Universality without symmetry

Pavel Kovtun,
University of Victoria

Non-equilibrium phenomena in cosmology and particle physics
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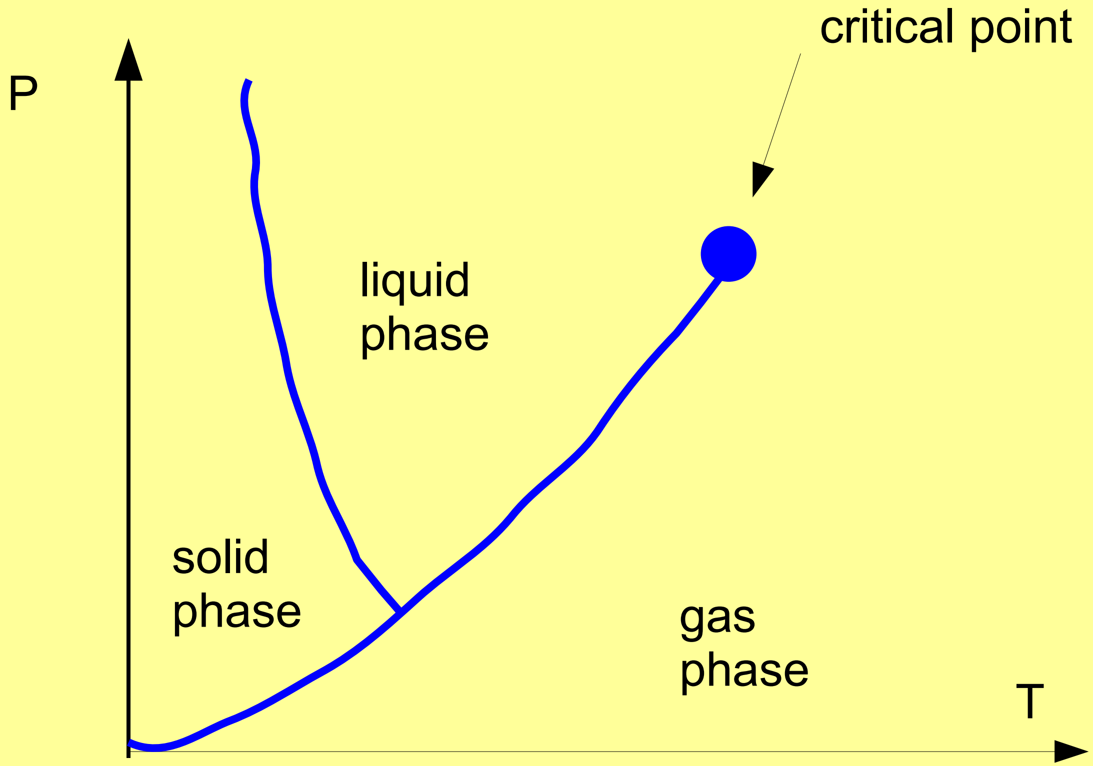
What questions we ask



