Cosmic strings - lessons from field theory

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G. Vincent & M. Sakellariadou [astro-ph/9612135]

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Cosmic strings observed in background radiation

11:46 21 January 2008
NewScientist.com news service
David Shiga

Traces of vast cosmic strings have been found in radiation from the early universe, a controversial new study says. If confirmed to exist, cosmic strings could offer an unprecedented window into the extreme physics of the infant universe.

Were cosmic strings seen in big bang afterglow?

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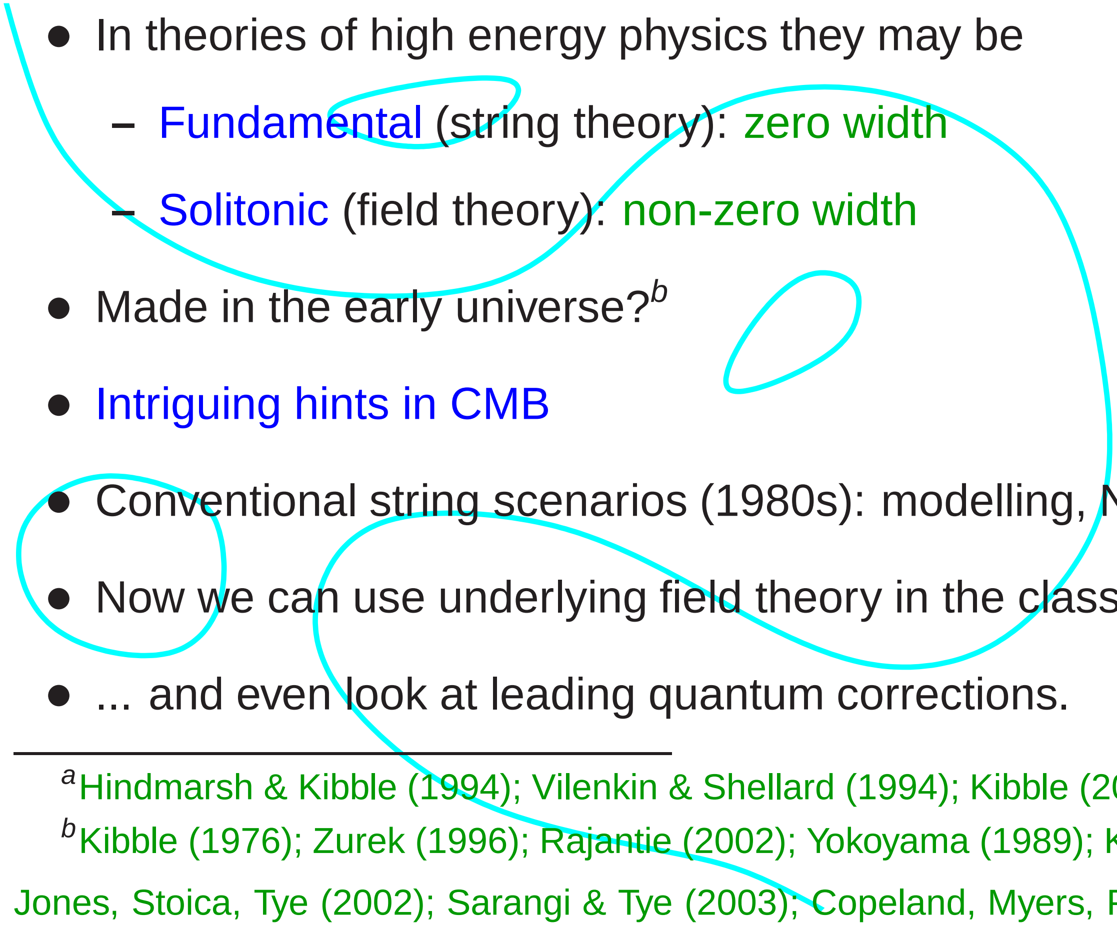
Introduction



- Cosmic strings[a] are linear distributions of mass-energy in the universe.
- In theories of high energy physics they may be
  - Fundamental (string theory): zero width
  - Solitonic (field theory): non-zero width
- Made in the early universe?[b]
- Intriguing hints in CMB
- Conventional string scenarios (1980s): modelling, Nambu-Goto simulations.
- Now we can use underlying field theory in the classical approximation ...
- ... and even look at leading quantum corrections.

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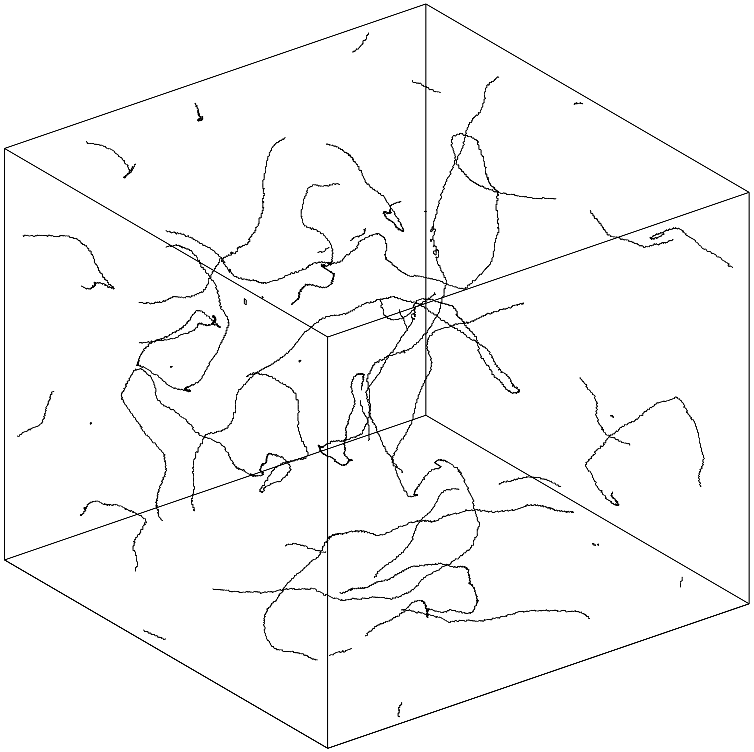
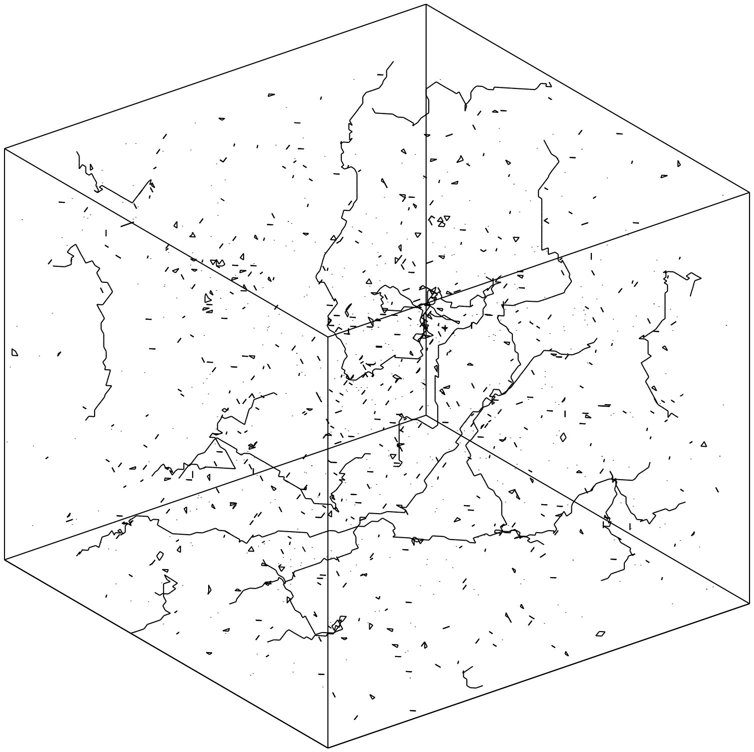
[a] Hindmarsh & Kibble (1994); Vilenkin & Shellard (1994); Kibble (2004)

