Status of Numerical Relativity Simulations With an Emphasis on Data Analysis Applications

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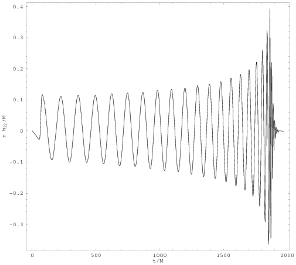
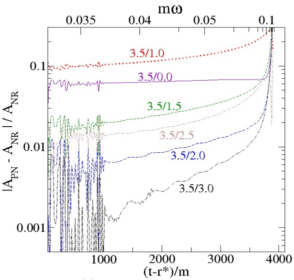
After 40 years of struggle, gravitational wave signals from the inspiral of black hole binaries can now be calculated numerically in full general relativity!
Complete waveforms can be obtained by matching to PN and QNMs.





Outline

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1. Intro: Recent History & Aims
2. Numerical Relativity in a Nutshell

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2. Numerical Relativity in a Nutshell
3. Toward Realistic Initial Data
4. Wave extraction
5. Results
   - equal masses, nonspinning
   - unequal masses, nonspinning
   - Equal mass, spinning
   - unequal mass + spins

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The gold rush . . .

NR plagued by instabilities – credibility was in danger as detectors went online!

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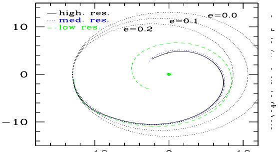

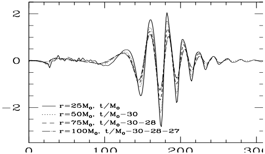

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- Generalized 2nd order harmonic evolution + constraint damping.
- Moving excision + full AMR + high resolution + lots of dissipation.

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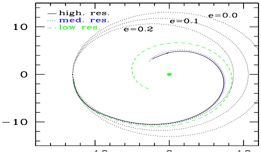

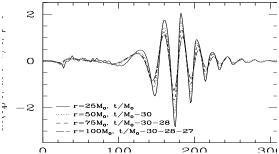

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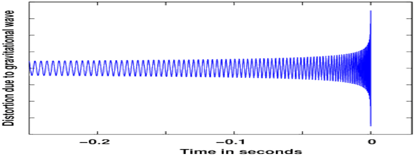
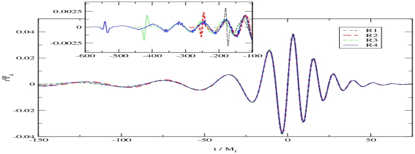
UTB and NASA Goddard, Nov. 2005: BBHs for the masses ($\approx$ 10 groups)!
Simplifying standard BSSN codes do the trick:

- Do not factor out 1/r singularity of the conformal factor, allow gauge to move BH through the grid!
- Interpretation: allow symmetry seeking gauge to find approximate stationary/helical KV, BH interior settles down to simple solution.

Source modeling for GW Data Analysis

Want: accurate signal templates from theoretical modeling!
Here: inspiral of compact objects (BHs)





Astrophysically most relevant case of inspiral: negligible eccentricity.

Parameter space is then 8-dimensional: mass ratio, spins, initial phase – results scale with total mass!

Complications for eccentric inspiral, 3-body interactions.

NR for DA: control "junk radiation" and eccentricity → get good phase accuracy → wise choice of # of orbits → estimate waves at infinity with error estimate.

Challenges: accurate matching to PN to produce "complete waveforms", waveform catalogues, recoil and final spin maps of parameter space, ...

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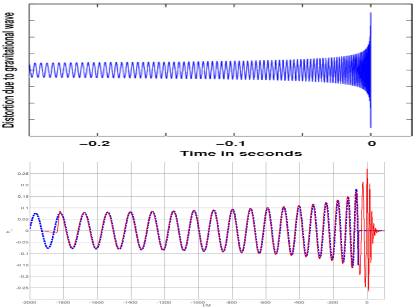

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Solving the Einstein equations numerically

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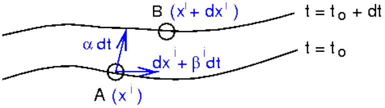
While our systematic understanding of “best practices in NR” is still very limited, current “recipes” for BBH coalescence prove surprisingly robust and accurate.

