Minimal integer automaton behind crystal plasticity

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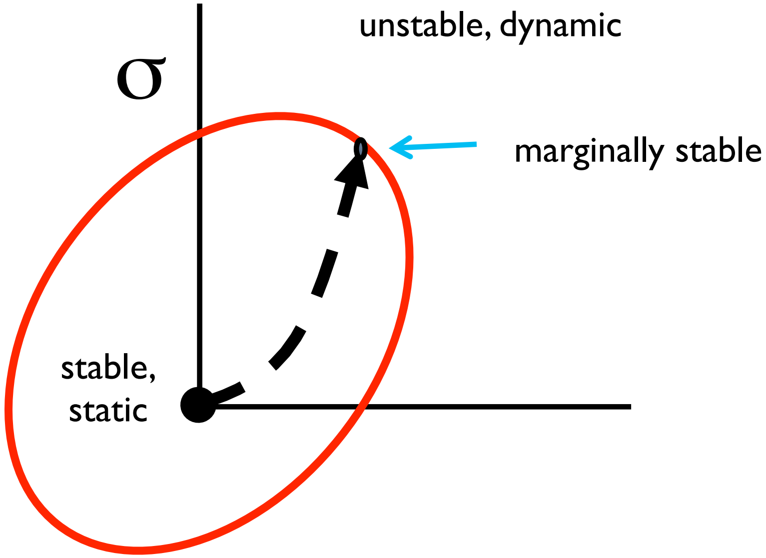
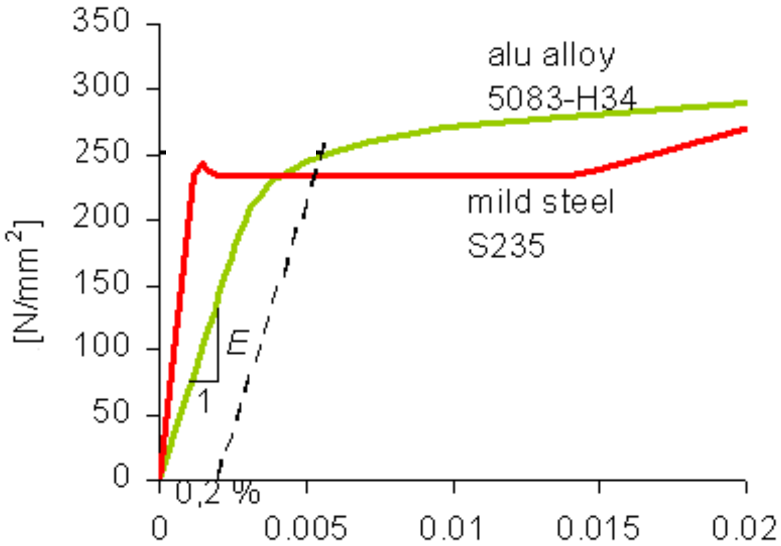
Ecole Polytechnique

Joint work with Umut Salman,
PRL 106, 175503 (2011)

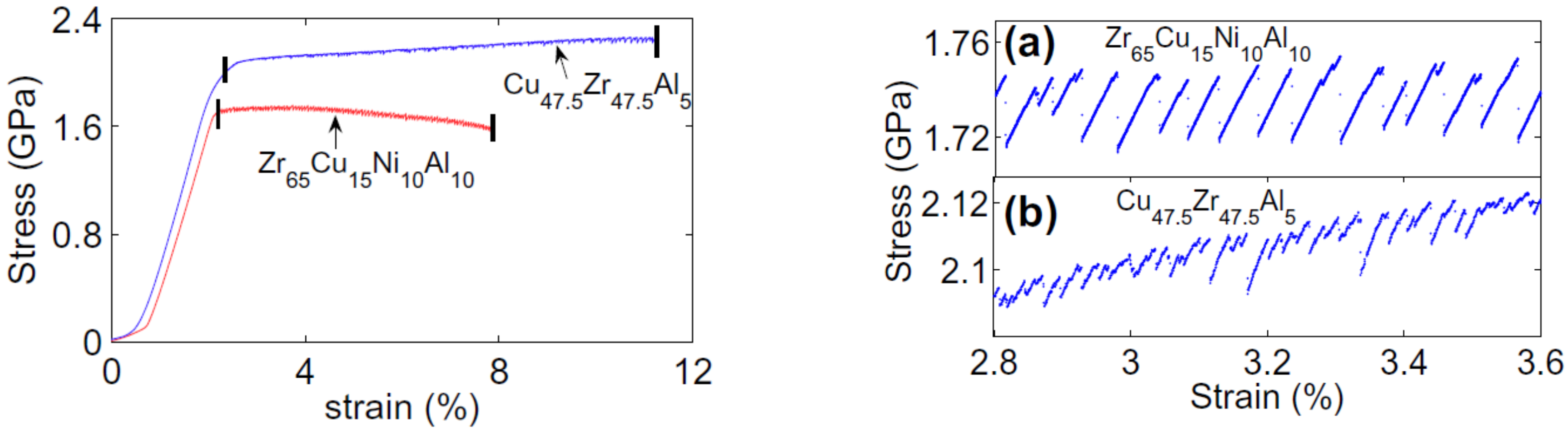
Engineering plasticity

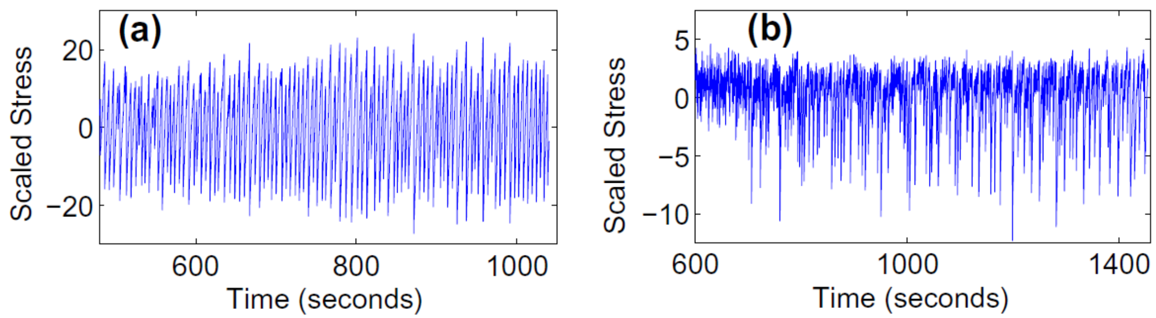




- Plastic yielding is associated with states that are only marginally stable
- Heat production at infinitely slow loading
- The area under the stress-strain curve has little to do with the stored energy

Fluctuations



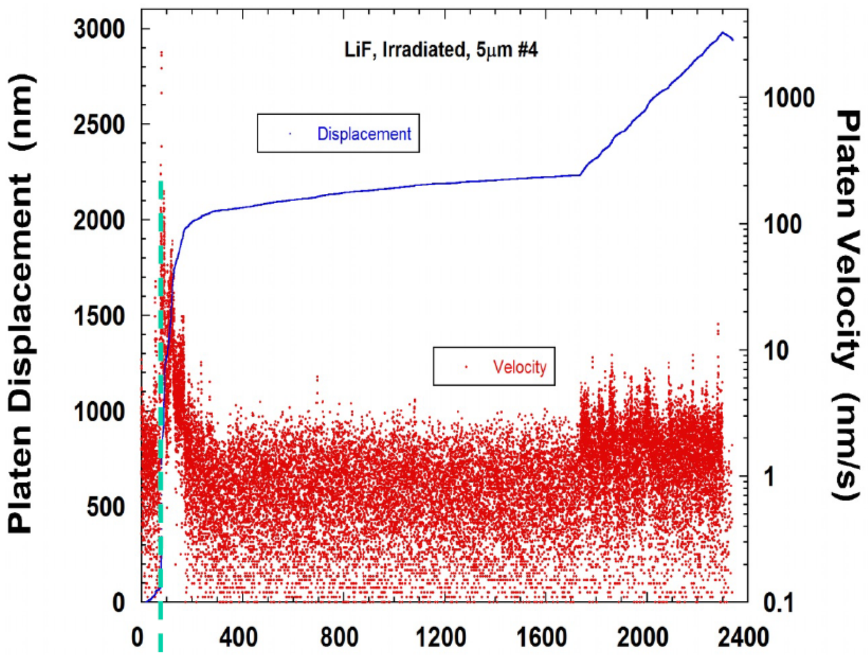


(a and b) Cleaned stress–time series for the less ductile $Zr_{65}Cu_{15}Ni_{10}Al_{10}$ BMG and the more ductile $Cu_{47.5}Zr_{47.5}Al_5$ BMG samples, respectively, for the region indicated in the original stress–strain plot.

