Jets in a Strongly Coupled N=4 Super Yang-Mills Plasma

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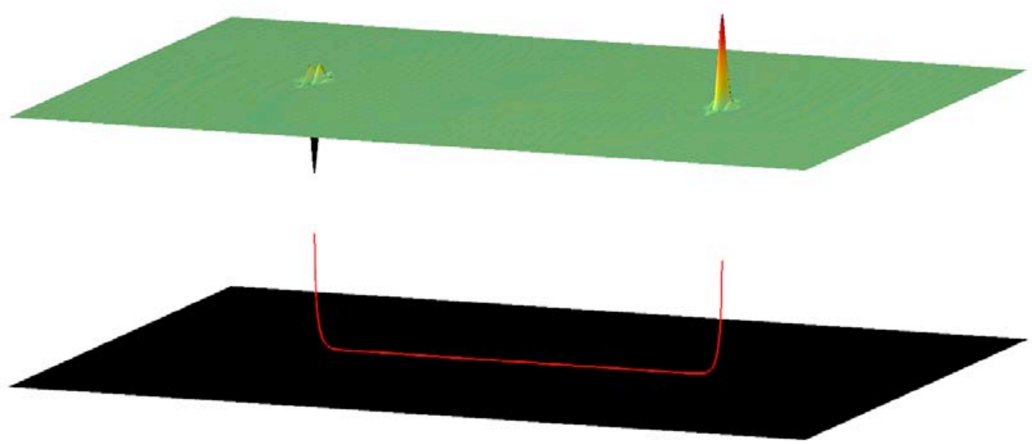
In collaboration with Laurence Yaffe, Kristan Jensen and Andreas Karch

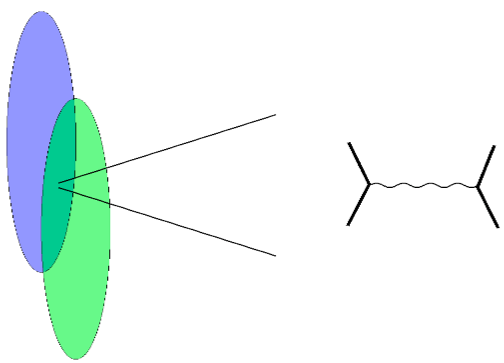


Outline

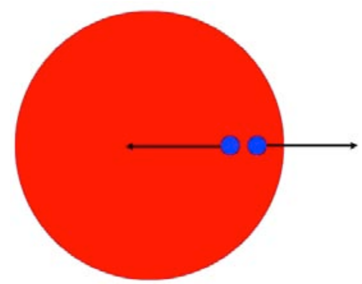
1) Motivation
2) Gravitational Description
    - String solutions
    - Boundary densities
3) Hydrodynamic Description
4) Baryon Density
5) Conclusions

Motivation: Jets at RHIC

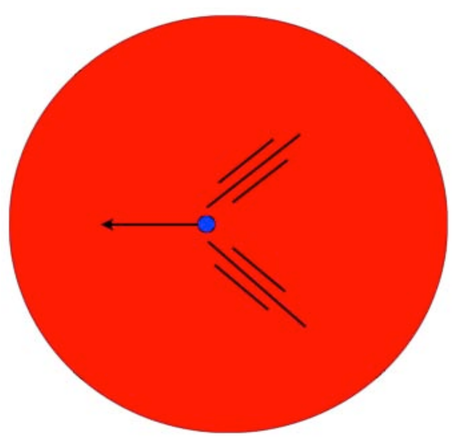
1) $q\bar{q}$ produced in early stages



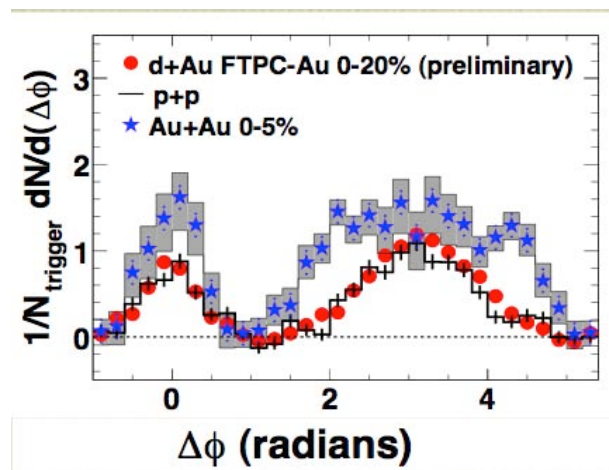
2) Thermalization



3) Expansion, cooling and quark energy loss, hadronization



4) Evidence for Mach cones in data? (STAR)

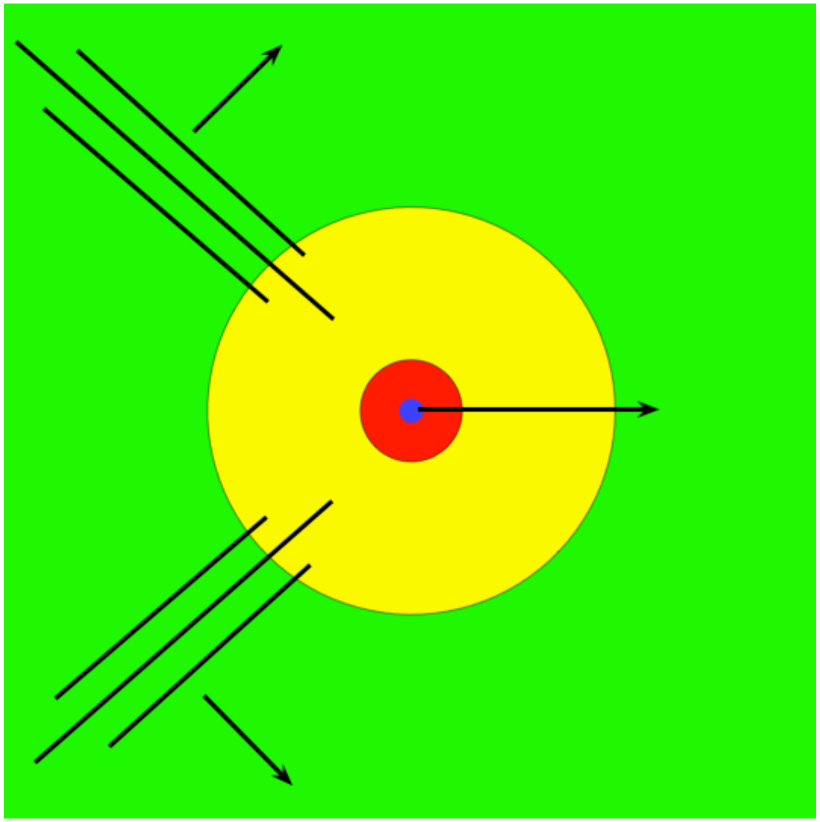


Jets in a Plasma

Quarks moving through a plasma will disturb the medium and deposit energy and momentum in the plasma.

The description of the physics depends on what scales questions are being asked at.

