Classical thermodynamics of gravitational collapse

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Outline

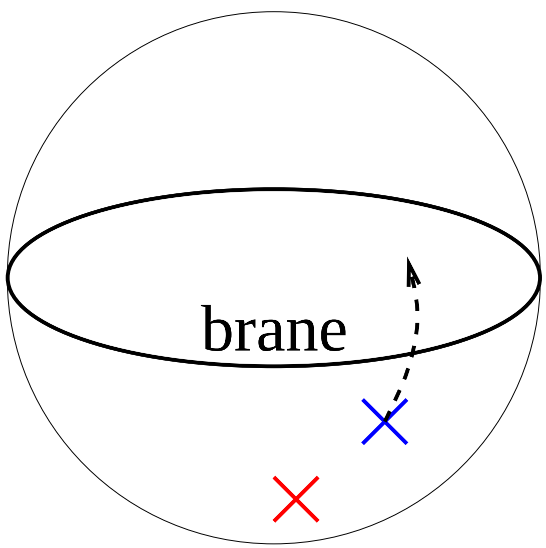
1. Motivation (braneworlds and the strong CP problem).
2. Energy vs. "free energy" in gravitational collapse.
3. A specific example: spherically-symmetric collapse of an instanton "particle" in 5 dimensions.
4. Numerical results.
5. Interpretation.
6. Brief comments on quantum mechanics.
7. Conclusion.

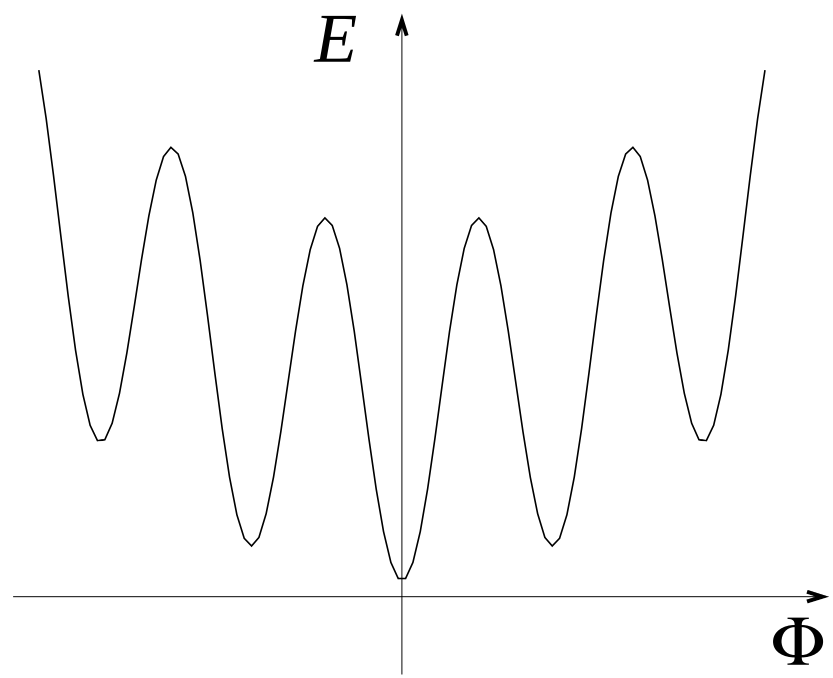
Motivation

Instantons on the brane = transport of instanton ‘‘particles’’ through the brane. A theta-angle is a steady flow of these "particles".

For certain topologies (e.g., a sphere), the "particles" cost energy (the space has a finite inductance). Then, the effective theta-angle becomes time-dependent and can relax to zero---solution to the strong CP problem (S.K. and M. Shaposhnikov, 2004).

