Axial-vector current in chiral effective field theory

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October 13, 2016

Outline

- Nuclear forces in chiral EFT
- Axial-vector current in chiral EFT
  - Unitary transformations for currents
  - Modified continuity equation and 4-vector relations
  - Matching to nuclear forces
  - Axial-vector current up to order Q

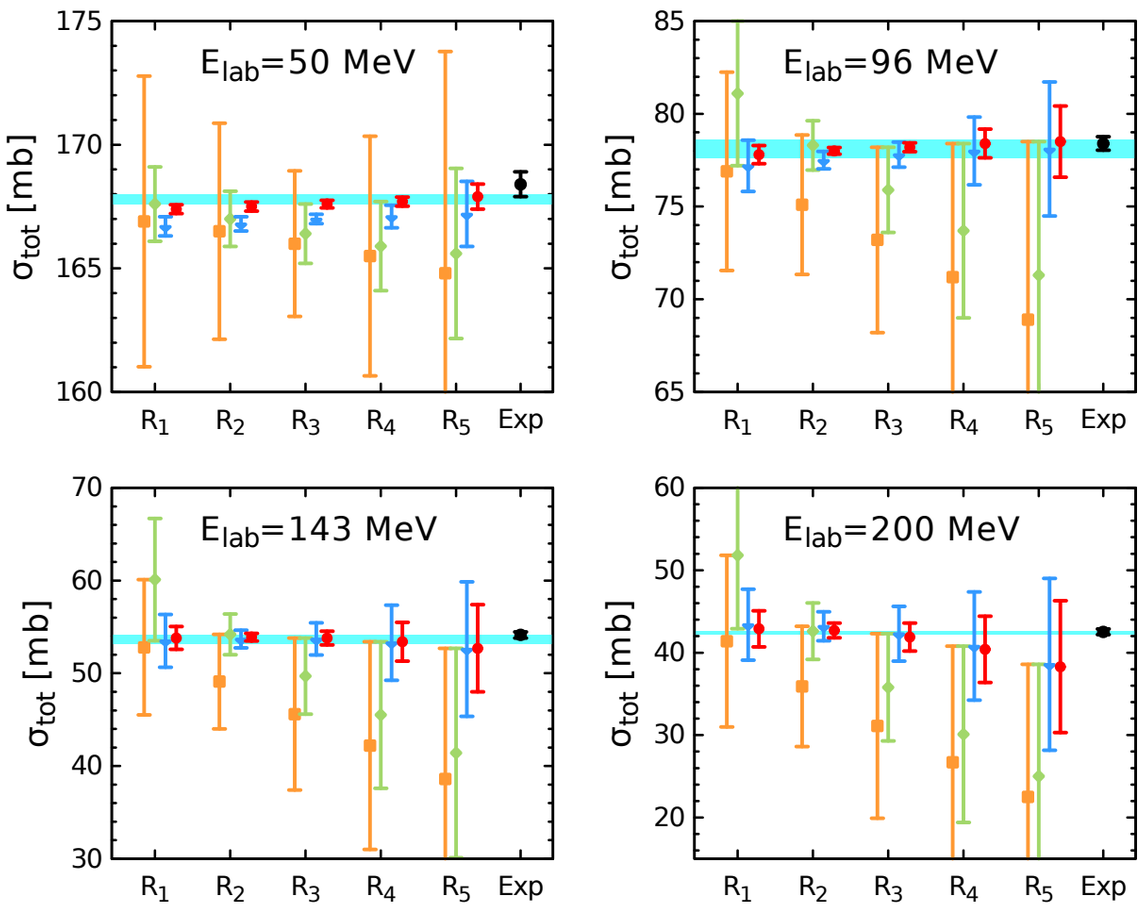
ChPT nuclear forces

	V_{NN}	V_{3N}	V_{4N}
Worked out up to the order	N^4LO	N^3LO N^4LO in progress	N^3LO
Regularization used	Dim. Reg In combination with semi-local regularization in Schrödinger eq.	Dim. Reg.	——

Novelties in NN sector (beside the construction of N^4LO NN)

- Local regularization in coordinate space: $V_{\mathrm{long-range}}(\vec{r}) \to V_{\mathrm{long-range}}(\vec{r}) \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^n$
  - ✓ By construction long - range physics is unaffected by this regulator
  - ✓ No additional SFR is needed
- Theoretical uncertainty estimation due to chiral expansion for every fixed cutoff R

Uncertainty due to chiral expansion



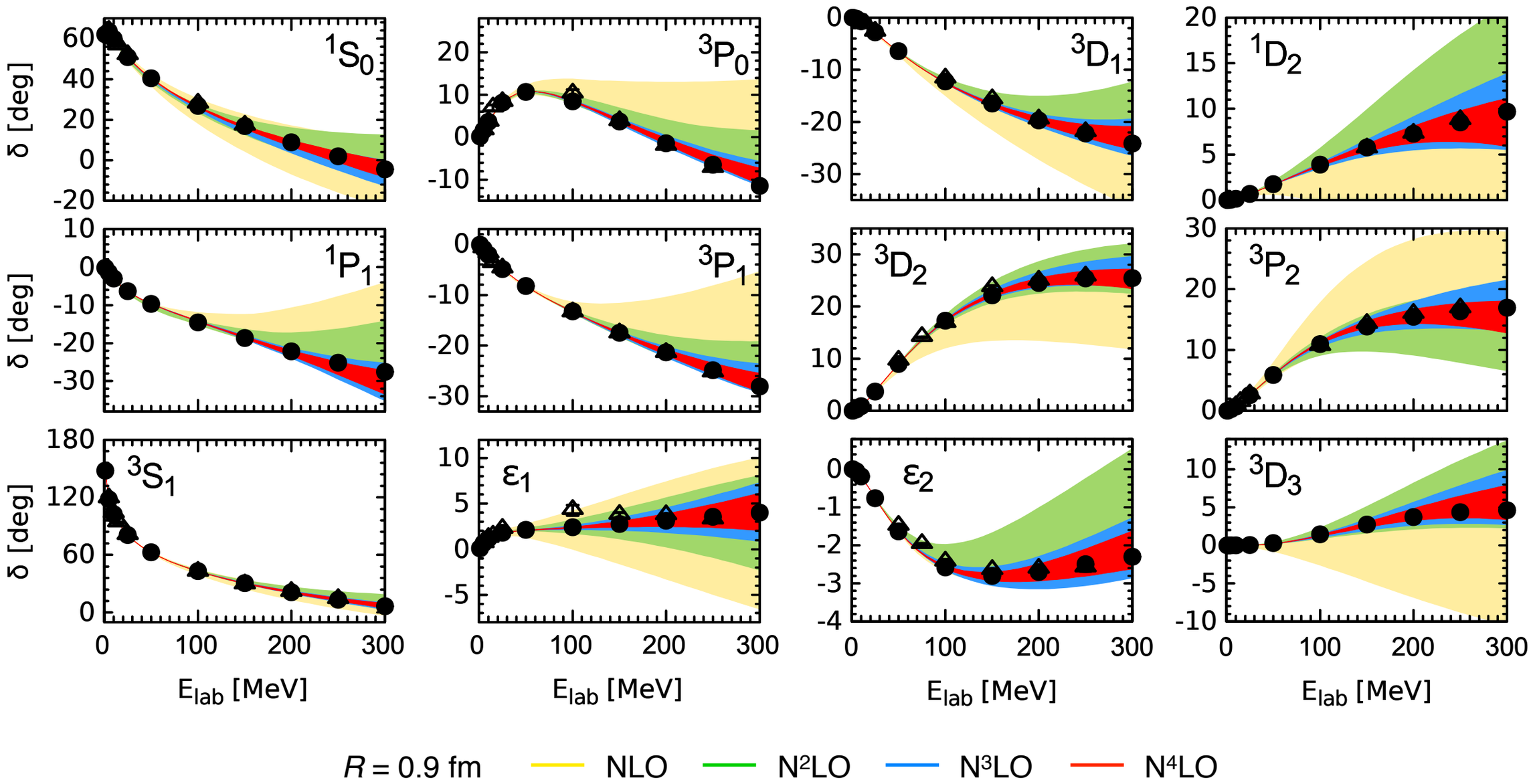
np total cross section at several cutoffs $R_i = (0.7 + i \times 0.1)\,\mathrm{fm}$

- Predictions based on different values of cutoff *R* are consistent with each other
- Results with larger value of *R* are less accurate due to larger cutoff artefacts
- Actual size of N⁴LO corrections is in good agreement with estimated uncertainty at N³LO

- The most accurate results (judging on the size of error bars) are for the cutoffs R_2=0.9 fm and R_3=1.0 fm
- At lowest energy the uncertainty due to cutoff variation of NLO results is underestimated. This pattern changes at higher energies

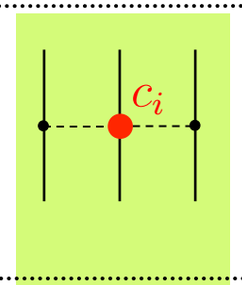
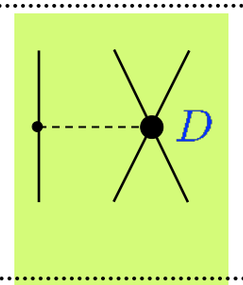
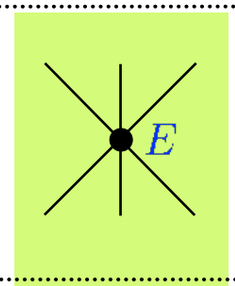
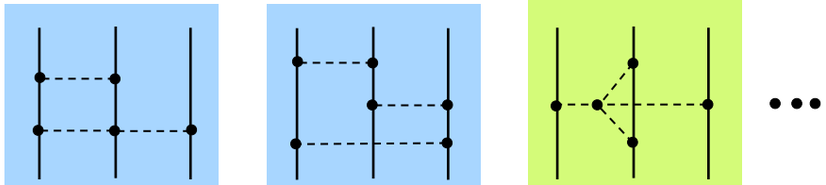
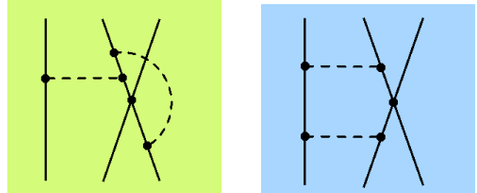
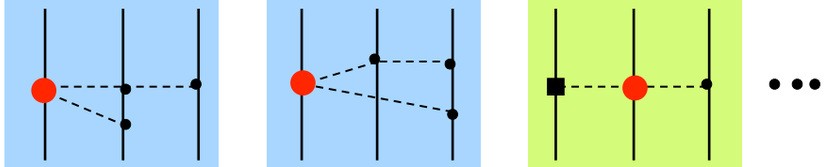
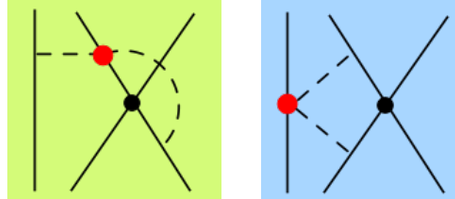
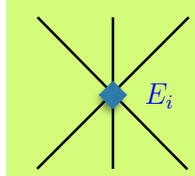
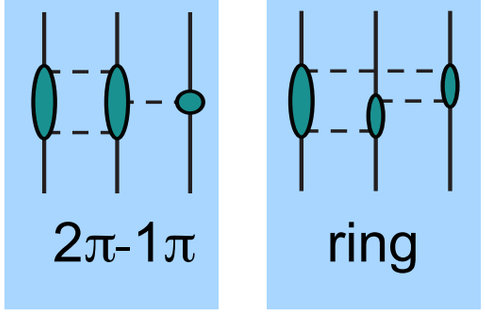
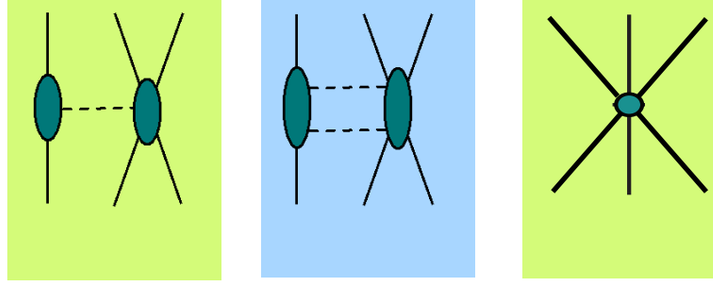
The suggested chiral error estimation is more reliable than the cutoff variation procedure

Phase shifts and mixing angles



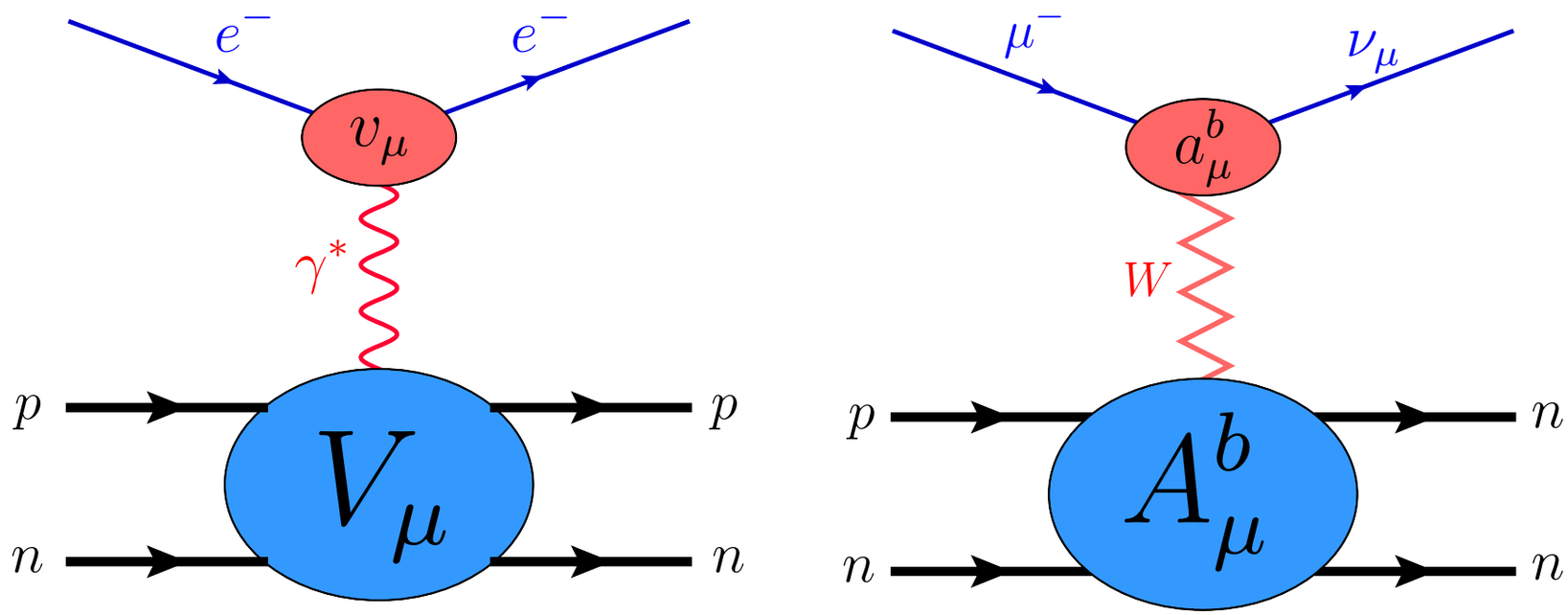
- Good convergence of chiral expansion
- Error bands are consistent with each other → strong support of chiral uncertainty estimation
- Excellent agreement with NPWA data

