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Formation of Gauged Cosmic Strings

Arttu Rajantie

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Outline

- Global symmetry: Kibble mechanism
- Gauge symmetry: Flux trapping
- Thermal phase transitions:
  - Vortex clusters
  - Long-range correlations
  - Cosmological consequences
- Quantum phase transitions

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Cosmic Strings

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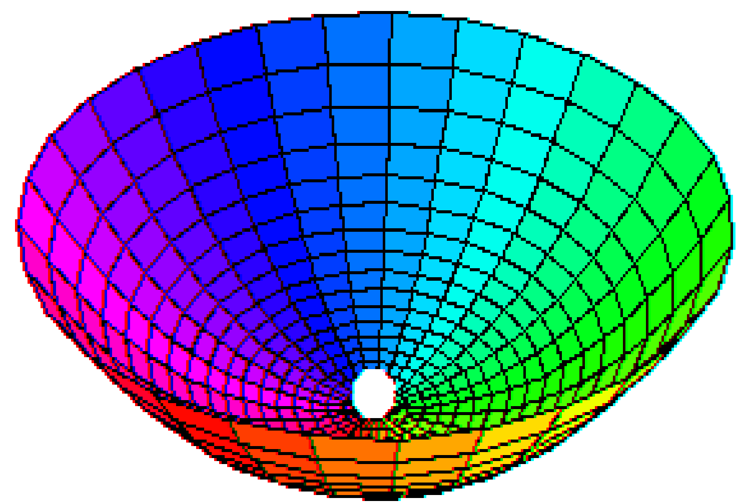
Kibble Mechanism

- Continuous global symmetry, e.g., $\phi \to e^{i\alpha}\phi$ in

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \lambda(\phi^* \phi)^2$$

- From $m^2 > 0$ to $m^2 < 0$:
  Spontaneous symmetry breaking
- Decreasing temperature,
  end of inflation,
  brane collision



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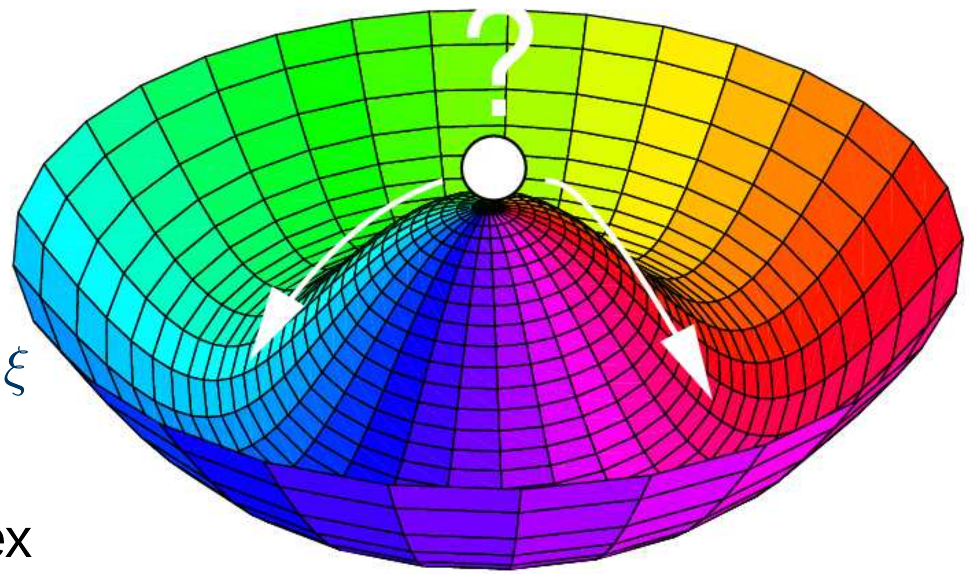
Kibble Mechanism



- Spontaneous symmetry breaking:
  Field has to "choose" a direction
- Finite correlation length ξ
  - Cosmology: $\xi < 1/H$
- Choice uncorrelated at distances $\gtrsim \xi$

$\Rightarrow$ Domains of size $\sim \xi$

  - Finite probability to form a vortex whenever three domains meet (Kibble 1976)
  - Roughly one vortex per domain
  - Number density per cross-sectional area:
    $n \sim \xi^{-2}$

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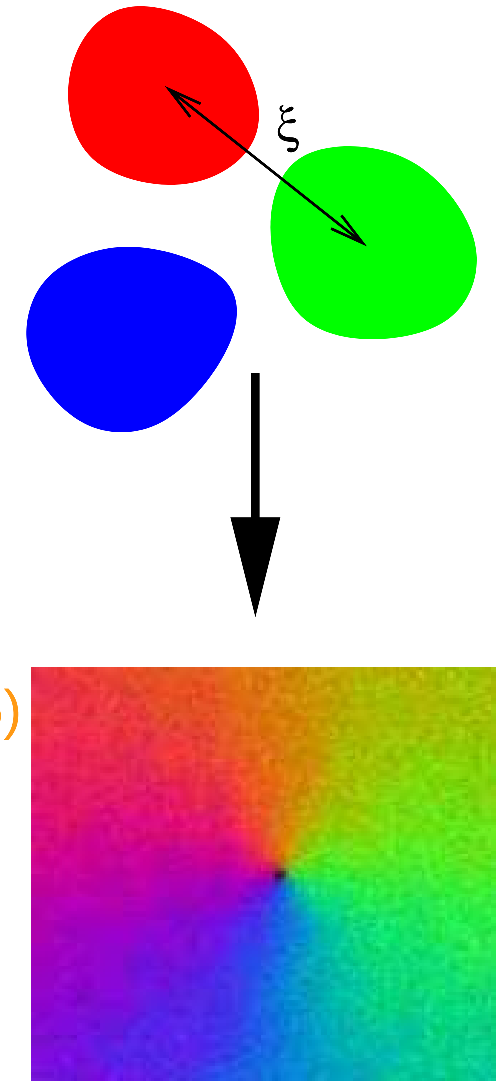

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Gauge Symmetry

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

Gauge Symmetry

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^* D^\mu\phi - m^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

- $D_\mu = \partial_\mu + ieA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- Invariant under $\phi(x) \to e^{i\alpha(x)}\phi(x)$, $A_\mu \to A_\mu - (1/e)\partial_\mu\alpha$
- Meissner effect in the broken phase:
  - Magnetic field confined into vortices
  - Flux quantum $\Phi_0 = 2\pi/e$ per vortex
- ”Direction” of symmetry breaking not a physical quantity
  - Can be rotated at any point independently of all others
  - Kibble mechanism based on it – Is it still valid?
  - Just fix the gauge and then it works?

