Viscous hydrodynamics for heavy-ion collisions*

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Work done in collaboration with Huichao Song

References:

U. Heinz, H. Song, A. K. Chaudhuri, Phys. Rev. C **73** (2006) 034904

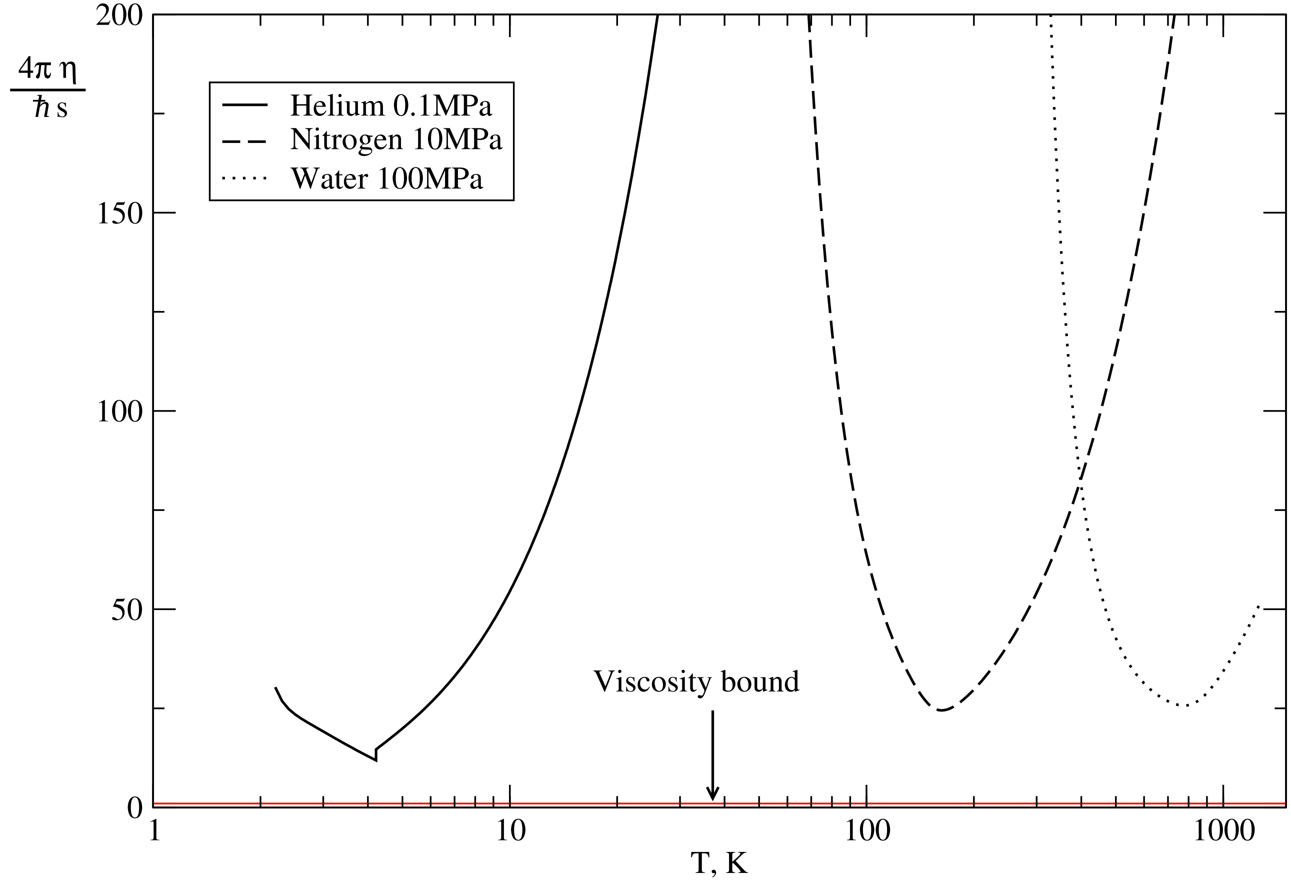
H. Song and U. Heinz, Phys. Lett. B **658** (2008) 279

H. Song and U. Heinz, arXiv:0712.3715 [nucl-th], Phys. Rev. C, in press

QGP – the most perfect fluid ever observed?

AdS/CFT universal lower viscosity bound conjecture: $\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$

Kovtun, Son, Starinets, PRL 94 (2005) 111601



Upper limit for QGP viscosity from various recent estimates are close to this bound!

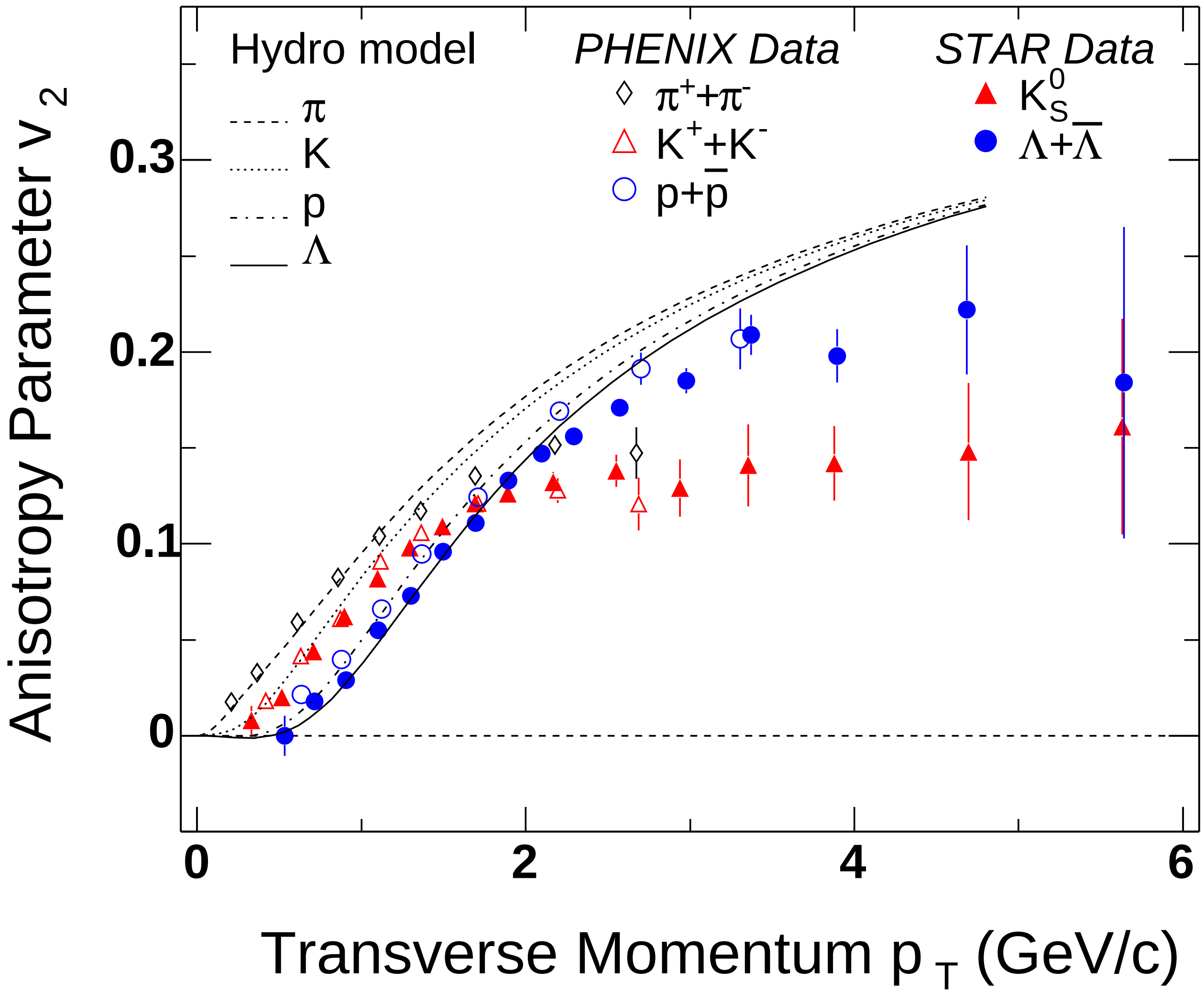
But: quantitative constraint on η/s requires viscous hydrodynamics code.

Ideal hydro works well at RHIC.

But there are also indications of non-zero viscosity:

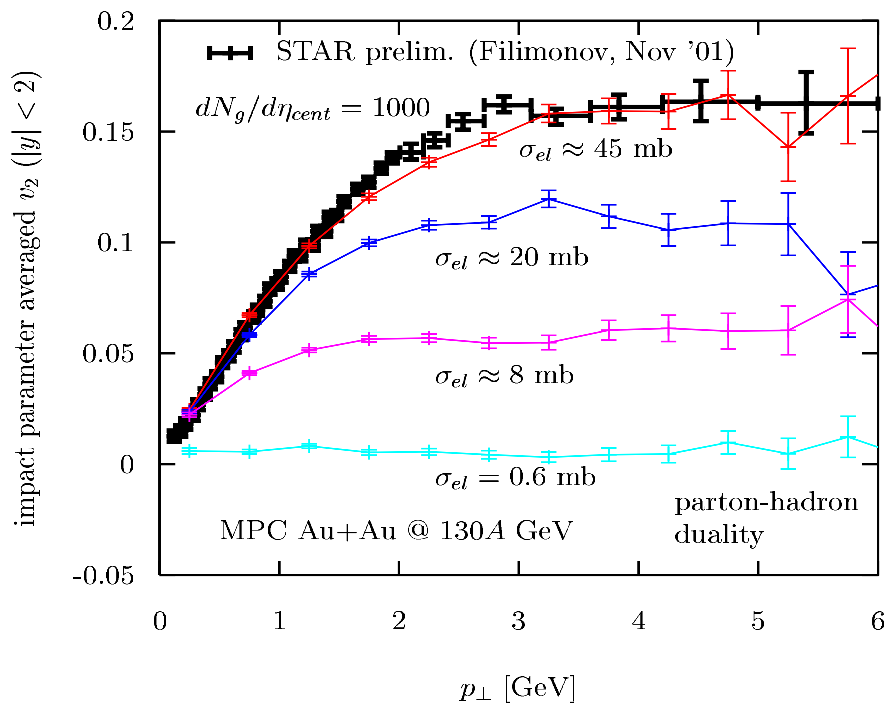
Ideal fluid dynamics breaks down at $p_T \gtrsim 1.5 - 2\,\mathrm{GeV}/c$:

STAR Coll., PRL 87, 182301 (2001) and PRL 92, 052302 (2004); PHENIX Coll., PRL 91, 182301 (2003)

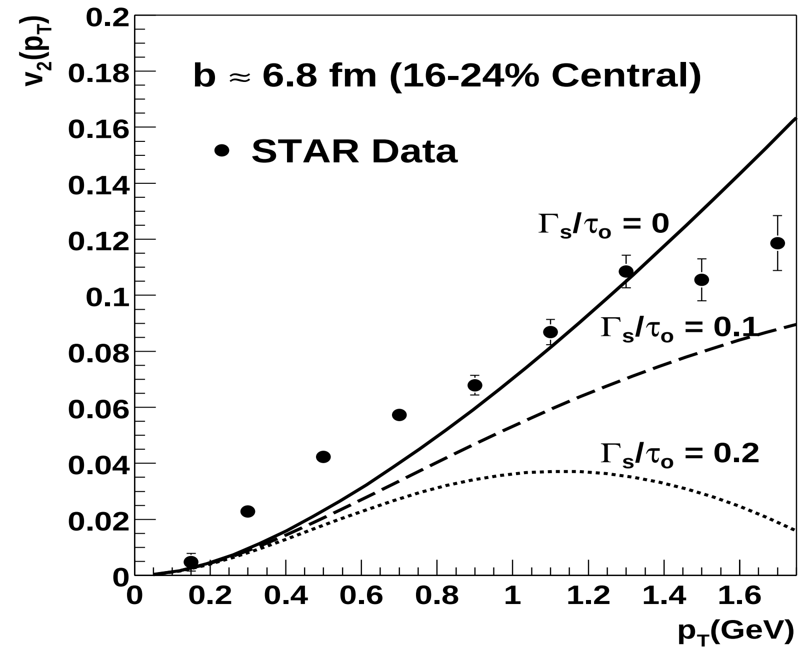


Breakdown of hydrodynamics at high $p_\perp$: upper limits for early QGP viscosity