1. Near zone: QFT
2. Intermediate zone: Nonlinear hydrodynamics and/or higher order derivative expansion?
3. Asymtotically far zone: linear hydrodynamics



Theoretical Challenge

- Ultimate goal: compute the angular distribution of radiated power and the corresponding distribution of particles associated with jet.
- Available tools include:

1) Kinetic theory
   - – Not valid at strong coupling
2) Hydrodynamics
   - + Universal long wavelength effective theory
   - – Unknown range of validity for wake: $\ell \gg \ell_{\rm mfp} \sim 1/T$
   - – Unknown sources: $\partial_\mu T^{\mu\nu} = F^\nu$
3) Gauge/string duality
   - + Complete description of phenomenon valid on all length scales
   - ± Useful only at large N_c large λ
   - – No known gravitational dual to QCD

Toy Problem

- Consider equilibrium SYM plasma of ∞ extent.
- Add fundamental quarks.
- Compute energy loss rates and $\langle T^{\mu\nu}(x)\rangle, \ \langle J^{\mu}(x)\rangle.$
- Some questions to consider:
  - What is the angular distribution of baryon density of a jet?
  - Where does energy and momentum lost by the quark go?
  - What are the sources for hydrodynamics?
  - At what distances from the quark does hydrodynamics apply?

Strongly coupled large N field theory: Gauge/String Duality

- $\mathcal{N}=4$ SYM
- $g^2_{\rm YM}$
- $\lambda = g^2_{\rm YM} N_c$
- Finite *T*
- Adding quarks
- Operators $\hat{\mathcal{O}}(x)$

- IIB strings on $AdS_5 \times S^5$
- $4\pi g_s$
- $\left(\frac{L}{\ell_s}\right)^4$
- Black brane
- Adding open strings
- Bulk Fields $\Phi(x,u)$

Weakly coupled string theory = strongly coupled field theory

(Maldecena), (Witten), (Gubser, Klebenov, Polyakov) and more

AdS-Black Hole Geometry

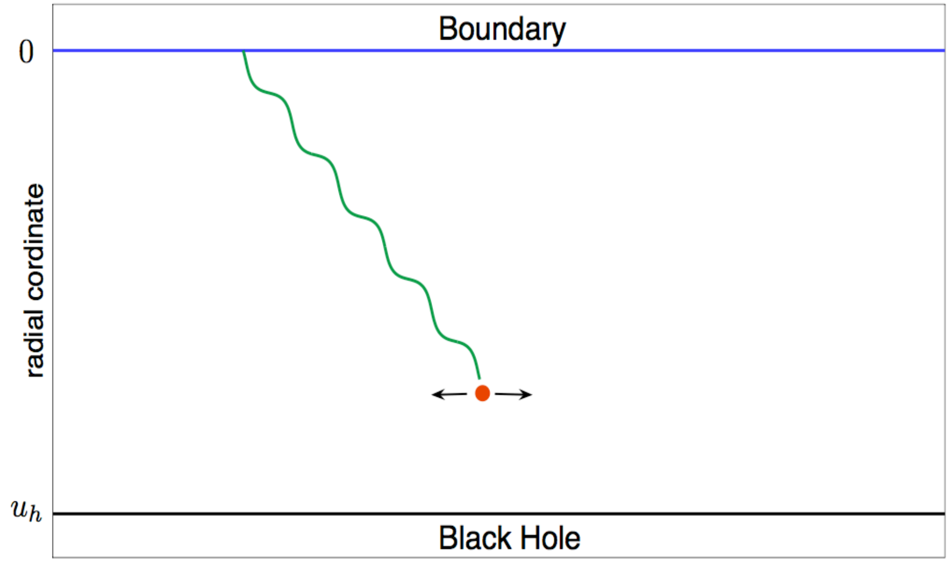
•Metric:

$$ds^2 = \frac{L^2}{u^2}\left(-f(u)dt^2 + d\mathbf{x}^2 + \frac{du^2}{f(u)}\right),$$
$$f(u) = 1 - \frac{u^4}{u_h^4}.$$

•Black hole at $u = u_h$.

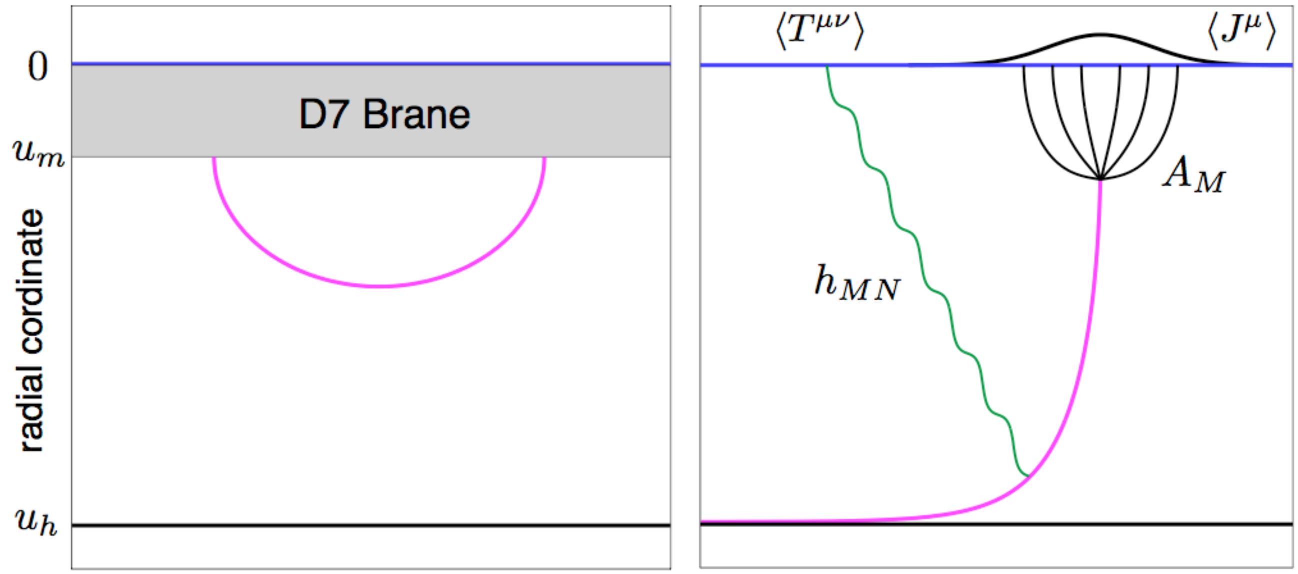
•Boundary at $u = 0$.

•Hawking Temperature $T_H = T = \frac{1}{\pi u_h}$.



Radial Coordinate ~ scale in the field theory.