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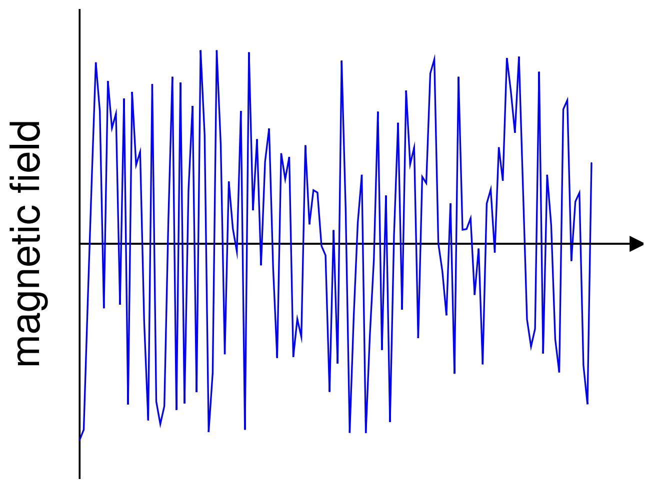
Flux Trapping

- Just fix the gauge and then it works?
- No! Gauge field plays an important role (Hindmarsh&Rajantie 2000)
  - "Magnetic" flux $\Phi = \int_C d\vec{S}\cdot\vec{B} = \oint_{\partial C} d\vec{x}\cdot\vec{A}$ conserved:

$$\frac{d}{dt}\Phi = -\oint_{\partial C} d\vec{x}\cdot\vec{E}$$

  - Long-wavelength fluctuations do not have time to decay:



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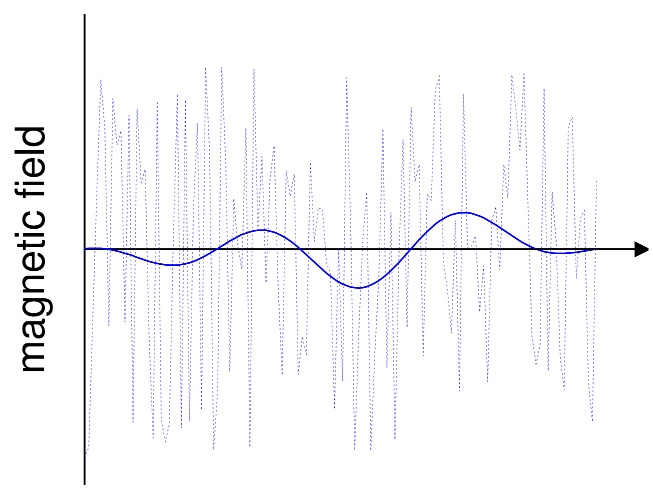

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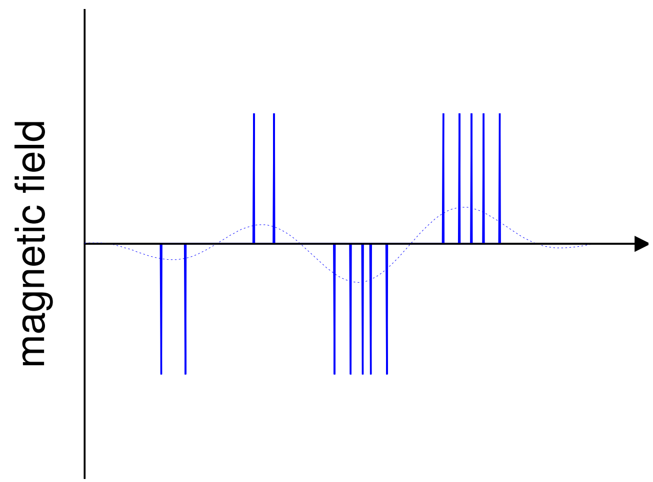


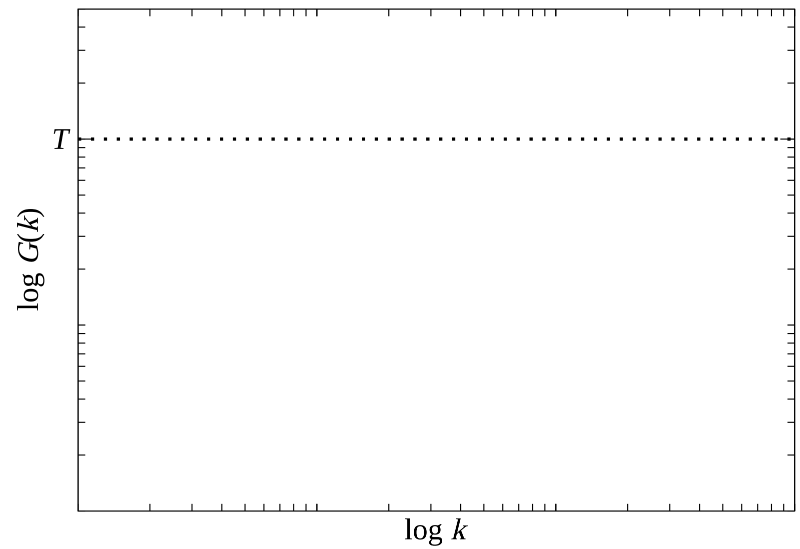
Flux Trapping

- Two-point correlator

$$\langle B_i(\vec{k})B_j(\vec{q})\rangle = (2\pi)^3\delta(\vec{k}+\vec{q})\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)G(k)$$