3NF up to N^{4}LO

	Long - range	Short - range
NLO	—	—
N^{2}LO	 van Kolck '94, Epelbaum et al. '02	 
N^{3}LO	 Ishikawa, Robilotta, PRC76 (07); Bernard, Epelbaum, HK, Meißner, PRC77 (08); PRC84 (11)	 Bernard, Epelbaum, HK, Meißner, PRC84 (11) — ...
N^{4}LO	 HK, Gasparyan, Epelbaum PRC85 (12); PRC87 (13)	 Work in progress  Girlanda, Kievsky, Viviani, PRC84 (11)
	 2π-1π ring	 2π

Nuclear currents in chiral EFT

Electroweak probes on nucleons and nuclei can be described by current formalism



Chiral EFT Hamiltonian depends on external sources

$$H[a, v, s, p]$$

Axial-vector source

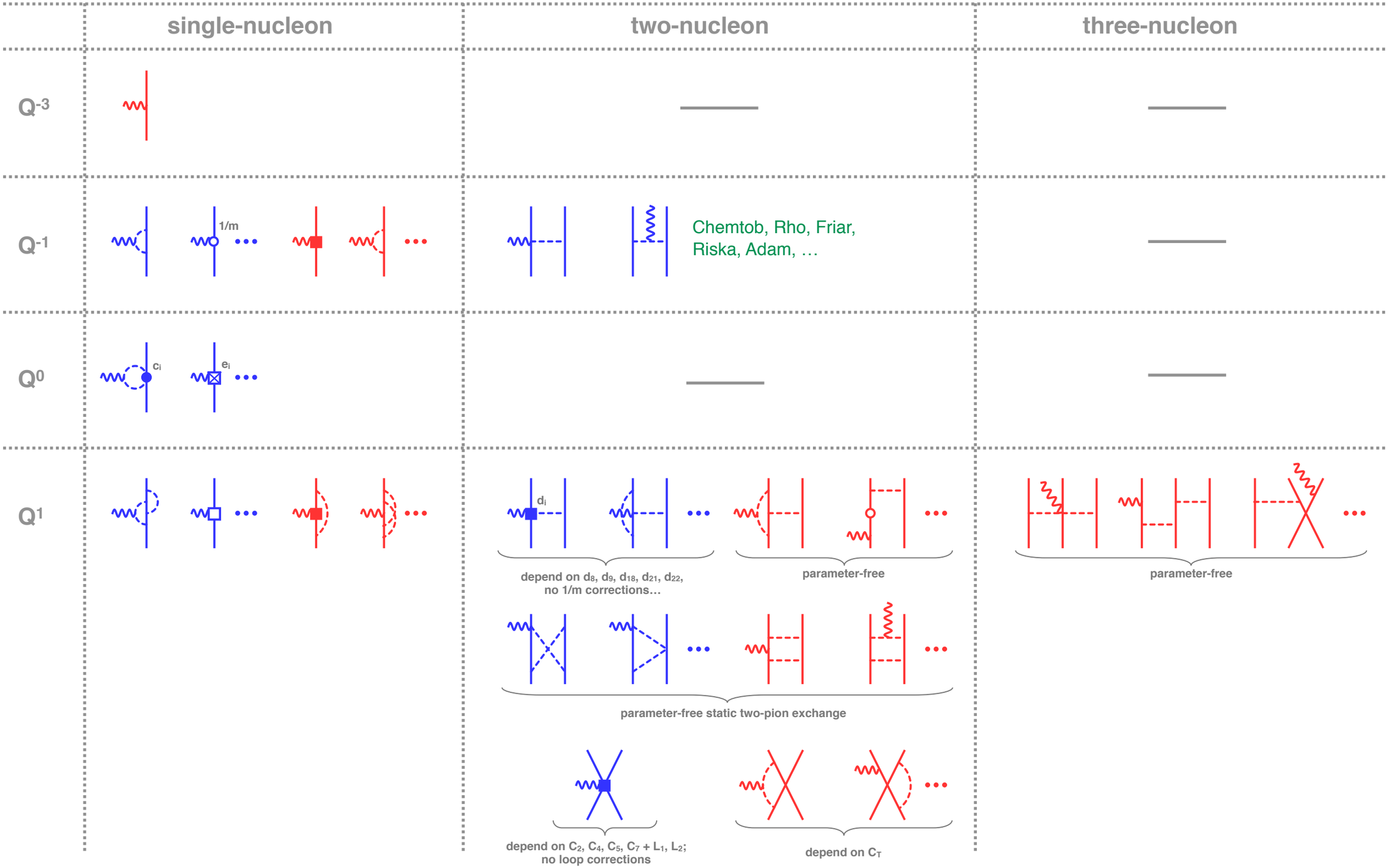
Vector source

Scalar source

Pseudoscalar source

Vector currents in chiral EFT

Chiral expansion of the electromagnetic **current** and **charge** operators



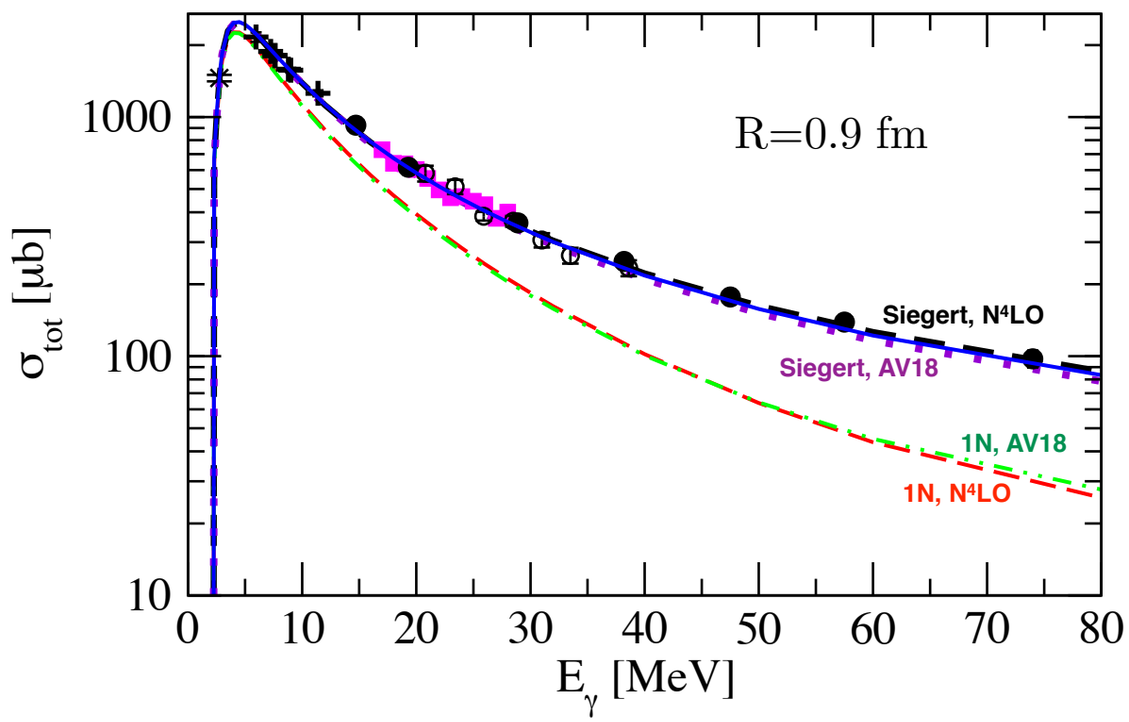
Park, Min, Rho, Kubodera, Song, Lazauskas (earlier works, incomplete, TOPT)
Pastore, Schiavilla et al. (TOPT), Kölling, Epelbaum, HK, Meißner (UT)

Siegert approach + N⁴LO

Skibinski, Golak, Topolniki, Witala, Edelbaum, HK, Kamada, Meißner, Nogga, arXiv:1605.02011

Generate longitudinal component of NN current by continuity equation

$\left[H_{\text{strong}}, \boldsymbol{\rho}\right] = \vec{k} \cdot \vec{\boldsymbol{J}} \leftarrow$ regularized longitudinal current (Siegert approach)

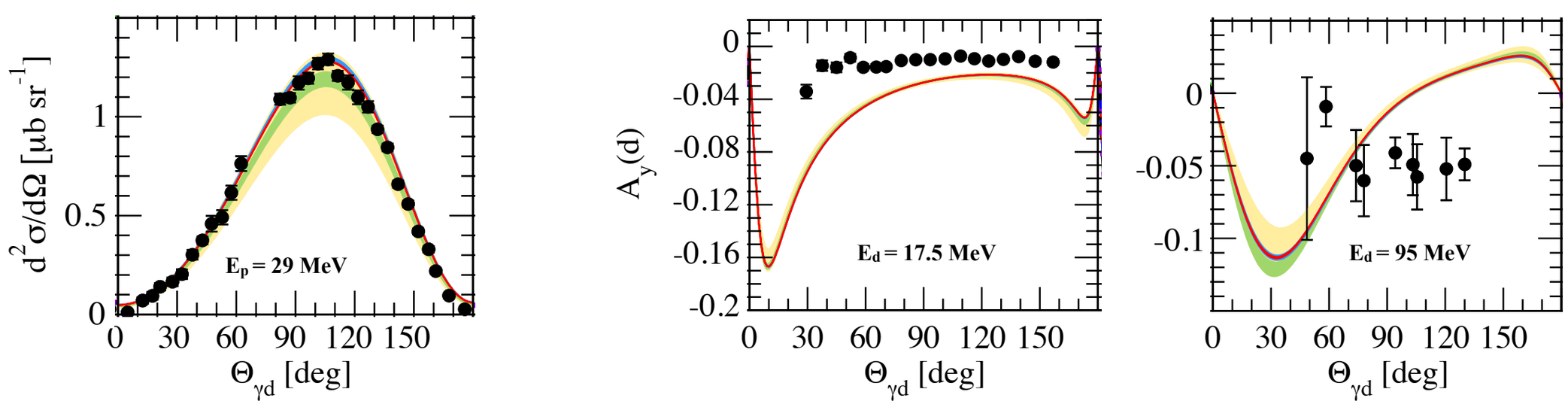


Deuteron photo-disintegration

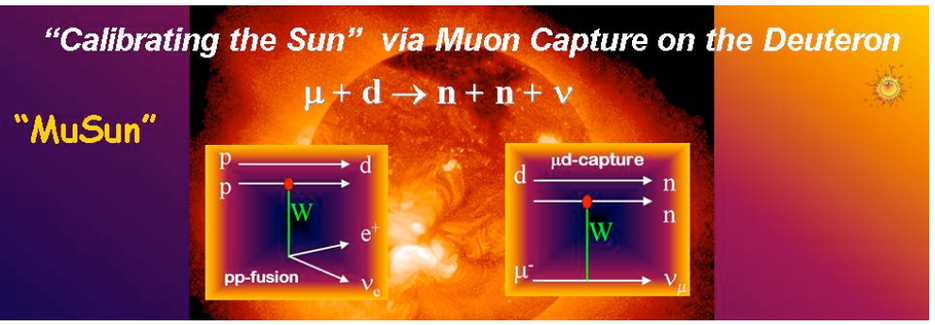
$$\gamma + d \rightarrow p + n$$

- consistent regularization via cont. eq.
- improvement by 1N+Siegert
- implementation of transverse part & exchange currents work in progress

Nucleon-deuteron radiative capture: $p(n) + d \rightarrow {}^3\mathrm{H}({}^3\mathrm{He}) + \gamma$