[b] Kibble (1976); Zurek (1996); Rajantie (2002); Yokoyama (1989); Kofman, Linde, Starobinski (1996); Jones, Stoica, Tye (2002); Sarangi & Tye (2003); Copeland, Myers, Polchinski (2003); Dvali & Vilenkin (2003)

Two models of a universe filled with string



Classical Abelian Higgs model



Nambu-Goto strings

Small-scale string dynamics: Significant uncertainty in predictions

Observational signals from strings

Small theoretical disagreement (factor < 10)

- Cosmic Microwave Background, density perturbations [a]

Large theoretical disagreement (factor $10^{\rm lots}$)

- Gravitational radiation[b]
- Cosmic rays[c]
- Gravitational lensing[d]

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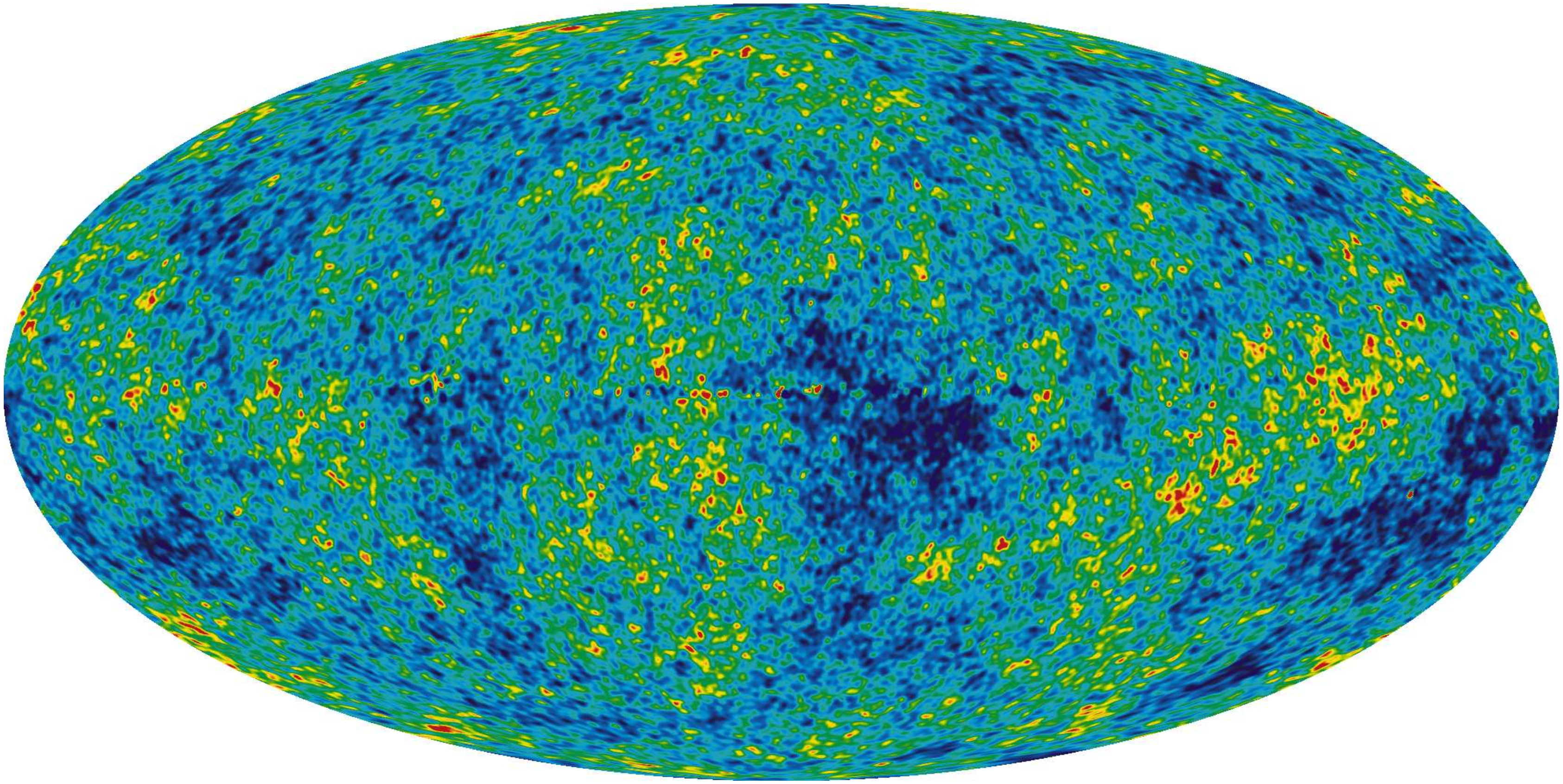
[a] Zel'dovich (1980); Vilenkin (1981); Kaiser & Stebbins (1984); Landriau & Shellard (2004); Wyman et al (2005); Bevis et al (2006,2007)

[b] Vachaspati & Vilenkin (1985); Hindmarsh (1990); Damour & Vilenkin (2000,2001,2005)

[c] Bhattarcharjee (1990); Sigl (1996); Protheroe (1996); Berezhinksi (1997); Vincent, M.H., Sakellariadou (1998); Wichowski, MacGibbon, Brandenberger (1998)

[d] Vilenkin (1984); Hindmarsh (1989); de Laix & Vachaspati (1996,1997)

Hints in the WMAP 3-Year Data?



$-200 < \Delta T < 200$ μK

Abelian Higgs string C_ℓs vs. WMAP3 and BOOMERanG

Multipole moments:

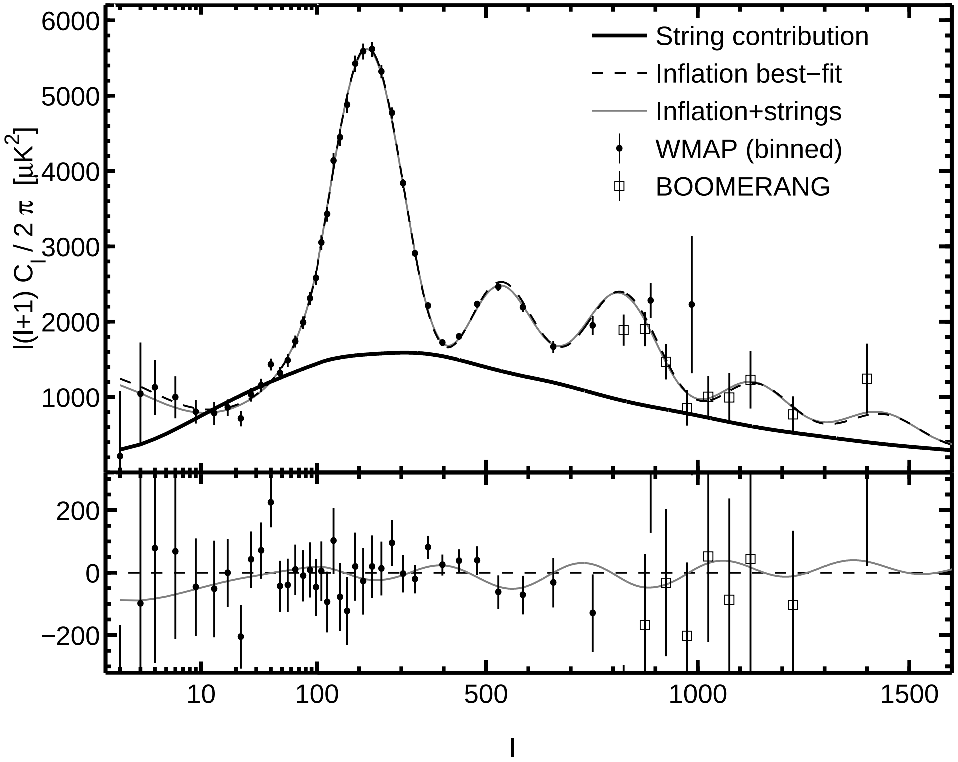
$$a_{lm} = \int d\Omega \Delta T(\mathbf{n}) Y^*_{lm}(\mathbf{n})$$