From Einstein to PDEs: York-ADM and beyond

Choose topology $(S \in R^3) \times I \rightarrow$ choose coordinates $\rightarrow$ PDEs:



$$ds^2 = -\alpha^2 dt^2 + \gamma_{ab}(dx^a + \beta^a dt)(dx^b + \beta^b dt)$$

$$K_{ab} := \frac{1}{2}\mathcal{L}_n \gamma_{ab}$$
$$K_{ab} = \frac{1}{2\alpha}\left(\dot\gamma_{ab} - D_a\beta_b - D_b\beta_a\right)$$

$$\begin{aligned} 8\pi j_a &= D_b K^b{}_a - D_a K^b{}_b, \\ 8\pi\rho &= \frac{1}{2}({}^3R + (K^a{}_a)^2 - K_{ab}K^{ab}). \end{aligned}$$

$$\dot K_{ab} = -D_a D_b\alpha + \beta^c D_c K_{ab} + K_{cb} D_a\beta^c - K_{bc} D_a \beta^c + \alpha({}^3R_{ab} + K_c{}^c K_{ab} + \text{matter}),$$

Bianchi-Id. $(\nabla^a G_{ab} = 0) \Rightarrow$ constraint propagation! (spoilt on discrete level!)

Free evolution schemes dominate – but weakly hyperbolic for ADM (frequency dependent growth) except for certain subclasses of data (e.g. spherical symm.).

Well-posed and numerically stable not enough! Robustness, well-conditioned!
BSSN formulation comes out as the winner so far, "Harmonic" elegant second.

Minimal list of ingredients

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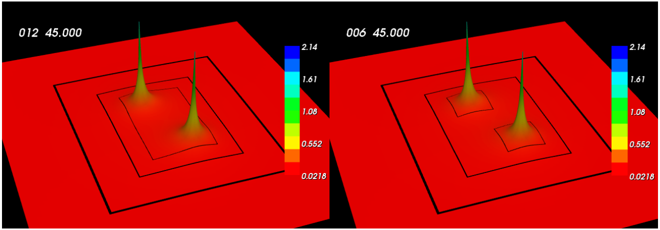
- Code! Community infrastructure or "hack your own"?

Remarks on infrastructure, mesh refinement

Free evolution currently typical: Constraints solved initially (elliptic, pseudo-spectral), later: monitor accuracy, only solve hyperbolic equations.
Inspiraling BHs allow simple mesh refinement strategy: boxes follow BHs:



Box-based mesh refinement resolves different scales: BHs, waves, $1/r$ falloff ...

Cartesian boxes over-resolve wave zone!

Memory scaling $1/\Delta x^3$: qualitative runs on workstation, high accuracy requires supercomputers, parallelized with MPI.

Numerics

vacuum GR: no shocks like in hydro – "smooth" solutions (unless screwed up by gauge) → standard high-order FD (4,6,8...), e.g. spectral, very successful!

@ "puncture" only *continuity* in 1D, direction dependent limits in 3D, but $\alpha = 0$!

Spatial discretization order dominates: use e.g. 4th order Runge-Kutta + spectral.

Complication: mesh refinement – many options to exchange information between grids, what is most efficient? Mathematical theory?

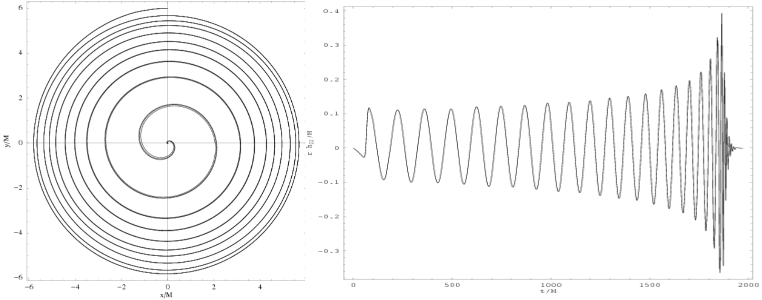
Convergence & Richardson extrapolation: Method consistency + error estimate!

$$\text{second order example:} \quad f(h) = f(0) + \frac{1}{2} f'' h^2 + O(h^m),$$

3 resolutions → consistency with 2nd order convergence, 2 resolutions to extrapolate to $f(0)$ when convergence order is known; analogous for other orders.

h is Δx or extraction radius, or initial separation!

Example: What can we do now with 6th order FD?