A (1+1)-dimensional Abelian example:



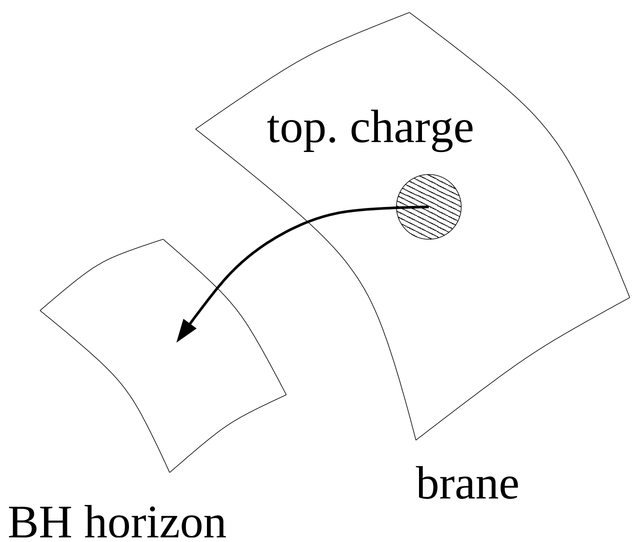


Motivation (continued)

A time-dependent theta is a ‘‘global axion’’---a single degree of freedom, not a particle, but is supposed to contribute correctly to the low-energy theorems of QCD (so that the solution to the U(1) problem is intact). It can be seen explicitly that it does in the 2-flavor massive Schwinger model with finite inductance (S.K., 2006).

Existence of such a mode is possible because of a weak violation of Lorentz invariance.

The same mechanism may work simply due to the presence of a bulk black hole.



‘‘Free energy’’ of a black hole

Is degeneracy of classical vacua determined by the energy E (mass) of the black hole or some ‘‘free energy’’?

A quantum state on the brane evolves with exp(iI), where I is the action (brane + bulk). Let canonical coordinates on the brane be q, and those in the bulk Q, and suppose q do not change.

The partial derivative with respect to time (of a brane observer) gives the energy:

$$\left.\frac{\partial I}{\partial t}\right|_{Q_f} = -E \ ,$$

while the total derivative gives the Lagrangian computed on the classical solution. Denote

$$\frac{dI}{dt} = -F \ .$$

Questions: Is F a "thermodynamic potential"? How does it behave during gravitational collapse?

Gravitational collapse of an instanton "particle"

This work (Z. Gecse and S.K., 2008):

1. Choice of theory: SU(2) Einstein-Yang-Mills in asymptotically flat 5d spacetime. [Not a realistic braneworld, but the question (about the time-dependence of F) can still be asked.]

2. Spherical symmetry, isotropic coordinates:

$$
\begin{aligned}
ds^2 &= -N^2(t,r)dt^2 + \Psi^2(t,r)(dr^2 + r^2 d\Omega_3^2) \ , \\
A_\mu^a &= \left(0, \eta_{ij}^a n_j \frac{f(t,r)}{r}\right) \ ,
\end{aligned}
$$

where spatial indices refer to the Cartesian coordinates built from r and the angles.

3. Initial conditions: a smooth $f(0,r)$ with weak gravity, e.g., an instanton

$$f_0(r) = \frac{2r^2}{\lambda_0^2 + r^2}$$

of a large size; zero or nonzero initial velocity.

Numerical evolution

Evolve the canonical pairs

$$(\Psi, K), (f, p) \; .$$

Update $N(t,r)$ from its ODE. Monitor the energy and momentum constraints.

A black hole forms when $N(t,r)$ as a function of r crosses zero.

