Hydrodynamics from gauge/gravity duality

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Plan

- Hydrodynamics
  - Relation to finite-temperature field theory
  - First-order hydrodynamics
  - Second-order hydrodynamics
- Gauge/gravity duality
  - Hydrodynamics as low-energy dynamics of black-brane horizons
  - Second-order transport coefficients from gauge/gravity duality

Refs: R. Baier, P. Romatschke, DTS, A. Starinets, M. Stephanov, arxiv:0712.2451

related work: Bhattacharyya, Hubeny, Minwalla, Rangamani, arxiv:0712.2456;
Loganayaram, arxiv:0801.3701

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  - Classical: bosonic modes at $\omega \ll T$
  - “Every cook has learned how to use hydrodynamics”

Why gauge/gravity duality

Practical consideration:

- Strong coupling, not treatable by other methods
- Simple calculations

Conceptual consideration:

- Deep connection between QFT and black-hole physics
- sharp contrast to weak coupling:
  weak coupling: QFT $\rightarrow$ kinetic theory $\rightarrow$ hydro
  strong coupling: QFT $\rightarrow$ hydro

Hope: more to be discovered, e.g., for second-order hydrodynamics.

Hydrodynamics as effective theory

Consider a finite-temperature interacting QFT
Real-time: close-time-path formalism

$$Z[g^1_{\mu\nu},\, g^2_{\mu\nu}] = \int D\psi_1\, D\psi_2\, \exp(iS[\psi_1, g^{(1)}_{\mu\nu}] - iS[\psi_2, g^{(2)}_{\mu\nu}])$$

$$\langle T^{\mu\nu}(x) T^{\alpha\beta}(y)\rangle = \frac{\delta^2 \ln Z}{\delta g_{\mu\nu}(x)\delta g_{\alpha\beta}(y)}$$

We want an effective field theory that gives correlators of $T^{\mu\nu}$ at low momenta.

Hydrodynamics: gives $\langle T_{\mu\nu}\rangle$ for any given smooth source $g^1_{\mu\nu} = g^2_{\mu\nu}$.

Validity: length scales $\gg$ mean free path

Degrees of freedom

- Chiral perturbation theory: d.o.f. = Goldstone modes
- Hydrodynamics: d.o.f = “collective coordinates” of thermal ensemble
  For a plasma with no conserved charge:
  - Temperature $T(x)$
  - Velocity (boost) $u^\mu(x)$, $u^2 = -1$

Optional:

- $\mu(x)$ for each conserved charge
- Phases of condensate (superfluid hydrodynamics)
- U(1) magnetic fields (magnetohydrodynamics)

Ideal and first-order

Ideal (zeroth order) hydrodynamics

$$\nabla_\mu T^{\mu\nu} = 0 \qquad T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu}$$

First-order hydrodynamics (relativistic Navier-Stokes)

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \underbrace{\Pi^{\mu\nu}}_{\text{viscous stress}}$$

Ambiguity of defining u^μ beyond leading order: fixed by $u_\mu \Pi^{\mu\nu} = 0$

$$\Pi^{\mu\nu} = -\eta \nabla^{\langle\mu} u^{\nu\rangle} - \zeta P^{\mu\nu} (\nabla \cdot u)$$

$$P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$A^{\langle\mu\nu\rangle} = \frac{1}{2} P^{\mu\alpha} P^{\nu\beta} (A_{\alpha\beta} + A_{\beta\alpha}) - \frac{1}{3} P^{\mu\nu} P^{\alpha\beta} A_{\alpha\beta}$$

Shear viscosity η and bulk viscosity ζ. Affect damping of shear and sound modes.

Second order: Müller-Israel-Stewart

Modified relationship between $\Pi^{\mu\nu}$ and $\nabla^\mu u^\nu$.

$$(\tau_\pi u^\lambda \nabla_\lambda + 1)\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu}$$

$\tau_\pi D \ll 1$: equivalent to keeping one next term in derivative expansion

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \eta\tau_\pi (u\cdot\nabla)\sigma^{\mu\nu},$$

Matching with AdS/CFT, $\mathcal{N}=4$ SYM

Bjorken flow: $\tau_\pi = \dfrac{1-\ln 2}{6\pi T}$

sound: $\tau_\pi = \dfrac{2-\ln 2}{2\pi T}$

This indicates not all second-order terms are taken into account.