Adding Open Strings/Quarks



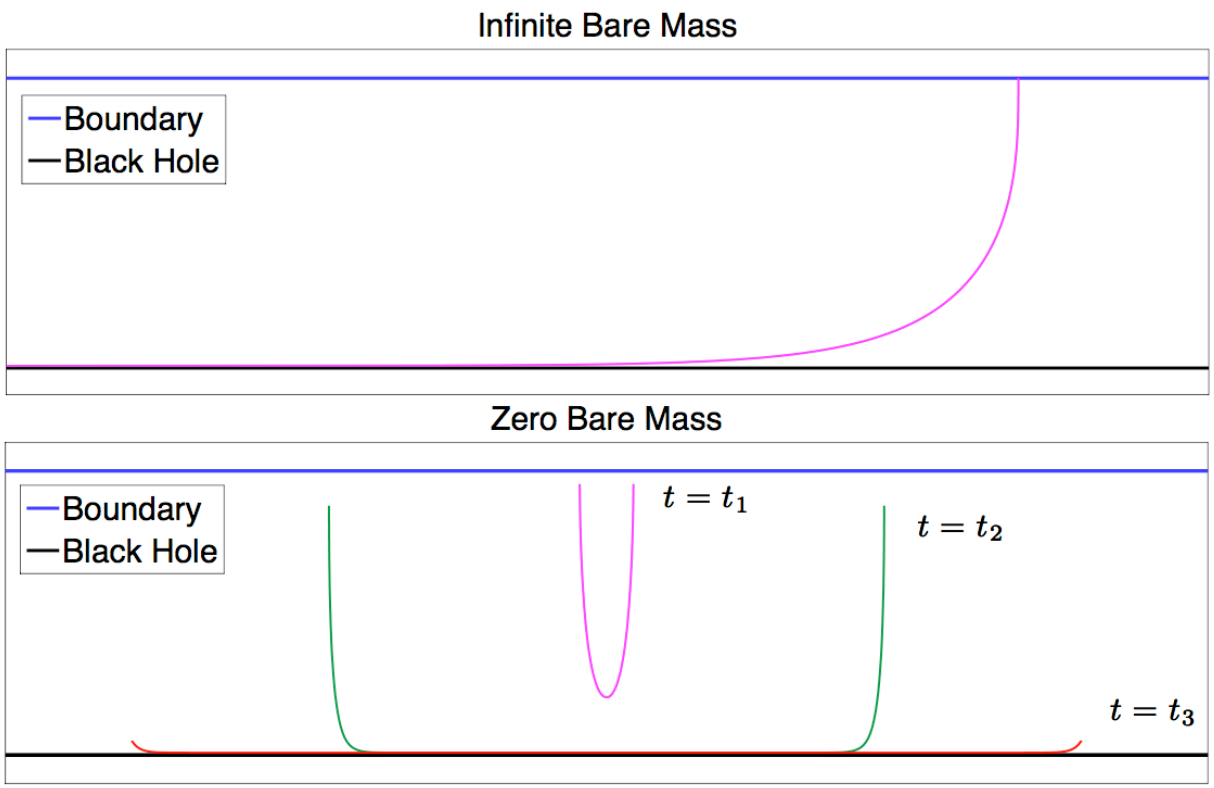
•Adding strings corresponds to adding a $q\bar{q}$ pair to the state.

(Karch, Katz)

•Strings end on D7 brane at $u = u_m$.

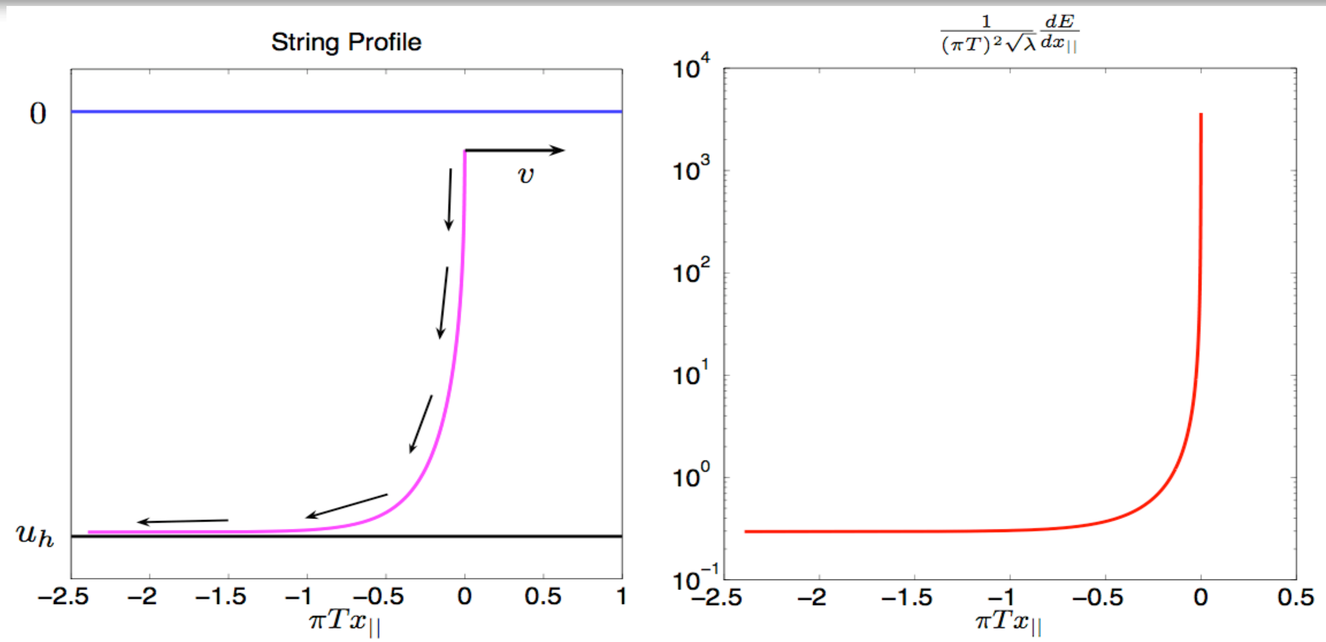
•Bare mass of quarks $M \sim \frac{\sqrt{\lambda}}{u_m}$.

Two limiting cases



- Infinite bare mass $\Rightarrow$ endpoints lies on the boundary.
- Zeros bare mass $\Rightarrow$ endpoints fall to horizon.

Heavy Quarks and Energy Flow



$$x(u) = \frac{v u_h}{2}\left[\tan^{-1}\left(\frac{u}{u_h}\right) + \frac{1}{2}\log\left(\frac{u_h - u}{u_h + u}\right)\right], \qquad \frac{dE_{\rm quark}}{dx} = -\frac{\sqrt{\lambda}}{2\pi}(\pi T)^2 \frac{v}{\sqrt{1-v^2}}$$

(Herzog, Karch, Yaffe, Kovtun, Kozcaz), (Gubser), (Teaney, Casalderrey-Solana)

How is the transfer of energy and momentum mapped to the boundary field theory?

Boundary Stress Tensor

- Presence of string perturbs geometry

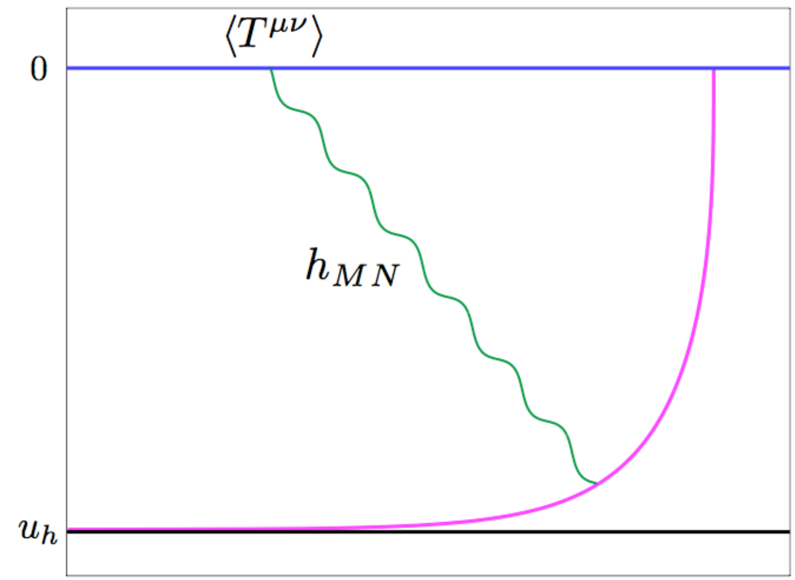
$$R_{MN} - \frac{1}{2} g_{MN} (R + 2\Lambda) = \kappa_5^2 t_{MN}.$$