Run	h_{min}	h_{max}	procs.	Mem/GByte	M/h
$\chi_{\eta=2}[5 \times \mathbf{56} : 5 \times 112 : 6]$	1/37.3	96/7	12	18. 2	11.6
$\chi_{\eta=2}[5 \times \mathbf{64} : 5 \times 128 : 6]$	1/42.7	12	12	22. 5	8.4
$\chi_{\eta=2}[5 \times \mathbf{72} : 5 \times 144 : 6]$	1/48.0	32/3	16	31. 3	3.5
$\chi_{\eta=2}[5 \times \mathbf{80} : 5 \times 160 : 6]$	1/53.3	48/5	24	45. 4	4.4

Phase Convergence

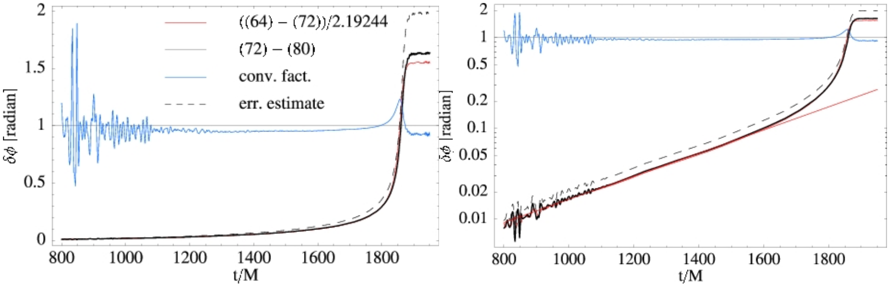


Figure: Convergence test for the gravitational wave phase. Plotted are the difference between the 72 and 80 runs, and the difference between the 64 and 72 runs rescaled for sixth-order convergence. $\delta\varphi \approx 0.0117 \exp 0.003 t/M$.

Wave phase & frequency

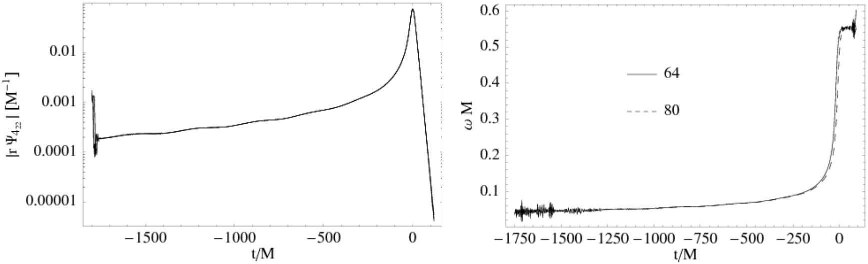


Figure: The $l = 2, m = 2$ mode of the wave signal is split into the absolute value of $\Psi_{4,22}$ (left panel) and the wave frequency ω (right panel). Both panels show the simulations $\{64, 72, 80\}$, aligned in time to coincide at the peak of $|\Psi_4|$.

Amplitude convergence

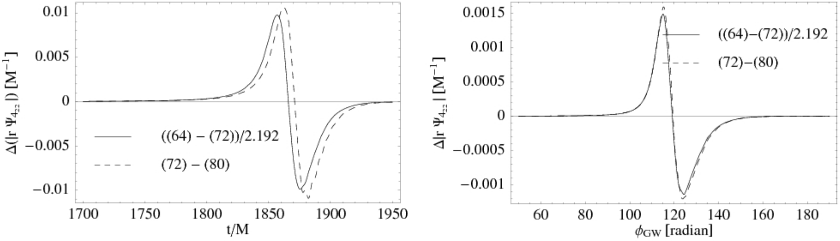


Figure: Convergence plot for the wave amplitude $|\psi_{4_{22}}|$ in the $l = 2, m = 2$ mode. Both panels show the difference between the 72 and 80 runs and differences between the 64 and 72 runs rescaled for sixth-order convergence.

What are those "puncture" initial data?

An application of conformal compactification! "puncture ID": wormhole topology & each **Euclidean** asymptotic end is compactified to a single point at the price of a coordinate singularity.

$K_{ab} = 0 \quad \Rightarrow \quad \Delta\psi = 0 \qquad \psi = 1 + \sum_i \frac{m_i}{2|\vec{r}-\vec{r}_i|}.$

ID for N BHs: add extra ∞'s $\rightarrow$ enforce minimal surfaces $\rightarrow$ trapped surfaces.