- Thermal initial state

$$G(k) \approx T$$



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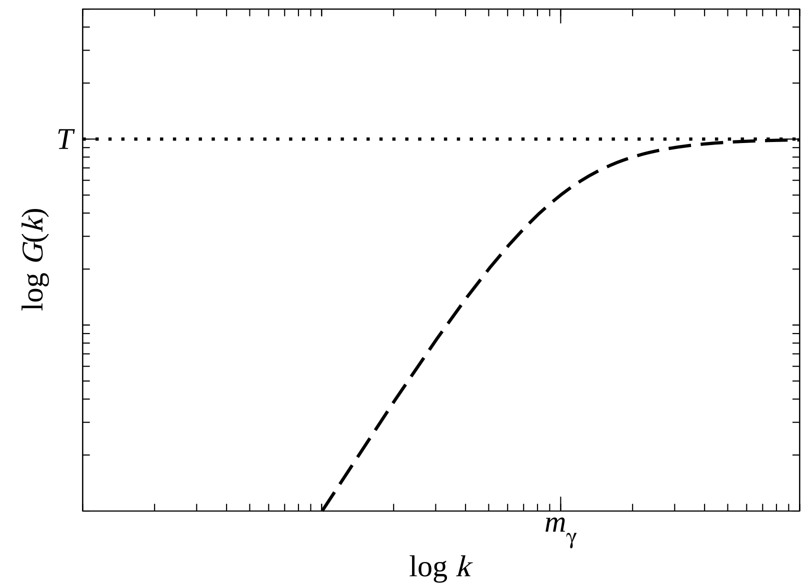
$\rightarrow$ Broken phase

$$G(k) \approx \frac{Tk^2}{k^2 + m_\gamma^2}$$



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Flux Trapping

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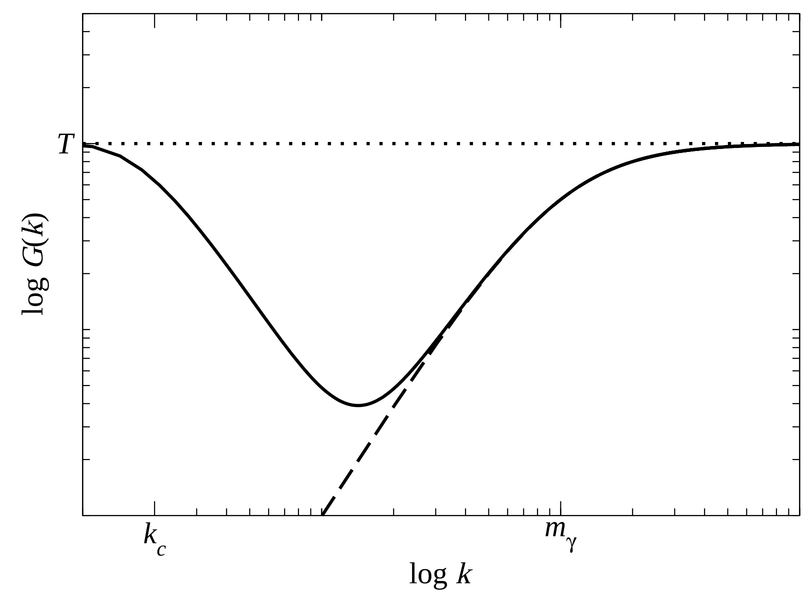
- Longest wavelengths $k \lesssim k_c$ freeze out
- Critical wavelength k_c depends on the cooling rate



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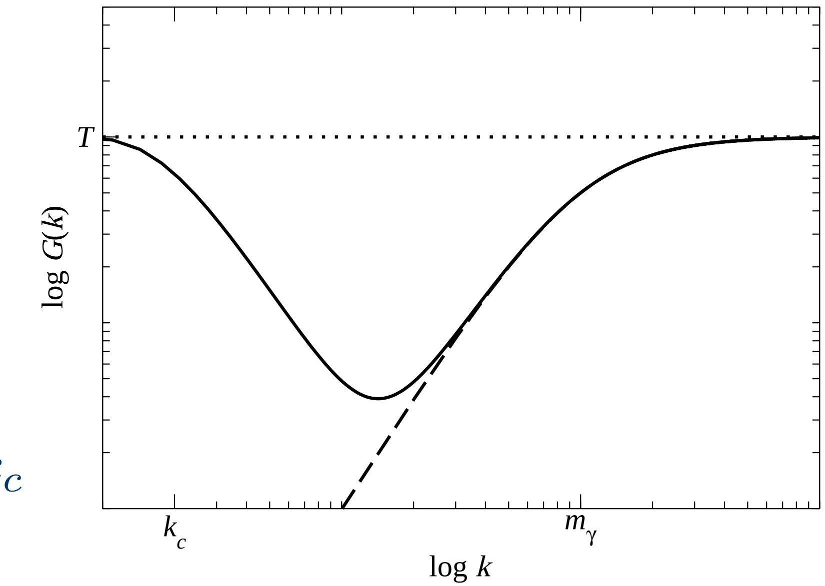
Flux Trapping



- Long-wavelength modes $k \lesssim k_c$ freeze out
- Meissner effect: Get trapped in strings

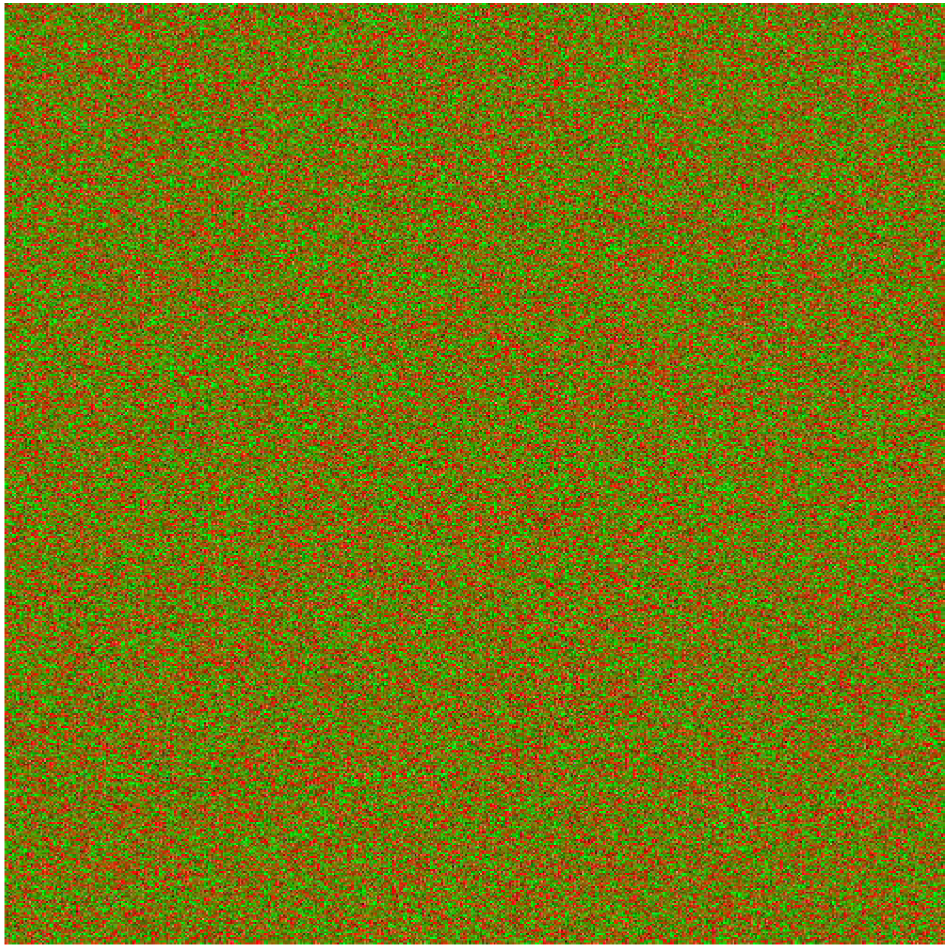
$\Rightarrow$ Winding number $\vec{w}(\vec{k}) = (e/2\pi)\vec{B}(\vec{k})$ at $k \ll k_c$