D. Molnár and M. Gyulassy, NPA 697 (2002) 495



D. Teaney, PRC 68 (2003) 034913

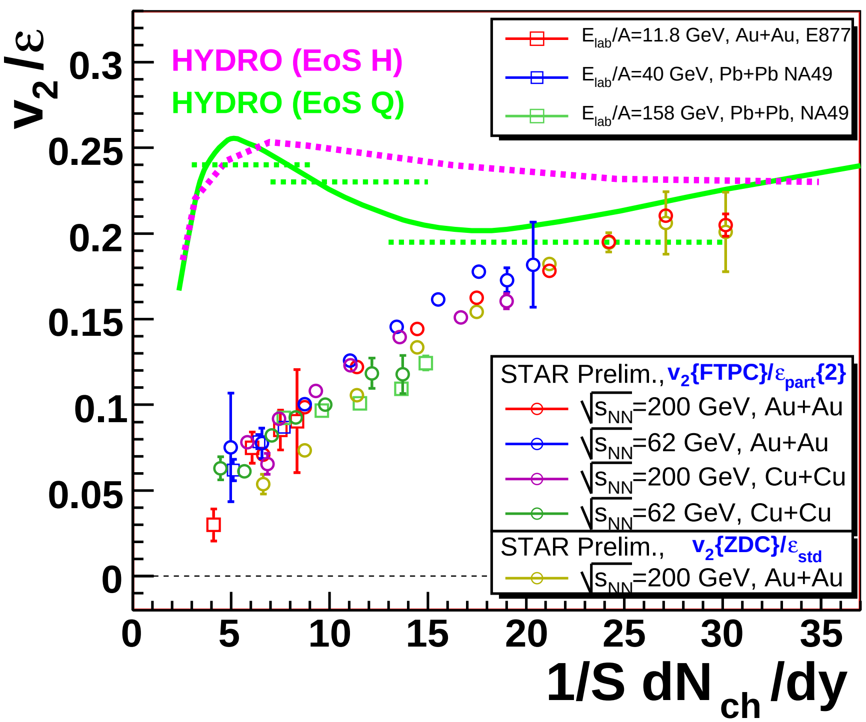


$$\Gamma_s = \tfrac{4}{3}\eta/(T{\cdot}s)$$

- For sufficiently (very) large $\sigma_{\rm el}$, $v_2(p_\perp)$ from covariant parton transport model MPC follows hydrodynamic curve at low $p_\perp$ and reproduces observed saturation at high $p_\perp$
- Similar pattern is seen in viscous hydrodynamics: viscous corrections increase $\sim p_\perp^2$
- v_2 data suggest $\frac{\Gamma_s}{\tau} < 0.1$, close to minimum viscosity $\frac{\eta}{s} = \frac{\hbar}{4\pi}$ (Son et al. 2002)

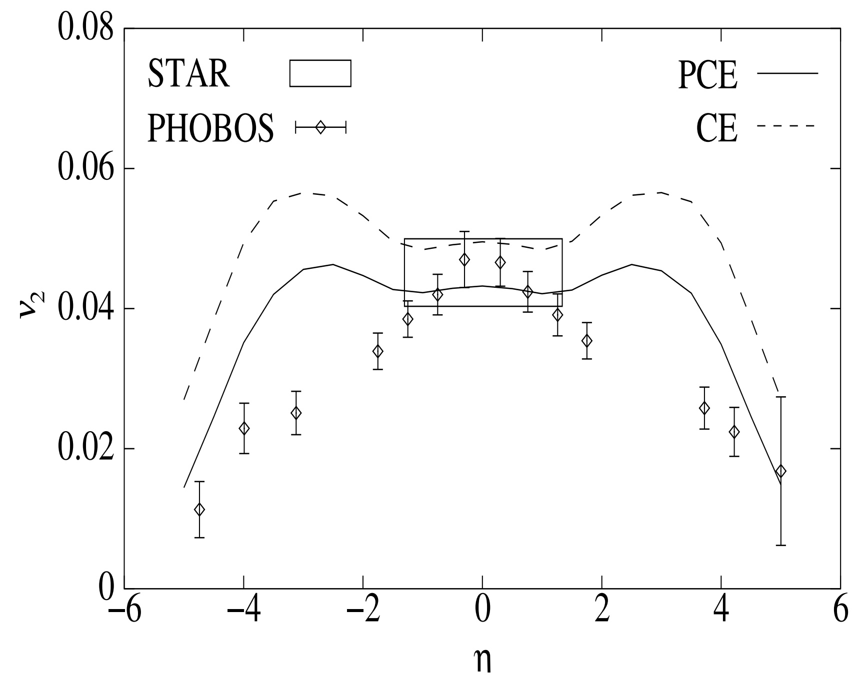
Smaller, less dense collision systems: late hadronic viscosity

S. Voloshin [STAR], JPG 34 (2007) S883



(3+1)-d hydrodynamics:

T. Hirano, PRC 65 ('02) 011901; 66 ('02) 054905



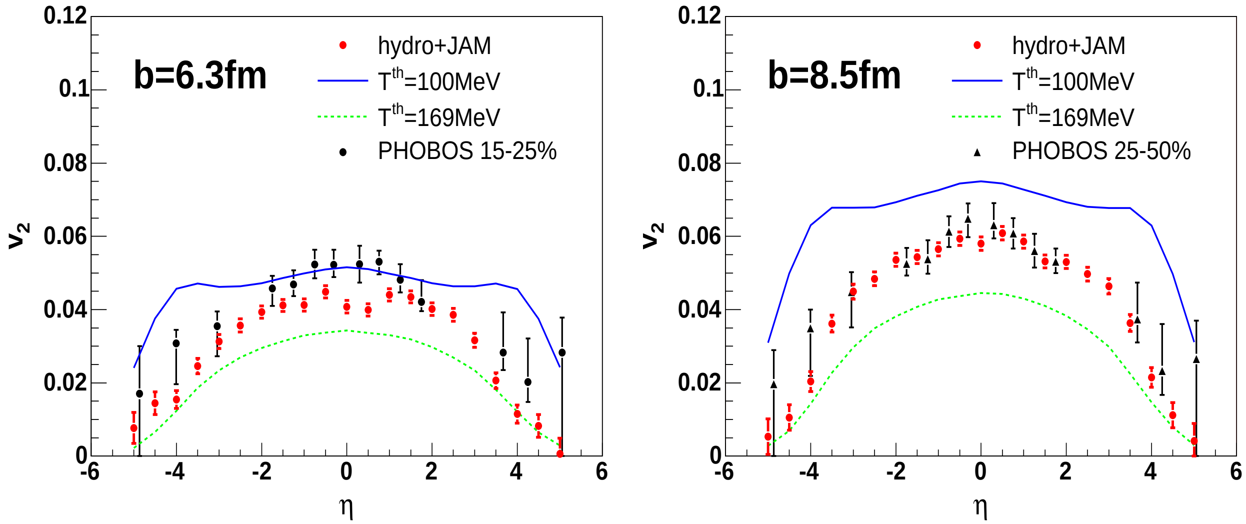
- $\frac{v_2^{\rm measured}}{v_2^{\rm hydro}}$ scales with $\frac{1}{S}\frac{dN_{\rm ch}}{dy} \propto s_{\rm init}$
- $e_{\rm init} > 10\,{\rm GeV/fm}^3$ needed for v_2 to saturate before hadronization and exhaust ideal hydro limit!
- hydrodynamics predicts non-monotonic v_2/ϵ: between AGS and RHIC it decreases, due to softening of EOS by quark-hadron transition (Kolb, Sollfrank, UH, PRC 62 (2000) 054909)
- data show instead monotonous increase of v_2/ϵ with $\sqrt{s}$!?

What's going on? Late hadronic viscosity! (Teaney, Shuryak 2001)

Late hadronic dissipation explains reduced v_2 at forward rapidity and in peripheral collisions:

3D Hydro+Cascade Model: Ideal fluid dynamics for QGP above T_c, hadronic cascade with realistic cross sections (JAM) below T_c

T. Hirano, U. Heinz, D. Kharzeev, R. Lacey, Y. Nara, PLB 636 (2006) 299



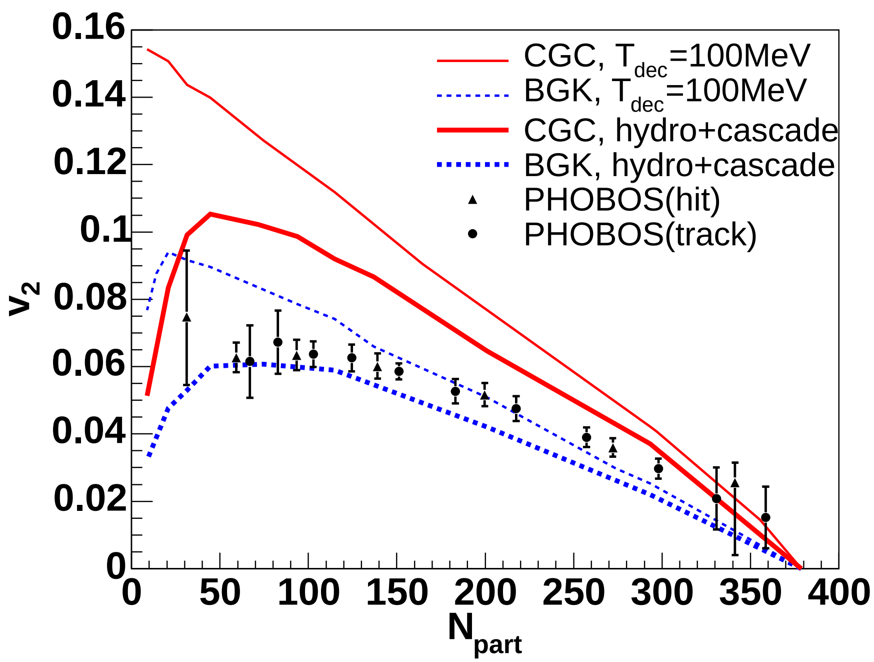
[Glauber model initial conditions (85% soft/15% hard)]

- Not enough elliptic flow from perfect QGP fluid – some hadronic contribution to v_2 is required
- Treating the hadronic stage as ideal fluid overpredicts v_2 in peripheral collisions and at forward rapidities
- Dissipation in hadronic cascade brings theory in line with data
- No need for QGP viscosity!?

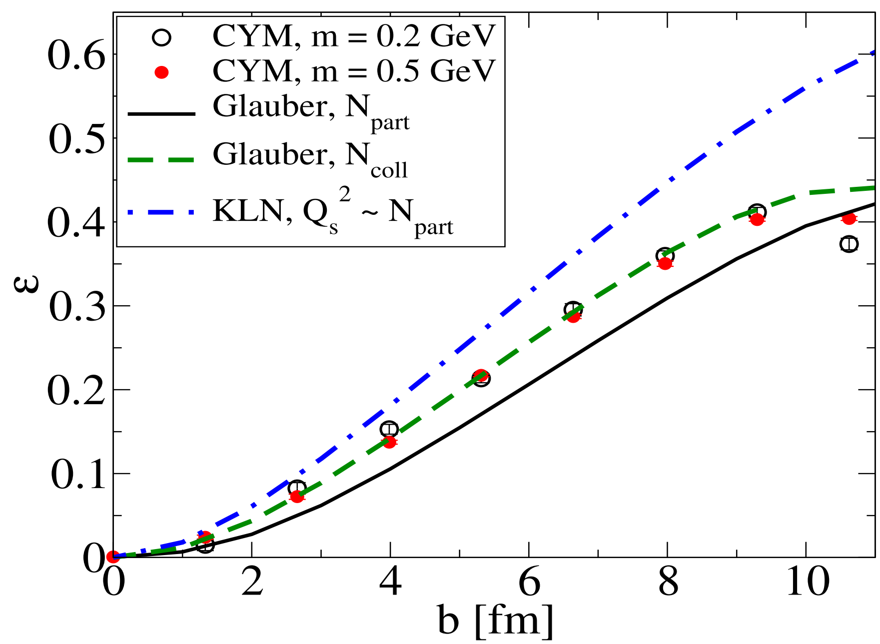
But: CGC gives larger initial eccentricity!

