Quantum Backreaction and Particle Production in QED and QCD in external fields
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I. INTRODUCTION

Nonequilibrium quantum field theory became a topic of interest as a result of several crucial questions facing physics in the 1980's. Before that time, people were mainly concerned with calculating cross sections (S-Matrix elements) for particles produced at colliders. In the early 80's the idea of the Inflationary universe was introduced by Guth, Linde and Starobinsky [1] and the mechanism for inflation was a scalar field (the inflaton) in a background (time dependent) Gravitational field. The initial calculations were done using a classical scalar field. It was natural for people to then try to understand how to solve time evolution problems in the quantum domain. In another area of Cosmological importance, Emil Mottola looked at the problem of particle production of "free" (apart from Gravity) scalar mesons in De Sitter Space as a possible mechanism for the solution of the Cosmological constant problem. [2]. These two problems were the basis for renewed interest in the problem of how one solves numerically initial value and backreaction problems in quantum field theory. At that time even the correct formalism for doing this was not well understood by particle physicists. Even though Schwinger and Keldysh [3] had introduced the correct formalism for treating this problem, they had not addressed for interacting relativistic quantum field theories how to do practical calculations which took into account questions of renormalization. Also at that time it was not known that if one used perturbation theory in the CTP (Closed Time Path) formalism, that the resulting perturbation series was secular. [4]. In 1985, Guth and PI [5] studied the problem of the quantum roll from an inverted harmonic oscillator in the free field limit to see to what extent classical ideas on inflation were modified by quantum effects. The formalism used to study this problem was the functional Schrodinger equation which was the generalization of the ordinary Schrodinger equation to quantum field theory. The Functional Schrodinger equation could be derived by the Dirac Action Principle and allowed people to study time the quantum evolution process using Gaussian trial wave functions which was related to solving the field theory in a Mean Field or Hartree approximation or a large-N approximation. The first conceptual problems that needed to be solved in the the O(N) scalar field theory in this approximation were how to disentangle the infiinities that arose from choosing unphysical initial data from those that were truly effects of renormalization. These issues and there solution was clarified in the work of Cooper and Mottola and Samiullah and Pi [6]. The initial value problems were solved in the Gaussian (leading order in large-N) approximation by choosing initial states that corresponded to finite energy density and number density. The renormalization issues were first understood by doing a WKB analysis of an adiabatic expansion of the Green's functions. This approach was related to the method of adiabatic regulation used to study free fields in background gravitational fields [7]. Later it was shown that one could study the renormalization group flow of the coupling constant with momentum and verify that a more standard renormalization which looked for the lattice answer converging to the continuum renormalization being the simplest approach. It was in order to better understand back reaction as a solution of the cosmological constant problem that Cooper and Mottola studied as a "toy" model, backreaction in the Electric Field case. However interest in particle production following Relativistic Heavy Ion collisions using a Flux Tube model, converted this toy problem into one with experimental consequences [8]. In light of the renewed interest in particle production from semi-classical gluonic fields we have recently undertaken a study of the quantum back reaction problem in SU(3) QCD in 3+1 dimensions in the hope of seeing what one can learn about jet production at RHIC and LHC and also what one can learn about the initial gluon condensate state.

II. PAIR PRODUCTION FROM A STRONG ELECTRIC FIELD

In order to pop a pair of fermions (or bosons) out of the vacuum one must supply an energy eEx in a Compton wave length $x \approx \hbar/mc$. Since this needs to produces (at) rest a pair with rest energy $2mc^2$ then it is clear that the critical value of the electric field for this to happen is of order

$$eE\hbar/mc = 2mc^2 \tag{1}$$

or

$$E_{critical} \approx 2m^2c^3/e\hbar. \tag{2}$$

In order to see that this is the correct variable to scale things, one can make the following simple tunneling picture of the non-perturbative process of pair production. One imagines that one has an electron bound in a potential well of order $|V_0| \approx 2mc^2$ and one then applies a constant electric field which leads to a one dimensional extra potential of eEx to the (say square well) potential of depth $2mc^2$. On then finds that the ionization probability is proportional to the WKB barrier penetration factor:

$$\exp\left[-\frac{2}{\hbar}\int_0^{V_0/eE} dx\left(2m(V_0-|eE|x)^{1/2}\right)\right] = e^{-(8m^2c^3/3\hbar eE)}. \tag{3}$$

In what follows we will set $\hbar = 1; c = 1$. We see that when $E > E_{critical}$ there is no exponential suppression of pair production.

A more careful calculation involves determining the imaginary part of the Action.

A. Constant Electric and Chromoelectric Field Results

In a classic paper in 1951 Schwinger derived the following one-loop non-perturbative formula

$$\frac{dW}{d^4x} = \frac{e^2E^2}{4\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\, e^{-\frac{n\pi m^2}{|eE|}} \tag{4}$$

for the probability of e^+e^- pair production per unit time per unit volume from a constant electric field E via vacuum polarization [9] by using proper time method. In case of charged scalar field theory the corresponding result is given by

$$\frac{dW}{d^4x} = \frac{e^2E^2}{8\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\, e^{-\frac{n\pi m^2}{|eE|}}. \tag{5}$$

The result of Schwinger was extended to QCD by Claudon, Yildiz and Cox [12]. However the p_T distribution of the e^+ (or e^-) production, $\frac{dW}{d^4xd^2p_T}$, could not be obtained by using proper time method of Schwinger. A WKB approximate method was used for this purpose by Casher et. al. [10], but an exact method to do this problem (of determining the transverse distribution of pairs was not found until recently [11]. For QED the WKB analysis gave the correct answer which depended only on the energy density of the Electric Field. However for QCD, the WKB answer was similar but incorrect in that the true answer depended on both

Casimir invariants of $SU(3)$. In the case of fermions in QED one finds for the transverse distribution of fermion pairs:

$$\frac{dW}{d^4xd^2p_T} = -\frac{|eE|}{4\pi^3}\mathrm{Log}[1 - e^{-\pi\frac{p_T^2+m^2}{|eE|}}]. \tag{6}$$

The corresponding result for the charged scalar production is given by

$$\frac{dW}{d^4xd^2p_T} = \frac{|eE|}{8\pi^3}\mathrm{Log}[1 + e^{-\pi\frac{p_T^2+m^2}{|eE|}}]. \tag{7}$$

In QCD the transverse distribution instead depends on two independent Casimir invariants: $C_1 = [E^aE^a]$ and $C_2 = [d_{abc}E^aE^bE^c]^2$ where E^a is the constant chromo-electric field with color index $a, b, c = 1, 2, ..8$[11]. Nayak obtained the following formula for the number of non-perturbative quarks (antiquarks) produced per unit time, per unit volume and per unit transverse momentum from a given constant chromo-electric field E^a

$$\frac{dN_{q,\bar{q}}}{dtd^3xd^2p_T} = -\frac{1}{4\pi^3}\sum_{j=1}^{3}|g\lambda_j|\ \ln[1 - e^{-\frac{\pi(p_T^2+m^2)}{|g\lambda_j|}}], \tag{8}$$

where m is the mass of the quark. This result is gauge invariant because it depends on the following gauge invariant eigenvalues

$$\begin{aligned}
\lambda_1 &= \sqrt{\frac{C_1}{3}}\ \cos\theta\,, \\
\lambda_2 &= \sqrt{\frac{C_1}{3}}\ \cos\,(2\pi/3-\theta)\,, \\
\lambda_3 &= \sqrt{\frac{C_1}{3}}\ \cos\,(2\pi/3+\theta)\,,
\end{aligned} \tag{9}$$

where θ is given by

$$\cos^2 3\theta = 3C_2/C_1^3\,. \tag{10}$$

The integration over p_T in eq. (32) yields

$$\frac{dW}{d^4x} = \frac{1}{4\pi^3}\sum_{j=1}^{3}g^2\lambda_j^2\sum_{n=1}^{\infty}\frac{e^{-\frac{n\pi m^2)}{|g\lambda_j|}}}{n^2}. \tag{11}$$

This result depends on both Casimirs, except for massless fermions when the series can be summed to give:

$$\frac{dW}{d^4x} = \frac{1}{4\pi^3} \sum_{j=1}^{3} g^2\lambda_j^2 \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{g^2}{4\pi^3} \sum_{1}^{8} E^a E^a \zeta(2) \tag{12}$$

which reproduces Schwinger's proper time result for massless fermions, extended to QCD, for the total production rate dN/d^4x [12]. The exact result in eq. (32) can be contrasted with the following formula obtained by the WKB tunneling method [28]

$$\frac{dN_{q,\bar{q}}}{dtd^3xd^2p_T} = \frac{-|gE|}{4\pi^3} \ln[1 - e^{-\frac{\pi(p_T^2+m^2)}{|gE|}}], \tag{13}$$

which does not reproduce the correct result for the p_T distribution of the quark (antiquark) production rate from a constant chromo-electric field E^a. For soft gluon production Nayak and van Nieuwenhuizen [11] found in the Feynman T'Hooft gauge [13]

$$\frac{dN_{gg}}{dtd^3xd^2p_T} = \frac{1}{4\pi^3} \sum_{j=1}^{3} |g\lambda_j| \ln[1 + e^{-\frac{\pi p_T^2}{|g\lambda_j|}}]. \tag{14}$$

This was shown to be indepent of the gauge fixing choice by Cooper and Nayak [15].

