What the Inflaton might tell us about RHIC

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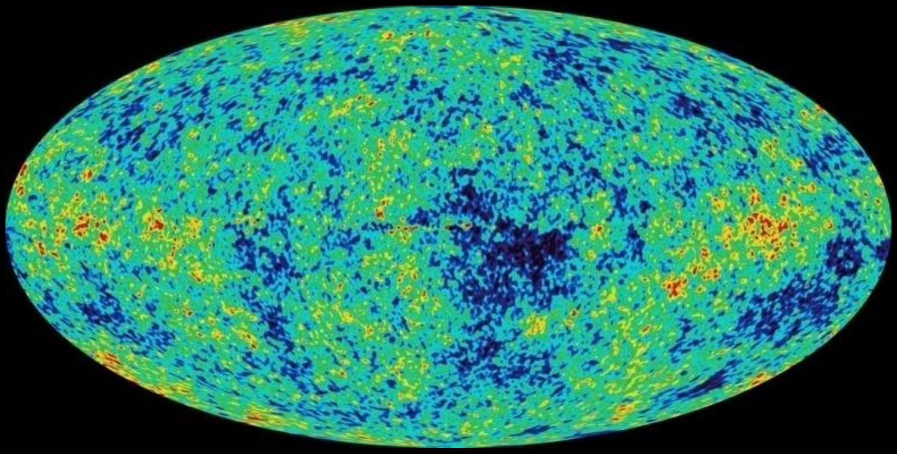
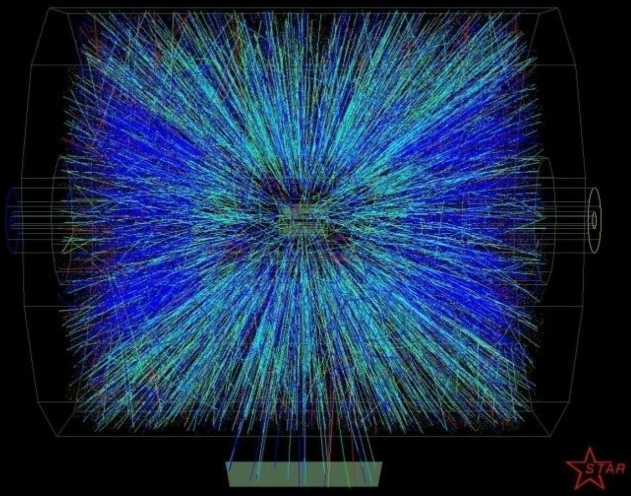




Content

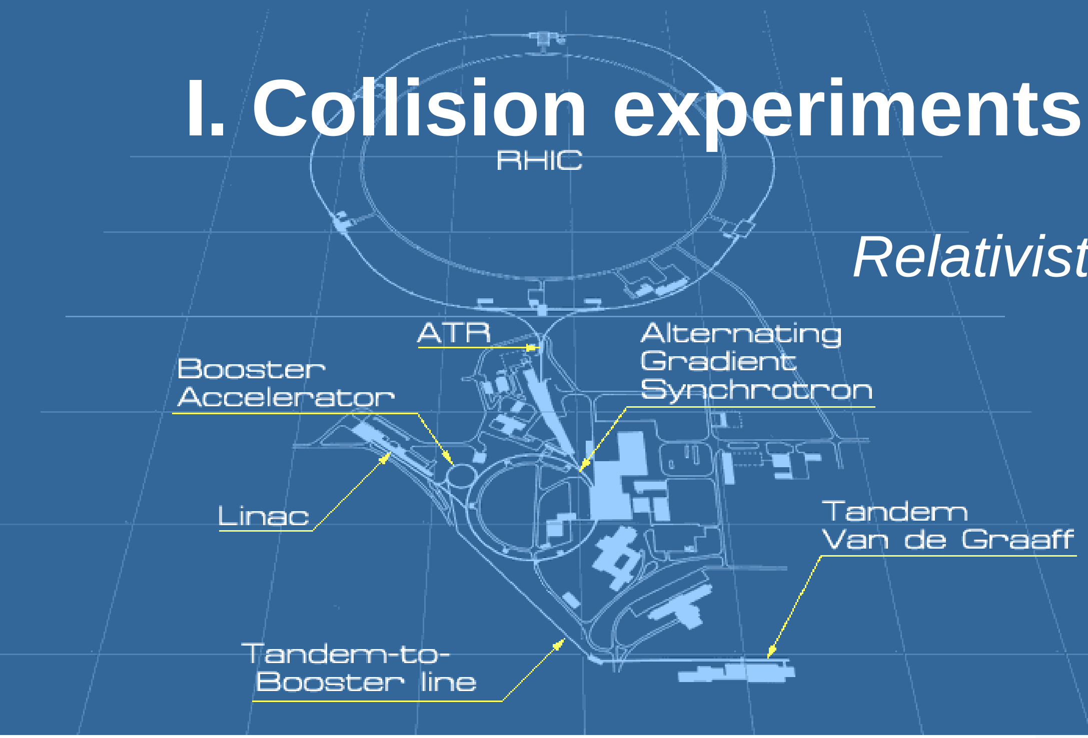
I. Collision experiments of heavy nuclei

II. Heating the universe after inflation

I & II:

- Instabilities and fast thermalization?
- Non-thermal fixed points: effective weak coupling
  in strongly correlated systems far from equilibrium

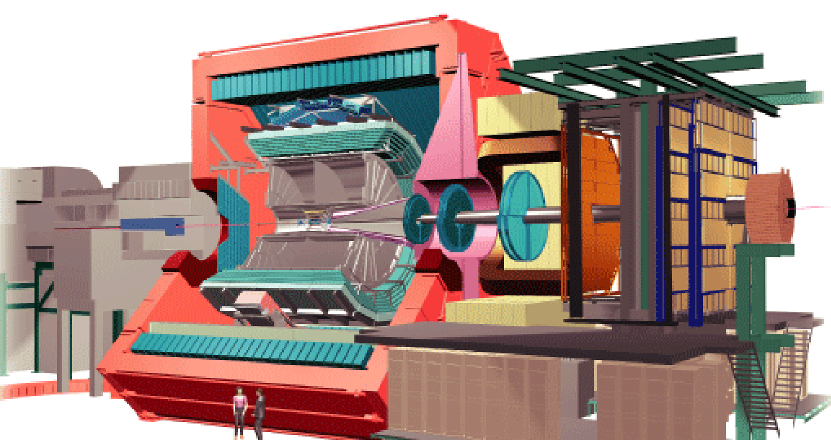
I. Collision experiments of heavy nuclei

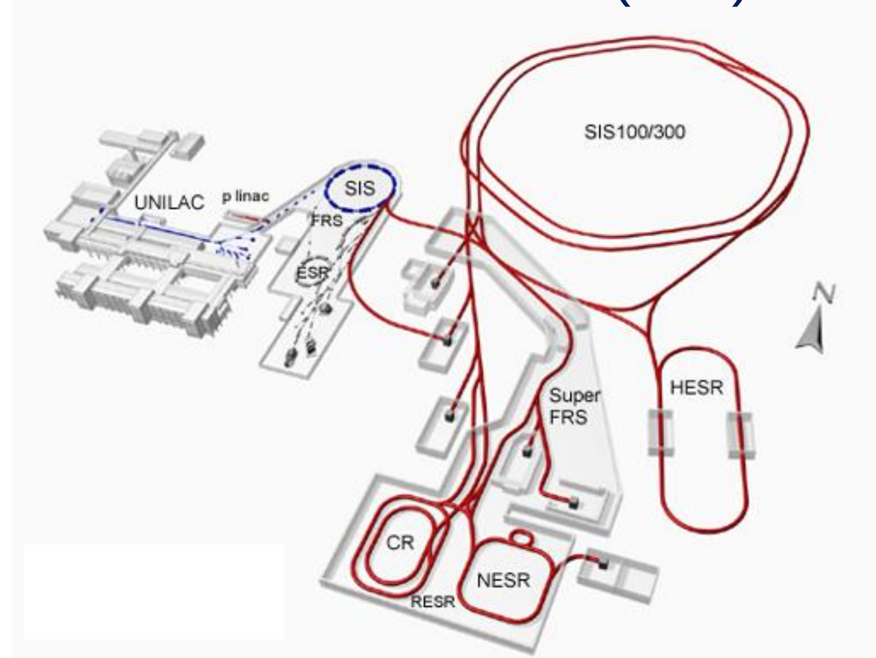


Relativistic Heavy Ion Collider (BNL)

Facility for Antiproton and Ion Research (GSI)

Large Hadron Collider (CERN)