Units (on the plots): energies and the Lagrangian are in units of the mass of a nongravitating (very large) instanton

$$E_{\rm inst} = \frac{8\pi^2}{g_{\rm YM}^2} \; ,$$

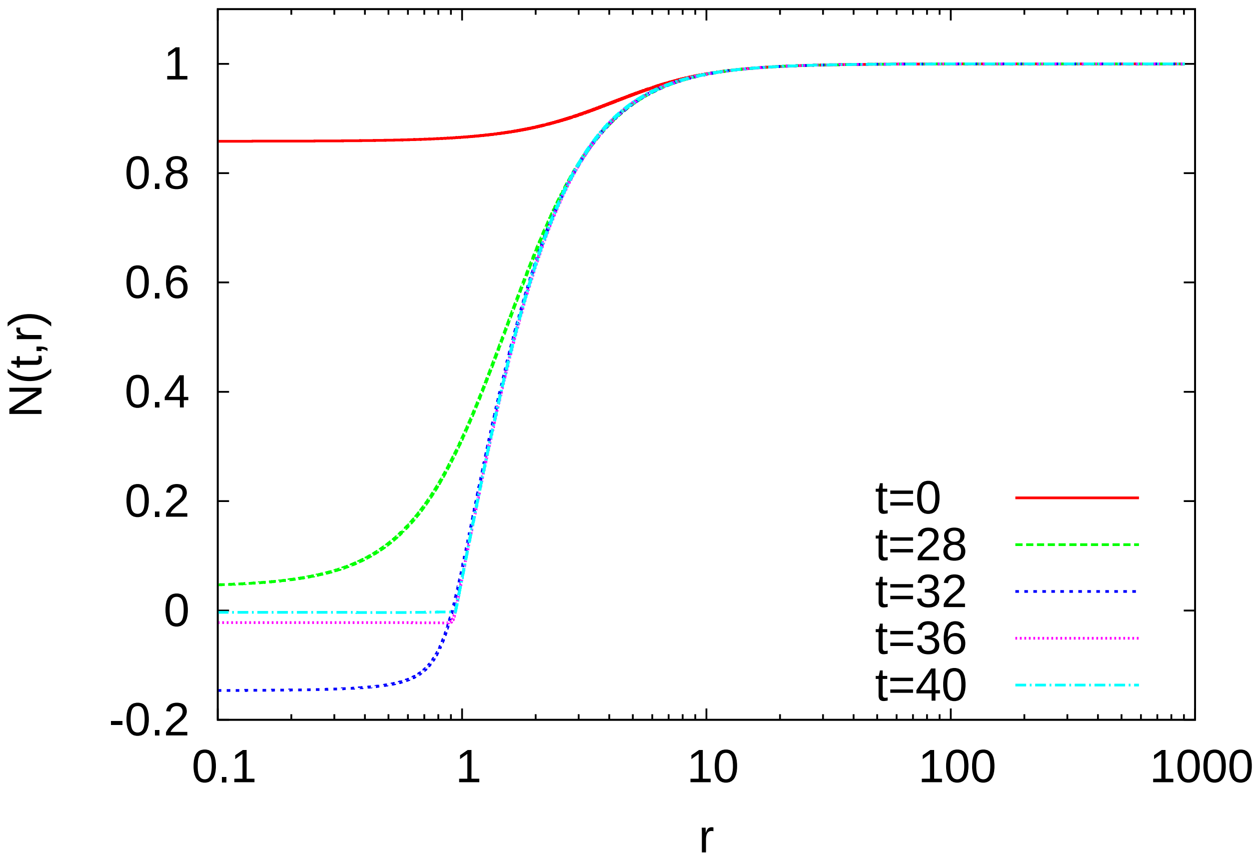
distances and time are in units of the gravitational radius corresponding to this mass.