B. Background Field Method and Schwinger Pair production in SU(3) Gauge Theory

In the background field method of QCD the gauge field is the sum of a classical background field and the quantum gluon field:

$$A_\mu^a \to A_\mu^a + Q_\mu^a \tag{15}$$

where in the right hand side A_μ^a is the classical background field and Q_μ^a is the quantum gluon field. The gauge field Lagrangian density is given by

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu\nu}^a[A+Q] F^{\mu\nu a}[A+Q]. \tag{16}$$

The background gauge fixing is given by by [13]

$$D_\mu[A] Q^{\mu a} = 0, \tag{17}$$

where the covariant derivative is defined by

$$D_\mu^{ab}[A] \;=\; \delta^{ab}\partial_\mu \;+\; g f^{abc} A_\mu^c. \tag{18}$$

The gauge fixing Lagrangian density is

$$\mathcal{L}_{\rm gf} \;=\; -\frac{1}{2\alpha}[D_\mu[A]Q^{\mu a}]^2 \tag{19}$$

where α is any arbitrary gauge parameter, and the corresponding ghost contribution is given by

$$\mathcal{L}_{ghost} \;=\; \overline{\chi}^a D_\mu^{ab}[A] D^{\mu,bc}[A+Q]\chi^c \;=\; \overline{\chi}^a \; K^{ab}[A,Q] \; \chi^b. \tag{20}$$

Now adding eqs. (16) and (19) and (20)we get the Langrangian density for gluons interacting with a classical background field

$$\begin{aligned}\mathcal{L}_{\rm gluon} \;&=\; -\frac{1}{4}F_{\mu\nu}^a[A+Q]F^{\mu\nu a}[A+Q] \;-\; \frac{1}{2\alpha}[D_\mu[A]Q^{\mu a}]^2 \\ &+ \overline{\chi}^a \; K^{ab}[A,Q] \; \chi^b.\end{aligned} \tag{21}$$

To discuss gluon pair production at the one-loop level on considers just the part of this Lagrangian which is quadratic in quantum fields. This quadratic Lagrangian is invariant under a restricted class of gauge transformations. The quadratic Lagrangian for a pair of gluon interacting with background field A_μ^a is given by

$$\mathcal{L}_{\rm gg} \;=\; \frac{1}{2}Q^{\mu a} M_{\mu\nu}^{ab}[A] Q^{\nu b} \tag{22}$$

where

$$M_{\mu\nu}^{ab}[A] \;=\; \eta_{\mu\nu}[D_\rho(A)D^\rho(A)]^{ab} \;-\; 2g f^{abc} F_{\mu\nu}^c \;+\; (\frac{1}{\alpha}-1)[D_\mu(A)D_\nu(A)]^{ab} \tag{23}$$

with $\eta_{\mu\nu} \;=\; (-1,+1,+1,+1)$.

For our purpose we write

$$M_{\mu\nu}^{ab}[A] \;=\; M_{\mu\nu,\alpha=1}^{ab}[A] \;+\; \alpha'[D_\mu(A)D_\nu(A)]^{ab} \tag{24}$$

where $\alpha' \;=\; (\frac{1}{\alpha}-1)$. The matrix elements for the gauge parameter α=1 is given by

$$M_{\mu\nu,\alpha=1}^{ab}[A] \;= \eta_{\mu\nu}[D_\rho(A)D^\rho(A)]^{ab} \;-\; 2g f^{abc} F_{\mu\nu}^c \tag{25}$$

which was studied in [11]. In this approximation the ghost Lagrangian density is given by

$$\mathcal{L}_{ghost} \;=\; \overline{\chi}^a D_\mu^{ab}[A] D^{\mu,bc}[A]\chi^c \;=\; \overline{\chi}^a\; K^{ab}[A]\; \chi^b \tag{26}$$

The vacuum-to-vacuum transition amplitude in pure gauge theory in the presence of a background field A_μ^a is given by:

$$_+ <0|0>_-^A \;=\; \int [dQ][d\chi][d\bar{\chi}]\; e^{i(S+S_{gf}+S_{\rm ghost})}. \tag{27}$$

For the gluon pair part this can be written by

$$_+ <0|0>_-^A \;=\; \frac{Z[A]}{Z[0]} \;=\; \frac{\int [dQ]\; e^{i\int d^4x\; Q^{\mu a} M_{\mu\nu}^{ab}[A] Q^{\nu b}}}{\int [dQ]\; e^{i\int d^4x\; Q^{\mu a} M_{\mu\nu}^{ab}[0] Q^{\nu b}}} \;=\; e^{iS^{(1)}_{eff}} \tag{28}$$

where $S^{(1)}_{eff}$, the one-loop effective action, is given by

$$S^{(1)}_{eff} \;=\; -\, i{\rm Ln}\frac{({\rm Det}[{\rm M}_{\mu\nu}^{\rm ab}[{\rm A}]])^{-1/2}}{({\rm Det}[{\rm M}_{\mu\nu}^{\rm ab}[{\rm A}]])^{-1/2}} \;=\; \frac{i}{2} Tr[{\rm Ln} M_{\mu\nu}^{ab}[A] - {\rm Ln} M_{\mu\nu}^{ab}[0]]. \tag{29}$$

The trace Tr contains an integration over d^4x and a sum over color and Lorentz indices. To the above action, we need to add the ghost action. The ghost action is gauge independent and eliminates the unphysical gluon degrees of freedom. The one-loop effective action for the ghost in the background field A_μ^a is given by

$$S^{(1)}_{ghost} \;=\; -\, i{\rm Ln}(Det\; K) \;=\; -\, i\; Tr \int_0^\infty \frac{ds}{s}[e^{is\;[K[0]+i\epsilon]} \;-\, e^{is\;[K[A]+i\epsilon]}] \tag{30}$$

where $K^{ab}[A]$ is given by (26). Since the total action is the sum of the gluon and ghost actions, the gauge parameter dependent part proportional to $(\frac{1}{\alpha} - 1)$ can be evaluated as an addition to the $\alpha = 1$.

The non-perturbative gluon pair production per unit volume per unit time is related to the imaginary part of this effective action via

$$\frac{dN}{dtd^3x} \;\equiv\; {\rm Im}\mathcal{L}_{eff} \;=\; \frac{{\rm Im} S^{(1)}_{eff}}{d^4x}. \tag{31}$$

This expression was evaluated for $\alpha = 1$ in [11] where for gluon pair production it was found

$$\frac{dN_{g,g}}{dtd^3xd^2p_T} \;=\; \frac{1}{4\pi^3} \;\sum_{j=1}^{3}\; |g\lambda_j|\; {\rm Ln}[1 \;+\; e^{-\frac{\pi p_T^2}{|g\lambda_j|}}]. \tag{32}$$

After this calculation was done in $\alpha = 1$ gauge, Cooper and Gouranga showed by explicit evaluation of the extra term proportion to $\alpha - 1$ that the result for the particle production rate was independent of the Gauge Fixing parameter α [15].

Recently there has also been some progress to extending this result to time dependent fields. Using a formal operator shift theorem [14], Nayak and Cooper were able to show:

$$\frac{dW}{d^4x d^2p_T} = \frac{|eE(t)|}{8\pi^3}\mathrm{Log}[1 + e^{-\pi\frac{p_T^2+m^2}{|eE(t)|}}]. \tag{33}$$

For Fermion pair production they obtained instead

$$\frac{dW}{d^4x d^2p_T} = -\frac{|eE(t)|}{4\pi^3}\mathrm{Log}[1 - e^{-\pi\frac{p_T^2+m^2}{|eE(t)|}}]. \tag{34}$$

These results came from evaluating the one loop Action. For the Boson case (elaborated on below) they found

$$S_B^{(1)} = \frac{i}{16\pi^3}\int_0^\infty \frac{ds}{s}\int d^4x \int d^2p_T e^{is(p_T^2+m^2+i\epsilon)}[\frac{1}{s} - \frac{eE(t)}{\sinh(seE(t))}]. \tag{35}$$

wheras in the fermion case they obtained

$$S^{(1)} = \frac{i}{8\pi^3}\int_0^\infty \frac{ds}{s}\int d^4x \int d^2p_T e^{is(p_T^2+m^2+i\epsilon)}[eE(t)\,\coth(seE(t)) \;-\; \frac{1}{s}]. \tag{36}$$

In the scalar case one uses:

$$M[A] = (\hat{p} - eA)^2 - m^2; \quad \hat{p}_\mu = i\frac{\partial}{\partial x^\mu}$$

The Action is then

$$S^{(1)} = i\mathrm{Trln}[(\hat{p} - eA)^2 - m^2] - i\mathrm{Trln}[\hat{p}^2 - m^2]$$
$$= -i\int_0^\infty \frac{ds}{s}\int d^4x < x|[e^{-is[(\hat{p}-eA)^2-m^2-i\epsilon]} - e^{-is(\hat{p}^2-m^2-i\epsilon)}]|x >$$

Choosing Axial gauge $A_3 = 0$ and $A_0 = -E(t)z$

$$\begin{aligned} S^{(1)} &= -i\int_0^\infty \frac{ds}{s}\int_{-\infty}^{+\infty} dt < t|\int_{-\infty}^{+\infty} dx < x|\int_{-\infty}^{+\infty} dy < y|\int_{-\infty}^{+\infty} dz \\ &< z|\; [e^{-is[(\hat{p}_0+eE(t)z)^2-\hat{p}_z^2-\hat{p}_T^2-m^2-i\epsilon]} - e^{-is(\hat{p}^2-m^2-i\epsilon)}]|z > |y > |x > |t > \end{aligned} \tag{37}$$

Inserting complete set of $|p_T>$ states $(\int d^2p_T|p_T><p_T| = 1)$ and using $<q|p>=\frac{1}{\sqrt{2\pi}}e^{iqp}$ one obtains

$$\begin{aligned}S^{(1)} &= \frac{-i}{(2\pi)^2}\int_0^\infty \frac{ds}{s}\int d^2x_T\int d^2p_T e^{is(p_T^2+m^2+i\epsilon)} \\ &[\int_{-\infty}^{+\infty} dt<t|\int_{-\infty}^{+\infty} dz<z|e^{-is[(-i\frac{d}{dt}+eE(t)z)^2-\hat{p}_z^2]}|z>|t>-\int dt\int dz\frac{1}{4\pi s}]\end{aligned} \quad (38)$$

This expression contains the noncommuting quantities $E(t)$ and $\frac{d}{dt}$ To evaluate these terms we derived a shift theorem

$$\begin{aligned}&\int_{-\infty}^{+\infty} dx<x|e^{-[(a(y)x+h\frac{d}{dy})^2+b(\frac{d}{dx})+c(y)]}|x>f(y) \\ &=\int_{-\infty}^{+\infty} dx<x-\frac{h}{a(y)}\frac{d}{dy}|e^{-[a^2(y)x^2+b(\frac{d}{dx})+c(y)]}|x-\frac{h}{a(y)}\frac{d}{dy}>f(y).\end{aligned} \quad (39)$$

