Preheating after inflation

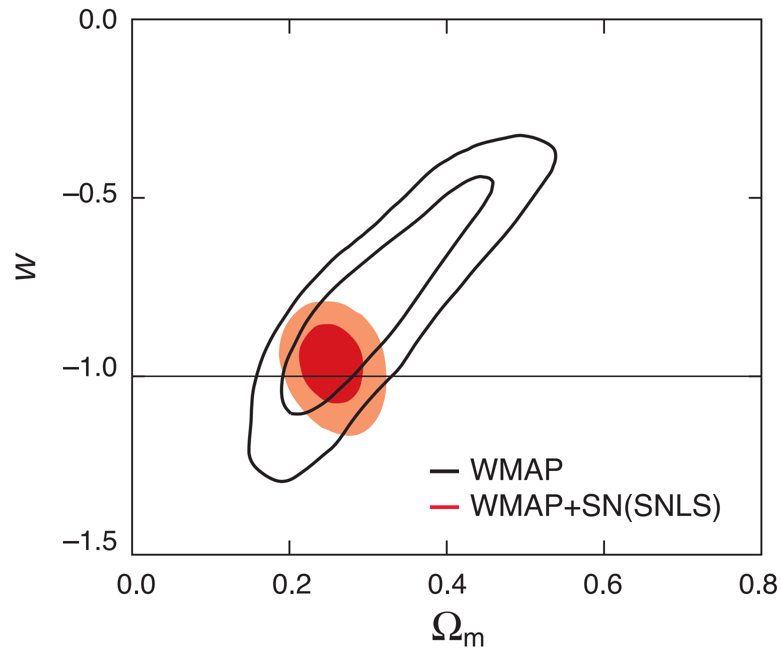
Marco Peloso, Minnesota

- Perturbative vs. nonperturbative inflaton decay
- Preheating $\longrightarrow$ thermalization
- Applications

History of the Universe

Clear knowledge

from BBN on



WMAP3

$\Omega_m = 0.249^{+0.024}_{-0.031}$ $w = -0.97^{+0.07}_{-0.09}$ $T_\gamma \simeq 2.7K$

Dark energy	$z \epsilon$	$[0, 0.4]$
Dark matter	$z \epsilon$	$\left[0.4, 10^4\right]$
Radiation	$z \epsilon$	$\left[10^4, ?\right]$

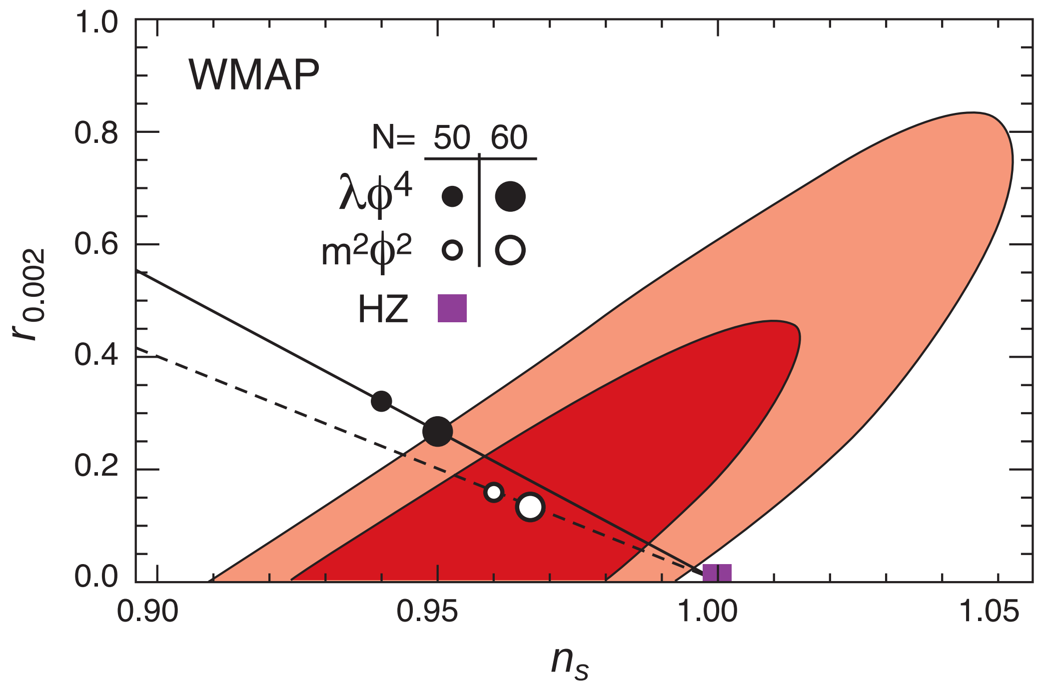
$z_{\rm BBN} \simeq 10^{10}$

Good theoretical control & data for inflation

Slow Roll : $\epsilon = \frac{M_p^2}{16\,\pi}\left(\frac{V'}{V}\right)^2 \ll 1 \quad , \quad \eta = \frac{M_p^2}{8\,\pi}\frac{V'}{V} \ll 1$

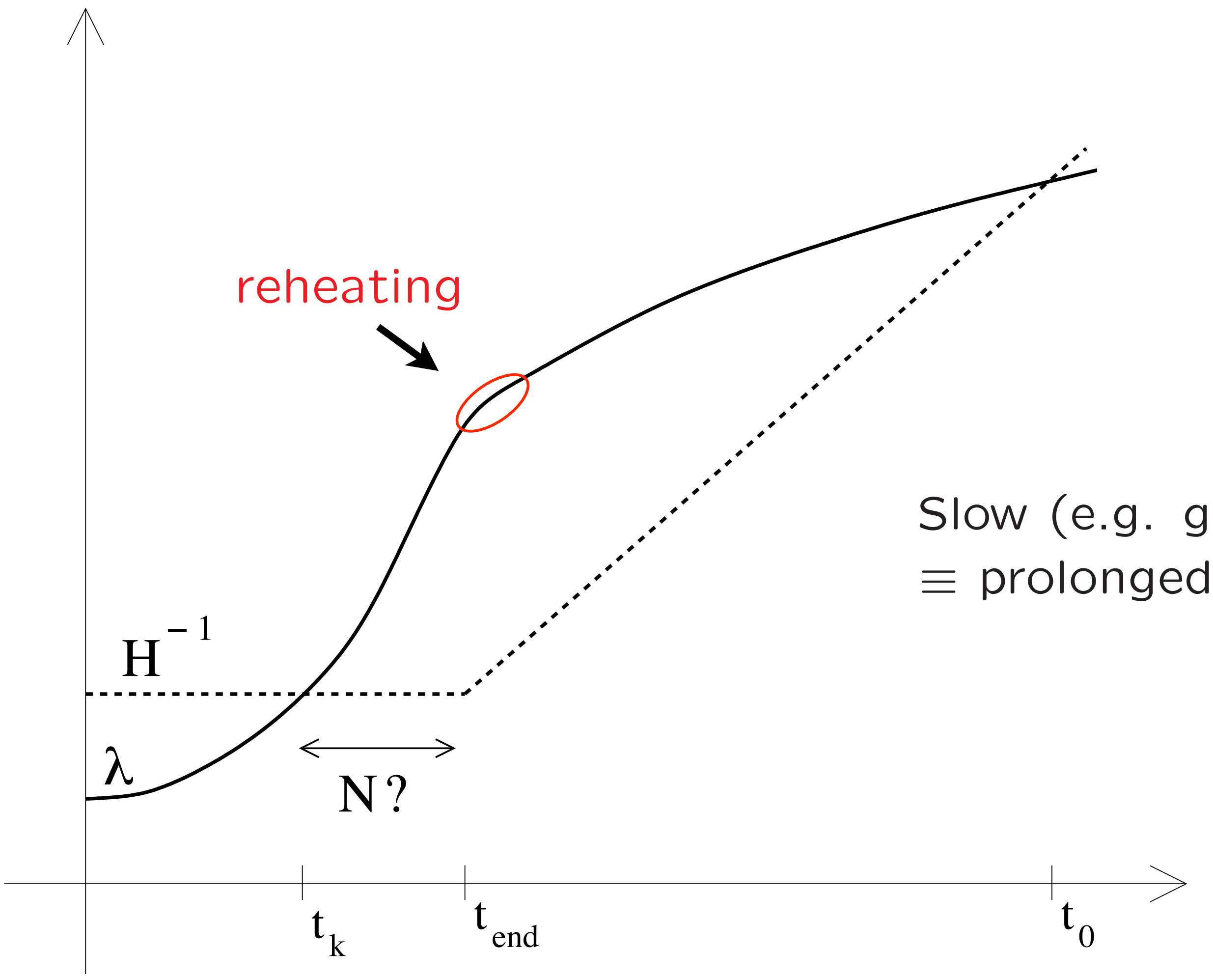
Large field models

$$V = \phi^\alpha \qquad n_s - 1 = -\frac{2+\alpha}{2\,N} \qquad r = \frac{4\,\alpha}{N}$$



WMAP3

Uncertainty on N



Slow (e.g. gravitational) inflaton decay
$\equiv$ prolonged MD stage

vs.