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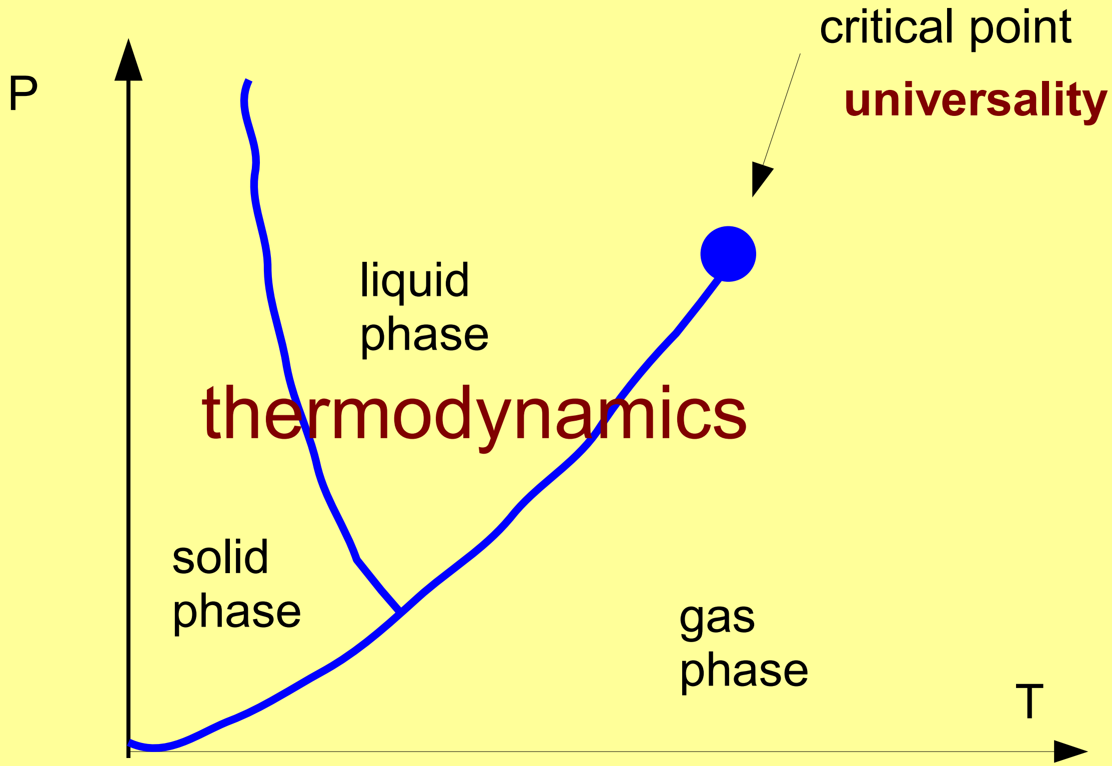
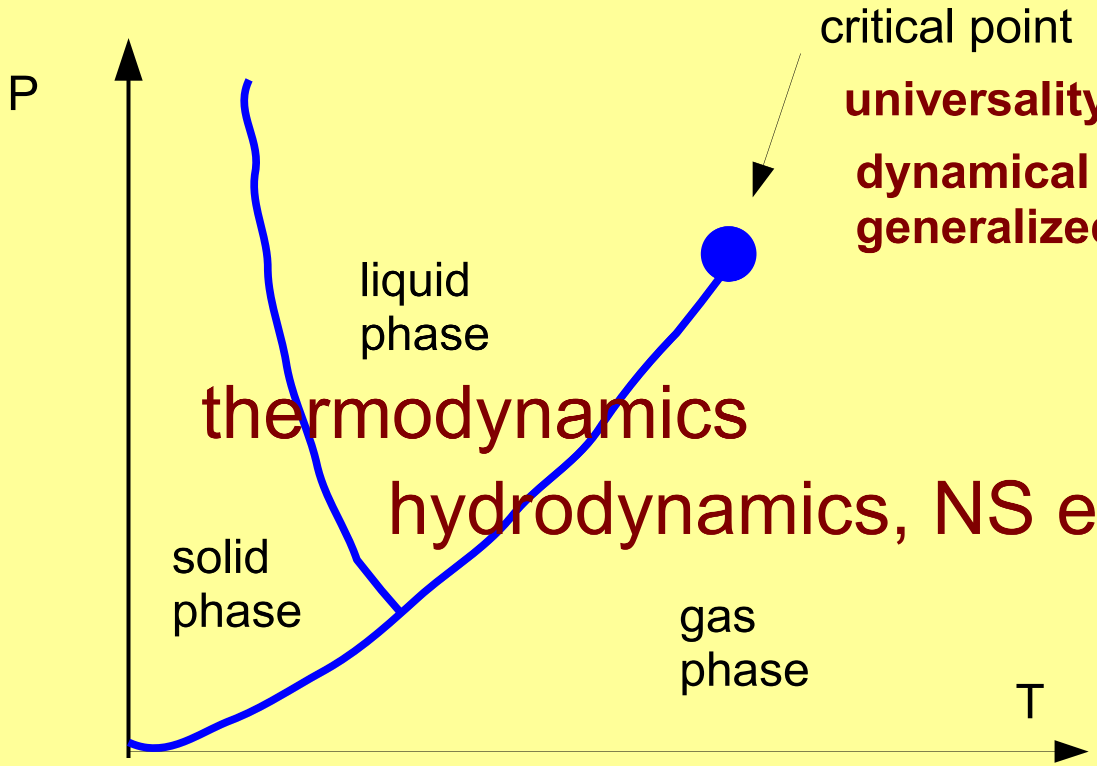