- Large N_c limit $\Rightarrow$ linearized equations

$$\Delta_{MN}^{PQ} h_{PQ} = \kappa_5^2 t_{MN}.$$

- The AdS-BH metric induces a metric on the boundary

$$g_{B\mu\nu}(x) = \lim_{u \to 0} \frac{u^2}{L^2} g_{\mu\nu}(x,u).$$



- The boundary stress tensor is $\langle T^{\mu\nu}(x) \rangle = \frac{2}{\sqrt{-g_B}} \frac{\delta S_G}{\delta g_{B\mu\nu}(x)}.$ (Witten)

Gauge Invariants

- $T^{\mu}_{\ \mu} = 0, \quad \partial_\mu T^{\mu\nu} = F^\nu$ so the SYM stress tensor contains 5 independent degrees of freedom.
- h_{MN} contains 15 degrees of freedom.
- GR is a gauge theory:

$$X_M \to X_M + \xi_M, \quad h_{MN} \to h_{MN} - D_M \xi_N - D_N \xi_M$$

- There are five independent gauge invariant degrees of freedom in metric perturbation.
- The correspondence suggests the bulk to boundary problem should be formulated in terms of gauge invariant degrees of freedom.

Using Gauge Invariants Makes the Problem Much Easier!

- Convenient gauge invariants can be constructed out of linear combinations of $h_{MN}(\omega, \mathbf{q}; u)$ and its radial derivatives. (Kovtun and Starinets 2005)
- Gauge invariants can be labeled by a spin under rotations about the $\hat{q}$ axis:

$$Z_0,\ Z_1^a,\ Z_2^{ab}$$

- Decoupled equations of motion:

$$Z_s'' + A_s Z_s' + B_s Z_s = S_s$$

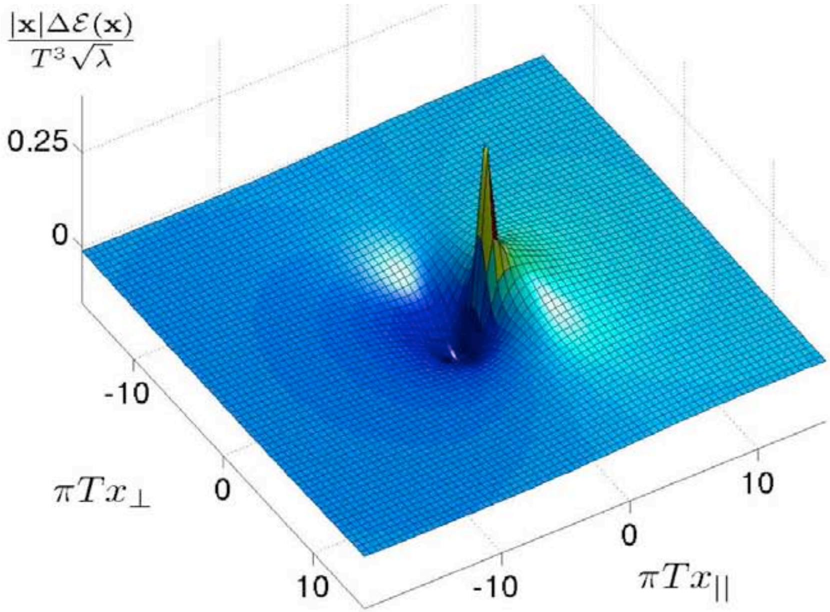
- Variation of on-shell gravitational action:

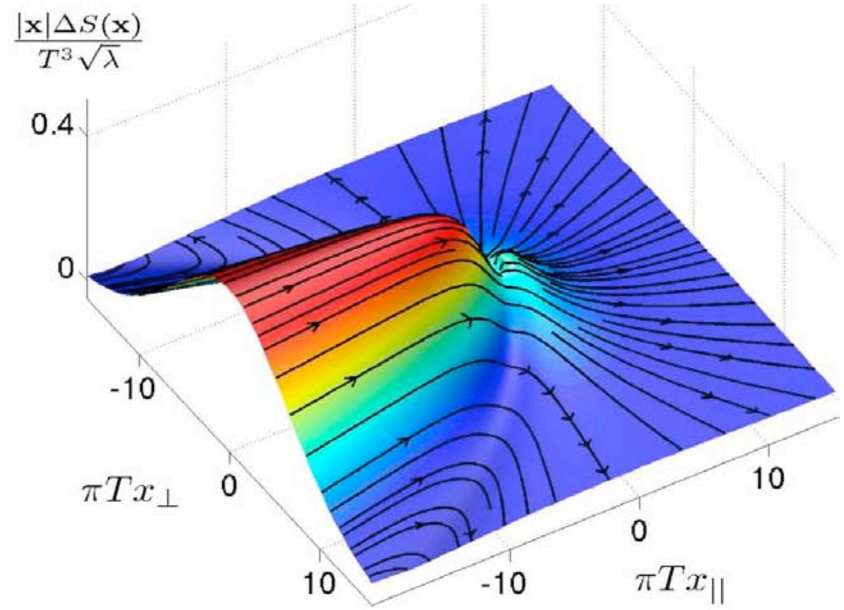
$$\delta S_{\mathrm{G}} = \int_{u=\epsilon} \frac{d^4 q}{(2\pi)^2} \left\{ \sum_s \mathcal{A}_s \delta Z_s^\dagger Z_s + \frac{1}{2} \delta H_{\mu\nu}^\dagger \mathcal{J}^{\mu\nu} + \frac{1}{2} \delta H_{\mu\nu}^\dagger T_{\mathrm{eq}}^{\mu\nu} \right\} + \delta S_{\mathrm{EM}}$$

where $iq_\mu \mathcal{J}^{\mu\nu} = F^\nu$.

Related work: (Gubser, Yarom and co.)