Need to include all second-derivative terms consistent with symmetry. After eliminating redundant ones: 16 independent terms

Conformal invariance

Assume fundamental theory is a CFT,

$$T^{\mu}{}_{\mu} = 0 \qquad \text{in flat space}$$

In curved space: Weyl anomaly

$$g_{\mu\nu}T^{\mu\nu} \sim R^2_{\mu\nu\alpha\beta} \qquad \text{in curved space}$$

But $R \sim \partial^2 g_{\mu\nu}$: Weyl anomaly reproduced in hydrodynamics only at fourth order in derivatives.

$$\Rightarrow g_{\mu\nu}T^{\mu\nu} = 0 \text{ for our purposes}$$

First order: $\zeta = 0$,

Second order: 8 possible structures in $\Pi^{\mu\nu}$

Conformal invariance (II)

Further constraint: $T^{\mu\nu}$ transforms simply under Weyl transformation

$$g_{\mu\nu} \to e^{2\omega} g_{\mu\nu}, \qquad T_{\mu\nu} \to e^{6\omega} T_{\mu\nu}$$

$8 \to 5$ possible structures in $\Pi^{\mu\nu}$

$$\Pi^{\mu\nu}_{\text{2nd order}} = \eta\tau_\pi \left[{}^{\langle} D\sigma^{\mu\nu\rangle} + \frac{1}{3}\sigma^{\mu\nu}(\nabla\cdot u) \right] + \kappa \left[R^{\langle\mu\nu\rangle} - 2u_\alpha R^{\alpha\langle\mu\nu\rangle\beta} u_\beta \right]$$
$$+ \lambda_1 \sigma^{\langle\mu}{}_\lambda \sigma^{\nu\rangle\lambda} + \lambda_2 \sigma^{\langle\mu}{}_\lambda \Omega^{\nu\rangle\lambda} + \lambda_3 \Omega^{\langle\mu}{}_\lambda \Omega^{\nu\rangle\lambda}$$

$$D \equiv u^\mu \nabla_\mu$$

$$\sigma^{\mu\nu} = 2\nabla^{\langle\mu} u^{\nu\rangle}$$

$$\Omega^{\mu\nu} = \frac{1}{2}(\nabla^{\langle\mu} u^{\nu\rangle} - \nabla^{\langle\nu} u^{\mu\rangle})$$ vorticity

κ only in curved space, but affects 2-point function of $T^{\mu\nu}$

λ_i nonlinear response

Hydrodynamics from AdS/CFT

Main philosophy:

Finite-T field theory in flat space $\Leftrightarrow$ black hole with flat horizon
Example: nonextremal D3 metric

$$ds^2 = \frac{r^2}{R^2}(-f dt^2 + d\vec{x}^2) + \frac{R^2}{r^2 f} dr^2, \qquad f = 1 - \frac{r^4}{r_0^4}$$

- Construct a family of configurations by changing $T \sim r_0/R^2$ and boosting along $\vec{x}$ directions by velocity $\vec{u}$

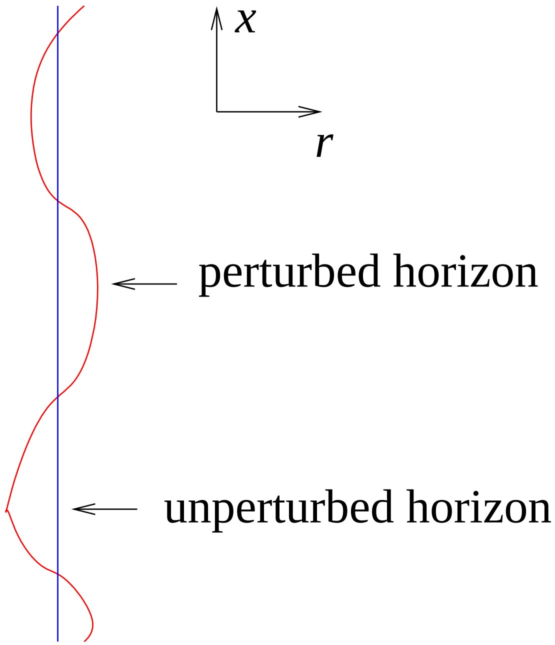
$$g_{\mu\nu} = g_{\mu\nu}(z; T, u^\mu)$$

- Promote T and $\vec{u}$ into fields.
- Require regularity away from $r = 0 \Rightarrow$ hydrodynamic equations