3D Hydro+Cascade Model: Ideal fluid dynamics for QGP above T_c, hadronic cascade with realistic cross sections (JAM) below T_c

Hirano et al., PLB 636 (2006) 299



Lappi & Venugopalan, PRC 74 (2006) 054905



- Hadronic dissipation reduces elliptic flow buildup in peripheral collisions
- Color Glass Condensate (CGC-KLN) model (McLerran & Venugopalan 1994; Kharzeev, Levin, Nardi 2001) produces steeper edge of initial distribution, resulting in larger eccentricities ϵ than in Glauber model
- Ideal hydrodynamics turns larger spatial eccentricity ϵ into larger elliptic flow v_2
- For Glauber model initial conditions, hadronic dissipation fully explains the data; for CGC/KLN initial conditions hadronic dissipation not enough – need additional QGP viscosity!

$\Longrightarrow$ Need better control over initial conditions!

Relativistic hydrodynamics for viscous fluids

Viscous relativistic hydrodynamics (Israel & Stewart 1979)

Include shear viscosity η, neglect bulk viscosity (massless partons) and heat conduction ($\mu_B \approx 0$); solve

$$\partial_\mu T^{\mu\nu} = 0$$

with modified energy momentum tensor

$$T^{\mu\nu}(x) = T_0^{\mu\nu}(x) + \pi^{\mu\nu} = \big(e(x){+}p(x)\big)u^\mu(x)u^\nu(x) - g^{\mu\nu}p(x) + \pi^{\mu\nu}.$$

$\pi^{\mu\nu}$ = traceless viscous pressure tensor which relaxes locally to 2η times the shear tensor $\sigma^{\mu\nu} \equiv \nabla^{\langle\mu}u^{\nu\rangle}$ on a microscopic kinetic time scale τ_π:

$$D\pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu}{-}2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) + \tfrac{1}{2}\pi^{\mu\nu}\left(5D\ln T{-}\theta\right) - 2\pi^{\alpha(\mu}\omega^{\nu)}{}_{\alpha} - \left(u^\mu\pi^{\nu\lambda}{+}u^\nu\pi^{\mu\lambda}\right)Du_\lambda$$

where $D \equiv u^\mu\partial_\mu$ is the time derivative in the local rest frame, $\theta = \partial\cdot u =$ local expansion rate, and $\omega^{\mu\nu} = \frac{1}{2}(\nabla^\nu u^\mu{-}\nabla^\mu u^\nu) =$ vorticity.

Kinetic theory relates η and τ_π, but for a strongly coupled QGP neither η nor this relation are known $\Longrightarrow$ treat η and τ_π as independent phenomenological parameters. For consistency: $\tau_\pi\theta \ll 1$ ($\theta = \partial^\mu u_\mu =$ local expansion rate).

(1+1)-d viscous hydrodynamic equations

(Muronga & Rischke 2004, Chaudhuri & Heinz 2005)

[For (2+1)-d viscous hydrodynamic equations see Heinz, Song & Chaudhuri, PRC 73 (2006) 034904]

Azimuthally symmetric transverse dynamics with long. boost invariance:

Use (τ, r, ϕ, η) coordinates and solve

- **hydrodynamic equations** for $T^{\tau\tau} = (e{+}\mathcal{P})\gamma_r^2{-}\mathcal{P}, \quad T^{\tau r} = (e{+}\mathcal{P})\gamma_r^2 v_r$ (with "effective pressure" $\mathcal{P} = p{-}r^2\pi^{\phi\phi}{-}\tau^2\pi^{\eta\eta}$) together with
- **kinetic relaxation equations** for $\pi^{\phi\phi},\ \pi^{\eta\eta}$:

$$\frac{1}{\tau}\partial_\tau\Big(\tau T^{\tau\tau}\Big) + \frac{1}{r}\partial_r\Big(r(T^{\tau\tau}+\mathcal{P})v_r\Big) = -\frac{p+\tau^2\pi^{\eta\eta}}{\tau},$$

$$\frac{1}{\tau}\partial_\tau\Big(\tau T^{\tau r}\Big) + \frac{1}{r}\partial_r\Big(r(T^{\tau r}v_r+\mathcal{P})\Big) = +\frac{p+r^2\pi^{\phi\phi}}{r},$$

$$\Big(\partial_\tau + v_r\partial_r\Big)\pi^{\eta\eta} = -\frac{1}{\gamma_r\tau_\pi}\left[\pi^{\eta\eta} - \frac{2\eta}{\tau^2}\left(\frac{\theta}{3}-\frac{\gamma_r}{\tau}\right)\right],$$

$$\Big(\partial_\tau + v_r\partial_r\Big)\pi^{\phi\phi} = -\frac{1}{\gamma_r\tau_\pi}\left[\pi^{\phi\phi} - \frac{2\eta}{r^2}\left(\frac{\theta}{3}-\frac{\gamma_r v_r}{r}\right)\right].$$

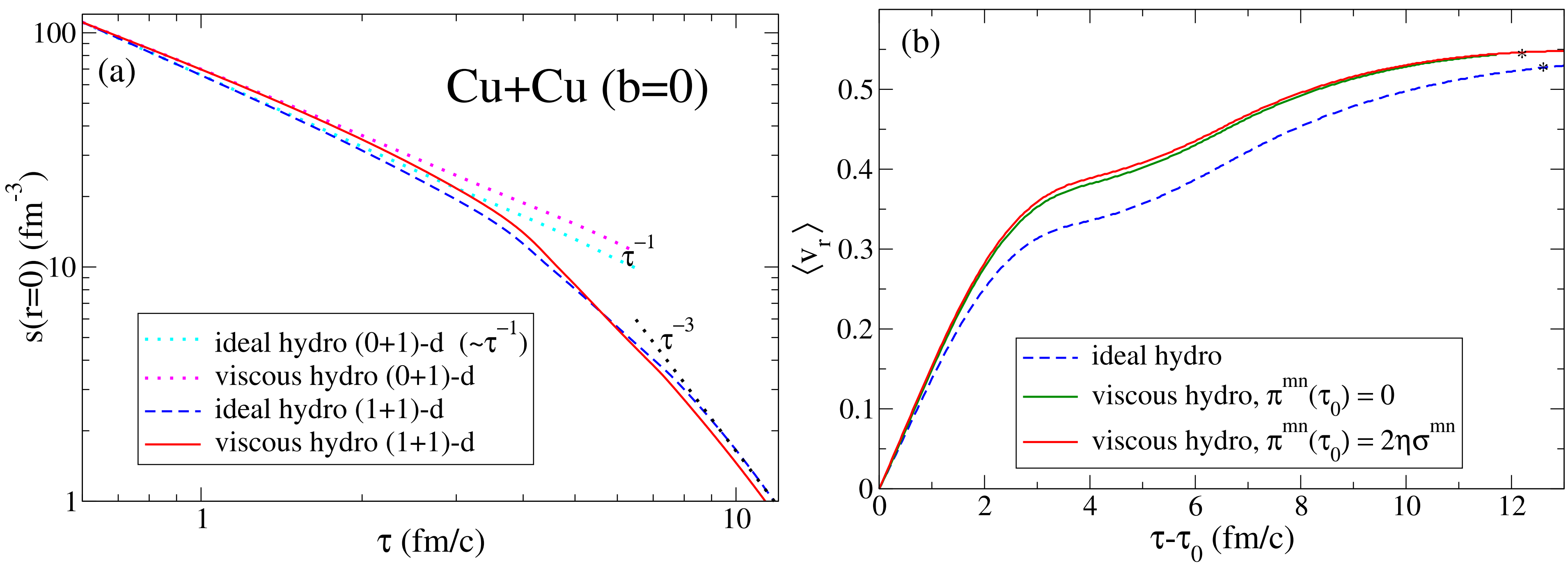
Close equations with EOS $p(e)$ where $e = T^{\tau\tau}{-}v_r T^{\tau r}$ and $v_r = T^{\tau r}/(T^{\tau\tau}{+}\mathcal{P})$.

(2+1)-d viscous hydro: less longitudinal work, more radial flow

Cu+Cu @ $b = 0$, EOS Q

$\tau_0 = 0.6\,\frac{\text{fm}}{c},\ e_0 = 30\,\frac{\text{GeV}}{\text{fm}^3},\ \frac{\eta}{s} = \frac{1}{4\pi},\ \tau_\pi = 0.24\left(\frac{200\,\text{MeV}}{T}\right)\frac{\text{fm}}{c},\ T_{\text{dec}} = 130\,\text{MeV}$

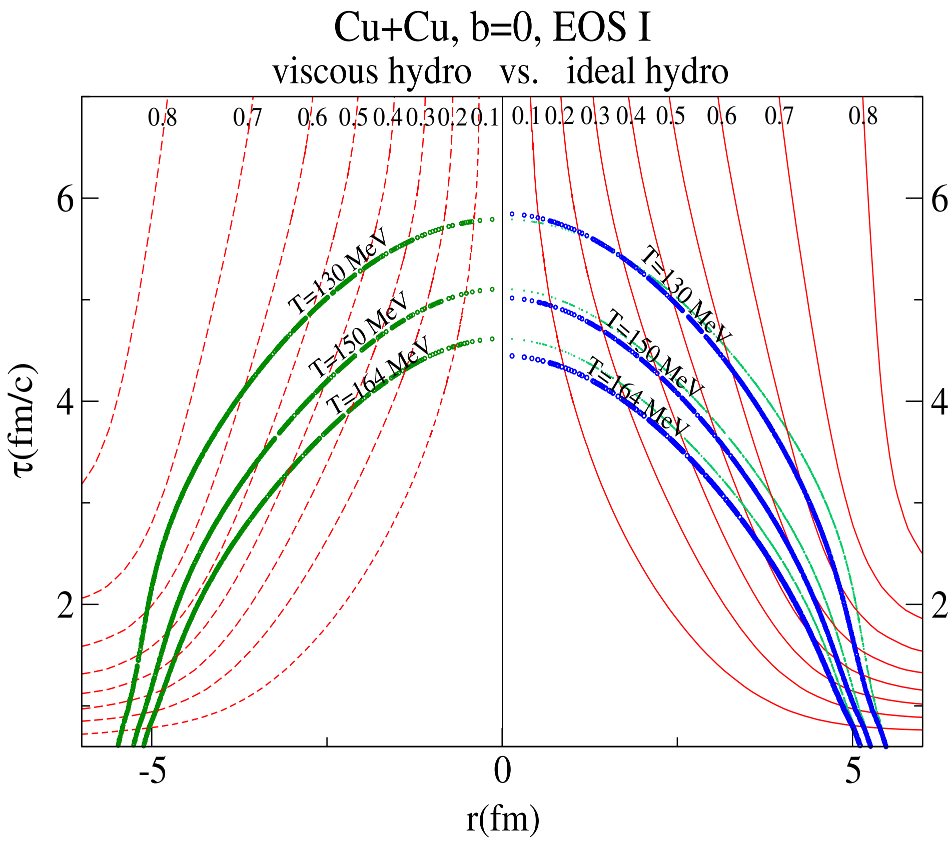
Entropy density vs. time　　　　avg. radial flow vs. time