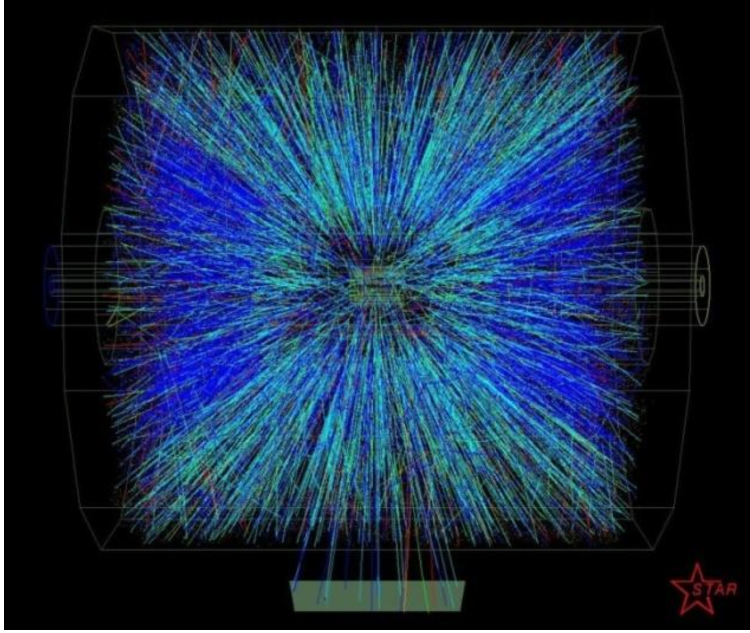




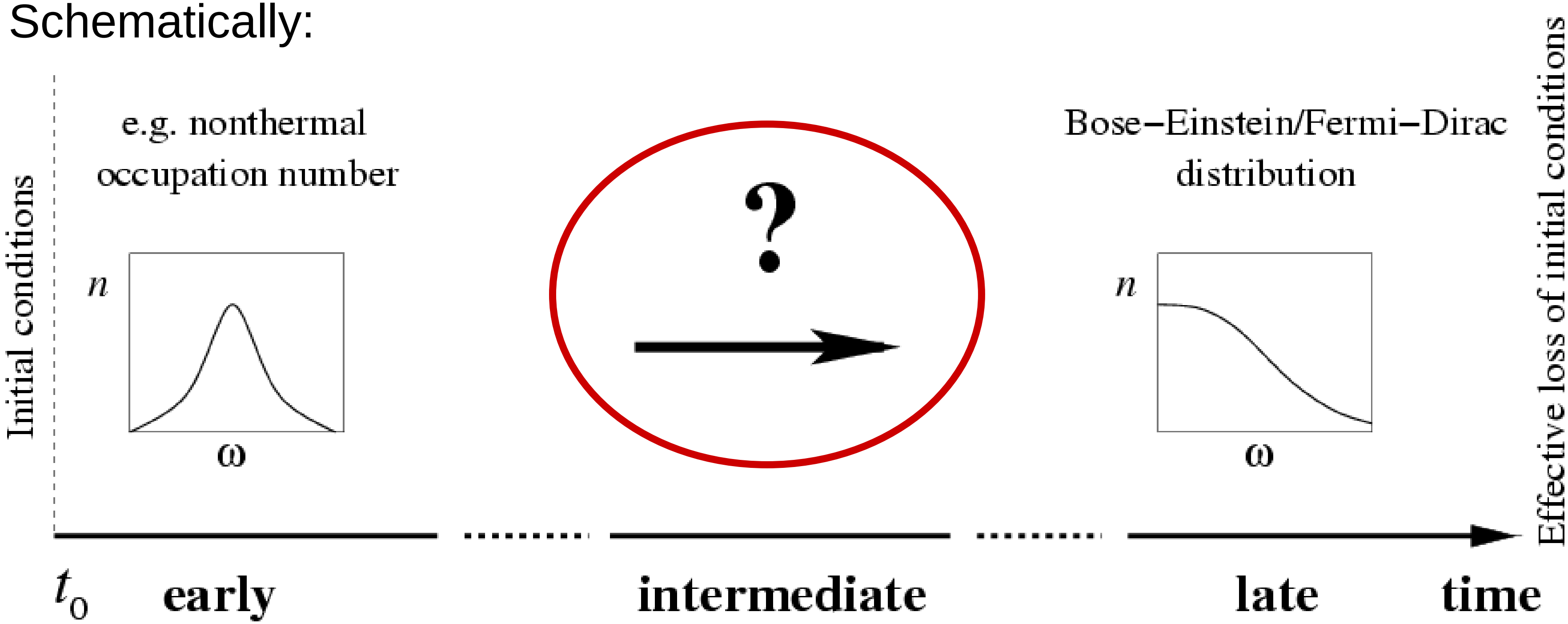
Nonequilibrium dynamics

Relativistic heavy-ion collisions explore strong interaction matter starting from a transient *nonequilibrium* state

Thermalization process?



Schematically:



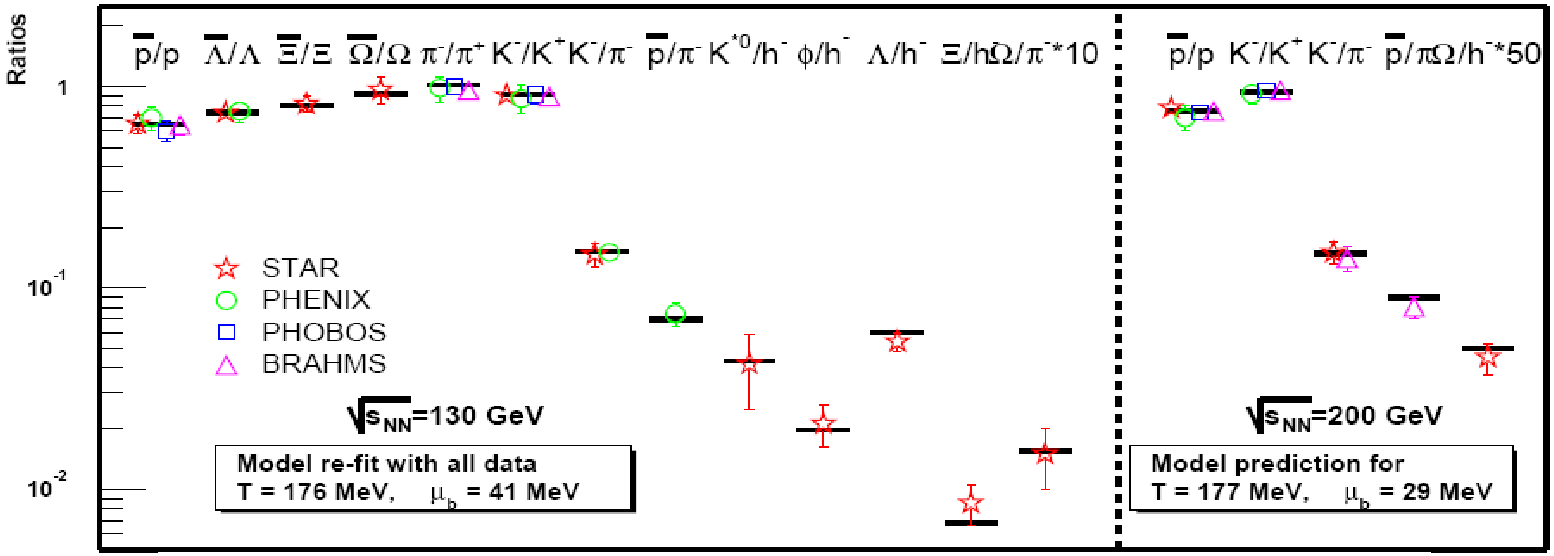
- Thermalization after $\gtrsim$ **10 fm/c?**

Properties of the *equilibrium phase diagram* of QCD?

RHIC: Measured relative particle abundancies consistent with

$$T \sim 10^{12}\ \mathrm{K} \sim 200\ \mathrm{MeV} \sim 1\ \mathrm{fm}^{-1}$$

from fit to statistical model … Braun-Munzinger, Redlich, Stachel

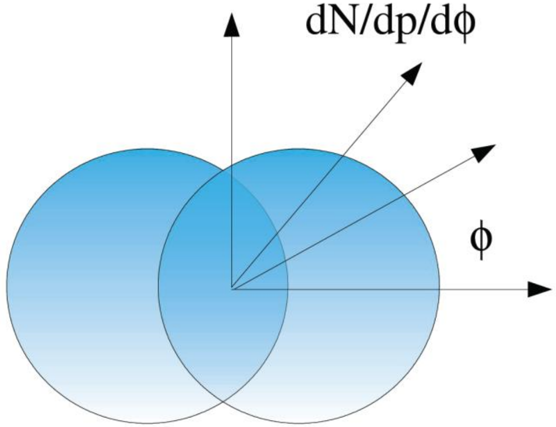


- Theoretical justification for *hydrodynamics after* $\lesssim$ **1 fm/c?**

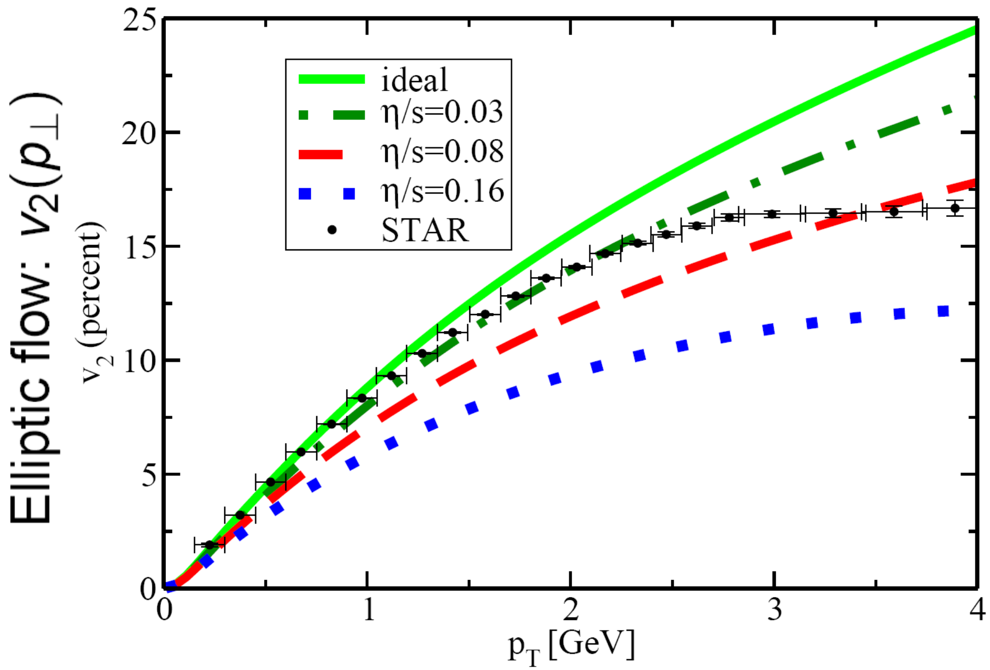
Early local thermal equilibrium ?

