# Physics of Time Dependent Black Holes

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## Gauge/Gravity Correspondence

**The problem of quantum gravity is solved in the framework of gauge/gravity correspondence.**

**Gauge Theory** ⟷ **Gravity**

**However we do not know the dictionary of the corres pondence very well.**

## AdS/ CFT Correspondence

Juan Maldacena 97

The N=4 SYM theory is dual to the type IIB string theory on $AdS_5 \times S_5$ geometry.

N=4 SYM theory ⟷ (1 to 1) IIB Strings on $AdS_5 \times S_5$

The YM coupling is related to the string coupling by

$$g_{YM}^2 = g_s$$

and

$$R^4/l_s^4 = g_{YM}^2 N_c = \lambda$$

## Strong weak coupling duality

The YM perturbative expansion is a small lambda expansi on. On the other hand the tension of the string

$$Tl_s^2 = \sqrt{\lambda}/(2\pi) = 1/(2\pi\hbar)$$

plays the role of $1/\hbar$ in the Nambu-Goto string action.

Hence, for the string $1/\sqrt{\lambda}$ expansion corresponds to the $\hbar$ expansion of the usual quantum mechanics.

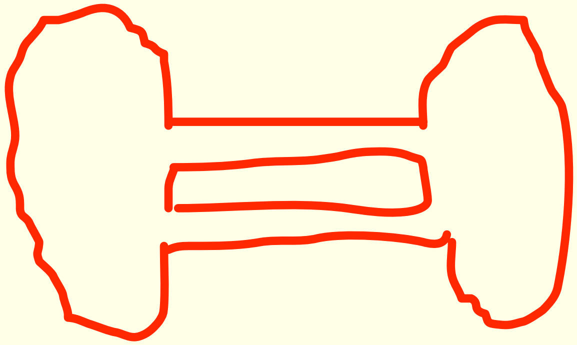
In this sense the duality here is a strong-weak coupling duality.

## 1/Nc expansion

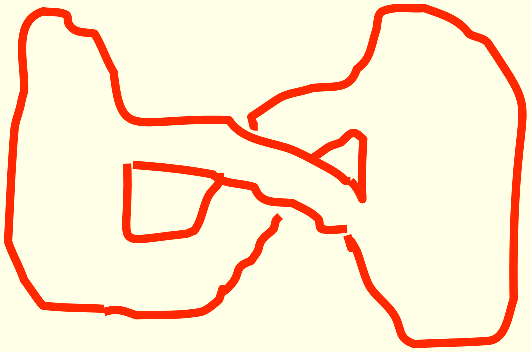
**1/Nc expansion is responsible for the higher genus contrib utions of the SYM theories and**

→

**the higher genus worldsheet string interactions.**



**planar**



**non-planar**

## Key points

**Large $N_c$ planar limit of N=4 SYM theory**

**dual to** ⟷

**Classical supergravity on $AdS_5 \times S_5$ geometry**

**$1/\sqrt{\lambda}$ expansions which controls the higher curvature corrections also belongs to the physics of classical geometries. As we shall see later on, this is responsible for all the essential nature of the classical gravity theories.**

**The quantum gravity requires the full interacting string theory. In the N=4 SYM side, one has to include the nonplanar corrections.**

**The key question now is whether one may have a more general framework including all possible gravity theories.**

**What is the corresponding dual of the gravity with zero or positive cosmological constant?**

**Do we have a dual of cosmological spacetime including big bang and big crunch? [DB] hep-th/0608030**

**For such general spacetimes, we do not know how to define string theories.**

## Today's topic

A controllable deformations of AdS/CFT correspondence.

It is on the recently constructed time dependent black hole solution with nontrivial scalar field turned on.

[DB, M. Gutperle and S. Hirano] [DB, M. Gutperle and A. Karch]

This solution reveals the nature of gravity and how the thermodynamic nature of the gravity appears in a clear manner.

**Thermalization and the second law of thermodynamics can be clearly understood.**

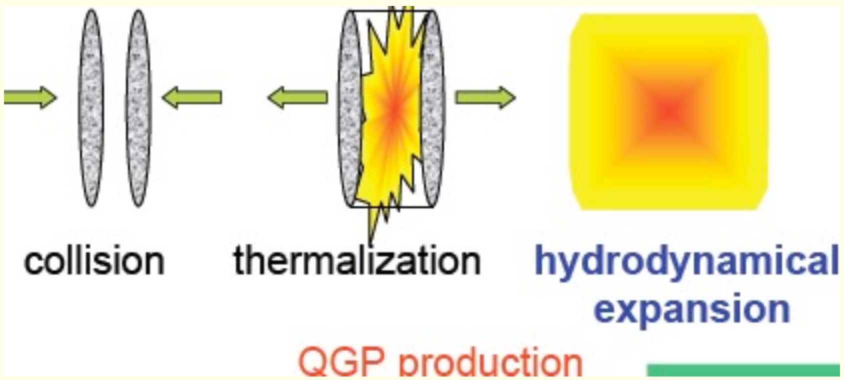
**Also the unitarity issue related to the Hawking's information paradox can be understood using the correspondence.**