Angular power spectrum:

$$C_l = \sum_{m=-l}^{l} |a_{lm}|^2$$

Anisotropy power:

$$l(l+1)C_l/(2\pi)$$



Top: Strings normalised to WMAP3 ($\ell = 10$)[a]

Bottom: Best fit strings+ΛCDM vs. ΛCDM only

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[a] Bevis, Hindmarsh, Kunz, Urrestilla (2006)

Results slide for string-o-philes

MCMC fit to CMB data (WMAP3, Boomerang, CBI, ACBAR, VSA)[a]

$$G\mu = (0.65 \pm 0.10) \times 10^{-6}$$

- Corresponds to $\mu \sim (10^{16}\ \text{GeV})^2$
- Fraction of CMB anisotropy power at $\ell = 10$ $f_{10} \simeq 0.1$

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[a] Classical Abelian Higgs model. See Urrestilla et al. 0704.3800 for other models.

Results slide for string-o-phobes

MCMC fit to CMB (7 parameters)[a]

- + Hubble Key Project ($H_0 = 72 \pm 8$ km s^{-1} Mpc^{-1})
- + D abundance & Big Bang Nucleosynthesis ($\Omega_{\mathrm{b}} h^2 = 2.14 \pm 0.20 \times 10^{-2}$)

$$G\mu < 0.7 \times 10^{-6} \text{ (95\%)}$$

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[a] Classical Abelian Higgs model. See J. Urrestilla's talk for other models.

Formation of strings (Kibble-Zurek) (2+1)D model

Real scalar field $\phi(\mathbf{x}, t)$, symmetry $\phi \to -\phi$. Lagrangian density:

$$\mathcal{L} = \frac{1}{2}\partial\phi^2 - V(\phi), \qquad V(\phi) = V_0 - \frac{1}{2}\mu^2(T)\phi^2 + \frac{1}{4!}\lambda\phi^4.$$

$T(t)$ is a control parameter

(e.g. temperature, inflaton)

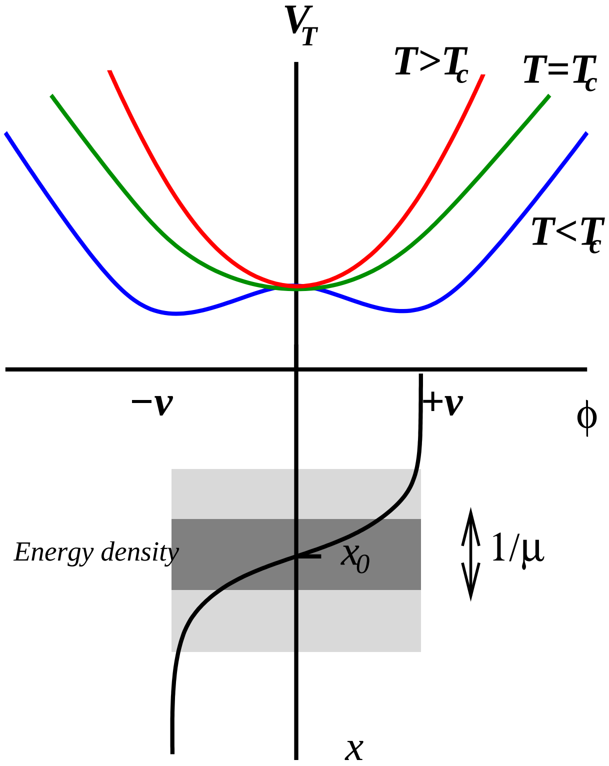
$\mu^2(T > T_c) < 0$, $\mu^2(T < T_c) > 0$

Phase transition at $T = T_c$.

Field eqn. (Minkowski space)

$$\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi - \mu^2(T)\phi + \frac{1}{3!}\lambda\phi^3 = 0$$

“String” solutions $\phi = v\tanh(\mu x)$



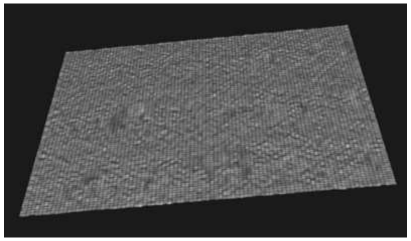
Formation of strings in 2D: numerical simulation

$$\ddot{\phi} + \eta(t)\dot{\phi} - \nabla^2\phi + (\phi^2 - \mu^2(t))\phi = 0$$

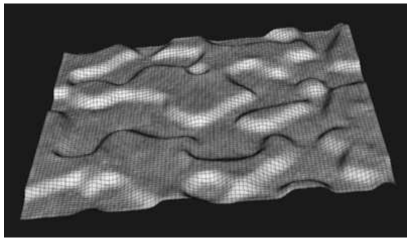
$\eta(t) = \theta(t_{\rm damp} - t)$ damping models cooling, expansion:[a]

$\mu^2(t) = \theta(t) - \theta(-t)$ models rapid transition

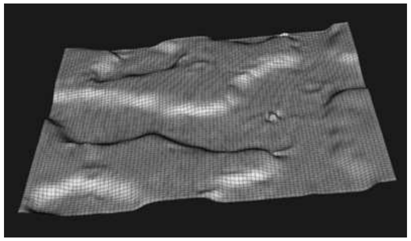
Initial conditions: $\phi(\mathbf{x})$ Gaussian random variable on each lattice site



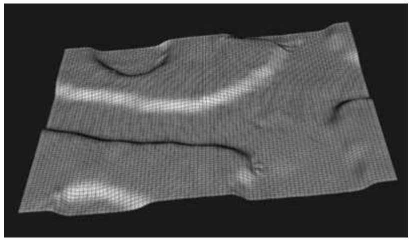
$t = 0\ \eta = 1$



$t = 30\ \eta = 0$



$t = 60\ \eta = 0$



$t = 90\ \eta = 0$

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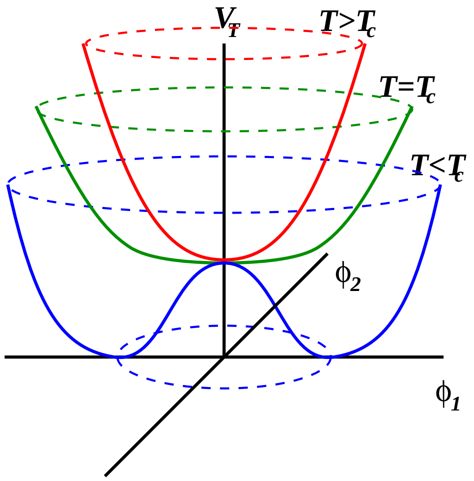
[a] Garagounis and Hindmarsh [hep-ph/0212359]

See next talk by A. Rajantie for strings in 3D.