This Shift Theorem also implies the important corollary

$$\int_{-\infty}^{+\infty} dxe^{-(f(y)x+\frac{d}{dy})^2}g(y)=\int_{-\infty}^{+\infty} dxe^{-f^2(y)x^2}g(y)=\sqrt{\pi}\frac{g(y)}{f(y)}$$

These results have the remarkable feature that they are equivalent to Schwinger's original expressions for the effective action with the substitution $E\to E(t)$. That is the adiabatic approximation appears to give the exact result for the action. Although this is initially surprising, it is not without precedent. A related result for the one Loop effective action was found recently by Fried and Woodard [21], using Fradkin's formulation of the Path Integral, for the case of an Electric field pointing in the z direction which arbitrarily depend on the light cone time coordinate $x^+=(x^0+x^3)$. Explicitly they found that the Action integrated over momentum was also equivalent to the Adiabatic result in the variable x^+, namely for the Fermion Action they obtained:

$$\Gamma_1[A] = -iL[A]\,, \quad (40)$$

$$= \frac{1}{8\pi^2}\int d^4x\int_0^\infty \frac{ds}{s^3}e^{-ism^2}\left\{eE(x^+)s\coth\big(eE(x^+)s\big)-1\right\}\,. \quad (41)$$

III. PARTICLE PRODUCTION AND BACK REACTION IN BOOST INVARIANT QED

We assume in what follows that the kinematics of ultrarelativistic high energy collisions results in a boost invariant dynamics in the longitudinal (z) direction (here z corresponds to the axis of the initial collision) so that all expectation values are functions of the proper time $\tau = \sqrt{t^2 - z^2}$. There are many ways of viewing this. In the Landau Hydrodynamical model, one realizes that as the discs of colliding Heavy Ions get thinner and thinner at higher energies due to Lorentz contraction effects, then the final result does not have a length scale in the z direction and velocities have to scale as z/t [18]. Bjorken phrased it a bit differently [19] in terms of the initial conditions of two heavy ions colliding becomes independent of longitudinal boosts and so the physics must reflect this boost invariance. Transcribed into the physics of particle production, we want the evolving chromoeletric (or Electric Field) to evolve a s a function of proper time τ so that energy densities depend only on proper time. A simple field theory model of colliding kinks in $1 + 1$ dimension also leads to this type of picture. [20]. Our model for the production of the quark-gluon plasma begins with the creation of a flux tube containing a strong color electric field. If the energy density of the chromoelectric field gets high enough (see below) the quark-anti quark pairs can be popped out of the vacuum by the Schwinger tunneling mechanism discussed earlier. First we discuss pair production (such as electron-positron pairs) from an abelian Electric Field and the subsequent quantum back-reaction on the Electric Field. The extension to quark anti-quark pairs produced from a chromoelectric field is discussed later. The back-reaction problem was first discussed [8] in a semi-classical approximation using a Vlasov Equation with a Schwinger source term in the adiabatic approximation. In 1+1 dimensions this leads to the equation for the phase space distribution function:

$$\begin{aligned} \frac{\partial f}{\partial \tau} + eF_{\eta\tau}(\tau)\frac{\partial f}{\partial p_\eta} \; &= \; \pm[1 \pm 2f(\mathbf{p},\tau)]e\tau|E(\tau)| \\ &\times \ln\left[1 \pm \exp\left(-\frac{\pi(m^2+\mathbf{p}_\perp^2)}{e|E(\tau)|}\right)\right]\delta(p_\eta). \end{aligned} \tag{42}$$

And the back reaction equation (Maxwell Equation)

$$-\tau\frac{dE}{d\tau} = j_\eta = j_\eta^{cond} + j_\eta^{pol}\,, \tag{43}$$

where j^{cond} is the conduction current and j_μ^{pol} is the polarization current due to pair creation [27]

$$\begin{aligned} j_\eta^{cond} &= 2e\int \frac{dp_\eta}{2\pi\tau p_\tau}p_\eta f(p_\eta,\tau) \\ j_\eta^{pol} &= \frac{2}{F^{\tau\eta}}\int\frac{dp_\eta}{2\pi\tau p_\tau}p^\tau\frac{Df}{D\tau} \\ &= \pm[1\pm 2f(p_\eta=0,\tau)]\frac{me\tau}{\pi}\mathrm{sign}[E(\tau)]\ln\left[1\pm\exp\left(-\frac{\pi m^2}{|eE(\tau)|}\right)\right]. \end{aligned} \tag{44}$$

Since solving in solving these equations one did not know the validity of either the semi-classical approximation or the adiabatic equation, it was important to actually solve the quantum back reaction problem to understand whether Nuclear Theorists using transport theory to model the quark gluon plasma were making reasonable assumptions. Remarkably, the transport theory gave a reasonable coarse grained time average of the exact field theory result. The assumption that the electric field can be treated "classically" gets translated into this approximation being the first term in a large-N approximation of QED, where N refers to having N identical electron flavors [31]. We assume in what follows that the kinematics of ultrarelativistic high energy collisions results in boost invariant dynamics in the longitudinal (z) direction so that all expectation values are functions of the proper time $\tau=\sqrt{t^2-z^2}$.We introduce the light cone variables τ and η, which will be identified later with fluid proper time and rapidity . These coordinates are defined in terms of the ordinary lab-frame Minkowski time t and coordinate along the beam direction z by

$$z=\tau\sinh\eta \quad , \quad t=\tau\cosh\eta\,. \tag{45}$$

The Minkowski line element in these coordinates has the form

$$ds^2=-d\tau^2+dx^2+dy^2+\tau^2 d\eta^2\,. \tag{46}$$

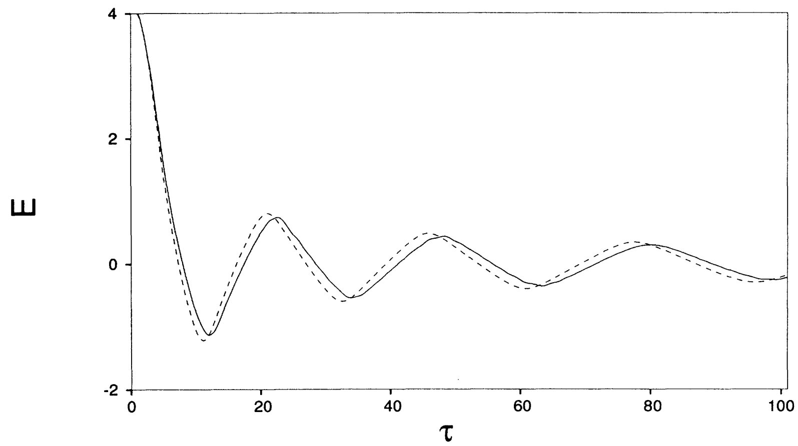

FIG. 1: Proper-time evolution of the electric field $E(\tau)$ for an initial $E = 4$.

The QED action in curvilinear coordinates is:

$$\begin{aligned} S &= \int d^{d+1}x\,(\det V)[-\frac{i}{2}\bar{\Psi}\tilde{\gamma}^{\mu}\nabla_{\mu}\Psi + \frac{i}{2}(\nabla^{\dagger}_{\mu}\bar{\Psi})\tilde{\gamma}^{\mu}\Psi \\ &\quad -im\bar{\Psi}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}], \end{aligned} \tag{47}$$

where

$$\nabla_{\mu}\Psi \equiv (\partial_{\mu} + \Gamma_{\mu} - ieA_{\mu})\Psi \tag{48}$$

Varying the action leads to the Heisenberg field equation:

$$\left[\gamma^0\left(\partial_\tau + \frac{1}{2\tau}\right) + \gamma_\perp \cdot \partial_\perp + \frac{\gamma^3}{\tau}(\partial_\eta - ieA_\eta) + m\right]\Psi = 0\,, \tag{49}$$

and the Maxwell equation: $E = E_z(\tau) = -\dot{A}_\eta(\tau)$

$$\frac{1}{\tau}\frac{dE(\tau)}{d\tau} = \frac{e}{2}\left\langle\left[\bar{\Psi}, \tilde{\gamma}^{\eta}\Psi\right]\right\rangle = \frac{e}{2\tau}\left\langle\left[\Psi^{\dagger}, \gamma^0\gamma^3\Psi\right]\right\rangle. \tag{50}$$

To solve the problem numerically we expand the fermion field in terms of Fourier modes at fixed proper time: τ, and used a grid in momentum space with a maximum momentum Λ.

$$\begin{aligned} \Psi(x) &= \int [d\mathbf{k}] \sum_s [b_s(\mathbf{k})\psi^{+}_{\mathbf{k}s}(\tau)e^{ik\eta}e^{i\mathbf{p}\cdot\mathbf{x}} \\ &\quad + d^{\dagger}_s(-\mathbf{k})\psi^{-}_{-\mathbf{k}s}(\tau)e^{-ik\eta}e^{-i\mathbf{p}\cdot\mathbf{x}}]. \end{aligned} \tag{51}$$

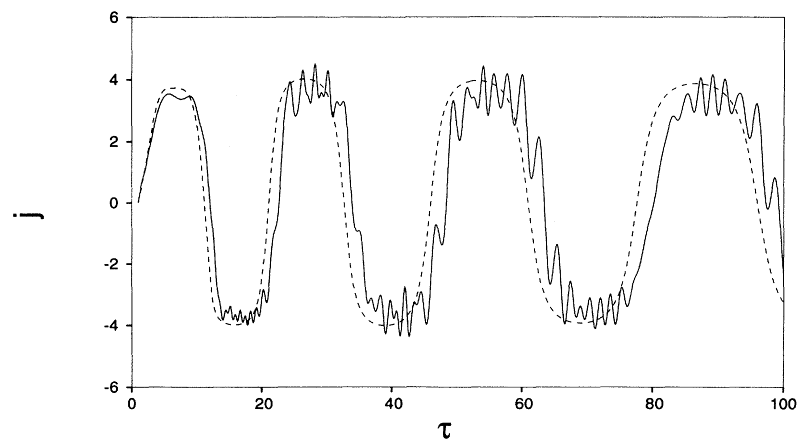

FIG. 2: Proper-time evolution of the fermionic current $j_\eta(\tau)$.