Numerical results

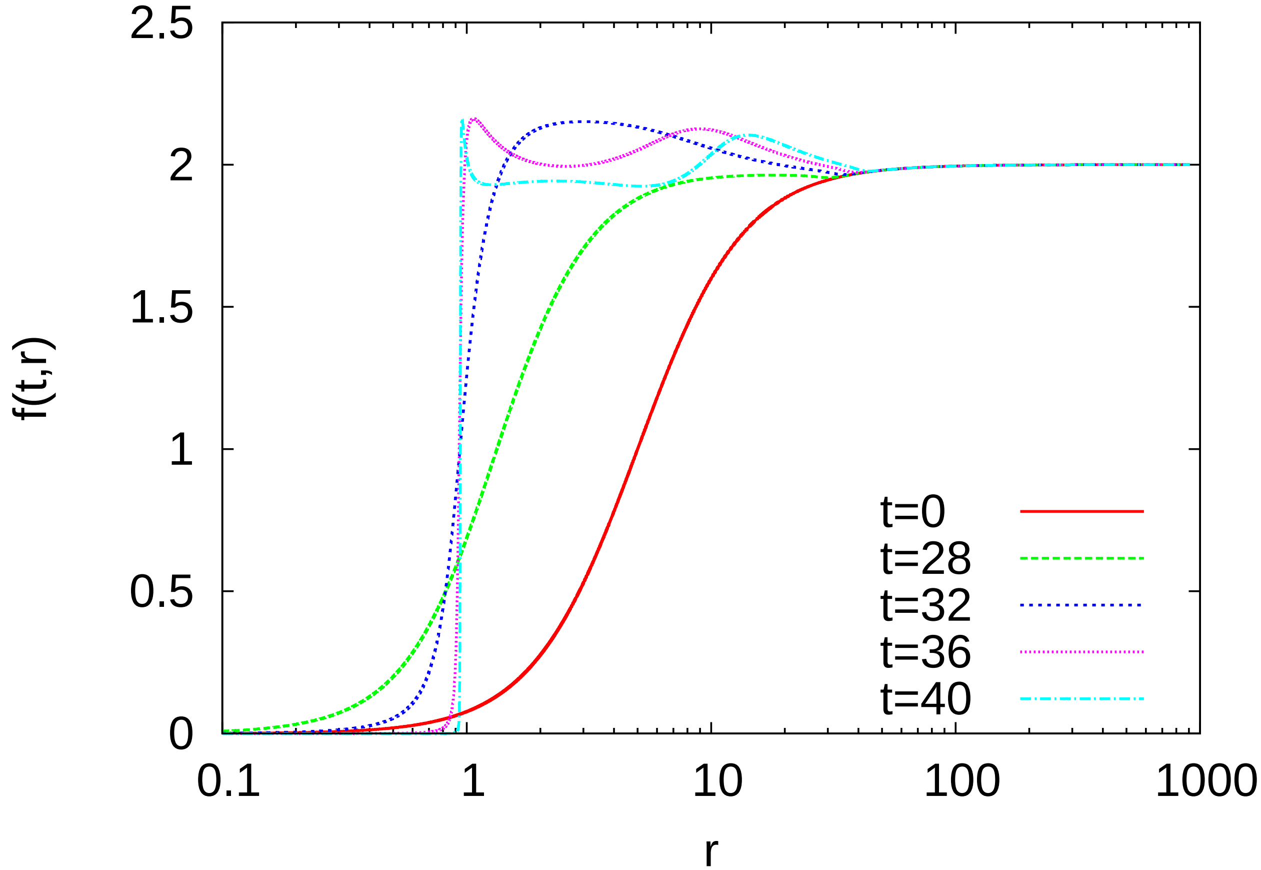
These are for $\lambda_0 = 5,\ p(0,r) = 0$.