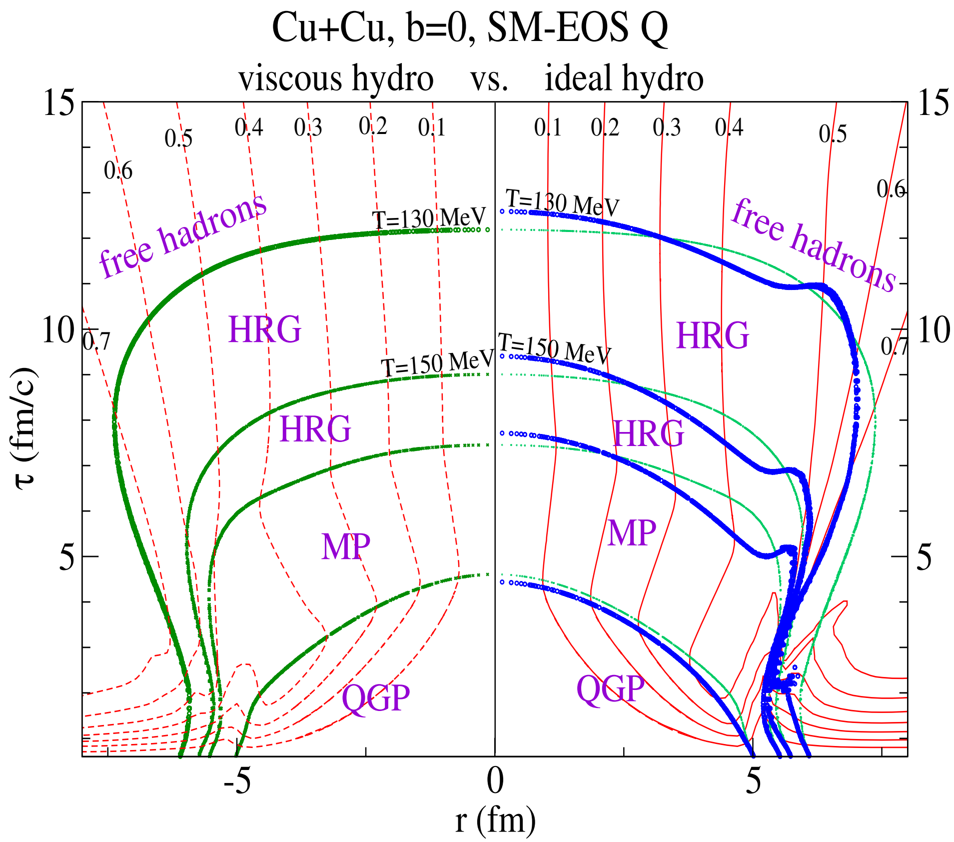


- Radial flow develops much faster, expansion turns 3-dimensional more abruptly
- Shear viscosity initially reduces the cooling due to longitudinal work, but then leads to faster cooling in the fireball center than for ideal fluid later, due to stronger radial flow
  (seen also by Teaney 2004, Chaudhuri 2006,2007; Romatschke et al. 2006,2007)

Central Cu+Cu (b=0): ideal vs. viscous hydro

$\tau_0 = 0.6\,\frac{\mathrm{fm}}{c},\ e_0 = 30\,\frac{\mathrm{GeV}}{\mathrm{fm}^3},\ \frac{\eta}{s} = \frac{1}{4\pi},\ \tau_\pi = 0.24\left(\frac{200\,\mathrm{MeV}}{T}\right)\frac{\mathrm{fm}}{c},\ T_{\mathrm{dec}} = 130\,\mathrm{MeV}$



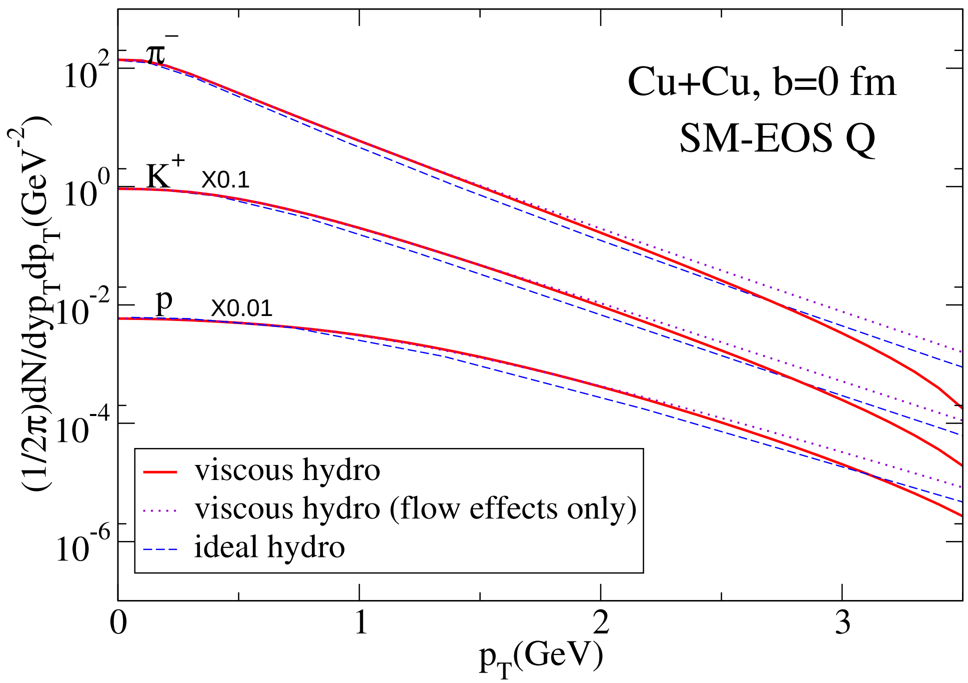


- Viscous hydro smoothes out phase transition structures
- Viscous hydro cools more slowly than ideal hydro, except for the center where cooling is accelerated at late times by faster radial expansion in the viscous case
- Viscous effects increase QGP lifetime, but viscous pressure gradients in the mixed phase shorten the mixed phase lifetime

(2+1)-d viscous hydro: more radial flow $\Longrightarrow$ flatter spectra

hadron p_T-spectra:

$$E\frac{dN}{d^3p} = \int_\Sigma \frac{p\cdot d^3\sigma(x)}{(2\pi)^3}\left[f_{\rm eq}(x,p) + \delta f(x,p)\right] = \int_\Sigma \frac{p\cdot d^3\sigma(x)}{(2\pi)^3}\, f_{\rm eq}(x,p)\left(1 + \frac{1}{2}\frac{p^\alpha p^\beta}{T^2(x)}\frac{\pi_{\alpha\beta}(x)}{(e+p)(x)}\right)$$



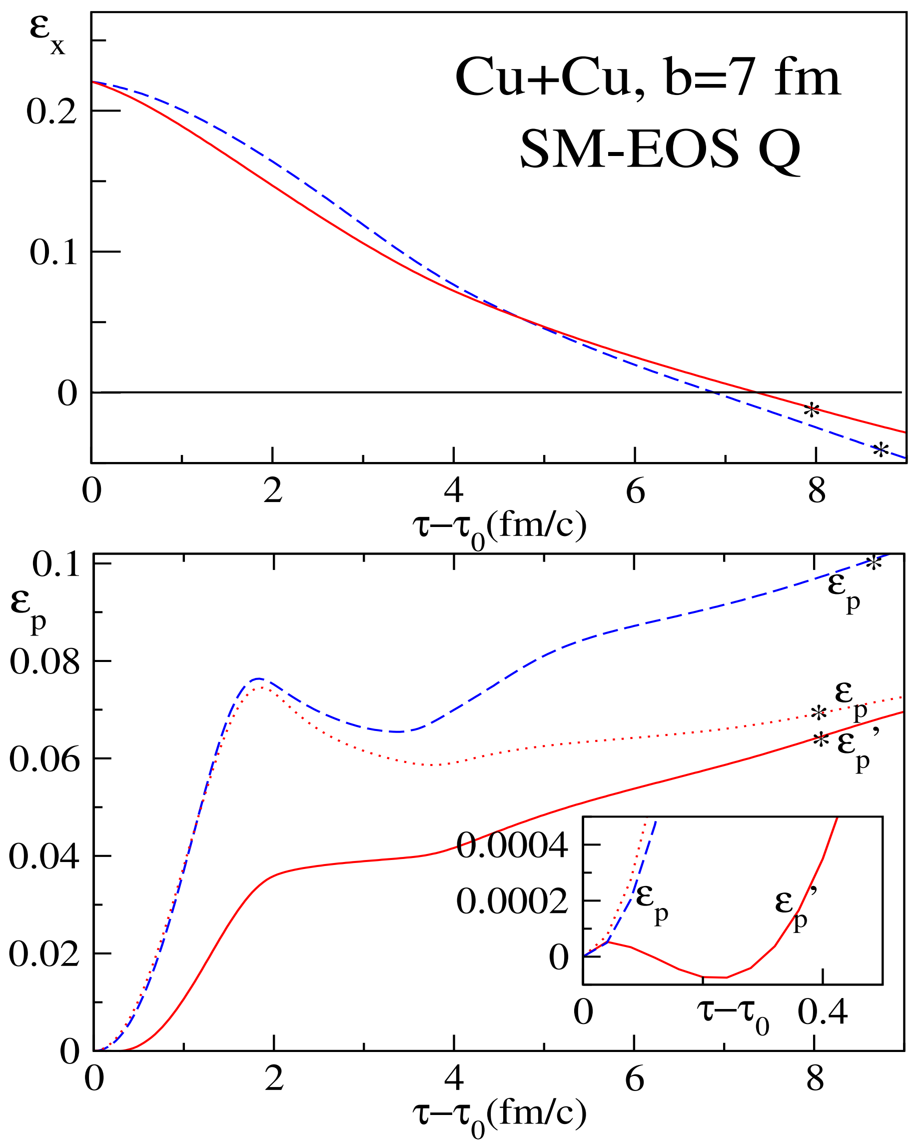
$$\tau_0 = 0.6\,\frac{\rm fm}{c},\ e_0 = 30\,\frac{\rm GeV}{\rm fm^3},\ \frac{\eta}{s} = \frac{1}{4\pi},\ \tau_\pi = 0.24\left(\frac{200\,\rm MeV}{T}\right)\frac{\rm fm}{c},\ T_{\rm dec} = 130\,\text{MeV}$$

- For identical initial and freeze-out conditions, viscous evolution yields more radial flow and flatter spectra (as previously seen by Chaudhuri 2006,2007; Romatschke 2007)
- Effect on $b = 0$ spectra can be largely absorbed by starting viscous hydro later with lower initial density (Romatschke et al., 2006,2007)