- Correlator $G_w(k)$:

$$\langle w_i(\vec{k}) w_j(\vec{q}) \rangle = (2\pi)^3 \delta(\vec{k}+\vec{q}) \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) G_w(k)$$

- Local dynamics (such as Kibble) can only give $G_w(k) \sim k^2$ at low k

2D Simulation



2D Simulation

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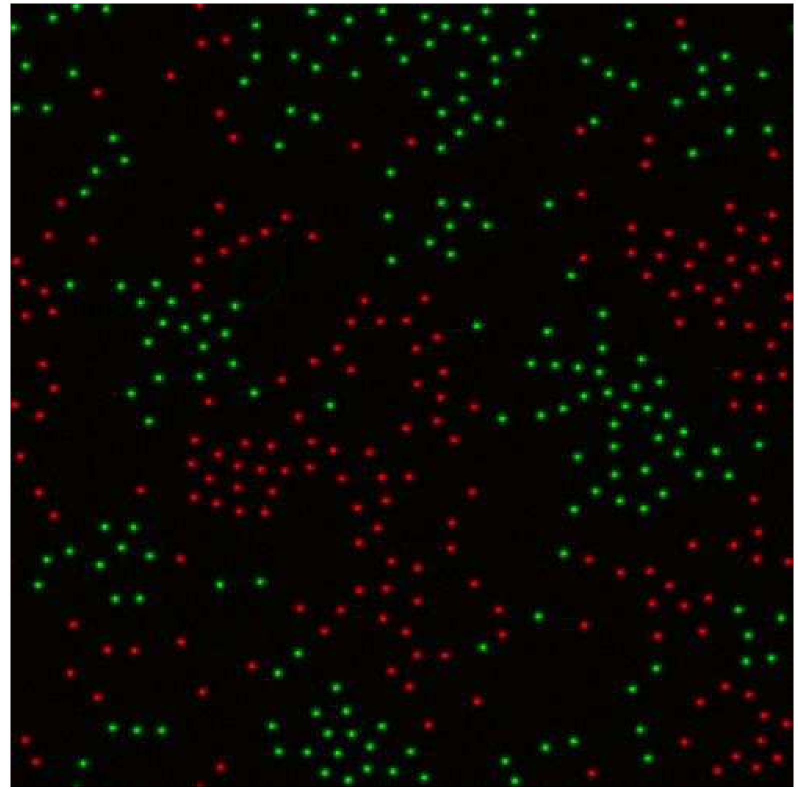
2D Simulation

- Clusters of equal-sign vortices:
  - $N \approx eT^{1/2}k_c^{D/2-2}$ per cluster
  - Kibble mechanism: No clusters
- Short-distance effect



gauge

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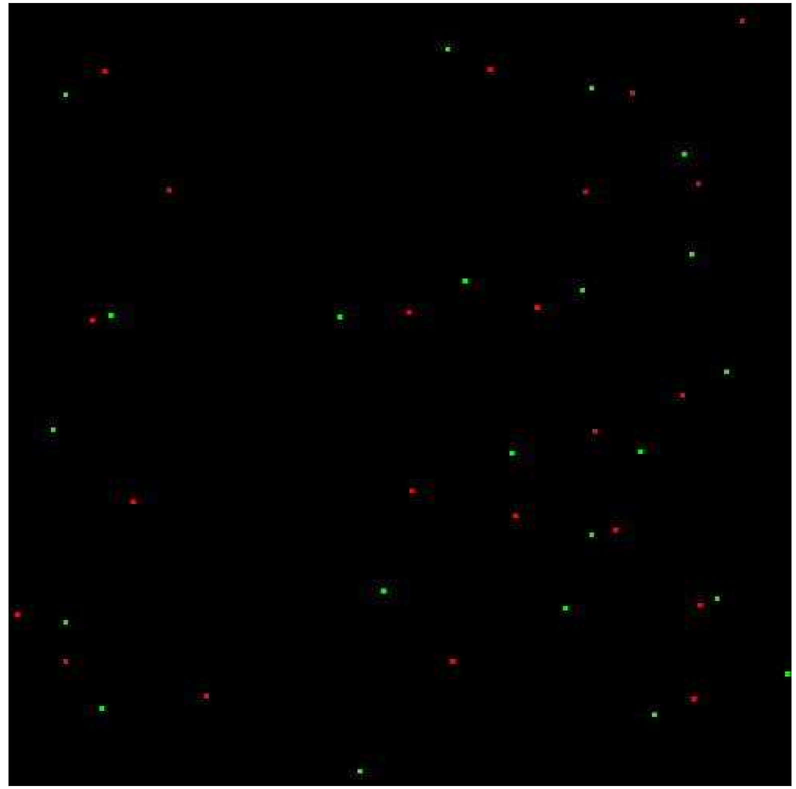