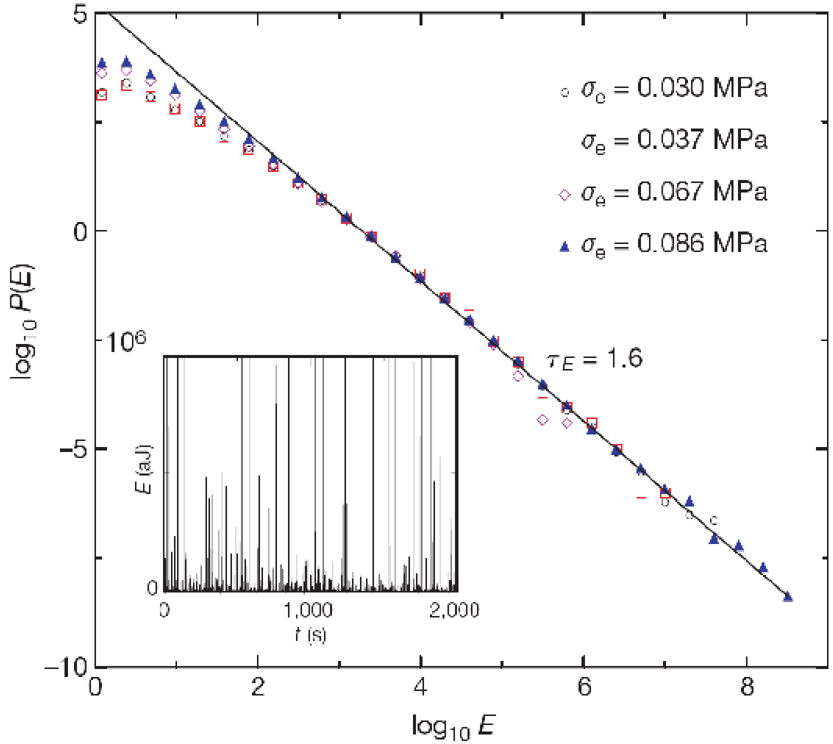
R. Sarmah et al. / Acta Materialia 59 (2011)

- Fluctuation statistics carry important information about the ‘health’ of the sample
- Fluctuations prevent precise control of small samples during forming

Plasticity experiment

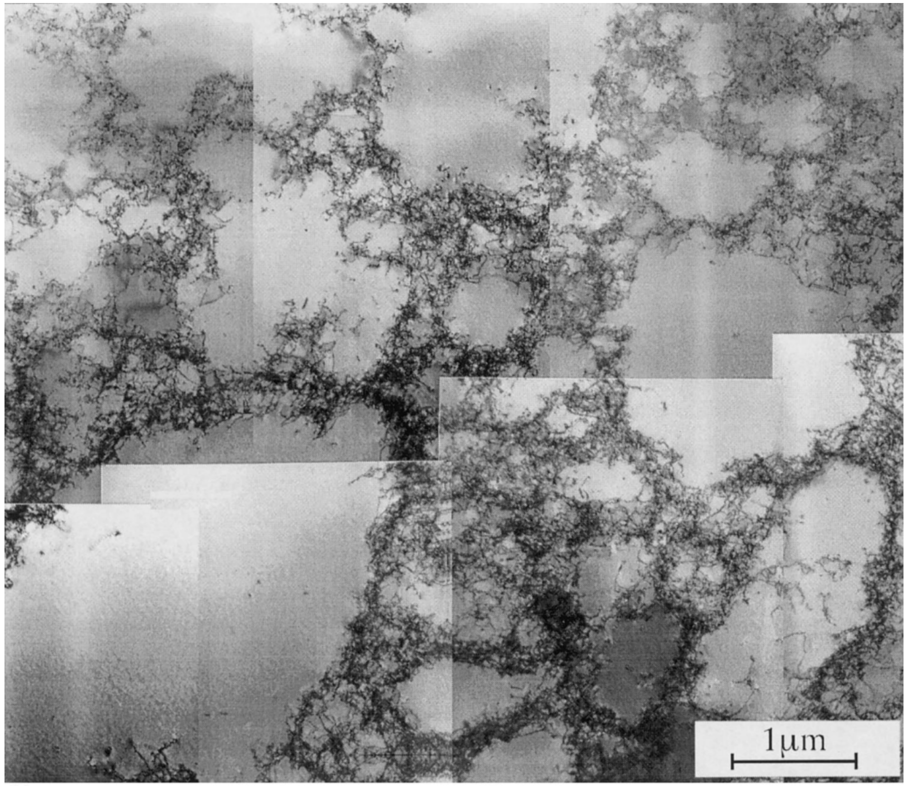


Dimiduk et al., Phil. Mag., 2010

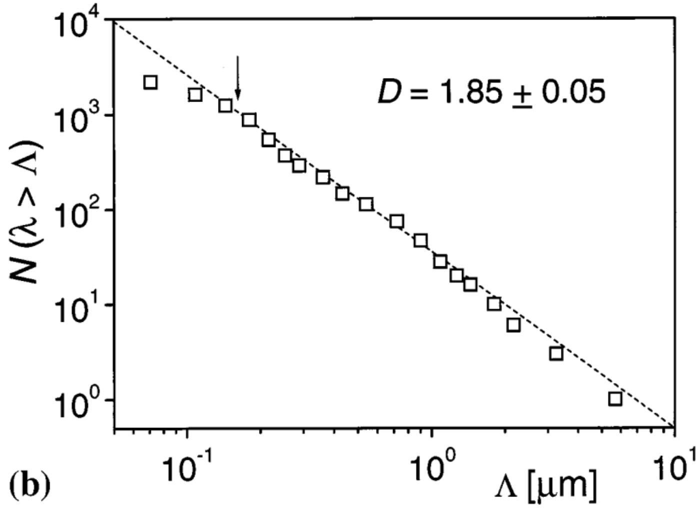


M.C. Miguel et al. Nature (2001)

Plasticity experiment



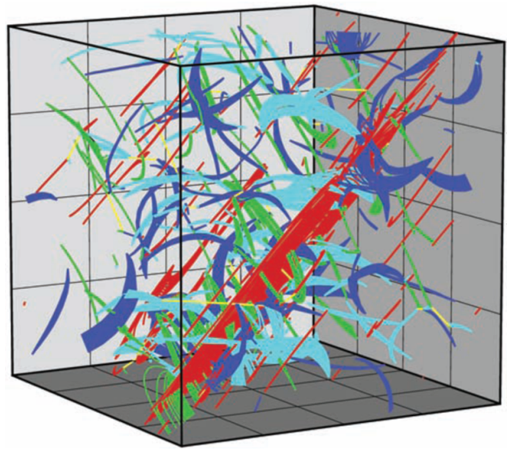
(a)

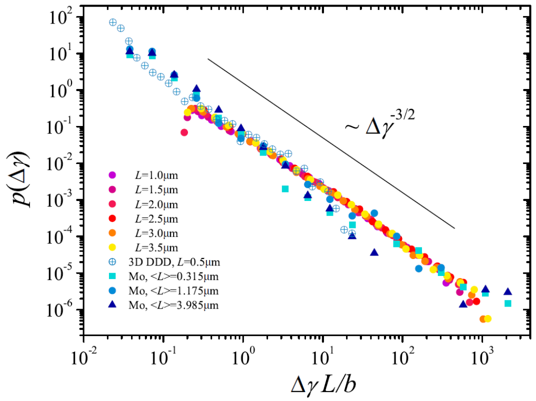


(b)

Zaiser and Hahner, 1999

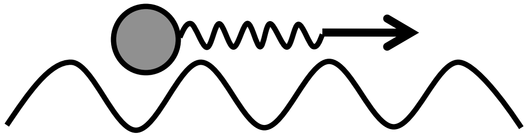
Discrete dislocation dynamics





M. Zaiser et al., 2011

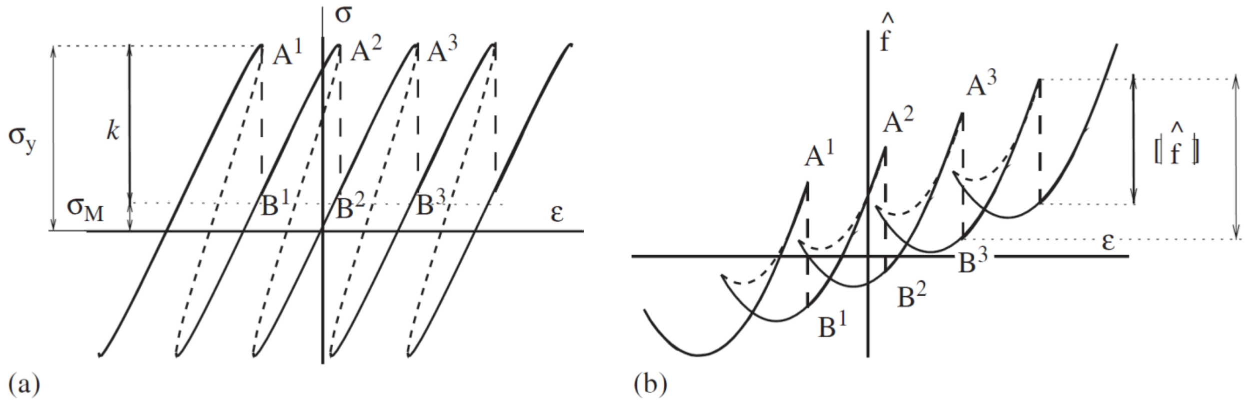
0D model



$$f(\varepsilon, \alpha) = \frac{E}{2}(\varepsilon - \alpha)^2 + \sigma_M \alpha - k\,\delta\,\cos\left(\frac{\alpha}{\delta}\right)$$

$$\nu\dot{\alpha} = E(\varepsilon - \alpha) - \sigma_M - k\sin\left(\frac{\alpha}{\delta}\right)$$

$$v = \dot{\varepsilon}$$



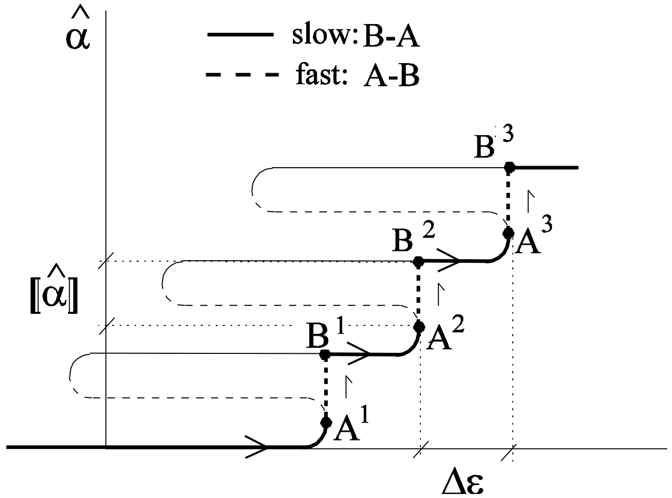
0D model

$$\mu\dot{\alpha} = -\frac{\partial f}{\partial \alpha}, \quad \dot{\varepsilon} = 1$$

$$\alpha = \hat{\alpha}(\varepsilon), \; f = \hat{f}(\varepsilon)$$

$$f(\varepsilon, \alpha) = \frac{E}{2}(\varepsilon - \alpha)^2 + \sigma_M \alpha - k\,\delta \cos\left(\frac{\alpha}{\delta}\right)$$