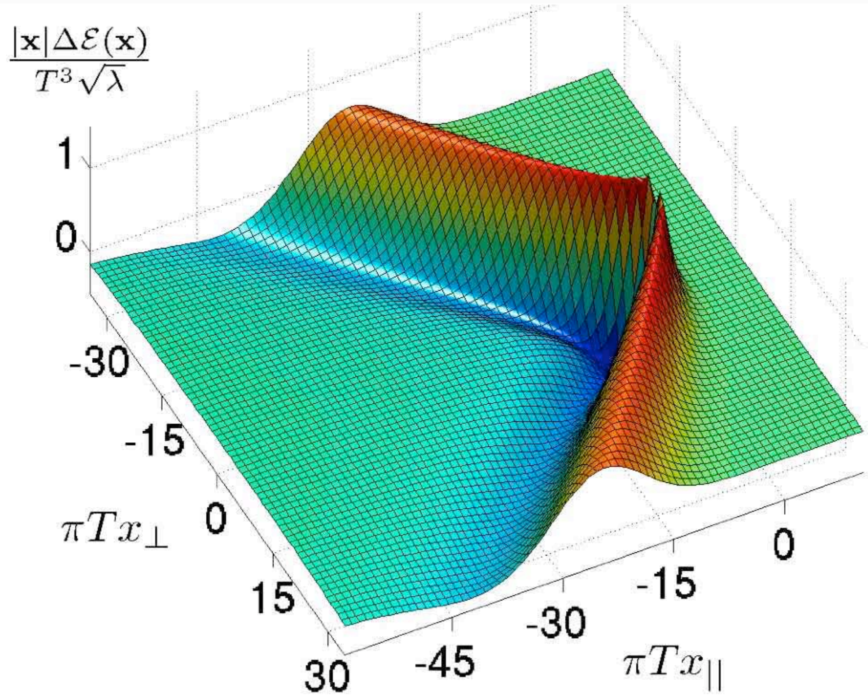
Subsonic Motion

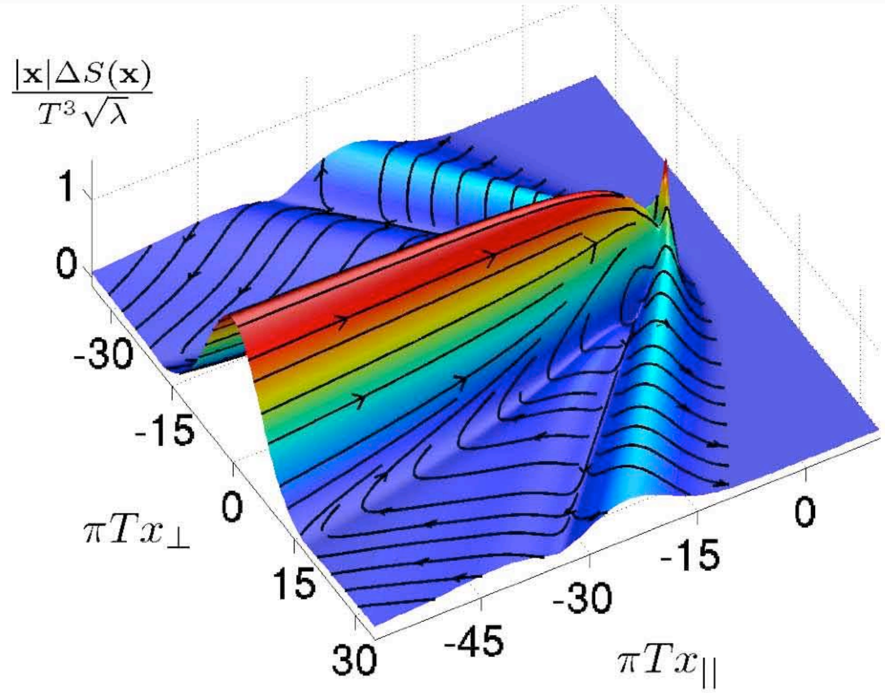




$$v = 1/4$$

Supersonic Motion





$$v = 3/4$$

Clear formation of Mach cone and laminar wake!

Hydrodynamics

$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq}} + \Delta T^{\mu\nu}$$

- In the large N_c limit $T^{\mu\nu}_{\text{eq}} = \mathcal{O}(N_c^2)$ while the perturbation due to the fundamental quark is $\Delta T^{\mu\nu} = \mathcal{O}(N_c^0)$.
- In the large N_c limit the hydrodynamic equations for the perturbation become linear *everywhere*!
  - This occurred in the gravitational calculation too.

Gradient Expansion

Trade $T^{0\mu}$ for fluid velocity u^μ and proper energy density ϵ.

$$\mathcal{E} \equiv \epsilon - \epsilon_{\rm eq}, \qquad \mathcal{P} \equiv p - p_{\rm eq},$$

$$\begin{aligned}
\Delta T^{00}_{\rm hydro} &= \mathcal{E}\,, \\
\Delta T^{0i}_{\rm hydro} &= (\epsilon_{\rm eq} + p_{\rm eq})\, u_i\,, \\
\Delta T^{ij}_{\rm hydro} &= \mathcal{P}\,\delta_{ij} - \eta\left(\nabla_i u_j + \nabla_j u_i - \tfrac{2}{3}\delta_{ij}\nabla\cdot\mathbf{u}\right) \\
&+ \Theta\left(\nabla_i\nabla_j - \tfrac{1}{3}\delta_{ij}\nabla^2\right)\mathcal{E} + \cdots\,,
\end{aligned}$$

Hydrodynamic Equations of Motion

- Equations of Motion follow from $\partial_\mu T^{\mu\nu}(x) = F^\nu(x)\,,$

  where $F^\nu(x) = f^\nu \delta^3(\mathbf{x} - \mathbf{v}t)$
- However, hydrodynamics is not valid in the near zone.
  - Gradients are large in near zone.
  - Non-hydrodynamic degrees of freedom are important!
- Take these issues into account with an effective source:

$$\partial_\mu T^{\mu\nu}(x) = J^\nu(x)\,.$$

Properties of Effective Source

1. Must be local.
   - Should have a gradient expansion in terms of derivatives of delta functions:

$$J^\nu = j^\nu_{(0)} \delta^3(\mathbf{x} - \mathbf{v}t) + \ldots$$

2. Must be consistent with quark energy loss.
   - If quark moves at constant velocity for time Δt, the total four-momentum transferred to the plasma is

$$\Delta t f^\mu = \int d^3x \, T^{0\mu}(\mathbf{x}, t)$$

   - For times much after the quark's motion has ceased this must be computable with hydrodynamics!

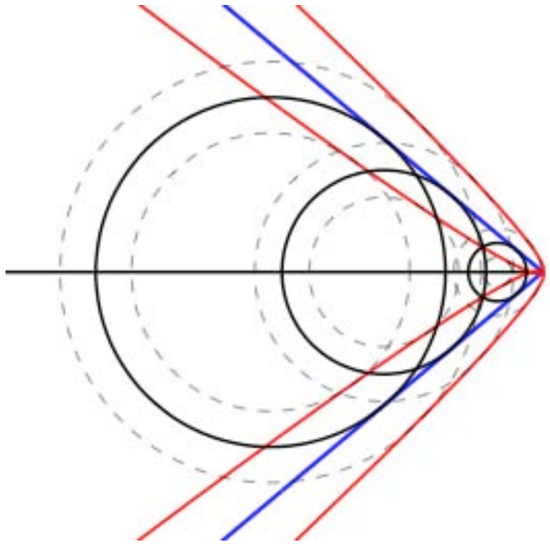
$$j^\nu_{(0)} = f^\nu$$

Hydrodynamic Modes

- Sound mode:

$$(-\partial_t^2 + c_s^2\nabla^2 + \gamma\nabla^2\partial_t)\mathcal{E} = \rho$$

$$(-\partial_t^2 + c_s^2\nabla^2 + \gamma\nabla^2\partial_t)\mathbf{S}_{\text{sound}} = \mathbf{J}_{\text{sound}}$$

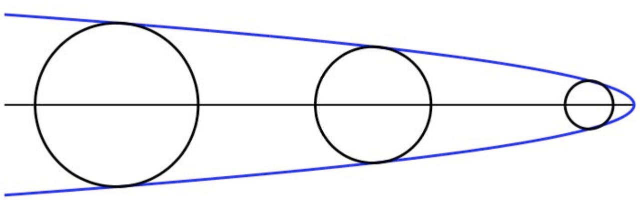


- Diffusion mode:

$$(\partial_t - D\nabla^2)\mathbf{S}_{\text{diffusion}} = \mathbf{J}_{\text{diffusion}}$$

- For strongly coupled SYM

$$\begin{aligned} c_s^2 &= 1/3, \\ \gamma &= 1/3\pi T, \\ D &= 1/4\pi T. \end{aligned}$$