Immediate decay
at preheating

$\Delta N \approx 5 - 10$

Inflation $\xrightarrow{\text{REHEATING}}$ Hot big-bang cosmology

Unknowns:

Scale of inflation

Inflaton ϕ

Coupling to matter

Require:

$T >$ MeV, for Nucleosynthesis

No gravitinos, $T < 10^5 - 10^7$ GeV

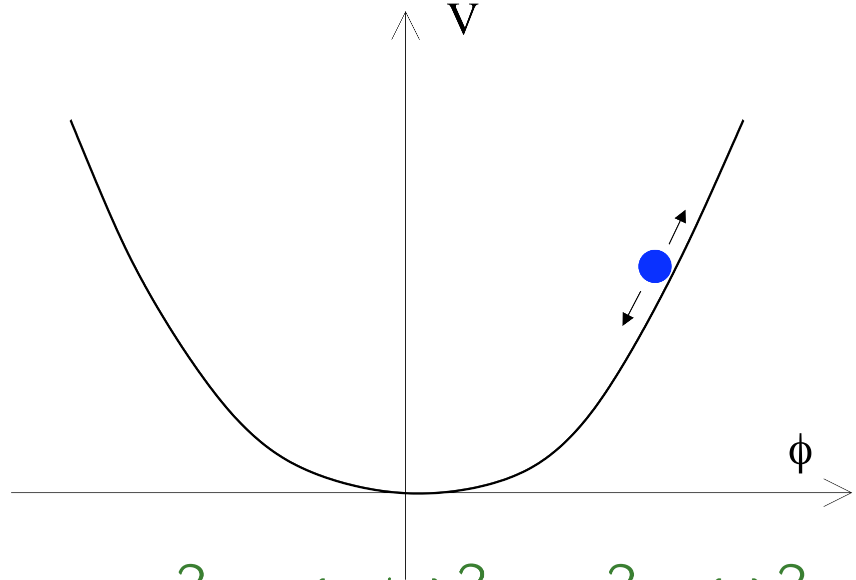
Baryon & dark–matter

1 Fast decay, slower thermalization

2 Slow decay, faster thermalization

Preheating

Kofman, Linde, Starobinsky '94; '97



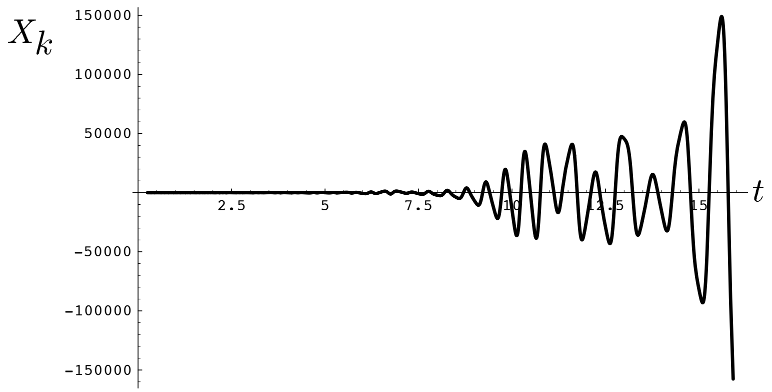
Resonant particle production due to coherent inflaton oscillations

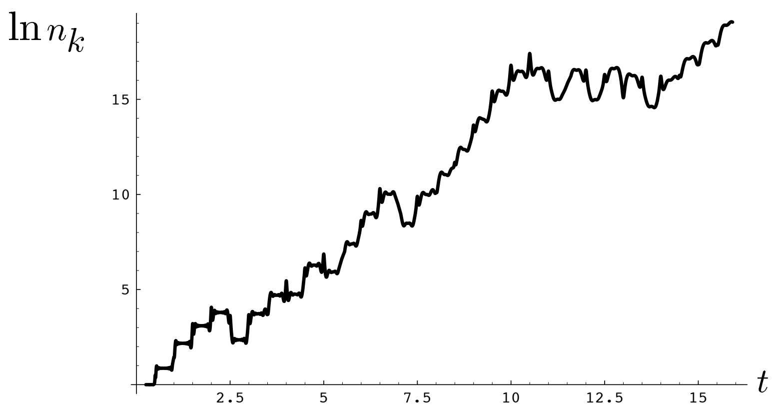
Parametric resonance

$$V = \frac{1}{2} m^2 \phi^2 + \frac{g^2}{2} \phi^2 \chi^2 \longrightarrow \omega_\chi^2 = (k/a)^2 + g^2 \phi(t)^2$$

- Nonadiabatic evolution of frequency $\omega' / \omega^2 \gg 1$ whenever $\phi \simeq 0$
- (Quasi) periodic effect $\rightarrow$ resonance

Large effect if $q \equiv \dfrac{g^2 \phi^2}{4 m^2} \sim 10^{10} g^2 > 1$

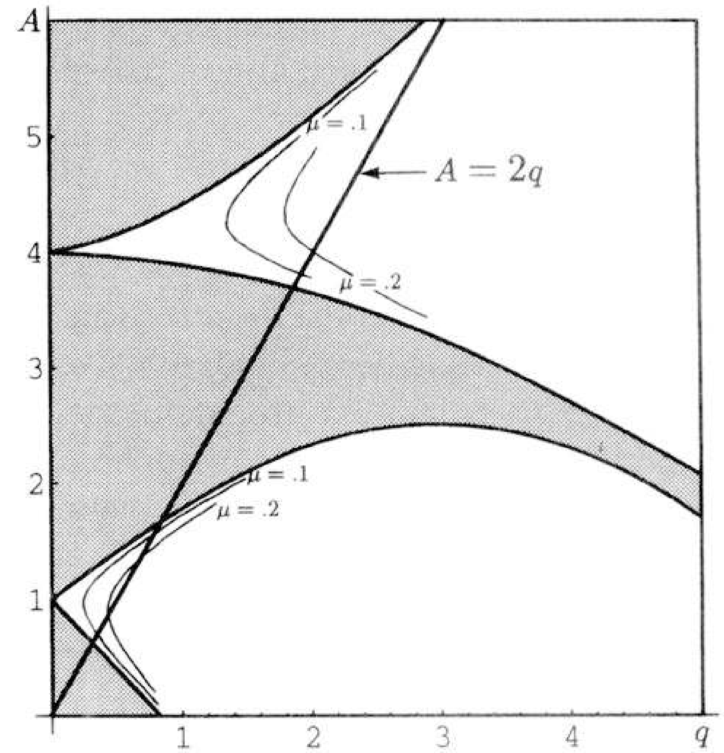




- Stimulated particle production
- Redshift of physical momenta:

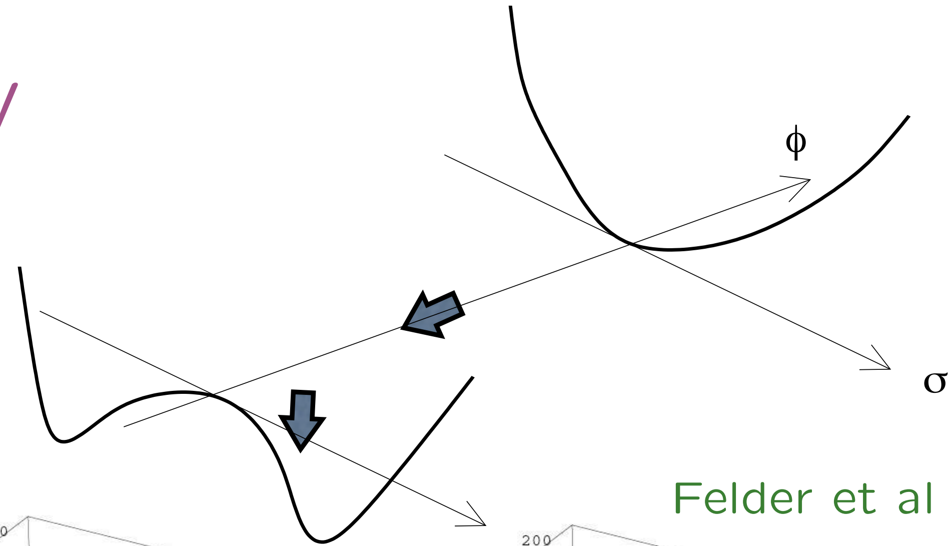
modes cross stability/instability bands

Stability / Instability chart



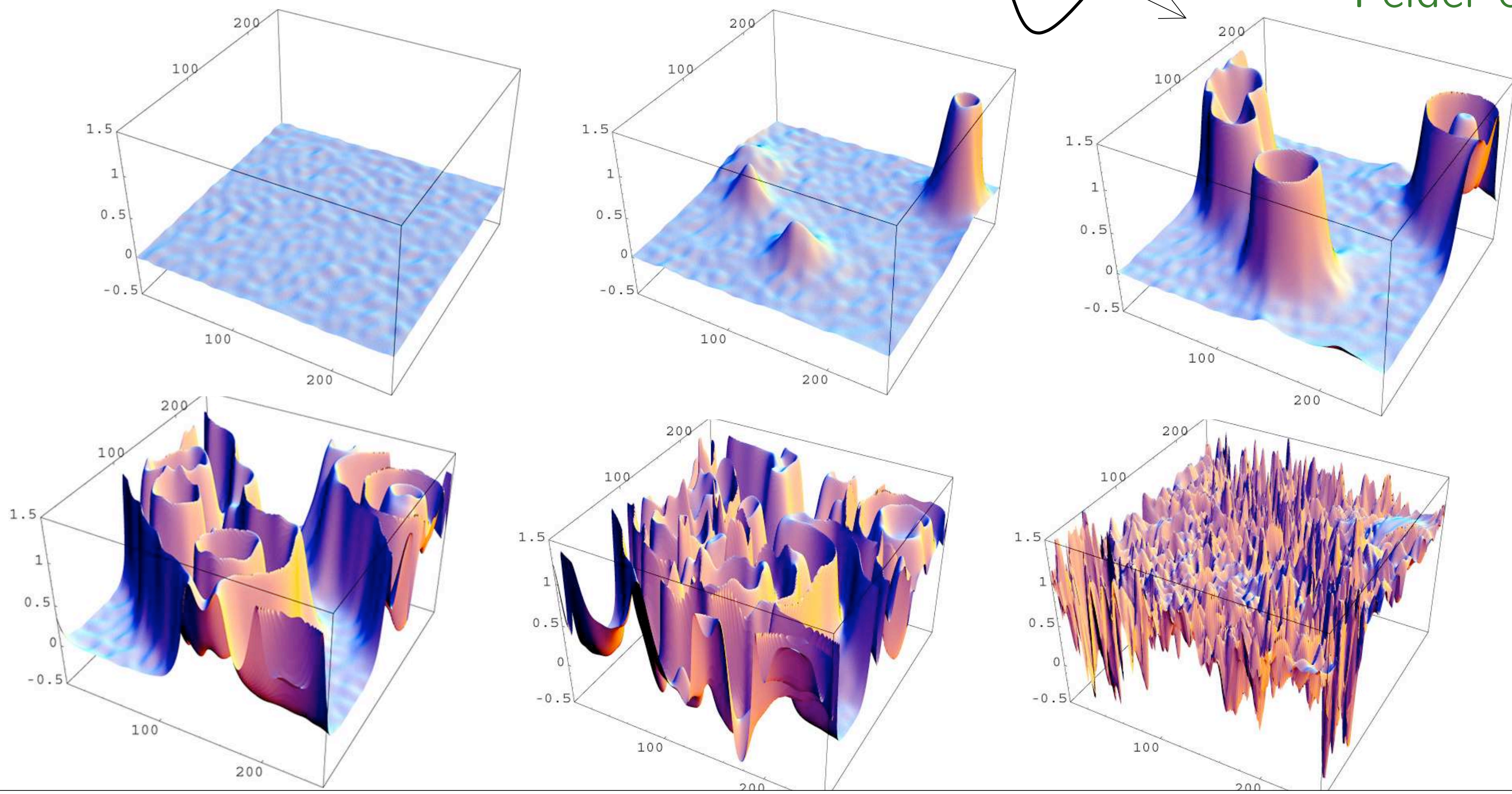
$$q \equiv \frac{g^2\,\phi^2}{4\,m^2} \quad , \quad A \equiv \frac{k^2}{m^2\,a^2} + 2\,q$$