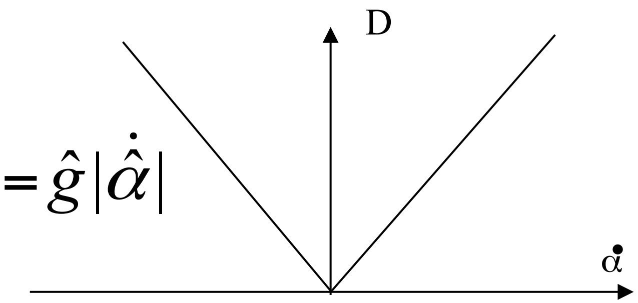
$$\mu = \frac{v}{\gamma} \to 0$$



$$D = v \lim_{\Delta\varepsilon \to 0} [\hat{f}] \frac{dn}{d\varepsilon} = v \lim_{\Delta\varepsilon \to 0} \frac{[\hat{f}]}{\Delta\varepsilon}$$

$$\hat{g} = -\lim_{\Delta\varepsilon \to 0} \frac{[\hat{f}]}{[\hat{\alpha}]}$$

$$D = \hat{g}\,|\dot{\hat{\alpha}}|$$



1D model

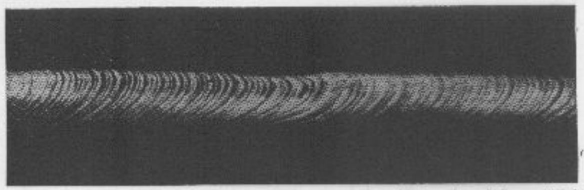
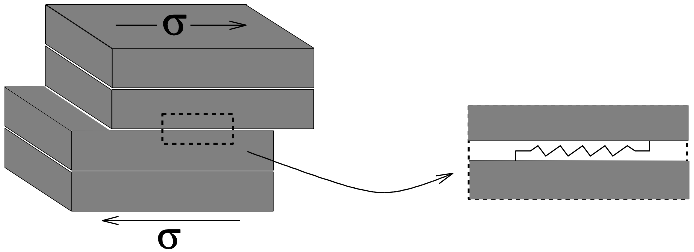
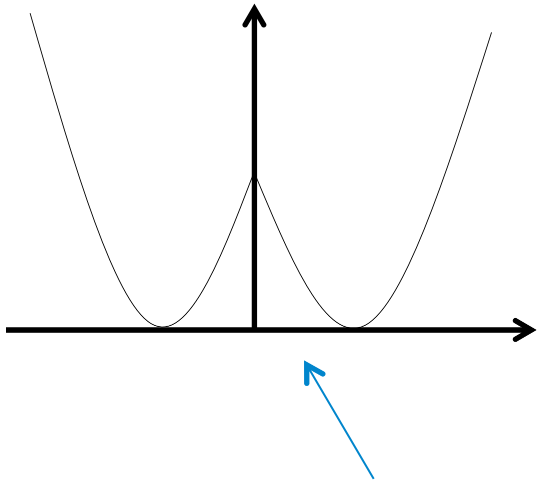
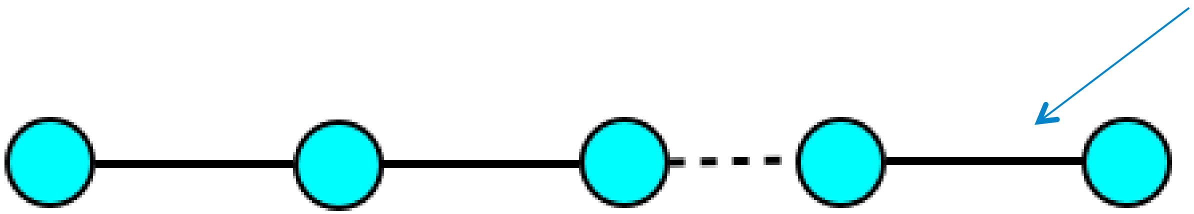


FIG. 7-4. Extended copper-aluminum single crystal. (*Reproduced from "Distortion of Metal Crystals," by C. F. Elam, see p. 51n*).