The $\psi^{\pm}_{\mathbf{k}s}$ then obey

$$\left[\gamma^0\left(\frac{d}{d\tau}+\frac{1}{2\tau}\right)+i\gamma_\perp\cdot\mathbf{k}_\perp+i\gamma^3\pi_\eta+m\right]\psi^{\pm}_{\mathbf{k}s}(\tau)=0, \tag{52}$$

Squaring the Dirac equation:

$$\psi^{\pm}_{\mathbf{k}s}=\left[-\gamma^0\left(\frac{d}{d\tau}+\frac{1}{2\tau}\right)-i\gamma_\perp\cdot\mathbf{k}_\perp-i\gamma^3\pi_\eta+m\right]\chi_s\frac{f^{\pm}_{\mathbf{k}s}}{\sqrt{\tau}}\,. \tag{53}$$

$$\gamma^0\gamma^3\chi_s=\lambda_s\chi_s \tag{54}$$

with $\lambda_s=1$ for $s=1,2$ and $\lambda_s=-1$ for $s=3,4$, we then get the mode equation:

$$\left(\frac{d^2}{d\tau^2}+\omega^2_{\mathbf{k}}-i\lambda_s\dot{\pi}_\eta\right)f^{\pm}_{\mathbf{k}s}(\tau)=0, \tag{55}$$

$$\omega^2_{\mathbf{k}}=\pi^2_\eta+\mathbf{k}^2_\perp+m^2;\quad \pi_\eta=\frac{k_\eta-eA}{\tau}. \tag{56}$$

The back-reaction equation in terms of the modes is

$$\frac{1}{\tau}\frac{dE(\tau)}{d\tau}=-\frac{2e}{\tau^2}\sum_{s=1}^{4}\int[d\mathbf{k}](\mathbf{k}^2_\perp+m^2)\lambda_s|f^{+}_{\mathbf{k}s}|^2, \tag{57}$$

Renormalization is done by realizing eE is renormalization invariant, so multiplying both sides of the Maxwell equation by e and recognizing that

$$e^2=Z^{-1}(\Lambda,m)e^2_R(m^2) \tag{58}$$

where

$$Z(\Lambda, m) = 1 - \frac{e_R^2(m^2)}{6\pi^2} \ln\left(\frac{\Lambda}{m}\right) \tag{59}$$

The finite result which is independent of Λ for large Λ [31] can be written as

$$\frac{e_R dE_R(\tau)}{d\tau} = -Z^{-1}(\Lambda, m)\frac{2e_R^2}{\tau}\sum_{s=1}^{4}\int [d\mathbf{k}](\mathbf{k}_\perp^2 + m^2)\lambda_s |f_{\mathbf{k}s}^+|^2, \tag{60}$$

This straightforward method of renormalization is to be compared with our original approach which was based on an adiabatic expansion of a WKB parameterization [7] [25]. Namely we can write:

$$f_{\mathbf{k}s}^+(\tau) = N_{\mathbf{k}s}\frac{1}{\sqrt{2\Omega_{\mathbf{k}s}}}\exp\left\{\int_0^\tau \left(-i\Omega_{\mathbf{k}s}(\tau') - \lambda_s \frac{\dot{\pi}_\eta(\tau')}{2\Omega_{\mathbf{k}s}(\tau')}\right) d\tau'\right\}, \tag{61}$$

where $\Omega_{\mathbf{k}s}$ obeys the real equation

$$\frac{1}{2}\frac{\ddot{\Omega}_{\mathbf{k}s}}{\Omega_{\mathbf{k}s}} - \frac{3}{4}\frac{\dot{\Omega}_{\mathbf{k}s}^2}{\Omega_{\mathbf{k}s}^2} + \frac{\lambda_s}{2}\frac{\ddot{\pi}_\eta}{\Omega_{\mathbf{k}s}} - \frac{1}{4}\frac{\dot{\pi}_\eta^2}{\Omega_{\mathbf{k}s}^2} - \lambda_s \frac{\dot{\pi}_\eta \dot{\Omega}_{\mathbf{k}s}}{\Omega_{\mathbf{k}s}^2} = \omega_{\mathbf{k}}^2(\tau) - \Omega_{\mathbf{k}s}^2\,. \tag{62}$$

One then finds that

$$2|f_{\mathbf{k}s}^+|^2 = \left[\omega_{\mathbf{k}}^2 + \Omega_{\mathbf{k}s}^2 + \left(\frac{\dot{\Omega}_{\mathbf{k}s} + \lambda_s \dot{\pi}_\eta}{2\Omega_{\mathbf{k}s}}\right)^2 + 2\lambda_s \pi_\eta \Omega_{\mathbf{k}s}\right]^{-1}. \tag{63}$$

Ω has an adiabatic expansion:

$$\Omega_s^2 = \omega^2 - \frac{1}{2\omega^2}\left[\pi\ddot{\pi} + \dot{\pi}^2\left(1 - \frac{\pi^2}{\omega^2}\right)\right] + \frac{3}{4}\frac{\pi^2\dot{\pi}^2}{\omega^4} + \frac{\dot{\pi}^2}{4\omega^2} + \frac{\lambda_s \dot{\pi}^2 \pi}{\omega^3} - \frac{\lambda_s \ddot{\pi}}{2\omega} + \cdots$$

Using this expansion we find

$$\sum_{s=1}^{4}(k_\perp^2 + m^2)(-2\lambda_s))\frac{|f_{\mathbf{k}s}^+|^2}{\tau} = \frac{2\pi_\eta}{\tau\omega_{\mathbf{k}}} - \left(\frac{\ddot{\pi}_\eta}{2\omega_{\mathbf{k}}^5} - \frac{5\dot{\pi}_\eta^2\pi_\eta}{4\omega_{\mathbf{k}}^7}\right)\frac{(\omega_{\mathbf{k}}^2 - \pi_\eta^2)}{\tau} - R_{\mathbf{k}}(\tau)\,,$$

where $R_{\mathbf{k}}(\tau)$ falls faster than ω^{-3}. This then yields

$$\begin{aligned} \frac{dE}{d\tau} &= \frac{e^2}{2\tau^2}\int [d\mathbf{k}]\ \frac{k_\perp^2 + m^2}{\omega_{\mathbf{k}}^5}\left\{\left(\ddot{A} - 2\frac{\dot{A}}{\tau}\right) + \frac{5\dot{A}\pi_\eta^2}{\tau\omega_{\mathbf{k}}^2}\right\} - e\int [d\mathbf{k}]R_{\mathbf{k}}(\tau) \\ &= -\frac{e^2}{6\pi^2}\ln\left(\frac{\Lambda}{m}\right)\frac{dE}{d\tau} - e\int [d\mathbf{k}]R_{\mathbf{k}}(\tau). \end{aligned} \tag{64}$$

where Λ is the cutoff in the transverse momentum integral which has been reserved for last.

Defining $\delta e^2 = (1/6\pi^2)\ln(\Lambda/m)$ as usual we obtain

$$e\frac{dE}{d\tau}(1+e^2\delta e^2) = -e^2\int[d\mathbf{k}]R_{\mathbf{k}}(\tau)\,, \tag{65}$$

after multiplying both sides of the equation by e. The renormalized charge is

$$e_R^2 = \frac{e^2}{(1+e^2\delta e^2)} = Ze^2. \tag{66}$$

Using $e_R E_R = eE$ we obtain

$$\frac{dE_R}{d\tau} = -e_R\int[d\mathbf{k}]R_{\mathbf{k}}(\tau), \tag{67}$$

where $R_{\mathbf{k}}(\tau)$ is defined by Eq. (64), and the integral is now completely convergent. This method of renormalizing was originally used for quantum fields in curved space, but we see that it is very cumbersome and unnecessary.

Our original simulations were in $1+1$ dimensions, and typical proper time evolution of E and j are shown in figs. 1 and 2. Here an initial value of $E = 4$ was chosen. The dotted line corresponds to the solution of the transport equations discussed above.

A. Spectrum of Particles

Although particle number is not conserved, at each τ on can diagonalize the Hamiltonian and define an effective particle number which is the adiabatic particle number which interpolates from the initial particle number to the final one if one chooses the initial state to be the appropriate solution of the free Dirac equation with no particles present at $\tau = \tau_0$. Namely,

$$\langle b_0^\dagger(k,\tau_0)b_0(k,\tau_0)\rangle = \langle b^\dagger(k)b(k)\rangle = 0. \tag{68}$$

with a similarly condition on d_0. Introducing the adiabatic bases for the fields via:

$$\begin{aligned}\Psi(x) \;&= \int[d\mathbf{k}]\sum_s[b_s^0(\mathbf{k};\tau)u_{\mathbf{k}s}(\tau)e^{-i\int\omega_{\mathbf{k}}d\tau} \\ &\quad + d_s^{(0)\dagger}(-\mathbf{k};\tau)v_{-\mathbf{k}s}(\tau)e^{i\int\omega_{\mathbf{k}}d\tau}]e^{i\mathbf{k}\cdot\mathbf{x}}.\end{aligned} \tag{69}$$

The operators $b_s(\mathbf{k})$ and $b_s^{(0)}(\mathbf{k};\tau)$ are related by a Bogolyubov transformation:

$$
\begin{aligned}
b_r^{(0)}(\mathbf{k};\tau) &= \sum \alpha_{\mathbf{k}r}^{s}(\tau) b_s(\mathbf{k}) + \beta_{\mathbf{k}r}^{s}(\tau) d_s^{\dagger}(-\mathbf{k}) \\
d_r^{(0)}(-\mathbf{k};\tau) &= \sum \beta_{\mathbf{k}r}^{*s}(\tau) b_s(\mathbf{k}) + \alpha_{\mathbf{k}r}^{*s}(\tau) d_s^{\dagger}(-\mathbf{k})
\end{aligned}
\tag{70}
$$