Tachyonic instability



$$m_\sigma^2 < 0$$

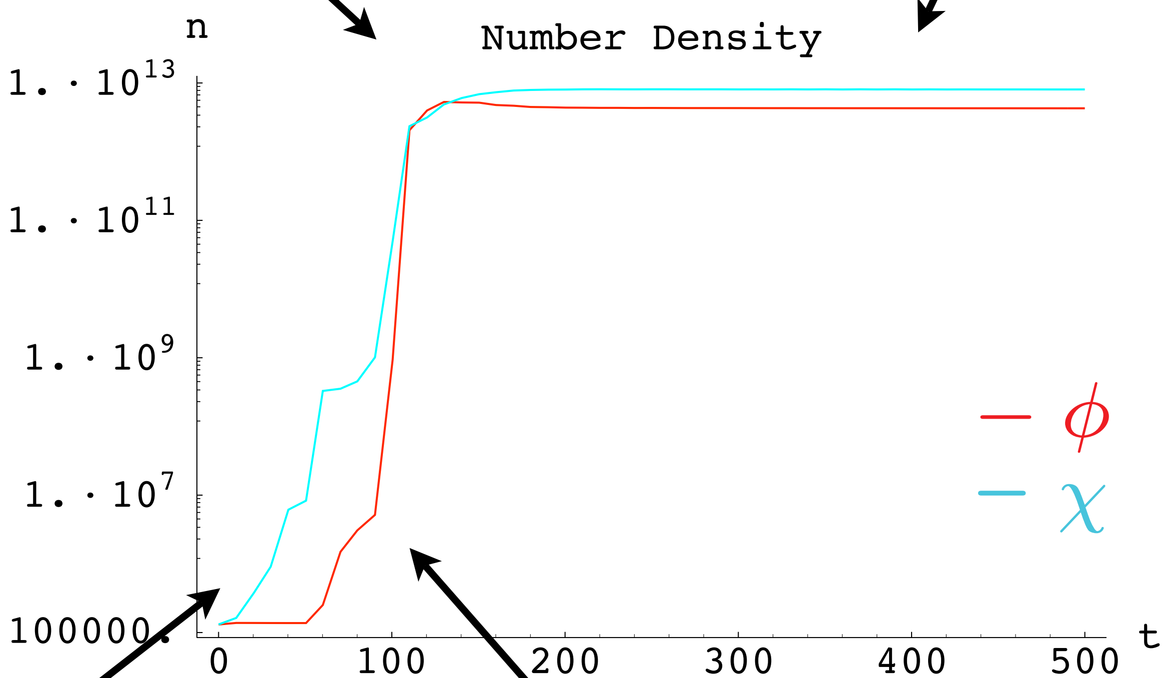
Felder et al 01



$$V = \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\chi^2$$

Nonlinear "bubbly" stage

Saturated distributions
(scaling behavior)



Thermalization

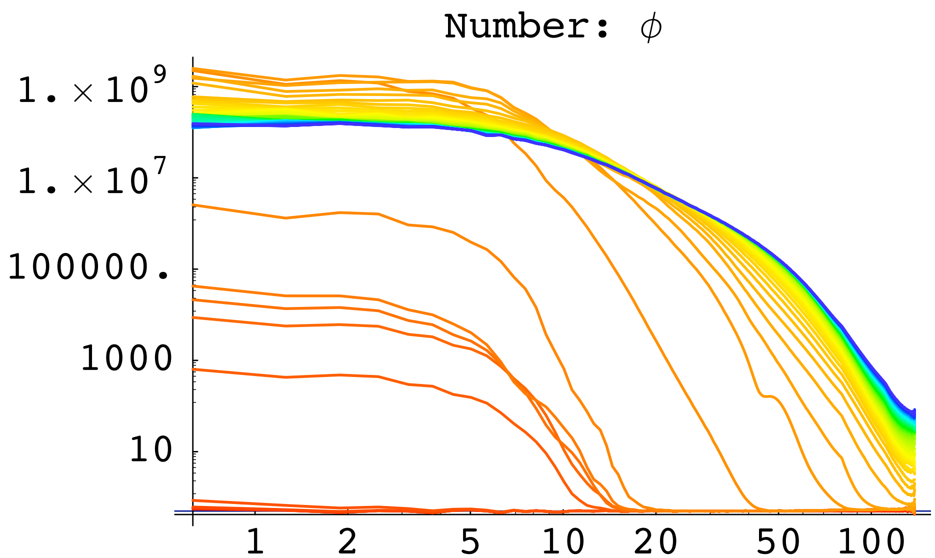
Preheating
(exp. production)

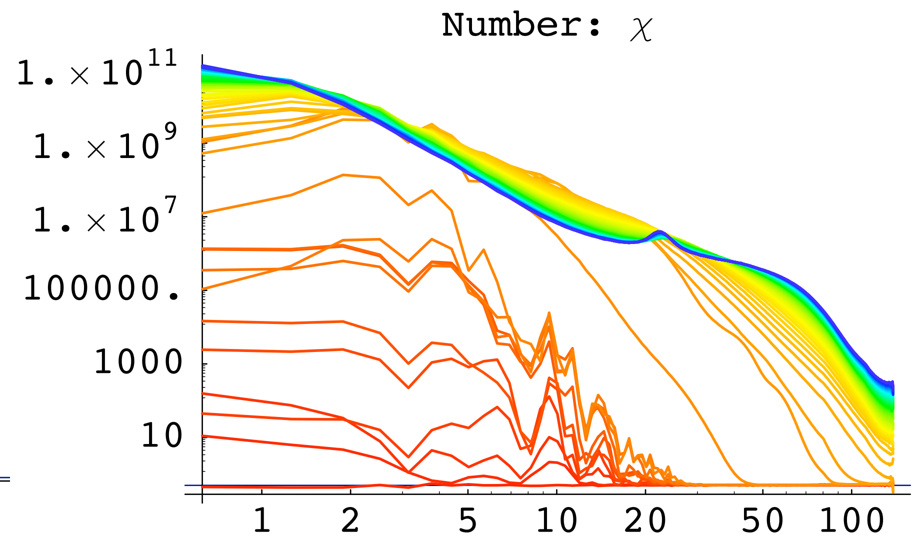
Rescattering
(destroys inflaton coherence)