Abelian Higgs model (FRW background)

$$S = -\int d^4x\,\sqrt{-g}\left(g^{\mu\nu}D_\mu\phi^* D_\nu\phi + V(\phi) + \frac{1}{4e^2}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\right),$$



Complex scalar field $\phi(\mathbf{x},t)$, vector field $A_\mu(\mathbf{x},t)$

Covariant derivative $D_\mu = \partial_\mu - iA_\mu$.

Potential $V(\phi) = \frac{1}{2}\lambda(|\phi|^2 - v^2)^2$.

Metric $ds^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)$

τ: conformal time, $\propto t^{\frac{1}{2}}, t^{\frac{1}{3}}$

Temporal gauge ($A_0 = 0$) field equations (index raised with Minkowski metric).

$$\begin{aligned}\ddot\phi + 2\frac{\dot a}{a}\dot\phi - D^2\phi + \lambda a^2(|\phi|^2 - v^2)\phi &= 0,\\ \partial^\mu\left(\frac{1}{e^2}F_{\mu\nu}\right) - ia^2(\phi^* D_\nu\phi - D_\nu\phi^*\phi) &= 0,\end{aligned}$$

Solutions in 3D

- Static (infinite energy) cylindrically symmetric:

  straight infinite string

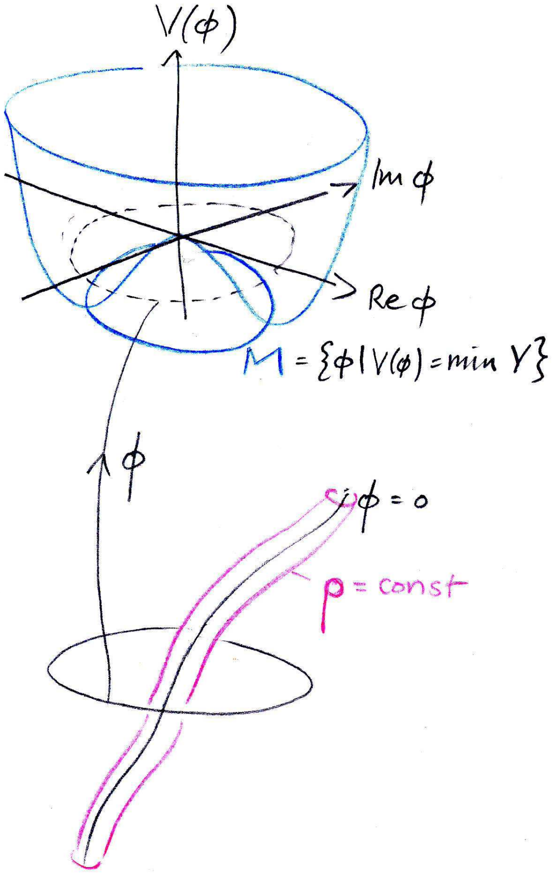
- Non-static:

  ∞ string with waves, oscillating loops.

- Radiation: scalar: $h = |\phi| - v$[a] vector: A_μ

$$(\Box - m_h^2)h = 0, \quad (\Box - m_A^2)A_\mu = 0$$

$$m_h = \sqrt{2\lambda}v, m_A = ev.$$



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[a] Unitary “gauge”

Parallel simulations of field theories: LATfield

- Public C++ library of objects for parallel classical lattice fields[a]
- Rewrite of MDP/FermiQCD[b]
- Objects:

  Lattice: Takes care of boundary conditions and domain decomposition

  Field: Template - can have real, complex, user-defined object.

  Site: Accesses elements of field
- Parallelisation by compiler switch

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[a] Bevis & Hindmarsh http://www.latfield.org/

[b] Massimo di Pierro et al., http://www.fermiqcd.net/

Visualising Abelian Higgs model simulations

(Loading ...)

Isosurfaces:

ρ (pink)

$\mathcal{L}$ (yellow)

Size: 192^3

$16 \leq \tau \leq 48$

Abelian Higgs model simulations: string length scale

Scaling: $L/V \propto \tau^{-2}$

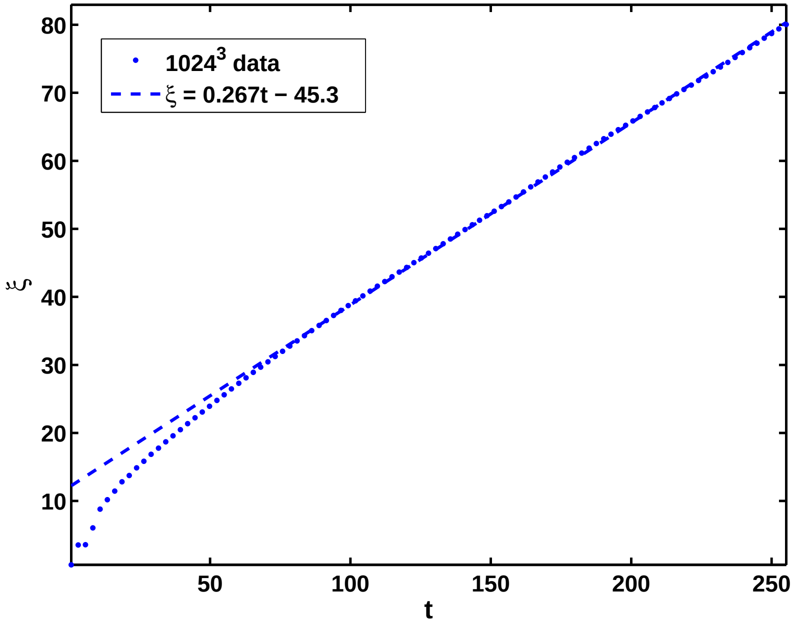
Network scale: $\xi = \sqrt{(V/L)}$

Hence $\xi \propto \tau$

Lattice spacing:	$\Delta x = 0.5$
Time step:	$\Delta t = 0.1$
Volume:	1024^3
Couplings:	$\lambda = 2, e = 1$
Masses:	$m_s = m_v$

Lagrangian length $\xi_{\rm Lag}$

Matter era



String network scaling hypothesis

We are interested in late-time dynamics

- String network characteristic scale ξ ($= \sqrt{V/L}$, i.e. average curvature radius)
- Network scaling hypothesis (a): $\boxed{\xi = x_* t}$ (x_* constant)
- Network scaling hypothesis (b): No trace of initial scale ξ_i
- $\xi_i = M^{-1} g(M\tau_Q)$ set by correlation length of fields, quench time τ_Q[a]
- String energy density: $\rho_s \simeq \mu/\xi^2$; total $\rho_t \sim 1/Gt^2$: $\boxed{\Omega_s \sim G\mu/x_*^2}$

Scaling: extrapolate from $t_i \sim 10^{-36}$ s to $t_0 \sim 3 \times 10^{17}$ s today

