non-Gaussianity from preheating

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questions

1. can detectable non-Gaussianity be generated during preheating?
2. if so, how much?

non-Gaussianity

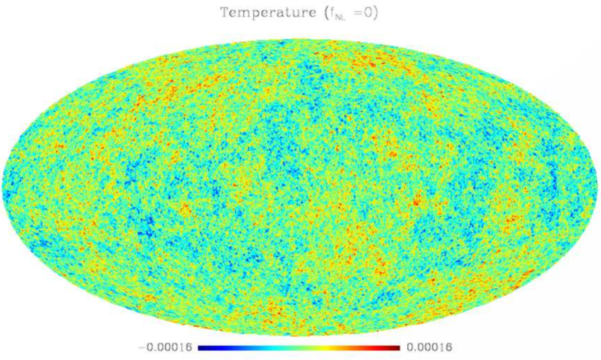
- non-Gaussianity is the difference from Gaussianity
- Gaussian perturbations are described entirely by the two-point correlation function (or its Fourier transform, the power spectrum)
- non-Gaussianity perturbations are not and we should include higher order *n*-point functions
- conventionally in cosmology we parameterise non-Gaussianity using the non-linearity parameter $f_{\rm NL}$,

$$\zeta = \zeta_0 - \frac{3}{5} f_{\rm NL} \left(\zeta_0^2 - \langle \zeta_0^2 \rangle \right) ,$$

where ζ is the curvature perturbation and ζ_0 is a Gaussian random field

non-Gaussianity

- Gaussian perturbations have $f_{\rm NL} = 0$
- single field, slow roll inflation predicts $|f_{\rm NL}| \ll 1$
- WMAP team endorsed result: $-36 < f_{\rm NL} < 100$
- Yadav and Wandelt (2007): $26.9 < f_{\rm NL} < 146.7$



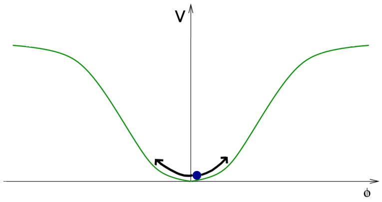
Liguori, Yadav, Hansen, Komatsu, Matarrese, Wandelt 2007

perturbative reheating

- inflaton into other particles
- process is very slow
- inflaton may not decay in time for big bang nucleosynthesis

preheating

- inflaton decays explosively into a second scalar particle
- process is very fast
- two popular models:
  - parametric resonance
  - tachyonic preheating



massless preheating

chaotic inflation + massless scalar field

$$V(\phi, \chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

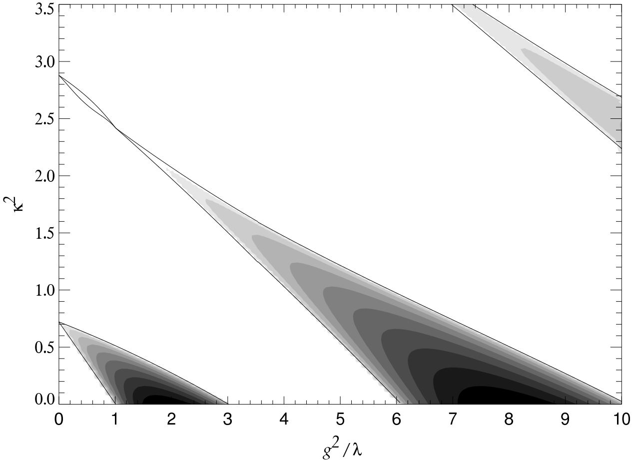
- inflaton oscillates: $\ddot{\phi} + 3H\dot{\phi} + \lambda\phi^3 = 0$
- universe is radiation dominated: $a \propto t^{\frac{1}{2}}$
- rescale fields: $\phi = a^{-1}\tilde{\phi}$, $\chi_k = a^{-1}\tilde{\chi}_k$
- rescale time: $d\tau = a^{-1}\lambda^{1/2}\tilde{\phi}_{\text{ini}}dt$