One finds that the interpolating phase space number density for the number of particles (or antiparticles) present per unit phase space volume at time τ is given by:

$$
n(\mathbf{k};\tau) = \sum_{r=1,2} \langle 0_{in} | b_r^{(0)\dagger}(\mathbf{k};\tau) b_r^{(0)}(\mathbf{k};\tau) | 0_{in} \rangle = \sum_{s,r} |\beta_{\mathbf{k}r}^{s}(\tau)|^2 \tag{71}
$$

This is an adiabatic invariant of the Hamiltonian dynamics governing the time evolution. At $\tau = \tau_0$ it is equal to our initial number operator. If at later times one reaches the out regime because of the decrease in energy density due to expansion it becomes the usual out state phase space number density. The phase space distribution of particles (or antipartcles) in light cone variables is

$$
n_{\mathbf{k}}(\tau) = f(k_\eta, k_\perp, \tau) = \frac{d^6 N}{\pi^2 dx_\perp^2 dk_\perp^2 d\eta dk_\eta}. \tag{72}
$$

We now need to relate this quantity to the spectra of electrons and positrons produced. We introduce the particle rapidity y and $m_\perp = \sqrt{k_\perp^2 + m^2}$ defined by the particle 4-momentum in the center of mass coordinate system

$$
k_\mu = (m_\perp \cosh y, k_\perp, m_\perp \sinh y) \tag{73}
$$

The boost that takes one from the center of mass coordinates to the comoving frame where the energy momentum tensor is diagonal is given by $\tanh \eta = v = z/t$, so that one can define the "fluid" 4-velocity in the center of mass frame as

$$
u^\mu = (\cosh \eta, 0, 0, \sinh \eta) \tag{74}
$$

We then find that the variable

$$
\omega_k = \sqrt{m_\perp^2 + \frac{k_\eta^2}{\tau^2}} \equiv k^\mu u_\mu \tag{75}
$$

has the meaning of the energy of the particle in the comoving frame. The momenta k_η that enters into the adiabatic phase space number density is one of two momenta canonical to the variables defined by the coordinate transformation to light cone variables. Namely the variables

$$\tau = (t^2 - z^2)^{1/2} \qquad \eta = \frac{1}{2} \ln \left(\frac{t+z}{t-z} \right)$$

have as their canonical momenta

$$k_\tau = Et/\tau - k_z z/\tau = k_\mu u^\mu \quad k_\eta = -Ez + t k_z. \tag{76}$$

The interpolating phase-space density f of particles depends on k_η, $\mathbf{k}_\perp$, τ, and is η-independent. In order to obtain the physical particle rapidity and transverse momentum distribution, we change variables from (η, k_η) to (z, y) at a fixed τ where y is the particle rapidity. We have

$$E\frac{d^3N}{d^3k} = \frac{d^3N}{\pi dy\, dk_\perp^2} = \int \pi dz\ dx_\perp^2\ J\ f(k_\eta, k_\perp, \tau) \tag{77}$$

where the Jacobian J is evaluated at a fixed proper time τ and is determined to be

$$J = \frac{m_\perp \cosh(\eta - y)}{\cosh \eta} = \frac{\partial k_\eta}{\partial z}|_\tau. \tag{78}$$

We also have

$$k_\tau = m_\perp \cosh(\eta - y); \quad k_\eta = -\tau m_\perp \sinh(\eta - y)\,. \tag{79}$$

Calling the integration over the transverse dimension the effective transverse size of the colliding ions $A_\perp$ we then obtain that:

$$\frac{d^3N}{\pi dy\, dk_\perp^2} = A_\perp \int dk_\eta f(k_\eta, k_\perp, \tau) \equiv \frac{d^3N}{\pi d\eta\, dk_\perp^2} \tag{80}$$

This quantity is independent of y which is a consequence of the assumed boost invariance. Note that we have proven using the property of the Jacobean, that the distribution of particles in particle rapidity is the same as the distribution of particles in fluid rapidity, verifying that in the boost-invariant regime that Landau's intuition based on a hydronamic picture was correct.

We now want to make contact with the hydrodynamic approach to calculating particle spectra, namely the Cooper- Frye formula [32]. First we note that the interpolating number density depends on k_η and $k_\perp$ only through the combination:

$$\omega_k = \sqrt{m_\perp^2 + \frac{k_\eta^2}{\tau^2}} \equiv k^\mu u_\mu \tag{81}$$

Thus $f(k_\eta, k_\perp) = f(k_\mu u^\mu)$ and so it depends on exactly the same variable as the comoving thermal distribution! We also have that a constant τ surface (which is the freeze out surface of Landau) is parametrized as:

$$d\sigma^\mu = A_\perp(dz, 0, 0, dt) = A_\perp d\eta(\cosh\eta, 0, 0, \sinh\eta) \tag{82}$$

We therefore find

$$k^\mu d\sigma_\mu = A_\perp m_\perp \tau \cosh(\eta - y) = A_\perp |dk_\eta| \tag{83}$$

Thus we can rewrite our expression for the field theory particle spectra as

$$\frac{d^3N}{\pi dy\, dk_\perp^2} = A_\perp \int dk_\eta f(k_\eta, k_\perp, \tau) = \int f(k^\mu u_\mu, \tau) k^\mu d\sigma_\mu \tag{84}$$

where in the second integration we keep y and τ fixed. Thus with the replacement of the thermal single particle distribution by the interpolating number operator, we get via the coordinate transformation to the center of mass frame the Cooper-Frye formula.

B. Hydrodynamic Variables

Our boost invariant kinematics leads to an Energy Momentum tensor which is diagonal in the$(\tau, \eta, x_\perp)$ coordinate system which is a comoving one. In that system one has:

$$T^{\mu\nu} = \text{diagonal}\ \{\varepsilon(\tau), p_\parallel(\tau), p_\perp(\tau), p_\perp(\tau)\} \tag{85}$$

We thus find in this approximation that there are two separate pressures, one in the longitudinal direction and one in the transverse direction which is quite different from the thermal equilibrium case. However only the longitudinal pressure enters into the "entropy" equation.

Only the longitudinal pressure enters into the "entropy" equation

$$\varepsilon + p_\parallel = Ts \tag{86}$$

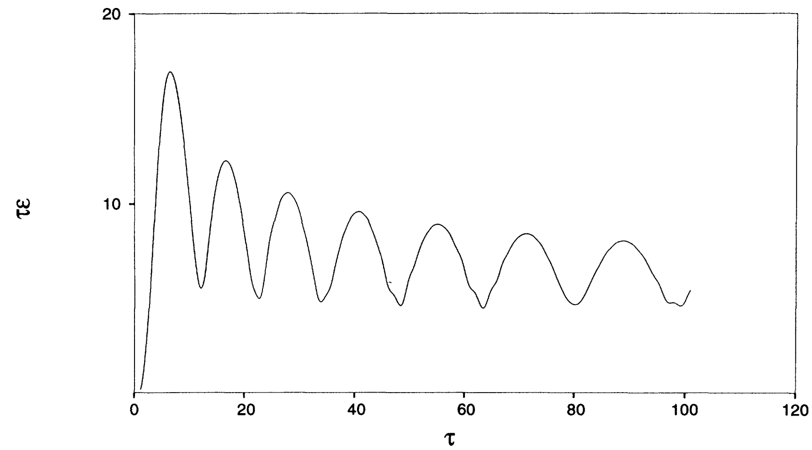

FIG. 3: Proper-time evolution of $\tau\,\epsilon(\tau)$.

$$\frac{d(\varepsilon\tau)}{d\tau} + p_{\|} = Ej_{\eta}$$
$$\frac{d(s\tau)}{d\tau} = \frac{Ej_{eta}}{T}$$

In the out regime we find as in the Landau Model

$$s\tau = constant$$

basis. The energy density as a function of proper time is shown in fig.3.

For our one-dimensional boost invariant flow we find that the energy in a bin of fluid rapidity is just:

$$\frac{dE}{d\eta} = \int T^{0\mu} d\sigma_{\mu} = A_{\perp}\tau \cosh\eta\varepsilon(\tau) \tag{87}$$

which is just the $(1 + 1)$ dimensional hydrodynamical result. Here however ε is obtained by solving the field theory equation rather than using an ultrarelativistic equation of state. Our result does not depend on any assumptions of thermalization. We can ask if we can directly calculate the particle rapidity distribution from the ansatz:

$$\frac{dN}{d\eta} = \frac{1}{m\cosh\eta}\frac{dE}{d\eta} = \frac{A_{\perp}}{m}\varepsilon(\tau)\tau. \tag{88}$$

We see from fig. 4. that this ansatz works well even in our case where we have ignored interactions between the fermions, so that we are not in thermal equilibrium.

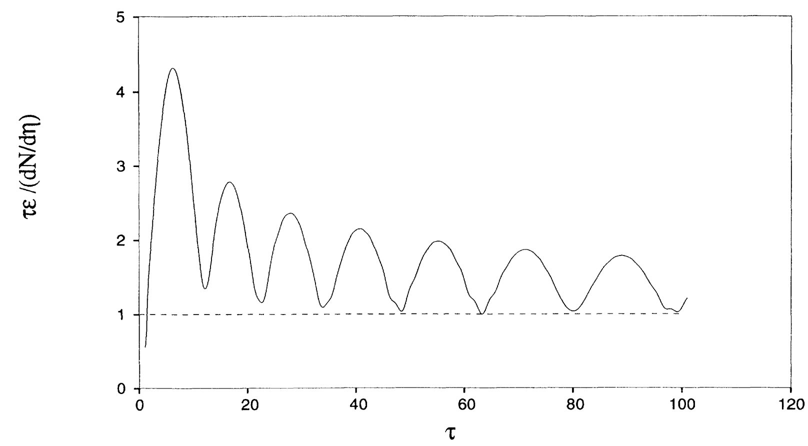

FIG. 4: Proper-time evolution of $\tau\epsilon/(dN/d\eta))$.