Early hydrodynamics

Hydrodynamics ‘works‘ from **∼ 1 fm/c !** Kolb, Heinz, *QGP3* (2004) 634; ...



$$\frac{dN}{dp_\perp d\phi dy} = \langle \frac{dN}{dp_\perp d\phi dy} \rangle_\phi \left(1 + 2v_2(p_\perp)\cos(2\phi) + \ldots\right)$$



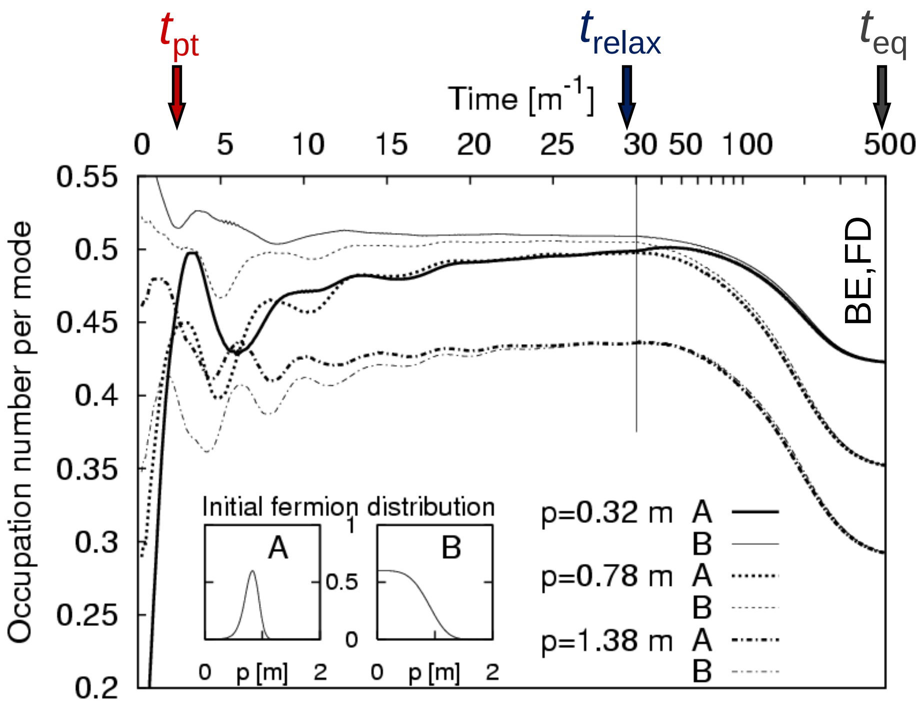
P. Romatschke, U. Romatschke, *PRL* 99 (2007) 172301

⇒ almost ideal hydrodynamics for $p_T \lesssim$ 1-2 GeV

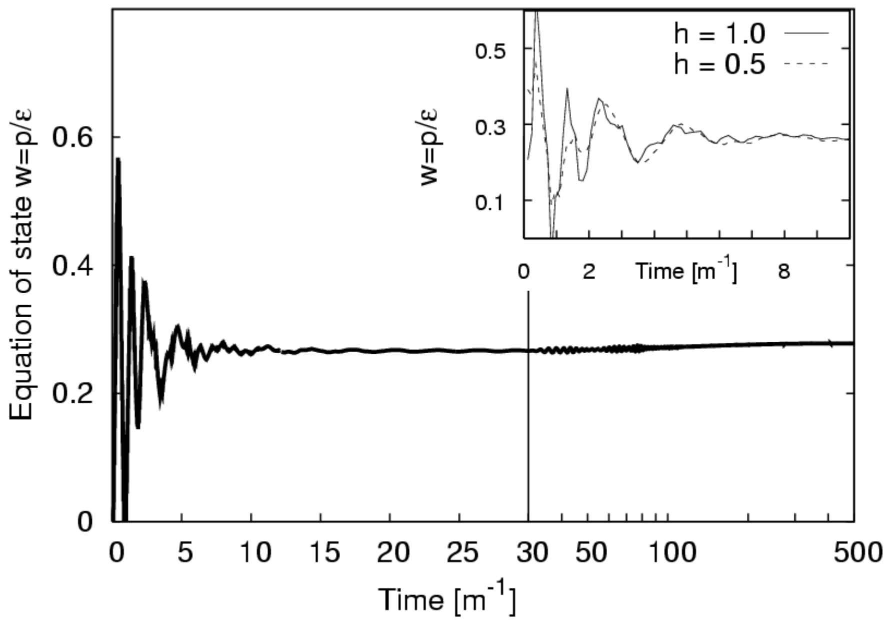
What are the essential assumptions for ideal fluid hydrodynamics?

• **Equation of state** relating pressure p to energy density ε

E.g. $SU_L(2) \times SU_R(2)$ *Yukawa model* in 3+1d with couplings $\sim \mathcal{O}(1)$, isotropy:



e.g.
$m \sim T = 200\text{–}700$ MeV $\simeq 1\text{–}3.5$ fm^{-1}



'Prethermalization' (dephasing) time for EOS: $T\, t_{pt} \sim \mathcal{O}(1)$

Consistent with early use of hydrodynamics –
far from equilibrium

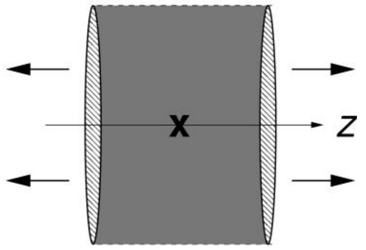
Berges, Borsanyi, Wetterich, *PRL* 93 (2004) 142002

- **Isotropy** of the stress tensor in the local fluid rest frame

$$T_{ij} \simeq P\,\delta_{ij}$$

Relativistic heavy-ion collisions:

anisotropy $\rightarrow$ $T_{xx} \sim T_{yy} \gg T_{zz}$



Isotropization time t_{iso}? In the absence of nonequilibrium instabilities:

$$t_{iso} \sim t_{relax} \sim \mathcal{O}(1/g^4 T)_{|\text{weak coupling QCD near equilibrium}}$$

Plasma instabilities: exponential growth of T_{zz} $\rightarrow$ isotropization?

Weibel, *PRL* 2 (1959) 83; Mrowczynski, *PRC* 49 (1994) 2191; ...

$$t_{iso} \overset{?}{\sim} \mathcal{O}(1/gT)_{|\text{weak coupling, } \mathcal{O}(1) \text{ anisotropy}}$$

Understanding early use of hydrodynamics means understanding fast isotropization due to plasma instabilities!

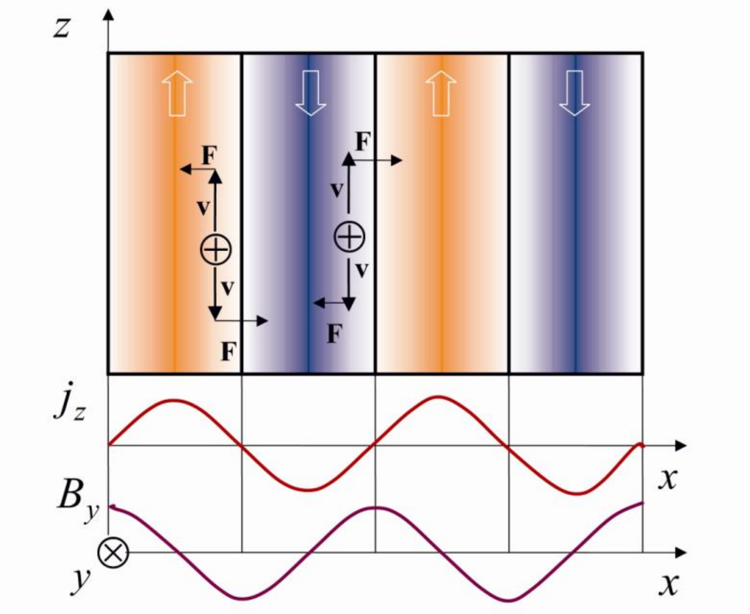
Arnold, Moore, Yaffe, *PRL* 94 (2005) 072302

Nonequilibrium instabilities

Large class of possible instabilities:

Spinodal, Parametric, Plasma (Weibel) …

E.g. Weibel instability in electrodynamics:



Initial fluctuating current:

$\mathbf{j}(x) = j \cos(kx)\, \mathbf{e}_z$

$\Rightarrow$ generated magnetic field:

$\mathbf{B}(x) = j \sin(kx)/k\, \mathbf{e}_y$

$\Rightarrow$ Lorentz force acts such that current grows:

$\mathbf{F}(x) = q\, \mathbf{v} \times \mathbf{B} = -\, q\, v_z\, j \sin(kx)/k\, \mathbf{e}_x$

$\Rightarrow$ B-field grows, etc.

Fast isotropization/thermalization due to instabilities?

Mrowczynski ’94; Romatschke, Strickland ‘03; Arnold, Lenaghan, Moore ‘03, Mrowczynski, Rebhan, Strickland ‘04; Rebhan, Romatschke, Strickland ’05; Dumitru, Nara ‘05; Romatschke, Venugopalan ‘06; Schenke, Strickland, Greiner, Thoma ‘06; Dumitru, Nara, Strickland ’07; Bödeker, Rummukainen ’07; Berges, Scheffler, Sexty ’08; Mrowczynski ’08 …

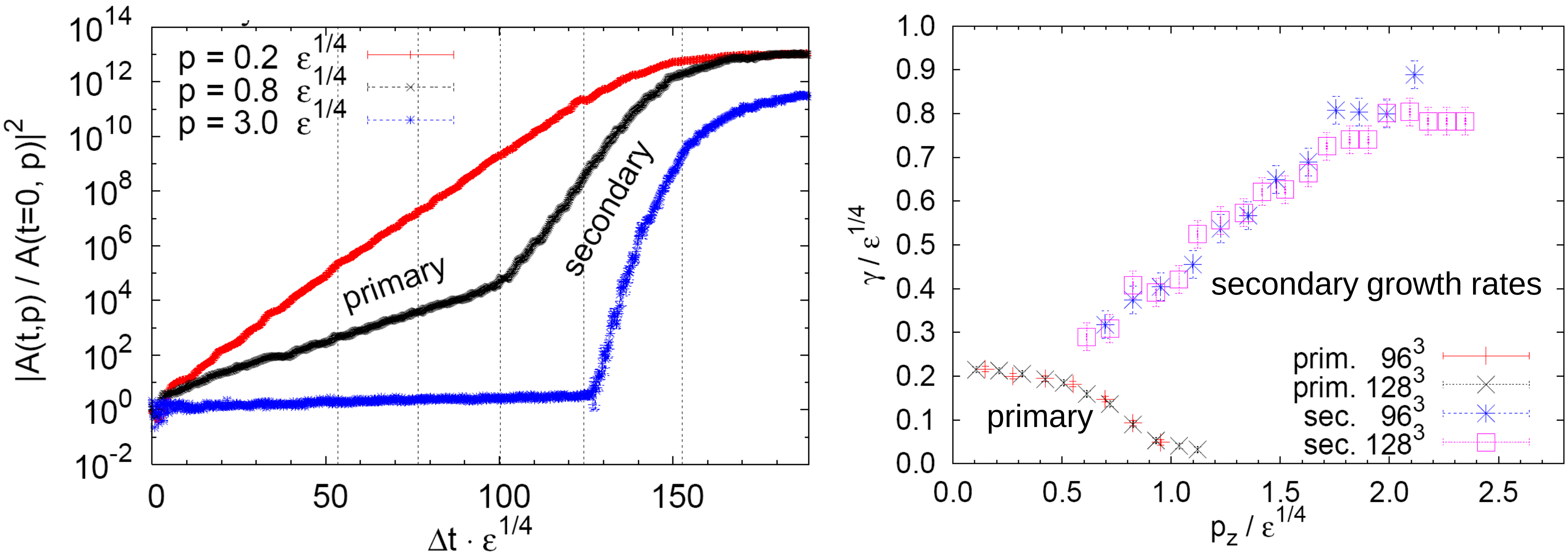
Characteristic time scales

A) ‘Soft‘ classical gauge fields + ‘hard‘ classical particles

Arnold, Moore, Yaffe; Rebhan, Romatschke, Strickland; Dumitru, Nara, Strickland; Bödeker, Rummukainen