$$\Rightarrow \tilde{\phi}'' + \lambda\tilde{\phi}^3 = 0 \Rightarrow \tilde{\phi}(\tau) = \tilde{\phi}_{\text{ini}}\text{cn}\left(\tau; \frac{1}{\sqrt{2}}\right) \quad \text{(Jacobi cosine)}$$

$$\tilde{\chi}_k'' + \left[\kappa^2 + \frac{g^2}{\lambda}\text{cn}^2\left(\tau; \frac{1}{\sqrt{2}}\right)\right]\tilde{\chi}_k = 0, \quad \kappa^2 = \frac{k^2}{\lambda\tilde{\phi}_{\text{ini}}^2}$$

massless preheating

Floquet's theorem $\Rightarrow \tilde{\chi}_k \sim e^{\mu\tau} p(\tau)$



Greene, Kofman, Linde, Starobinsky 1997

curvature perturbations

- horizon size is small during preheating
- preheating cannot create observable perturbations

however

- during inflation: $\phi \gg M_{Pl}$ and $\chi \sim 0$
- if the χ field is light ($m_\chi = g\phi \ll H$), then it contains Gaussian scale invariant perturbations at the end of inflation

$$\mathcal{P}_\chi \approx \mathcal{P}_\phi \approx \frac{H^2}{4\pi^2} \approx 10^{-12} M_{\rm Pl}^2$$

curvature perturbations

we can now refine our original questions to:

1. can perturbations in a secondary light scalar field at the end of inflation be converted into super-horizon curvature perturbations by preheating?
2. how non-Gaussian are these perturbations?

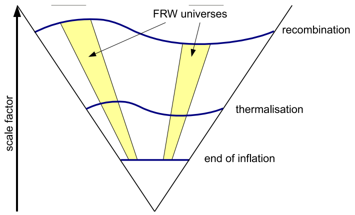
Jokinen and Mazumdar (JCAP 0604 (2006) 003; astro-ph/0512368) said:

1. yes
2. $|f_{NL}| \sim O(1000)$

separate universes

alternative approach: separate universes approximation

- maintain full non-linear field equations
- assume universe to be made up of many causally disconnected FRW universes
- leads to: $\zeta = \delta \ln a \,|_H$
- solve field equations numerically from the end of inflation to thermalisation

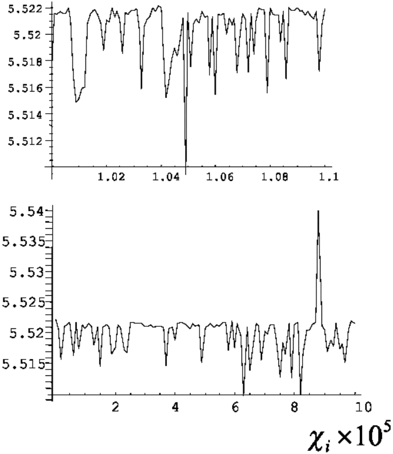


equations to solve:

$$\begin{aligned}\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\vec{\nabla}^2\phi + \lambda\phi^3 + g^2\chi^2\phi &= 0 \\ \ddot{\chi} + 3H\dot{\chi} - \frac{1}{a^2}\vec{\nabla}^2\chi + g^2\phi^2\chi &= 0\end{aligned}$$

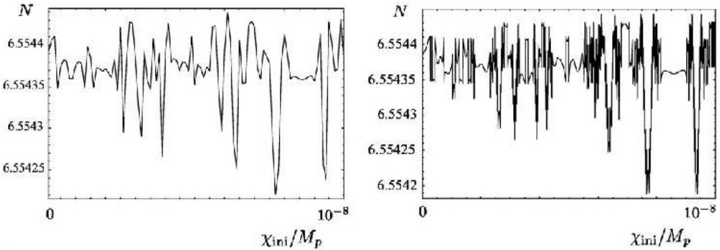
$$H^2 = \frac{1}{3M_{\mathrm{Pl}}^2}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}(\vec{\nabla}\chi)^2 + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2\right)$$