In the field theory calculation the expectation value of the stress tensor must be renormalized since the electric field undergoes charge renormalization and one needs to subtract terms renormalize the cosmological constant. Here we follow the adiabatic method but this approach is not necessary. We can determine the two pressures and the energy density as a function of τ. Explicitly we have in the fermion case.

$$\varepsilon(\tau) =< T_{\tau\tau} >= \tau\Sigma_s \int [dk] R_{\tau\tau}(k) + E_R^2/2$$

where

$$\begin{aligned}
&R_{\tau\tau}(k) = 2(p_\perp^2 + m^2)(g_0^+|f^+|^2 - g_0^-|f^-|^2) - \omega \\
&-(p_\perp^2 + m^2)(\pi + e\dot{A})^2/(8\omega^5\tau^2) \\
&p_\parallel(\tau)\tau^2 =< T_{\eta\eta} >= \tau\Sigma_s \int [dk]\lambda_s \pi R_{\eta\eta}(k) - \frac{1}{2}E_R^2\tau^2
\end{aligned} \tag{89}$$

where

$$\begin{aligned}
R_{\eta\eta}(k) \quad &= 2|f^+|^2 - (2\omega)^{-1}(\omega + \lambda_s\pi)^{-1} - \lambda_s e\dot{A}/8\omega^5\tau^2 \\
&-\lambda_s e\dot{E}/8\omega^5 - \lambda_s\pi/4\omega^5\tau^2 + 5\pi\lambda_s(\pi + e\dot{A})^2/(16\omega^7\tau^2)
\end{aligned} \tag{90}$$

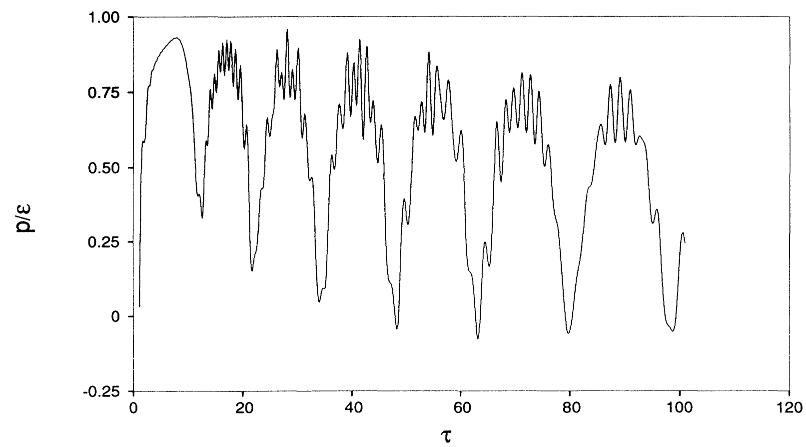

FIG. 5: Proper-time evolution of p/ϵ.

and

$$
\begin{aligned}
&p_\perp(\tau) =< T_{yy} >=< T_{xx} > \\
&= (4\tau)^{-1} \sum_s \int [dk]\{p_\perp^2 (p_\perp^2 + m^2)^{-1} R_{\tau\tau} - 2\lambda\pi p_\perp^2 R_{\eta\eta}\} \\
&+ E_R^2/2. \qquad (91)
\end{aligned}
$$

Thus we are able to numerically determine the effective time dependent equation of state $p_i = p_i(\varepsilon)$ as a function of τ. A typical result is shown in fig. 5.

IV. QCD BACK REACTION PROBLEM WITH CYLINDRICAL SYMMETRY

Earlier we have seen that pair production from constant External fields in QCD in the one loop approximation was was not only gauge covariant and given in terms of the Casimirs (as gauranteed by the Background field method used) but independent of the gauge fixing parameter α. Also, since the results suggest that event by event the transverse distribution of jets might depend on the values of the casimirs and not just the initial energy density, one might be able to find an experimentally observable effect at RHIC and LHC for those jets coming from the semi-classical quark- gluon plasma produced at RHIC following the collision of heavy Ions. Knowing that the one loop result is independent of the gauge fixing parameter, one can with confidence study the Back reaction problem in an Axial gauge. Since we are concerned with collisions with azimuthal symmetry, we have chosen to solve the Dirac equation in a coordinate system with both cylindrical symmetry and in fluid rapidity and proper time coordinates.

For brevity here we will just discuss pair production of quarks, described by a field $\psi(x)$ and satisfying Dirac's equation:

$$\big[\,\gamma^\mu\,\big(\,\partial_\mu - g\,A_\mu(x)\,\big) + m\,\big]\,\psi(x) = 0\,, \tag{92}$$

interacting with a classical Yang-Mills field $A_\mu(x) = A^a_\mu(x)\,T^a$, where T^a are the generators of the $SU(3)$ algebra, and satisfying a back-reaction equation given by:

$$D^{ab}_\mu\,F^{b,\mu\nu}(x) = g\,\langle\,[\,\hat{\bar{\psi}}(x), \tilde{\gamma}^\nu(x)\,T^a\,\hat{\psi}(x)\,]_{\_}\,\rangle/2\,, \tag{93}$$

with $D^{ab}_\mu = \delta^{ab}\partial_\mu + g\,f^{abc}\,A^c_\mu(x)$.

We want to consider a 3+1 dimensiona case with cylindrical symmetry when the plasma starts out with all the energy density in the semiclassical gluonic state and then system of classical glue and produced quarks and gluons expands in a boost invariant manner into the vacuum. We use the variables $x^\mu = (\tau, r, \theta, \eta)$, which are related to Cartesian coordinates by:

$$t = \tau\cosh\eta\,, \qquad z = \tau\sinh\eta\,, \qquad x = r\cos\theta\,, \qquad y = r\sin\theta\,. \tag{94}$$

For a boost invariant expansion, the classical gauge fields are restricted to be in the η-direction and depend only on τ. We also consider only the $a = 3$ and $a = 8$ gauge fields which carry all colors. Then, using the Gell-Mann representation for the λ^a matrices,

$$\begin{aligned}\tilde{\gamma}^\mu(x)A_\mu(x) =& \frac{1}{2}\,\tilde{\gamma}^\eta(x)\,\big[\,A^3_\eta(\tau)\,\lambda^3 + A^8_\eta(\tau)\,\lambda^8\,\big]\\ =& \frac{1}{2}\,\tilde{\gamma}^\eta(x)\begin{pmatrix} A^3_\eta(\tau) + A^8_\eta(\tau)/\sqrt{3} & 0 & 0 \\ 0 & -A^3_\eta(\tau) + A^8_\eta(\tau)/\sqrt{3} & 0 \\ 0 & 0 & -2\,A^8_\eta(\tau)/\sqrt{3}\end{pmatrix}\,,\end{aligned} \tag{95}$$

We choose an axial gauge, so that only the electric field terms:

$$E^a_\eta(\tau) = -\frac{\partial A^a_\eta(\tau)}{\partial\tau}\,, \tag{96}$$

for $a=3$ and $a=8$ contribute. Then, in our coordinate system, Eq. (93) becomes:

$$\frac{1}{\tau}\frac{\partial}{\partial\tau}\Big(\tau\, E^a_\eta(\tau)\Big) = -g\,\langle\,[\,\hat{\bar{\psi}}(x), \tilde{\gamma}^\eta(x)\, T^a\,\hat{\psi}(x)\,]_-\,\rangle/2\,. \tag{97}$$

Eqs. (92) and (97) are the equations we want to solve.

The two Casimir invariants for $SU(3)$ are given by:

$$C_1 = E^a E^a\,, \quad \text{and} \quad C_2 = \big[\,d^{abc}\,E^a\,E^b\,E^c\,\big]^2\,, \tag{98}$$

where d^{abc} are the symmetric $SU(3)$ structure factors. Choosing E as arbitary linear combination of the two diagonal directions 3 and 8 allows one to cover the range of possible Casimir invariants.

In Cylindrical Coordinates the canonical quark fields obey (suppressing all SU(3) indices):

$$[\,\hat{\phi}_\alpha(\tau,\rho,\theta,\eta), \hat{\phi}^\dagger_\beta(\tau,\rho',\theta',\eta')\,]_+ = \delta_{\alpha,\beta}\,\frac{\delta(\rho-\rho')}{\sqrt{\rho\rho'}}\,\delta(\theta-\theta')\,\delta(\eta-\eta')\,, \tag{99}$$

We can write the Dirac field operator in the cylindrical coordinate system in terms of solutions of the Dirac equation in cylindrical coordinate times appropriate creation and annihilation operators.

$$\begin{aligned}\hat{\phi}(\tau,\rho,\theta,\eta) = \int_{-\infty}^{\infty}\frac{\mathrm{d}k_\eta}{2\pi}\int_0^\infty \frac{k_\perp\,\mathrm{d}k_\perp}{2\pi}\sum_{h=\pm1}\sum_{m=-\infty}^{+\infty} \\ \times\Big\{\,\hat{b}^{(h)}_{k_\eta,k_\perp,m}\,\phi^{(h,+)}_{k_\eta,k_\perp,m}(\tau,\rho,\theta,\eta) + \hat{d}^{(h)\,\dagger}_{k_\eta,k_\perp,m}\,\phi^{(-h,-)}_{-k_\eta,k_\perp,-m}(\tau,\rho,\theta,\eta)\,\Big\}\,.\end{aligned} \tag{100}$$

where:

$$\phi^{(h)}_{k_\perp,m}(\tau,\rho,\theta,\eta) = \begin{pmatrix}\phi^{(h)}_{(+);k_\perp}(\tau,\eta)\,\chi^{(h)}_{k_\perp,m}(\rho,\theta)\\[6pt] \phi^{(h)}_{(-);k_\perp}(\tau,\eta)\,\chi^{(-h)}_{k_\perp,m}(\rho,\theta)\end{pmatrix}\,, \tag{101}$$

with $\lambda = hk_\perp$, and where $h=\pm1$.