Good enough until now: conformal flatness: $\bar{h}_{ab} = \delta_{ab} \rightarrow$ analytic solution of momentum constraint for spinning and boosted BHs ("Bowen-York" K_{ab}).

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Alternative: use cylindrical asymptotic ends – infinitely large slice fits into upper left corner of Penrose diagram, compatible with stationary representation!

Alternative: specify boundary conditions at apparent horizon or trapped surface, evolve using excision or rely on robustness of stationary "trumpet solution".

GW templates: reduce eccentricity!

Eccentricity in typical ID setup ironically due to the use of "quasicircular orbit" initial momenta.

Options to reduce eccentricity: iteration based on short simulations (Caltech, excellent results) or approximate inspiral from large distance.

Want: inspiral from large distance – can do this with PN, then use flexibility of puncture method.

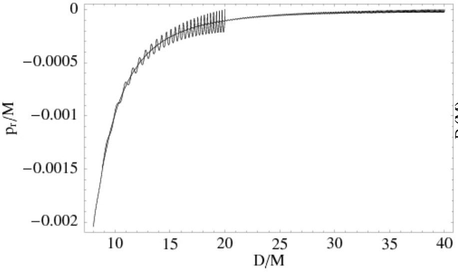
$$\frac{dx^i}{dt} = \frac{\partial H}{\partial p_i}, \tag{1}$$

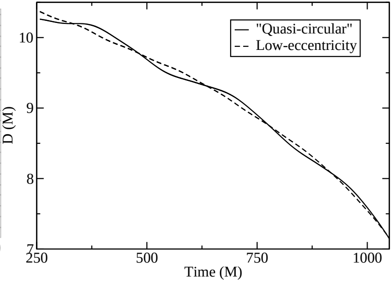
$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial x^i} + F_i, \tag{2}$$

First case: equal masses, no spins: 3 PN accuracy for H *and* 3.5 PN for F_i (orbit averaged)!

Integration with Mathematica takes several seconds for several hundred orbits.

reducing eccentricity: PN method equal masses results





Configuration	P_x/M	P_y/M	e_D	e_ω
QC11	∓ 0.0899395	0	0.012	0.01
E11	∓ 0.0900993	$\mp 7.09412 \times 10^{-4}$	0.002	0.002

See arXiv:0706.0904 [gr-qc] for analytical fits and a table of initial parameters.
Challenge: general case.

Wave extraction

Wave-zone: adopt transverse-traceless (TT) gauge, all the information about the radiative degrees of freedom contained in h_{ij}:

$$h_{ij} = h_+(\mathbf{e}_+)_{ij} + h_\times(\mathbf{e}_\times)_{ij}, \tag{3}$$

$$(\mathbf{e}_+)_{ij} = \hat{\iota}_i\hat{\iota}_j - \hat{\phi}_i\hat{\phi}_j\,, \qquad \text{and} \qquad (\mathbf{e}_\times)_{ij} = \hat{\iota}_i\hat{\phi}_j + \hat{\iota}_j\hat{\phi}_i\,. \tag{4}$$

Newman-Penrose scalar method, in wave zone: $\mathrm{h} = h_+ - \mathrm{i}h_\times$ as

$$\mathrm{h} = \lim_{r\to\infty}\int_0^t \mathrm{d}t' \int_0^{t'} \mathrm{d}t''\Psi_4, \qquad \Psi_4 = -R_{\alpha\beta\gamma\delta}n^\alpha\bar{m}^\beta n^\gamma\bar{m}^\delta, \tag{5}$$

Null-tetrad ℓ (in), n (out), m, $\bar{m}$,

$$-\ell\cdot n = 1 = m\cdot\bar{m}, \tag{6}$$

Spin-weight -2 fields represent symmetric trace-free tensor fields on a sphere (in our case $R_{\alpha\beta\gamma\delta}n^\alpha n^\gamma$) in terms of a complex scalar field. Freedom in the choice of tetrad used in defining Ψ_4!

Spherical harmonics decomposition

Project onto spin-weighted $s = -2$ spherical harmonics $Y^{-2}_{\ell m}$, e.g.

$$Y^{-2}_{2-2} \equiv \sqrt{\frac{5}{64\pi}}\,(1 - \cos\iota)^2\, e^{-2i\phi}, \quad Y^{-2}_{22} \equiv \sqrt{\frac{5}{64\pi}}\,(1 + \cos\iota)^2\, e^{2i\phi}. \tag{7}$$

$$A_{\ell m} = \langle Y^{-2}_{\ell m}, \Psi_4 \rangle = \int_0^{2\pi} \int_0^{\pi} \Psi_4 \overline{Y^{-2}_{\ell m}} \sin\theta d\theta d\phi \tag{8}$$

Spectral decompositions converge very fast, filter out HF noise!