(2+1)-d viscous hydro: less momentum anisotropy

Cu+Cu @ $b = 7\,\text{fm}$, EOS Q, same initial and final conditions

spatial eccentricity and momentum anisotropy

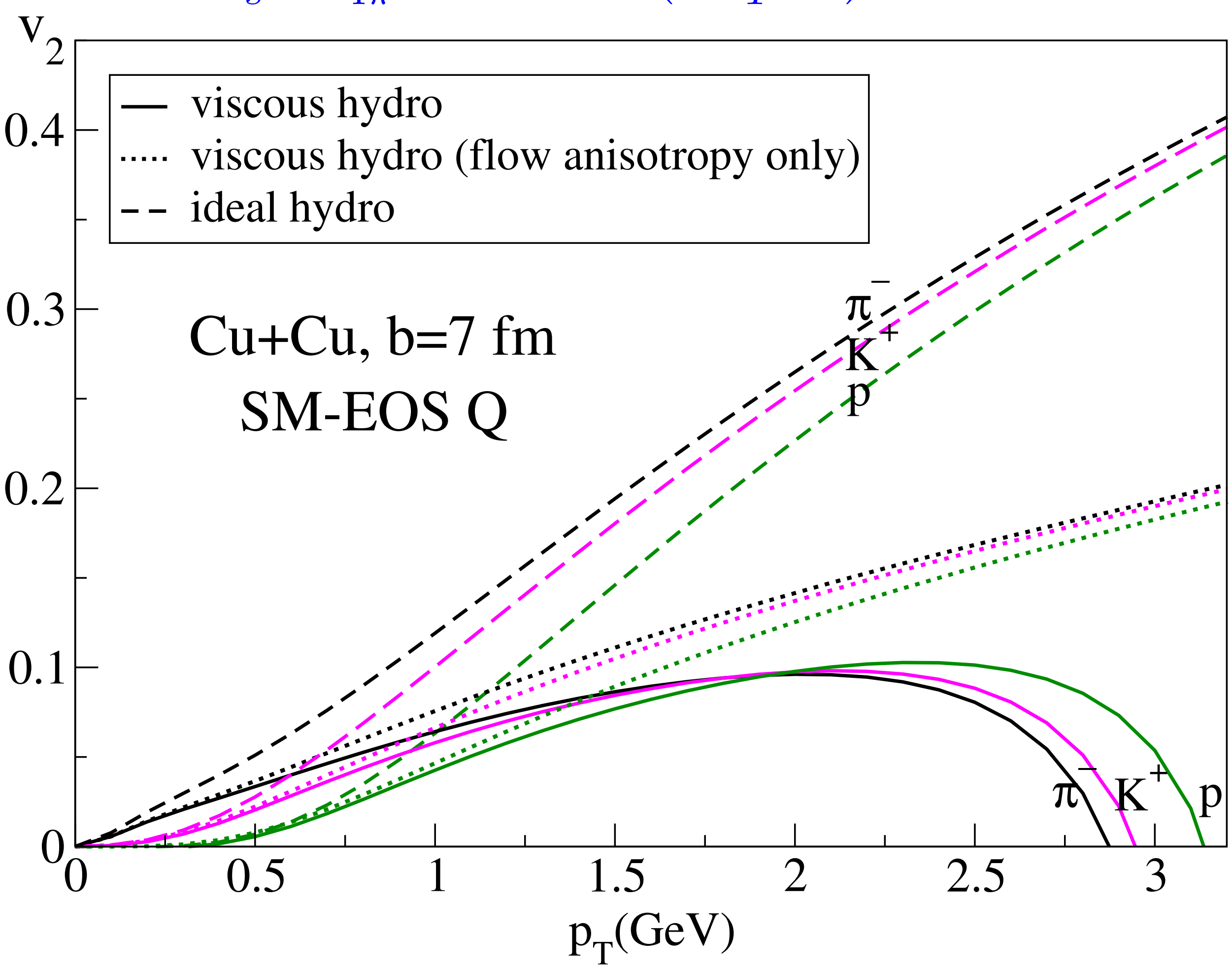


- - - ideal hydro, ——, · · · viscous hydro

- Source eccentricity $\epsilon_x = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$ decays initially faster, but later more slowly;
- Flow anisotropy $\epsilon_p = \frac{\langle T_0^{xx} - T_0^{yy} \rangle}{\langle T_0^{xx} + T_0^{yy} \rangle}$ develops faster initially, but soon drops significantly below ideal fluid values;
- during the first 3-4 fm$/c$ viscous pressure components $\pi^{\mu\nu}$ contribute strong out-of-plane (i.e. negative) momentum anisotropy in the local fluid rest frame; inhibits build-up of flow anisotropy and delays local momentum isotropization
- Total momentum anisotropy $\epsilon_p' = \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$ is reduced by almost 50% relative to ideal fluid.

Elliptic flow from (2+1)-dim. viscous hydrodynamics

$$\frac{\eta}{s} = \frac{1}{4\pi},\ \tau_\pi = 0.24\left(\frac{200\,\mathrm{MeV}}{T}\right)\ \mathrm{fm}/c$$

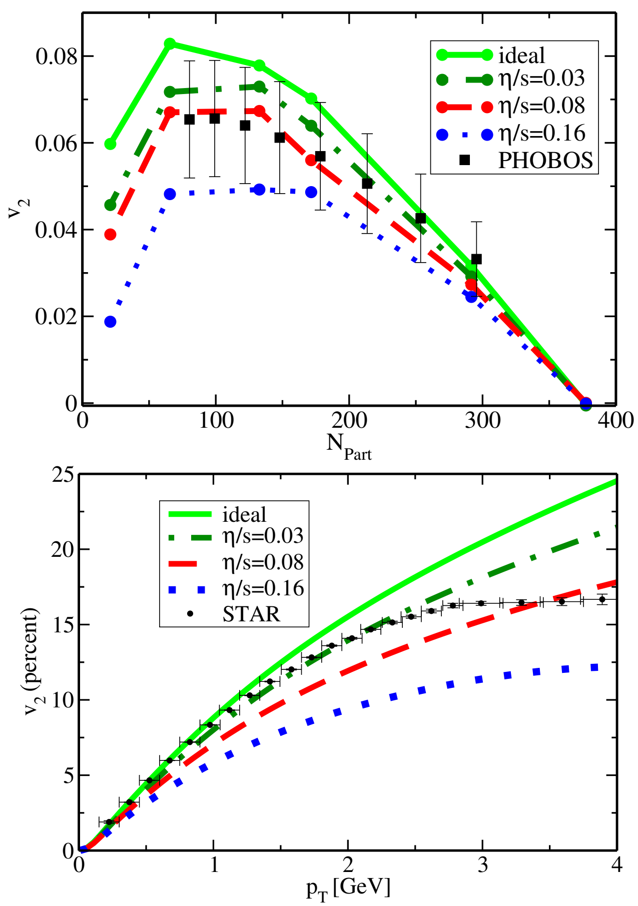


- **Elliptic flow very sensitive to even minimal shear viscosity!**
- Viscous corrections to equilibrium distribution fct. have significant effect on v_2 (Teaney 2003), but at low p_T the effects from the reduced hydrodynamic flow anisotropy are larger

Elliptic flow from (2+1)-d viscous hydro: other results

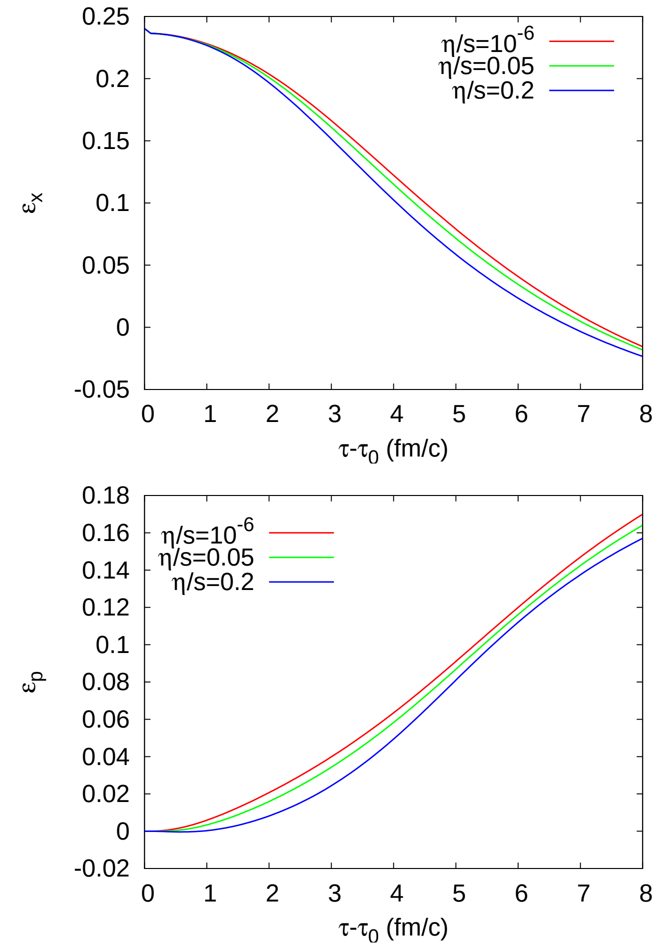
P.&U.Romatschke, PRL99 (2007) 172301

Israel-Stewart



Dusling & Teaney, arXiv:0710.5932

Öttinger-Grmela



• Both groups find much weaker viscous effects on v_2, but also solve different sets of equations

Sensitivity to parameters

$(\tau_\pi,\ \pi^{\mu\nu}(\tau_0))$

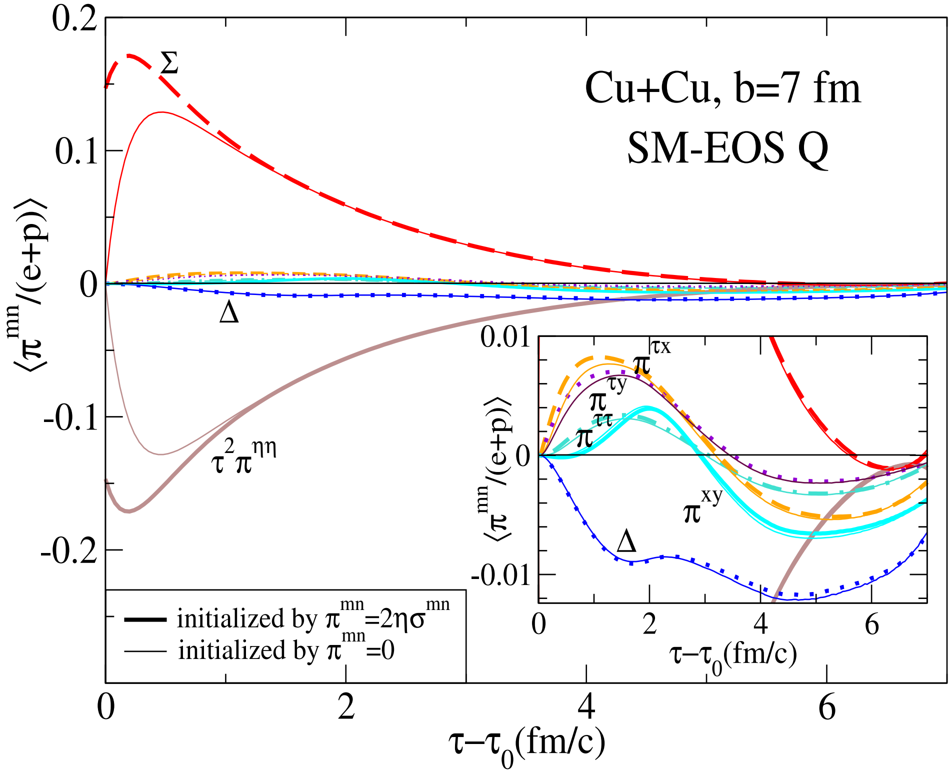
Sensitivity to initial values for viscous pressure tensor

Thin lines: $\pi_0^{mn} = 0$; Thick lines: $\pi_0^{mn} = 2\eta\sigma^{mn} \equiv 2\eta\nabla^{\langle m}u^{n\rangle}$.

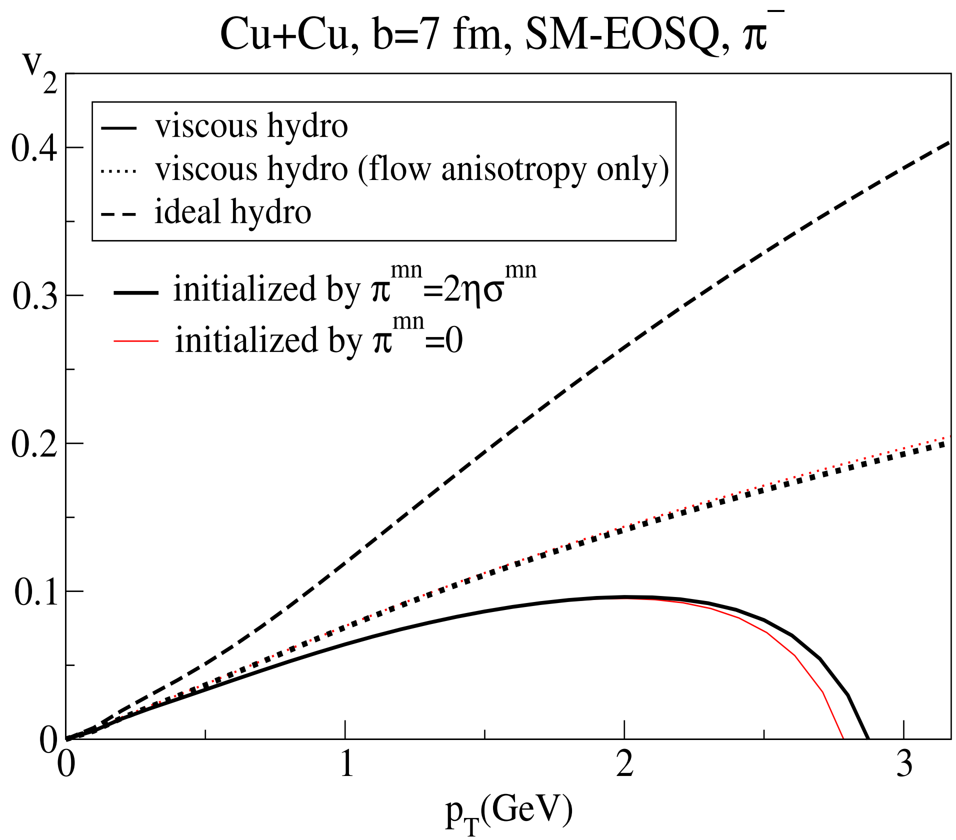
$\tau_0 = 0.6\,\frac{\text{fm}}{c},\ e_0 = 30\,\frac{\text{GeV}}{\text{fm}^3},\ \frac{\eta}{s} = \frac{1}{4\pi},\ \tau_\pi = 0.24\left(\frac{200\,\text{MeV}}{T}\right)\frac{\text{fm}}{c},\ T_{\text{dec}} = 130\,\text{MeV}$