Saturated distributions

with scaling behavior

$$n_k \sim 1/g^2 \gg 1$$

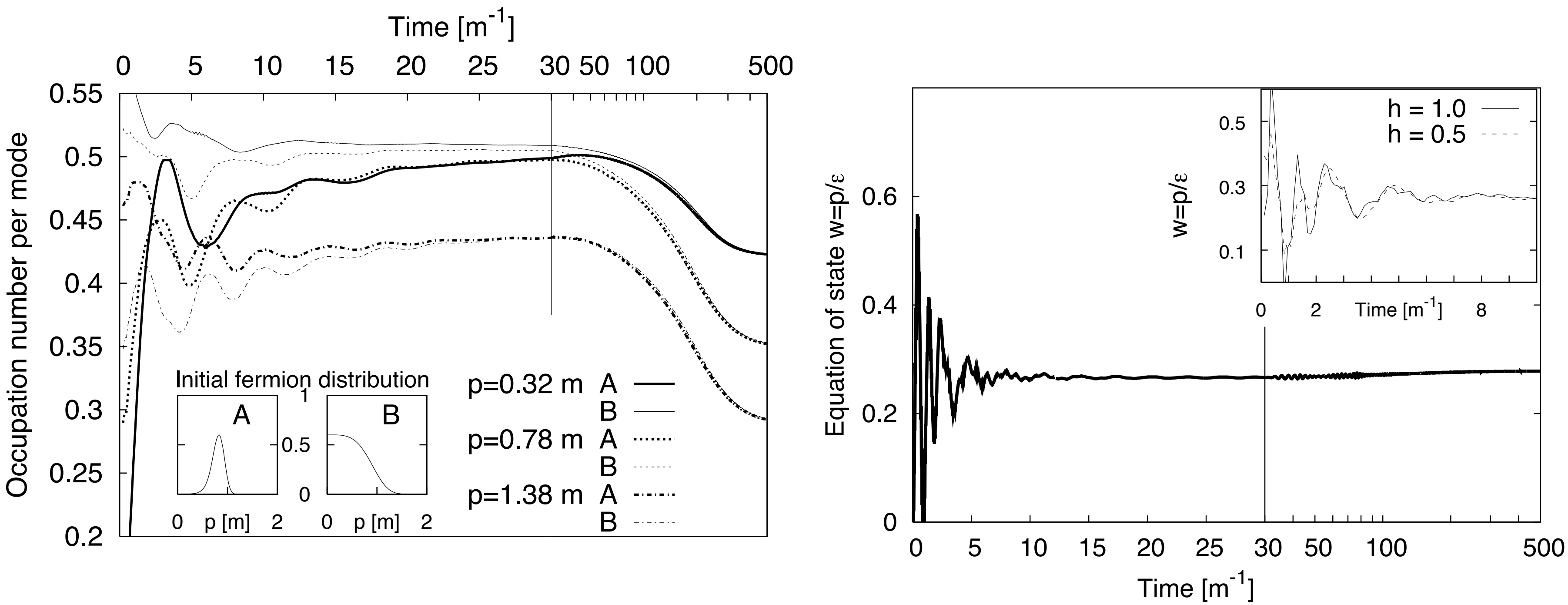




- In many models of inflation, mechanism that exponentially amplifies long–wavelength modes. Highly IR spectrum
- Excitation of χ $\rightarrow$ excitation of all other fields coupled to it (even if not directly coupled to inflaton)
- Once amplified, fields approach thermal equilibrium by scattering energy into higher momentum modes
- Nontrivial dependence on couplings (e.g., perturbative decay $h\,\phi\,\bar{X}\,X$, $\Gamma \propto h^2 \Rightarrow T_{\mathrm{rh}} \propto h$)
- Equation of state evolves towards radiation domination long before thermalization is established

Pre–thermalization

Two–flavors quarks $\leftrightarrow \sigma, \vec{\pi}$



Berges, Borsanyi, Wetterich '04

Intermediate equation of state

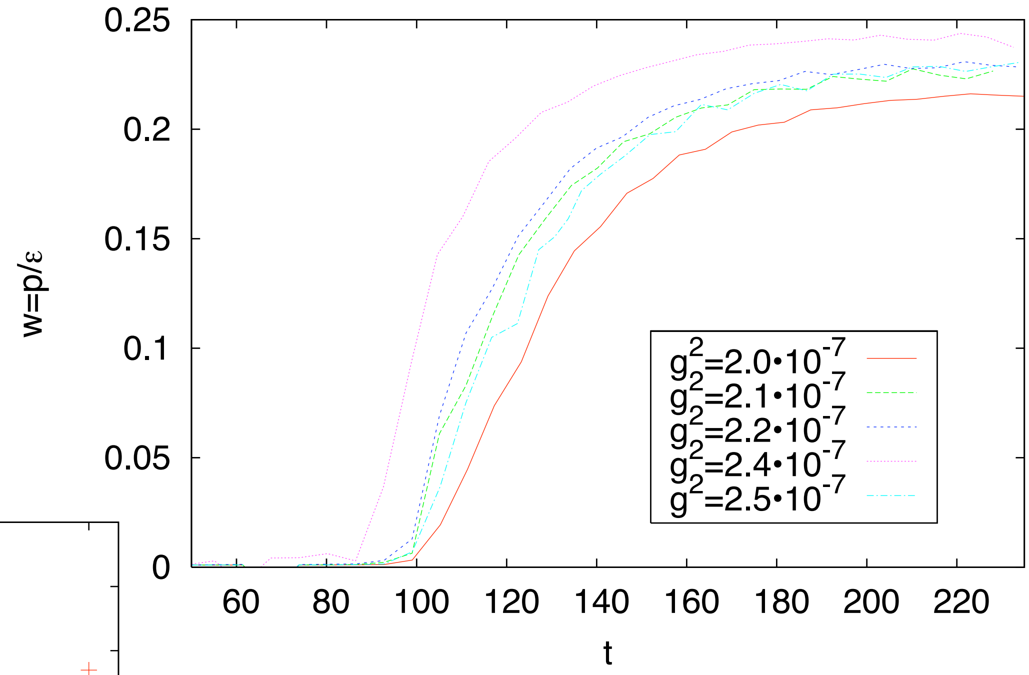
Podolsky, Felder, Kofman, MP '05

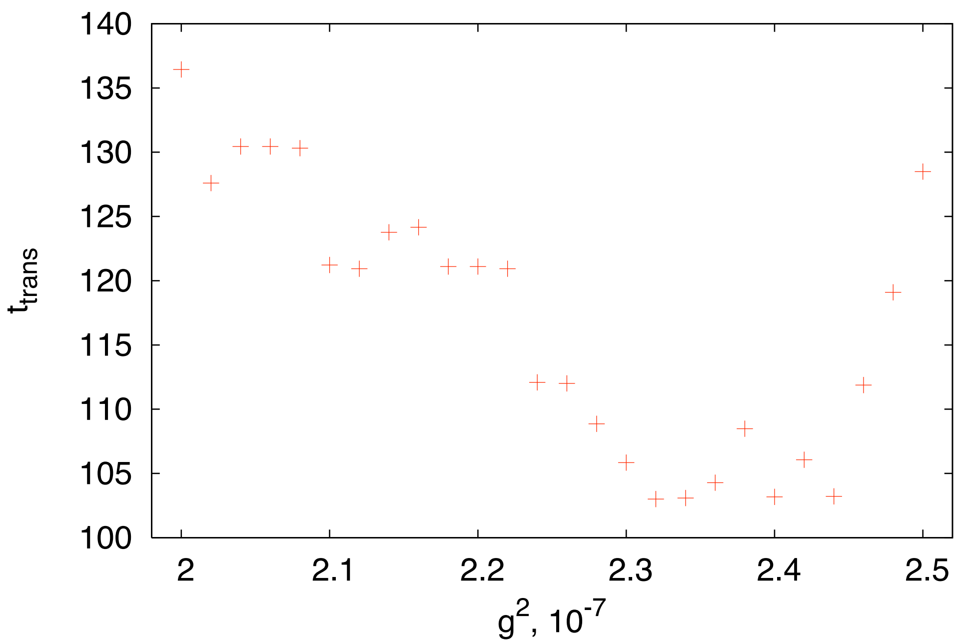
$$V = \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\chi^2$$

Between matter ($w = 0$)

and radiation ($w = 1/3$)

Expansion : $a = t^{\frac{2}{3(1+w)}}$





Transition time

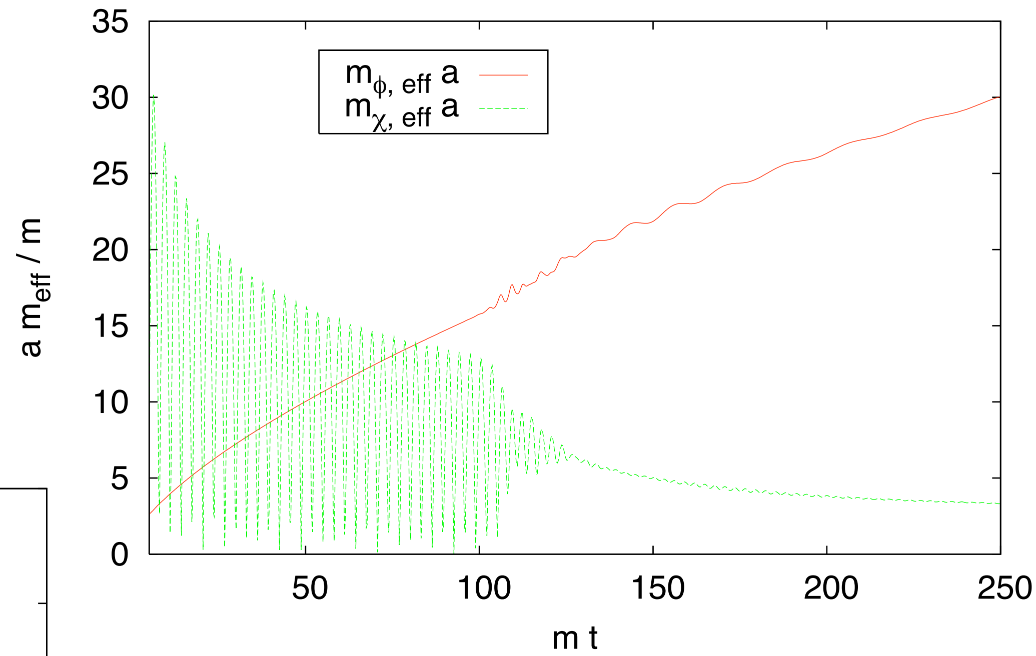
non monotonic in g^2

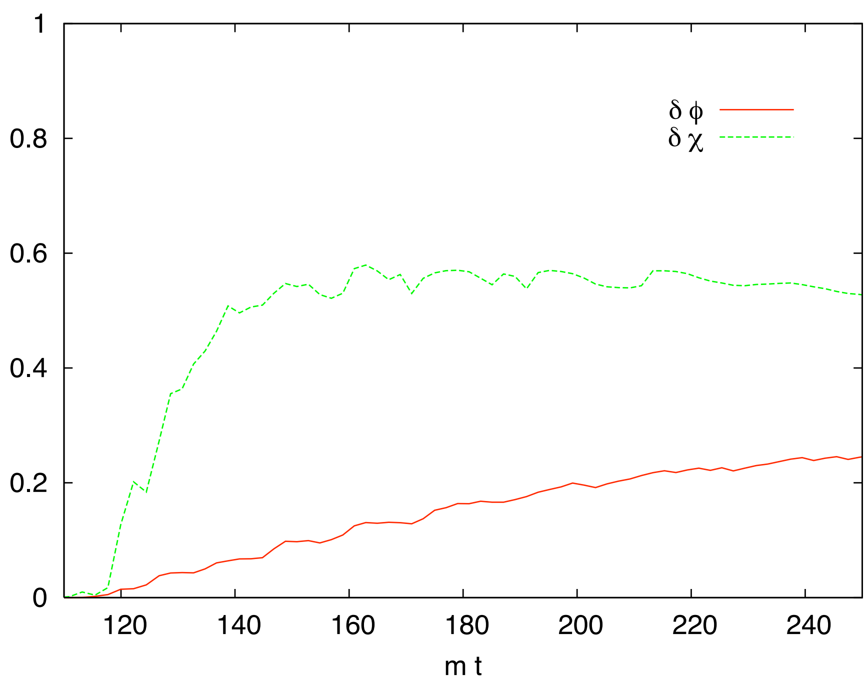
Effective masses

$$V = \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\chi^2$$

$$m^2_{\phi,\text{eff}} = m^2 + g^2\langle\chi^2\rangle$$

$$m^2_{\chi,\text{eff}} = g^2\langle\phi^2\rangle$$





Fraction of relativistic quanta

$$k/a > m_{\text{eff}}$$

$$\Gamma\,(\phi \to \chi\,\chi) \simeq \frac{g^4\,\Phi^2}{8\,\pi\,m} \propto a^{-3} \quad \text{while} \quad H \propto a^{-3/2}$$