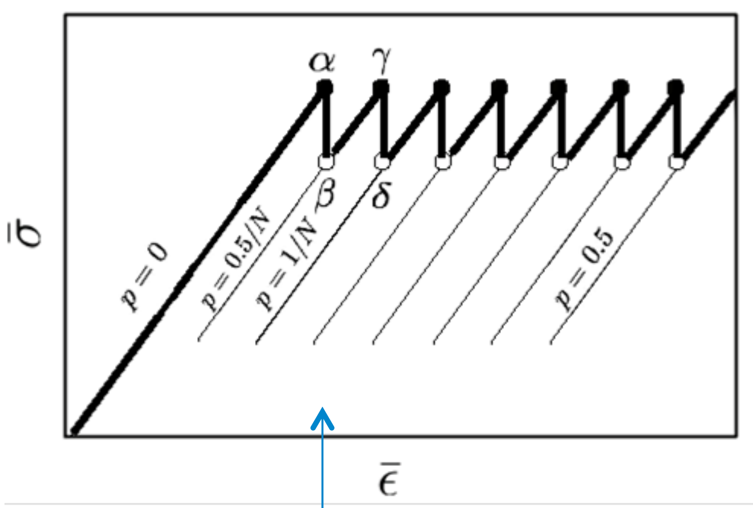


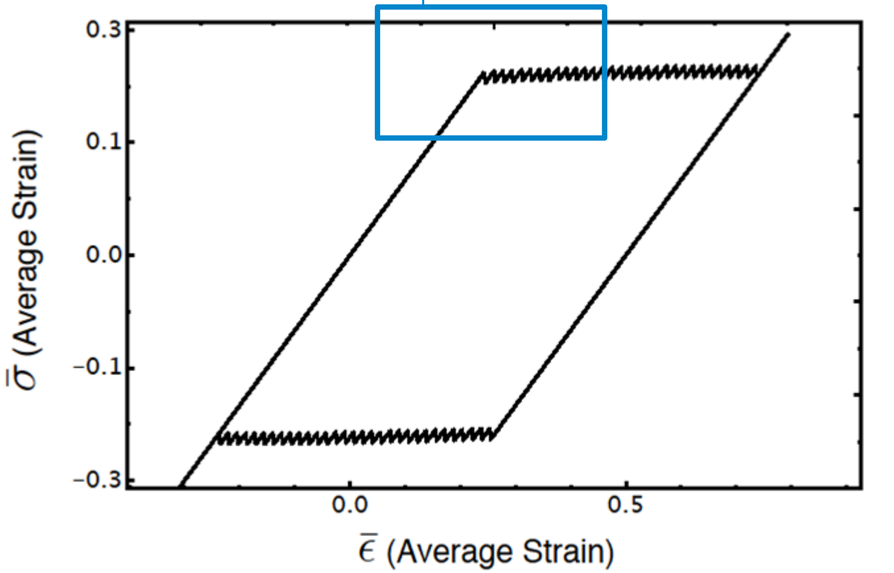


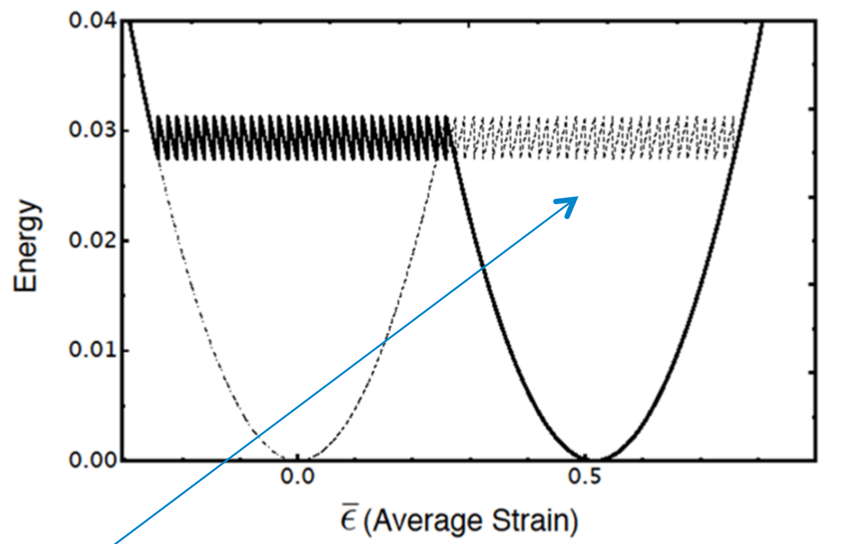
snap-spring



1D model

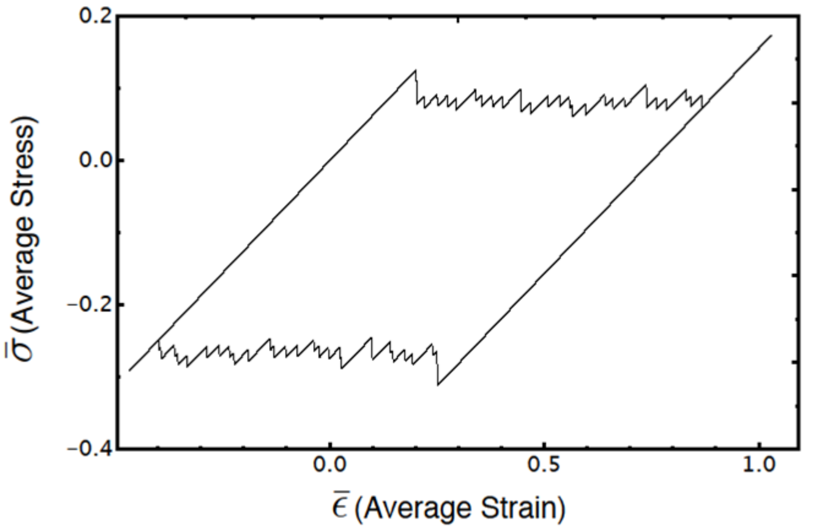


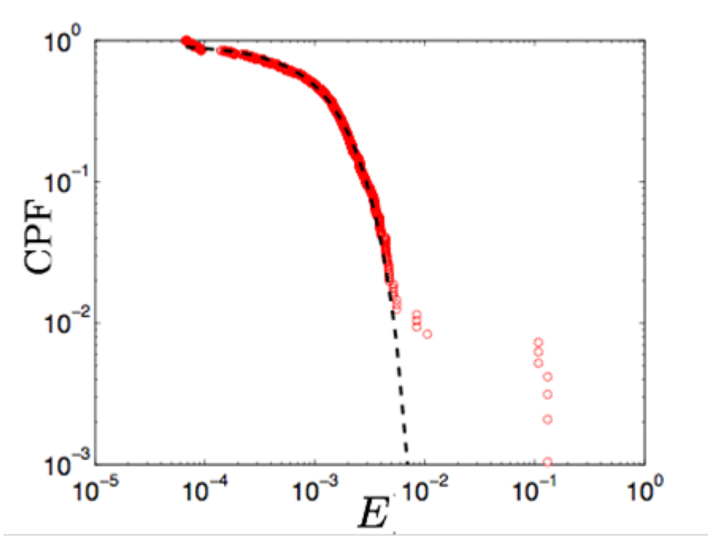




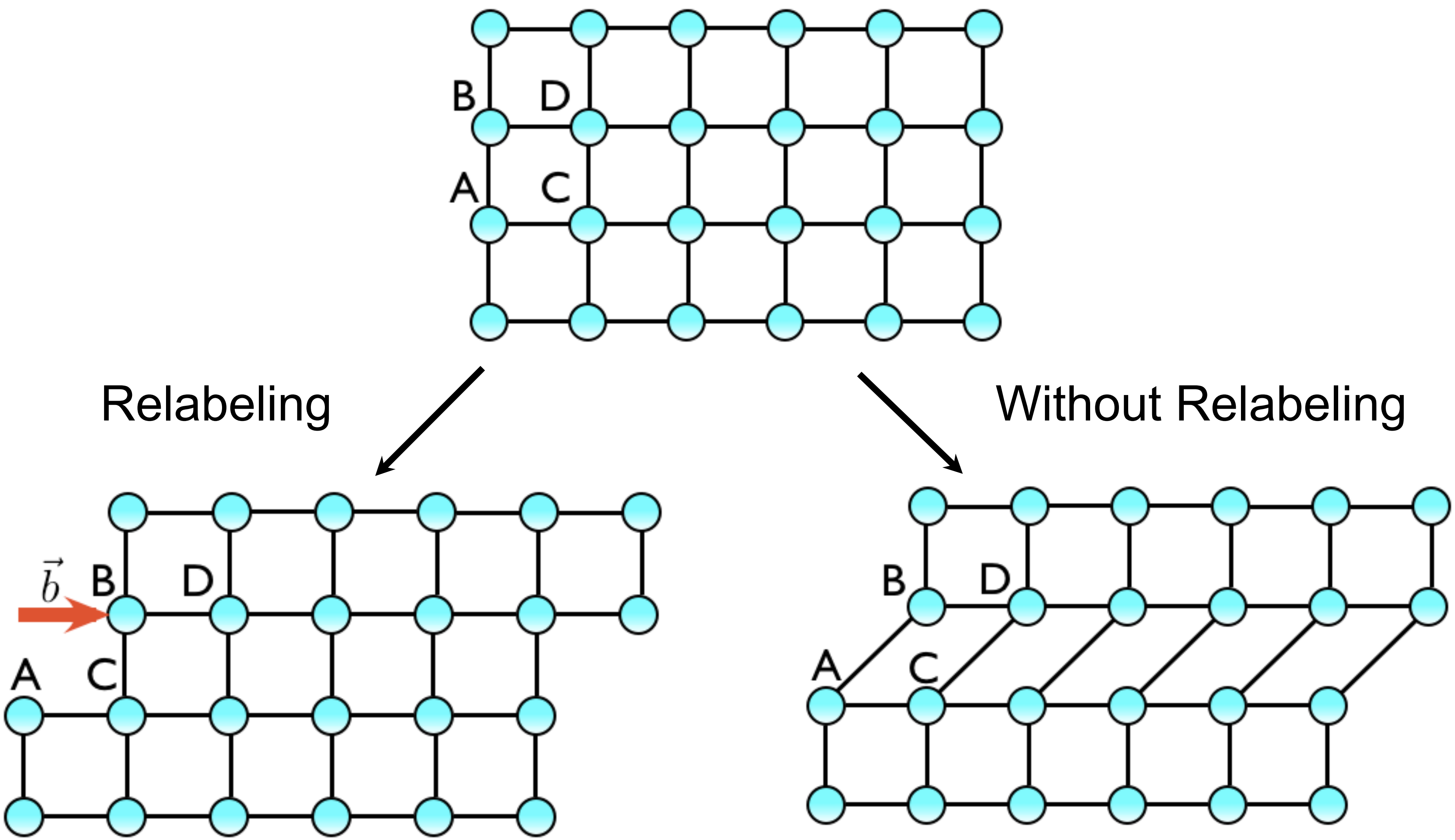
Marginally stable states

1D model (RFIM)



