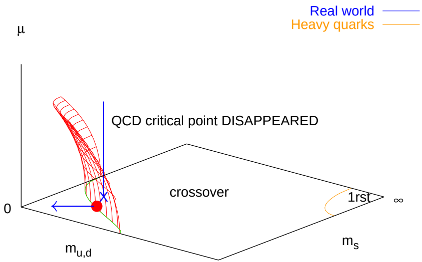




A critical point at “small” μ (ie. $\mu/T \lesssim 1$) would require curvature to

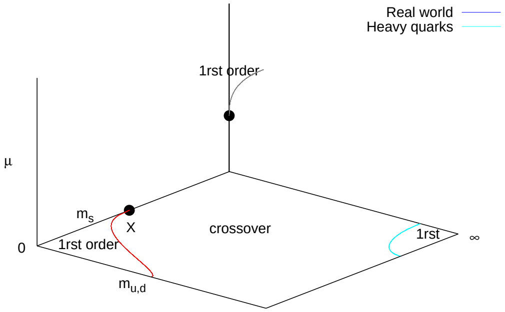
change sign **and** become large

as $a \longrightarrow 0$

Arguments for standard wisdom?

- $O(4)$ transition for 2 massless flavors Pisarski & Wilczek

$\Rightarrow$ tricritical points $(m_{u,d}=0, m_s=\infty, \mu=\mu^*)$ and $(m_{u,d}=0, m_s=m_s^*, \mu=0)$



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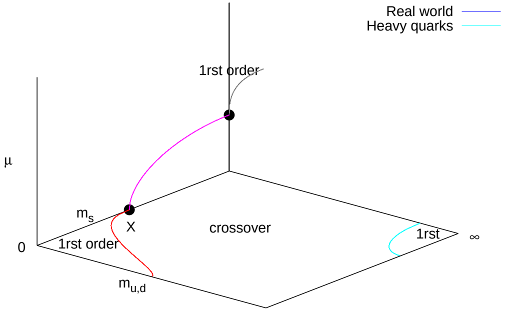

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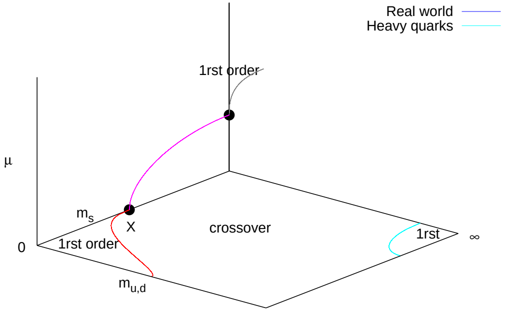

Critique:

- $O(4)$ if strong enough $U_A(1)$ anomaly, otherwise first-order

Chandrasekharan & Mehta

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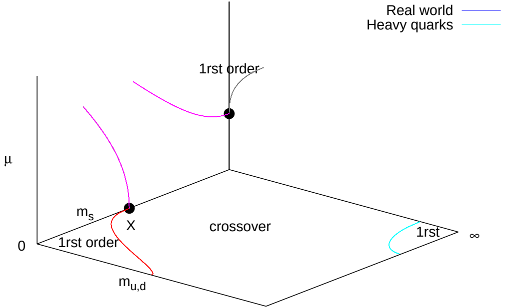

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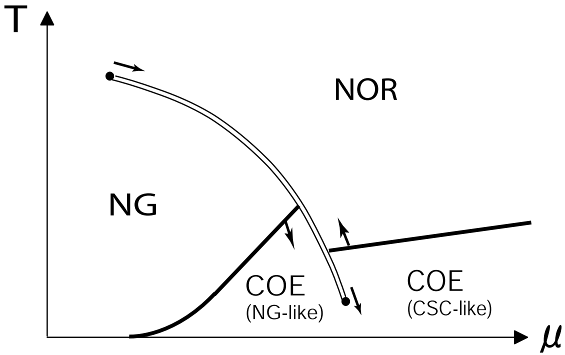
Chandrasekharan & Mehta

- $N_f=2$ and $N_f=2+1$ need not be connected

Conclusions

- Tough problem, but steady progress
- Cutoff error: $\mu = 0 \rightarrow \mathcal{O}(10\%)$ and $\mu \neq 0 \rightarrow \mathcal{O}(100\%)$
  work in progress
- **Keep open mind:**
  - critical point at small μ or not?
  - second critical point at small T? — Baym, Hatsuda et al.
  - "quarkyonics" at large N_c? — McLerran & Pisarski
- Phase diagram may be very different in next review

A second QCD critical point?

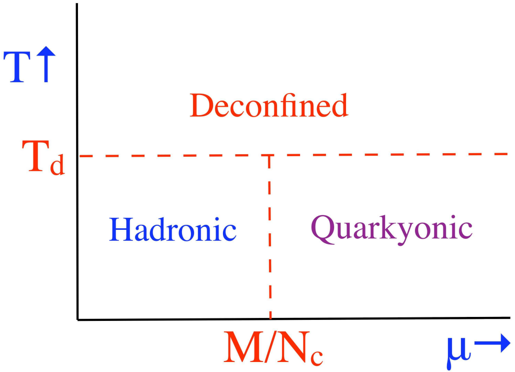


Baym, Hatsuda et al.

- Ginzburg-Landau analysis with two condensates: $\langle\bar{\psi}\psi\rangle$ and $\langle\psi\psi\rangle$
- Mapping from coeffs of V_{eff} to (T,μ) ??

2nd critical point could require, eg, $T<0$

Quarkyonics?



McLerran & Pisarski

- $N_c \to \infty, N_f$ fixed: $T_c(\mu) =$ const.
  pressure ~ 1 (hadronic), $\sim N_c^2$ (deconfined)
  "quarkyonic" phase: pressure $\sim N_c$
- Also for N_f/N_c fixed arXiv:0803.0279
  relevance for $N_c = 3, N_f = 2+1$??