$$\chi^{(h)}_{k_\perp,m}(\rho,\theta) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{im\theta}\,J_m(k_\perp\rho)\\[6pt] h\,e^{i(m+1)\theta}\,J_{m+1}(k_\perp\rho)\end{pmatrix}\,, \tag{102}$$

with eigenvalues $\lambda = hk_\perp$ and the helicity $h = \pm 1$ Orthogonality is given by the relation:

$$\int_0^{+\infty} \rho\, \mathrm{d}\rho \int_0^{2\pi} \mathrm{d}\theta\; \chi^{(h)\dagger}_{k_\perp,m}(\rho,\theta)\, \chi^{(h')}_{k'_\perp,m'}(\rho,\theta)$$
$$= \pi\, \delta_{m,m'} \int_0^{+\infty} \rho\, \mathrm{d}\rho\, \{\, J_m(k_\perp\rho)\, J_m(k'_\perp\rho) + hh'\, J_{m+1}(k_\perp\rho)\, J_{m+1}(k'_\perp\rho)\,\}$$
$$= \delta_{h,h'}\, \delta_{m,m'}\, (2\pi)\, \frac{\delta(k_\perp - k'_\perp)}{\sqrt{k_\perp k'_\perp}}\,. \quad (103)$$

The Dirac equation acting on the quantum field leads to the following matrix equation for the functions $\phi(\tau,\eta)$:

$$\begin{pmatrix} i\partial_\tau + 1 & (\, i\partial_\eta + g\, A(\tau)\,)/\tau - ihk_\perp \\ (\, i\partial_\eta + g\, A(\tau)\,)/\tau + ihk_\perp & i\partial_\tau - 1 \end{pmatrix} \begin{pmatrix} \phi^{(h)}_{(+);k_\perp}(\tau,\eta) \\ \phi^{(h)}_{(-);k_\perp}(\tau,\eta) \end{pmatrix} = 0\,, \qquad (104)$$

which is independent of m. Introducing the Fourier transform:

$$\phi^{(h)}_{(\pm);k_\perp}(\tau,\eta) = e^{ik_\eta\eta}\, \phi^{(h)}_{(\pm);k_\eta,k_\perp}(\tau)\,, \qquad (105)$$

gives an equation involving τ alone:

$$\begin{pmatrix} i\partial_\tau + 1 & -\pi_{k_\eta}(\tau) - ihk_\perp \\ -\pi_{k_\eta}(\tau) + ihk_\perp & i\partial_\tau - 1 \end{pmatrix} \begin{pmatrix} \phi^{(h)}_{(+);k_\eta,k_\perp}(\tau) \\ \phi^{(h)}_{(-);k_\eta,k_\perp}(\tau) \end{pmatrix} = 0\,, \qquad (106)$$

where we have defined $\pi_{k_\eta}(\tau)$ by:

$$\pi_{k_\eta}(\tau) = (\, k_\eta - g\, A(\tau)\,)/\tau\,. \qquad (107)$$

Eq. (106) is the equation we want to solve numerically as a function of τ for some given initial spinor at $\tau = \tau_0$. Since we have scaled all variable with the fermion mass m we choose $\tau_0 = 1$. Initially in Axial Gauge the electomagnetic field A can be chosen to be zero. This allows us to use a complete set of solutions to the "free" Dirac equation corresponding to the vacuum state as initial conditions. These solutions are also chosen to be adiabatic in that they will be assumed to hold near $\tau = 1$ also. That is *near* $\tau = 1$, Eq. (106) becomes:

$$\begin{pmatrix} i\partial_\tau + 1 & -k_\eta - ihk_\perp \\ -k_\eta + ihk_\perp & i\partial_\tau - 1 \end{pmatrix} \begin{pmatrix} \phi^{(h)}_{0\,(+);k_\eta,k_\perp}(\tau) \\ \phi^{(h)}_{0\,(-);k_\eta,k_\perp}(\tau) \end{pmatrix} = 0\,, \qquad (108)$$

which have positive and negative frequency solutions of the form:

$$\phi^{(h,+)}_{0\,;k_\eta,k_\perp}(\tau) = \sqrt{\frac{\omega_{0;k_\eta,k_\perp} - 1}{2\omega_{0;k_\eta,k_\perp}}} \begin{pmatrix} 1 \\ +\dfrac{k_\eta - ihk_\perp}{\omega_{0;k_\eta,k_\perp} - 1} \end{pmatrix} \exp[\,-i\omega_{0;k_\eta,k_\perp}(\tau - 1)\,] \,, \tag{109a}$$

$$\phi^{(h,-)}_{0\,;k_\eta,k_\perp}(\tau) = \sqrt{\frac{\omega_{0;k_\eta,k_\perp} - 1}{2\omega_{0;k_\eta,k_\perp}}} \begin{pmatrix} -\dfrac{k_\eta - ihk_\perp}{\omega_{0;k_\eta,k_\perp} - 1} \\ 1 \end{pmatrix} \exp[\,+i\omega_{0;k_\eta,k_\perp}(\tau - 1)\,] \,, \tag{109b}$$

where $\omega_{0;k_\eta,k_\perp} = \sqrt{k_\eta^2 + k_\perp^2 + 1}$. These solutions are orthogonal:

$$\sum_{\alpha=\pm} \phi^{(h,\lambda)\,*}_{0\,(\alpha);k_\eta,k_\perp}(\tau)\, \phi^{(h,\lambda')}_{0\,(\alpha);k_\eta,k_\perp}(\tau) = \delta_{\lambda,\lambda'} \,, \tag{110}$$

and complete:

$$\sum_{\lambda=\pm 1} \phi^{(h,\lambda)}_{0\,(\alpha);k_\eta,k_\perp}(\tau)\, \phi^{(h,\lambda)\,*}_{0\,(\beta);k_\eta,k_\perp}(\tau) = \delta_{\alpha,\beta} \,. \tag{111}$$

So at $\tau = 1$, we choose our solutions of Eq. (106) so that:

$$\phi^{(h,\lambda)}_{(\alpha);k_\eta,k_\perp}(1) = \phi^{(h,\lambda)}_{0\,(\alpha);k_\eta,k_\perp}(1) \,, \tag{112}$$

for $\alpha = \pm$ and where $\lambda = \pm 1$ labels the initial positive and negative frequency solutions of Eq. (108). The τ-dependent solutions will then be numerically stepped out from the values at $\tau = 1$.

Maxwell's equation becomes:

$$\partial_\tau E(\tau) = -\frac{g}{\tau} \int_{-\infty}^{\infty} \frac{\mathrm{d}k_\eta}{2\pi} \int_0^{\infty} \frac{k_\perp\, \mathrm{d}k_\perp}{2\pi} \sum_{h=\pm 1} j^{(h)}_{k_\eta,k_\perp}(\tau) \,, \tag{113}$$

where $j^{(h)}_{k_\eta,k_\perp}(\tau)$ is given by the positive energy solutions of the Dirac equation only:

$$\begin{aligned} j^{(h)}_{k_\eta,k_\perp}(\tau) &= \phi^{(h,+)\,*}_{(+);k_\eta,k_\perp}(\tau)\, \phi^{(h,+)}_{(-);k_\eta,k_\perp}(\tau) + \phi^{(h,+)\,*}_{(-);k_\eta,k_\perp}(\tau)\, \phi^{(h,+)}_{(+);k_\eta,k_\perp}(\tau) \,, \\ &= \phi^{(h,+)\,\dagger}_{k_\eta,k_\perp}(\tau)\, \sigma_x\, \phi^{(h,+)}_{k_\eta,k_\perp}(\tau) \,. \end{aligned} \tag{114}$$

Here, $\phi^{(h,+)}_{k_\eta,k_\perp}(\tau)$ is the two-component positive energy spinor:

$$\phi^{(h,+)}_{k_\eta,k_\perp}(\tau) = \begin{pmatrix} \phi^{(h,+)}_{(+);k_\eta,k_\perp}(\tau) \\ \phi^{(h,+)}_{(-);k_\eta,k_\perp}(\tau) \end{pmatrix} \,, \tag{115}$$

and σ_x the Pauli matrix. Dirac's Eq. (106) and Maxwell's Eq. (113), are the update equations we want to solve simultaneously.

V. INTERPOLATING NUMBER OPERATOR

When we previously solved the back reaction problem for QED we squared the Dirac equation and used adiabatic regularization. Now that a simpler renormalization procedure is being used, a simpler definition of the interpolating number operator can be determined directly from the Dirac equation. We choose to define our interpolating wave functions in terms of the exact solutions of the Dirac equation in the *absence* of external fields. These zeroth order spinors are given by:

$$\phi^{(h,\lambda)}_{0;k_\eta,k_\perp,m}(\tau,\rho,\theta,\eta) = e^{ik_\eta\eta} \begin{pmatrix} \phi^{(h,\lambda)}_{0\,(+);k_\eta,k_\perp}(\tau)\,\chi^{(h)}_{k_\perp,m}(\rho,\theta) \\ \phi^{(h,\lambda)}_{0\,(-);k_\eta,k_\perp}(\tau)\,\chi^{(-h)}_{k_\perp,m}(\rho,\theta) \end{pmatrix} , \tag{116}$$

where $\phi^{(h,\lambda)}_{0;k_\eta,k_\perp}(\tau)$ given by Eqs. (109). These spinors are also orthogonal and complete. Expansion of the field operator in the *zeroth order* spinors then requires that the creation and annihilation operators $\hat{A}^{(h,\lambda)}_{0;k_\eta,k_\perp,m}(\tau)$ become time-*dependent*. That is:

$$\hat{\phi}(\tau,\rho,\theta,\eta) = \int_{-\infty}^{\infty}\frac{\mathrm{d}k_\eta}{2\pi}\int_0^\infty \frac{k_\perp\,\mathrm{d}k_\perp}{2\pi}\sum_{h=\pm1}\sum_{\lambda=\pm1}\sum_{m=-\infty}^{+\infty}\hat{A}^{(h,\lambda)}_{0;k_\eta,k_\perp,m}(\tau)\,\phi^{(h,\lambda)}_{0;k_\eta,k_\perp,m}(\tau,\rho,\theta,\eta) , \tag{117}$$