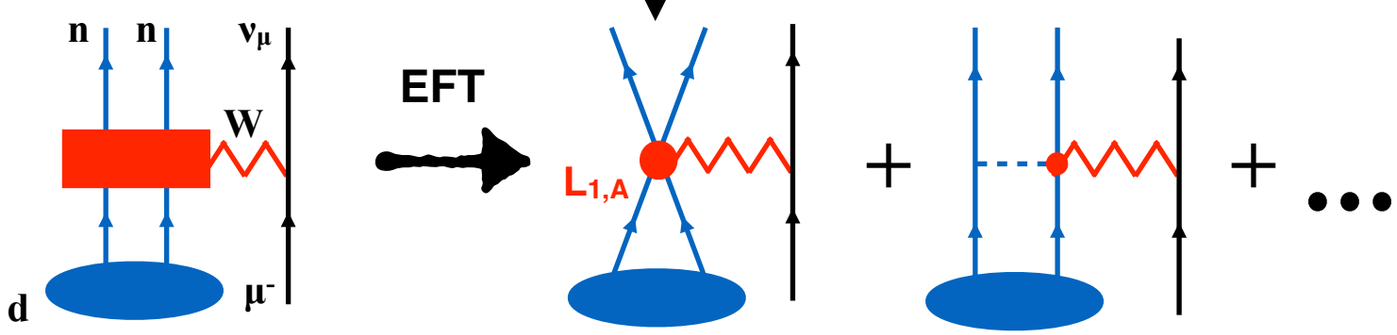


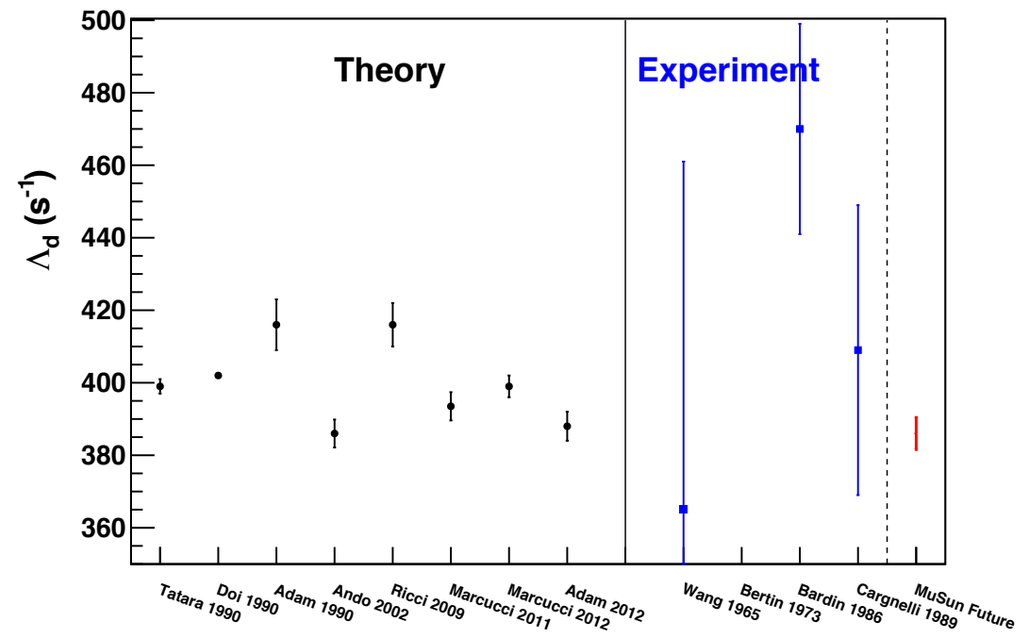
MuSun experiment at PSI



Main goal: measure the doublet capture rate Λ_d in $\mu^- + d \rightarrow \nu_\mu + n + n$ with the accuracy of ~ 1.5%

This will strongly constrain the short-range axial current





The resulting axial exchange current can be used to make precision calculations for

- triton half life, $fT_{1/2} = 1129.6 \pm 3.0$ s, and the muon capture rate on ^{3}He, $\Lambda_0 = 1496 \pm 4\ s^{-1}$ → precision tests of the theory
- weak reactions of astrophysical interest such as e.g. the pp chain of the solar burning:

$$p + p \rightarrow d + e^+ + \nu_e$$
$$p + p + e^- \rightarrow d + \nu_e$$
$$p + {}^3He \rightarrow {}^4He + e^+ + \nu_e$$
$${}^7Be + e^- \rightarrow {}^7Li + \nu_e$$
$${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$$

- $L_{1,A}$ governs the leading 3NF

Historical remarks

- Meson-exchange theory, Skyrme model, phenomenology, …
  Brown, Adam, Mosconi, Ricci, Truhlik, Nakamura, Sato, Ando, Kubidera, Riska, Sauer, Friar, …

- First derivation within chiral EFT to leading 1-loop order using TOPT
  Park, Min, Rho Phys. Rept. 233 (1993) 341; Park et al., Phys. Rev. C67 (2003) 055206

  – only for the threshold kinematics

  – pion-pole diagrams ignored

  – box-type diagrams neglected

  – renormalization incomplete

- Leading one-loop expressions using TOPT including pion-pole terms for general kinematics (still incomplete, e.g. no 1/m corrections)

  Baroni, Girlanda, Pastore, Schiavilla, Viviani, PRC93 (2016) 015501, Erratum: PRC 93 (2016) 049902

  Complete derivation to leading one-loop order using the method of UT

  HK, Epelbaum, Meißner, arXiv:1610.03569

Diagonalization via Okubo

- Decomposition of the Fock space $\mathcal{H}$

Projector operators: $\eta + \lambda = 1$

$$\mathcal{H}_M \xleftarrow{\eta} \mathcal{H} = \mathcal{H}_M \oplus \mathcal{H}_R \xrightarrow{\lambda} \mathcal{H}_R$$

Model-space including only pure nucleon states

Remainder-space including states with at least one pion

$$H|\Psi\rangle = (H_0 + H_I)|\Psi\rangle = E|\Psi\rangle \Longleftrightarrow \begin{pmatrix} \eta\, H\, \eta & \eta\, H\, \lambda \\ \lambda\, H \eta & \lambda\, H \lambda \end{pmatrix} \begin{pmatrix} \eta|\Psi\rangle \\ \lambda|\Psi\rangle \end{pmatrix} = E \begin{pmatrix} \eta|\Psi\rangle \\ \lambda|\Psi\rangle \end{pmatrix}$$

- Block-diagonalization by applying unitary transformation

$$\tilde{H} = U^\dagger H\, U = \begin{pmatrix} \eta\, \tilde{H}\, \eta & 0 \\ 0 & \lambda\, H \lambda \end{pmatrix}$$

$$V_{\text{eff}} = \eta(\tilde{H} - H_0)\eta$$

V_{eff} is E - indep. ➡ important for few-nucleon simulations

Possible parametrization by Okubo '54

$$U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger(1 + AA^\dagger)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + AA^\dagger)^{-1/2} \end{pmatrix}$$

With decoupling eq. $\lambda(H - [A, H] - AHA)\eta = 0$

Can be solved perturbatively within ChPT

Epelbaum, Glöckle, Meißner, ´98

Unitary transformations for currents

- Step 1: $\tilde{H} \to \tilde{H}[a,v,s,p] = U^\dagger H[a,v,s,p] U$

  Okubo transf. or further strong unitary transf. are not enough to renormalize the currents

- Step 2: additional (time-dependendent) unitary transformations

$$i\frac{\partial}{\partial t}\Psi = H\Psi \longrightarrow i\frac{\partial}{\partial t}U(t)U^\dagger(t)\Psi = U(t)i\frac{\partial}{\partial t}U^\dagger(t)\Psi + \left(i\frac{\partial}{\partial t}U(t)\right)U^\dagger(t)\Psi = HU(t)U^\dagger(t)\Psi$$

$$\Psi' = U^\dagger(t)\Psi \longrightarrow i\frac{\partial}{\partial t}\Psi' = \left[U^\dagger(t)HU(t) - U^\dagger(t)\left(i\frac{\partial}{\partial t}U(t)\right)\right]\Psi'$$

Explicit time-dependence through source terms

$$\tilde{H}[a,v,s,p] \to \underbrace{U^\dagger[a,v]\tilde{H}[a,v,s,p]U[a,v] + \left(i\frac{\partial}{\partial t}U^\dagger[a,v]\right)U[a,v]}_{=:\, H_{\text{eff}}[a,\dot{a},v,\dot{v}]}$$

$$A^b_\mu(\vec{x},t) := \frac{\delta}{\delta a^{\mu,b}(\vec{x},t)} H_{\text{eff}}[a,\dot{a},v,\dot{v}]\Big|_{a=v=0}$$

Due to time-derivatives ($\dot{a}$,$\dot{v}$) the currents depend on energy transfer if transformed into momentum space

Chiral symmetry constraints

Chiral symmetry transformations on the path integral level

Gasser, Leutwyler Ann. Phys. (1984) 142: $v_\mu = \frac{1}{2}(r_\mu + l_\mu)$ and $a_\mu = \frac{1}{2}(r_\mu - l_\mu)$

$$\langle 0_{\rm out}|0_{\rm in}\rangle_{a,v,s,p} = \exp\left(i\,Z[a,v,s,p]\right) = \exp\left(i\,Z[a',v',s',p']\right) = \langle 0_{\rm out}|0_{\rm in}\rangle_{a',v',s',p'}$$

$$\begin{aligned} r_\mu &\to r'_\mu = R\,r_\mu R^\dagger + iR\,\partial_\mu R^\dagger\ , \\ l_\mu &\to l'_\mu = L\,l_\mu L^\dagger + iL\,\partial_\mu L^\dagger\ , \\ s + i\,p &\to s' + i\,p' = R(s+i\,p)L^\dagger\ , \\ s - i\,p &\to s' - i\,p' = L(s-i\,p)R^\dagger\ . \end{aligned}$$

Chiral $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ rotation does not change the generating functional $\longrightarrow$ Ward identities

Chiral symmetry transformations on the Hamiltonian level

- There exists a unitary transformation $U(R,L)$ such that from Schrödinger eq.

$$i\frac{\partial}{\partial t}\Psi = H_{\rm eff}[a,v,s,p]\Psi \quad \text{takes the form} \quad i\frac{\partial}{\partial t}U^\dagger(R,L)\Psi = H_{\rm eff}[a',v',s',p']U^\dagger(R,L)\Psi$$

Transformed Hamiltonian is unitary equivalent to the untransformed one

$$H_{\rm eff}[a',\dot a',v',\dot v',s',p'] = U^\dagger(R,L)H_{\rm eff}[a,\dot a,v,\dot v,s,p]U(R,L) + \left(i\frac{\partial}{\partial t}U^\dagger(R,L)\right)U(R,L)$$

Continuity equation

Infinitesimally we have $R = 1 + \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\epsilon}_R(x)$ and $L = 1 + \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\epsilon}_L(x)$

Expressed in $\boldsymbol{\epsilon}_V = \frac{1}{2}(\boldsymbol{\epsilon}_R + \boldsymbol{\epsilon}_L)$ and $\boldsymbol{\epsilon}_A = \frac{1}{2}(\boldsymbol{\epsilon}_R - \boldsymbol{\epsilon}_L)$ we have

$$\begin{aligned} \boldsymbol{v}_\mu &\to \boldsymbol{v}'_\mu = \boldsymbol{v}_\mu + \boldsymbol{v}_\mu \times \boldsymbol{\epsilon}_V + \boldsymbol{a}_\mu \times \boldsymbol{\epsilon}_A + \partial_\mu \boldsymbol{\epsilon}_V \\ \boldsymbol{a}_\mu &\to \boldsymbol{a}'_\mu = \boldsymbol{a}_\mu + \boldsymbol{a}_\mu \times \boldsymbol{\epsilon}_V + \boldsymbol{v}_\mu \times \boldsymbol{\epsilon}_A + \partial_\mu \boldsymbol{\epsilon}_A \end{aligned} \quad \longrightarrow \quad \begin{aligned} \dot{v}_\mu &\to \dot{v}'_\mu = \partial_\mu \dot{\epsilon}_V + \ldots \\ \dot{a}_\mu &\to \dot{a}'_\mu = \partial_\mu \dot{\epsilon}_A + \ldots \end{aligned}$$

$$H_{\text{eff}}[a', \dot{a}', v', \dot{v}', s', p'] = U^\dagger(R,L) H_{\text{eff}}[a, \dot{a}, v, \dot{v}, s, p] U(R,L) + \left(i \frac{\partial}{\partial t} U^\dagger(R,L) \right) U(R,L)$$

- $H_{\text{eff}}[a', \dot{a}', v', \dot{v}', s', p']$ is a function of $\boldsymbol{\epsilon}_V, \dot{\boldsymbol{\epsilon}}_V, \ddot{\boldsymbol{\epsilon}}_V, \boldsymbol{\epsilon}_A, \dot{\boldsymbol{\epsilon}}_A, \ddot{\boldsymbol{\epsilon}}_A$

$$\longrightarrow \; U = \exp\left(i \int d^3x \left[\boldsymbol{R}_0^v(\vec{x}) \cdot \boldsymbol{\epsilon}_V(\vec{x},t) + \boldsymbol{R}_1^v(\vec{x}) \cdot \dot{\boldsymbol{\epsilon}}_V(\vec{x},t) + \boldsymbol{R}_0^a(\vec{x}) \cdot \boldsymbol{\epsilon}_A(\vec{x},t) + \boldsymbol{R}_1^a(\vec{x}) \cdot \dot{\boldsymbol{\epsilon}}_A(\vec{x},t) \right] \right)$$

Expanding both sides in $\vec{\epsilon}_V, \vec{\epsilon}_A$, comparing the coefficients and transforming to momentum space we get the continuity equation

$$\left[H_{\text{strong}}, \boldsymbol{A}_0(\vec{k},0) \overbrace{- \frac{\partial}{\partial k_0} \vec{k} \cdot \vec{\boldsymbol{A}}(\vec{k},k_0) + \frac{\partial}{\partial k_0} \left[H_{\text{strong}}, \boldsymbol{A}_0(\vec{k},k_0) \right] + i \frac{\partial}{\partial k_0} m_q \boldsymbol{P}(\vec{k},k_0)}^{\text{new terms}} \right]$$

$$= \vec{k} \cdot \vec{\boldsymbol{A}}(\vec{k},0) - i\, m_q \boldsymbol{P}(\vec{k},0)$$

Continuity equation

The continuity eq. is invariant under the two discussed classes of unitary transformations

$$\left[H_{\text{strong}}, \boldsymbol{A}_0(\vec{k},0) - \frac{\partial}{\partial k_0}\vec{k}\cdot\vec{\boldsymbol{A}}(\vec{k},k_0) + \frac{\partial}{\partial k_0}\left[H_{\text{strong}}, \boldsymbol{A}_0(\vec{k},k_0)\right] + i\frac{\partial}{\partial k_0} m_q \boldsymbol{P}(\vec{k},k_0)\right]$$

$$= \vec{k}\cdot\vec{\boldsymbol{A}}(\vec{k},0) - i\, m_q \boldsymbol{P}(\vec{k},0)$$