Orthonormality:

$$\frac{dE}{dt} = \lim_{r\to\infty} \left[\frac{r^2}{16\pi} \sum_{l,m} \left| \int_{-\infty}^{t} A_{\ell m} d\tilde{t} \right|^2 \right]. \tag{9}$$

Amplitude-phase split often very useful:

$$A_{\ell m}(t) = A(t) e^{i\varphi(t)}, \quad \omega(t) = \frac{\partial \varphi(t)}{\partial t}. \tag{10}$$

Radiated energy, linear and angular momentum

Radiated energy and linear & angular momentum:

$$\frac{dE}{dt} = \lim_{r\to\infty}\left[\frac{r^2}{16\pi}\int_\Omega \left|\int_{-\infty}^{t}\Psi_4 d\tilde{t}\right|^2 d\Omega\right], \tag{11}$$

$$\frac{dP_i}{dt} = -\lim_{r\to\infty}\left[\frac{r^2}{16\pi}\int_\Omega \ell_i \left|\int_{-\infty}^{t}\Psi_4 d\tilde{t}\right|^2 d\Omega\right], \tag{12}$$

$$\begin{aligned}\frac{dJ_z}{dt} = & -\lim_{r\to\infty}\left\{\frac{r^2}{16\pi}\mathrm{Re}\left[\int_\Omega \left(\partial_\phi \int_{-\infty}^{t}\Psi_4 d\tilde{t}\right)\right.\right. \\ & \left.\left.\left(\int_{-\infty}^{t}\int_{-\infty}^{\hat{t}}\overline{\Psi_4} d\tilde{t} d\hat{t}\right) d\Omega\right]\right\},\end{aligned} \tag{13}$$

At finite extraction radius we need to perform "double Richardson extrapolation" – in extraction radius and grid spacing.

Total energy, linear and angular momentum

Define surface integrals (ADM type integrals)

$$
\begin{aligned}
E(r) &= \frac{1}{16\pi}\int_{S_r} \sqrt{g} g^{ij} g^{kl} \left(g_{ik,j} - g_{ij,k}\right) dS_l, \quad M_{ADM} = \lim_{r\to\infty} E(r), \\
P_j(r) &= \frac{1}{8\pi}\int_{S_r} \sqrt{g}\left(K^i_j - \delta^i_j K\right) dS_i, \quad P_j = \lim_{r\to\infty} P_j(r), \\
J_j(r) &= \frac{1}{8\pi}\epsilon_{jl}{}^m \int_{S_r} \sqrt{g} x^l \left(K^i_m - K\delta^i_m\right) dS_i, \quad J_j = \lim_{r\to\infty} J_j(r),
\end{aligned}
$$

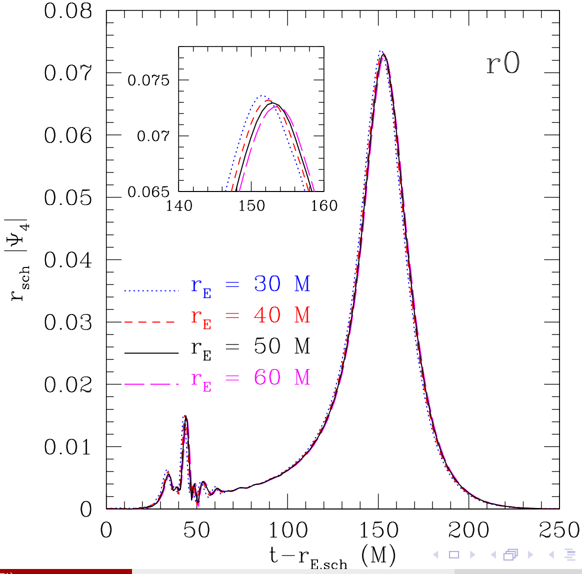
which have to be evaluated in an asymptotically Cartesian coordinate system.

Bondi quantities: take values at fixed retarded time.

(Gauge) problems check: balance laws!!!