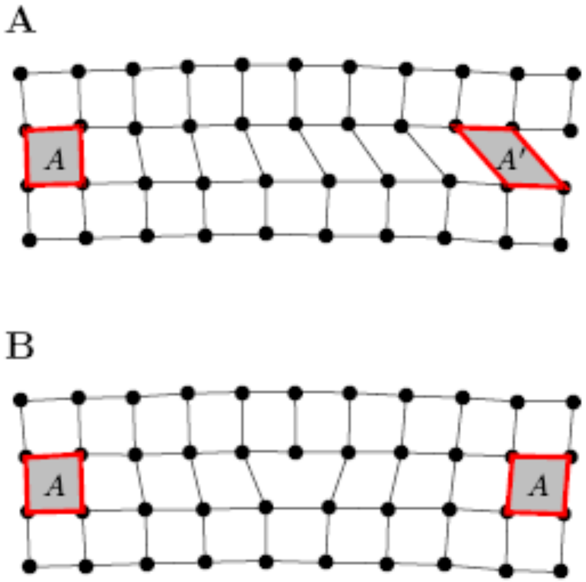


2D model



Two representations of a dislocation

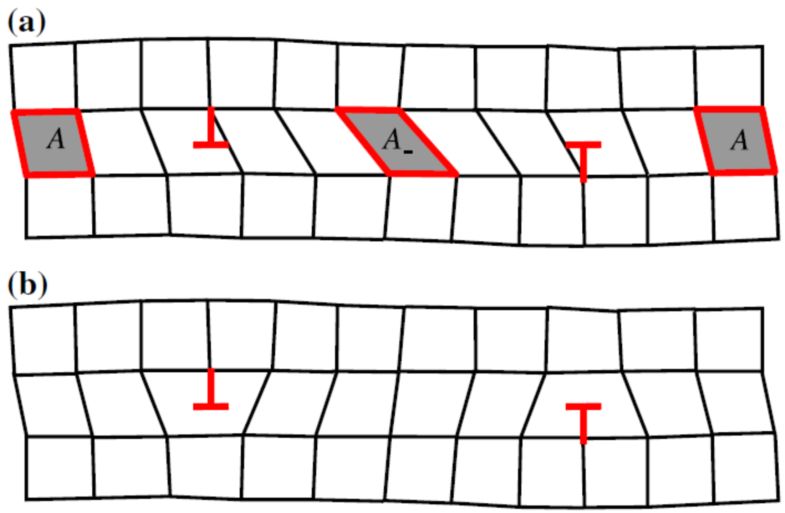
One dislocation



no relabeling

relabeling

Two dislocations

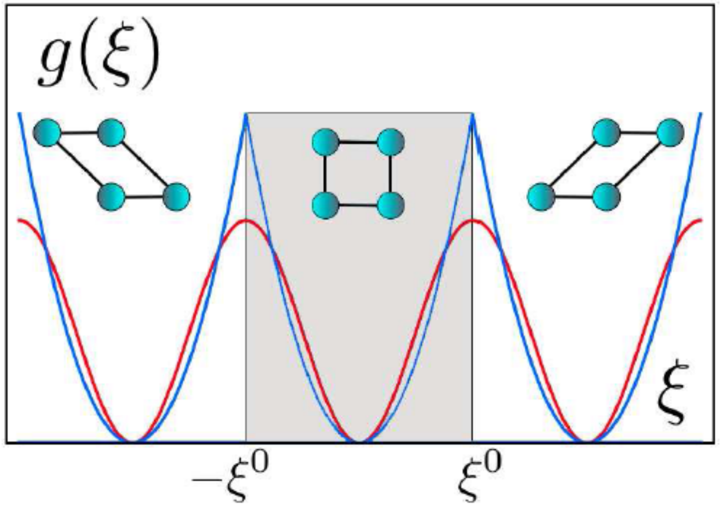


A simple discrete model

$$\nu \dot{u} = -\partial \Phi(u)/\partial u$$

$$\Phi(u) = \sum_{i,j} \phi(\theta, \xi)$$

$$\theta(i,k) = u(i+1,k) - u(i,k)$$
$$\xi(i,k) = u(i,k+1) - u(i,k)$$



$$\phi(\theta, \xi) = g(\xi) + \frac{K}{2}(\theta)^2 - h_1\xi - h_2\theta$$

Loading

$$\sum_{k=0}^{N-1} \xi(i,k) = t \qquad \sum_{i=0}^{N-1} \theta(i,k) = 0$$

Numerical solution

$$\dot{u}_{i,j} = K(D_x^- D_x^+ u_{i,j}) + D_y^- \mu(g(D_y^+ u_{i,j})).$$

$$c\hat{u}(\mathbf{q}) = \left\{ \frac{D_x^- D_x^+ u_{i,j}}{\Delta x^2} \right\}_{\mathbf{q}} = 2(\cos(q_i) - 1)\hat{u}(\mathbf{q}),$$

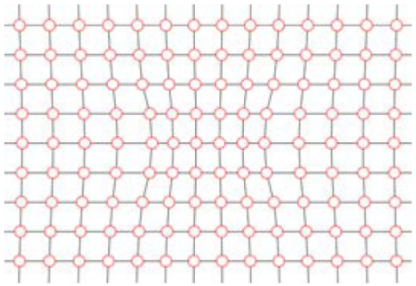
$$s_y^+ \hat{u}(\mathbf{q}) = \left\{ \frac{D_y^+ u_{i,j}}{\Delta x} \right\}_{\mathbf{q}} = (\cos(q_j) + I\sin(q_j) - 1)\hat{u}(\mathbf{q})$$

$$s_y^- \hat{u}(\mathbf{q}) = \left\{ \frac{D_y^- u_{i,j}}{\Delta x} \right\}_{\mathbf{q}} = (1 - \cos(q_j) + I\sin(q_j))\hat{u}(\mathbf{q}).$$

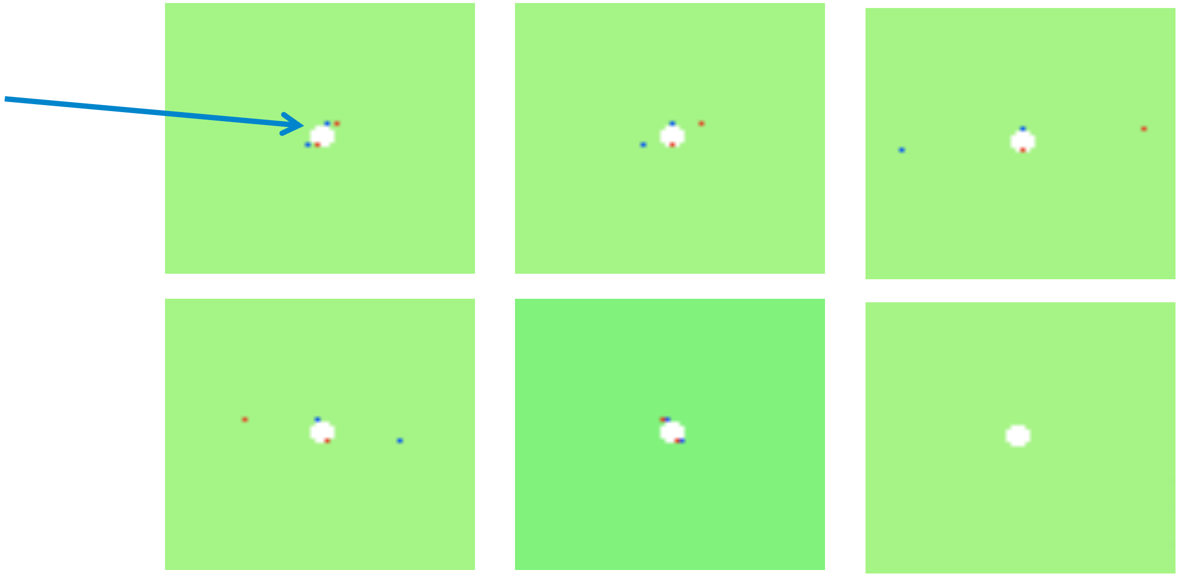
$$\dot{\hat{u}}(\mathbf{q}) = Kc\hat{u}(\mathbf{q}) + \mu s^- \left\{ g(\{s^+\hat{u}(\mathbf{q})\}_{\mathbf{q}}^{-1}) \right\}_{\mathbf{q}}$$

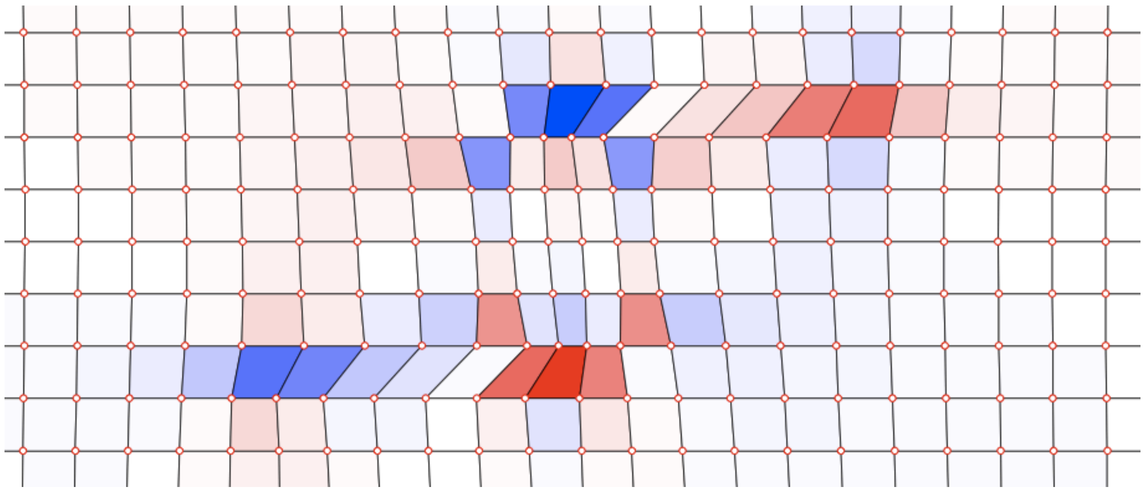
$$\hat{u}^{t+1}(\mathbf{q}) = \frac{\hat{u}^t(\mathbf{q}) + \Delta t \mu s^- \left\{ g(\{s^+\hat{u}^t(\mathbf{q})\}_{\mathbf{q}}^{-1}) \right\}_{\mathbf{q}}}{1 + \Delta t K c(\mathbf{q})},$$