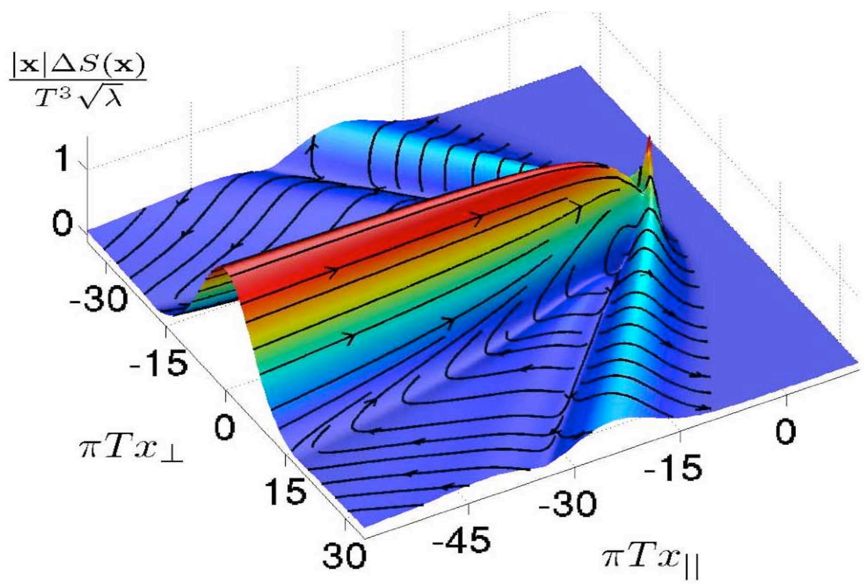


(Son, Policastro and Starinets 2001)

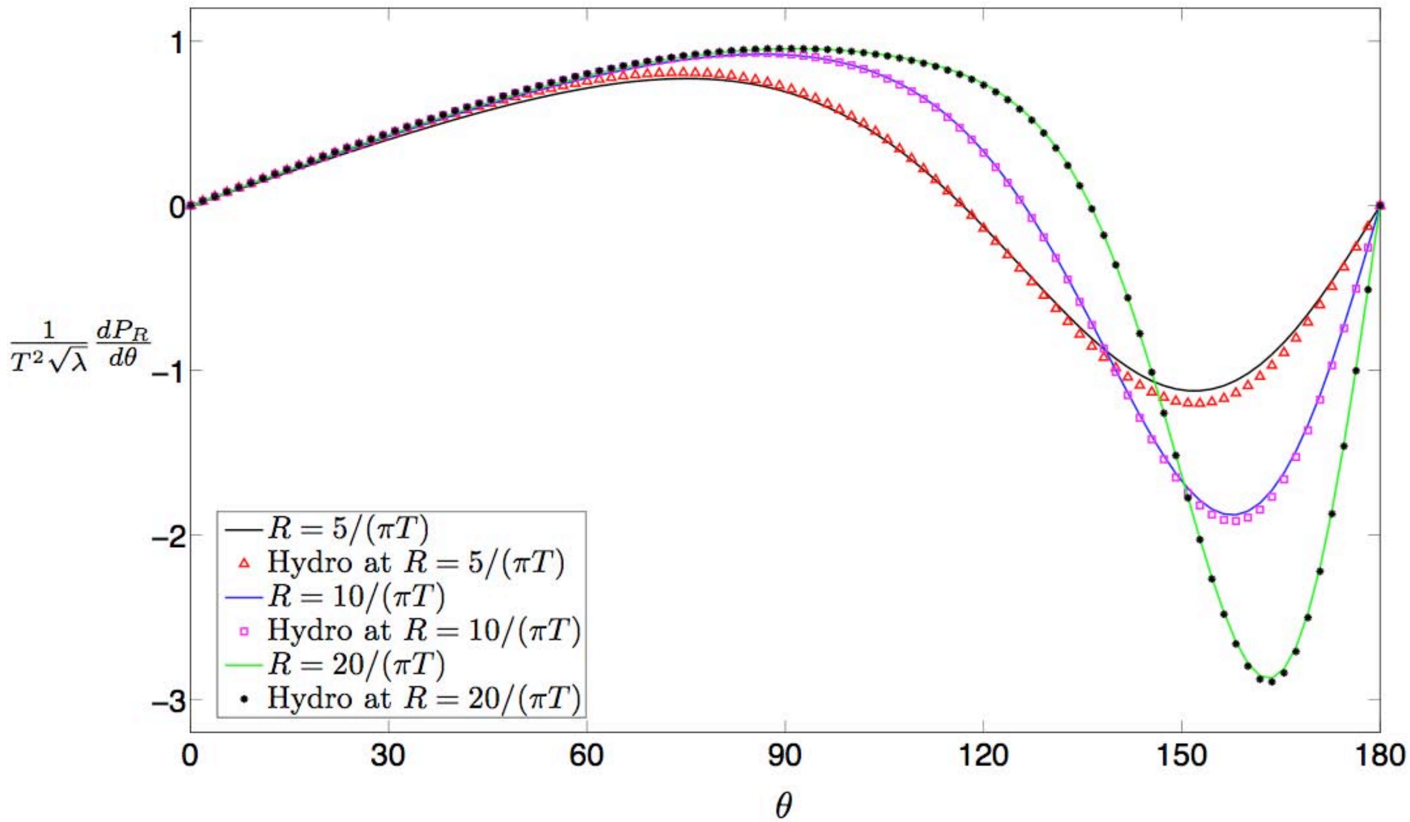
Comparing to Complete Result

Simple quantity which involves both sound and diffusion modes is the angular distribution of power

$$\frac{dP_R}{d\theta} = 2\pi R^2 \sin\theta \left(\cos\theta \Delta S_{||} + \sin\theta \Delta S_{\perp}\right).$$