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[a] Kibble (1976); Zurek (1985); Rajantie & Hindmarsh (1999), Hindmarsh & Rajantie (2000)

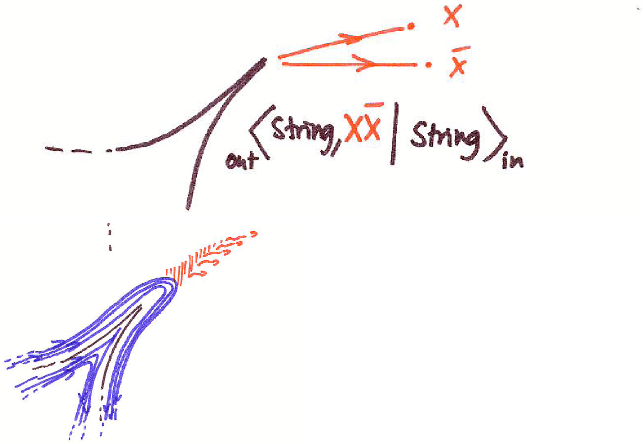
Energy loss (without gravity)

String curvature radius ξ

String mass/unit length $\mu \sim v^2 \sim M^2/e^2$

String width $w \sim M^{-1}$

String energy density $\rho_s \simeq \mu/\xi^2$



- Perturbative particle production[a]

  Power/length $\sim M/\xi^2$

- Cusp annihilation[b]

  Power/length $\sim M^{\frac{1}{3}}/\xi^{\frac{4}{3}}$

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[a] Srednicki & Theisen (1987)

[b] Brandenberger (1987)

Scaling: Power/length $\sim \dot{\rho}_s/(L/V) \simeq -(\mu/\xi^3)(\xi^2)$: requires $\sim M^2/\xi$

There is another non-perturbative mechanism[a]

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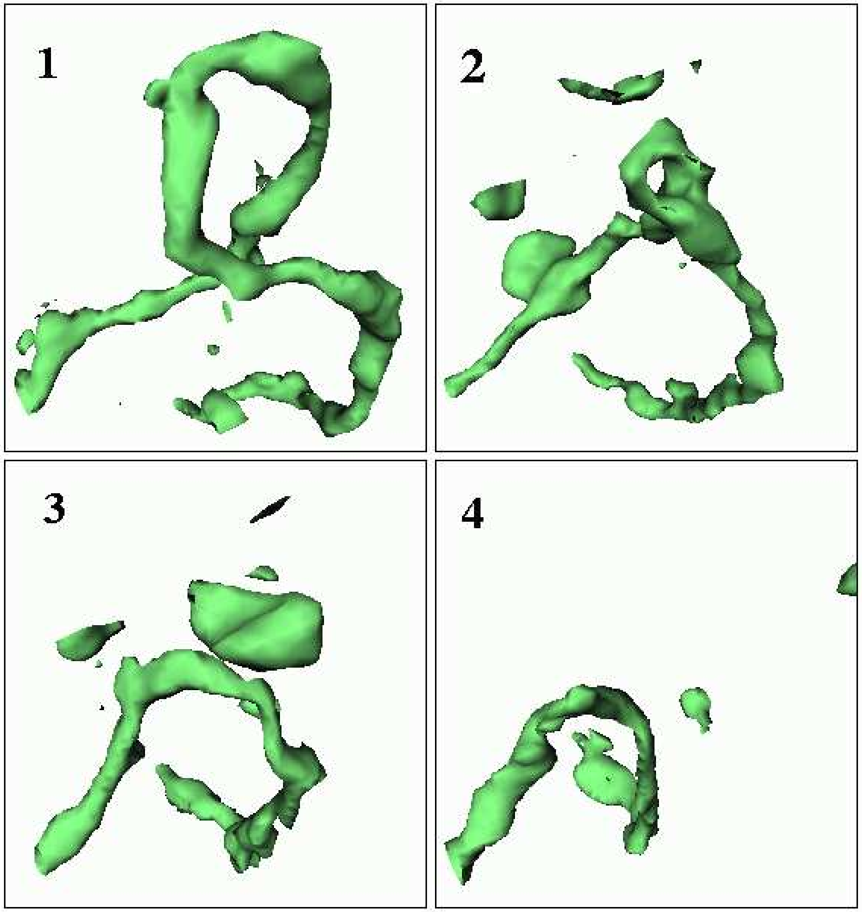
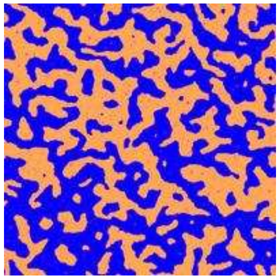
[a] Vincent, Antunes, Hindmarsh (1998)

Radiation or “core” or “proto”-loops?

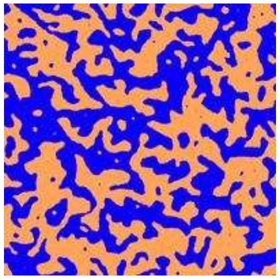


(Moore, Shellard, Martins (2001)

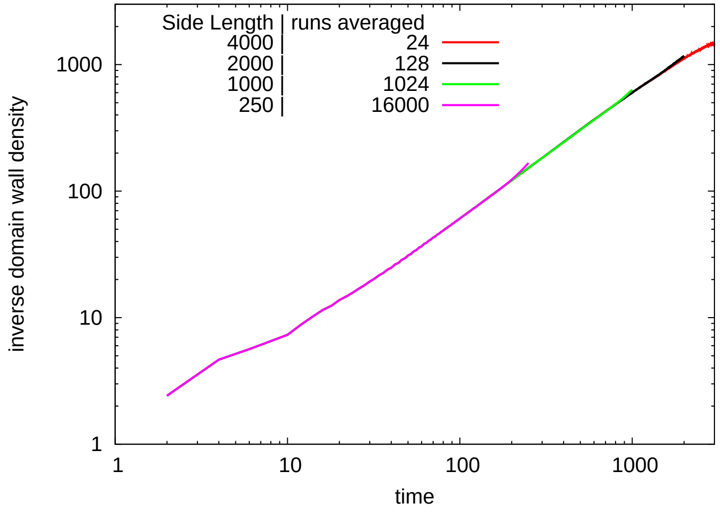
Strings (domain walls) in 2D



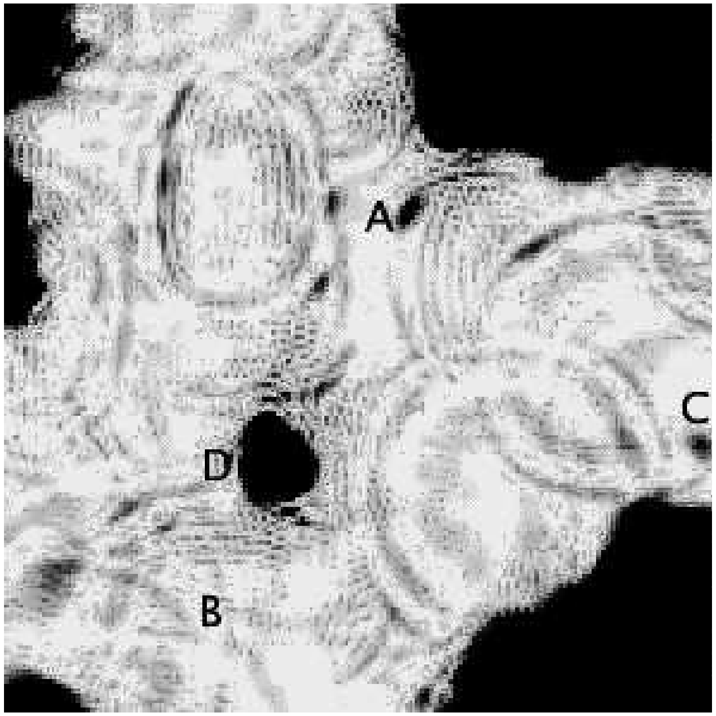
$t = 50$



$t = 100$



Scaling beyond $\xi M \sim 10^3$[a]