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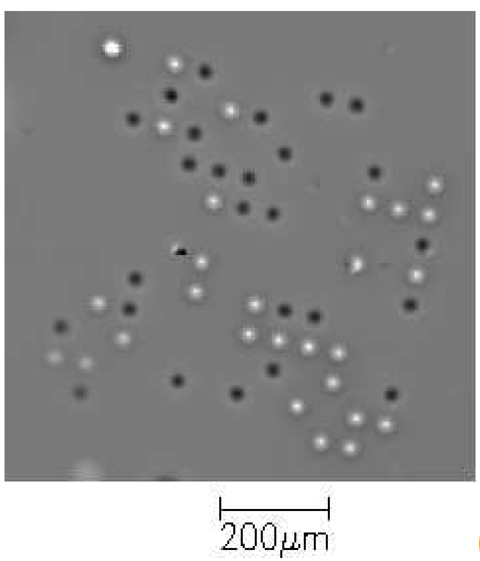
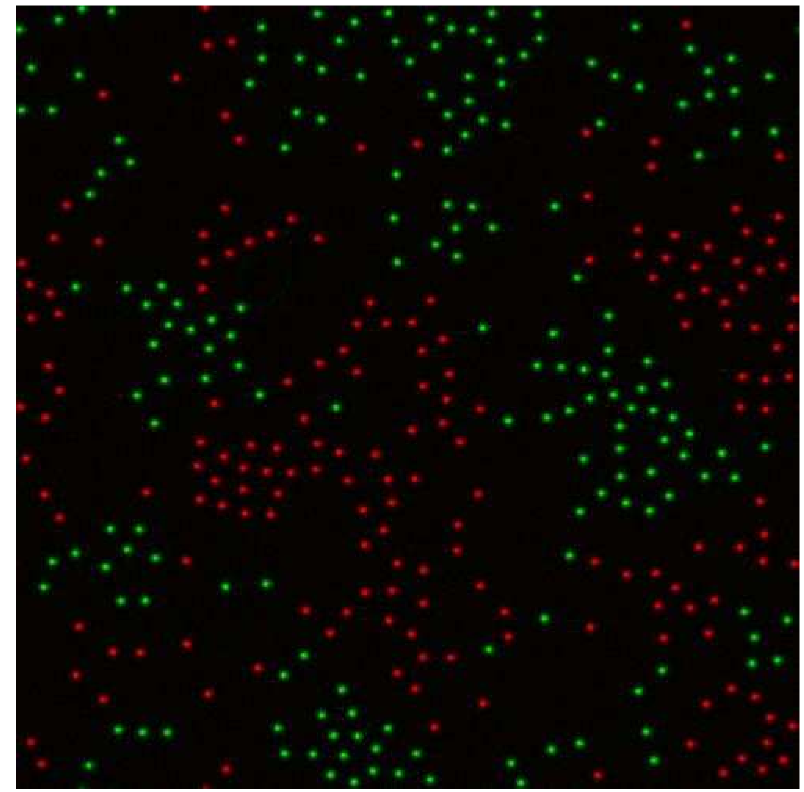
2D Simulation

- Clusters of equal-sign vortices:
  - $N \approx eT^{1/2} k_c^{D/2-2}$ per cluster
  - Kibble mechanism: No clusters
- Short-distance effect
- Can be tested with superconductors



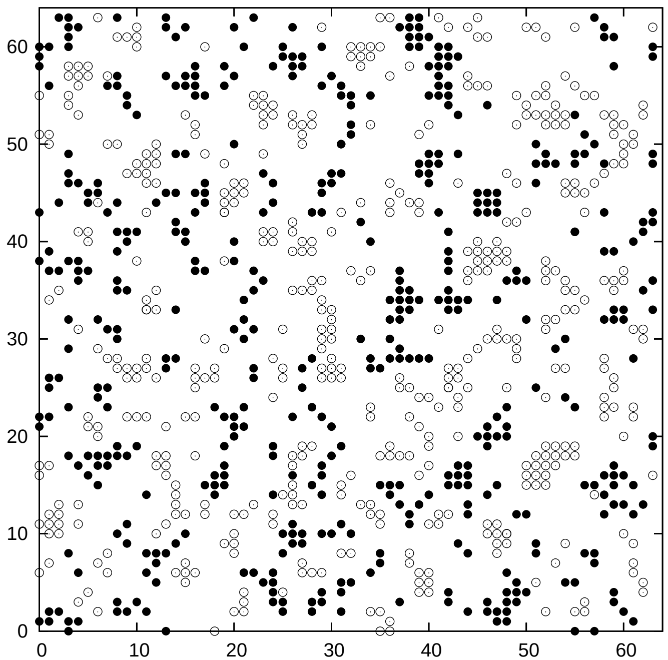
(Kirtley et al 2003)

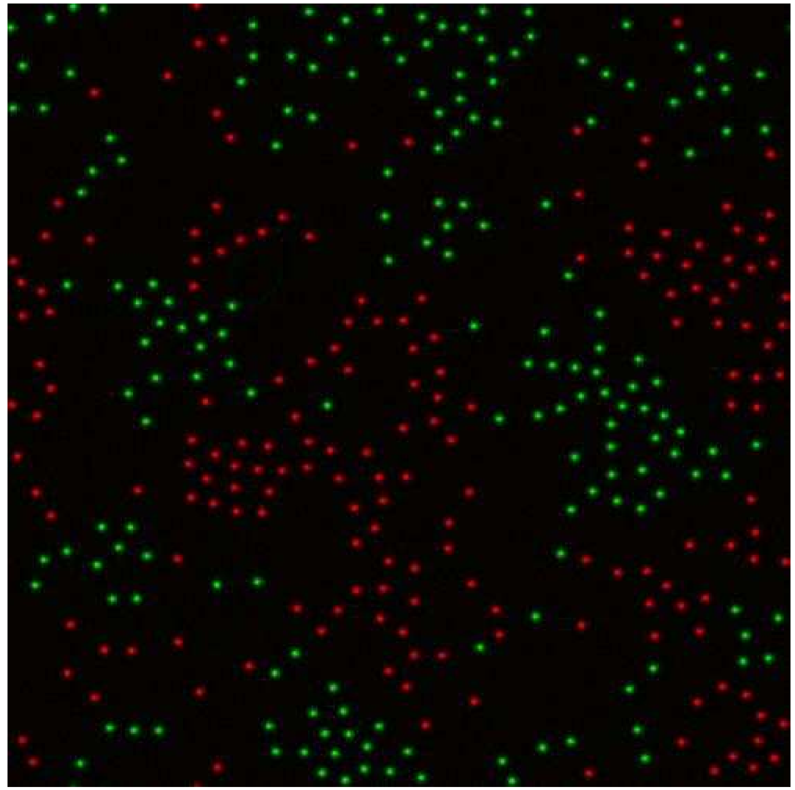


gauge

2D Simulation

- Clusters of equal-sign vortices:
  - $N \approx eT^{1/2}k_c^{D/2-2}$ per cluster
  - Kibble mechanism: No clusters
- Short-distance effect
- Also present in 3D





gauge

(Blanco-Pillado et al 2007)