- Class I: time-independent unitary transformations

$$\boldsymbol{A}_\mu(k) \to U^\dagger \boldsymbol{A}_\mu(k) U, \quad \boldsymbol{P}(k) \to U^\dagger \boldsymbol{P}(k) U, \quad H_{\text{strong}} \to U^\dagger H_{\text{strong}} U$$

✓

- Class II: time-dependent unitary transformations $U[a] = \exp\left(i\int d^3x \boldsymbol{Y}_\mu(\vec{x})\cdot \boldsymbol{a}^\mu(\vec{x},x_0)\right)$

$$\boldsymbol{A}_\mu(k) \to \boldsymbol{A}_\mu(k) + i\left[H_{\text{strong}}, \boldsymbol{Y}_\mu(\vec{k})\right] - i\, k_0 \boldsymbol{Y}_\mu(\vec{k})$$

$$\to \begin{cases} \text{lhs of cont. eq.} & \to \quad \text{lhs of cont. eq.} + i\left[H_{\text{strong}}, \vec{k}\cdot\vec{\boldsymbol{Y}}(\vec{k})\right] \\ \text{rhs of cont. eq.} & \to \quad \text{rhs of cont. eq.} + i\left[H_{\text{strong}}, \vec{k}\cdot\vec{\boldsymbol{Y}}(\vec{k})\right] \end{cases}$$

✓

- Class II: time-dependent unitary transformations $U[p] = \exp\left(i\int d^3x \boldsymbol{Z}(\vec{x})\cdot \boldsymbol{p}(\vec{x},x_0)\right)$

$$\boldsymbol{P}(k) \to \boldsymbol{P}(k) + i\left[H_{\text{strong}}, \boldsymbol{Z}(\vec{k})\right] - i\, k_0 \boldsymbol{Z}(\vec{k})$$

$$\to \begin{cases} \text{lhs of cont. eq.} & \to \quad \text{lhs of cont. eq.} + m_q\left[H_{\text{strong}}, \boldsymbol{Z}(\vec{k})\right] \\ \text{rhs of cont. eq.} & \to \quad \text{rhs of cont. eq.} + m_q\left[H_{\text{strong}}, \boldsymbol{Z}(\vec{k})\right] \end{cases}$$

✓

Four-vector constraint

Boost operator Heisenberg picture Lorentz transformation

$$\exp\left(-i\,\vec{e}\cdot\vec{K}\theta\right)\boldsymbol{A}_\mu^H(x)\exp\left(i\,\vec{e}\cdot\vec{K}\theta\right)=\Lambda_\mu{}^\nu(\theta)\boldsymbol{A}_\nu^H\left(\Lambda^{-1}(\theta)x\right)$$

Infinitesimally we get $\Lambda(\theta)x = x + \theta\, x^\perp + \mathcal{O}(\theta^2)$ with $x^\perp = (\vec{e}\cdot\vec{x}, \vec{e}\,x_0)$

Linear expansion of the four-vector relation in θ leads in momentum space to

$$2\pi\delta(E_\alpha - E_\beta - k_0)\langle\alpha|\left[i\,\vec{e}\cdot\vec{K}, \boldsymbol{A}_\mu(k)\right] + \boldsymbol{A}_\mu^\perp(k) - \vec{e}\cdot\vec{\nabla}_k\left[H_{\text{strong}}, \boldsymbol{A}_\mu(k)\right] - \vec{e}\cdot\vec{k}\frac{\partial}{\partial k_0}\boldsymbol{A}_\mu(k)|\beta\rangle = 0$$

$$\rightarrow \quad \left[i\,\vec{e}\cdot\vec{K}, \boldsymbol{A}_\mu(k)\right] + \boldsymbol{A}_\mu^\perp(k) - \vec{e}\cdot\vec{\nabla}_k\left[H_{\text{strong}}, \boldsymbol{A}_\mu(k)\right] - \vec{e}\cdot\vec{k}\frac{\partial}{\partial k_0}\boldsymbol{A}_\mu(k) + i\left[H_{\text{strong}}, \boldsymbol{X}_\mu\right] - ik_0\boldsymbol{X}_\mu = 0$$

$\boldsymbol{X}_\mu$ is an arbitrary operator satisfying $\lim\limits_{k_0\to E_\beta - E_\alpha}(k_0 + E_\alpha - E_\beta)\langle\beta|\boldsymbol{X}_\mu|\alpha\rangle = 0.$

To check the four-vector relation we need to block-diagonalize it via unitary transf.

Poincare algebra gets block-diagonalized by Okubo unitary transformation

Glöckle, Müller, PRC23 (1981) 1183; Krüger, Glöckle nucl-th:9712043 ← *special model*

HK, Epelbaum, Meißner, arXiv:1691217 ← *general proof*

Straightforward calculation of effective boost operator

Four-vector constraint

Four-vector constraint is unaffected by the two discussed classes of unitary transformations

$$\left[i\,\vec{e}\cdot\vec{K},\boldsymbol{A}_\mu(k)\right]+\boldsymbol{A}_\mu^\perp(k)-\vec{e}\cdot\vec{\nabla}_k\left[H_{\rm strong},\boldsymbol{A}_\mu(k)\right]-\vec{e}\cdot\vec{k}\frac{\partial}{\partial k_0}\boldsymbol{A}_\mu(k)+i\left[H_{\rm strong},\boldsymbol{X}_\mu\right]-ik_0\boldsymbol{X}_\mu=0$$

- Class I: time-independent unitary transformations

$$\boldsymbol{A}_\mu(k)\to U^\dagger\boldsymbol{A}_\mu(k)U,\quad \vec{K}\to U^\dagger\vec{K}U,\quad H_{\rm strong}\to U^\dagger H_{\rm strong}U,\quad \boldsymbol{X}_\mu\to U^\dagger\boldsymbol{X}_\mu U$$ ✓

- Class II: time-dependent unitary transformations $U[a]=\exp\left(i\int d^3x\boldsymbol{Y}_\mu(\vec{x})\cdot\boldsymbol{a}^\mu(\vec{x},x_0)\right)$

$$\boldsymbol{A}_\mu(k)\to\boldsymbol{A}_\mu(k)+i\left[H_{\rm strong},\boldsymbol{Y}_\mu(\vec{k})\right]-i\,k_0\boldsymbol{Y}_\mu(\vec{k})$$

$$\boldsymbol{X}_\mu\to\boldsymbol{X}_\mu-i\left[\vec{e}\cdot\vec{K},\boldsymbol{Y}_\mu(\vec{k})\right]+\vec{e}\cdot\vec{\nabla}_k\left[H_{\rm strong},\boldsymbol{Y}_\mu(\vec{k})\right]-\boldsymbol{Y}_\mu^\perp$$

Using the Poincaré algebra relation $\left[\vec{e}\cdot\vec{K},H_{\rm strong}\right]=i\,\vec{e}\cdot\vec{P}$ we can rewrite

$$\left[i\,\vec{e}\cdot\vec{K},i\left[H_{\rm strong},Y_\mu(\vec{k})\right]\right]=i\left[H_{\rm strong},\left[i\,\vec{e}\cdot\vec{K},Y_\mu(\vec{k})\right]\right]-i\,\vec{e}\cdot\vec{k}\,Y_\mu(\vec{k})$$

Multiplying out we realize that the four-vector constraint is unaffected ✓

Four-vector constraint is unitary unambiguous and should be satisfied for any four-current

Unitary ambiguities

34 different unitary transformations are possible at the order Q

$$U_i(a) = \exp\left(S_i^{\rm ax} - h.c.\right)$$

$$S_1^{ax} = \alpha_1^{ax}\eta A_{2,0}^{(0)}\eta H_{2,1}^{(1)}\lambda^1\frac{1}{E_\pi^3}H_{2,1}^{(1)}\eta,$$

$$S_2^{ax} = \alpha_2^{ax}\eta H_{2,1}^{(1)}\lambda^1\frac{1}{E_\pi^2}A_{2,0}^{(0)}\lambda^1\frac{1}{E_\pi}H_{2,1}^{(1)}\eta$$

$$\ldots$$

Vertices without axial source are denoted by $H_{n,p}^{(\kappa)}$

Vertices with one axial source are denoted by $A_{n,p}^{(\kappa)}$

n — number of nucleons

p — number of pions

a — number of axial sources

$\kappa = d + \frac{3}{2}n + p + a - 4$ ← inverse mass dimension

High unitary ambiguity is related to appearance of the axial-vector-one-pion interaction $A_{0,1}^{(-1)}$ (30 out of 34 transformations depend on it)

Reasonable constraints come from

- Perturbative renormalizability of the current

$$l_i = l_i^r(\mu) + \gamma_i\lambda =: \frac{1}{16\pi^2}\bar{l}_i + \gamma_i\lambda + \frac{\gamma_i}{16\pi^2}\ln\left(\frac{M_\pi}{\mu}\right),$$

$$d_i = d_i^r(\mu) + \frac{\beta_i}{F^2}\lambda =: \bar{d}_i + \frac{\beta_i}{F^2}\lambda + \frac{\beta_i}{16\pi^2F^2}\ln\left(\frac{M_\pi}{\mu}\right)$$

$$\gamma_3 = -\frac{1}{2},$$

$$\gamma_4 = 2,$$

$$\beta_2 = -2\beta_5 = \frac{1}{2}\beta_6 = -\frac{1}{12}(1+5g_A^2),$$

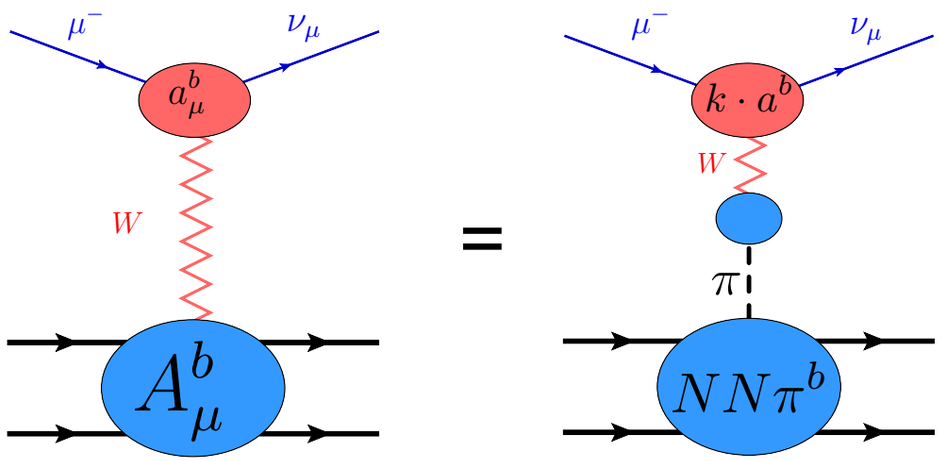
$$\beta_{15} = \beta_{18} = \beta_{22} = \beta_{23} = 0,$$

$$\beta_{16} = \frac{1}{2}g_A + g_A^3.$$

After renormalizing LECs l_i from $\mathcal{L}_\pi^{(4)}$ and d_i from $\mathcal{L}_{\pi N}^{(3)}$ and using well known β- and γ-functions (*Gasser et al. Eur. Phys. J. C26 (2002), 13*) we require the current to be finite

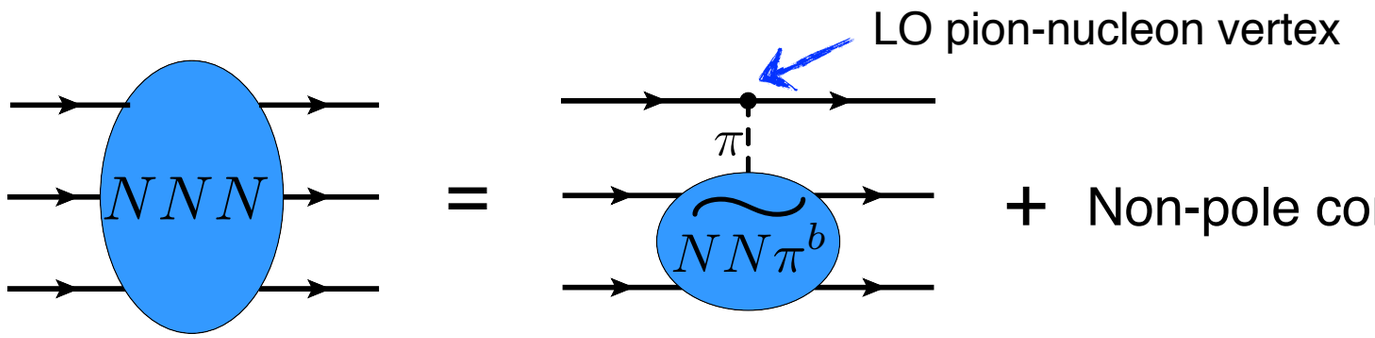
Matching to nuclear forces

Dominance of the pion production operator at the pion-pole (axial-vector current)



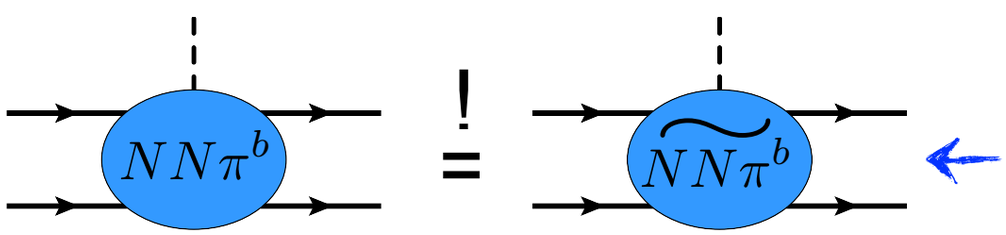
+ Non-pole contributions

Dominance of the pion production operator at the pion-pole (three-nucleon force)



+ Non-pole contributions

Consistent regularization of nuclear forces and currents calls for matching requirement between pion-production operators in different processes



Matching requirement is fulfilled only for particular choice of unitary phases

After renormalizability and matching requirement there are no further unitary ambiguities!