**One can compute the thermalization time scale rather precisely even for the large perturbations, which is almost impossible by any other means for such strongly coupled systems.**

## Fast equilibration time of strongly coupled QGP

**In RHIC experiment, the relaxation occurs very fast.**

**It is less than 0.6 fm/c.**




**We shall show that the thermalization time scale is given by**

$$\tau_{therm.} = \frac{1}{2\pi T}$$

**For** $T = 300\ \mathrm{MeV},$ **this would give** $\tau_{therm.} \sim 0.1\ \mathrm{fm/c}.$

# Physics of the time dependent black holes

**Ansatz to solve type IIB supergravity**

**Step I: Compactification on S5**

$$ds^2 = g_{ab}dx^a dx^b + ds^2_{S^5} .$$

$$\phi = \phi(x_a) ,$$

$$F_5 = *\omega_{S^5} + \omega_{S^5} ,$$

**leads to the 5d Einstein scalar system with negative cosmological constant:**

$$I = \frac{1}{16\pi G_d} \int \sqrt{g_d} \Big( R_d - g_d^{ab} \partial_a \phi \partial_b \phi + (d-1)(d-2) \Big) .$$

# The Einstein equations become

$$R_{ab} = \partial_a\phi\partial_b\phi - (d-1)g_{ab} \ ,$$
$$\nabla^2\phi = 0 \ .$$