universality classes, critical exponents

What questions we ask



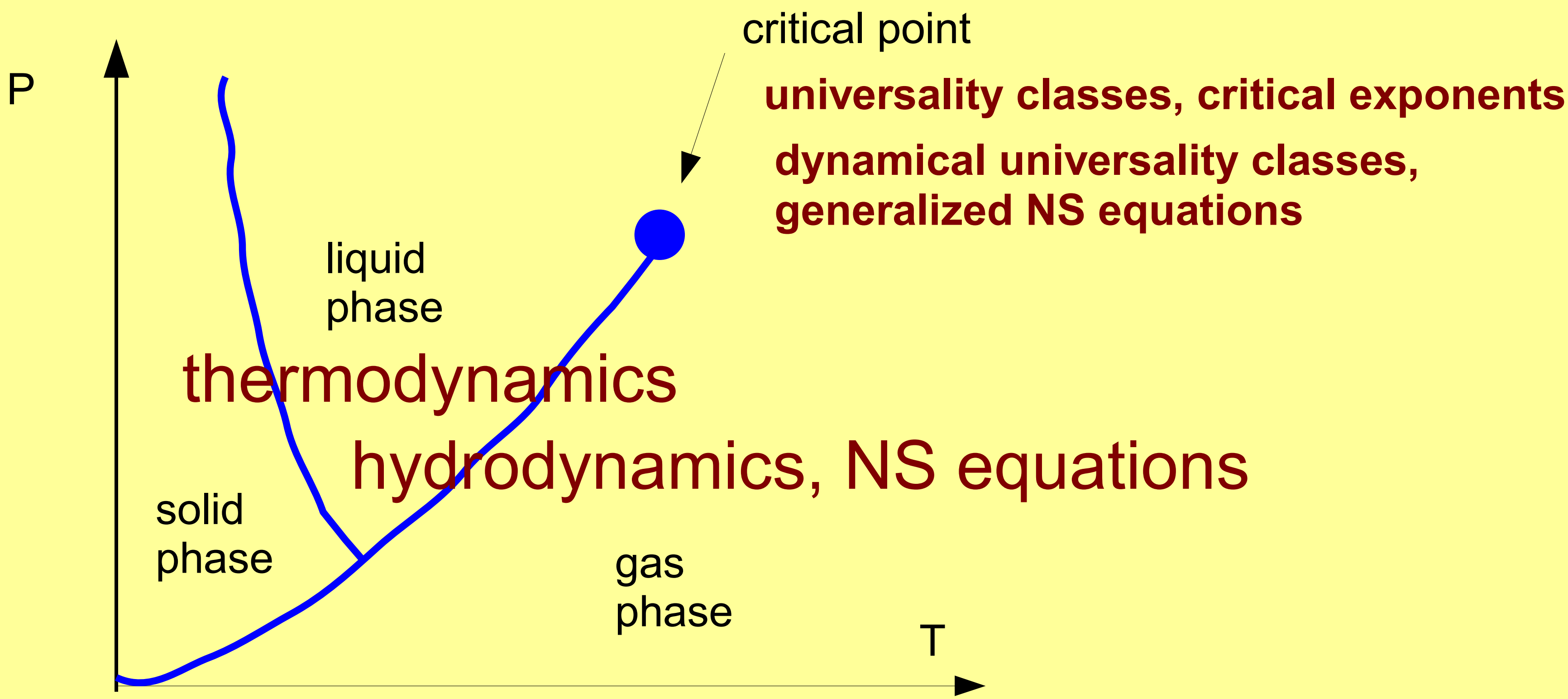
universality classes, critical exponents

dynamical universality classes, generalized NS equations

thermodynamics

hydrodynamics, NS equations

What questions we ask



The notion of universality pertains to the **functional form** of the basic observables, e.g. $C \sim |T-T_c|^{-\alpha}$ or the form of the NS equations