AEI: compare ψ_4 with Zerilli wave extraction, "peeling".
LSU: detailed studies of validity and errors of wave extraction algorithms.

Peeling, junk radiation



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  Many ideas, implementation will come with the need!
- Precision work and high spins require better modeling of the physics: beyond conformal flatness, PN matching

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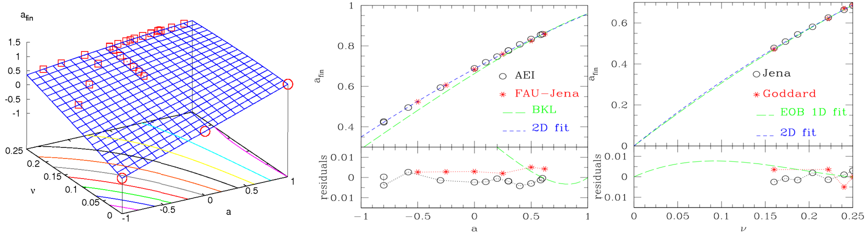
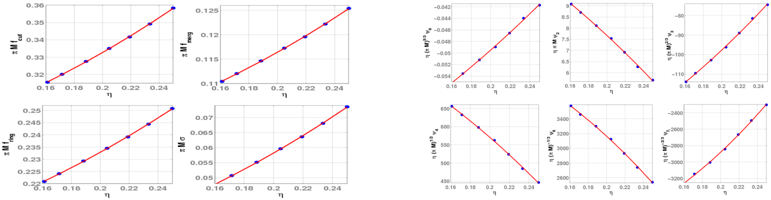
- data analysis implications

Toward efficient exploration of parameter space

Much effort to create fitting formulas for recoil, final spin, waveforms ...



AEI: detailed study of spins orthogonal to orbital plane, 2D fitting formulas (spin + mass-ratio) matched to test-particle predictions.



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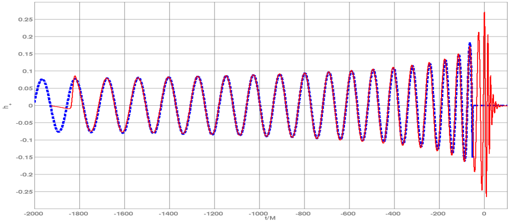
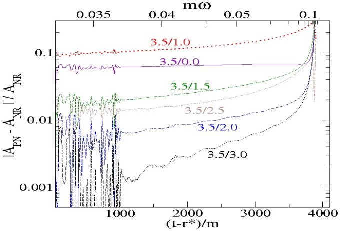
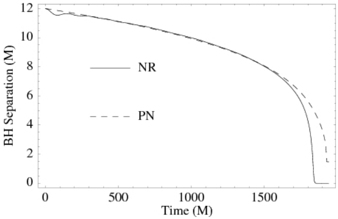
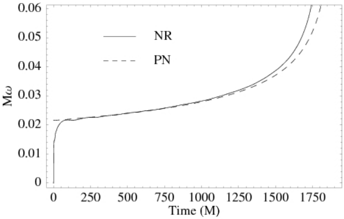
- Matching to PN: "wild" spin configurations?

Equal masses, no spins: precision work and PN comparison









Pioneered by Goddard; Amplitude error: restricted PN ≈ 7%, 2.5 PN ≈ 2%.

Error balance: Caltech 15 orbits evolution

Table: Summary of uncertainties in the comparison between numerical relativity and post-Newtonian expansions. Quoted error estimates ignore the junk-radiation noise at $t \lesssim 1000m$ and apply to times before the numerical waveform reaches gravitational wave frequency $m\omega = 0.1$.

Effect	$\delta\phi$ (radians)	$\delta A/A$
Numerical truncation error	0.003	0.001
Finite outer boundary	0.005	0.002
Extrapolation $r \to \infty$	0.005	0.002
GW extraction at r_{areal}=const?	0.002	10^{-4}
Drift of mass m	0.002	10^{-4}
Coordinate time = proper time?	0.002	10^{-4}
Lapse spherically symmetric?	0.01	4×10^{-4}
residual eccentricity	0.02	0.004
residual spins	0.03	0.001
root-mean-square sum	**0.04**[1]	**0.005**

Symmetry breaking: Gravitational radiation recoil

Unequal masses or spins: asymmetric beaming of radiation causes a recoil of the final black hole.