Therefore, add trilinear term

$$V = \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\chi^2 + \frac{\sigma}{2}\phi\,\chi^2 + \frac{\lambda}{4}\chi^4$$

Dufaux, Felder, Kofman, MP, Podolsky '06

- Speeds up thermalization ($\chi + \chi \to \phi, \ldots$)
- New preheating, $m^2_{\text{eff},\chi} < 0$ at periodically recurring times

Quartic interaction contrasts this, but

Preheating mostly at $\phi \simeq 0$, where cubic term important

Expansion reduces the amplitude of ϕ

Tachyonic resonance

$$\omega_k^2 = k^2 + \sigma\,\phi$$

WKB approx.

$$\chi_k(t) \simeq \chi_k^j(t) = \frac{\alpha_k^j}{\sqrt{2\omega_k(t)}}\,\exp\left(-i\int_{t_0}^{t}\omega_k(t')dt'\right) + \frac{\beta_k^j}{\sqrt{2\omega_k(t)}}\,\exp\left(i\int_{t_0}^{t}\omega_k(t')dt'\right) \qquad \omega_k^2(t) > 0$$

$$\chi_k(t) \simeq \frac{a_k^j}{\sqrt{2\Omega_k(t)}}\,\exp\left(-\int_{t_{kj}^-}^{t}\Omega_k(t')dt'\right) + \frac{b_k^j}{\sqrt{2\Omega_k(t)}}\,\exp\left(\int_{t_{kj}^-}^{t}\Omega_k(t')dt'\right) \qquad \Omega_k^2(t) = -\omega_k^2(t) > 0$$

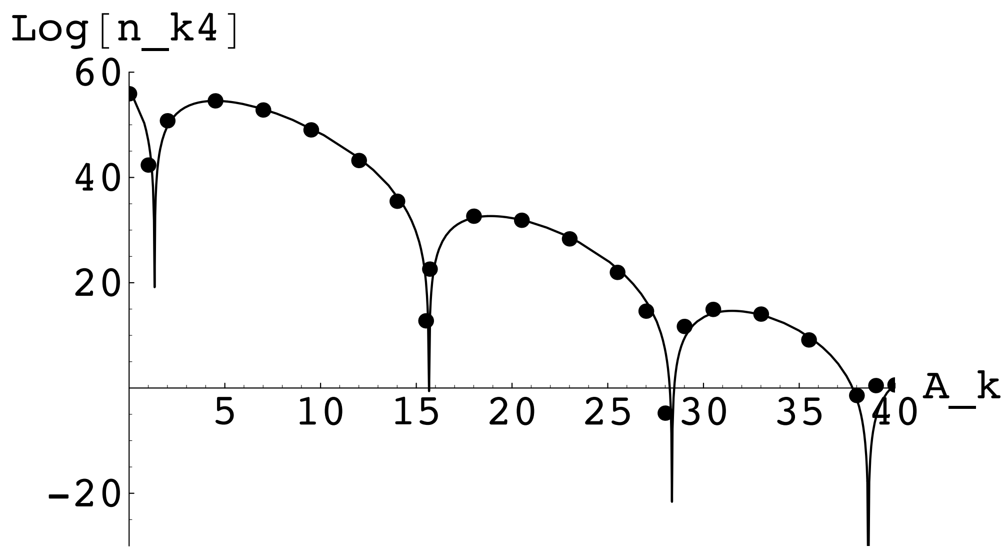
Parametric resonance: instability bands where $\phi\,(t)$ "resonate" with ω

Tachyonic resonance:

stability bands where

resonance counterbalances

the tachyonic instability

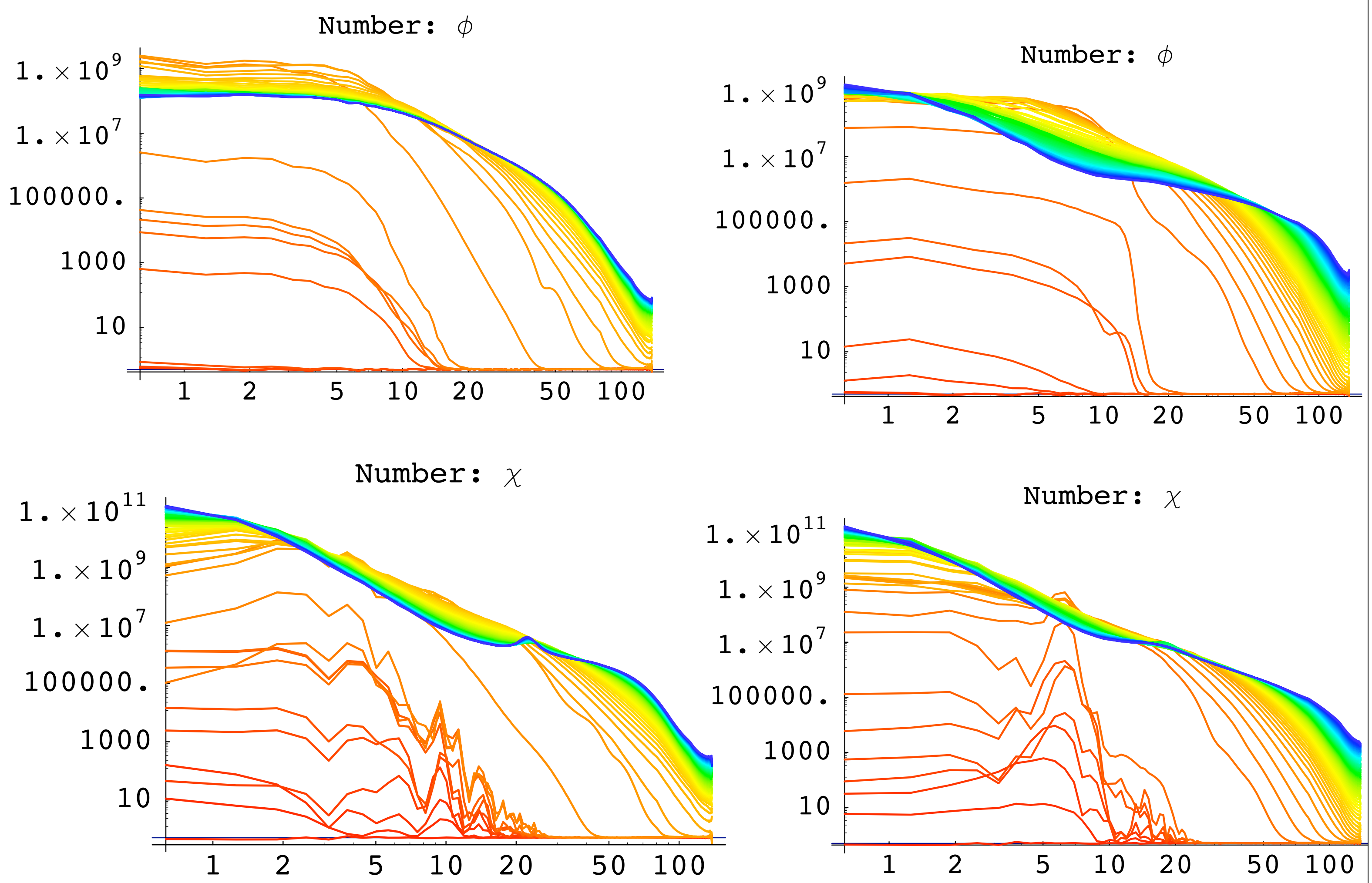


$g^2 \sim 10^{-7}$

$\sigma \sim 10^{-2}\, m_\phi$

$$\frac{g^2}{2}\phi^2\chi^2$$

$$\frac{g^2}{2}\phi^2\chi^2 + \sigma\,\phi\,\chi^2$$

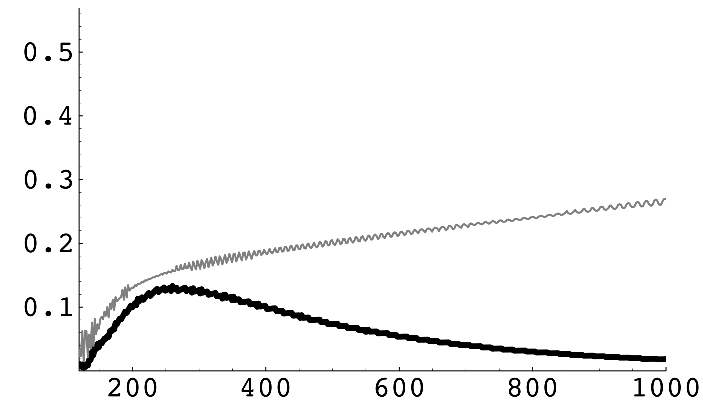


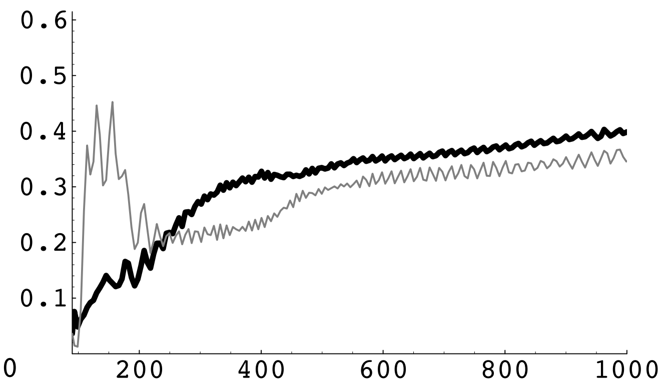
$g^2 \sim 10^{-7}$

$\sigma \sim 10^{-2}\, m_\phi$

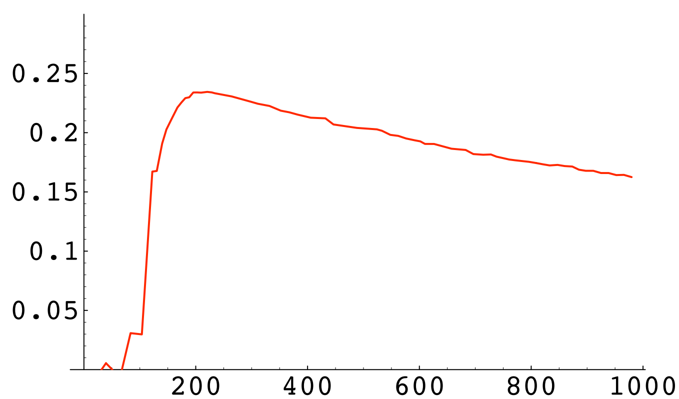
$$\frac{g^2}{2}\phi^2\chi^2 \qquad\qquad \frac{g^2}{2}\phi^2\chi^2 + \sigma\,\phi\,\chi^2$$

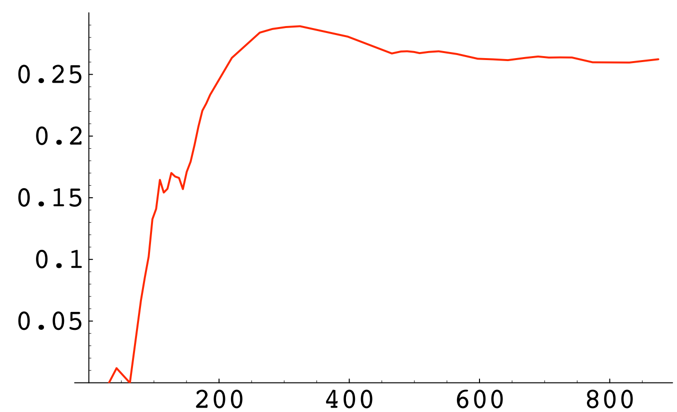
Fraction of relativistic modes





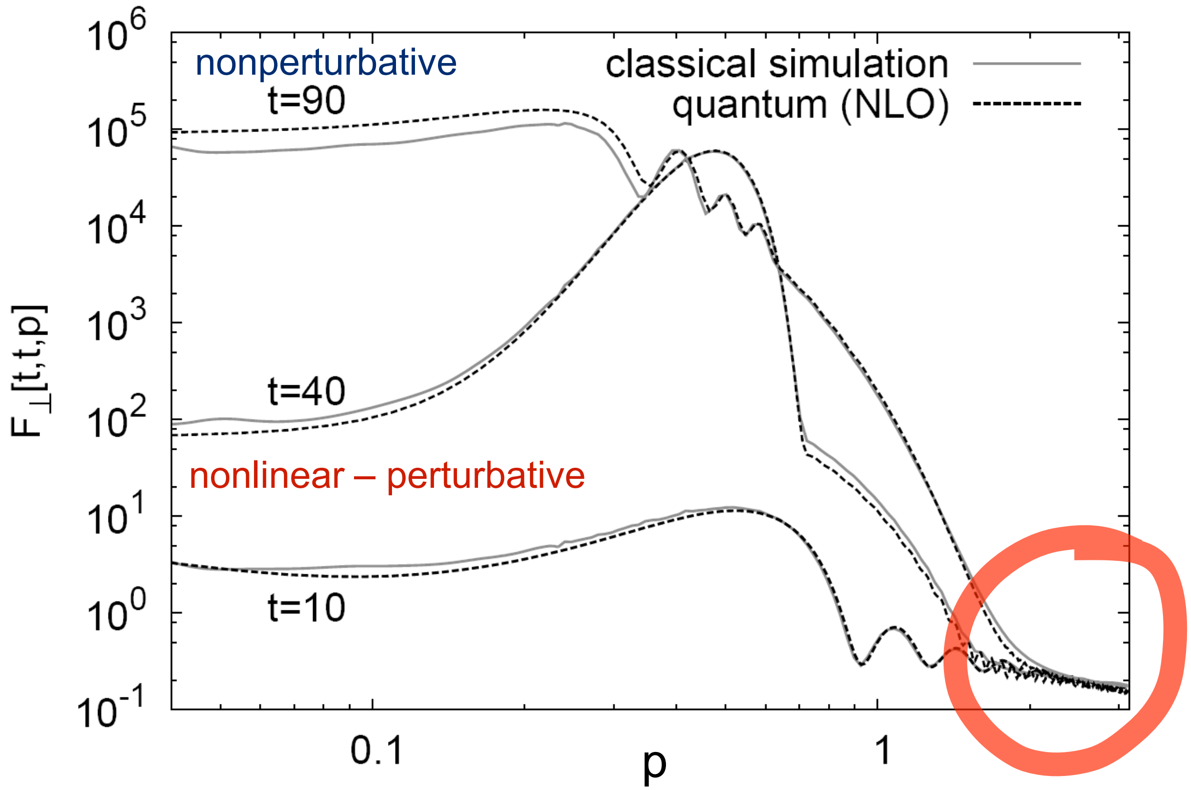
Equation of state





Comparison quantum/classical dynamics

Classical-statistical simulations: Khlebnikov, Tkachev '96; Prokopec,Roos '97; Tkachev, Khlebnikov, Kofman, Linde '98; …



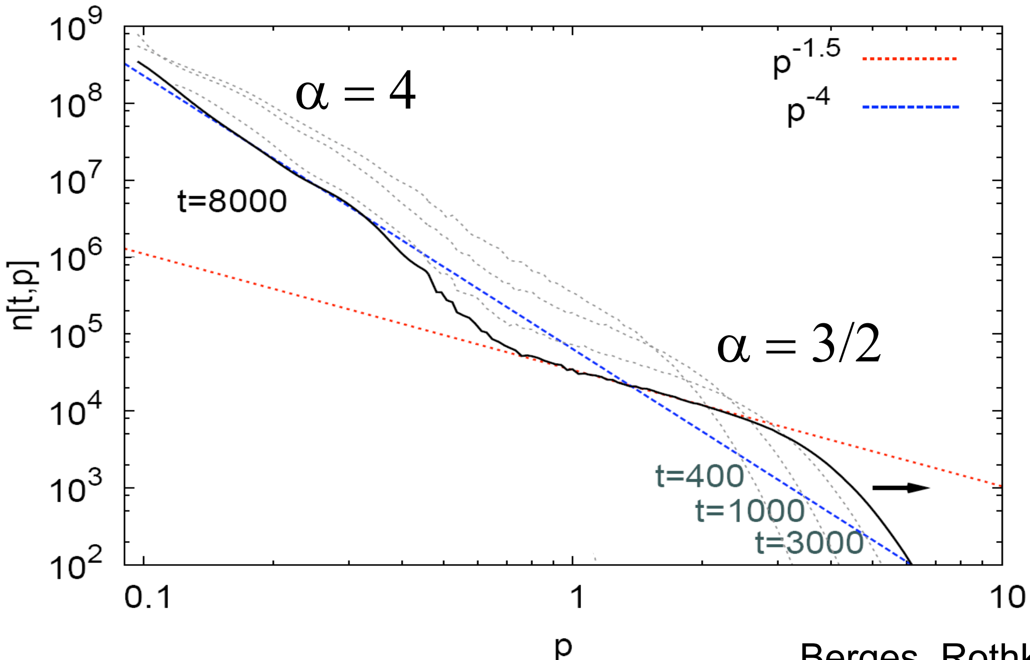
Berges, Rothkopf, Schmidt '08

Practically no quantum corrections at the end of preheating

Accurate nonperturbative description by 2PI 1/*N* to NLO

(Berger's talk @ KITP)

Comparison analytical/simulation results



Berges, Rothkopf, Schmidt '08

Late-time behavior well characterized by non-thermal fixed points!

UV: α = 3/2 coincides with perturbative (Boltzmann) analysis exponent

a) local four-leg interaction $\Rightarrow$ α = 0, 1, 4/3, 5/3
b) local three-leg interaction $\Rightarrow$ α = 1, 3/2

Micha, Tkachev '04

(Berger's talk @ KITP)

- Nonthermal fix points; “stuck” there if small couplings

- Thermalization in real life (gauge couplings, fermions).

  What timescale ? Reheating temperature ?