Because of the orthogonality of the initial spinors, we see that the $\hat{A}^{(h,\lambda)}_{0;k_\eta,k_\perp,m}(\tau)$ operators obey the same commutation relations as the time-*independent* ones at equal time:

$$[\,\hat{A}^{(h,\lambda)}_{0;k_\eta,k_\perp,m}(\tau),\hat{A}^{(h',\lambda')\,\dagger}_{0;k'_\eta,k'_\perp,m'}(\tau)\,]_+ = \delta_{\lambda,\lambda'}\,\delta_{h,h'}\,\delta_{m,m'}\,(2\pi)^2\,\delta(k_\eta-k'_\eta)\,\frac{\delta(k_\perp-k'_\perp)}{\sqrt{k_\perp k'_\perp}} , \tag{118}$$

and are a reasonable interpolating number operators at time τ. The interpolating particle and anti-particle operators at time τ are

$$\hat{A}^{(h,+)}_{0;k_\eta,k_\perp,m}(\tau) = \hat{b}^{(h)}_{0;k_\eta,k_\perp,m}(\tau) , \qquad \text{and} \qquad \hat{A}^{(h,-)}_{0;k_\eta,k_\perp,m}(\tau) = \hat{d}^{(-h)\,\dagger}_{0;-k_\eta,k_\perp,-m}(\tau) . \tag{119}$$

As before we can determine the adiabatic number operator from the Bogoliubov transformation. The overlap between the adiabatic wave functions and the exact ones is : $C^{(h;\lambda,\lambda')}_{k_\eta,k_\perp}(\tau)$ is given by:

$$C^{(h;\lambda,\lambda')}_{k_\eta,k_\perp}(\tau) = \phi^{(h,\lambda)\,\dagger}_{0;k_\eta,k_\perp}(\tau)\,\phi^{(h,\lambda')}_{k_\eta,k_\perp}(\tau) , \tag{120}$$

and is independent of m. So the creation and annihilation operators are related by the expression:

$$\hat{A}^{(h,\lambda)}_{0;k_\eta,k_\perp,m}(\tau) = \sum_{\lambda=\pm} C^{(h;\lambda,\lambda')}_{k_\eta,k_\perp}(\tau)\, \hat{A}^{(h,\lambda')}_{k_\eta,k_\perp,m}\,, \tag{121}$$

which is a Bogoliubov transformation of the operators.

Calling $n^{(h)}_{k_\perp,\phi,k_\eta}(\tau)$ be the phase space number density

$$n^{(h)}_{k_\eta,k_\perp,\phi}(\tau) = \frac{\mathrm{d}^6 N(\tau)}{\mathrm{d}k_\eta\, k_\perp\, \mathrm{d}k_\perp\, \mathrm{d}\phi\, \mathrm{d}\eta\, \rho\, \mathrm{d}\rho\, \mathrm{d}\theta}\,. \tag{122}$$

$n^{(h)}_{k_\eta,k_\perp,\phi}(\tau)$ is determined from the interpolating number operators using

$$\langle\, \hat{a}^{(h)\dagger}_{0\,k_\eta,k_\perp,\phi}(\tau)\, \hat{a}^{(h')}_{0\,k'_\eta,k'_\perp,\phi'}(\tau)\,\rangle = n^{(h)}_{k_\eta,k_\perp,\phi}(\tau)\, \delta_{h,h'}\, (2\pi)^3\, \delta(k_\eta - k'_\eta)\, \frac{\delta(k_\perp - k'_\perp)}{\sqrt{k_\perp k'_\perp}}\, \delta(\phi-\phi')\,, \tag{123}$$

$$n^{(h)}_{k_\eta,k_\perp,\phi}(\tau) = |\, C^{(h;+,-)}_{k_\eta,k_\perp}(\tau)\,|^2 = 1 - |\, C^{(h;+,+)}_{k_\eta,k_\perp}(\tau)\,|^2\,, \tag{124}$$

and is *independent* of ϕ. Explicitly, $|\, C^{(h;+,+)}_{k_\eta,k_\perp}(\tau)\,|^2$ is

$$|\, C^{(h;+,+)}_{k_\eta,k_\perp}(\tau)\,|^2 = \frac{\omega_{k_\eta,k_\perp}-1}{2\omega_{k_\eta,k_\perp}} \left|\, \phi^{(h,+)}_{(+);k_\eta,k_\perp}(\tau) + \frac{k_\eta + ih\, k_\perp}{\omega_{k_\eta,k_\perp}-1}\, \phi^{(h,+)}_{(-);k_\eta,k_\perp}(\tau)\,\right|^2\,, \tag{125}$$

which has unit value at $\tau = 1$, as required. That is, no particles are produced at $\tau = 1$. Right now we are in the process of doing these calculations. The renormalization method we are using for the back reaction equation will mimic the direct method of using the cutoff value for the the multiplicative charge renormalization $Z(\Lambda, m)$, in analogy the QED result found in Eq. [60]

VI. DOES THE PLASMA THERMALIZE?

In order to discuss whether the plasma thermalizes, one needs to have a robust enough approximation which leads to thermalization for non expanding plasmas. It has been shown that the 2-PI 1/N approximation does have that property. One then needs to discover whether the expansion rate will preclude or slow-down the thermalization of the quarks and gluons produced. To include interactions among the quarks and gluons one would solve the coupled Schwinger Dyson equations using the CTP formalism and a 2-PI Action expanded in 1/N. [31]. Here one would need to keep the background field formalism also to handle the background Chromoelectric Field. Below we sketch some features of the calculation that we are about to begin in order to answer the important question of whether the interactions will drastically change the transverse distribution of jets from that predicted in the case of noninteracting fermions and gluons. This formalism has already been used in QCD to determine transport coefficients by Aarts and Resco [33] and we follow their notation here. The action for N_f identical fermion fields ψ_a $(a = 1, \ldots, N_f)$ then reads

$$S = \int_x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}_a \left(i \not{D} - m \right) \psi_a \right] + S_{\rm gf} + S_{\rm gh}, \tag{126}$$

with

$$\not{D} = \gamma^\mu D_\mu, \qquad D_\mu = \partial_\mu + \frac{ie}{\sqrt{N_f}} A_\mu, \tag{127}$$

and we use the notation

$$\int_x = \int_{\mathcal{C}} dx^0 \int d^3x, \tag{128}$$

where $\mathcal{C}$ refers to the CTP contour in the complex-time plane. We follow the closed time path formalism of Schwinger where all the Green's function can be thought of as ordered according to the closed time path or equivalently as 2×2 matrix Green's functions. The 2PI effective action is an effective action for the contour-ordered two-point functions

$$D_{\mu\nu}(x,y) = \langle | T_{\mathcal{C}} \left(A_\mu(x) A_\nu(y) \right) | \rangle \qquad S_{ab}(x,y) = \langle | T_{\mathcal{C}} \left(\psi_a(x) \bar{\psi}_b(y) \right) | \rangle, \tag{129}$$

and can be written schematically as

$$\Gamma[S,D] = \frac{i}{2} \mathrm{tr} \ln D^{-1} + \frac{i}{2} \mathrm{tr} D_0^{-1} (D - D_0) \tag{130}$$

$$- i \mathrm{tr} \ln S^{-1} - i \mathrm{tr} S_0^{-1} (S - S_0) + \Gamma_2[S,D] + \text{ghosts}, \tag{131}$$

where D_0^{-1} and S_0^{-1} are the free inverse propagators.

For the gauge theory the NLO Schwinger Dyson equations that result from varying the 2-PI action are:

$$S^{-1} = S_0^{-1} - \Sigma, \qquad D^{-1} = D_0^{-1} - \Pi, \tag{132}$$

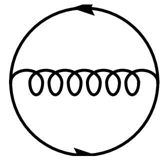

FIG. 6: NLO contribution to the 2PI effective action in the $1/N_f$ expansion.

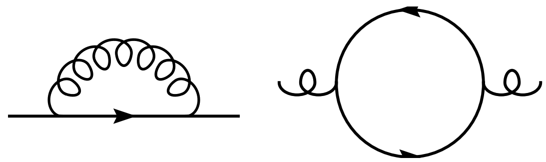

FIG. 7: Self energies at at NLO in large N_f QCD.

with S the fermion and D the gauge field propagator. The self energies, depending on full propagators, are shown in Fig 7. The Back Reaction equation is given by:

$$\nabla_\mu F^{\mu\nu} = \langle j^\nu \rangle = -ig^2 Tr\gamma^\nu S \tag{133}$$

To determine the interpolating number densities of quarks and antiquarks one can follow the procedure of Berges, Borsanyi and Serreau [34] and define these from the current. Namely the associated 4–current for each given flavor is $\sim \bar{\psi}\gamma^\mu\psi$. Fourier transforming with respect to spatial momenta, the expectation value of the latter can be written as $J^\mu_f(t,p) = \mathrm{tr}[\gamma^\mu S^<(t,t,p)]$, In terms of the equal-time two point function, its temporal and spatial components are

$$\begin{aligned} J^0_f(t,p) &= 2\,[1 - 2\,F^0_V(t,t;p)]\,, \\ \vec{J}_f(t,p) &= -4\,F_V(t,t;p)\,. \end{aligned}$$

To obtain an effective particle number, Berges et. al. identify these expressions with the corresponding ones in a quasi-particle description with free-field expressions. These are given by

$$\begin{aligned} J^{0\,(\mathrm{QP})}_f(t,p) &= 2\,[1 + Q_f(t,p)]\,, \\ \vec{J}^{(\mathrm{QP})}_f(t,p) &= -2\,[1 - 2N_f(t,p)]\,, \end{aligned}$$

where $Q_f(t,p) = n_f - \bar{n}_f$ is the difference between particle and anti-particle effective number densities and $N_f(t,p) = (n_f + \bar{n}_f)/2$ is their half-sum. The physical content of these expressions is simple: the temporal component J^0 directly represents the net-charge density per mode $Q_f(t,p)$, whereas the spatial part $\vec{J}$ is the net current density per mode and is therefore sensitive to the sum of particle and anti-particle number densities. Identifying the above expressions, theydefine

$$\frac{1}{2}\, Q_f(t,p) \;=\; -F_V^0(t,t;p)\,, \tag{134}$$

$$\frac{1}{2} - N_f(t,p) \;=\; F_V(t,t;p)\,. \tag{135}$$

Using these definitions and solving the backreatction problem to NLO in 2-PI 1/N we would also be able to discover if there is time for the produced quarks and antiquarks to thermalize before hadronization time scale and to see if the constant field result for the transverse distribution will be modified by the interactions.

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