A Large amplitude wave front

B Waves emitted earlier by D

C Stationary oscillon

D Collapsing domain

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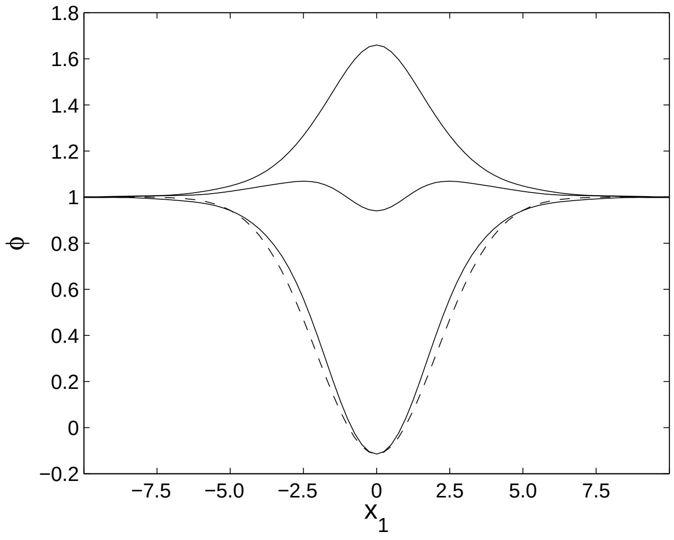
[a] Borsanyi & Hindmarsh (2007)

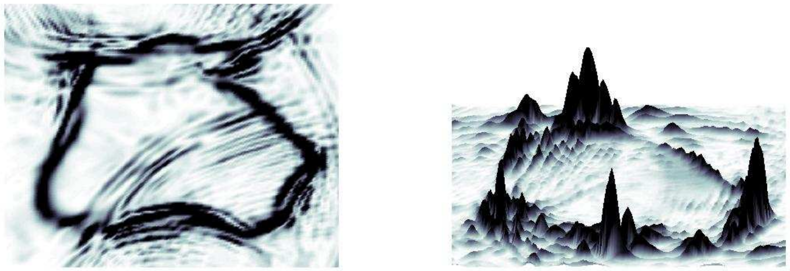
Oscillon production: relics of non-perturbative oscillations

- large-amplitude
- localised
- long-lived
- exist in Abelian Higgs[a]

$$V(\phi) = \frac{1}{4}\lambda(\phi^2 - 1)^2$$



Production by domain wall network[a]



Produced with high velocities ($\gamma > 1$)

Large amplitude oscillon $\equiv$ small loop of wall

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[a] Gleiser & Thorarinson (2007)

[a] Hindmarsh & Salmi (2007)

Summary - non-perturbative energy loss from strings

A problem of two scales:

string curvature radius ξ string width M^{-1}

- Operates over more than 3 orders of magnitude: $\xi M > 10^3$
- Produces classical radiation with $\lambda \sim M^{-1}$
- Large amplitude perturbations can be counted as small loops (long-lived in 2D)

More clues from small-scale structure?[a]

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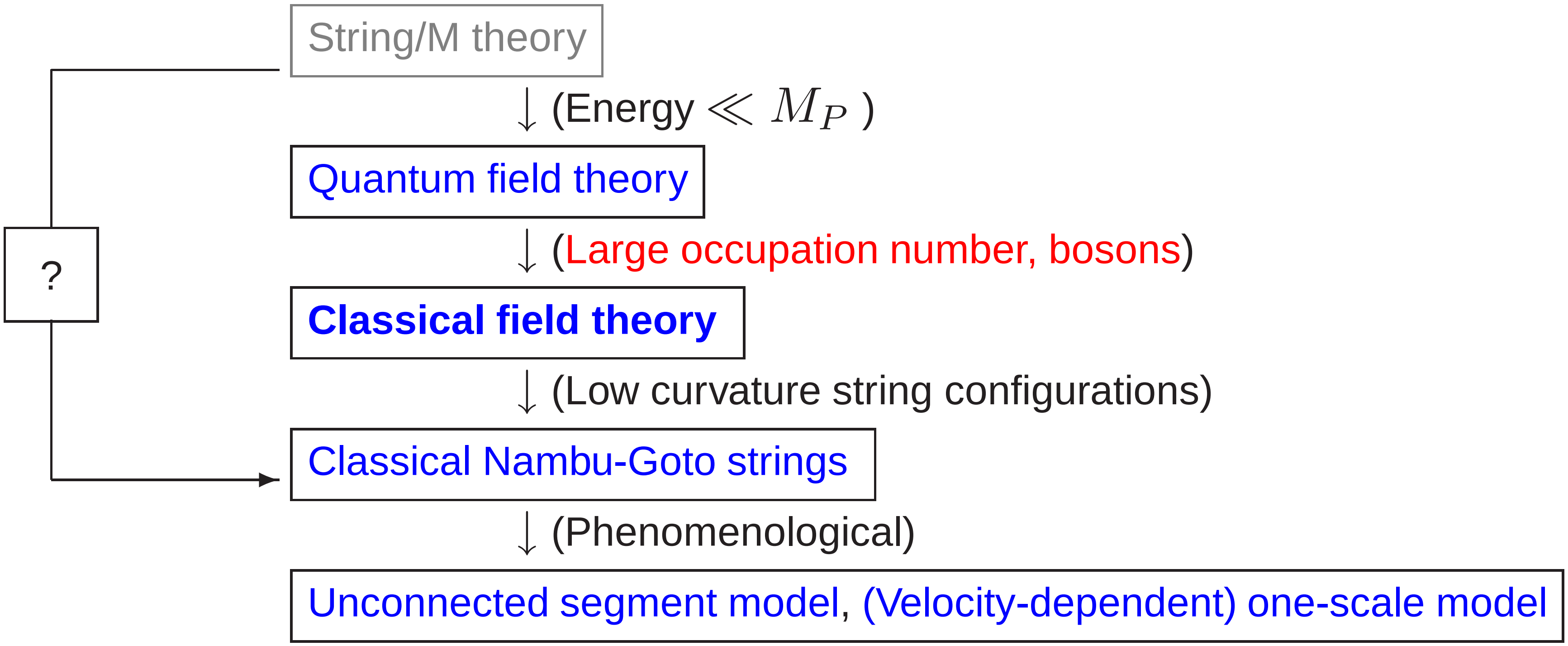
[a] Hindmarsh, Stuckey, Bevis in preparation.

Cosmic strings: Approximations



String/M theory

↓ (Energy $\ll M_P$)

Quantum field theory

↓ (Large occupation number, bosons)

Classical field theory

↓ (Low curvature string configurations)

Classical Nambu-Goto strings

↓ (Phenomenological)

Unconnected segment model, (Velocity-dependent) one-scale model

Semiclassical decay of strings in 2D

Is classical field theory really a good approximation?[a]

$$(\partial^2 + m^2)\hat{\phi}(x) + \frac{\lambda}{6}\hat{\phi}^3(x) = 0\,.$$

Quantum expectation value: $\hat{\phi}(x) = \bar{\Phi}(x) + \hat{\varphi}(x)$, with $\langle\hat{\varphi}(x)\rangle \equiv 0$.