3D Simulation

- 256x256x256 lattice, spacing $\delta x = 1$
- Thermal initial conditions with $T_{\rm ini} = 0.5$
- Radiation dominated with $H_{\rm ini} = 0.1$: Evolve until $a = 2$
- Scalar coupling $\lambda = 1$,
  m^2 such that transition takes place at $a = \sqrt{2}$

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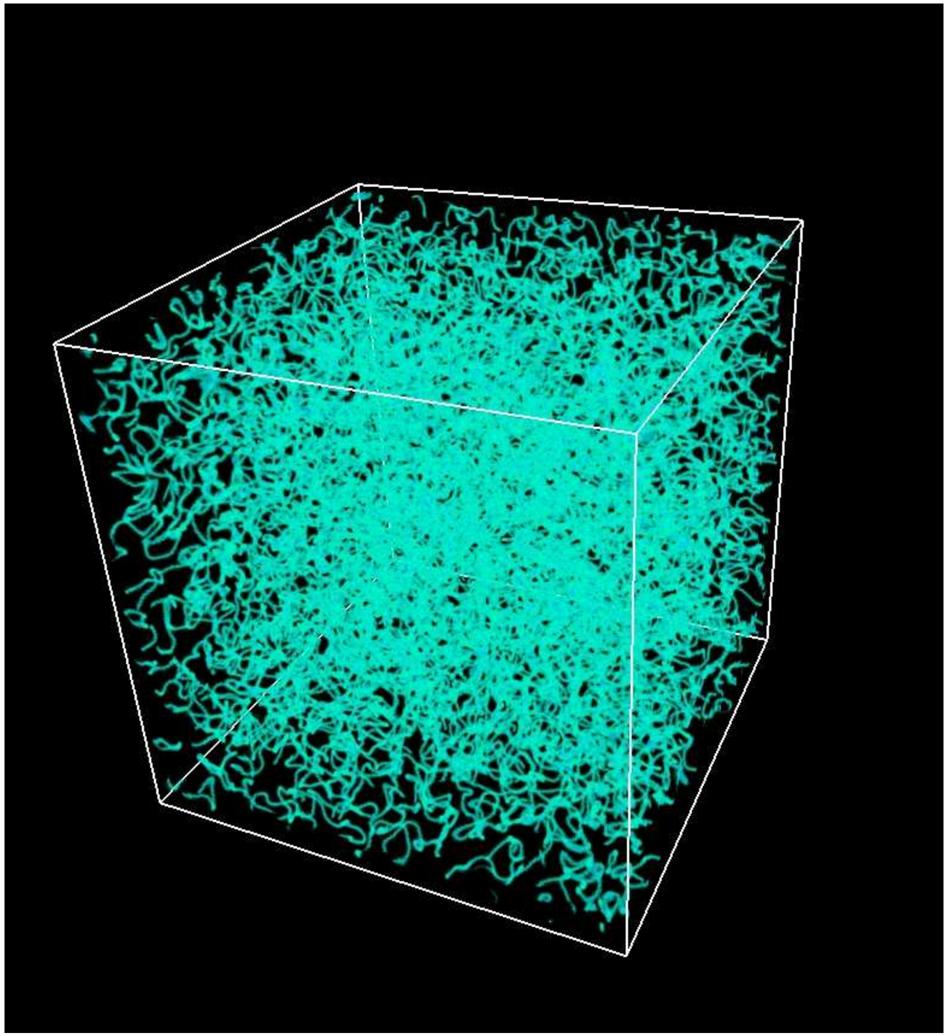
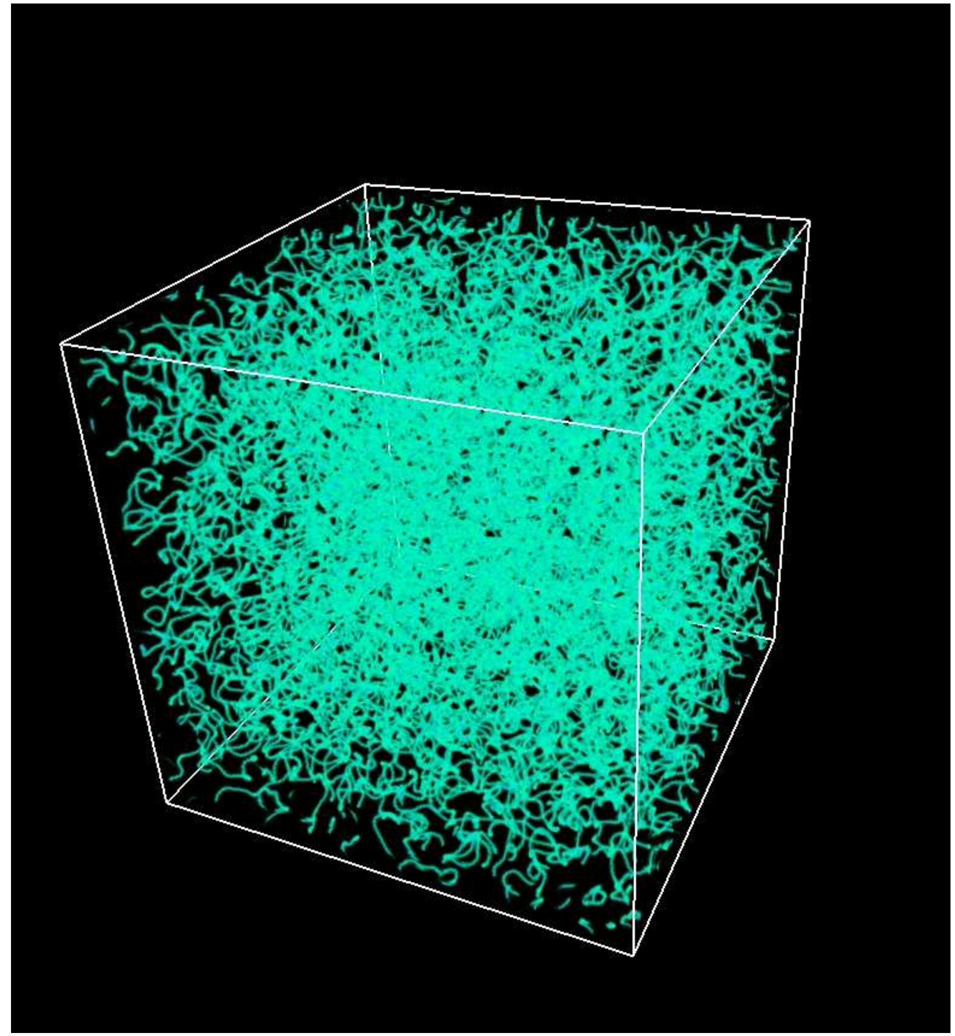
3D Simulation