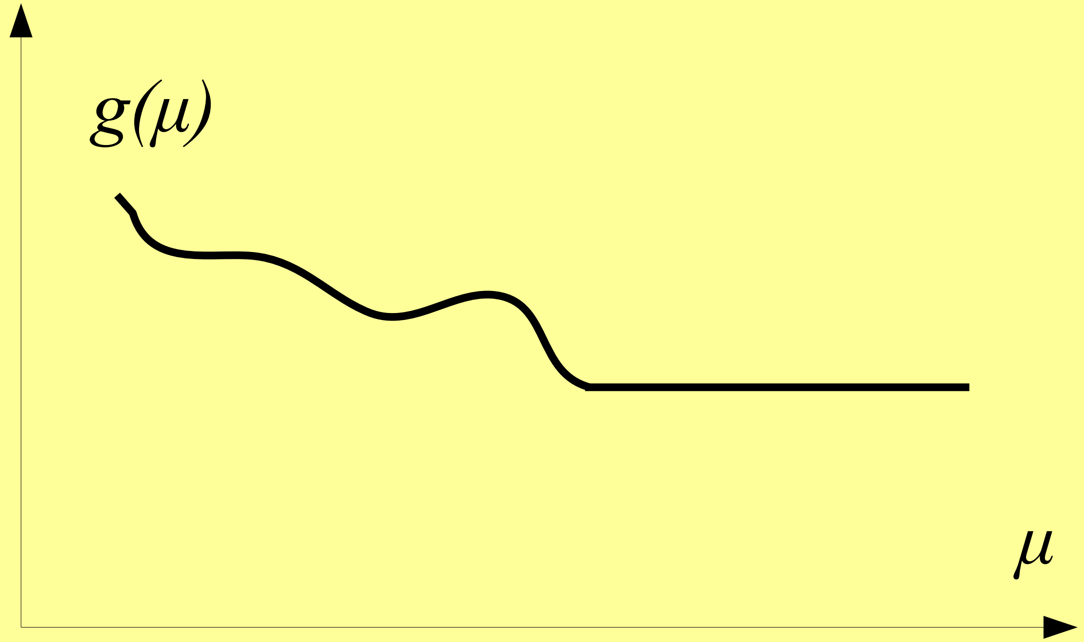
In this talk: a novel notion of universality which pertains not to the functional form, but to **thermodynamic/transport coefficients**.

Outline

- Universality in thermodynamics
- Universality in charge transport
- Universality in momentum transport

Universality in Thermodynamics

- Need to find dimensionless quantities in thermodynamics
- Look at field theories which are either CFT or relevant deformations of a CFT (fixed point in the UV)



E.g., QCD is a (marginally) relevant deformation of a non-interacting CFT

or think about Wilson-Fisher fixed point

Universality in Thermodynamics

Take T=0 first. At short distances:

$$\langle T_{\mu\nu}(x)\, T_{\alpha\beta}(0)\rangle = \frac{c}{x^{2d}}\, \frac{P_{\mu\nu,\alpha\beta}}{\mathrm{Vol}(S^{d-1})}$$

$$P_{\mu\nu,\alpha\beta} = (\delta_{\mu\alpha} - \frac{2x_\mu x_\alpha}{x^2})(\delta_{\nu\beta} - \frac{2x_\nu x_\beta}{x^2}) + (\delta_{\mu\beta} - \frac{2x_\mu x_\beta}{x^2})(\delta_{\nu\alpha} - \frac{2x_\nu x_\alpha}{x^2}) - \frac{2}{d}\delta_{\mu\nu}\delta_{\alpha\beta}$$

At short distances $<T_{\mu\nu}T_{\alpha\beta}>$ is uniquely fixed by one dimensionless number c — the central charge

Universality in Thermodynamics

Now take $T \neq 0$. Free energy:

$$F = -V\, T^d\, f(m/T)$$

As $T \to \infty$, $f(m/T) \to c' = const$

The high-temperature thermodynamics is uniquely specified by one dimensionless number, c'.

Universality in Thermodynamics

So the short-distance properties are characterized by c, while the long-distance (thermodynamic) properties by c'.

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But scale invariance means that there is no distinction between short and long distance. Are c and c' related to each other?

In d=2, zero temperature and finite temperature are related by a symmetry transformation, and one finds:

Blote+Cardy+Nightingale, 1986

Affleck, 1986

$$c'/c = \pi/6$$

Universality in Thermodynamics

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however...

It turns out that there is a large class of CFTs in d>2 where thermodynamics **is** uniquely determined by the central charge (just like in d=2).

$c'/c = \pi/6$,	d=2
$c'/c = \pi^3/162$,	d=3
$c'/c = \pi^2/80$,	d=4
...	

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$c'/c = \pi/6,$	d=2	These are field theories with dual description in terms of classical gravity
$c'/c = \pi^3/162,$	d=3	
$c'/c = \pi^2/80,$	d=4	
...		