previous work



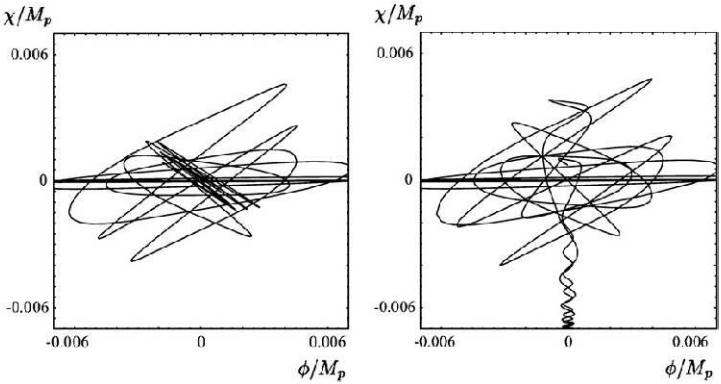
Bassett and Tanaka 2003

previous work



Suyama and Yokoyama 2006

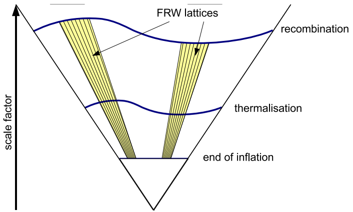
previous work



Suyama and Yokoyama 2006

our approach

- solve the full non-linear equations on many FRW lattices
- Friedmann equation gives growth by integrating over density in the lattice at every time-step
- vary the initial mean value (zero-mode) of χ field and observe relationship to $\delta \ln a \,|_H$



our approach

equations to solve:

$$\begin{aligned}\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\vec{\nabla}^2\phi + \lambda\phi^3 + g^2\chi^2\phi &= 0 \\ \ddot{\chi} + 3H\dot{\chi} - \frac{1}{a^2}\vec{\nabla}^2\chi + g^2\phi^2\chi &= 0\end{aligned}$$

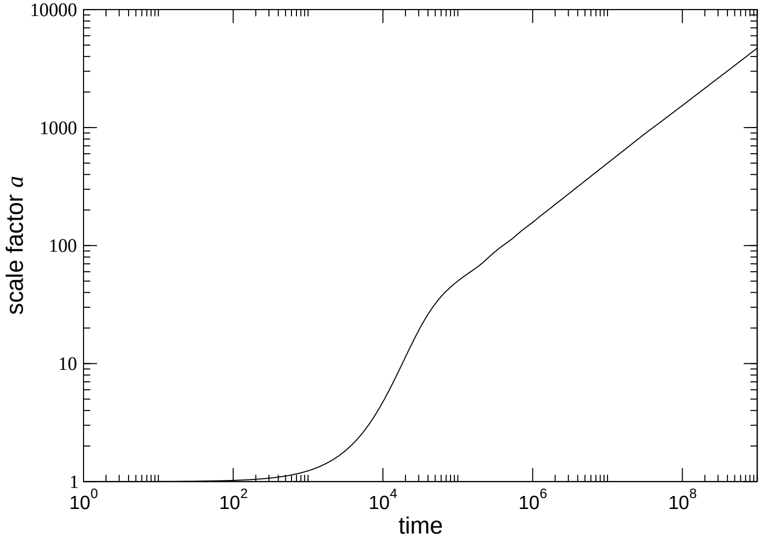
$$H^2 = \frac{1}{3M_{\rm Pl}^2 V}\int d^3x \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}(\vec{\nabla}\chi)^2 + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2\right)$$

our approach

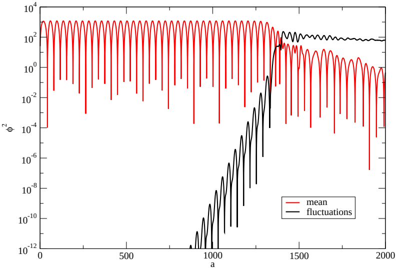
some details:

- $\delta x = 1.25 \times 10^5 M_{Pl}$, lattice size$= 32^3$
- $\phi_{\text{ini}} = 5M_{Pl}$, $\lambda = 7 \times 10^{-14}$
- 60 – 240 runs for each χ_{ini}
- quantum initial conditions: Gaussian fluctuations with the same two-point function as the quantum vacuum
$$\overline{|\chi_k|^2} = \frac{1}{V}\frac{1}{2\omega_k}$$
- subsequent evolution of lattice is classical