Dislocation nucleation

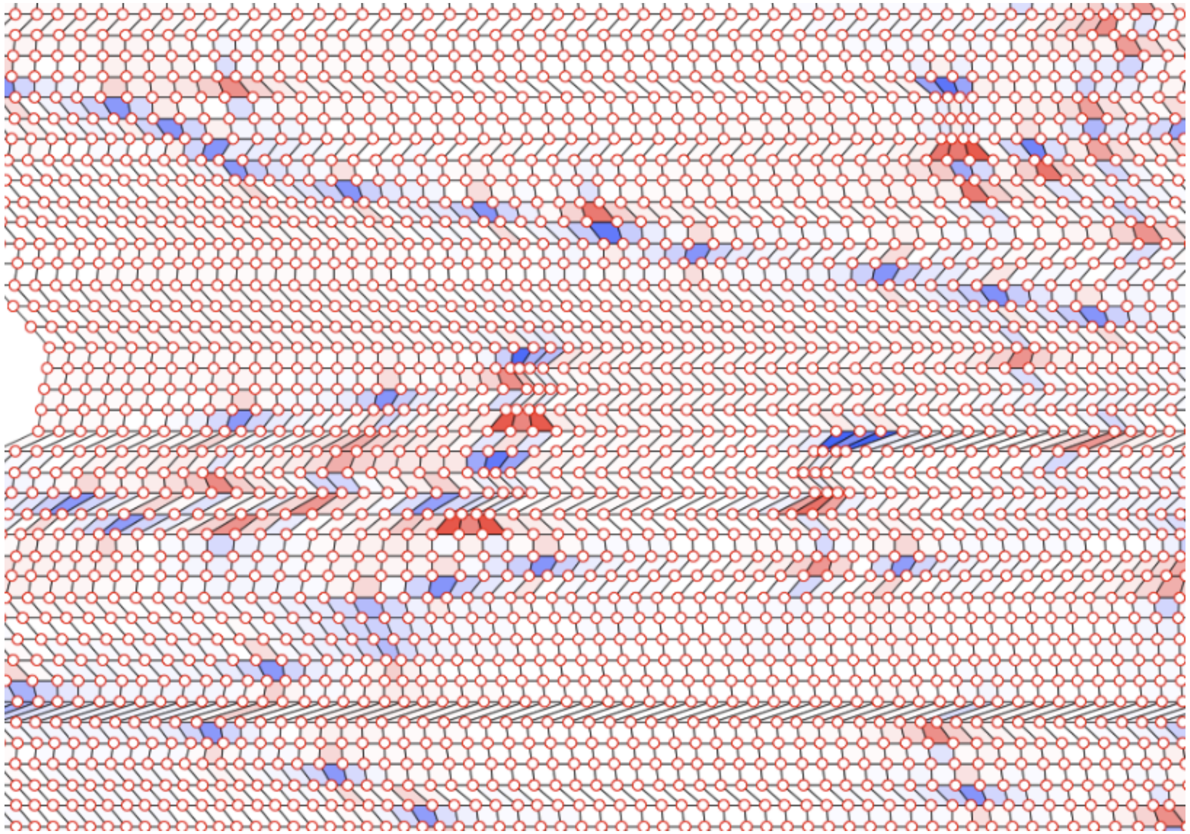


Typical defect

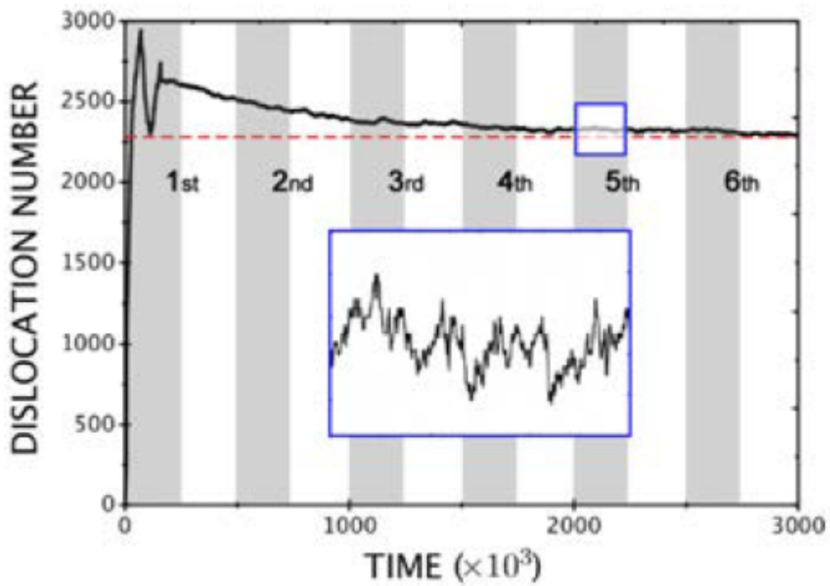


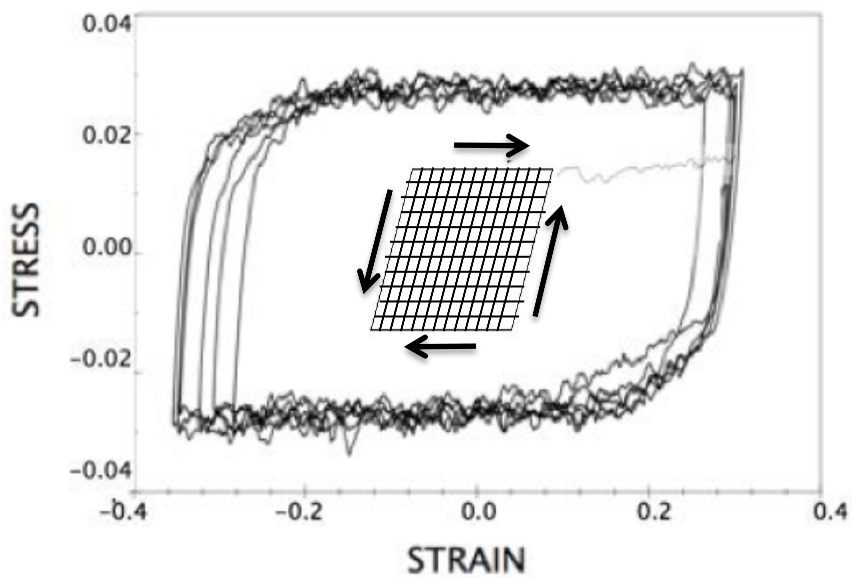


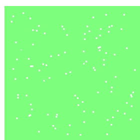
Typical configuration of a dislocated body