**AdS3 x S3 x M4 can be treated similarly and the dimensional reduction leads to the 3d Einstein scalar theory.**

**Step II: Janus ansatz by AdS$_{d-1}$ slicing**

$$ds^2 = f(\mu)(d\mu^2 + ds^2_{d-1})$$
$$\phi = \phi(\mu)$$

**The d-1 metric satisfies the AdS d-1 equation,**

$$\bar{R}_{pq} = -(d-2)\bar{g}_{pq} \ .$$

## The scalar field can be determined from

$$\phi'(\mu) = \frac{c}{f^{\frac{d-2}{2}}(\mu)}$$

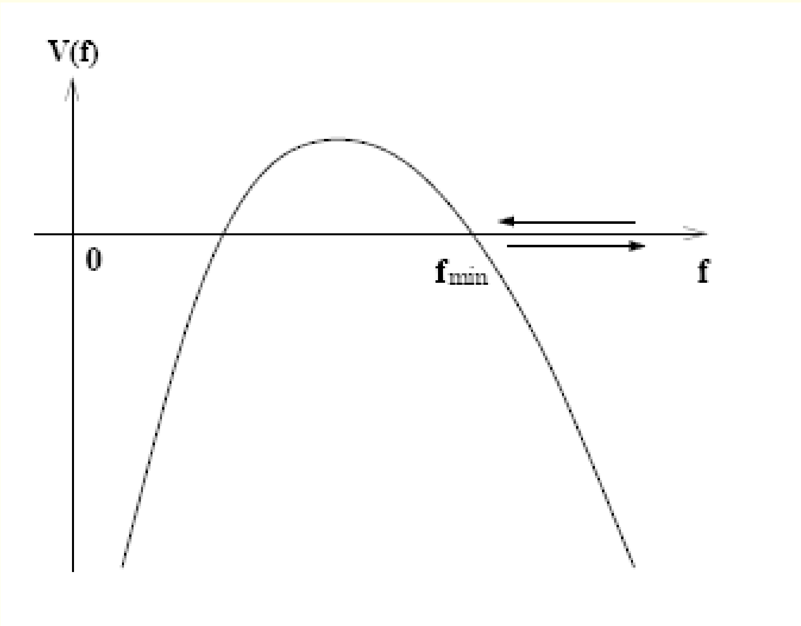
**and**

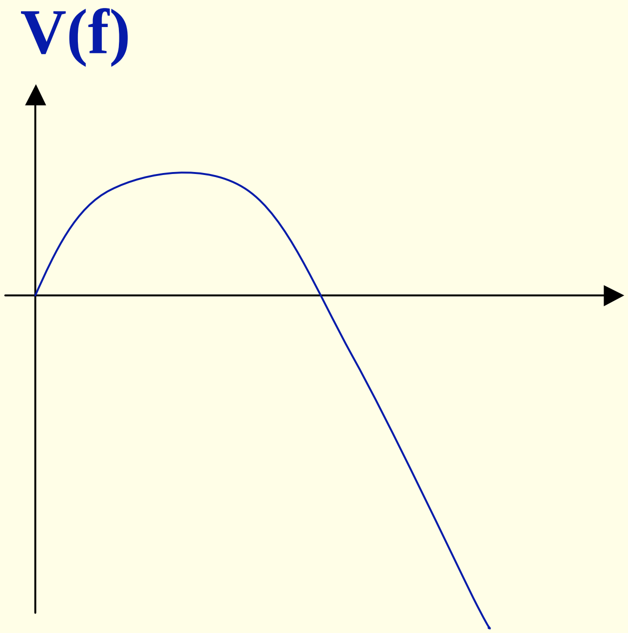
$$f'f' = 4(f^3 - f^2 + \frac{c^2}{f^{d-4}}) \ .$$

**The scale factor equation is reduced to a zero energy particle dynamics with a potential:**

$$V(f) = -4(f^3 - f^2 + \frac{c^2}{f^{d-4}})$$

## There are three cases depending on magnitude of the d eformation parameter c.



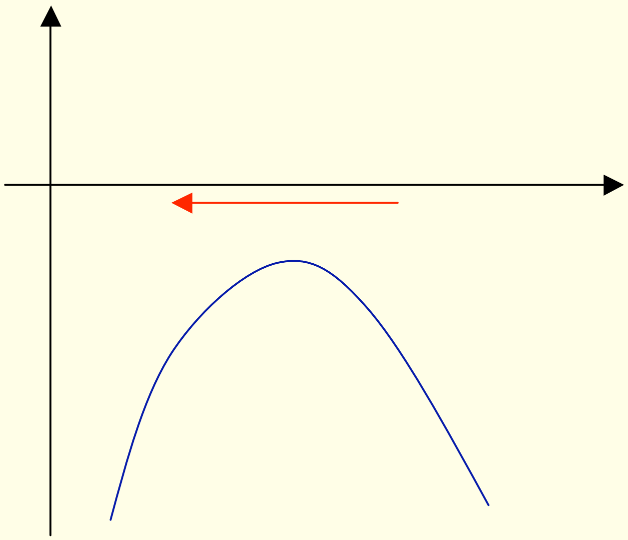




**I.** $c^2 < c_c^2$ **with**

$$c_c^2 = (d-2)\left(\frac{d-2}{d-1}\right)^{d-2}$$

**c=0 case**

## II. overcritical case of $c^2 \geq c_c^2$



**The singularity can be covered with horizon. For simplicity, we shall not discuss this case any further here.**

## The particle motion is solved by

$$\mu_0 \pm \mu = \int_f^\infty \frac{dx}{2\sqrt{x^3 - x^2 + \frac{c^2}{x^{d-4}}}} \,;$$

**The slicing coordinate is ranged over** $[-\mu_0, \mu_0]$ **where**

$$\mu_0 = \int_{f_t}^\infty \frac{dx}{2\sqrt{x^3 - x^2 + \frac{c^2}{x^{d-4}}}} \geq \frac{\pi}{2} \,.$$