Wightman propagator $G^{<}(x,y) = \langle\hat{\varphi}(y)\hat{\varphi}(x)\rangle$

Hartree (Gaussian) approximation

$$\begin{aligned}
\left[\partial_x^2 + m^2 + \frac{\lambda}{2}G^{<}(x,x)\right]\bar{\Phi}(x) + \frac{\lambda}{6}\bar{\Phi}^3(x) &= 0\,,\\
\left[\partial_x^2 + m^2 + \frac{\lambda}{2}\bar{\Phi}^2(x) + \frac{\lambda}{2}G^{<}(x,x)\right]G^{<}(x,y) &= 0\,,
\end{aligned}$$

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[a] Borsanyi & Hindmarsh (2007)

Hartree ensemble

N_E classical trajectories $\varphi_i(\mathbf{x}, t)$, solutions of the equation[a]

$$\left(\partial_x^2 + m^2 + \frac{\lambda}{2}\left[\bar{\Phi}^2(\mathbf{x}, t) + \left\langle \varphi^2(\mathbf{x}, t)\right\rangle_E\right]\right)\varphi_i(\mathbf{x}, t) = 0\,.$$

Wightman propagator from ensemble average: $G^<(x, y) = \langle \phi(\mathbf{x}, t)\hat{\varphi}(\mathbf{y}, t)\rangle_E$

Initial conditions for quantum "particles":

$$\begin{aligned}
\langle \varphi(\vec{x}, 0)\varphi(\vec{y}, 0)\rangle &= \hbar \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k}(\mathbf{x}-\mathbf{y})} \frac{1}{\omega_k}\left(n^0_{\vec{k}} + \frac{1}{2}\right)\,, \\
\langle \dot{\varphi}(\mathbf{x}, 0)\dot{\varphi}(\mathbf{y}, 0)\rangle &= \hbar \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k}(\mathbf{x}-\mathbf{y})} \omega_k \left(n^0_{\vec{k}} + \frac{1}{2}\right)\,, \\
\langle \varphi(\mathbf{x}, 0)\dot{\varphi}(\mathbf{y}, 0)\rangle &= 0\,.
\end{aligned}$$

Vacuum: $n^0_{\vec{k}} = 0$ initialised after domain walls formed

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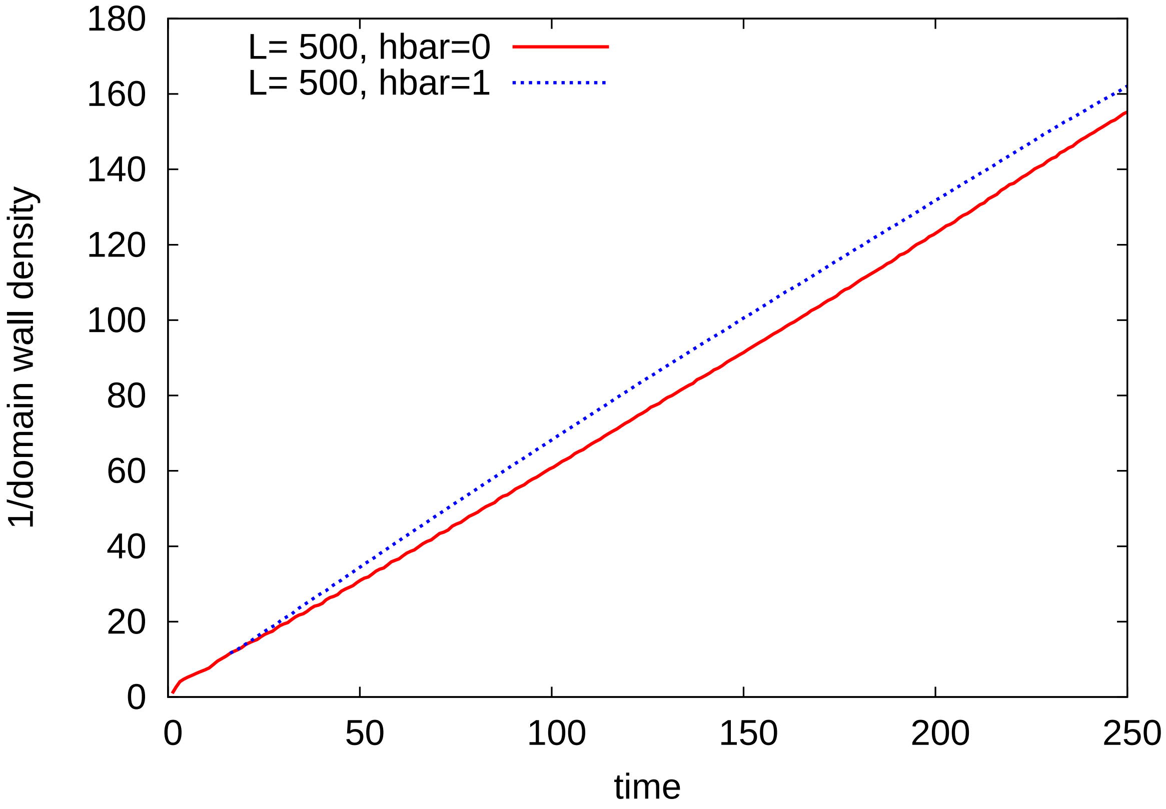
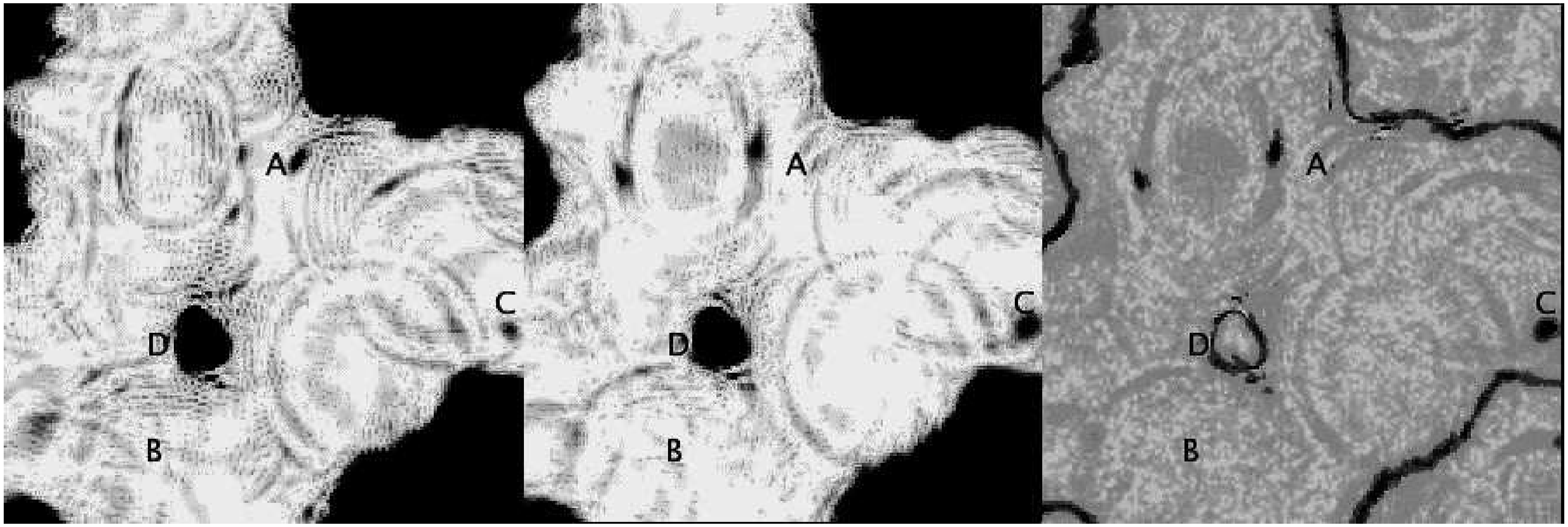
[a] C.f Sallé, Smit & Vink (2001), Salleé (2004)