largest viscous pressure components vs. time

$\Sigma = \pi^{xx}+\pi^{yy},\quad \Delta = \pi^{xx}-\pi^{yy}$

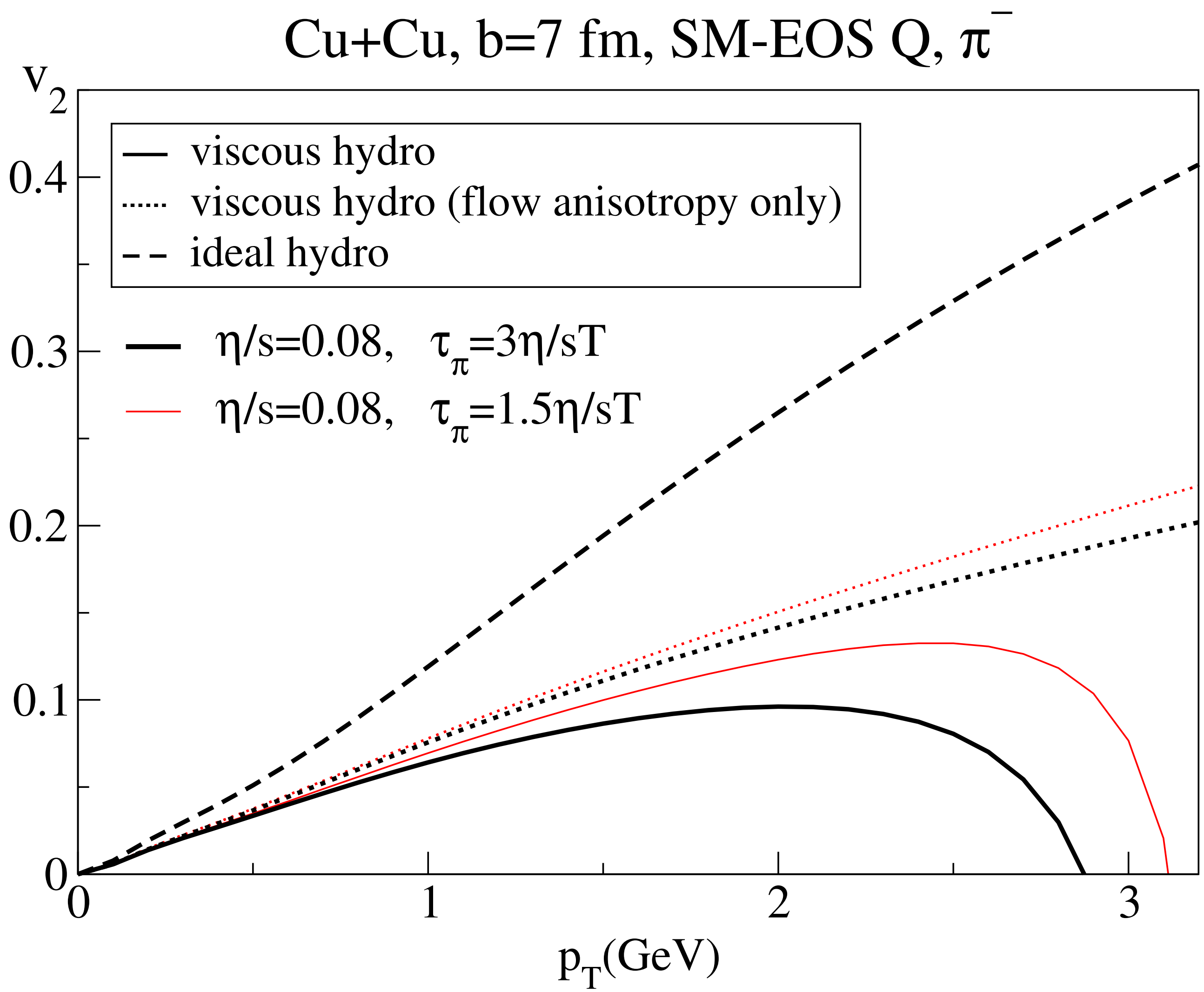


pion elliptic flow



- For fixed η/s, viscous pressure components become small at late times $\longrightarrow$ ideal hydro
- After $\tau \sim 1\,\text{fm}/c \sim 5\tau_\pi$, viscous pressure tensor has lost all memory of initial conditions!
- Effects of initial π^{mn} on final v_2 are small

Sensitivity to kinetic relaxation time τ_π:



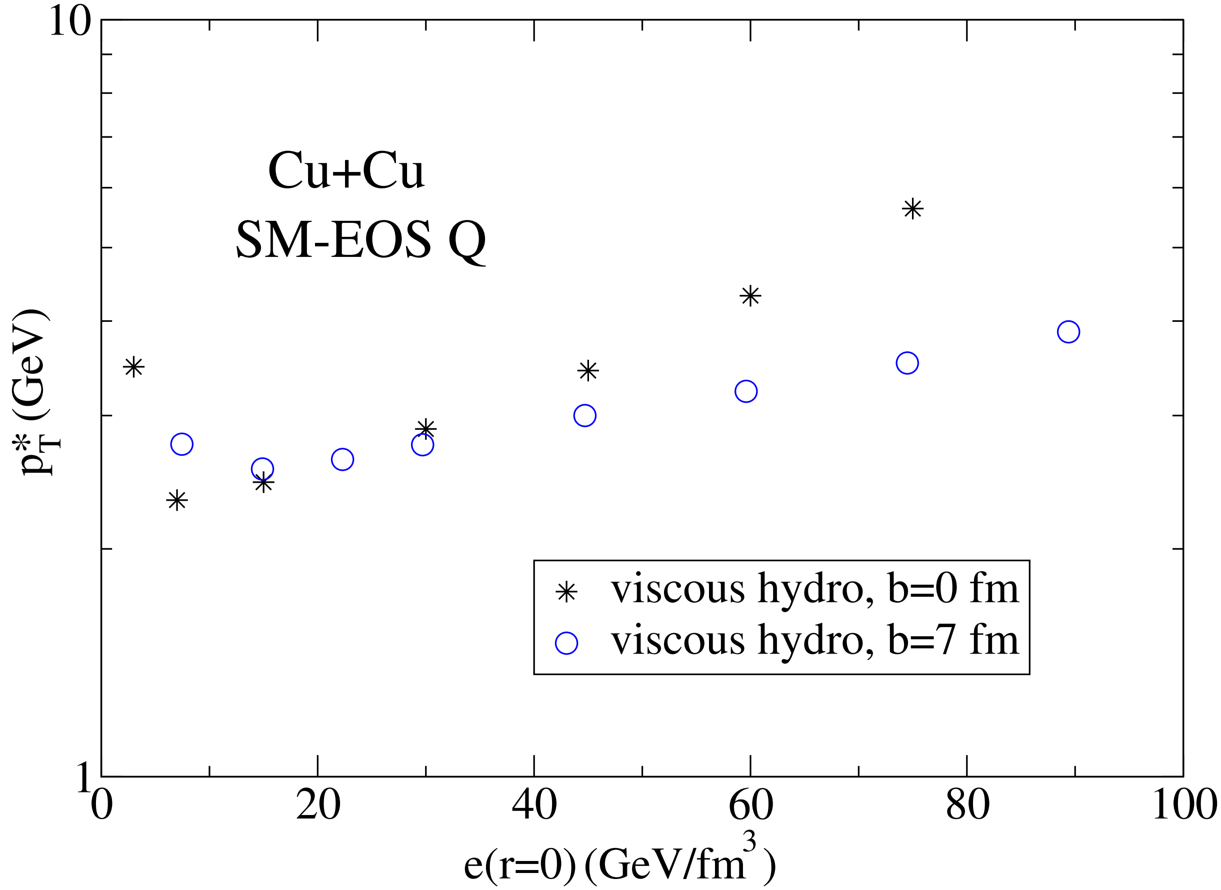
- **Faster kinetic relaxation at fixed η/s reduces viscous effects $\longrightarrow$ Janik 2007**
- **larger τ_π $\rightarrow$ larger $\frac{\pi^{\mu\nu}}{e+p}$ at early times, and more deviation from ideal hydro!**

Limits of viscous hydrodynamics:

The limits of viscous hydrodynamics

At sufficiently large p_T, viscous corrections become large even if η/s is small.

$|\delta N(p)| > \frac{1}{2}|N_0(p)|$ indicates breakdown of the assumptions:

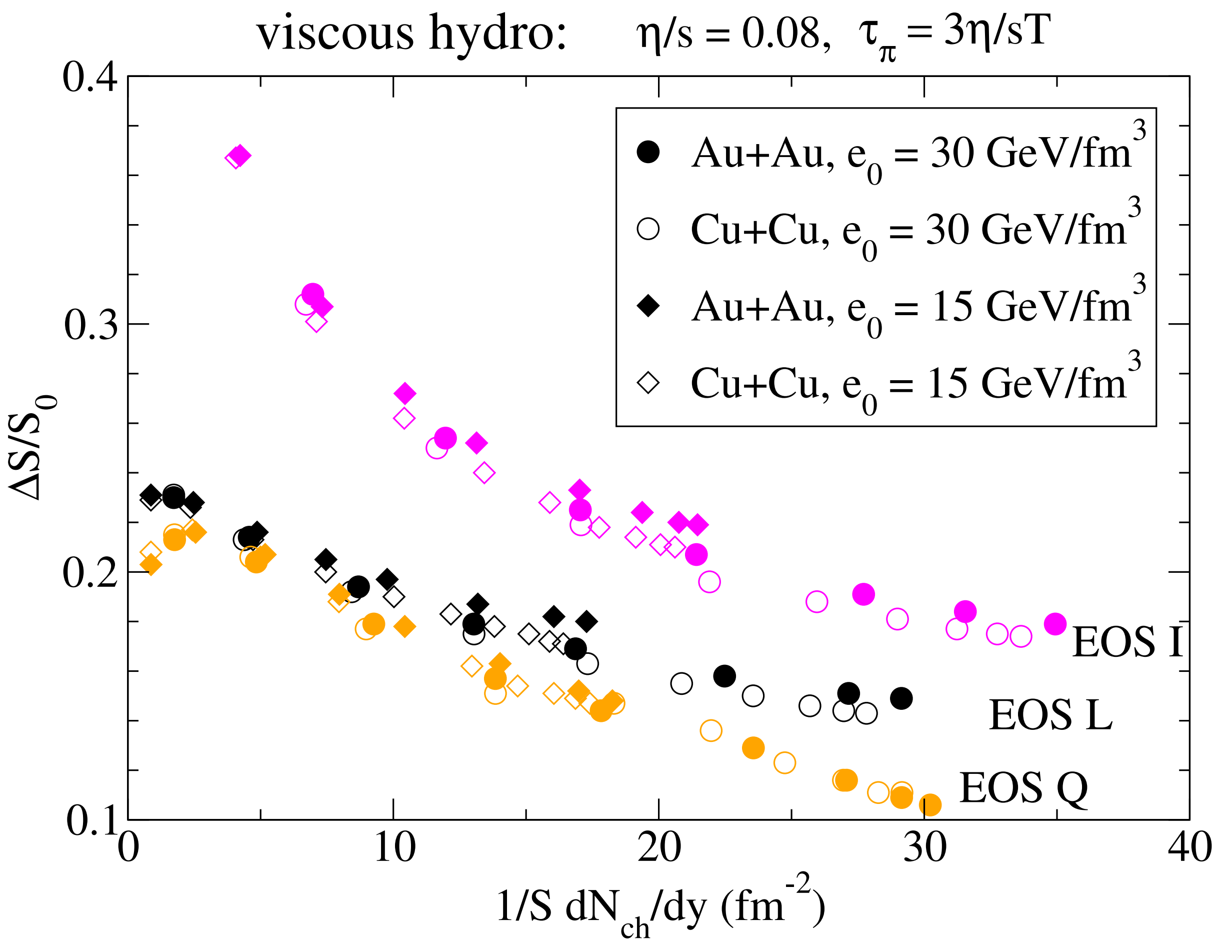


- **For larger initial energy densities, p_T-range increases where viscous hydro can be applied to describe hadron spectra.**