- Several issues, for which T_{rh} not essential

After all, huge energy density stored in these distributions, while

$\ll$ energy density at the time of thermal equilibrium

(dramatic shift from more traditional perturbative reheating)

Production of super–heavy particles

- Many models of baryogenesis require heavy masses;

thermal production can be in conflict with bounds on

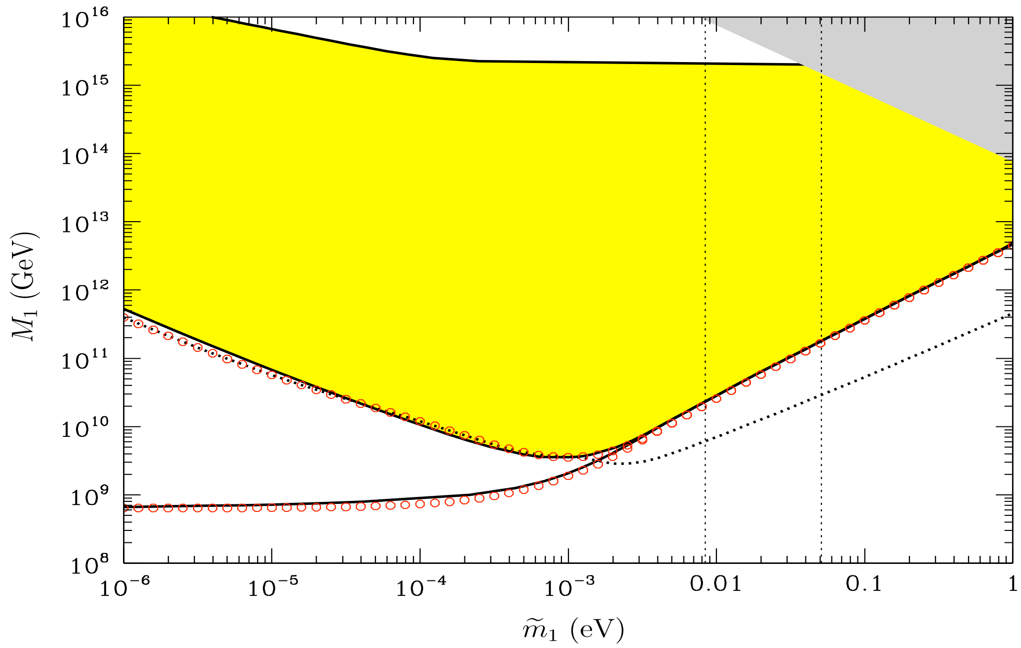
T_{rh} from gravitino problem

E.g. Thermal leptogenesis

from r.h. neutrinos

Buchmuller, Di Bari, Plumacher '04

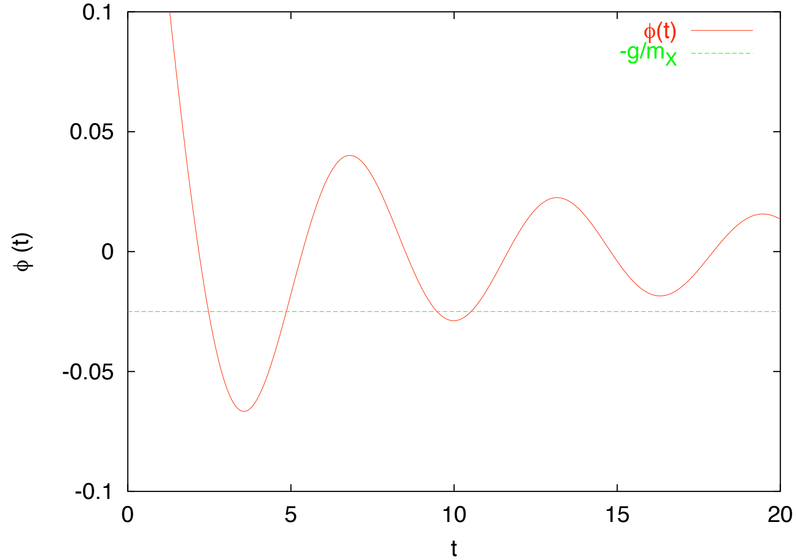
(main reason, $\delta_{\mathrm{CP}} \propto m_{N_1}$)



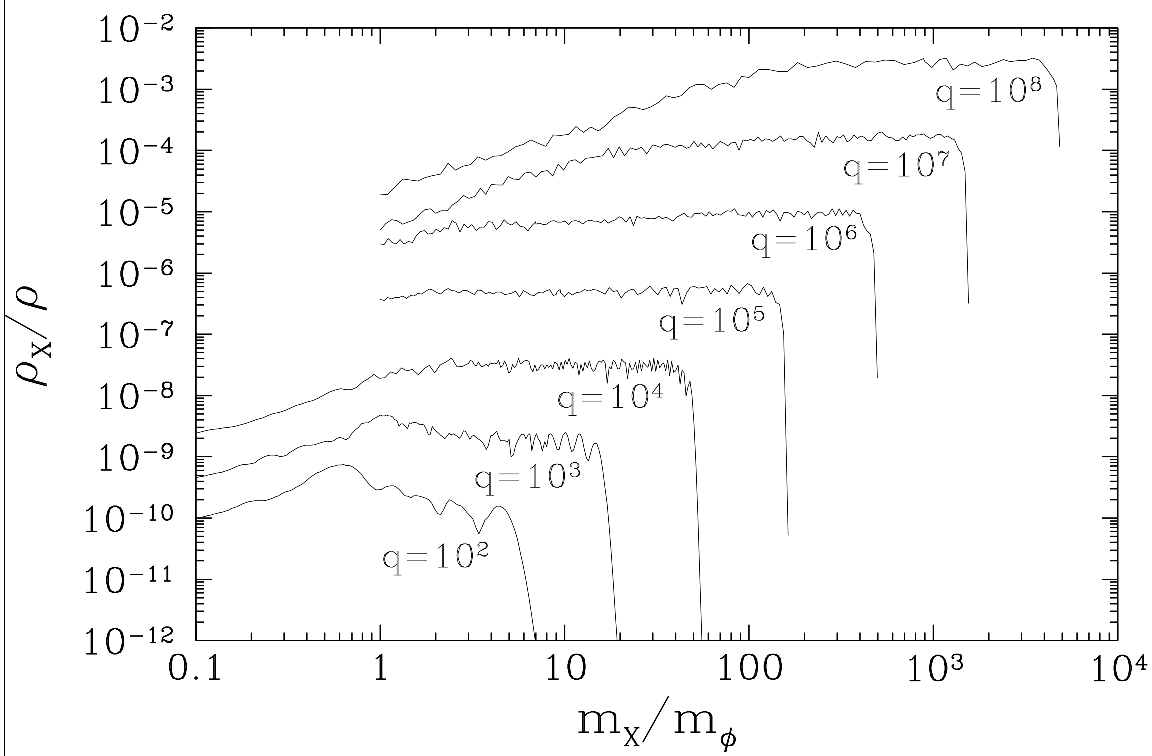
- GUT Baryogenesis $M_{\mathrm{GUT}} \gtrsim 10^{14}\,\mathrm{GeV}$

$$\mathcal{L}_{\phi X} = (m_X + g\,\phi)\,\bar{X}\,X$$

Preheating $\leftrightarrow \dot{\omega}_k \gtrsim \omega_k^2 \leftrightarrow \dot{m} \gtrsim m^2$



Giudice, M.P., Riotto, Tkachev, '99



$$(m_X)_{\rm max} \sim q^2 = g\,\phi_0/2$$

$$\frac{\rho_X}{\rho_\phi} \propto q\,m_X^{1/2}\left[\log\frac{q^{1/2}}{m_X}\right]^{3/2}$$

M.P., Sorbo '00

Greene, Kofman '98, '00

Gravitational waves

Khlebnikov, Tkachev '97; Easther, Lim '06; Easther, Giblin, Lim '06;

Garcia–Bellido, Figueroa '07; Garcia–Bellido, Figueroa, Sastre '07;

Dufaux, Bergman, Felder, Kofman, Uzan '07

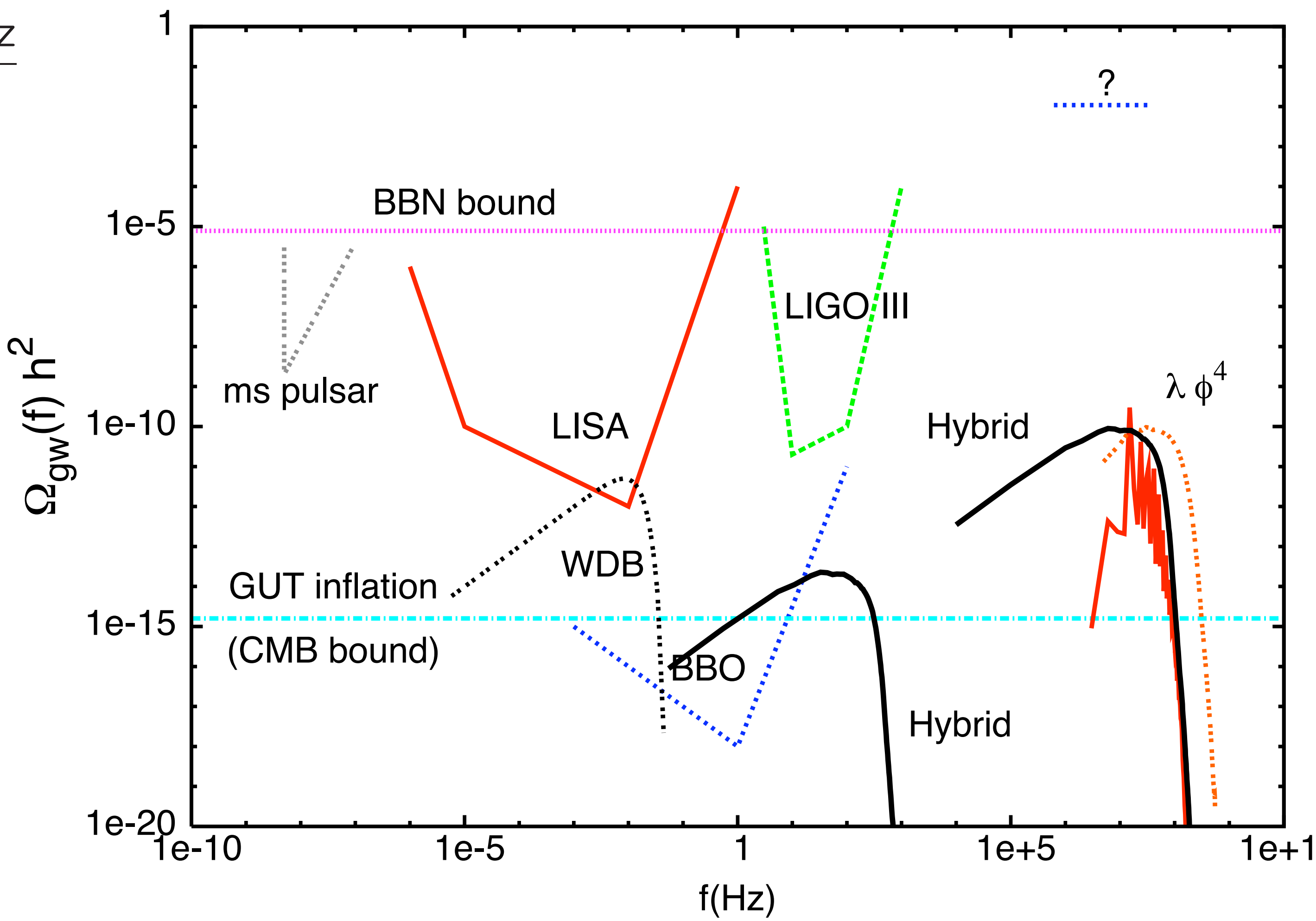
$$h''_{ij} + 2\,\frac{a'}{a}\,h'_{ij} - \nabla^2 h_{ij} = 16\,\pi\,G\left(T_{ij} - \langle p\rangle\,g_{ij}\right)^{TT}$$