For zero initial momentum and large $\lambda_0,$ Newtonian gravity lasts until

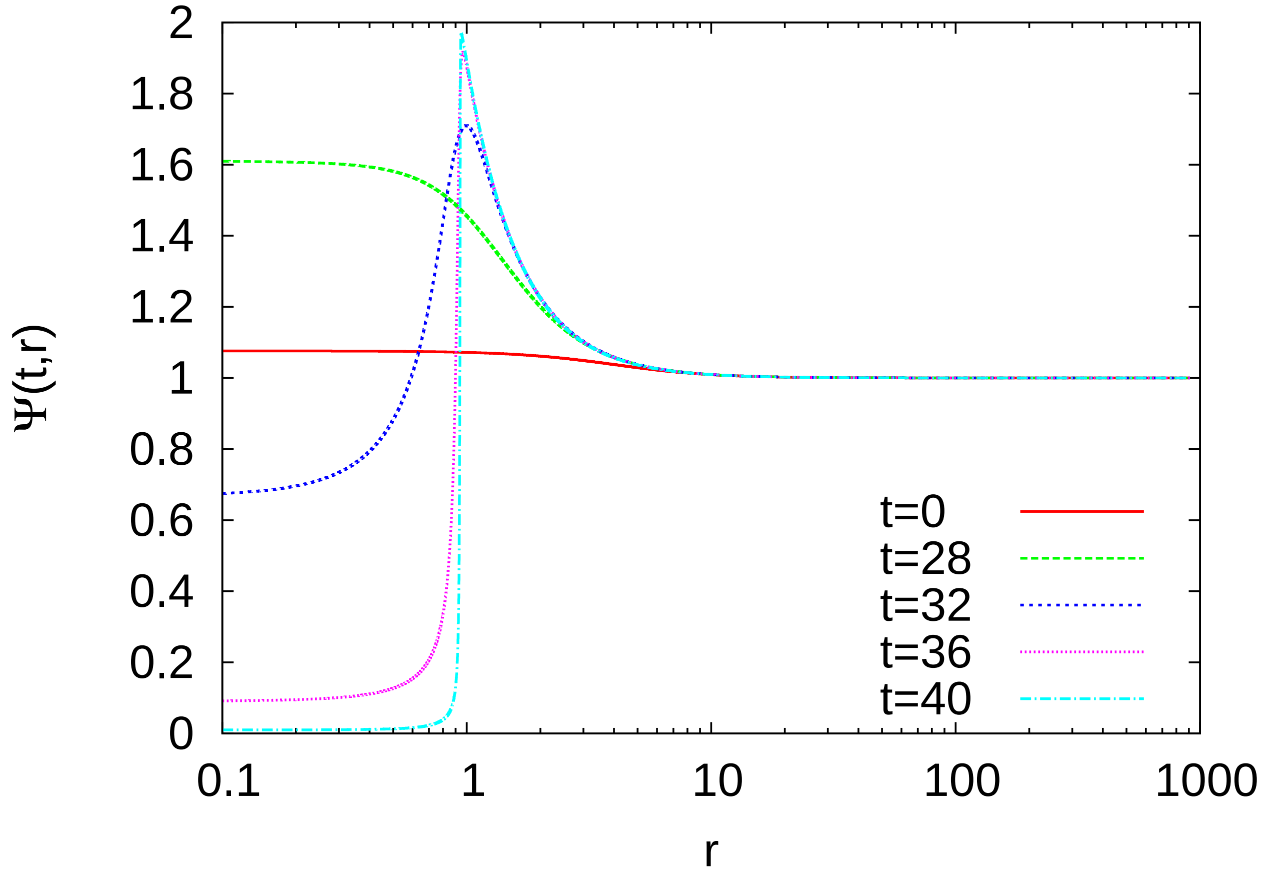
$$t \approx \frac{\sqrt{5}}{2}\lambda_0^2 \approx 1.1\lambda_0^2 \ .$$



Numerical results (the gauge field)