Tests of the viscous hydro code VISH2+1

- $\eta \to 0 \quad \longrightarrow$ ideal fluid code AZHYDRO (test hydro evolution algorithm)
- $\nabla_\perp p = 0,\ \tau_\pi \to 0 \quad \Longrightarrow$ reproduce analytic soln. of boost-invariant Navier-Stokes
- $\eta,\ \tau_\pi$ small $\quad \Longrightarrow$ Israel-Stewart $\mapsto$ Navier-Stokes (tests kinetic evolution algorithm for $\pi^{\mu\nu}$)
- $\pi^\mu_\mu = 0,\ u_\mu \pi^{\mu\nu} = 0$ to better than 2%
- Evolution of e, u^μ, $\pi^{\mu\nu}$ by VISH2+1 tested against Romatschkes' code:
  - excellent agreement for identical initial conditions, EOS, kinetic evolution equations
  - large difference in published $v_2(p_T)$ due to extra terms in $D\pi^{\mu\nu} = \ldots$ used by the Romatschkes

Viscous entropy production



EOS I: ideal gas of massless partons

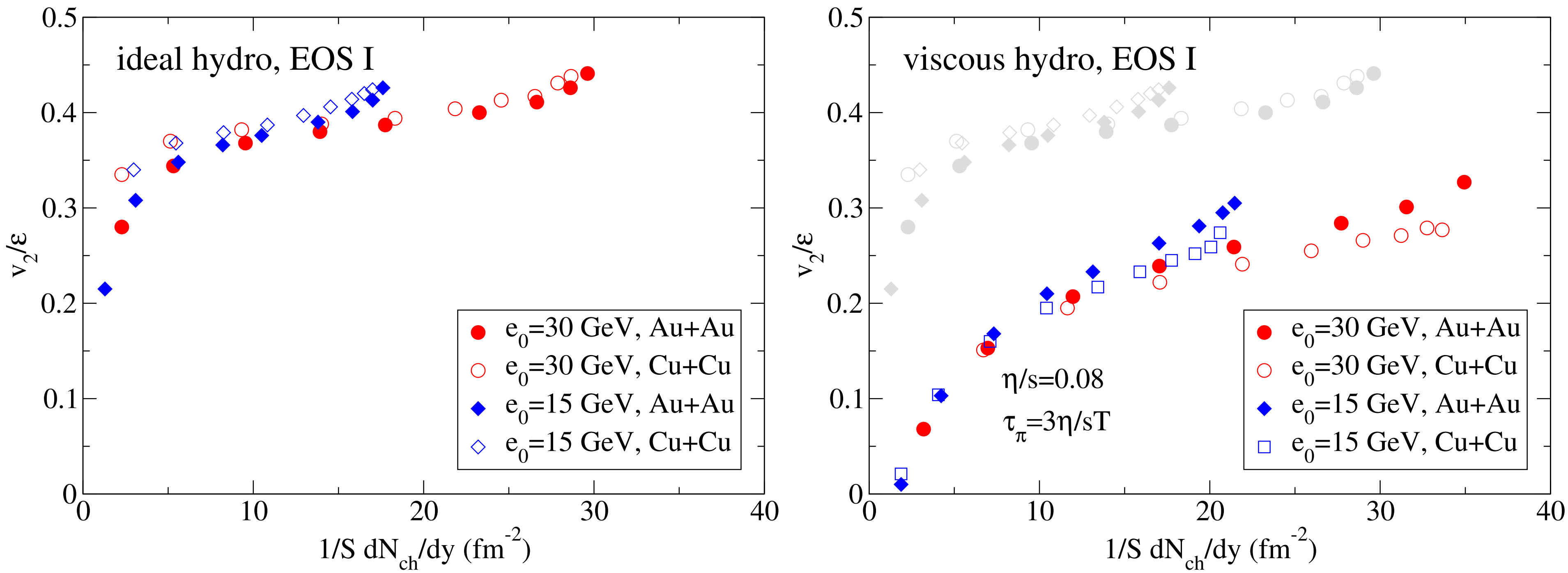
EOS Q: 1st order QGP-HRG phase transition

EOS L: smooth crossover from lattice QCD data above T_c to HRG below T_c.

- Viscous entropy production larger for faster-expanding fireballs
- Entropy production scales approximately with charged multiplicity density per unit area, $\frac{1}{S}\frac{dN_{ch}}{dy}$
- Entropy production fraction is larger for smaller $\frac{1}{S}\frac{dN_{ch}}{dy}$ (lower-energy and more peripheral collisions)
- At the same $\frac{1}{S}\frac{dN_{ch}}{dy}$, collisions between larger nuclei take longer to freeze out, generating slightly more entropy

Multiplicity scaling of the normalized elliptic flow v_2/ϵ_x (I)

Preliminary!

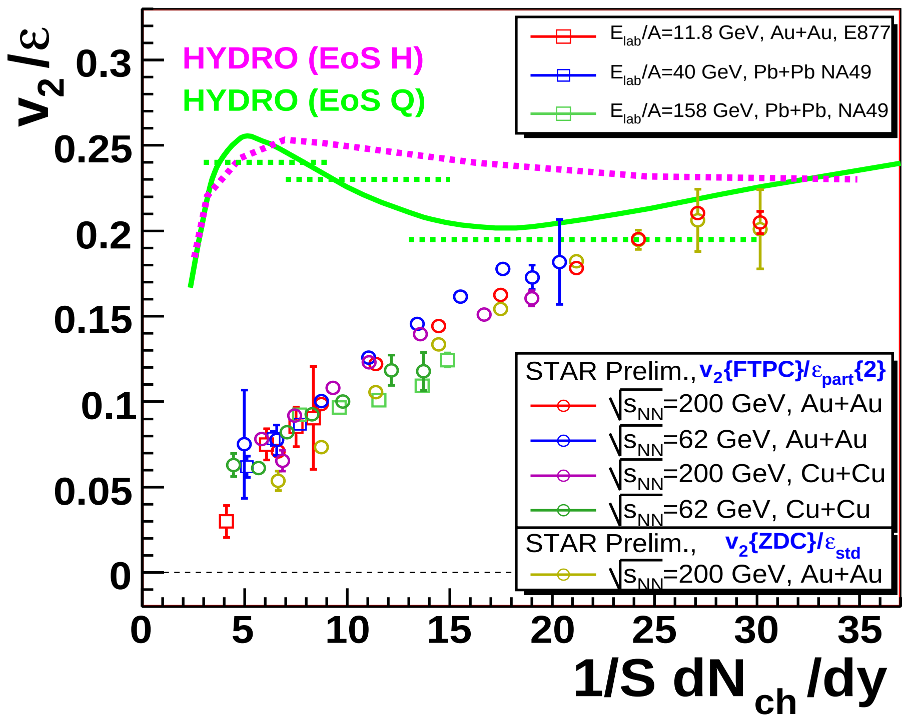


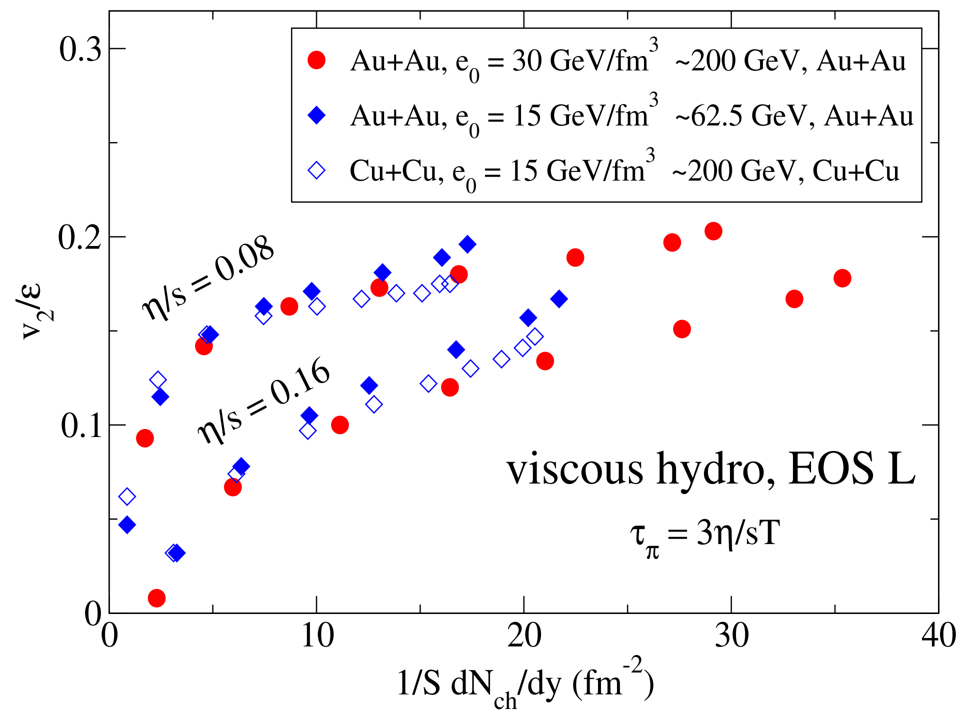
- Freeze-out at constant $e_{\rm dec}$ introduces time scale, breaking the scale invariance of ideal hydro and cutting short the build-up of elliptic flow before it saturates
- At the same $\frac{1}{S}\frac{dN_{\rm ch}}{dy}$, collisions between smaller nuclei and more peripheral collisions freeze out earlier, with less elliptic flow v_2/ϵ_x
- This breaks the multiplicity scaling with $\frac{1}{S}\frac{dN_{\rm ch}}{dy}$ even for ideal hydro
- This scaling is broken even more strongly in viscous hydro!

  At fixed $\frac{1}{S}\frac{dN_{\rm ch}}{dy}$, smaller collision systems and more peripheral collisions show more viscous suppression of v_2/ϵ_x than more central collisions or collisions of larger nuclei

Multiplicity scaling of the normalized elliptic flow v_2/ϵ_x (II)

A case study with fixed specific viscosity η/s:





- General tendency of experimental data consistent with viscous effects
- Data require more than minimal shear viscosity (due to highly viscous late hadron gas stage!)
- Search for scale-breaking effects requires more accurate data
- Realistic modeling must account for T-dependence of shear and bulk viscosity, especially near T_c

Summary

- Shear viscosity reduces the longitudinal pressure but increases the transverse pressure in heavy ion collision
  $\Longrightarrow$ slower cooling by longitudinal work initially, but faster cooling by stronger transverse expansion later

- While viscous pressure effects on angle-averaged p_T-spectra (radial flow) can be largely absorbed by changing the initial conditions (starting the transverse expansion later and with lower initial energy density), this increases the destructive effects of shear viscosity on the buildup of elliptic flow.

- The effects of shear viscosity on elliptic flow are large; even Son's minimal viscosity $\eta/s = 1/4\pi$ seems almost incompatible with RHIC data $\Longrightarrow$ needs more checking (resolve ambiguities in Israel-Stewart approach!).

- Shorter kinetic relaxation times for the viscous pressure tensor reduce the effects from shear viscosity; only weak sensitivity to initial values for $\pi^{\mu\nu}$.