Universality in Thermodynamics

- In every CFT with a classical gravity dual, free energy (or entropy) is uniquely fixed by the central charge.
- Theories in this class may have different gauge groups, different representation of matter fields, different global symmetries — and still have **the same** c'/c=const.
- If one assumes AdS/CFT + string landscape, the number of such models is huge, 10^{500} or so.

In the critical d=3 O(N) model at large N:

Sachdev, 1993

PK+Ritz, 2008

$$c'/c = (\pi^3/162)\ 1.0662...$$

Outline

- Universality in thermodynamics
- Universality in charge transport
- Universality in momentum transport

Universality in charge transport

- Look at conductivity in linear response
- Kubo formula: σ ~ real-time <JJ> in equilibrium

$$C^{\mathrm{ret}}_{\mu\nu}(\omega,\boldsymbol{k}) = P^T_{\mu\nu}\Pi^T(\omega,\boldsymbol{k}^2)+P^L_{\mu\nu}\Pi^L(\omega,\boldsymbol{k}^2)$$

$$P^T_{ij}=\delta_{ij}-\frac{k_i k_j}{\boldsymbol{k}^2},\quad P^T_{0\mu}=0,\quad P^L_{\mu\nu}=(\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2})-P^T_{\mu\nu}$$

σ(ω) = Im Π(ω,k=0) /ω

Universality in charge transport

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It turns out that there is a large class of CFTs in $d \geq 2+1$ where d.c. conductivity ($\omega=0$) **is** uniquely determined by the equilibrium susceptibility:

$\sigma/\chi = 3/4\pi$, d=2+1

$\sigma/\chi = 1/2\pi$, d=3+1

...

where $$\chi = \frac{\langle Q^2 \rangle}{V\,T^2}$$

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These are field theories with dual description in terms of classical gravity

where $$\chi = \frac{\langle Q^2 \rangle}{V\,T^2}$$

Universality in charge transport

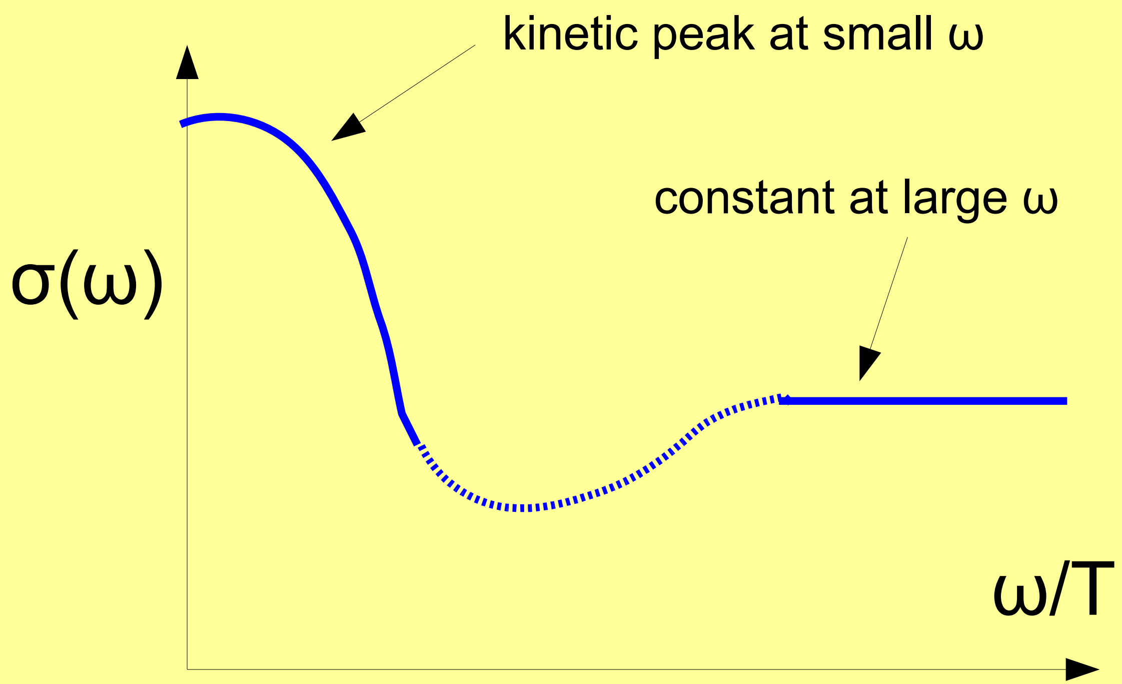
- In every CFT with a classical gravity dual, d.c. conductivity in uniquely determined by the static charge susceptibility
- Theories in this class may have different gauge groups, different representation of matter fields, different global symmetries — and still have **the same** σ/χ=const
- In these models, dynamic conductivity $\sigma(\omega)$ is in general a non-trivial function of ω/T ... unless we are in 2+1 dim:

$$\sigma(\omega)/\chi = 3/4\pi, \text{ for all } \omega$$

Herzog+PK+Sachdev+Son, 2007

Universality in charge transport

This ω-independence is unusual. At weak coupling:



From S.Sachdev's book «Quantum Phase Transitions», p.188:

... the distinct physical interpretation of σ(ω→0) and σ(ω→∞) make it clear that, in general, there is no reason for them to have equal values (we can not of course rule out the existence of exotic models or symmetries that may cause these two to be equal).

The "exotic models" are provided by AdS/CFT, where σ(ω)=const

Universality in charge transport

In fact, the universality in 2+1 dim includes more than $\sigma(\omega)$=const. In every CFT with a classical gravity dual,

$$\Pi^L(\omega,\mathbf{k})\ \Pi^T(\omega,\mathbf{k}) = -\chi^2\ (\omega^2-\mathbf{k}^2)\ (3/4\pi)^2$$

This is a strong constraint because $\Pi^L(\omega,\mathbf{k})$ and $\Pi^T(\omega,\mathbf{k})$ are in general completely independent (except at $\mathbf{k}=0$).

This is a universal relation for **full correlation functions**, not just for transport coefficients.

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Universality in momentum transport

- Look at momentum relaxation in linear response
- Kubo formula: η ~ real-time <TT> in equilibrium
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Universality in momentum transport

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- Kubo formula: η ~ real-time <TT> in equilibrium
- Viscosity is in general determined by the microscopic physics such as l_{mfp}. Thermodynamics does not fix η.

however...

It turns out that there is a large class of models in $d \geq 2+1$ where viscosity **is** uniquely determined by the equilibrium entropy:

$$\eta/s = 1/4\pi$$

PK+Son+Starinets, 2004

Buchel, 2004

...

Universality in momentum transport

- In every field theory with a classical gravity dual, shear viscosity in uniquely determined by the equilibrium entropy

these models:

- May be either conformal or confining
- May have different gauge groups
- May have different representation of matter fields
- May have different global symmetries
- May be at zero or finite chemical potential
- May not even have a fixed point in the UV
- May live in different number of dimensions

and still all of them have $\eta/s=1/4\pi$

Some corollaries

- There is a large class of CFTs in d≥2+1 where viscosity is uniquely determined by the central charge
- In QCD, $\frac{\eta}{s} \neq \frac{1}{4\pi}$ at all temperatures. Therefore, QCD does not have a classical gravity dual (in spite of that, AdS/CFT is definitely useful)

Summary

- There is a large class of critical models in d≥2+1 whose thermodynamics is universal: $\frac{c'}{c} = \frac{\pi^3}{162}$ etc
- There is a large class of critical models in d=2+1 whose response function is universal: $\Pi^L \Pi^T \sim (\omega^2 - \mathbf{k}^2)$
- There is a large class of critical models in d≥2+1 whose conductivity is universal: $\frac{\sigma}{\chi} = \frac{3}{4\pi}$ etc
- There is a large class of models in d≥2+1 whose viscosity is universal, $\frac{\eta}{s} = \frac{1}{4\pi}$ in any dimension

Summary

These universalities are **very unusual** from a conventional point of view. What do all these models have in common?

- They all arise in some sort of a large-N limit
- They are all strongly interacting
- They all have a dual description in terms of classical gravity

thus...

- Universalities of σ/χ etc reflect universality of classical gravity
- Quantum corrections to classical gravity will modify universal relations such as $\eta/s=1/4\pi$

Questions

- Are there other families of non-Gaussian fixed points where free energy is uniquely determined by the central charge?
- Are there other families of non-Gaussian fixed points where conductivity is uniquely determined by susceptibility or is frequency-independent?
- Are there other families of non-Gaussian fixed points where viscosity is uniquely determined by entropy?
- Are there analogous universalities far from equilibrium?

THE END!