## We thus find $f(\mu)$ and $\phi(\mu)$ exactly

**with one deformation parameter turned on.**

**Now the trick is to replace d-1 D metric by the AdS cosmological type solution,**

$$ds^2_{d-1} = -d\tau^2 + \cos^2\tau\, ds^2_{\Sigma} \quad ,$$

**where we make the hyperbolic space Sigma smooth and compact by an appropriate quotient.**

**Then this final form of the solution describes the time depend ent BH.**

$$ds^2 = f(\mu)(d\mu^2 - d\tau^2 + \cos^2\tau\, ds^2_{\Sigma})$$

$$\phi = \phi(\mu)\,.$$

**In d=3 case, the solution describes a deformation of BTZ black hole.**

**The c=0 case of general dimensions corresponds to a static topological black hole solutions given by D. Birm ingham.**

**In this case, the scale factor f is given by**

$$f(\mu) = \frac{1}{\cos^2 \mu}$$

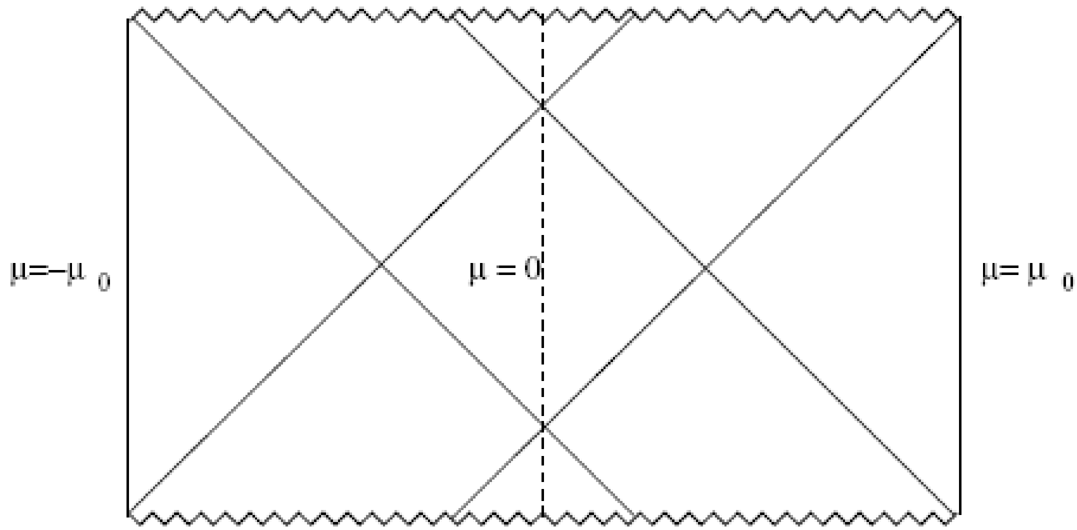
**By the transformation,**

$$r = \cos\tau / \cos\mu, \qquad \tanh t = \sin\tau / \sin\mu\ ,$$

**one recovers the usual form**

$$ds^2_{eq} = -(r^2-1)dt^2 + \frac{dr^2}{r^2-1} + r^2 ds^2_{\Sigma}\ .$$

## Penrose diagram




**The Penrose diagram is elongated horizontally and this is the first explicit construction of such structure.**

**The conformal time coordinate tau runs**

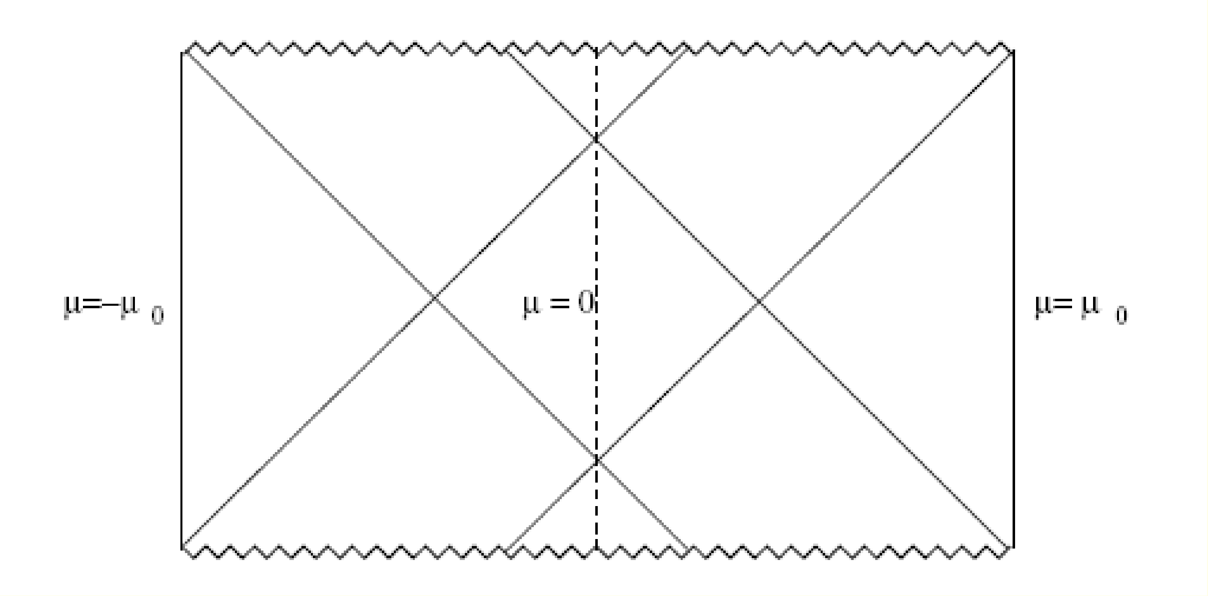
$$\tau \in [-\pi/2, \pi/2]$$

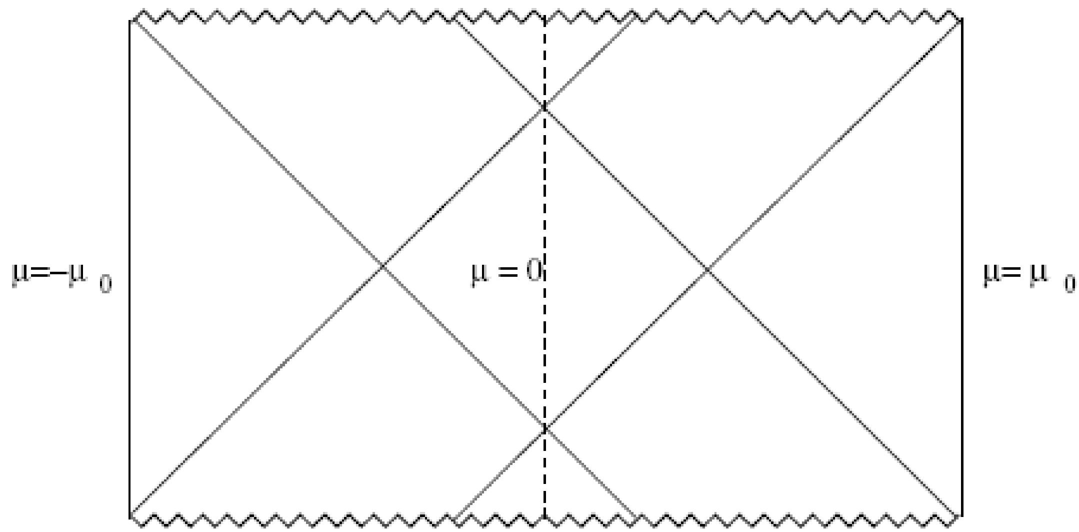
**Here coordinate mu runs from** $\mu \in [-\mu_0, \mu_0]$ **where**

$$\mu_0 \geq \pi/2$$

The dilaton runs from $\phi_-$ to $\phi_+$ as a function of mu and the difference of the coupling is related to the deformation parameter.