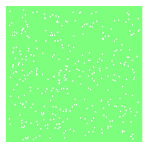


Stress-strain curves, shakedown





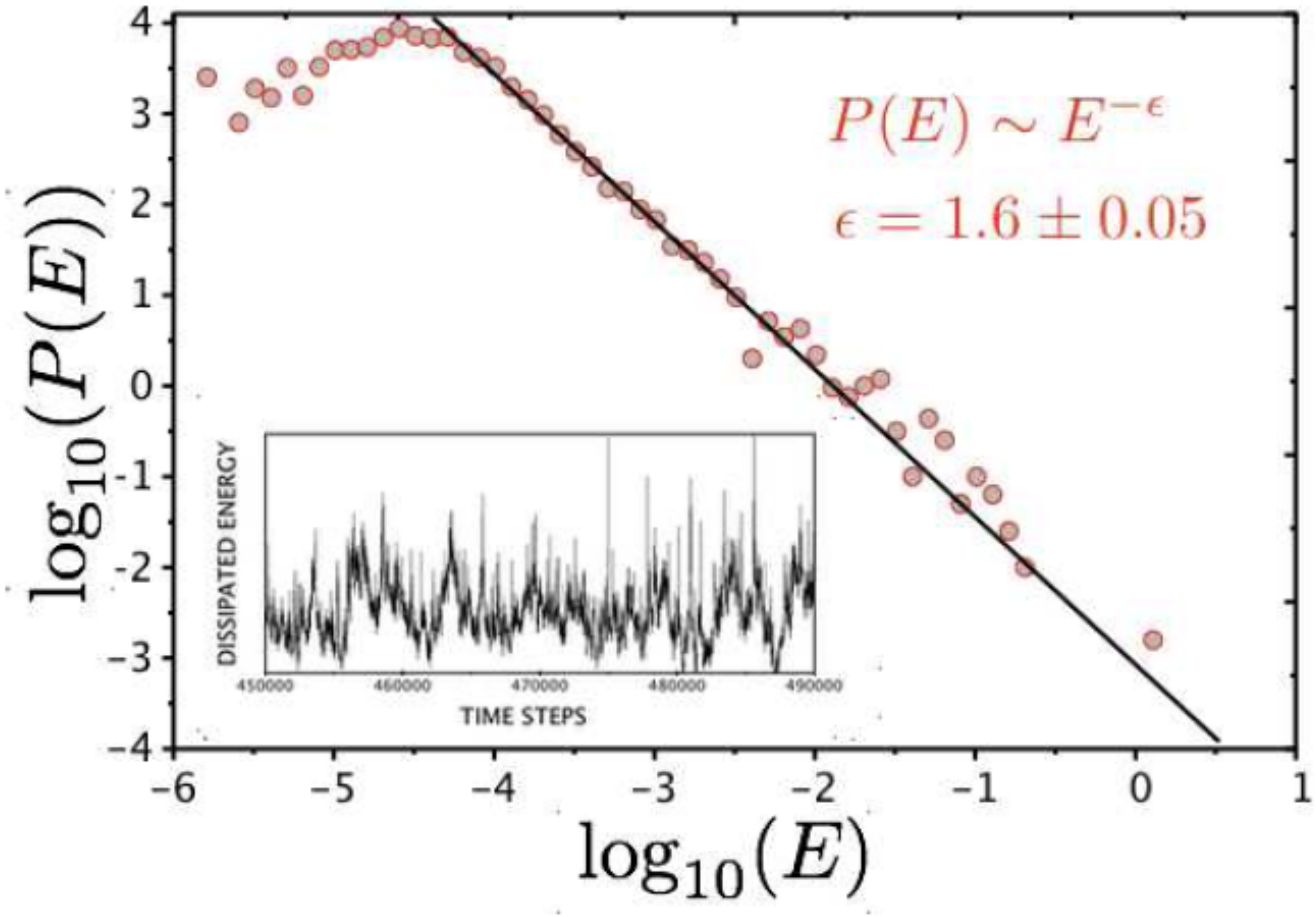




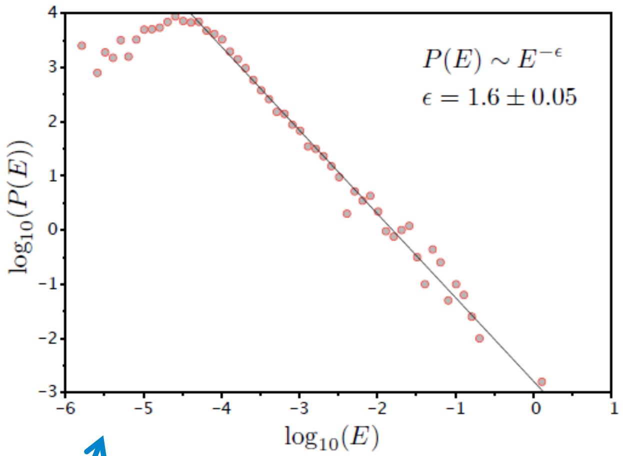
Fast time movie

Slow time movie

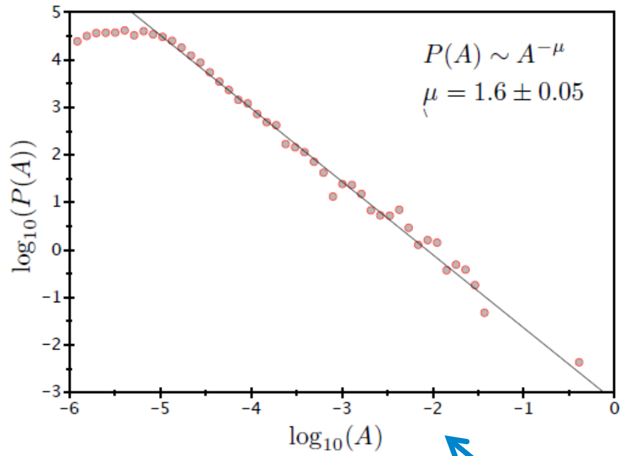
Temporal correlations: power law



Temporal correlations



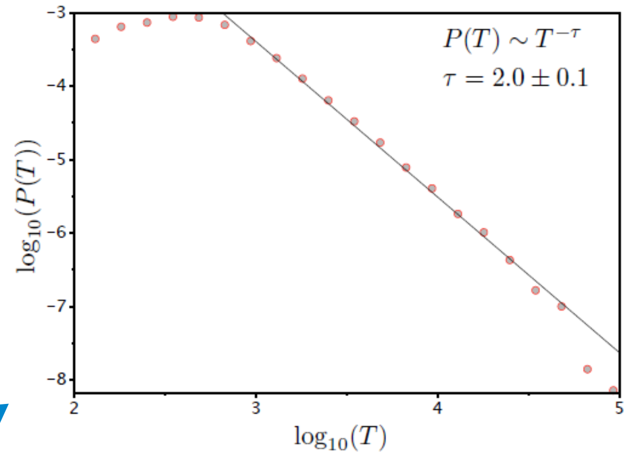
(a)



(b)

Dissipated energy

Slip area

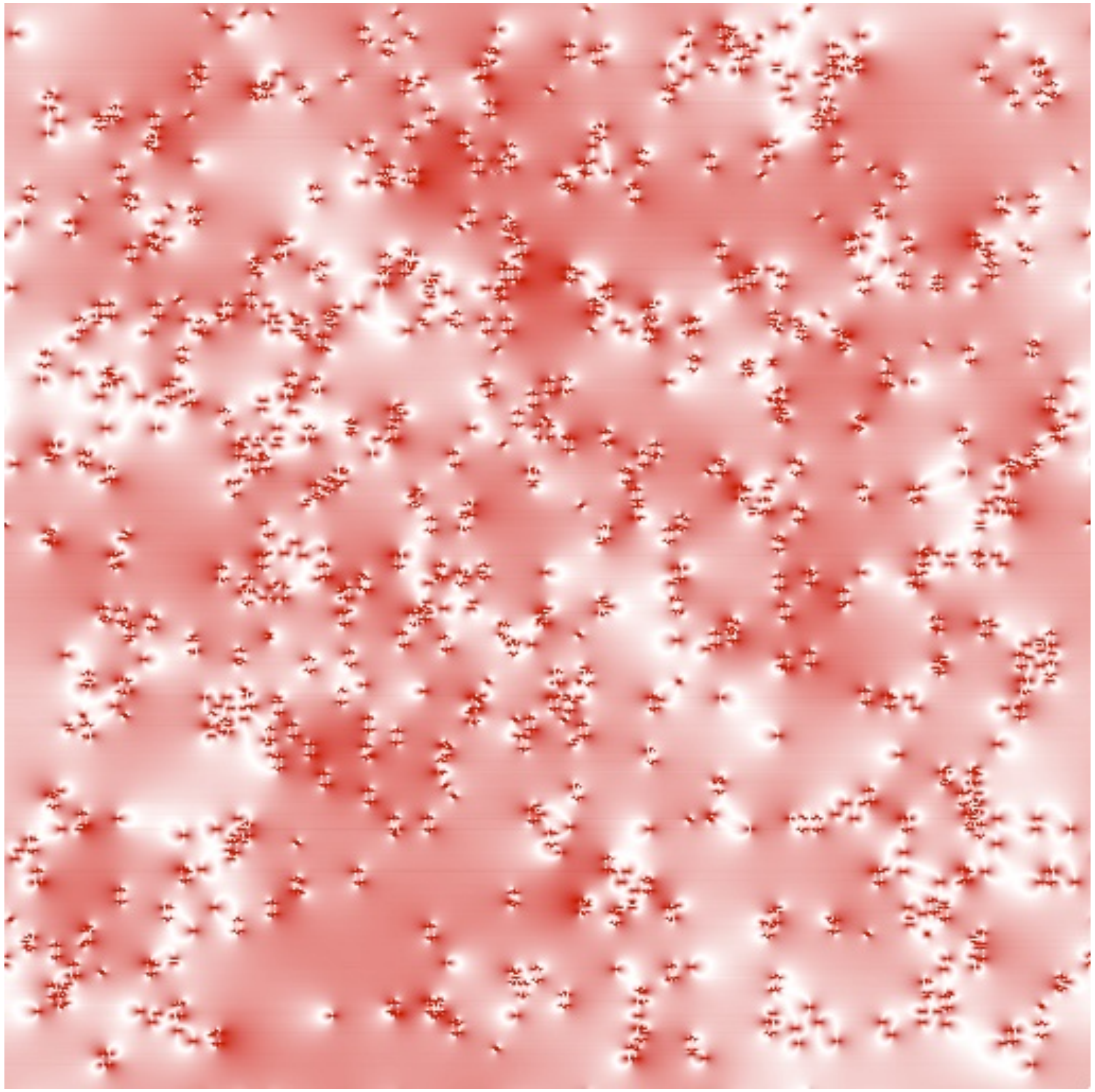


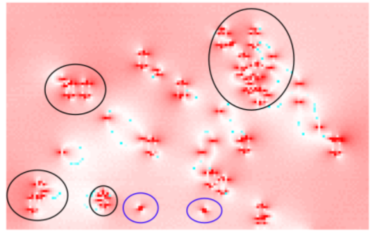
(c)