- Astrophysics: recoil ($>$ 2000 km/s could kick remnant out of even a giant elliptical galaxy.
- Fitchett (1983): maximum kick could be 100's or 1000's of km/s
- post-Newtonian estimates: Maximum between 50 and 500 km/s.
- Data-analysis implications?
- Largest kicks?

  (Hindsight) intuition: Asymmetry breaking in quadrupole radiation – breaking of north/south symmetry results in recoil in z-direction.

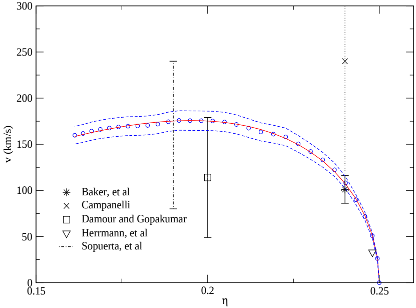
  Requires spins parallel to orbital plane.

  Potentially strong dependence on spin-angle at merger/initial spin angle!

Recoil from unequal-mass nonspinning binaries

Quasicircular inspiral of unequal mass non-spinning BHs is a natural candidate for a first parameter study of inspiral and GWs!

First: PSU, Goddard; Jena: mass ratio up to 1:4 (31 steps), $\approx$ 150000 CPUh.



$\approx$ 2.5 orbits

extraction radius 40 M

Gonzalez et al. report 1:10 at GR18

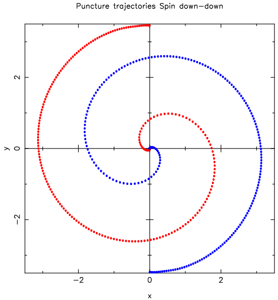
Ajith et al.: phenomenological template family

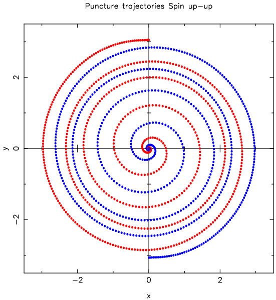
Jena: waveforms $>$ 10 cycles for $M_1/M_2 = 2, 3$.

- full GR: $V_{max} = 175 \pm 11$ km/s for $\eta = \frac{m_1 m_2}{(m_1+m_2)^2} = 0.195 \pm 0.005$.

Spin effects 101

Brownsville results for orbital hangup for (- -) and (+ +) spins with $S = 0.75m^2$.





$\vec{S}_1 = -\vec{S}_2$ dynamics does not change to leading order in spins!

But radiated linear momentum proportional to spin difference!

Precession $\rightarrow$ numerical challenge, interesting multipole structure!

There is a *maximal* spin S/M^2!

A new discovery: huge kicks from spinning binaries

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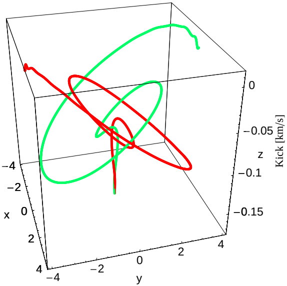
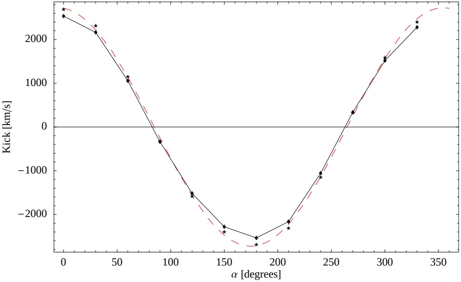
A new discovery: huge kicks from spinning binaries

- Larger kicks should be possible when spin is in the orbital plane (RIT)
- Jena: for initial $S_i/M^2 = 0.8$ find kick of 2500 km/s!
  RIT: extrapolation to extreme Kerr yields $\approx$ 4000 km/s.





- Energy mostly radiated in $l = 2, m = \pm 2$, kick due to $m = \pm 2$ asymmetry.
- Kicks $> 1000 km/s$ possible for other configurations (FAU).
- Main kick contribution occurs *after* standard PN expressions break down.