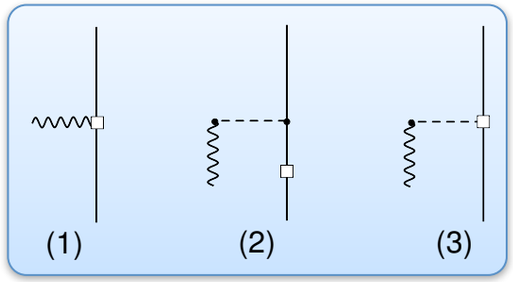
Single nucleon current up to order Q

Up to 1/m - corrections one can parametrize axial-vector current by form factors

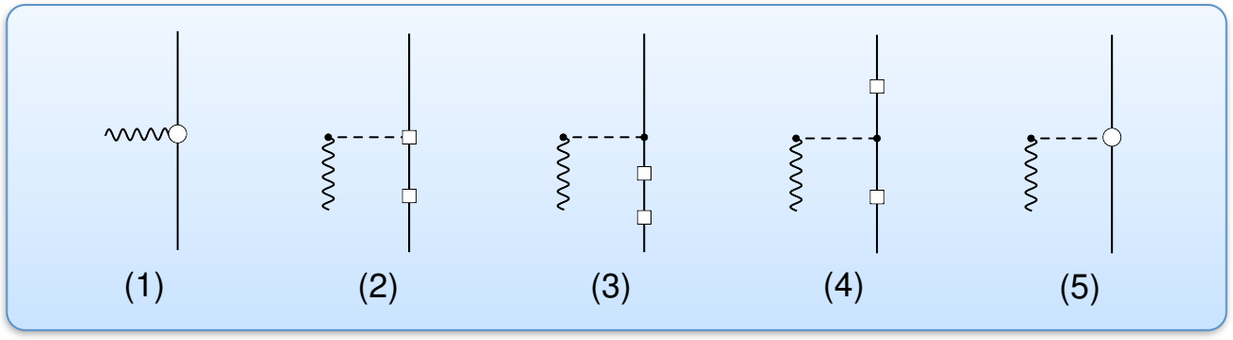
$$
\begin{aligned}
A^{0,a}_{\rm 1N} &= -\frac{G_A(-k^2)}{2m}\tau^a_i \vec{k}_i\cdot\vec{\sigma}_i + \frac{G_P(-k^2)}{8m^2}\tau^a_i k_0\,\vec{k}\cdot\vec{\sigma}_i\,,\\
\vec{A}^a_{\rm 1N} &= -\frac{G_A(-k^2)}{2}\tau^a_i\vec{\sigma}_i + \frac{G_P(-k^2)}{8m^2}\tau^a_i\vec{k}\,\vec{k}\cdot\vec{\sigma}_i + \vec{A}^{a\,(Q)}_{\rm 1N:\,1/m,UT'} + \vec{A}^{a\,(Q)}_{\rm 1N:\,1/m^2}
\end{aligned}
$$

- Axial and pseudoscalar formfactors are known up to two-loop order: *Kaiser PRC67 (2003) 027002*

$$
\vec{A}^{a\,(Q)}_{\rm 1N:\,1/m,UT'} = -\frac{g_A k_0}{8m}\frac{\vec{k}}{k^2+M_\pi^2}\tau^a_i\left(2(1+2\,\bar\beta_9)\vec{\sigma}_i\cdot\vec{k}_i - (1+2\,\bar\beta_8)\vec{k}\cdot\vec{\sigma}_i\frac{p_i'^{\,2}-p_i^2}{k^2+M_\pi^2}\right)
$$

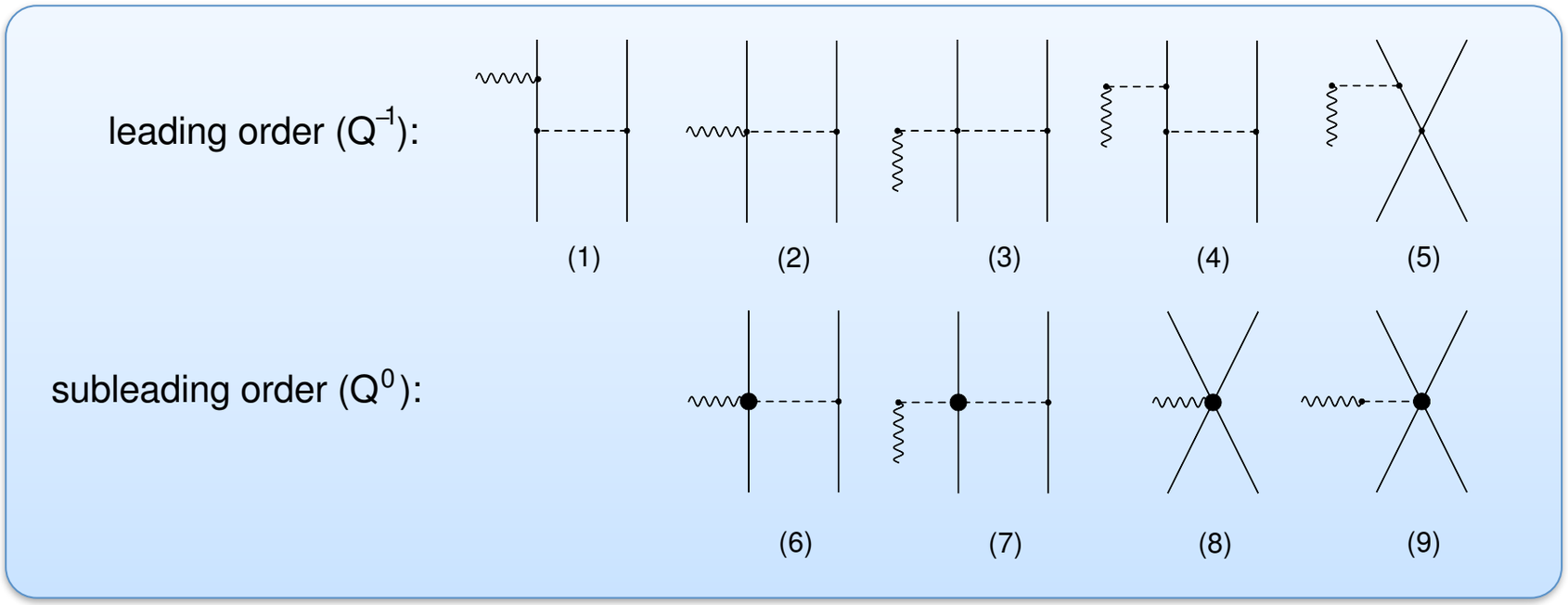


$k_0/m \sim Q^4/\Lambda_b^4$ due to adopted counting for 1/m-corrections



$$
\begin{aligned}
\vec{A}^{a\,(Q)}_{\rm 1N:\,1/m^2} &= \frac{g_A}{16m^2}\tau^a_i\left(\vec{k}\,\vec{k}\cdot\vec{\sigma}_i(1-2\bar\beta_8)\frac{(p_i'^{\,2}-p_i^2)^2}{(k^2+M_\pi^2)^2} - 2\vec{k}\frac{(p_i'^{\,2}+p_i^2)\vec{k}\cdot\vec{\sigma}_i - 2\bar\beta_9(p_i'^{\,2}-p_i^2)\vec{k}_i\cdot\vec{\sigma}_i}{k^2+M_\pi^2}\right.\\
&\left. +\ 2\,i\,[\vec{k}\times\vec{k}_i] + \vec{k}\,\vec{k}\cdot\vec{\sigma}_i - 4\,\vec{k}_i\,\vec{k}_i\cdot\vec{\sigma}_i + \vec{\sigma}_i\Big(2(p_i'^{\,2}+p_i^2)-k^2\Big)\right).
\end{aligned}
$$

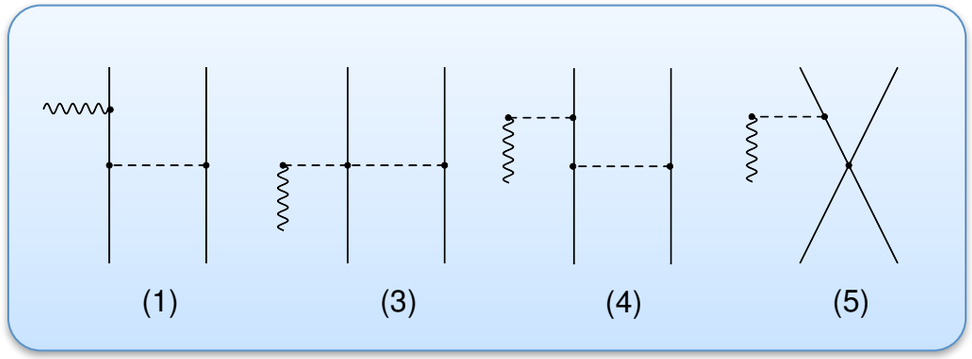
NN current at order Q^{-1} & Q^0



Well known results for axial NN current at Q^{-1} and Q^0 - order

$$
\begin{aligned}
A^{0,a\,(Q^{-1})}_{\text{2N: }1\pi} &= -\frac{i g_A \vec{q}_1 \cdot \vec{\sigma}_1 [\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]^a}{4F_\pi^2\,(q_1^2 + M_\pi^2)} \;+\; 1 \leftrightarrow 2\,, \\
\vec{A}^{a\,(Q^{-1})}_{\text{2N: }1\pi} &= 0, \\
\vec{A}^{a\,(Q^{0})}_{\text{2N: }1\pi} &= \frac{g_A}{2F_\pi^2}\frac{\vec{\sigma}_1 \cdot \vec{q}_1}{q_1^2 + M_\pi^2}\Bigg\{\tau_1^a\Bigg[-4c_1 M_\pi^2 \frac{\vec{k}}{k^2 + M_\pi^2} + 2c_3\left(\vec{q}_1 - \frac{\vec{k}\,\vec{k}\cdot\vec{q}_1}{k^2 + M_\pi^2}\right)\Bigg] + c_4[\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]^a\left(\vec{q}_1 \times \vec{\sigma}_2 - \frac{\vec{k}\,\vec{k}\cdot\vec{q}_1 \times \vec{\sigma}_2}{k^2 + M_\pi^2}\right) \\
&\quad - \frac{\kappa_v}{4m}[\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]^a \vec{k} \times \vec{\sigma}_2\Bigg\} + \;1 \leftrightarrow 2\,, \\
\vec{A}^{a\,(Q^{0})}_{\text{2N: cont}} &= -\frac{1}{4} D\,\tau_1^a\left(\vec{\sigma}_1 - \frac{\vec{k}\,\vec{\sigma}_1 \cdot \vec{k}}{k^2 + M_\pi^2}\right) + \;1 \leftrightarrow 2\,,
\end{aligned}
$$

NN current at order Q



Tree-level diagrams contribute to energy-transfer dependent contributions

$$A^{0,a\ (Q)}_{\mathrm{2N:\,1\pi,UT'}} = 0\,,$$

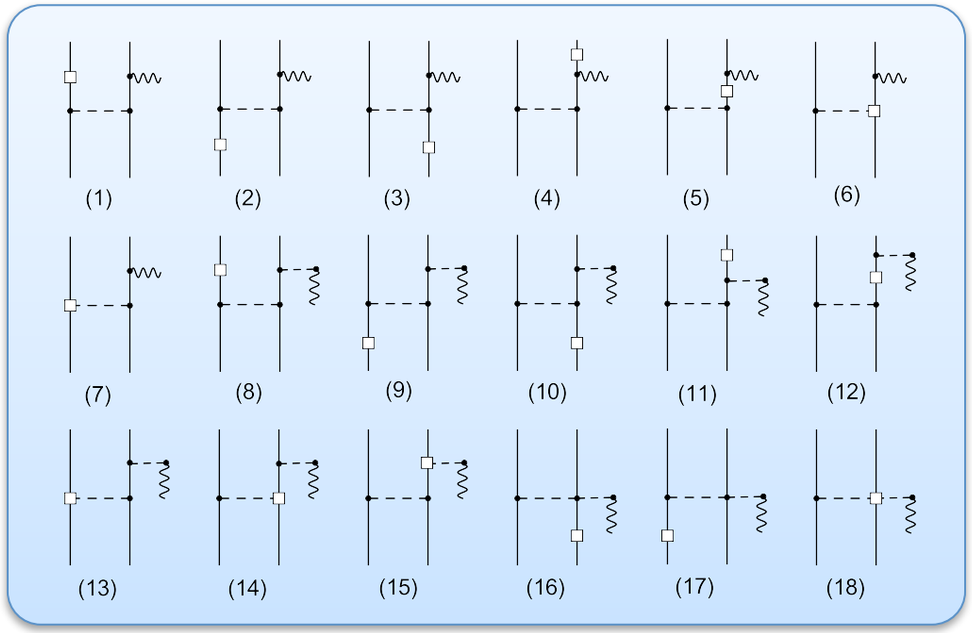
$$\vec{A}^{a\ (Q)}_{\mathrm{2N:\,1\pi,UT'}} = -i\frac{g_A}{8F_\pi^2}\frac{k_0\,\vec{k}\,\vec{q}_1\cdot\vec{\sigma}_1}{(k^2+M_\pi^2)(q_1^2+M_\pi^2)}\left([\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\left(1-\frac{2g_A^2\vec{k}\cdot\vec{q}_1}{k^2+M_\pi^2}\right)-\frac{2g_A^2\tau_1^a\vec{k}\cdot[\vec{q}_1\times\vec{\sigma}_2]}{k^2+M_\pi^2}\right)+\ 1\leftrightarrow 2\,.$$

$$A^{0,a\ (Q)}_{\mathrm{2N:\,cont,\,UT'}} = 0,$$

$$\vec{A}^{a\ (Q)}_{\mathrm{2N:\,cont,\,UT'}} = -i\,k_0\vec{k}\frac{g_A C_T\vec{k}\cdot\vec{\sigma}_1[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a}{(k^2+M_\pi^2)^2}\ +\ 1\leftrightarrow 2\,.$$