There are two boundaries as the case of usual AdS black holes but now the couplings and Hamiltonians become different.






**The time dependent nature of the solution can be seen clearly by looking at the horizon area along the future horizon.**

**The horizon area is time dependent now:**

$$A(\tau) = \mathcal{A}_{\Sigma}(\cos(\tau) f^{\frac{1}{2}}(\mu_0 + \tau - \pi/2))^{d-2} \, .$$

## Boundary N=4 SYM theory

**The boundary time t is defined by the relation**

$$\sin\tau = \tanh t$$

**so the time t runs from – infinity to + infinity.**

**And the boundary N=4 SYM theory is defined on the boundary spacetime of T x Sigma space,**

$$ds_B^2 = -dt^2 + ds_\Sigma^2 .$$

**To talk about the correspondence further, one has to specify t he initial density matrix of the quantum system.**

**We shall use so called thermofield dynamics which is a mathematical technique to describe the real time thermodynamics.**

**The density matrix can be constructed from the thermofield state,**

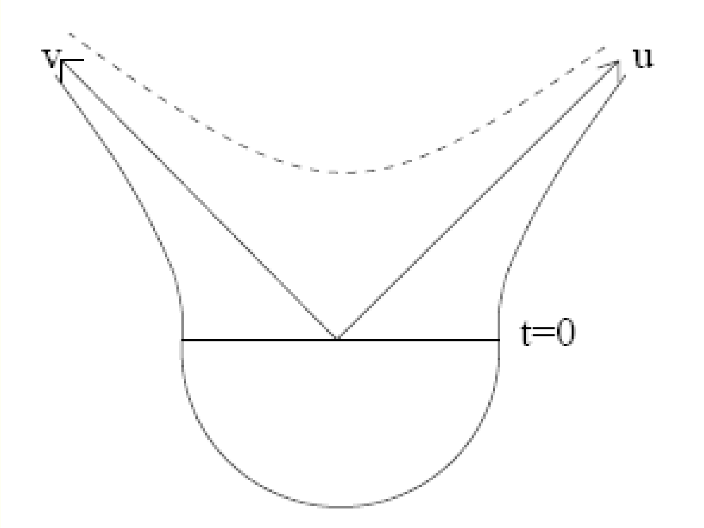
$$|\Psi\rangle = \frac{1}{\sqrt{Z}} \sum_{mn} \langle E_m^+|U|E_n^-\rangle \; |E_m^+\rangle \times |E_n^-\rangle$$

**defined on the two product Hilbert spaces.**

**In the usual thermofield case, we simply double a single Hilbert space.**

**But, in the present case, it is a slight generalization because the geometry now involves two different Hamiltonians H+ an d H-.**

**Now like the case of the instanton physics of bubble nucleation, we patch the lower half of the Euclidean geometry to the upper half of the Minkowski geometry as in this figure.**