B) Classical-statistical gauge field evolution (here)

Romatschke, Venugopalan; Berges, Scheffler, Sexty



Inverse primary growth rate:

$\Rightarrow$ **$1/\gamma_{max} \simeq$ 1.1 fm/c** for $\varepsilon = 30$ GeV/fm^3

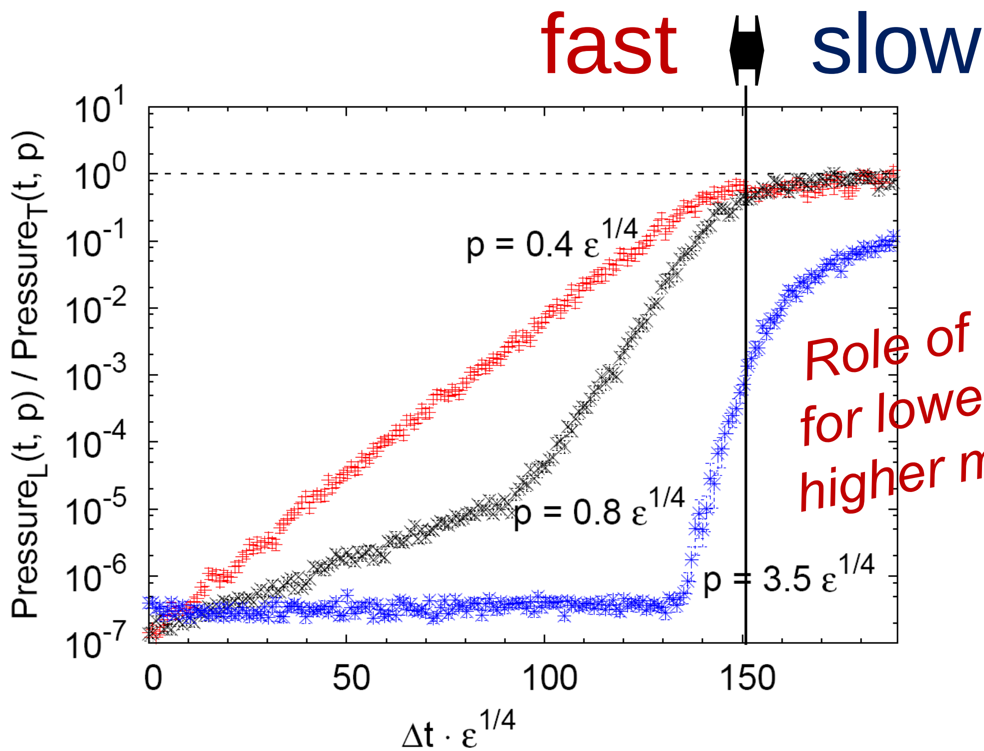
What energy density would be required to get $1/\gamma_{max} \simeq$ 0.1 fm/c ?

$\Rightarrow$ ε = 300 TeV/fm^3 (!)

Bottom-up isotropization of pressure

Spatial Fourier transform of the energy-momentum tensor $T^{\mu\nu}(x)$:

$P_L(t,\mathrm{p})$ for $\mu=\nu=3$, $P_T(t,\mathrm{p})$ from transversal components



Role of quantum corrections for lower occupied modes at higher momenta?

For what p is $P_L(p)/P_T(p) \gtrsim 0.6$ at end of exponential growth? $\Rightarrow p_z \lesssim 1.4\ \varepsilon^{1/4}$

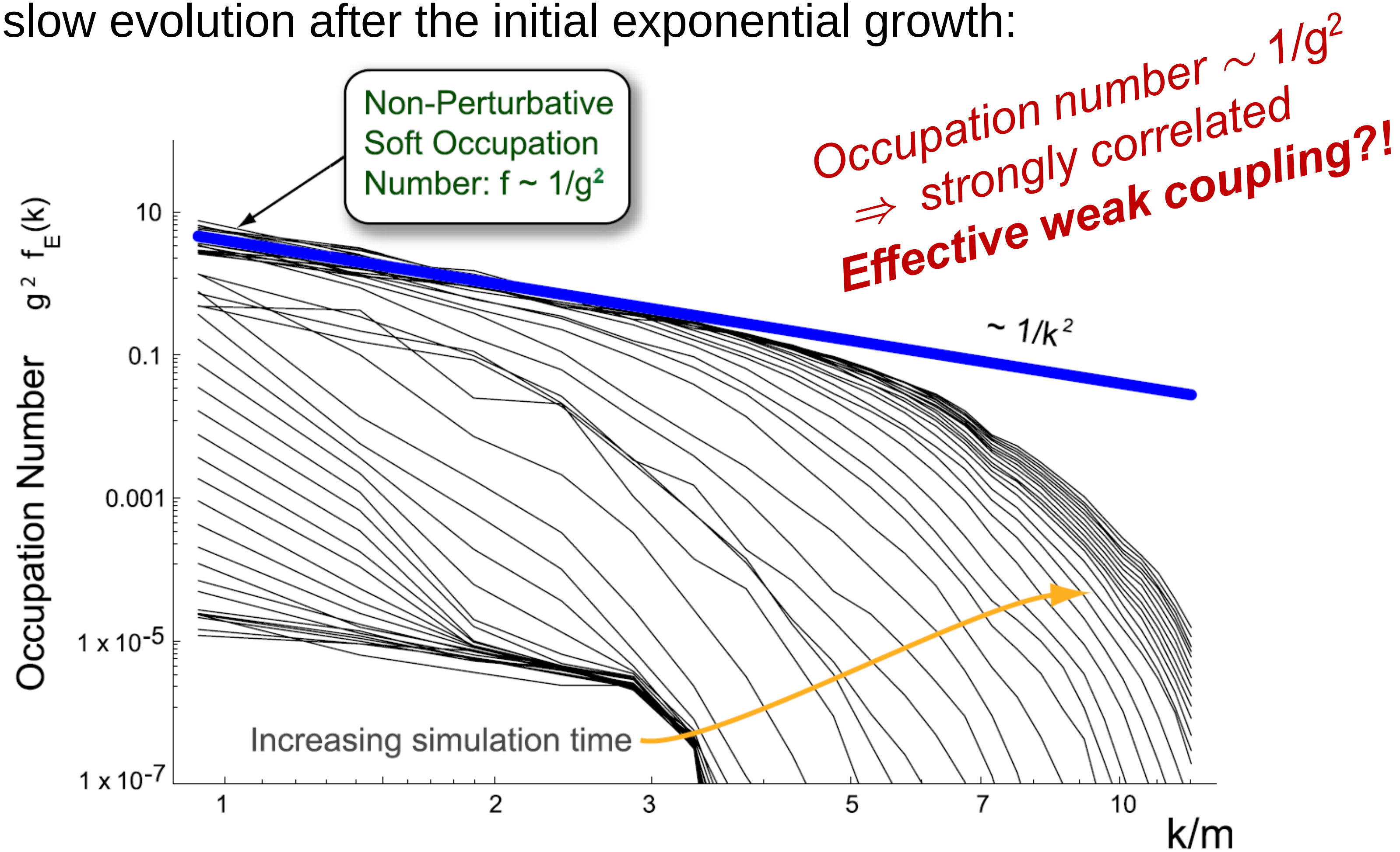
$p_z \lesssim 1$ GeV for $\varepsilon = 30\ \mathrm{GeV/fm}^3$

'enough' for hydro?

BUT: Isotropization time of dominant higher momenta consistent with 'infinity'

Evolution towards turbulent-type spectrum?

Very slow evolution after the initial exponential growth:



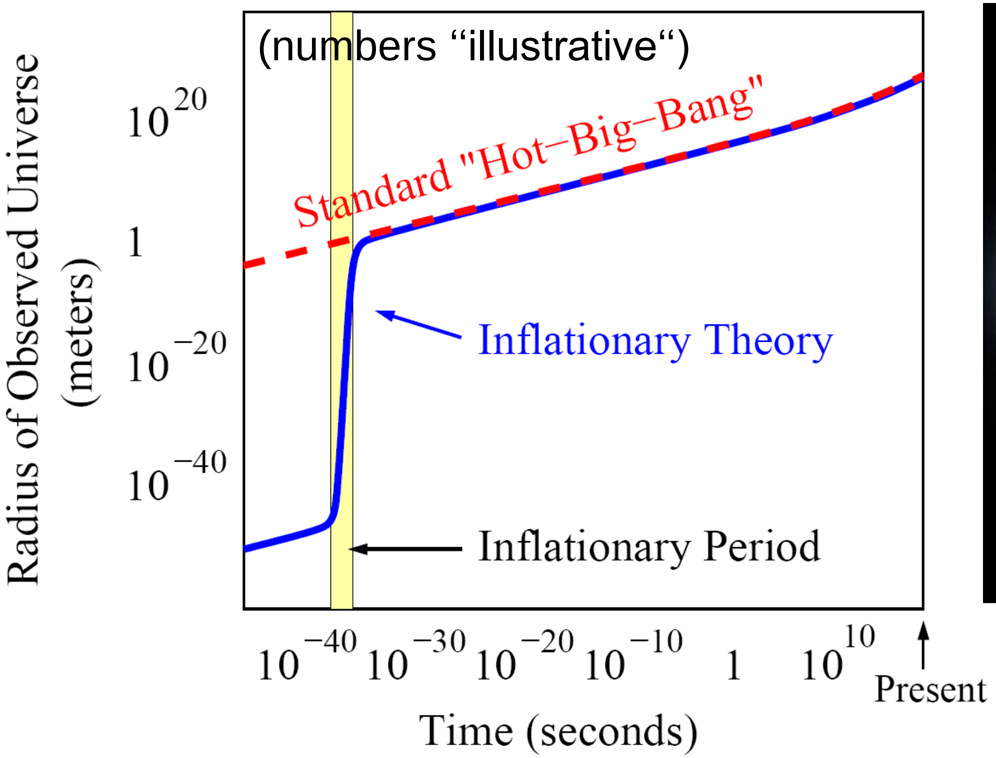
Strickland, *J Phys G*34 (2007) S429

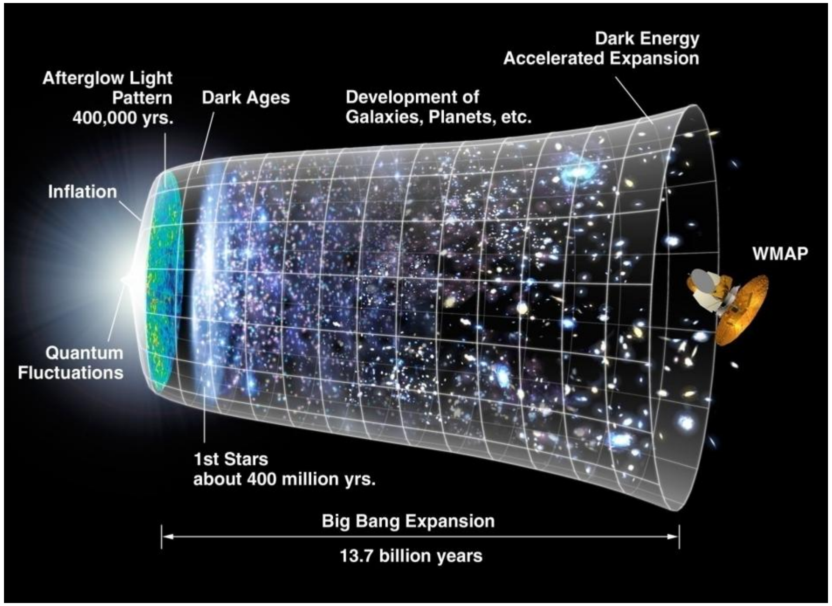
Arnold, Moore, *PRD* 73 (2006) 025006

See, however: Bödeker, Rummukainen, JHEP 0707 (2007) 022 (Vlasov equations)

II. Heating the Universe after inflation: scalar inflaton dynamics as a *quantum* example

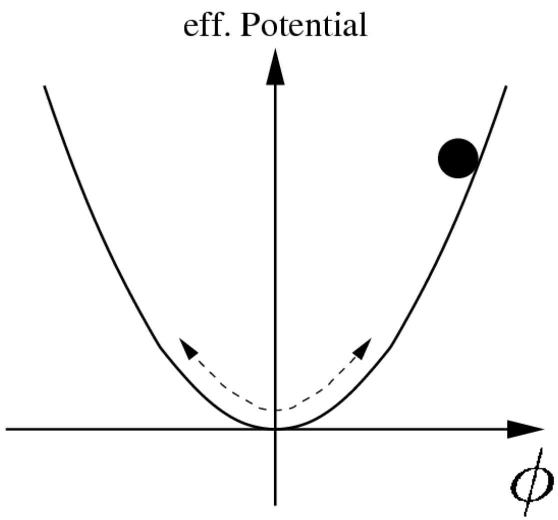
Schematic evolution:





- Energy density of matter ($\sim a^{-3}$) and radiation ($\sim a^{-4}$) decreases
- Enormous heating after inflation to get 'hot-big-bang' cosmology!

Parametric resonance preheating



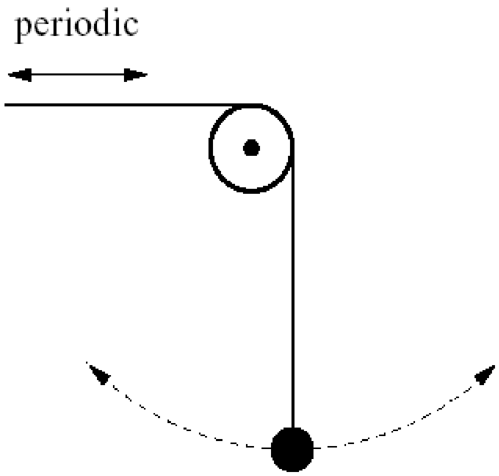
Kofman, Linde, Starobinsky, *PRL* 73 (1994) 3195

E.g. scalar $\lambda\Phi^4$ inflaton dynamics:

- Field expectation value $\phi = \langle\Phi\rangle$
- Fluctuation $F \sim \langle\{\Phi,\Phi\}\rangle$

parametric resonance: $F(t) \sim e^{\gamma t}$

Classical oscillator analogue (exact early): $\omega(t) \leftrightarrow \phi(t)$, $x(t) \leftrightarrow F(t)$



$\ddot{x} + \omega^2(t)x = 0\,, \quad \omega(t+T) = \omega(t)$

invariant under $t \to t + T$:

$\rightsquigarrow \quad x(t + T) = c\,x(t)$, i.e.

$x(t) = c^{t/T}\Pi(t)\,, \quad \Pi(t+T) = \Pi(t)$

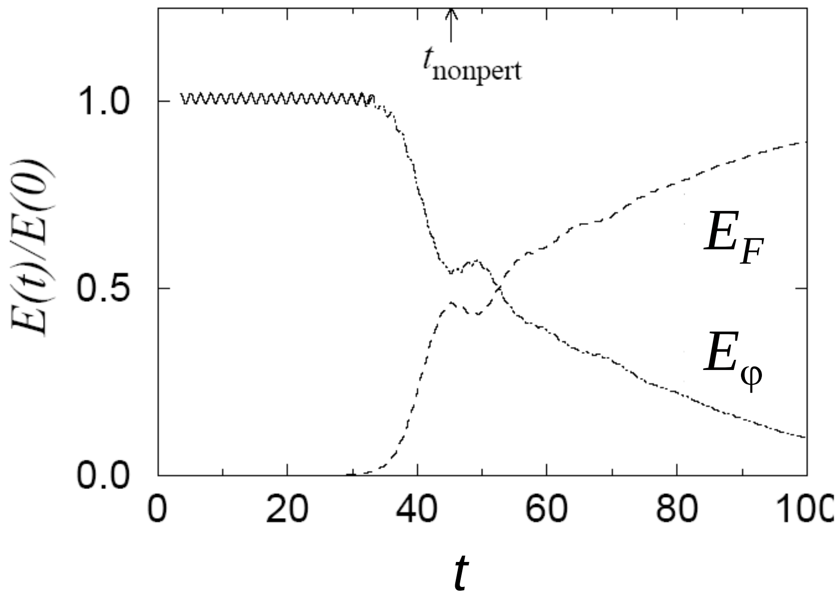
for real $c > 1$:

instability with *exponential growth!*

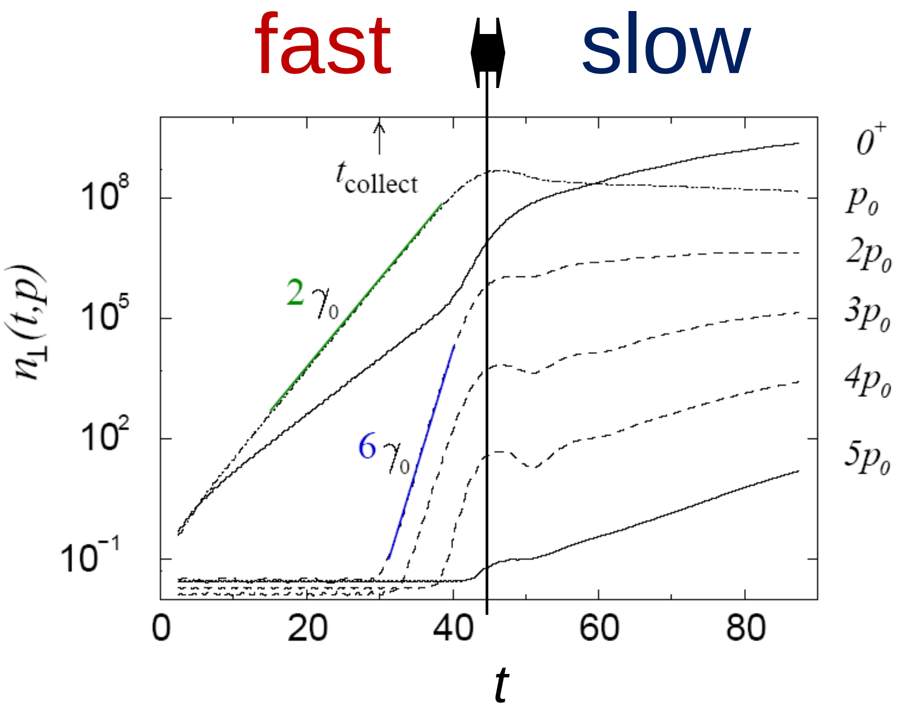
Inflaton dynamics in the quantum theory

<u>(N=4)-component $\lambda(\Phi_a\Phi_a)^2$ quantum field theory</u>

Energy:



Occupation numbers:



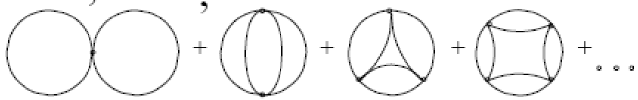
(Approximation: 2PI 1/N to NLO)

Berges, Serreau, *PRL* 91 (2003) 111601

Tachyonic preheating: Arrizabalaga, Smit, Tranberg, *JHEP* 0410 (2004) 017 time

nonperturbative regime: quasistationary evolution

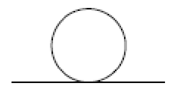
(IV) $\sim N, N^0$; $\sim N^0$



$\sim O(\lambda^{-1})$

$t_{nonpert} \sim \ln(\lambda^{-1})/2\,\gamma_0$

$F_\perp \sim O(N^0 \lambda^{-1})$



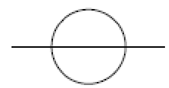
$\sim O(N^0 \lambda^0)$

nonlinear regime: source induced amplification

(III)