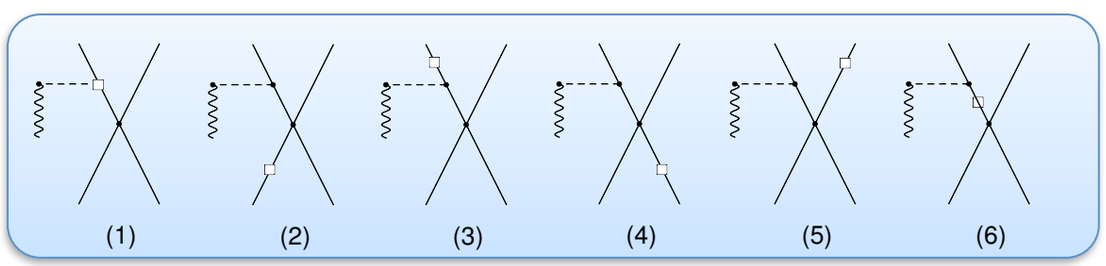
Off-shell effects proportional to energy transfer are important for frame-independent investigations and also for checking the continuity equation and four-current relations

1/m-corrections to axial NN current



$$
\begin{aligned}
\vec{B}_1 &= g_A^2 \vec{q}_1\cdot\vec{\sigma}_1[-2(1+2\bar{\beta}_8)\vec{q}_1\,\vec{k}_1\cdot\vec{q}_1-(1-2\bar{\beta}_8)(2\vec{q}_1\,\vec{k}_2\cdot\vec{q}_1-i\,\vec{q}_1\times\vec{\sigma}_2\,\vec{k}\cdot\vec{q}_1],\\
\vec{B}_2 &= (1-2\bar{\beta}_8)g_A^2\vec{k}\,\vec{k}\cdot\vec{q}_1\vec{q}_1\cdot\vec{\sigma}_1[2\vec{k}\cdot\vec{k}_2-i\vec{k}\cdot\vec{q}_1\times\vec{\sigma}_2],\\
\vec{B}_3 &= 2\vec{k}\Big[-g_A^2((1+2\bar{\beta}_9)\vec{k}\cdot\vec{q}_1\vec{k}_1\cdot\vec{\sigma}_1+(1-2\bar{\beta}_9)\vec{q}_1\cdot\vec{\sigma}_1(\vec{k}\cdot\vec{k}_2+\vec{k}_2\cdot\vec{q}_1))\\
&\quad+\vec{q}_1\cdot\vec{\sigma}_1(\vec{k}\cdot\vec{k}_2+i\,\vec{k}\cdot\vec{q}_1\times\vec{\sigma}_2-\vec{k}_1\cdot\vec{q}_1+\vec{k}_2\cdot\vec{q}_1)\Big],\\
\vec{B}_4 &= g_A^2[2(1+2\bar{\beta}_9)\vec{q}_1\vec{k}_1\cdot\vec{\sigma}_1+(1-2\bar{\beta}_9)\vec{q}_1\cdot\vec{\sigma}_1(2\vec{k}_2-i\vec{k}\times\vec{\sigma}_2)]-2\vec{q}_1\cdot\vec{\sigma}_1(i\vec{q}_1\times\vec{\sigma}_2-i\vec{k}\times\vec{\sigma}_2+2\vec{k}_2),\\
\vec{B}_5 &= g_A^2\vec{q}_1\cdot\vec{\sigma}_1\Big[(1-2\bar{\beta}_8)(\vec{q}_1\,\vec{k}\cdot\vec{q}_1-2i\,\vec{q}_1\times\vec{\sigma}_2\vec{k}_2\cdot\vec{q}_1)-2i(1+2\bar{\beta}_8)\vec{q}_1\times\vec{\sigma}_2\,\vec{k}_1\cdot\vec{q}_1\Big],\\
\vec{B}_6 &= -(1-2\bar{\beta}_8)g_A^2\vec{k}\,\vec{q}_1\cdot\vec{\sigma}_1[(\vec{k}\cdot\vec{q}_1)^2-2i\,\vec{k}\cdot\vec{k}_2\vec{k}\cdot\vec{q}_1\times\vec{\sigma}_2],\\
\vec{B}_7 &= g_A^2\vec{k}\Big[(1-2\bar{\beta}_9)\vec{q}_1\cdot\vec{\sigma}_1(-2i(\vec{k}\cdot\vec{k}_2\times\vec{\sigma}_2+\vec{k}_2\cdot\vec{q}_1\times\vec{\sigma}_2)+k^2+q_1^2)-2i(1+2\bar{\beta}_9)\vec{k}_1\cdot\vec{\sigma}_1\vec{k}\cdot\vec{q}_1\times\vec{\sigma}_2\Big],\\
\vec{B}_8 &= -g_A^2[(1-2\bar{\beta}_9)\vec{q}_1\cdot\vec{\sigma}_1(\vec{k}-2i\,\vec{k}_2\times\vec{\sigma}_2)-2i(1+2\bar{\beta}_9)\vec{q}_1\times\vec{\sigma}_2\,\vec{k}_1\cdot\vec{\sigma}_1].
\end{aligned}
$$

$$
\begin{aligned}
\vec{A}^{a\,(Q)}_{\text{2N: }1\pi,\,1/m} &= \frac{g_A}{16F_\pi^2 m}\Bigg\{i[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\left[\frac{1}{(q_1^2+M_\pi^2)^2}\left(\vec{B}_1-\frac{\vec{k}\,\vec{k}\cdot\vec{B}_1}{k^2+M_\pi^2}\right)+\frac{1}{q_1^2+M_\pi^2}\left(\frac{\vec{B}_2}{(k^2+M_\pi^2)^2}+\frac{\vec{B}_3}{k^2+M_\pi^2}+\vec{B}_4\right)\right]\\
&\quad+\tau_1^a\left[\frac{1}{(q_1^2+M_\pi^2)^2}\left(\vec{B}_5-\frac{\vec{k}\,\vec{k}\cdot\vec{B}_5}{k^2+M_\pi^2}\right)+\frac{1}{q_1^2+M_\pi^2}\left(\frac{\vec{B}_6}{(k^2+M_\pi^2)^2}+\frac{\vec{B}_7}{k^2+M_\pi^2}+\vec{B}_8\right)\right]\Bigg\}+1\leftrightarrow 2
\end{aligned}
$$

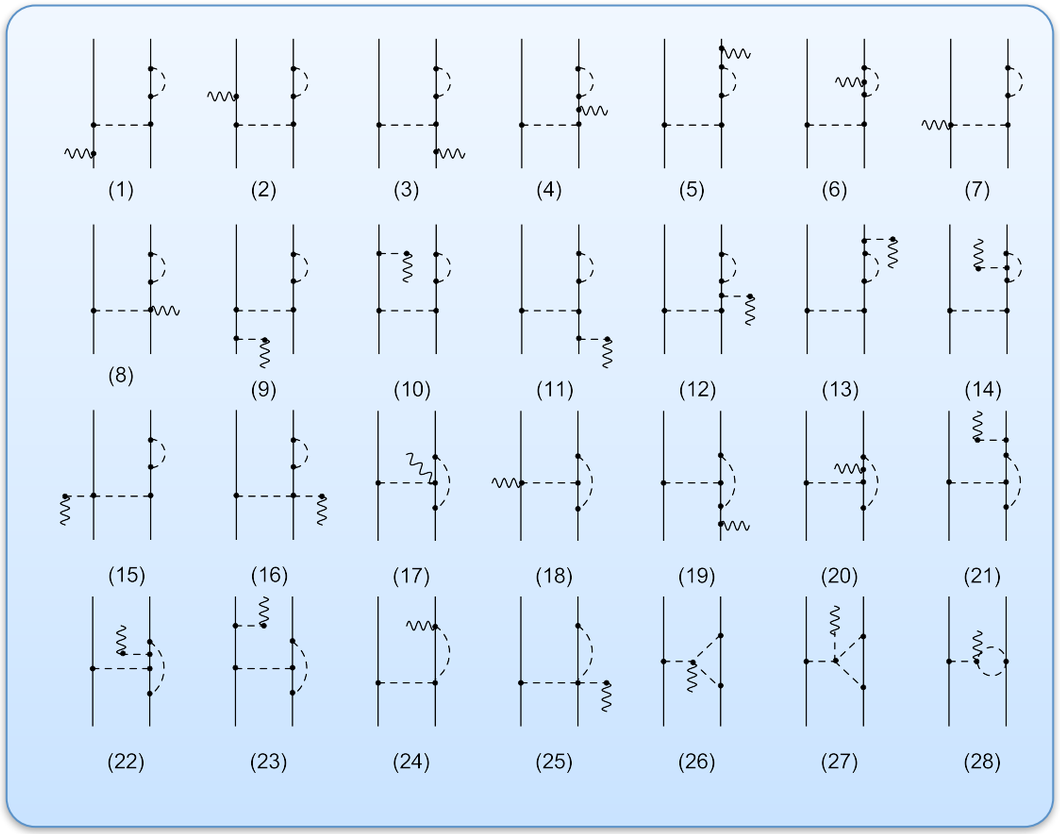


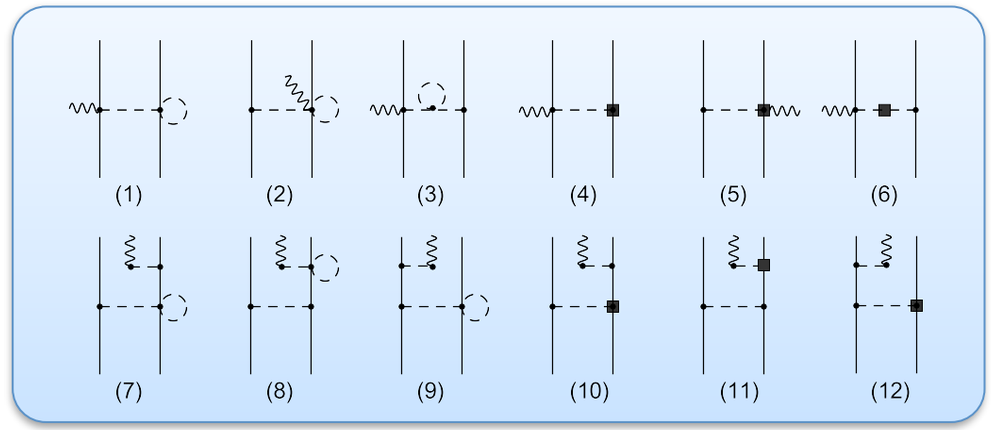
$$
\begin{aligned}
\vec{A}^{a\,(Q)}_{\text{2N: cont, }1/m} &= -\frac{g_A}{4m}\frac{\vec{k}}{k^2+M_\pi^2}\tau_1^a\Big\{(1-2\bar{\beta}_9)\Big(C_S\vec{q}_2\cdot\vec{\sigma}_1+C_T(\vec{q}_2\cdot\vec{\sigma}_2+2i\,\vec{k}_1\cdot\vec{\sigma}_1\times\vec{\sigma}_2)\Big)\\
&\quad-\frac{1-2\bar{\beta}_8}{k^2+M_\pi^2}\Big(C_S\vec{k}\cdot\vec{q}_2\vec{k}\cdot\vec{\sigma}_1+C_T(\vec{k}\cdot\vec{q}_2\vec{k}\cdot\vec{\sigma}_2+2i\,\vec{k}\cdot\vec{k}_1\vec{k}\cdot\vec{\sigma}_1\times\vec{\sigma}_2)\Big)\Big\}+1\leftrightarrow 2\,.
\end{aligned}
$$

No relativistic corrections to the axial NN charge

NN current at order Q

One-pion exchange contributions match to 2π − exchange 3NF at N³LO





$$
\begin{aligned}
h_1(q_2) &= -\frac{g_A^6 M_\pi}{128\pi F_\pi^6}, \\
h_2(q_2) &= \frac{g_A^4 M_\pi}{256\pi F_\pi^6} + \frac{g_A^4 A(q_2)\left(4M_\pi^2+q_2^2\right)}{256\pi F_\pi^6}, \\
h_3(q_2) &= \frac{g_A^4\left(g_A^2+1\right)M_\pi}{128\pi F_\pi^6} + \frac{g_A^4 A(q_2)\left(2M_\pi^2+q_2^2\right)}{128\pi F_\pi^6}, \\
h_4(q_2) &= \frac{g_A^4}{256\pi F_\pi^6}\left(A(q_2)\left(2M_\pi^4+5M_\pi^2 q_2^2+2q_2^4\right)+\left(4g_A^2+1\right)M_\pi^3+2\left(g_A^2+1\right)M_\pi q_2^2\right), \\
h_5(q_2) &= -\frac{g_A^4}{256\pi F_\pi^6}\left(A(q_2)\left(4M_\pi^2+q_2^2\right)+\left(2g_A^2+1\right)M_\pi\right), \\
h_6(q_2) &= \frac{g_A^2\left(3\left(64+128g_A^2\right)M_\pi^2+8\left(19g_A^2+5\right)q_2^2\right)}{36864\pi^2 F_\pi^6} - \frac{g_A^2}{768\pi^2 F_\pi^6}L(q_2)\left(\left(8g_A^2+4\right)M_\pi^2+\left(5g_A^2+1\right)q_2^2\right) \\
&\quad + \frac{\bar{d}_{18} g_A M_\pi^2}{8F_\pi^4} - \frac{g_A^2(2\bar{d}_2+\bar{d}_6)\left(M_\pi^2+q_2^2\right)}{16F_\pi^4} - \frac{\bar{d}_5 g_A^2 M_\pi^2}{2F_\pi^4}, \\
h_7(q_2) &= \frac{g_A^2(2\bar{d}_2-\bar{d}_6)}{16F_\pi^4}, \\
h_8(q_2) &= -\frac{g_A^2(\bar{d}_{15}-2\bar{d}_{23})}{8F_\pi^4}.
\end{aligned}
$$