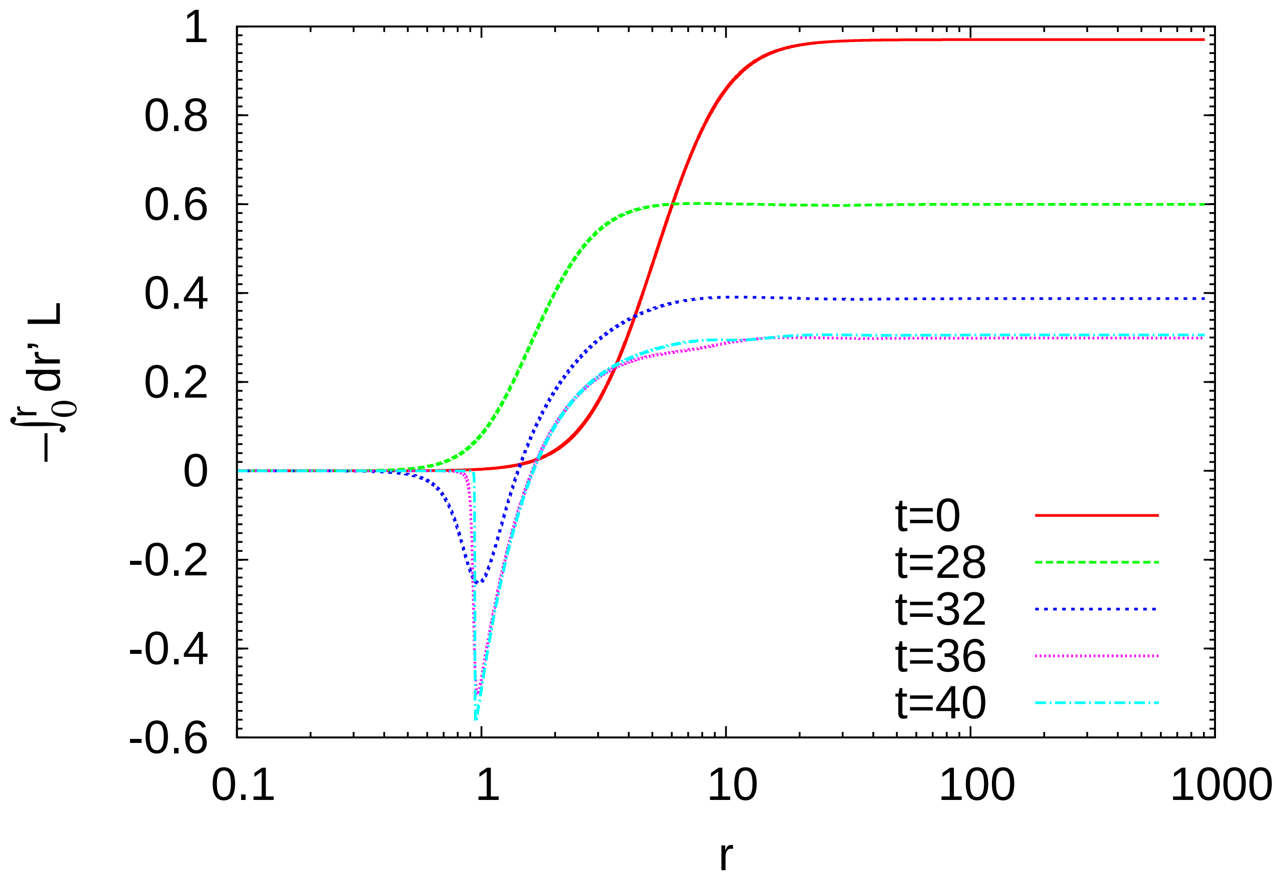


Numerical results (the conformal factor)



Numerical results (accumulation plot)