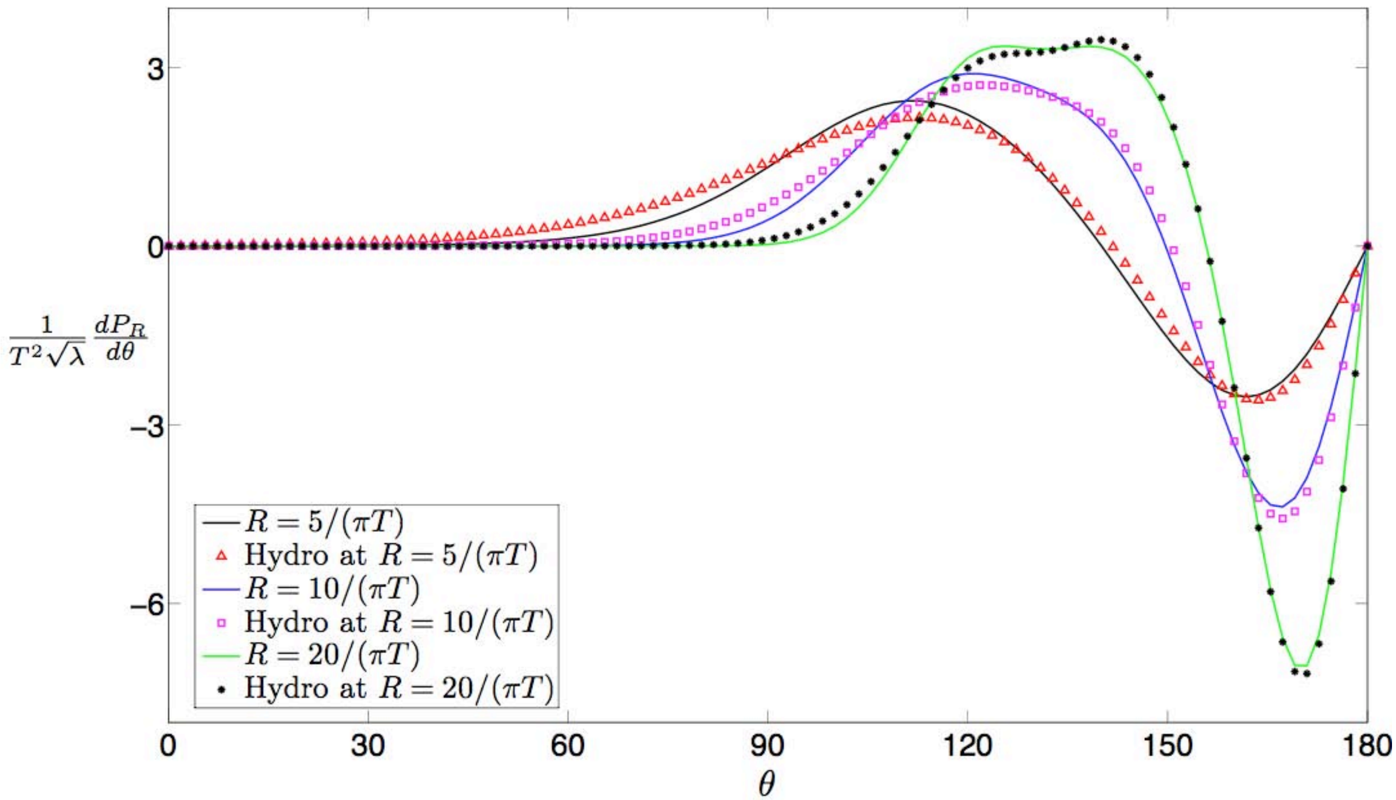


Subsonic Motion



$$v = 1/4$$

Supersonic Motion



$$v = 3/4$$

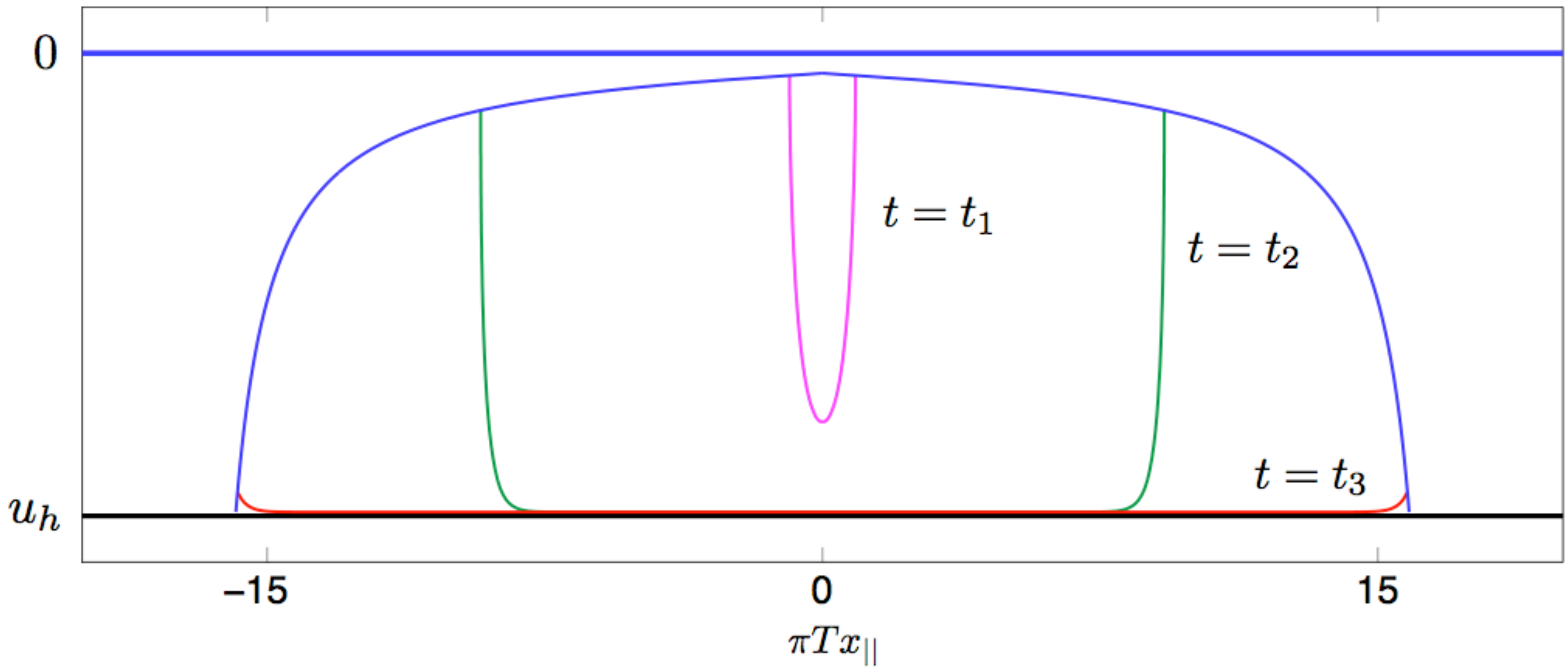
What Do We Learn?

- Hydrodynamics works very well all the way down to $d \gtrsim 1.5/\pi T$.
  - Should be contrasted with weak coupling results.
- Effect of viscosity is important!
  - Neglecting viscosity will yield discontinuities on Mach cone and a diffusion wake with zero width!

$$\mathcal{E} \sim \frac{x_{||}}{\left(x_{||}^2 + (1 - 3v^2)\, x_{\perp}^2\right)^{\frac{3}{2}}} \theta\left(-x_{||} - x_{\perp}\sqrt{3v^2 - 1}\right)$$

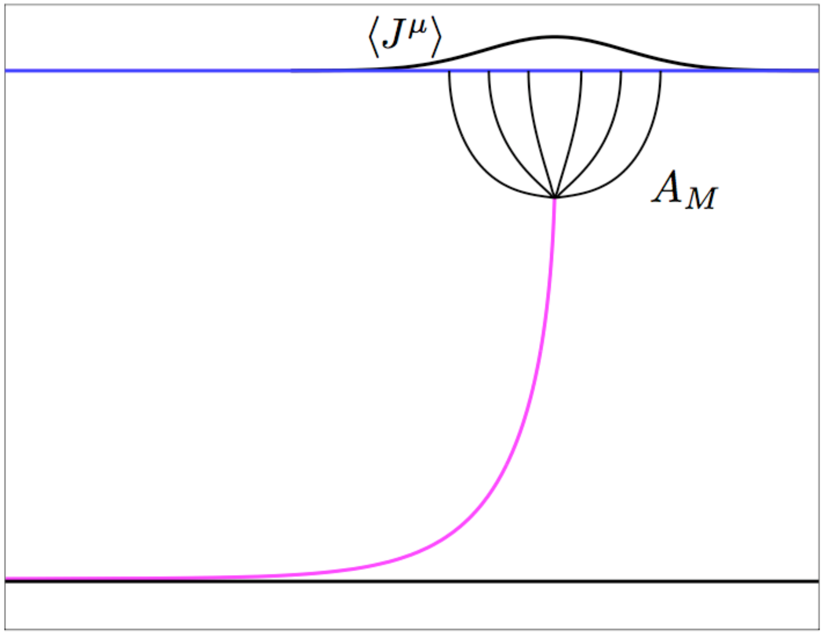
$$\mathbf{S}_{\text{diffusion}} \sim \delta^2(\mathbf{x}_{\perp})\theta(-x_{||})$$

Light Quarks



- How is the baryon density on the boundary correlated with endpoint motion?
- For given quark energy, how far does the quark propagate through the plasma?