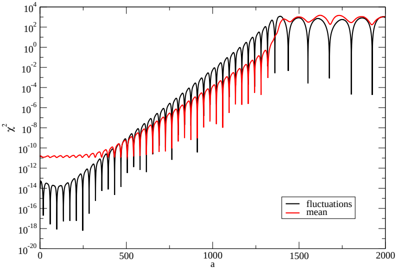
results



results

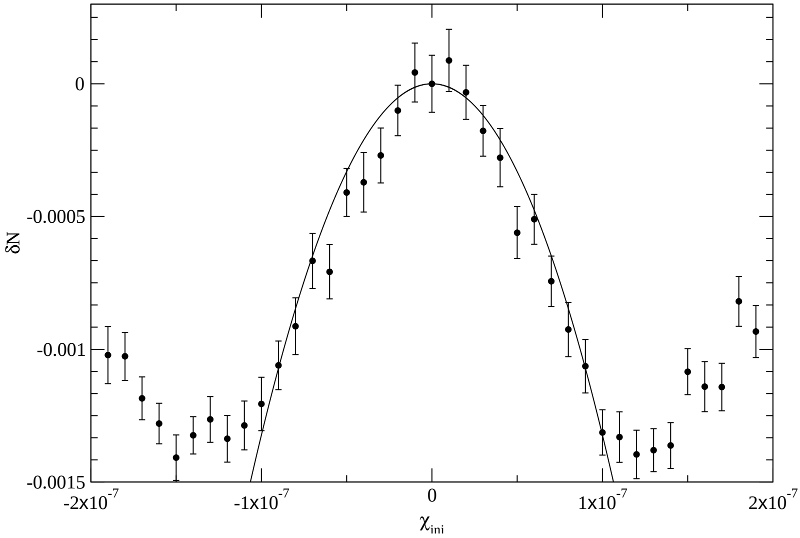


ϕ



χ

results



results

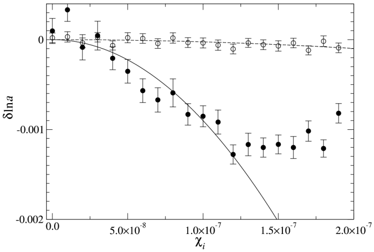
fit: $\ln a(\chi) = \ln a(0) + c\chi_i^2$

$$2c = \frac{\partial^2 \delta \ln a|_H}{\partial^2 \chi_i}$$

$$\frac{g^2}{\lambda} = 1.875 \quad \Rightarrow \quad c = 10^{11.26 \pm 0.05}$$

$$\frac{g^2}{\lambda} = 2.700 \quad \Rightarrow \quad c = 10^{14.01 \pm 0.10}$$

$$\frac{g^2}{\lambda} = 1.050 \quad \Rightarrow \quad c = 10^{4.30 \pm 0.05}$$



results

calculate f_{NL}:

- WMAP-3 bound: $-54 < f_{NL} < 134$
- use $\zeta = \delta \ln a \,|_H$ to calculate f_{NL}
- Boubekeur and Lyth (2006) use: $\zeta = \zeta_0 + c\left(\chi^2 - \langle\chi^2\rangle\right)$
- $\Rightarrow f_{NL} = -\frac{5}{6} c^3 \frac{\mathcal{P}_\chi^3}{\mathcal{P}_\zeta^2} \ln \frac{k}{H}$
- substitute: $\mathcal{P}_\chi = (H^2/4\pi^2)$ and $\mathcal{P}_\zeta = (V/24\pi^2 \epsilon M_{\rm Pl}^4)$

$$\Rightarrow \quad f_{NL} = -\frac{40}{9\pi^2} c^3 \lambda M_{\rm Pl}^6 \ln \frac{k}{H} \sim 10^{20}$$