$t_{collect} \sim 2\, t_{nonpert}/3 + \ln(N)/\,6\,\gamma_0$

$F_\perp \sim O(N^{1/3} \lambda^{-2/3})$ for $N \lesssim \lambda^{-1}$



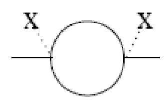
$\sim O(N^0 \lambda^0)$

rate: $6\,\gamma_0$ for $F_\perp(p \neq p_0)$

(II)

$t_{source} \sim t_{nonpert}/2$

$F_\perp \sim O(N^0 \lambda^{-1/2})$



$\sim O(N^0 \lambda^0)$

rate: $4\,\gamma_0$ for $F_\parallel(p \lesssim 2p_0)$

linear regime: parametric resonance

(I)

$F_\perp(t,t;p_0) \sim \exp(2\,\gamma_0\, t)$

rate: $2\,\gamma_0$

$t = 0$, $F_\perp \sim O(N^0 \lambda^0)$

$\phi \sim (N/\lambda)^{1/2}$

slow

Nonperturbative: saturated occupation numbers $\sim 1/\lambda$

$\rightarrow$ *universal:* λ drops out

$\rightarrow$ *all processes* $\mathcal{O}(1)$

Effective weak coupling!

fast

Nonlinear – perturbative: occupation numbers $< 1/\lambda$

secondary growth rates $c\,(2\gamma_0)$ with $c = 2,3,\ldots$

Classical/linear: primary growth rate

slow:

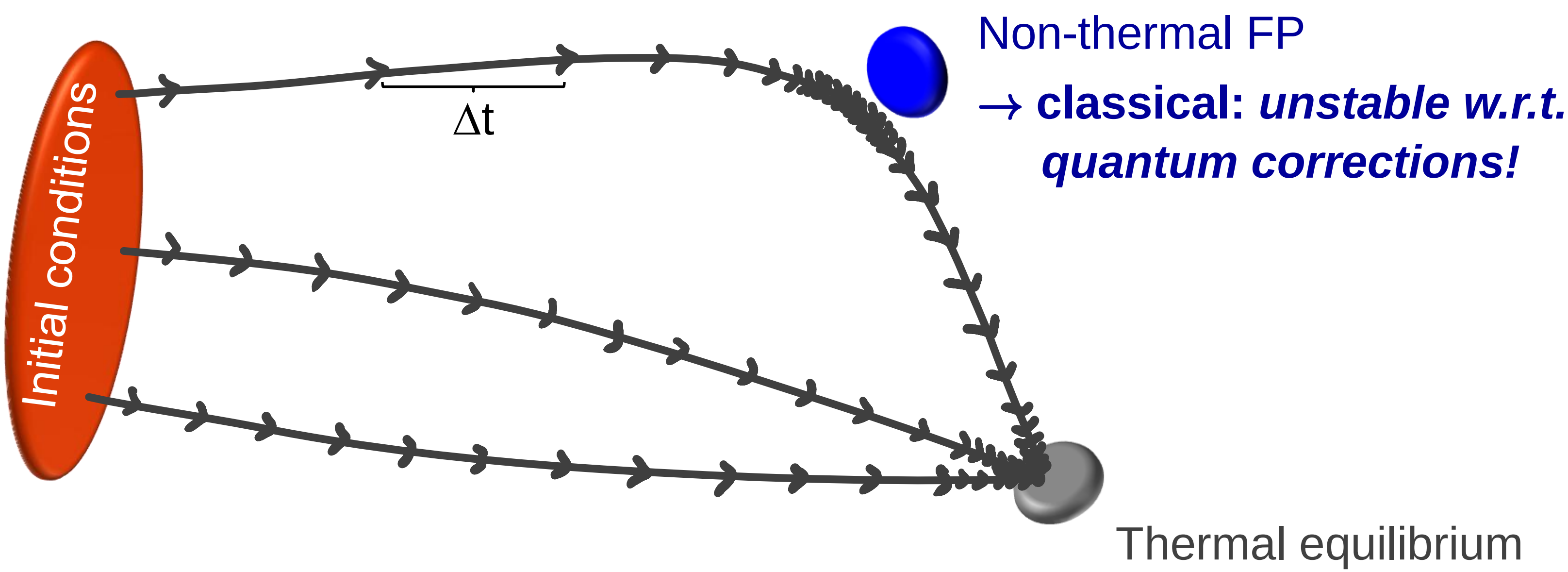
Non-thermal fixed points

Time-translation invariant non-thermal solutions?

No, thermal equilibrium unique (H-theorem)

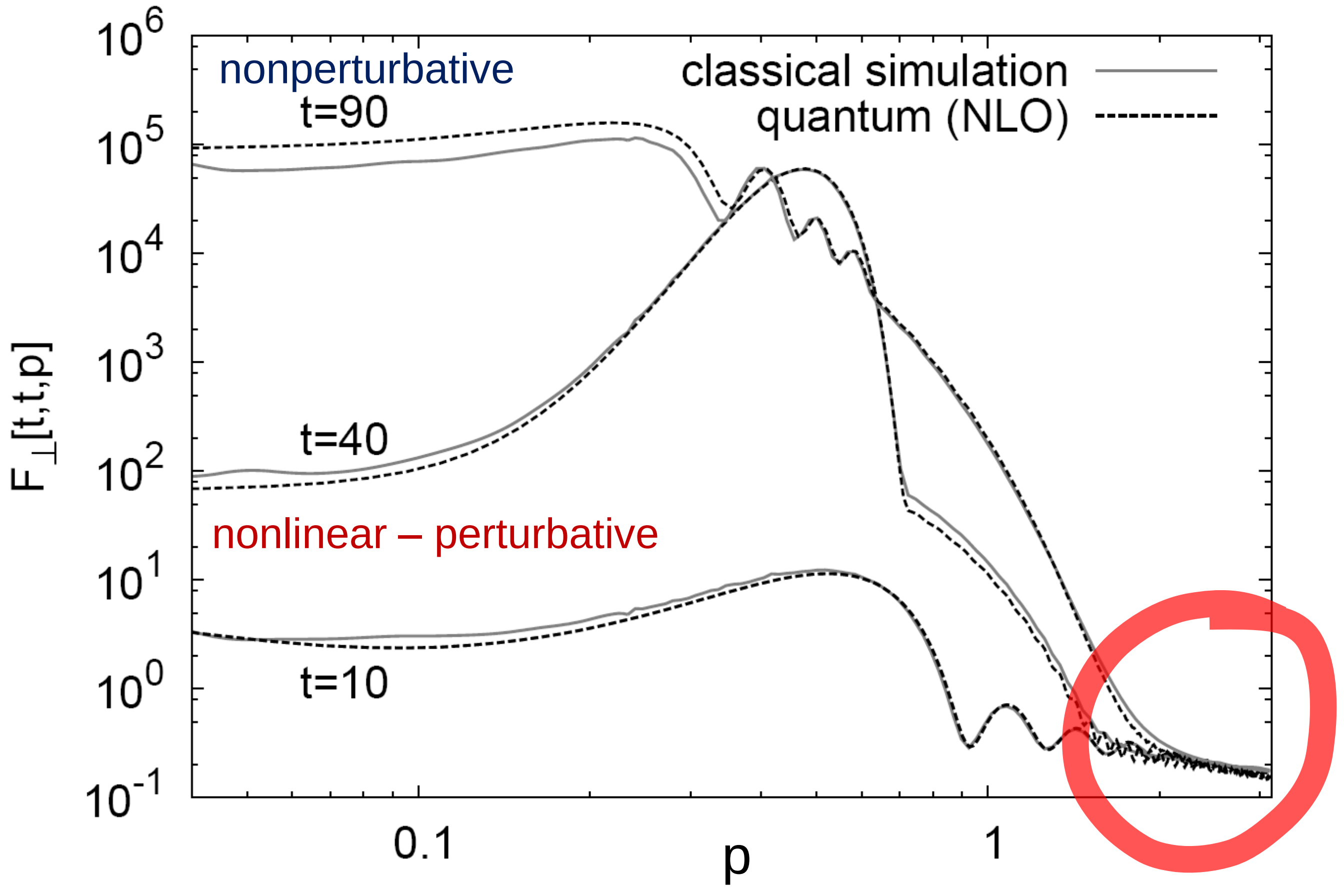
But: Slow dynamics after saturation governed by approximate non-thermal FP of the corresponding ***classical-statistical*** theory

Cartoon: ‘Space of correlation functions‘



Comparison quantum/classical dynamics

Classical-statistical simulations: Khlebnikov, Tkachev ‘96; Prokopec,Roos ‘97; Tkachev, Khlebnikov, Kofman, Linde ’98; …



Berges, Rothkopf, Schmidt ‘08

Practically no quantum corrections at the end of preheating

Accurate nonperturbative description by 2PI 1/*N* to NLO

Nonequilibrium evolution equations

Propagator: spectral function $\sim \langle[\Phi,\Phi]\rangle$

$$G(x,y) = F(x,y) - \frac{i}{2}\rho(x,y)\,\mathrm{sign}_{\mathscr{C}}(x^0 - y^0)$$

statistical propagator $\sim \langle\{\Phi,\Phi\}\rangle$

Tremendous simplification if thermal equilibrium $G^{(eq)}(x,y)=G^{(eq)}(x-y)$ with

$$F^{(\mathrm{eq})}(\omega,\mathbf{p}) = -i\left(\frac{1}{2}+n_{\mathrm{BE}}(\omega)\right)\rho^{(\mathrm{eq})}(\omega,\mathbf{p})$$ *" fluctuation-dissipation relation"*

Nonequilibrium:

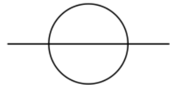
$$F \not\sim \rho$$

$$\left[\Box_x\delta_{ac}+M^2_{ac}(x)\right]\rho_{cb}(x,y) = -\int_{y^0}^{x^0}\mathrm{d}z\,\Sigma^{\rho}_{ac}(x,z)\rho_{cb}(z,y)$$

$$\left[\Box_x\delta_{ac}+M^2_{ac}(x)\right]F_{cb}(x,y) = -\int_{0}^{x^0}\mathrm{d}z\,\Sigma^{\rho}_{ac}(x,z)F_{cb}(z,y) + \int_{0}^{y^0}\mathrm{d}z\,\Sigma^{F}_{ac}(x,z)\rho_{cb}(z,y)$$