Boundary Baryon Density



•Curved space Maxwell

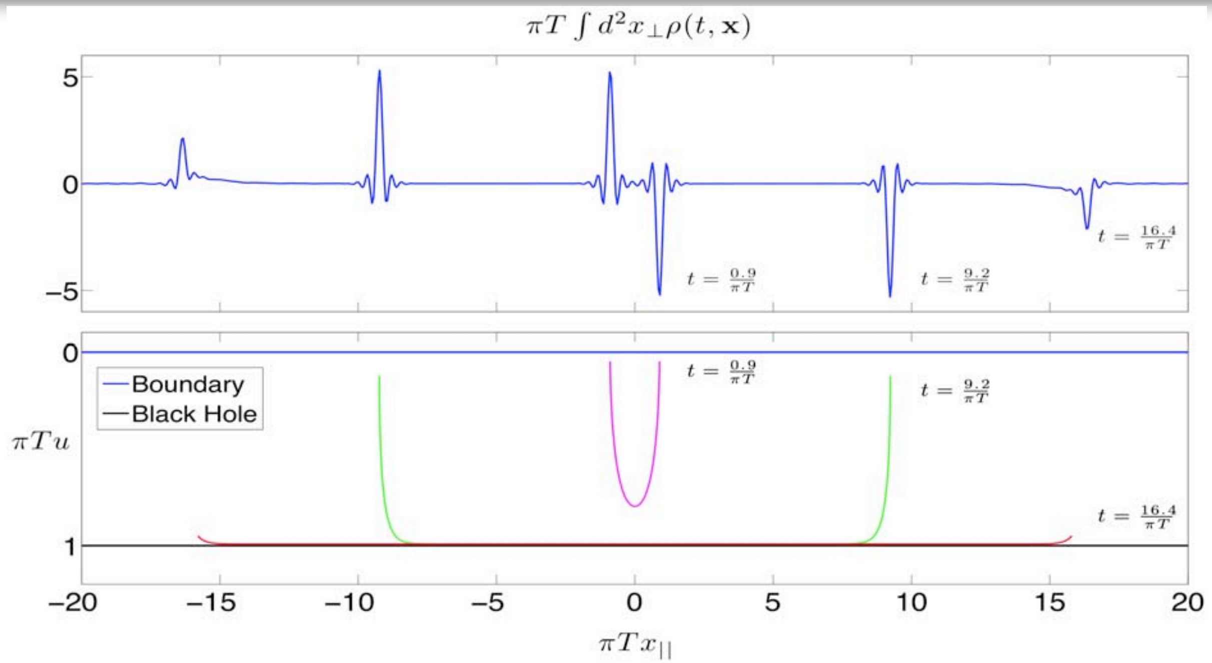
$$D_M F^{MN} = j^N.$$

•Boundary behaves as a conductor.

•Boundary baryon current

$$\langle J^\mu(x) \rangle = \lim_{u \to 0} \sqrt{-g(u)} F^{5\mu}(x, u).$$

Boundary Baryon Density II



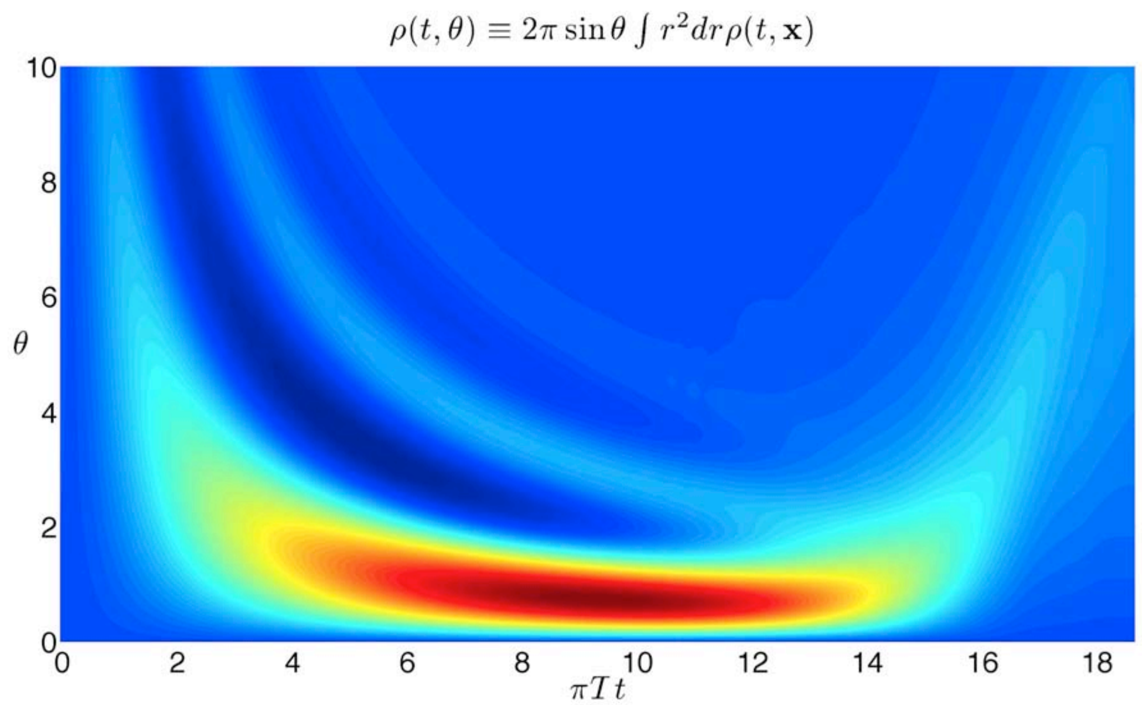
•Center of charge closely follows endpoint motion.

•Total distance traveled can by made arbitrarily large.

$\Rightarrow$ Good quasiparticles.

•Explicitly see the process of thermalization of baryon density.

Angular Distribution



Intermediate time behavior:

$$\langle \theta \rangle \approx \frac{\pi}{2\gamma}$$

Asymptotic late time, long wavelength behavior:

$$\rho(t,\mathbf{x}) = 2\Delta x \frac{\partial G_{\text{diffusion}}(t,\mathbf{x})}{\partial x_{||}}$$

Future Directions

- Light quark energy loss rate and penetration length.
- Light quark wake.
  - Does hydro work well at short distances?
- Jets at zero temperature.
  - Can zero temperature jets in SYM shine light on QCD?

Conclusions

- Using gauge/string duality, we computed stress tensor of a heavy quark moving through a strongly coupled SYM plasma.
  - Calculation valid on all length scales.
  - The formation of sound and diffusion modes was clearly evident.
- We compared the complete result to viscous hydrodynamics and found remarkable agreement at distances down to $d \gtrsim 1.5/\pi T$ from the quark.
- We computed the baryon density of a light quark and found a highly focused jet.
  - Explicitly see process of thermalization of baryon density.