- h_i are related to TPE 3NF functions A & B

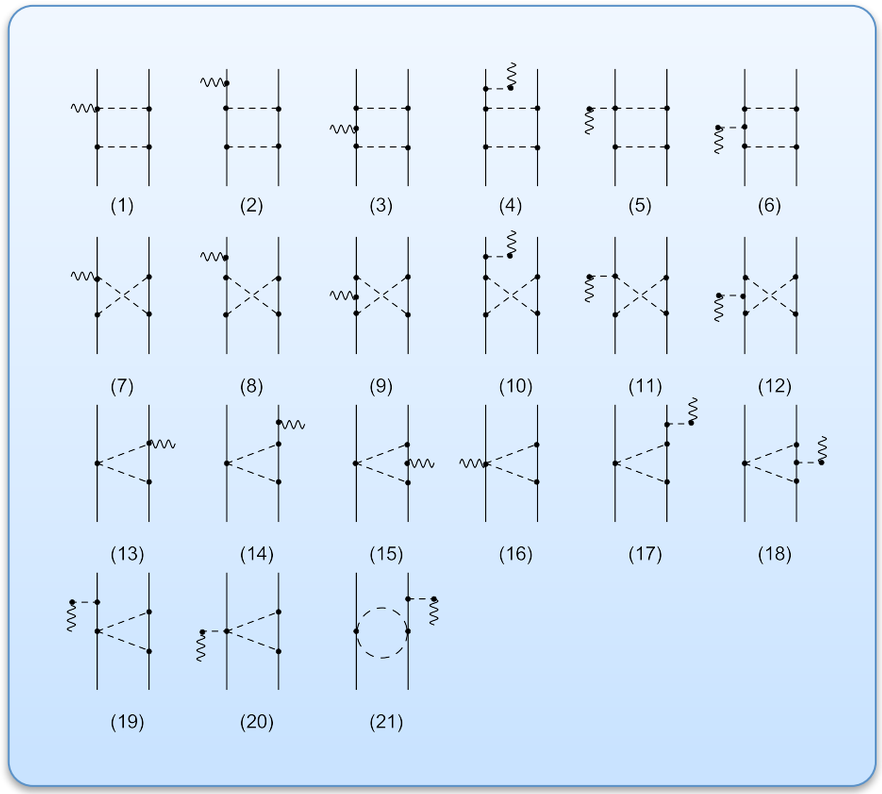
$$
h_4(q_2) = \mathcal{A}^{(4)}(q_2), \quad h_5(q_2) = \mathcal{B}^{(4)}(q_2)
$$

$$
\begin{aligned}
\vec{A}^{a\,(Q)}_{\text{2N: }1\pi} &= \frac{4F_\pi^2}{g_A}\frac{\vec{q}_1\cdot\vec{\sigma}_1}{q_1^2+M_\pi^2}\Big\{[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\Big([\vec{q}_1\times\vec{\sigma}_2]\,h_1(q_2)+[\vec{q}_2\times\vec{\sigma}_2]\,h_2(q_2)\Big)+\boldsymbol{\tau}_1^a\big(\vec{q}_1-\vec{q}_2\big)h_3(q_2)\Big\} \\
&+ \frac{4F_\pi^2}{g_A}\frac{\vec{q}_1\cdot\vec{\sigma}_1\,\vec{k}}{(k^2+M_\pi^2)(q_1^2+M_\pi^2)}\Big\{\boldsymbol{\tau}_1^a h_4(q_2)+[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\vec{q}_1\cdot[\vec{q}_2\times\vec{\sigma}_2]h_5(q_2)\Big\} + 1\leftrightarrow 2,
\end{aligned}
$$

$$
A^{0,a\,(Q)}_{\text{2N: }1\pi} = i\frac{4F_\pi^2}{g_A}\frac{\vec{q}_1\cdot\vec{\sigma}_1}{q_1^2+M_\pi^2}\Big\{[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\left(h_6(q_2)+k^2h_7(q_2)\right)+\tau_1^a\,\vec{q}_1\cdot[\vec{q}_2\times\vec{\sigma}_2]\,h_8(q_2)\Big\} + 1\leftrightarrow 2\,,
$$

NN current at order Q

Two-pion exchange contributions match to $2\pi - 1\pi$ 3NF at N³LO



$$
\begin{aligned}
g_1(q_1) &= \frac{g_A^4 A(q_1)\left(\left(8g_A^2-4\right)M_\pi^2+\left(g_A^2+1\right)q_1^2\right)}{256\pi F_\pi^6 q_1^2} - \frac{g_A^4 M_\pi\left(\left(8g_A^2-4\right)M_\pi^2+\left(3g_A^2-1\right)q_1^2\right)}{256\pi F_\pi^6 q_1^2\left(4M_\pi^2+q_1^2\right)}\\
g_2(q_1) &= \frac{g_A^4 A(q_1)\left(2M_\pi^2+q_1^2\right)}{128\pi F_\pi^6} + \frac{g_A^4 M_\pi}{128\pi F_\pi^6},\\
g_3(q_1) &= -\frac{g_A^4 A(q_1)\left(\left(8g_A^2-4\right)M_\pi^2+\left(3g_A^2-1\right)q_1^2\right)}{256\pi F_\pi^6} - \frac{\left(3g_A^2-1\right)g_A^4 M_\pi}{256\pi F_\pi^6},\\
g_4(q_1) &= -\frac{g_A^6 A(q_1)}{128\pi F_\pi^6},\\
g_5(q_1) &= -q_1^2\, g_4(q_1),\\
g_6(q_1) &= g_8(q_1) = g_{10}(q_1) = g_{12}(q_1) = 0,\\
g_7(q_1) &= \frac{g_A^4 A(q_1)\left(2M_\pi^2+q_1^2\right)}{128\pi F_\pi^6} + \frac{\left(2g_A^2+1\right)g_A^4 M_\pi}{128\pi F_\pi^6},\\
g_9(q_1) &= \frac{g_A^6 M_\pi}{64\pi F_\pi^6},\\
g_{11}(q_1) &= -\frac{g_A^4 A(q_1)\left(4M_\pi^2+q_1^2\right)}{512\pi F_\pi^6} - \frac{g_A^4 M_\pi}{512\pi F_\pi^6},\\
g_{13}(q_1) &= -\frac{g_A^6 A(q_1)}{128\pi F_\pi^6},\\
g_{14}(q_1) &= \frac{g_A^4 A(q_1)\left(\left(8g_A^2-4\right)M_\pi^2+\left(g_A^2+1\right)q_1^2\right)}{256\pi F_\pi^6 q_1^2} + \frac{g_A^4 M_\pi\left(\left(4-8g_A^2\right)M_\pi^2+\left(1-3g_A^2\right)q_1^2\right)}{256\pi F_\pi^6 q_1^2\left(4M_\pi^2+q_1^2\right)}\\
g_{15}(q_1) &= \frac{g_A^4 A(q_1)\left(\left(8g_A^2-4\right)M_\pi^2+\left(3g_A^2-1\right)q_1^2\right)}{256\pi F_\pi^6} + \frac{\left(3g_A^2-1\right)g_A^4 M_\pi}{256\pi F_\pi^6},
\end{aligned}
$$

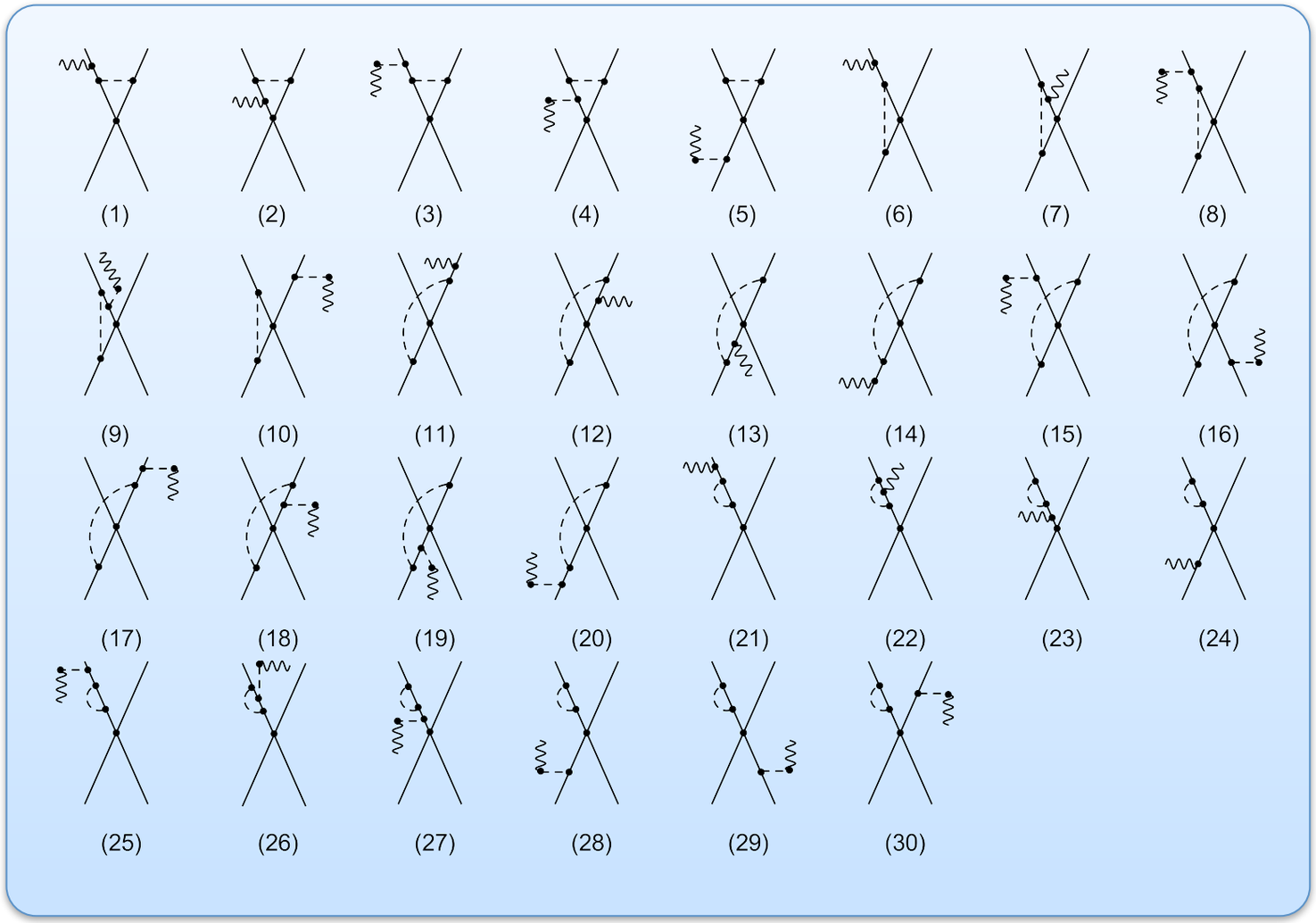
$$
\begin{aligned}
g_{16}(q_1) &= \frac{g_A^4 A(q_1)\left(2M_\pi^2+q_1^2\right)}{64\pi F_\pi^6} + \frac{g_A^4 M_\pi}{64\pi F_\pi^6},\\
g_{17}(q_1) &= -\frac{g_A^6 q_1^2 A(q_1)}{128\pi F_\pi^6},\\
g_{18}(q_1) &= \frac{g_A^2 L(q_1)\left(\left(4-8g_A^2\right)M_\pi^2+\left(1-3g_A^2\right)q_1^2\right)}{128\pi^2 F_\pi^6\left(4M_\pi^2+q_1^2\right)},\\
g_{19}(q_1) &= \frac{g_A^4 L(q_1)}{32\pi^2 F_\pi^6}.
\end{aligned}
$$

$$
\begin{aligned}
\vec{A}^{a\,(Q)}_{2\mathrm{N}:\,2\pi} &= \frac{2F_\pi^2}{g_A}\frac{\vec{k}}{k^2+M_\pi^2}\Big\{\tau_1^a\Big(-\vec{q}_1\cdot\vec{\sigma}_2\,\vec{q}_1\cdot\vec{k}\,g_1(q_1)+\vec{q}_1\cdot\vec{\sigma}_2\,g_2(q_1)-\vec{k}\cdot\vec{\sigma}_2\,g_3(q_1)\Big) + \tau_2^a\Big(-\vec{q}_1\cdot\vec{\sigma}_1\,\vec{q}_1\cdot\vec{k}\,g_4(q_1)\\
&- \vec{k}\cdot\vec{\sigma}_1\,g_5(q_1)-\vec{q}_1\cdot\vec{\sigma}_2\,\vec{q}_1\cdot\vec{k}\,g_6(q_1)+\vec{q}_1\cdot\vec{\sigma}_2\,g_7(q_1)+\vec{k}\cdot\vec{\sigma}_2\,\vec{q}_1\cdot\vec{k}\,g_8(q_1)-\vec{k}\cdot\vec{\sigma}_2\,g_9(q_1)\Big)\\
&+ [\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\Big(-\vec{q}_1\cdot[\vec{\sigma}_1\times\vec{\sigma}_2]\,\vec{q}_1\cdot\vec{k}\,g_{10}(q_1)+\vec{q}_1\cdot[\vec{\sigma}_1\times\vec{\sigma}_2]\,g_{11}(q_1)-\vec{q}_1\cdot\vec{\sigma}_2\,\vec{q}_1\cdot[\vec{q}_2\times\vec{\sigma}_1]\,g_{12}(q_1)\Big)\Big\}\\
&+ \frac{2F_\pi^2}{g_A}\Big\{\vec{q}_1\Big(\tau_2^a\,\vec{q}_1\cdot\vec{\sigma}_1\,g_{13}(q_1)+\tau_1^a\,\vec{q}_1\cdot\vec{\sigma}_2\,g_{14}(q_1)\Big)-\tau_1^a\,\vec{\sigma}_2\,g_{15}(q_1)-\tau_2^a\,\vec{\sigma}_2\,g_{16}(q_1)-\tau_2^a\,\vec{\sigma}_1\,g_{17}(q_1)\Big\} + 1\leftrightarrow 2\,,
\end{aligned}
$$