**The Euclidean boundary evolution**

$$U = Texp\Big[ - \int_{s_-}^{s_+} ds\, H(s) \Big] = e^{-\frac{\beta_{eq}}{4} H_+} e^{-\frac{\beta_{eq}}{4} H_-}$$

**fixes the thermofield initial state.**

**One then defines the + boundary density matrix by**

$$\rho_+ = \mathrm{tr}_- |\Psi\rangle\langle\Psi|$$

**tracing over the – boundary Hilbert space.**

**Without the deformation, the two Hamiltonians become the same and we recover the usual thermal density matrix at finite temperature:**

$$\rho = e^{-\beta H_+}$$

**One can further define the one point correlation functions**

$$\langle O_+ \rangle = \mathrm{tr}_+ \rho_+ O_+$$

**with help of the density matrix.**

**Standard SUGRA dictionary** **[Witten, Gubser, Klebanov,Polyakov]**

**On-shell supergravity action with given boundary data gives a generating functional of the correlation functions:**

$$e^{-I_{SUGRA}|_{\phi_B}} = \langle e^{-\int \phi_B \hat{O}} \rangle \, .$$

**From these one can calculate expectation values of operators such as energy momentum tensor or Lagrange density.**

**The results of computation for the energy density, pressure**

$$\langle \mathcal{E} \rangle = \frac{1}{8\pi G} \left(\frac{d-2}{d-1}\right) \left(\frac{d-3}{d-1}\right)^{\frac{d-3}{2}} = (d-2)\, p$$

**and the expectation value of Lagrange density**

$$\langle \mathcal{L} \rangle = \langle \mathcal{K} - \mathcal{V} \rangle = \frac{c}{8\pi G} \frac{1}{\cosh^{d-1} t},$$

**which corresponds to the kinetic minus potential energy densities.**

## Interpretation

$$\langle \mathcal{L} \rangle = \frac{c}{8\pi G} \frac{1}{\cosh^{d-1} t}$$

**The results from both sides agree precisely as it should be.**

**Since we are dealing with a homogeneous state over a compact space, the energy density should be constant by the energy momentum conservation.**

**Then the exponential fall-off of the kinetic minus potential energy describes the physics of thermalization.**

**At t=0, the system starts from an out-of equilibrium state with an excess kinetic energy.**

**As time goes by, the system equilibriates and the initial difference disappears exponentially, which is nothing but the physics of thermalization.**

## Black hole is a fast thermalizer!

One can compute the thermalization time scale,

$$\tau_{therm.} = \frac{1}{2\pi T}$$

even for the large perturbations.

For high enough temperature, the thermalization is very fast. For example if we use the typical temperature T= 300 MeV of RHIC experiments,

$$\tau_{therm.} \sim 0.1\ fm/c$$

which is fast enough to explain the thermailzation time of RHIC. Such fast thermalization has been quite puzzling from the view point of the perturbative QFT.

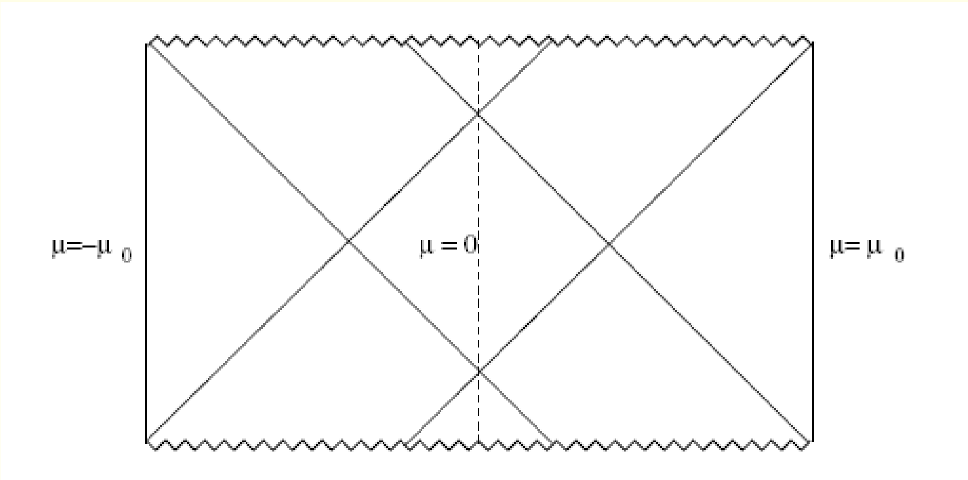
**The usual form of the first law is not working here for the system;**

$$TdS \neq dE + pdV$$

**since dV=0 and dE =0 but dS is nonvanishing.**

**But this simply implies that the system is not in a quasi equilibrium state.**

**Even the second law is not working either due to the $Z_2$ symmetry. It begins with a maximal value, decreases to a minimal value and then grows to the maximal value.**