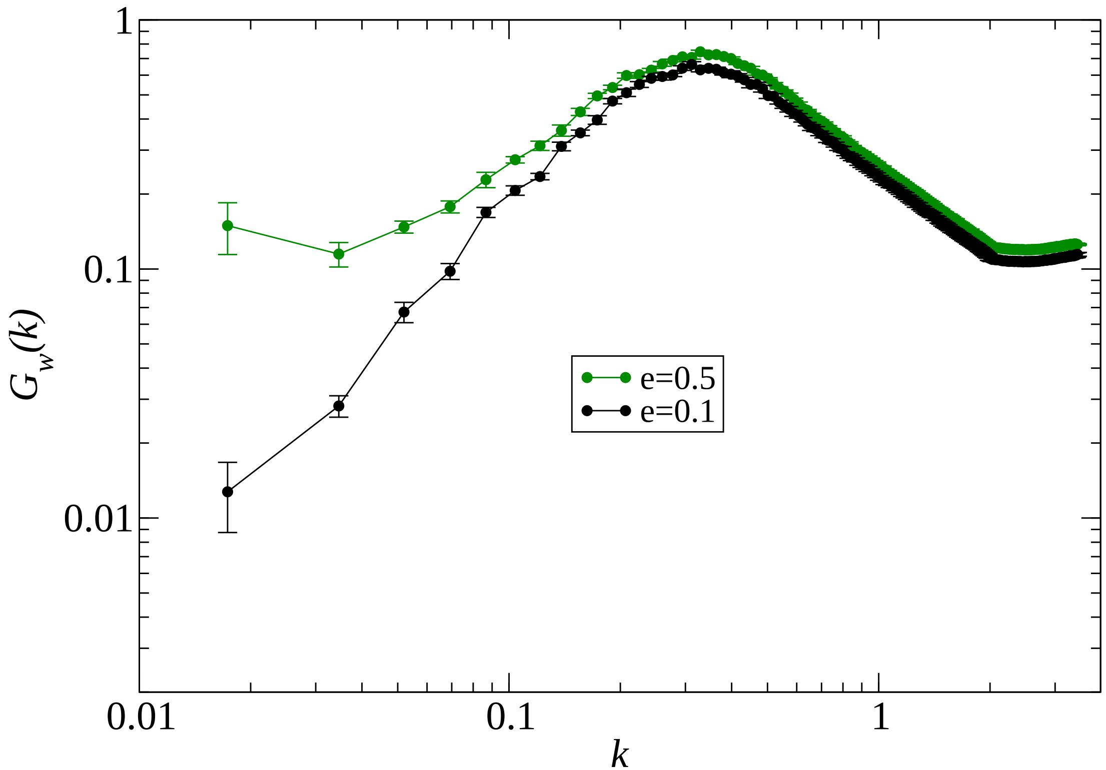


$e = 0.1$



$e = 0.5$

3D Simulation



- Winding number correlator (smoothed with gradient flow):

$$\langle w_i(\vec{k}) w_j(\vec{q}) \rangle = (2\pi)^3 \delta(\vec{k} + \vec{q}) \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) G_w(k)$$