- Viscous entropy production roughly scales with multiplicity per transverse area; larger viscous effects for smaller collision systems and larger impact parameters

- Multiplicity scaling of normalized elliptic flow v_2/ϵ_x weakly broken by freeze-out in ideal hydro and slightly more strongly broken by shear viscosity in viscous hydro

Supplements

(2+1)-d viscous hydrodynamic equations

Heinz, Song & Chaudhuri, PRC 73 (2006) 034904

Transverse dynamics w/o azimuthal symmetry, but with long. boost invariance:
Use (τ, x, y, η) coordinates and solve

- hydrodynamic equations for $T^{\tau\tau} = (e{+}p)\gamma_r^2 - p + \pi^{\tau\tau}$, $T^{\tau x} = (e{+}p)\gamma_\perp^2 v_x + \pi^{\tau x}$, $T^{\tau y} = (e{+}p)\gamma_\perp^2 v_y + \pi^{\tau y}$:

$$\frac{1}{\tau}\partial_\tau\left(\tau T^{\tau\tau}\right) + \partial_x\left(v_x T^{\tau\tau}\right) + \partial_y\left(v_y T^{\tau\tau}\right) = \mathcal{S}^{\tau\tau}[v_x, v_y, \pi^{\eta\eta}, \pi^{\tau\tau}, \pi^{\tau x}, \pi^{\tau y}]$$

$$\frac{1}{\tau}\partial_\tau\left(\tau T^{\tau x}\right) + \partial_x\left(v_x T^{\tau x}\right) + \partial_y\left(v_y T^{\tau x}\right) = \mathcal{S}^{\tau x}[v_x, v_y, \pi^{xx}, \pi^{xy}, \pi^{\tau x}]$$

$$\frac{1}{\tau}\partial_\tau\left(\tau T^{\tau y}\right) + \partial_x\left(v_x T^{\tau y}\right) + \partial_y\left(v_y T^{\tau y}\right) = \mathcal{S}^{\tau y}[v_x, v_y, \pi^{yy}, \pi^{xy}, \pi^{\tau y}]$$

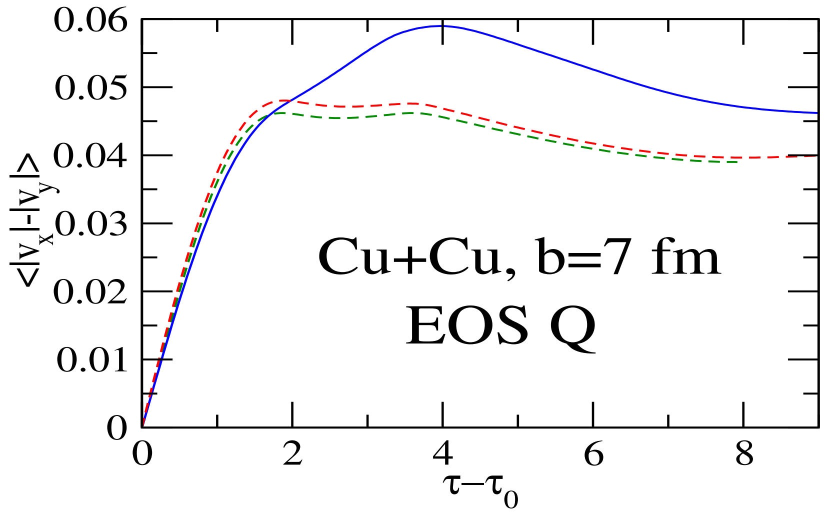
- kinetic relaxation equations for $\pi^{\tau\tau}$, $\pi^{\tau x}$, $\pi^{\tau y}$, and $\pi^{\eta\eta}$ (4, not 3!).

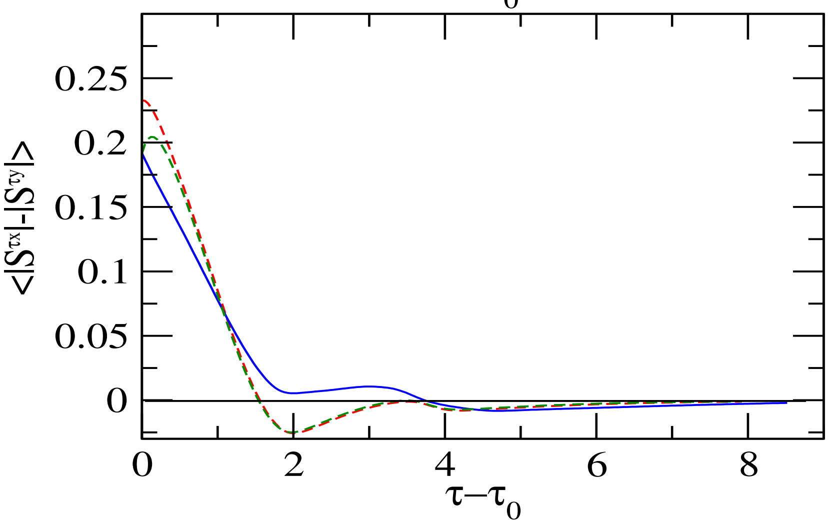
Close equations with EOS $p(e)$ where $e = M_0 - v_\perp M$ and $v_\perp = M/(M_0 + p(e))$ (again one implicit scalar equation!), with the definitions
$(M_0, M_x, M_y) \equiv (T^{\tau\tau} - \pi^{\tau\tau},\ T^{\tau x} - \pi^{\tau x},\ T^{\tau y} - \pi^{\tau y})$ and $M = \sqrt{M_x^2 + M_y^2}$,
and the relations $v_x = M_x/M,\ v_y = M_y/M$.

Sensitivity to initial values for viscous pressure tensor

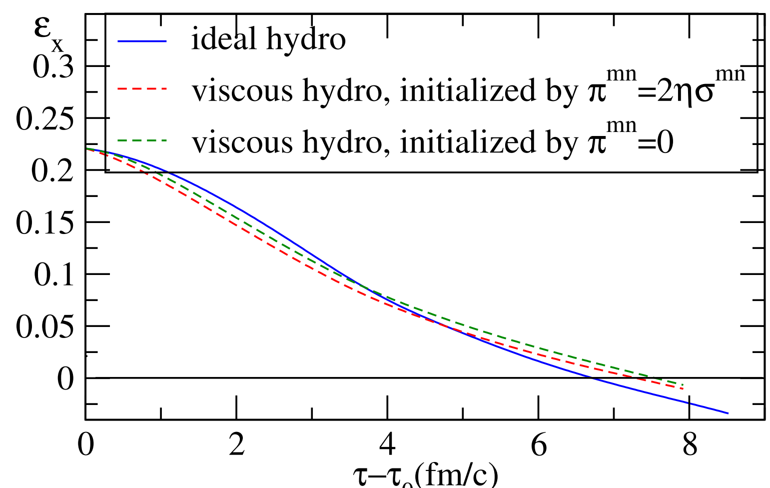
Romatschke & Romatschke 2007 seem to find much smaller viscous effects than we do. But they initialize their evolution with $\pi^{mn} = 0$. Could this be the origin of the discrepancy? **No!**

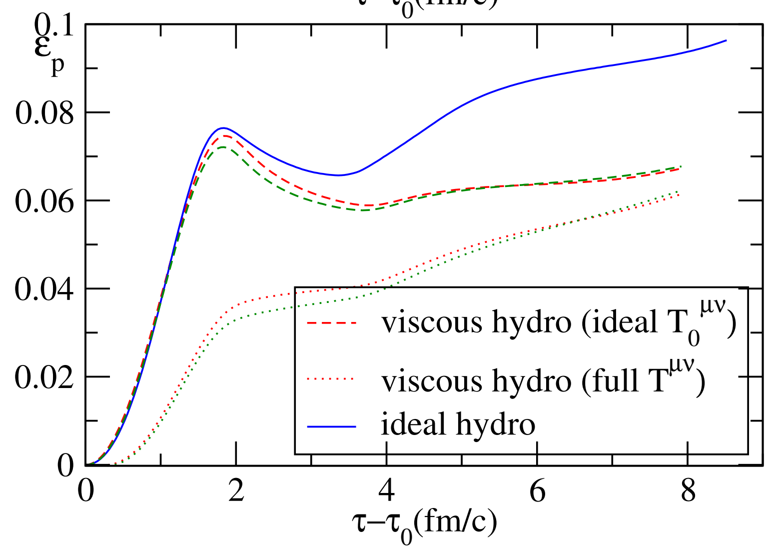
flow anisotropy vs. time





spatial eccentricity and momentum anisotropy

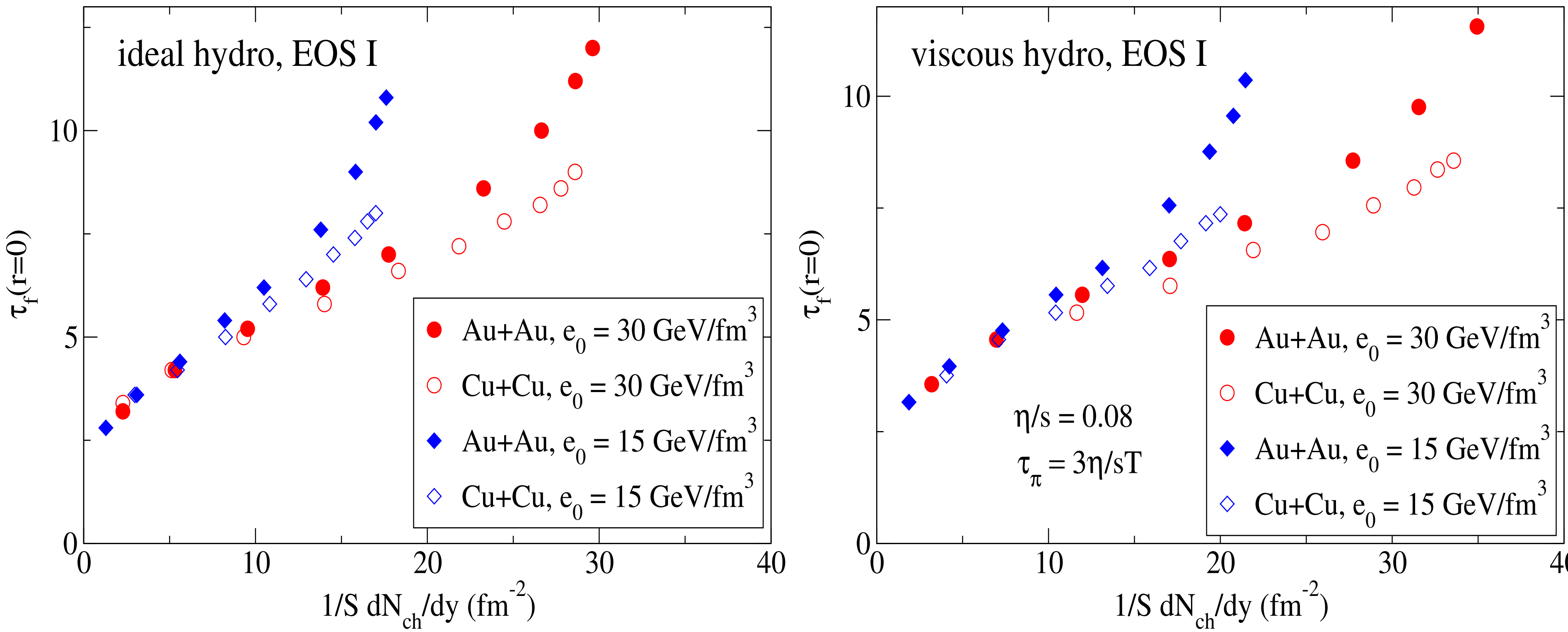




Green lines show results for $\pi_0^{mn} = 0$, with otherwise identical parameters

$\Longrightarrow$ weak sensitivity to initial conditions for viscous pressure tensor.

Central freeze-out times for different collisions systems and centralities



- At the same $\frac{1}{S}\frac{dN_{\rm ch}}{dy}$, collisions between larger nuclei and more central collisions take longer to freeze out

Comparison between VISH2+1 and Romatschkes' code

Evolution of total momentum anisotropy ϵ'_p, Au+Au with EOS I