**This is a consequence of the incredible initial fine tuning.**

## Issue of the unitarity

**The exponential fall-off of the operator expectation value is contradicting with the quantum Poincare recurrence theorem.**

**Quantum Poincare recurrence theorem**

**For any quantum system with discrete eigenvalues, any state will return to its initial values arbitrarily closely in a finite amount of time.**

**hep-th/0208013 [Dyson Lidsay Susskind]**

**It follows immediately that the expectation value of operators should also return arbitrarily closely.**

## Large $N_c$ planar limit of the thermal SYM theory

**This contradiction is basically due to the large $N_c$ limit and we conclude that the planar limit of thermal SYM theory is not unitary.**

**This violation comes from the intrinsically nonperturbative nature of the large time limit of the thermal planar YM theory as argued by**

**G. Festuccia and H. Liu, hep-th/0611098.**

## Future directions

**What remains is clear.**

**1. Computation of the thermalization time for general cases.**

**2. Check of the AdS/CFT for the non-planar case.**

**3. Generalization to more general backgrounds including co smological spacetime will be also very interesting.**

## Janus deformation

DB, M. Gutperle, S. Hirano

It is a simple nonsupersymmetric deformation of

$$AdS_5 \times S^5$$

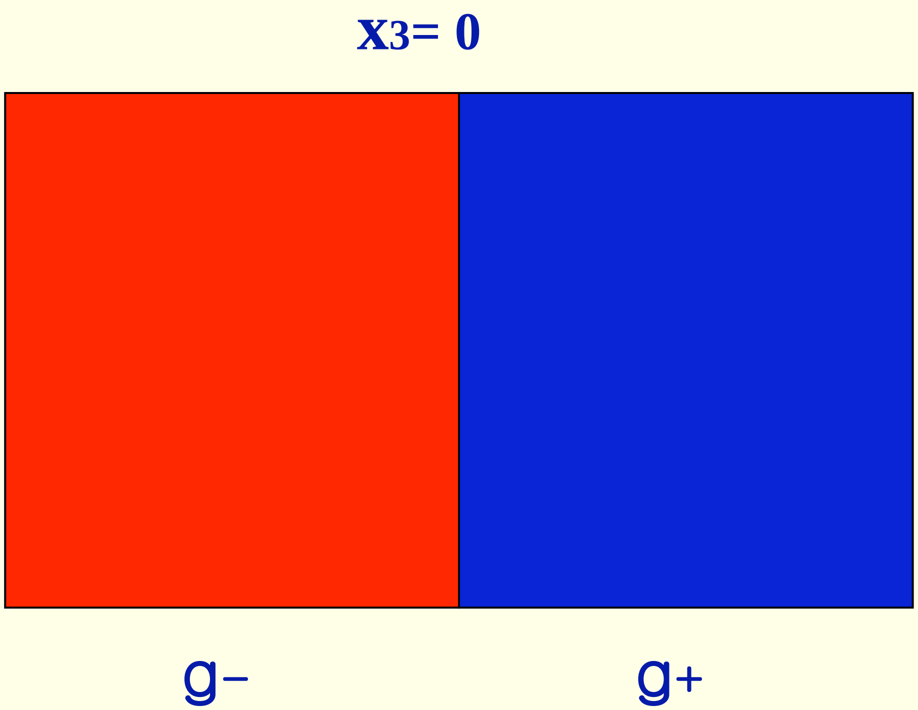
geometry.

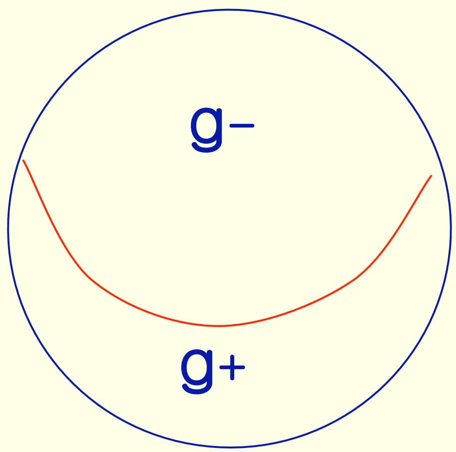
In this deformation, we like to keep the SO(2,3) x SO(6) part of the SO(2,4) x SO(6) symmetries.

We have found an exact solution of IIB sugra with one extra deformation parameter corresponding to the spatial change of the coupling constant.