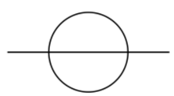
$$\left(\left[\Box_x+\frac{\lambda}{6N}\phi^2(x)\right]\delta_{ab}+M^2_{ab}(x;\phi=0,F)\right)\phi_b(x) = -\int_0^{x^0}\mathrm{d}y\,\Sigma^{\rho}_{ab}(x,y;\phi=0,F,\rho)\,\phi_b(y)$$

Quantum- vs. classical-statistical contributions

Example: Quantum  (Similar for $1/N$ to NLO and $\phi \neq 0$)

$$\Sigma^F(t,t';\mathbf{p}) = -\frac{\lambda^2}{6}\int_{\mathbf{q},\mathbf{k}} F(t,t';\mathbf{p}-\mathbf{q}-\mathbf{k}) \left[F(t,t';\mathbf{q})F(t,t';\mathbf{k}) - \frac{3}{4}\rho(t,t';\mathbf{q})\rho(t,t';\mathbf{k})\right]$$

$$\Sigma^\rho(t,t';\mathbf{p}) = -\frac{\lambda^2}{2}\int_{\mathbf{q},\mathbf{k}} \rho(t,t';\mathbf{p}-\mathbf{q}-\mathbf{k}) \left[F(t,t';\mathbf{q})F(t,t';\mathbf{k}) - \frac{1}{12}\rho(t,t';\mathbf{q})\rho(t,t';\mathbf{k})\right]$$

Classical 

$$\Sigma^F_{\mathrm{cl}}(t,t';\mathbf{p}) = -\frac{\lambda^2}{6}\int_{\mathbf{q},\mathbf{k}} F(t,t';\mathbf{p}-\mathbf{q}-\mathbf{k})F(t,t';\mathbf{q})F(t,t';\mathbf{k})$$

$$\Sigma^\rho_{\mathrm{cl}}(t,t';\mathbf{p}) = -\frac{\lambda^2}{2}\int_{\mathbf{q},\mathbf{k}} \rho(t,t';\mathbf{p}-\mathbf{q}-\mathbf{k})F(t,t';\mathbf{q})F(t,t';\mathbf{k})$$

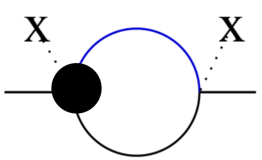
Fixed point condition

Time and space translation invariant solutions require:

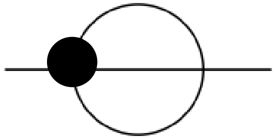
$$\Sigma^{\rho}_{ab}(\phi,p)F_{bc}(p) - \Sigma^{F}_{ab}(\phi,p)\rho_{bc}(p) = J^{(3)}_{ac}(\phi,p) + J^{(4)}_{ac}(\phi,p) \equiv 0$$

Neglecting quantum corrections and $F_{ab} \sim \delta_{ab} F$, $\rho_{ab} \sim \delta_{ab}\rho$, $1/N$ to NLO:

$$J^{(3)}_{aa}(\phi,p) = \frac{\lambda\phi^2}{18N(2\pi)^4}\int \mathrm{d}^4k\,\mathrm{d}^4q\,\delta^4(p-q-k) \\ [\lambda_{\rm eff}(k)+\lambda_{\rm eff}(q)+\lambda_{\rm eff}(p)]\,[\rho(k)F(q)F(p) \\ +F(k)\rho(q)F(p) - F(k)F(q)\rho(p)]$$



$$J^{(4)}_{aa}(\phi,p) = \frac{\lambda}{18(2\pi)^8}\int \mathrm{d}^4k\,\mathrm{d}^4q\,\mathrm{d}^4r\,\delta^4(p+k-q-r) \\ \lambda_{\rm eff}(p+k)\Big\{[F(p)\rho(k)+\rho(p)F(k)]F(q)F(r) \\ -F(p)F(k)[F(q)\rho(r)+\rho(q)F(r)]\Big\} \\ (1-\Delta(\phi,p))$$

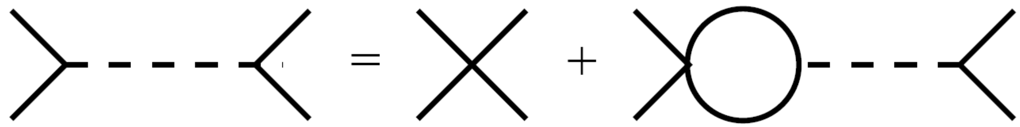


$\times\ (1-\Delta(\phi))$

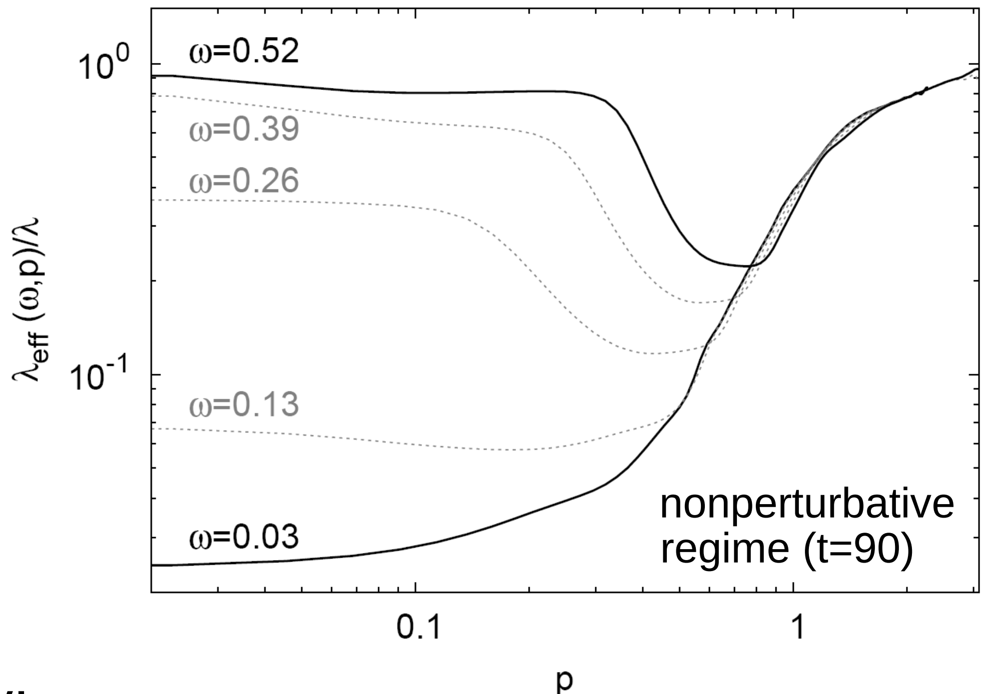
Berges, Rothkopf, Schmidt ‘08

Effective weak coupling

Graphically:



$$\lambda_{\text{eff}}(p) = \frac{\lambda}{|1+\Pi_R(p)|^2}$$



'One-loop' retarded self-energy:

$$\Pi_R(p) = \frac{\lambda}{3(2\pi)^4}\int \mathrm{d}^4q\, F(q) G_R(p-q) \quad ; \quad \Delta(\phi,p) = \frac{2\lambda\phi^2}{3N}\mathrm{Re}\left[\frac{G_R(p)}{1+\Pi_R(p)}\right]$$

Scaling solutions

$$\begin{aligned} F(p) &= s^{2+\alpha} F(sp) \\ \rho(p) &= s^2 \rho(sp) \\ \lambda_{\rm eff}(p) &= s^\gamma \lambda_{\rm eff}(sp) \end{aligned}$$

$\Rightarrow \quad \Pi_R(p) = s^\alpha \Pi_R(sp)$, i.e. $\lambda_{\rm eff}$ scales differently in UV and IR:

$\mathrm{I}: \quad \gamma = 0 \;\text{ for }\; \Pi_R(p) \ll 1$ **(UV)**

$$\begin{aligned} J^{(3)}_{aa}(\phi, p) &= s^{2\alpha} J^{(3)}_{aa}(s\phi, sp) \\ J^{(4)}_{aa}(0, p) &= s^{3\alpha} J^{(4)}_{aa}(0, sp) \end{aligned}$$

← dominates UV for $\alpha > 0$

$$J^{(3)}_{aa}(\phi, p) = 0 \;\Rightarrow\; \boxed{\alpha = 1 \,,\; \alpha = \frac{3}{2}}$$

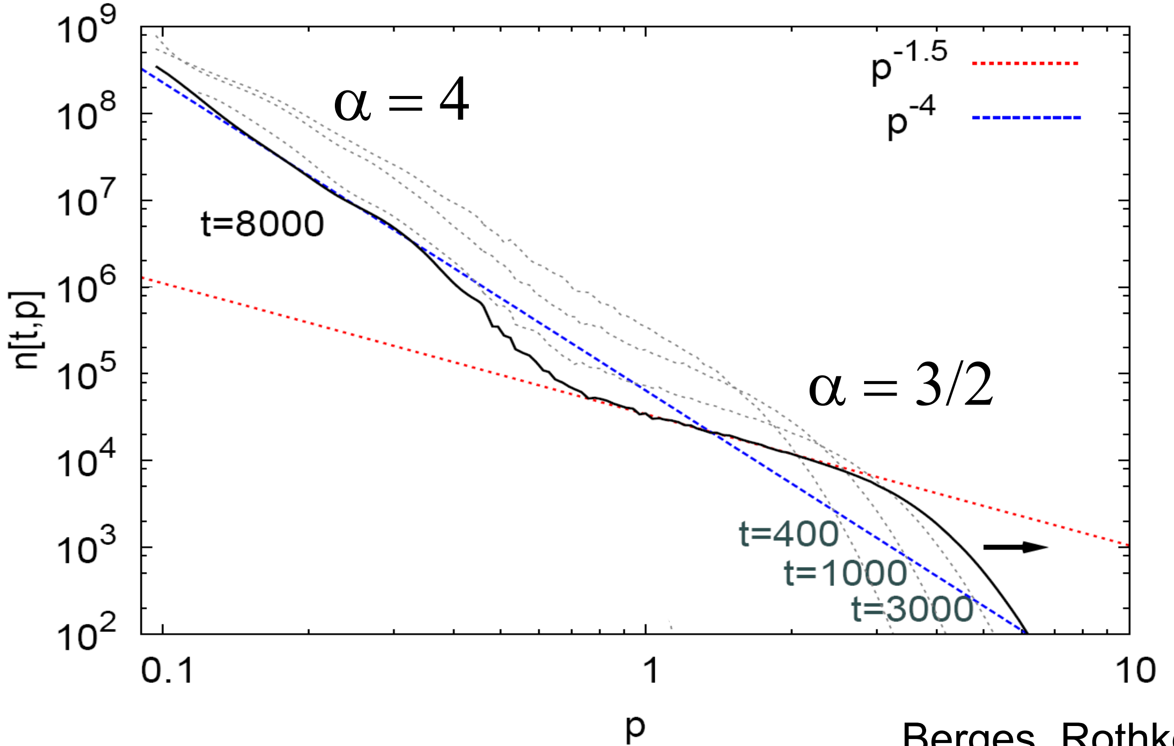
$\mathrm{II}: \quad \gamma = -2\alpha \;\text{ for }\; \Pi_R(p) \gg 1$ **(IR)**