$$T_{\mu\nu} = \partial_\mu\phi_a\,\partial_\nu\phi_a - g_{\mu\nu}\,\left(\frac{1}{2}\,g^{\rho\sigma}\,\partial_\rho\phi_a\,\partial_\sigma\phi_a + V\right)$$

$$\rho_{\rm gw} = \frac{1}{32\pi G a^4}\,\langle\bar{h}'_{ij}(\tau,\mathbf{x})\,\bar{h}'_{ij}(\tau,\mathbf{x})\rangle_V = \frac{1}{32\pi G a^4}\,\frac{1}{V}\,\int d^3\mathbf{k}\,\bar{h}'_{ij}(\tau,\mathbf{k})\,\bar{h}'^{*}_{ij}(\tau,\mathbf{k})$$

Evaluated through realizations on the lattice

$$f_{\text{peak}} \simeq \frac{4 \times 10^{10}\,\text{Hz}}{R\,\rho^{1/4}}$$

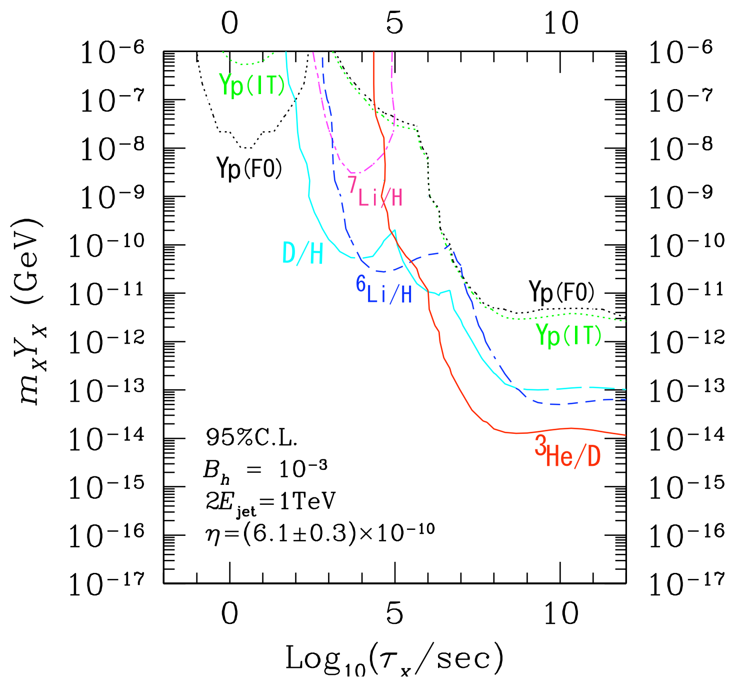


Garcia–Bellido, Figueroa '07

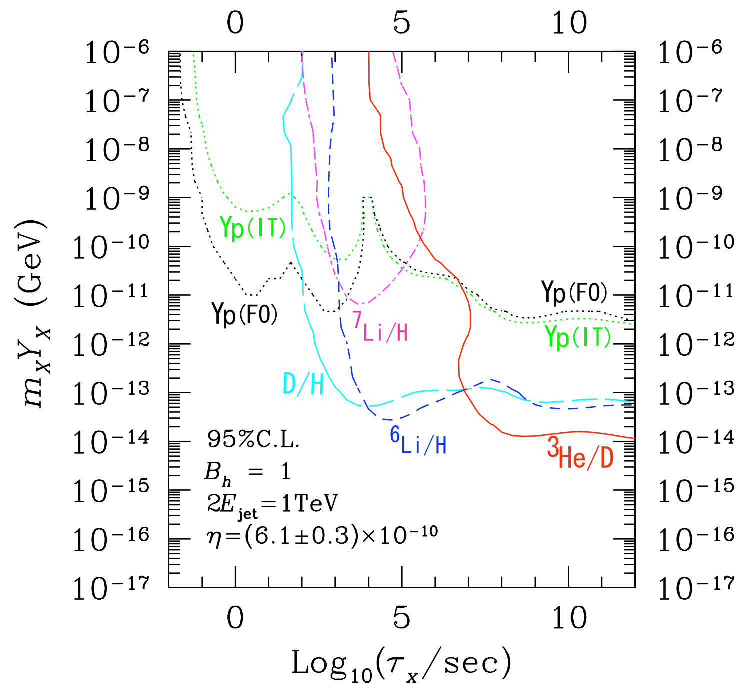
The gravitino problem

Heavy late decaying particles alter the BBN abundances

Radiative decay



Hadronic decay

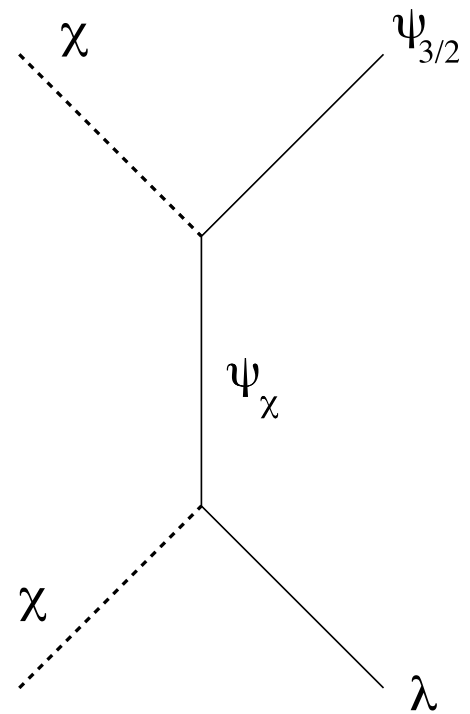


Kawasaki, Kohri, Moroi '04

$$\tau_{3/2} = 4 \times 10^5\,\text{sec} \times N_G^{-1} \left(m_{3/2}/\text{TeV}\right)^3$$

$$m_{3/2} = \text{TeV} \;\Rightarrow\; Y_X \lesssim 10^{-14}(\text{radiative})\;,\; Y_X \lesssim\Rightarrow 10^{-17}(\text{hadronic})$$

Gravitino thermal production



$$\frac{dn_{3/2}}{dt} + 3H\cancel{n_{3/2}} = \langle \sigma \, |v| \rangle \, n_T^2$$

$$\underbrace{\langle \sigma \, |v| \rangle}_{\frac{10^{-2}}{M_p^2}} \qquad \underbrace{n_T^2}_{T^6}$$

Most of the production in the first Hubble time ($H^{-1} \sim M_p/T^2$)

$$n_{3/2} \approx \frac{10^{-2}\,T^6}{M_p^2} \times \frac{M_p}{T^2} \;\Rightarrow\; \frac{n_{3/2}}{s} \approx 10^{-2}\,\frac{T}{M_p} \;\Rightarrow\; T \lesssim 10^{5-7}\,\text{GeV}$$

- Small $T_{\rm rh}$ $\equiv$ late inflaton decay; energy "frozen" in the coherent oscillations until diluted by expansion

- Rescattering $\rightarrow$ distributions far from thermal; but $\rho \gg \left(10^9\,\text{GeV}\right)^4$. Compute gravitino production when they form, $t_* \simeq 120/m$.

$$V = \frac{1}{2} m^2 \phi^2 + \frac{g^2}{2} \phi^2 \chi^2 + h\, \chi\, \bar{\psi}_\chi\, \psi_\chi$$

Yukawa interaction as 2nd vertex

Rough estimates suggest overproduction for $h \gtrsim 10^{-7}$

(limit on g in concrete models)

Other applications

- Non–gaussianity (Chambers' talk)
- Modulated perturbations
- Primordial magnetic fields (Smit's talk)
- Electroweak baryogenesis
- Nonthermal symmetry restoration

Lot of fun !

Conclusions

- Cosmology $\equiv$ nonequilibrium physics

- We know that our universe thermalized, but

  Only direct evidence $T_{\mathrm{rh}} > \mathrm{MeV}$

  Theoretical input needed (model $\to T_{\mathrm{rh}}$)

- Several questions asked / answered even without this knowledge