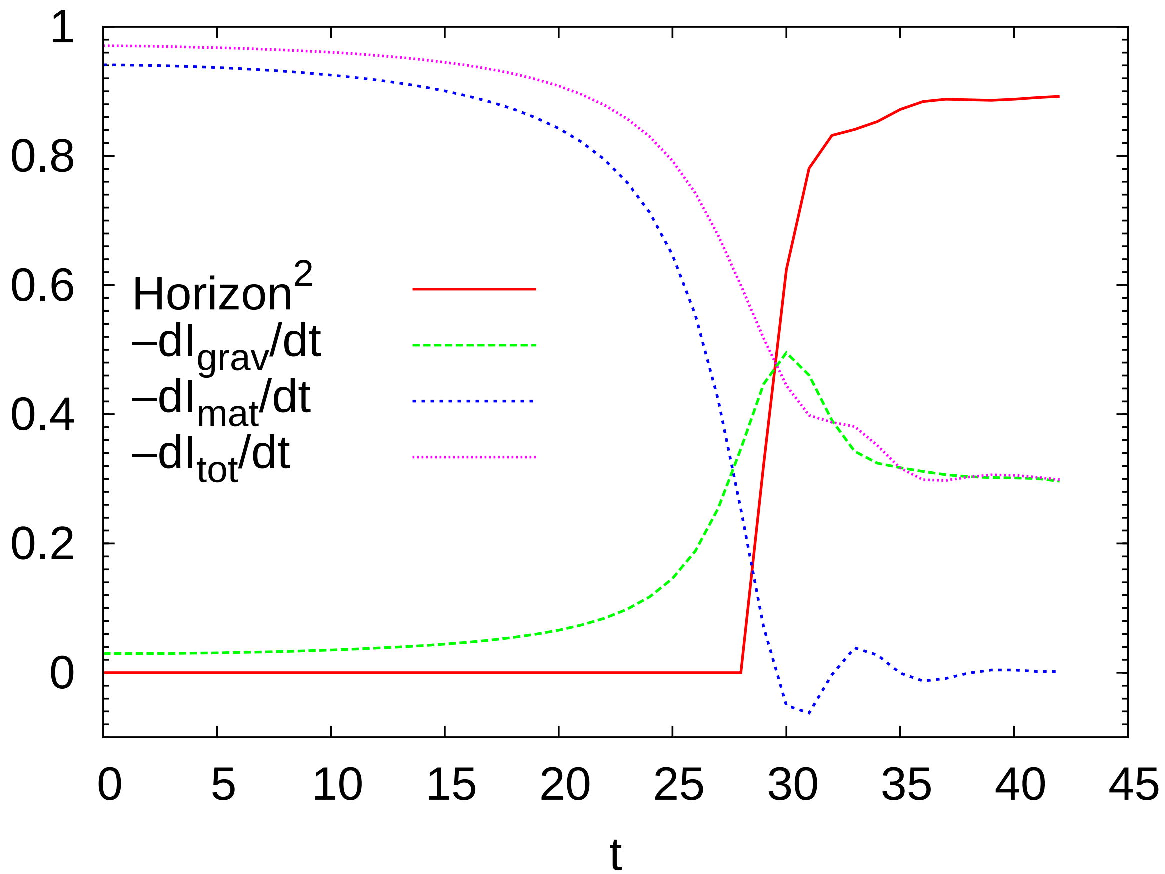
Need to separate the Lagrangian of the black hole from that of the outgoing wave.



In this case,

$$\frac{dI}{dt} \approx \frac{dI_{\rm tot}}{dt} \ .$$

Numerical results (the Lagrangian)



At late times,

$$-\frac{dI}{dt} \approx \frac{1}{3} E = \frac{1}{3} r_H^2 \ .$$

Interpretation

1. In a sense, E-F is the energy associated with evolution that goes unnoticeably to a distant observer. (If we set all time derivatives to zero, we would have $F = E$.) This is similar to statistical equilibrium.

2. $F = E/3$ agrees with the standard black-hole thermodynamics:

$$F_{\rm thermo} = E - T_H S_{BH} = \frac{1}{3} E \ .$$

While the temperature and entropy each contain a power of the Planck constant, these cancel in the product, which means that the free energy may have a classical interpretation. Our results suggest that it is simply minus the Lagrangian.

3. The coincidence is nontrivial: the thermal free energy is computed from the vacuum exterior alone, while our F from a time-dependent solution with collapsing matter.

4. The difference E-F is accumulated in a thin shell near the horizon.

Conclusion

- Numerical studies suggest a classical interpretation of the free energy of a black hole. Our definition of the free energy is complementary to the usual thermal (Euclidean) definition. Unlike the latter, it explicitly refers to a time-dependent ("nonequilibrium") metric.

- Although classical solutions do not allow one to compute the overall normalization of the black-hole entropy, they can tell where (in space) the entropy is coming from.