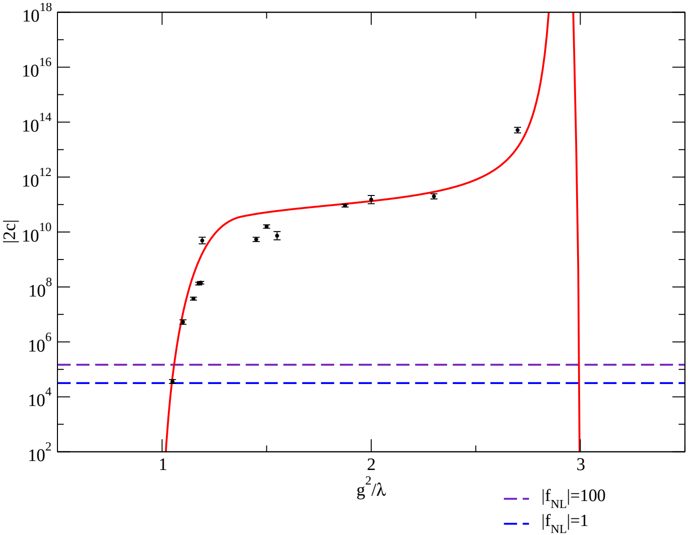
analytic estimate

derive an analytic estimate for c (and therefore f_{NL}) by solving for the zero-mode and inhomogeneous modes separately:

- $\langle \tilde{\chi}^2 \rangle(a) = \overline{\chi}^2(a) + \langle \delta\tilde{\chi}^2 \rangle(a)$
- model zero-mode: $\overline{\chi}^2(a) = \overline{\chi}^2(1) e^{2\tilde{\mu}_0(a-1)}$
- model inhomogeneous modes: $\langle \delta\tilde{\chi}^2 \rangle(a) = m^2 e^{2\tilde{\mu}_k(a-1)}$
- where $m^2 = \lambda\tilde{\phi}^2_{\text{ini}} = g^2\langle \tilde{\chi}^2 \rangle$ is the amplitude where the system becomes non-linear
- make ansatz $N = N_0 + c\chi^2_{\text{ini}}$

$$\Rightarrow \ldots \Rightarrow \log|c| = -2\frac{\tilde{\mu}_0}{\tilde{\mu}_k}\log g + \log\left(\frac{1}{2\tilde{\phi}^2_{\text{ini}}\log\frac{1}{g}}\right)$$

analytic estimate



conclusions

our questions were:

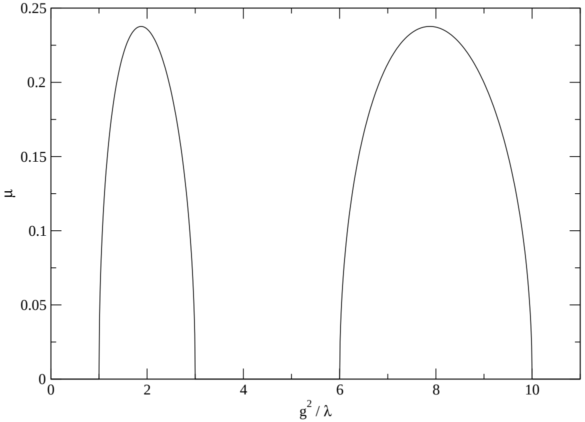
1. can perturbations in a secondary light scalar field at the end of inflation be converted into super-horizon curvature perturbations by preheating?
2. how non-Gaussian are these perturbations?

our answers are:

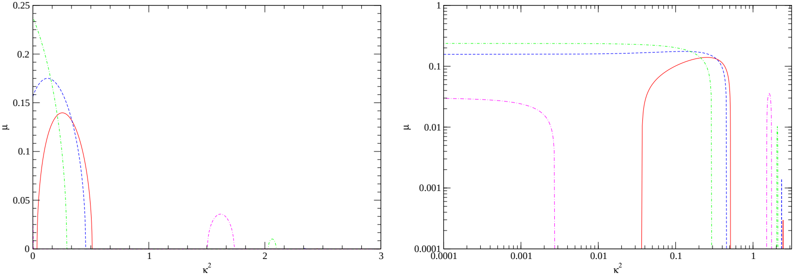
1. yes, in the chaotic inflation model if a light scalar field is present at the end of inflation and if its zero-mode is in resonance during preheating
2. $f_{NL} \gg O\left(10^5\right)$ which is outside of observational bounds

and we use nonequilibrium classical field theory simulations to arrive at these answers.

extra slide



extra slide



0.25
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