Concretely realized by

Janik, Peschanski, Heller
Bhattacharyya, Hubeny, Minwalla, Rangamani

Dynamics of the horizon



$$T \sim r_0 = r_0(\vec{x})$$

Generalizing black hole thermodynamics M, Q,...
to black brane hydrodynamics

$$T = T_H(\vec{x}), \qquad \mu = \mu(\vec{x})$$

Dissipation in QFT $\Leftrightarrow$ dissipative behavior of black hole horizon $\sim$ "black-hole membrane paradigm" Damour; Thorne, Price, McDonald

Kinetic coeffcients from AdS/CFT

One strategy to find τ_π and κ:

- Within hydro: compute some $\langle T^{\mu\nu} T^{\alpha\beta} \rangle$ from linear response theory: response to gravitational perturbations $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$
- Compare with AdS/CFT calculations

Example: for momentum ω, $\vec{k} = (0, 0, k)$

$$\langle T^{xy} T^{xy} \rangle (\omega, k) = P - i\eta\omega + \eta\tau_\pi\omega^2 - \frac{\kappa}{2}(\omega^2 + k^2)$$

from that

$$\eta = \frac{s}{4\pi}, \qquad \text{universal result}$$

$$\tau_\pi = \frac{2 - \ln 2}{2\pi T}, \qquad \kappa = \frac{\eta}{\pi T}$$

Sound-wave dispersion: Re $\omega = c_s k + \# k^3 \Rightarrow$ the same value for τ_π

Nonlinear coefficients $\lambda_{1,2,3}$

One needs to look beyond small perturbations around thermal equilibrium.
λ_1: can be found from long-time tail of a boost-invariant solution (Janik, Peschanski, Heller):

$$\epsilon(\tau) \sim \frac{1}{\tau^{4/3}} - \frac{2\eta}{\tau^2} + \frac{\#}{\tau^{8/3}} \tag{0}$$

Maching the coefficient of $\tau^{-8/3}$ term:

$$\lambda_1 = \frac{\eta}{2\pi T}$$

Bhattacharyya et al. also found

$$\lambda_1 = \frac{\eta}{2\pi T}\,, \qquad \lambda_2 = -\frac{2\ln 2}{2\pi T}\eta, \qquad \lambda_3 = 0$$

Comparision with Israel-Stewart formalism

- Israel-Stewart equation valid only in hydro regime.
- Frequently terms required by Weyl invariance are thrown away,

$$^{\langle}D\Pi^{\mu\nu\rangle} + \frac{4}{3}\Pi^{\mu\nu}(\nabla\cdot u)$$

  (equivalent to ones used by Romatschke & Romatschke). Such terms may be numerically important (U. Heinz's talk)
- In addition, $\lambda_1 = \lambda_3 = 0$ in IS theory; in $\mathcal{N} = 4$ SYM $\lambda_1 \neq 0$ (but $\lambda_3 = 0$).
- Additional terms nonlinear: not important for sound wave propagation, but important for Bjorken expansion

Entropy current

Loganayaram

- One is unable to force the IS Ansatz $s^\mu = u^\mu + (s + \#\Pi_{\alpha\beta}\Pi^{\alpha\beta})$ to have explicitly positive derivative $\partial_\mu s^\mu \sim \Pi^2$
- More generally, s^μ has to be a expressed in terms of u^μ and its derivatives,

$$s^\mu = su^\mu + \#u^\mu\Pi^2 + u^\mu\omega^2 + O(u\nabla^2 u)$$

One can construct a current so that

$$\partial_\mu s^\mu = \frac{\eta}{2T}\sigma^{\mu\nu}\sigma_{\mu\nu} + \frac{1}{4}(\kappa - 2\lambda_1)\sigma^\mu{}_\nu\sigma^\nu{}_\lambda\sigma^\lambda{}_\mu$$

Generally not explicitly positive, but positive in the hydrodynamic regime: $\sigma^3 \ll \sigma^2$.

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Suprise: in $\mathcal{N}=4$ SYM $\kappa = 2\lambda_1$!

Conclusion

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To be done:

- Gravitational loop effects ($1/N_c$): thermal noise + nonlinearity of hydro equation
  - Hydrodynamic long-time tail as quantum gravity Kovtun, Yaffe
- Breaking conformal invariance Buchel
- Second-order transport coefficient at weak coupling, large-N_c QCD, $\mathcal{N} = 4$ SYM   can one use some kinetic theory?
- Implications for elliptic flow in heavy ion collisions