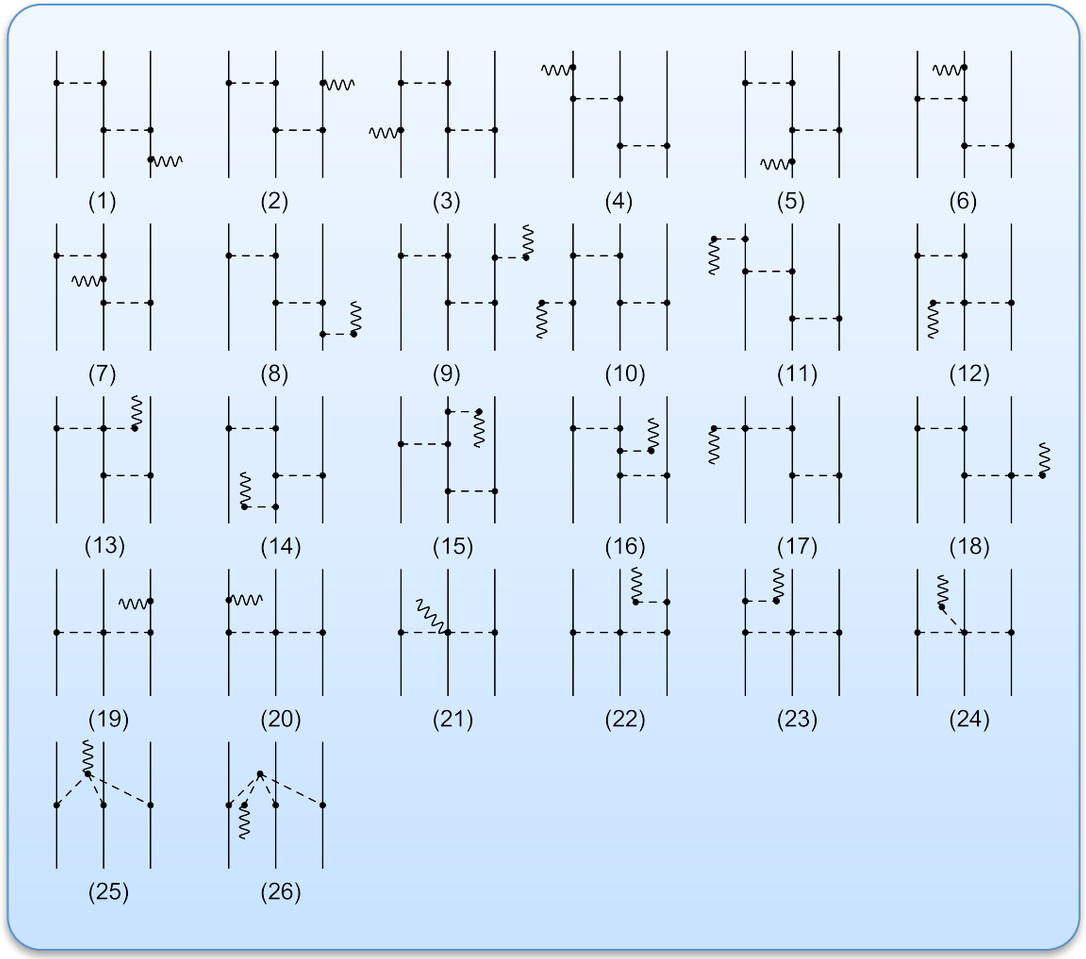
$$
A^{0,a\,(Q)}_{2\mathrm{N}:\,2\pi} = i\frac{2F_\pi^2}{g_A}\Big\{[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\vec{q}_1\cdot\vec{\sigma}_2\,g_{18}(q_1)+\tau_2^a\vec{q}_1\cdot[\vec{\sigma}_1\times\vec{\sigma}_2]g_{19}(q_1)\Big\} + 1\leftrightarrow 2\,,
$$

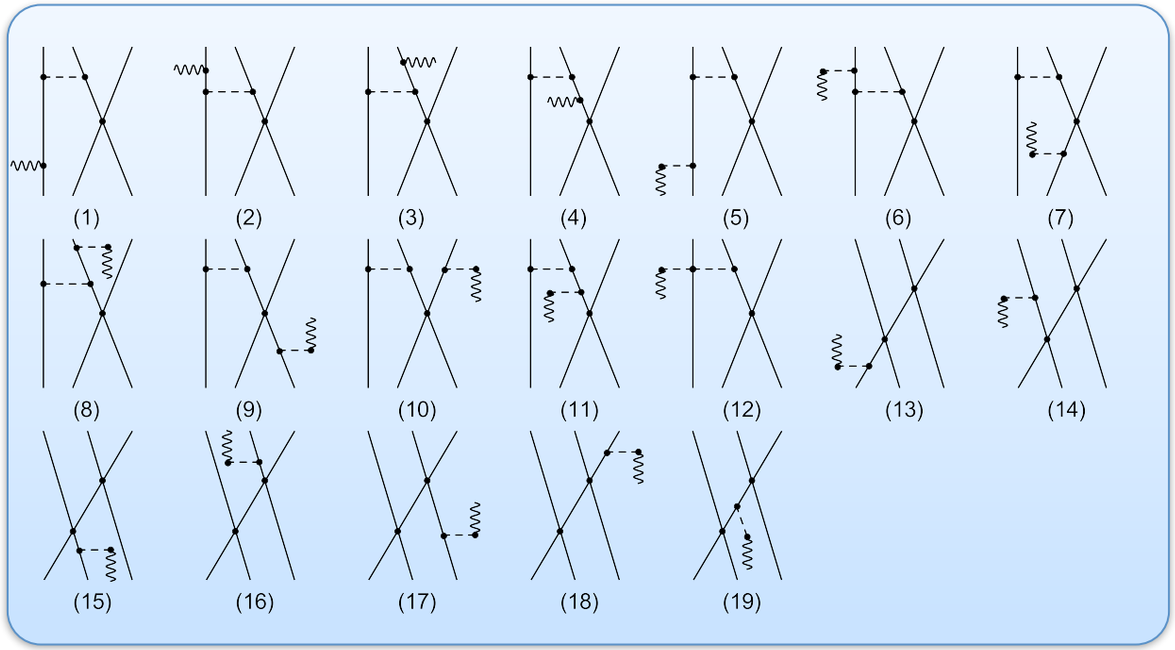
NN current at order Q

Vanishing short-range contributions for the current, after antisymmetrization



Three-nucleon current





- First complete calculation of axial 3N currents
  - Lengthy expression for current: *HK, Epelbaum, Meißner, arXiv:1610.03569*
  - Vanishing charge operator
- Pion-pole terms match to 4NF

Compare with Baroni et al.

Baroni et al. arXiv:1605.01620; PRC93 (2016) no. 1, 015501; Erratum PRC93 (2016) no. 4, 049902

At zero momentum transfer the result of Baroni et al. is

$$\mathbf{j}_{\pm}^{\mathrm{N4LO}}(\mathrm{OPE};\mathbf{k}) = \frac{g_A^5\, m_\pi}{256\,\pi\, f_\pi^4}\left[\,18\,\tau_{2,\pm}\,\mathbf{k} - (\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_\pm\,\boldsymbol{\sigma}_1\times\mathbf{k}\,\right]\boldsymbol{\sigma}_2\cdot\mathbf{k}\,\frac{1}{\omega_k^2} + (1 \rightleftharpoons 2)\ , \tag{5}$$

$$\begin{aligned}\mathbf{j}_{\pm}^{\mathrm{N4LO}}(\mathrm{MPE};\mathbf{k}) = {}& \frac{g_A^3}{32\,\pi f_\pi^4}\,\tau_{2,\pm}\left[W_1(k)\,\boldsymbol{\sigma}_1 + W_2(k)\,\mathbf{k}\;\boldsymbol{\sigma}_1\cdot\mathbf{k} + Z_1(k)\left(2\,\mathbf{k}\;\boldsymbol{\sigma}_2\cdot\mathbf{k}\frac{1}{\omega_k^2} - \boldsymbol{\sigma}_2\right)\right]\\ & + \frac{g_A^5}{32\,\pi f_\pi^4}\,\tau_{1,\pm}\,W_3(k)\,(\boldsymbol{\sigma}_2\times\mathbf{k})\times\mathbf{k} - \frac{g_A^3}{32\,\pi f_\pi^4}\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_\pm\,Z_3(k)\,\boldsymbol{\sigma}_1\times\mathbf{k}\\ & \times\boldsymbol{\sigma}_2\cdot\mathbf{k}\,\frac{1}{\omega_k^2} + (1\rightleftharpoons 2)\ ,\end{aligned} \tag{6}$$

$$\begin{aligned}
W_1(k) &= \frac{M_\pi}{2}\left(1 + g_A^2\left(-9 + \frac{4M_\pi^2}{k^2+4M_\pi^2}\right)\right) + \frac{1}{2}\left((1-5g_A^2)k^2 + 4(1-2g_A^2)M_\pi^2\right)A(k),\\
W_2(k) &= \frac{M_\pi}{2k^2(k^2+4M_\pi^2)}\left(\left(1+3g_A^2\right)k^2 + 4\left(1+2g_A^2\right)M_\pi^2\right) - \frac{1}{2k^2}\left((-1+g_A^2)k^2 + 4(1+2g_A^2)M_\pi^2\right)A(k),\\
W_3(k) &= -\frac{1}{6M_\pi} - \frac{4}{3}A(k),\\
Z_1(k) &= 2M_\pi + 2(k^2+2M_\pi^2)A(k),\\
Z_3(k) &= \frac{M_\pi}{2} + \frac{1}{2}(k^2+4M_\pi^2)A(k).
\end{aligned}$$

The current of Baroni et al. does not exist in the chiral limit!

$$\begin{aligned}\vec{j}_a^{\,\mathrm{N4LO}}(\mathrm{MPE},\vec{q}_1) - \vec{A}^{a\,(Q)}_{2\mathrm{N:}\,2\pi} - \vec{A}^{a\,(Q)}_{2\mathrm{N:}\,1\pi}\Big|_{\alpha_1^{ax}=0} &= -\frac{g_A^5 A(q_1)\left(\vec{\sigma}_2\tau_1^a q_1^4 + 2\vec{q}_1\left(6M_\pi^2+q_1^2\right)\vec{q}_1\cdot\vec{\sigma}_2\tau_1^a\right)}{96\pi F_\pi^4 q_1^2}\\ &+ \ \text{rational function of } \vec{q}_1 \ + \ 1\leftrightarrow 2\ .\end{aligned}$$

Two currents have different long range parts!

order	single-nucleon	two-nucleon	three-nucleon
LO (Q^{-3})	$\vec{A}^a_{\mathrm{1N:\,static}}$,	—	—
NLO (Q^{-1})	$\vec{A}^a_{\mathrm{1N:\,static}}$,	—	—
N^{2}LO (Q^0)	—	$\vec{A}^a_{\mathrm{2N:\,1\pi}}$, ✓ $+\ \vec{A}^a_{\mathrm{2N:\,cont}}$, ✓	—
N^{3}LO (Q)	$\vec{A}^a_{\mathrm{1N:\,static}}$, $+\ \vec{A}^a_{\mathrm{1N:\,1/m,UT'}}$, $+\ \vec{A}^a_{\mathrm{1N:\,1/m^2}}$,	$\vec{A}^a_{\mathrm{2N:\,1\pi}}$, $+\ \vec{A}^a_{\mathrm{2N:\,1\pi,UT'}}$, ✖ $+\ \vec{A}^a_{\mathrm{2N:\,1\pi,\,1/m}}$, ✖ $+\ \vec{A}^a_{\mathrm{2N:\,2\pi}}$, $+\ \vec{A}^a_{\mathrm{2N:\,cont,\,UT'}}$, ✖ $+\ \vec{A}^a_{\mathrm{2N:\,cont,\,1/m}}$, ✖	$\vec{A}^a_{\mathrm{3N:\,\pi}}$, $+\ \vec{A}^a_{\mathrm{3N:\,cont}}$, ✖

Baroni et al. considered only irr. diagrams of 3N current

✖ terms not discussed by Baroni et al. ´16 ✓ terms on which we agree with Baroni et al. ´16

order	single-nucleon	two-nucleon	three-nucleon
LO (Q^{-3})	—	—	—
NLO (Q^{-1})	$A^{0,a}_{\mathrm{1N:\,UT'}}$, $+\ A^{0,a}_{\mathrm{1N:\,1/m}}$,	$A^{0,a}_{\mathrm{2N:\,1\pi}}$, ✓	—
N^{2}LO (Q^0)	—	—	—
N^{3}LO (Q)	$A^{0,a}_{\mathrm{1N:\,static,\,UT'}}$, $+\ A^{0,a}_{\mathrm{1N:\,1/m}}$,	$A^{0,a}_{\mathrm{2N:\,1\pi}}$, $+\ A^{0,a}_{\mathrm{2N:\,2\pi}}$, ✓ $+\ A^{0,a}_{\mathrm{2N:\,cont}}$, ✓	—

Pseudoscalar current

order	single-nucleon	two-nucleon	three-nucleon
LO (Q^{-4})	$P^a_{\mathrm{1N:\,static}}$,	—	—
NLO (Q^{-2})	$P^a_{\mathrm{1N:\,static}}$,	—	—
N^{2}LO (Q^{-1})	—	$P^a_{\mathrm{2N:\,1\pi}}$, $+\ P^a_{\mathrm{2N:\,cont}}$,	—
N^{3}LO (Q^0)	$P^a_{\mathrm{1N:\,static}}$, $+\ P^a_{\mathrm{1N:\,1/m,UT'}}$, $+\ P^a_{\mathrm{1N:\,1/m^2}}$,	$P^a_{\mathrm{2N:\,1\pi}}$, $+\ P^a_{\mathrm{2N:\,1\pi,UT'}}$, $+\ P^a_{\mathrm{2N:\,1\pi,\,1/m}}$, $+\ P^a_{\mathrm{2N:\,2\pi}}$, $+\ P^a_{\mathrm{2N:\,cont,\,UT'}}$, $+\ P^a_{\mathrm{2N:\,cont,\,1/m}}$,	$P^a_{\mathrm{3N:\,\pi}}$, $+\ P^a_{\mathrm{3N:\,cont}}$,

Continuity equations are verified for all currents

Summary

- Axial-vector current is analyzed up to order Q
- There is a high degree of unitary ambiguity
- Modified continuity equation and 4-vector relations are satisfied for any choice of unitary phases
- Renormalizability and matching to nuclear forces conditions lead to unique current
- Differences in long range part between our results and Baroni et al.

Outlook

- Numerical implementations
- Axial-vector current up to order Q^2