Brightness asymmetry

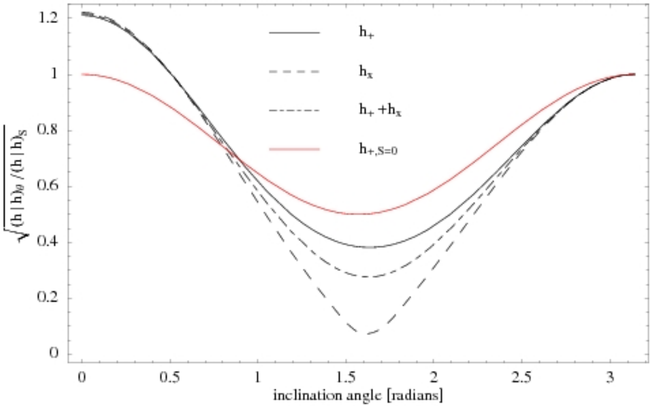


Figure: SNR relative to SNR at south pole plotted as a function of the inclination angle θ for h_+, $h_\times$ and $h_+ - ih_\times$ for the near extremal member of the α-series $\alpha = 0$.

2007: 1-D equal mass families with spin

2006: spins pioneered by UTB (orbital hangup, precession).

Several parameter studies with short waveforms:

RIT, Jena: “Superkick configuration”,

FAU: 8 selected cases at $|S_1|/M_i^2 = 0.8$, confirm large kicks.

PSU: $+-$ configuration: kicks, data analysis: need higher modes for detection!

$S_1 = -S_2$, vary angle; S_1 aligned, vary inclination S_2. $|S_1|/M_i^2 = 0.6$ good agreement of spin dynamics with 2PN.

AEI: parallel to orbital angular momentum, no precession: very detailed study. 3 new misaligned cases to test fitting formula.

Jena: PN comparison for long > 10 orbits waveforms for orbital hangup case, $S/M^2 = \{0.25, 0.5, 0.75, 0.85\}$.

Unequal mass + spins: First steps

RIT: pioneer one case (1:2), early kick result $\approx 500km/s$.

Berti et al.: anti-aligned spins, superkick configuration: multipolar analysis, Schwarzschild remnant.

$\rightarrow$ Emanuele slides in discussion.

Big focus in 2008!

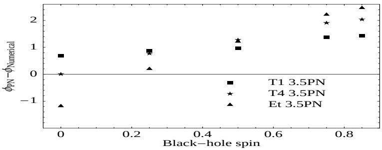
Long waveforms (What is that really?)

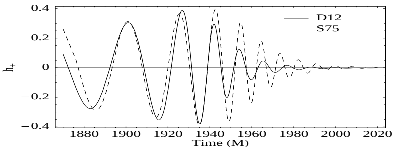
PN matching should work well, initial junk can be neglected, need more input from DA requirements!

- Equal mass, no spins (Goddard, Jena, Caltech, AEI)
- Uneqal mass, no spins: up to 1:3 (Jena).
- Equal mass, aligned spins, up to $S/M^2 = 0.85$ (Jena+AEI).
- Equal mass, eccentricity (PSU).

Expensive, main driver data analysis applications!

2008: Expect lots of results, in particular comparing to PN!





Open problems

- Need to drop conformal flatness assumption in order to reduce junk radiation (in particular for large spins) and gauge dynamics.
- Matching of NR and PN waveforms is in progress – how many orbits are required to answer specific questions to desired accuracy for general situations?
- Dominant error comes from wave extraction/outer boundaries and initial data! Different niches for different numerical codes?
- Significant numerical error also comes from mesh refinement boundaries.
- Establish high-bandwith feedback between NR and DA communities.

Where will we be in 20XX ?

- 2008: digest, increase understanding, longer waveforms become more typical, take larger bites out of spin parameter space.
- Next two years: conformal flatness, standard puncture ID fall!
- Next two years: better boundary conditions become standard.
- Next two years: characteristic extraction becomes standard tool.
- Next 5 years: compactify or do something very clever that is practically as good.
- Next 5 years: Evolutions become cheap!
- Next 5 years: Methods for large mass ratios, ultra-relativistic, ...
- Next 5 years: Alternatives to GR, focus on matter.

Conclusions

- PN looks good and is being improved! Matching to PN inspiral is an intrinsic part of getting the physics right and producing complete waveforms!
- Numerical simulations of strong-field GR *do* give surprising results: superkicks! Much work expected in 2008 on "interesting" configurations!
- The numrel community is ready for large scale parameter studies! Will this be a community effort?
- First application to searches: break inspiral searches with injections!
- Will data analysts, astrophysicists and relativists eventually work together to understand gravitational wave observations on a case-by-case basis?

Great Expectations . . .