$$\begin{aligned} J^{(3)}_{aa}(\phi, p) &= s^{0} J^{(3)}_{aa}(s\phi, sp) \\ J^{(4)}_{aa}(0, p) &= s^{\alpha} J^{(4)}_{aa}(0, sp) \end{aligned}$$

← dominates IR for $\alpha > 0$

$$J^{(4)}_{aa}(0, p) = 0 \;\Rightarrow\; \boxed{\alpha = 0 \,,\; \alpha = 1 \,,\; \alpha = 4 \,,\; \alpha = 5}$$

Comparison analytical/simulation results



Berges, Rothkopf, Schmidt '08

Late-time behavior well characterized by non-thermal fixed points!

UV: $\alpha = 3/2$ coincides with perturbative (Boltzmann) analysis exponent

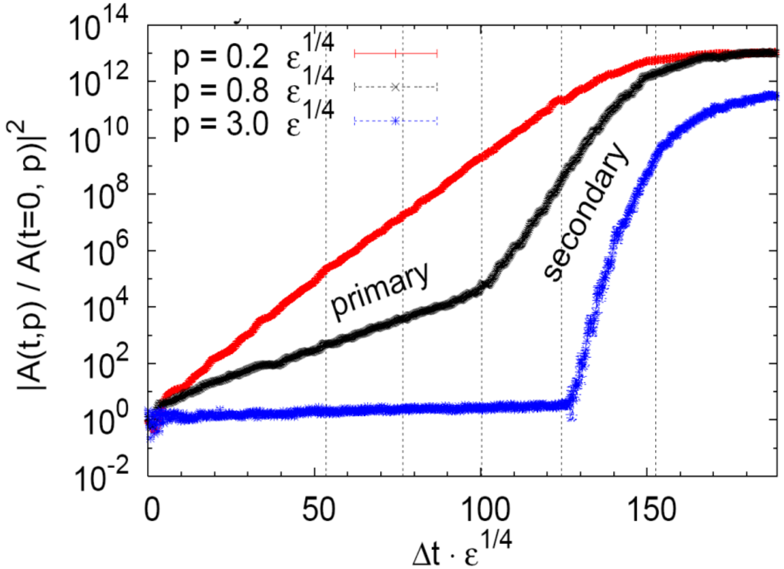
a) local four-leg interaction $\Rightarrow$ $\alpha = 0, 1, 4/3, 5/3$
b) local three-leg interaction $\Rightarrow$ $\alpha = 1, 3/2$

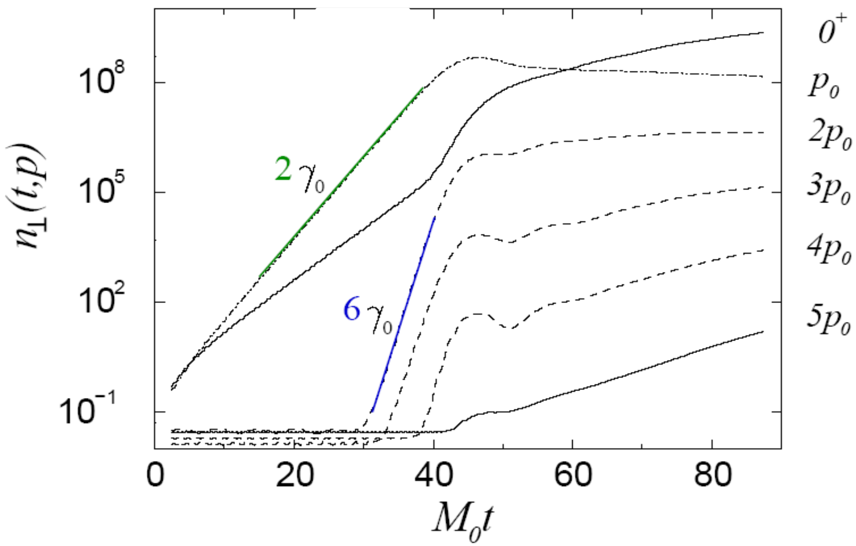
Micha, Tkachev '04

QCD

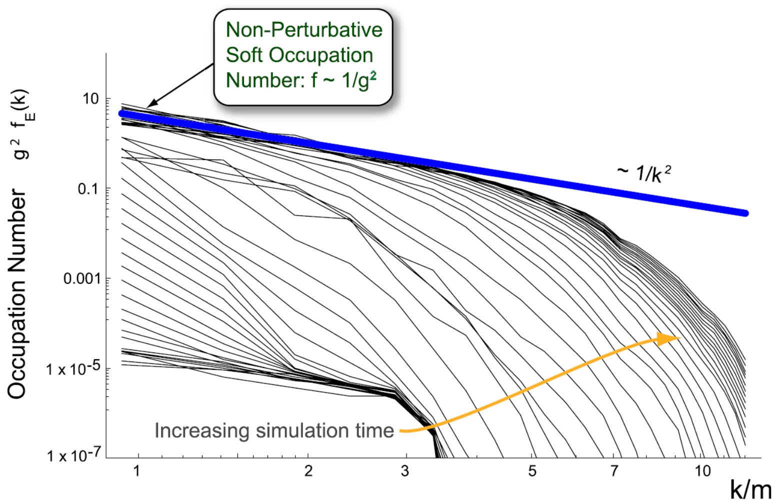
Inflaton

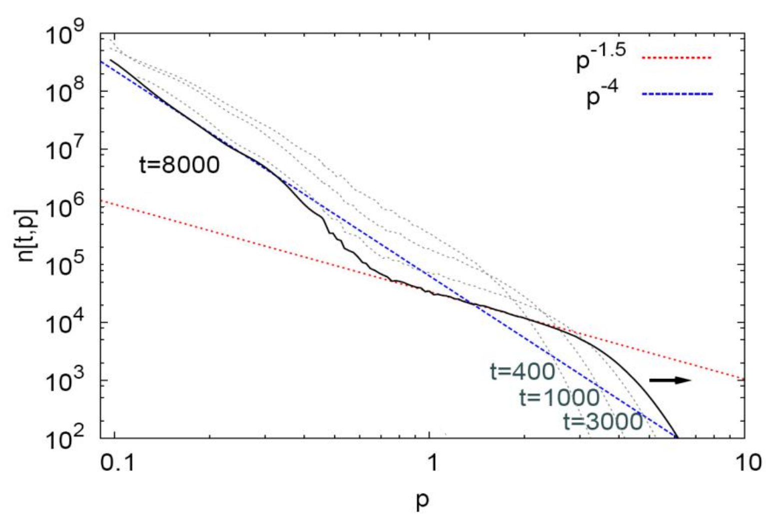
Early: fast dynamics driven by instabiltities





Late: slow dynamics governed by non-thermal fixed points

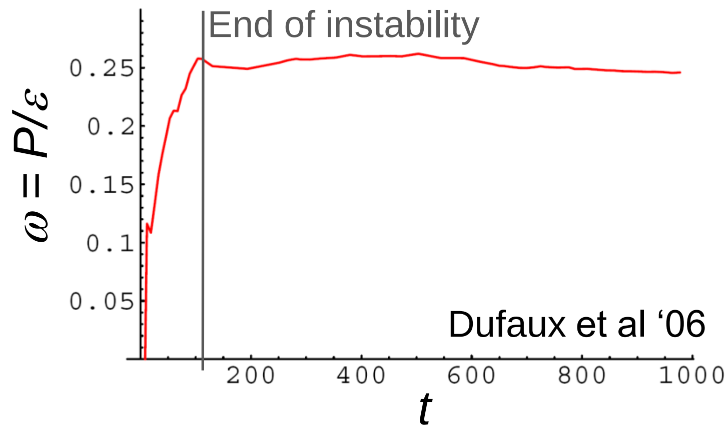




Inflaton: Quantum evolution available (2PI 1/*N* to NLO)

- **Instabilities do not lead to fast thermalization**

But: lead to fast prethermalization of some 'bulk' quantities, e.g. EOS



- **Non-thermal fixed points** govern late-time behavior

  - *nonperturbative: all processes* $\mathcal{O}(1)$
  - *universal*
  - ***effective weak coupling!***

Unstable w.r.t. quantum corrections:

$\rightarrow$ *small corrections only if occupation numbers* $\gg \lambda$

QCD: Classical evolution available

- Characteristic time scale from **plasma instabilities**:

$$\mathbf{1/\gamma_{max} \sim 1\ fm/c} \quad \text{for} \quad \varepsilon = 30\ \text{GeV/fm}^3$$

- **'Bottom-up' isotropization** of stress tensor for

$$\mathbf{p \lesssim 1\ GeV} \quad \text{for} \quad \varepsilon = 30\ \text{GeV/fm}^3$$

i.e. (optimistically) about the range where hydro 'works'

No isotropization for dominant UV momenta seen yet!

- **Quantum corrections** for lower occupied high momenta?
- Can slow late-time behavior be understood in terms of non-thermal fixed points? **Effective weak coupling?!** Viscosity? ...