Classical versus Hartree dynamics 1



Classical versus Hartree dynamics 2



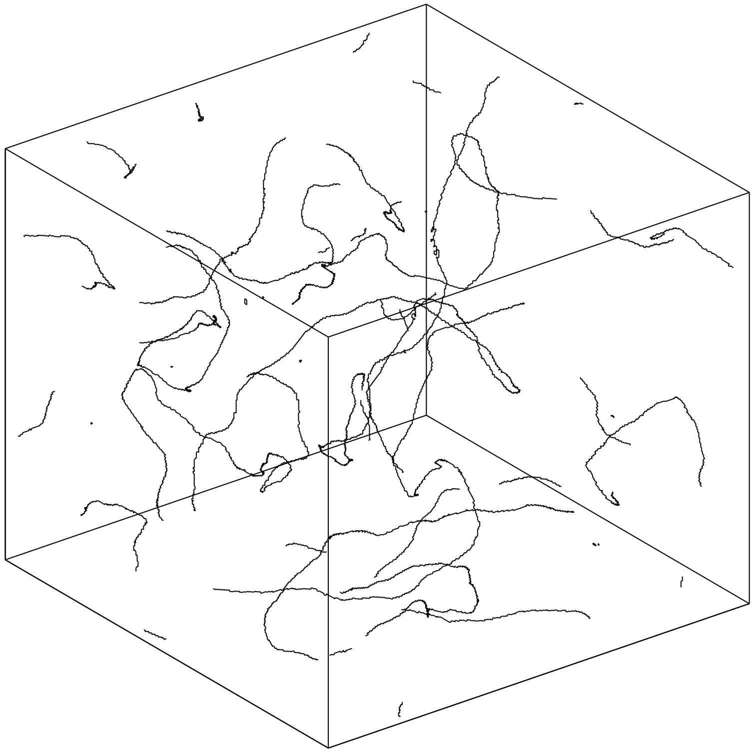
Classical field Hartree mean field particle number density

quantum corrections to evolution small

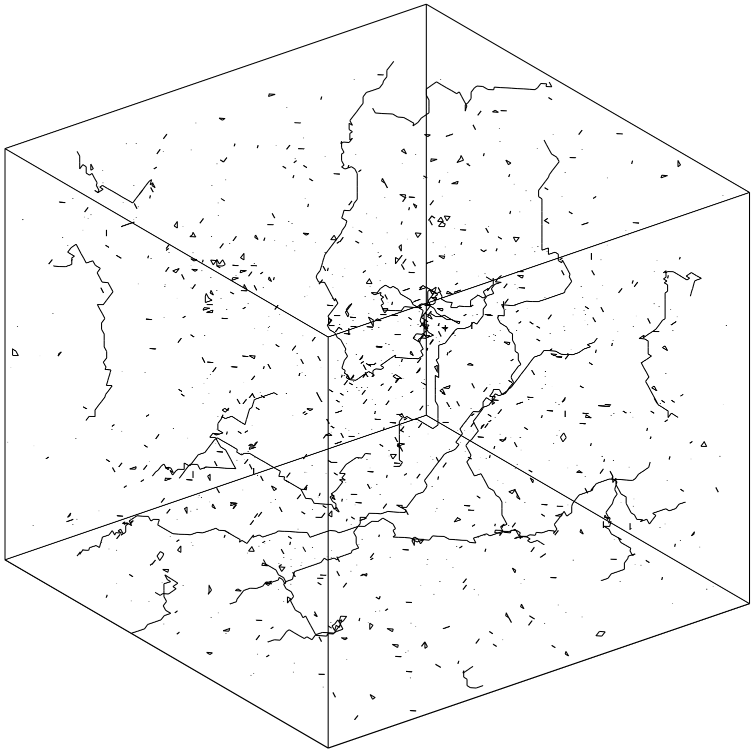
$n^0_{\vec{k}} \ll 1$

Two models of a universe filled with string

Classical Abelian Higgs model



Nambu-Goto strings



Can we reconcile these pictures?

Energy loss from strings: towards a solution

Field theory: classical radiation

Nambu-Goto: small loops, small-scale structure

Small-scale structure $\longrightarrow$ loop production peaked at UV cut-off[a]

- Synthesis: $\langle \ell_{\rm prod} \rangle \sim w$:[b]
- Small loops $\equiv$ large-amplitude field oscillations
- Energy goes into UV theory (scalar & gauge fields, closed strings $E \sim M_s$)
- Scaling without gravitational radiation reaction

New: Small-scale structure on Abelian Higgs strings[c]

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[a] Polchinski, Rocha (2006); Dubath, Polchinski, Rocha (2007).

[b] Conventional picture: gravitational radiation reaction $\langle \ell_{\rm prod} \rangle \sim (G\mu)^p t$

[c] Stuckey, Hindmarsh, Bevis in preparation.

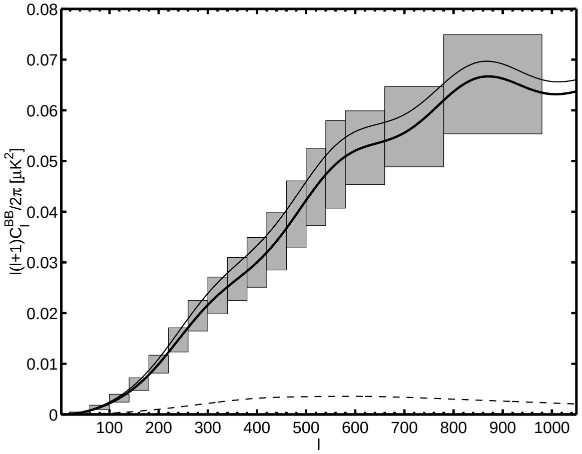
Conclusions

- Field theory: underlies solitonic cosmic strings
- Field theory: less modelling, more physics
- Energy loss: non-perturbative classical radiation
- Quantum corrections: small
- Scale separation problem: how does theory transport energy?
- Hints of strings at $G\mu \simeq 0.7 \times 10^{-6}$ (GUT scale)?
- Motivates D-term, F-term inflation models.

Future prospects

- Tighter constraints:
  - WMAP5 & ACBAR (higher ℓ)
  - Planck - improved TT, TE, EE:

    strings v. tensors
  - Polarisation (QUAD, CLOVER, . . .)
  - (Improved H_0)



String fraction $f_{10} = 0.0035$

- Other observational signals highly sensitive to energy loss mechanism
  - Field theory: classical radiation ⟶ cosmic rays
  - Conventional: gravitational waves

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