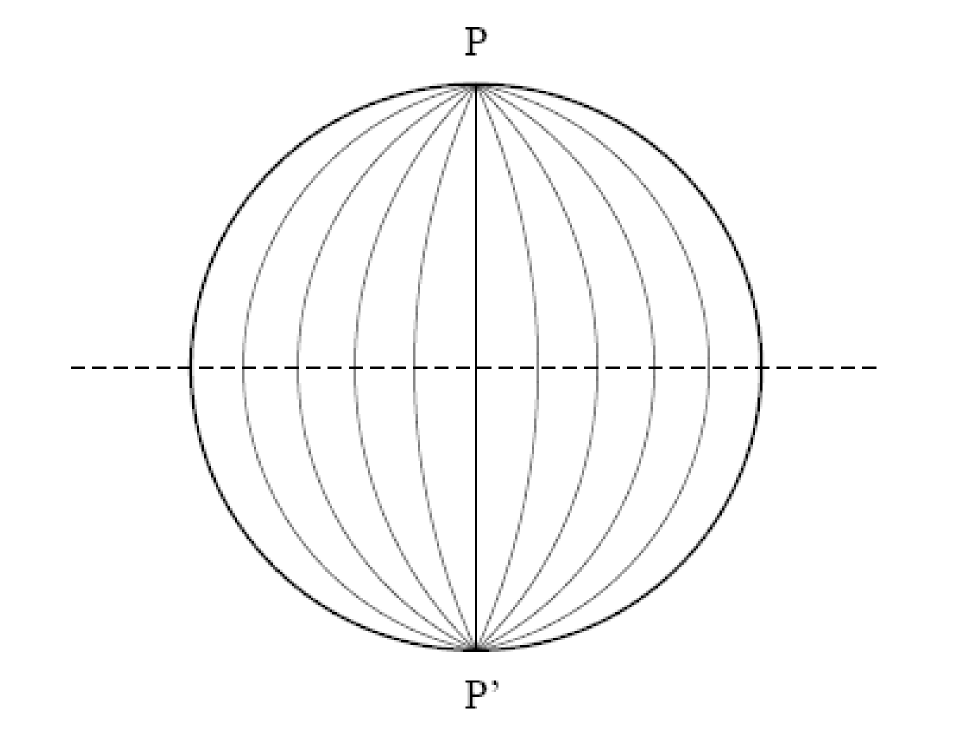
## Interface conformal field theory in 4d

**preserves 3d conformal symmetries for fixed $x_3$ .**



## The conformal shape of the Euclidean geometry is depicted here.




$[-\pi/2, \pi/2]$ $[\pi/2, 3\pi/2]$

$H_-$ $H_+$

**Lorenzian boundary time**

$$\tanh t = \pm \sin \tau$$

**Euclidean boundary time**

$$\tan t_E = \pm \sinh \tau_E .$$

**at** $\mu = \pm \mu_0 .$

## Standard dictionary

Witten, Gubser, Klebanov,Polyakov

On-shell supergravity action with given boundary sources gives a generating functional for the connected field theory correlators:

$$e^{-I_{SUGRA}|_{\phi_B}} = \langle e^{-\int \phi_B \hat{O}} \rangle \, .$$

This implies that, for the near boundary behavior of the supergravity field,

$$\phi(y,h) = a(y) h^{-\Delta+d} + b(y) h^{\Delta} + \cdots \, .$$

`a(y)' is interpreted as the source term and `b(y)' is the corr esponding expectation value of the operator.

**Note that** $\mathcal{L}_\pm \propto (g_s^\pm)^{-1} = e^{-\tilde{\phi}_\pm}\,.$

**The dilaton has a near boundary expansion as**

$$\phi \sim \phi_\pm \mp \frac{c}{d-1}|\mu \mp \mu_0|^{d-1} + \text{h. o. t}\,.$$

**Using the fact** $h \sim |\mu - \mu_0|/|\cos\tau| = |\mu - \mu_0|\cosh t$

**The near boundary behavior of the scalar can be presented as**

$$\phi \sim \phi_+ - \frac{c}{(d-1)\cosh^{d-1} t} h^{d-1} + \text{h. o. t}\,.$$

**from the view point of the r.h.s. boundary.**

**Then** $\langle \mathcal{L} \rangle = \frac{\tilde{c}}{16\pi G}\frac{1}{\cosh^{d-1} t}$

**The failure is of the geometrical description is of order**

$$e^{-cS_{eq}}$$

**that is the scale of the Poincare recurrence time:**

$$t \sim cS_{eq} l.$$

**Since** $S_{eq} \propto N_c^2 = \lambda/(16\pi^2 g_s^2)$ **,**

**the failure is nonperturbative.**

**This may indicate that the large N_c limit is as some kind of the thermodynamic limit.**