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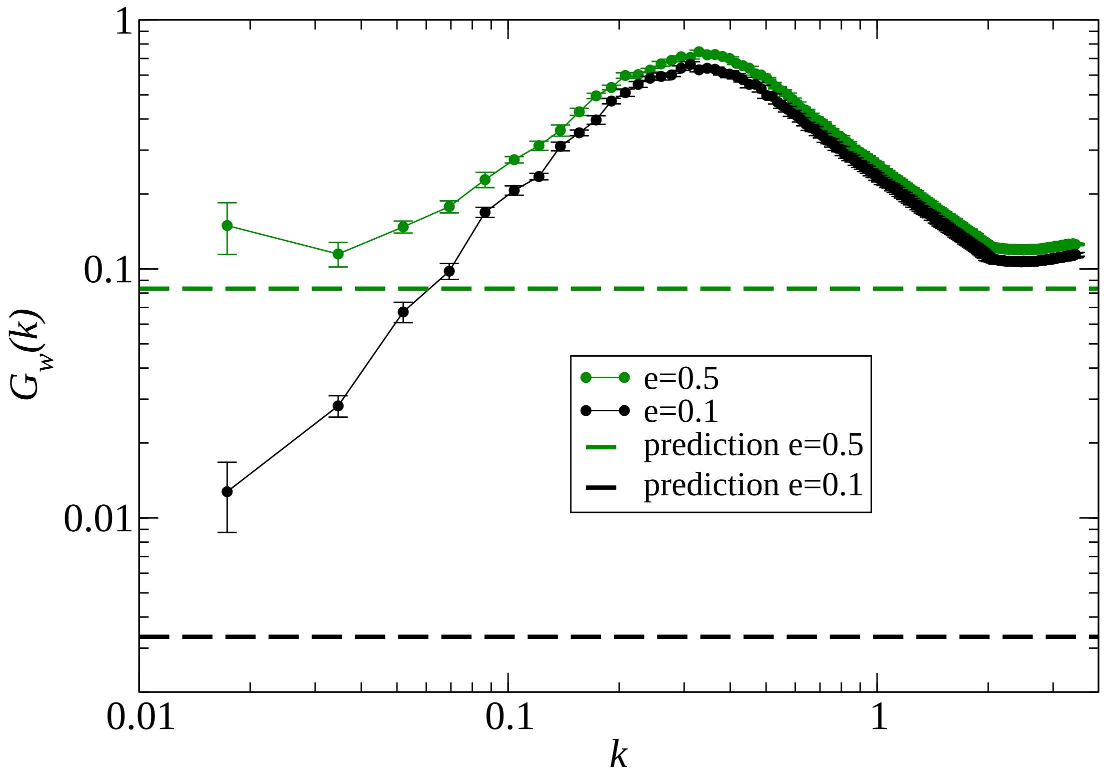
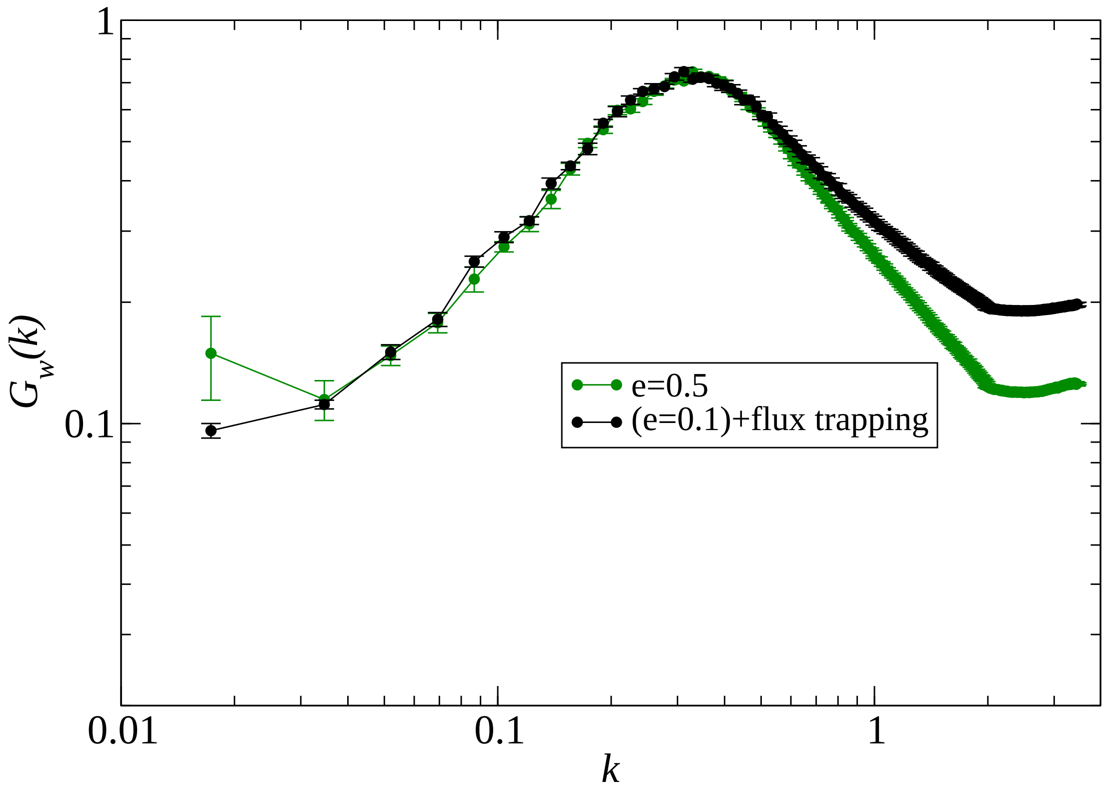
3D Simulation



- Flux trapping prediction:

$$\langle w_i(\vec{k}) w_j(\vec{q}) \rangle = \left(\frac{e}{2\pi}\right)^2 \langle B_i(\vec{k}) B_j(\vec{q}) \rangle_{\rm ini}$$

3D Simulation



- Total=Kibble+Flux trapping

Long-Range Correlations

- Pre-existing flux correlations survive in string network
- Thermal initial state:
  - Massless photon $\Rightarrow$ Infinite correlation length

    (cf. Kibble: Exponentially decaying correlations)
- Cosmology: Super-horizon correlations frozen until they enter horizon
  - Acoustic peaks?
  - Net number of strings

$$N_{\rm FT} = \sqrt{\frac{G_w(0)}{aH}} = \sqrt{\frac{e^2 T_c}{2\pi^2 aH}}$$

  - Changes as the universe expands: No scaling
  - Today: $N_{\rm FT} \approx \sqrt{T_{\rm CMB}/H_0} \approx 10^{15}$, $\Omega_{\rm strings} \approx N_{\rm FT} G\mu$
  - 70 e-foldings of inflation needed to dilute these away

Zero-Temperature Phase Transition

- End of (long) period of inflation: Hybrid, brane etc.
  - Superhorizon modes after reheating
- Classical approximation:

  $\vec{B}$ is a Gaussian random field with $G(k) = \frac{k}{2}$
- Flux trapping: $G_w(k) = (e/2\pi)^2 G(k) \sim k$
- Local dynamics (such as Kibble) can only give $\sim k^2$
- Apparently different long-distance behaviour
  - Is this correct: Where did these correlations come from?
  - Naively $N_{\rm FT} \approx (e/2\pi)\sqrt{\ln(T_c/aH)} \approx 10e$ today
  - Potentially observable if $e \approx 1$
- Proper quantum field theory calculation needed!
  - Hard: 2PI does not work with defects! (Rajantie&Tranberg 2006)

Conclusions

- Global strings: Kibble mechanism
  - Local process $\Rightarrow$ No long-range correlations
- Gauged strings: Kibble+Flux trapping
  - Short distance: Clusters
  - Flux trapping dominates at long distances: Long-range correlations
    - Thermal: Ruled out!
    - Pre-inflation: Potentially observable
    - Quantum: Are they real?
- Also valid in
  - Brane collisions (Dvali&Vilenkin 2004)
  - Superconductors