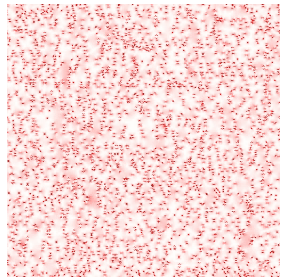
Avalanche durations

Salman, LT, PRL, 2011

Stress field

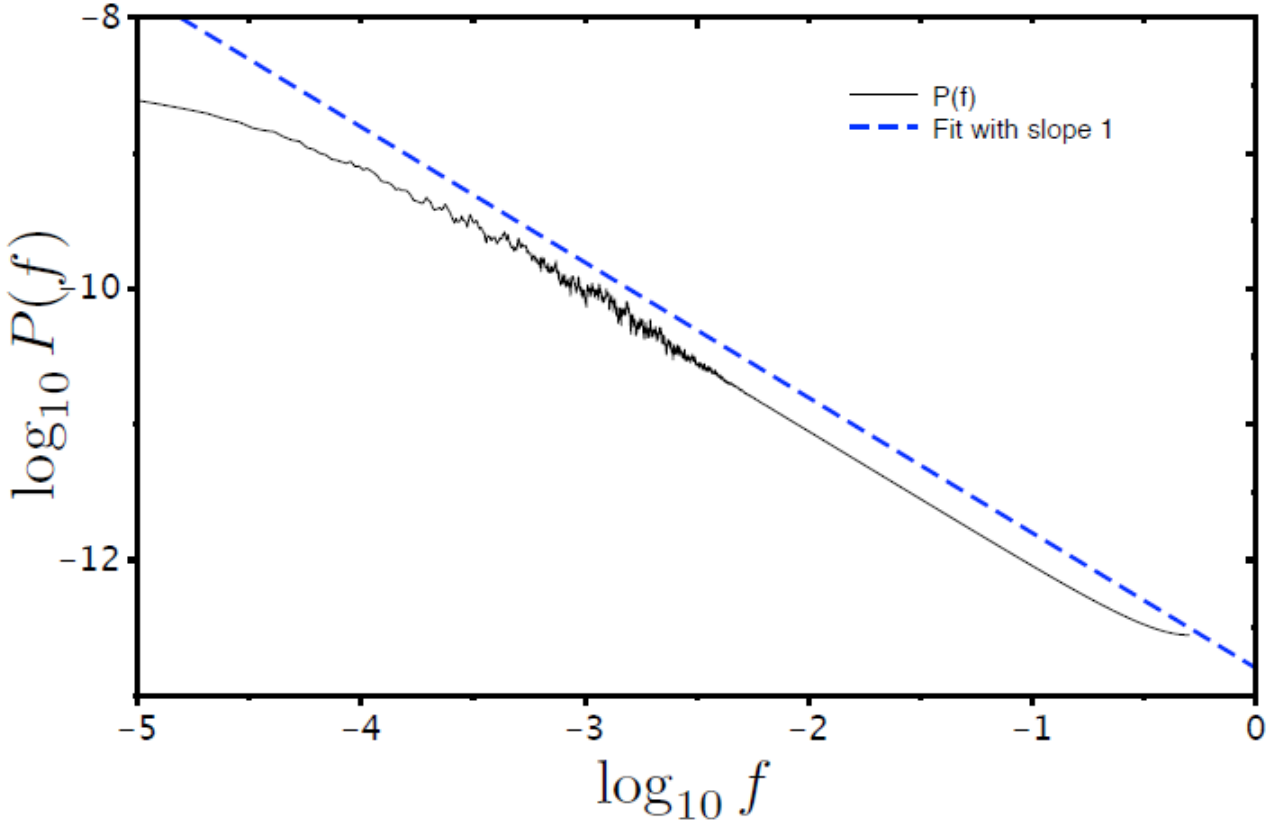




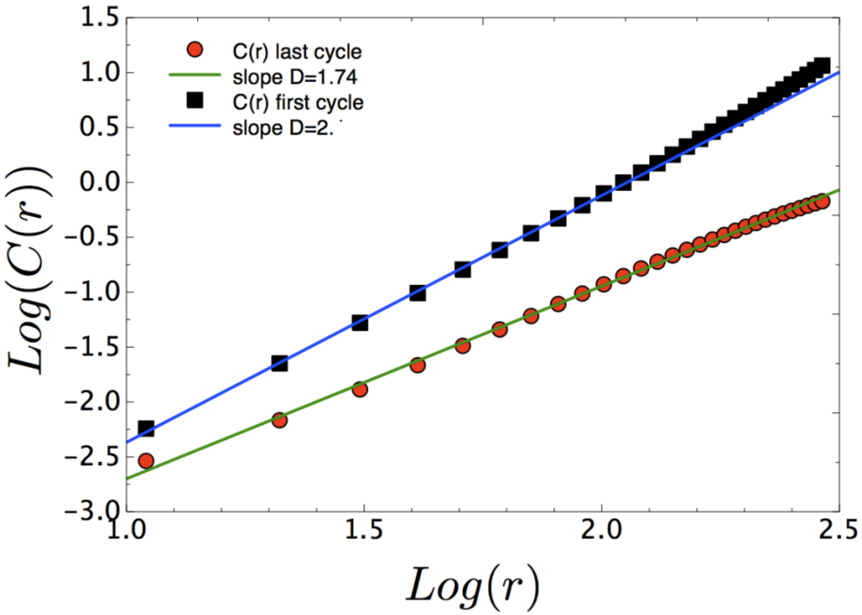


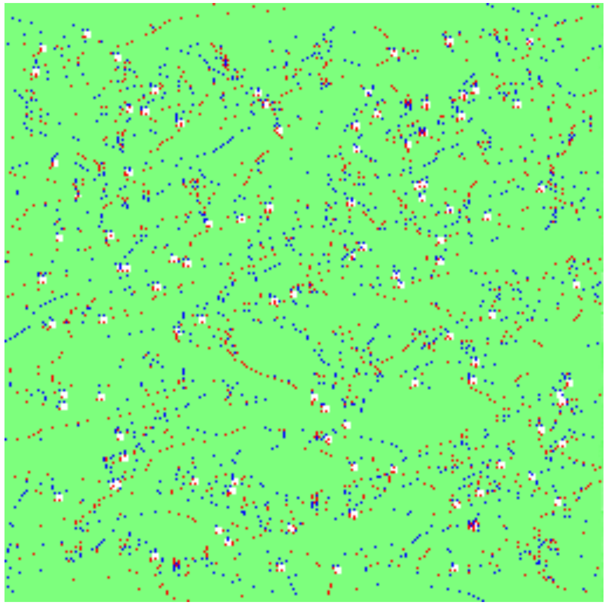
Slow time movie

Power spectrum: 1/f noise

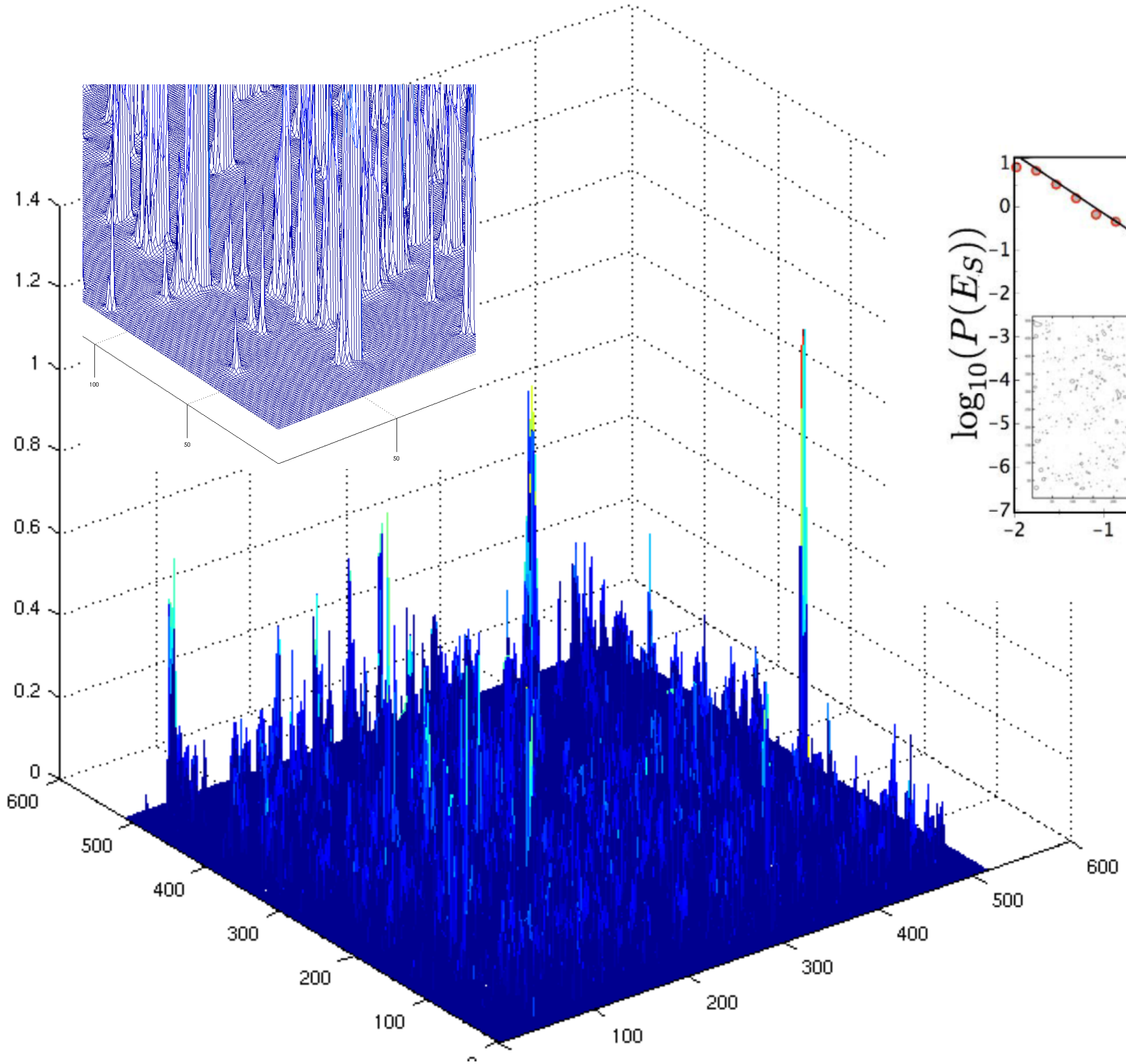


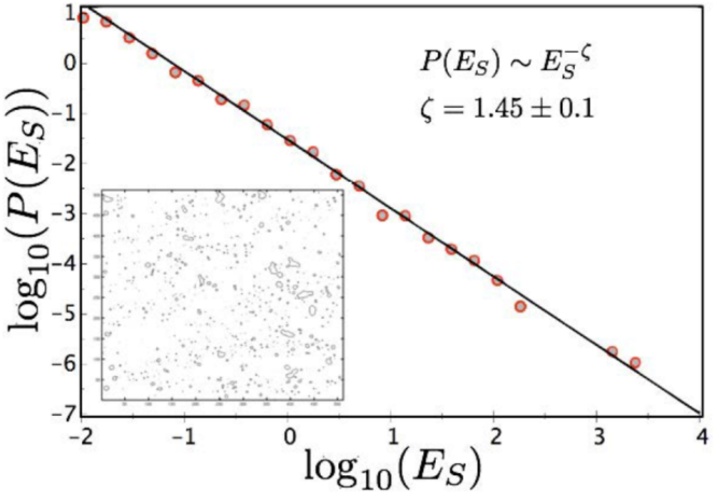
Spatial correlations



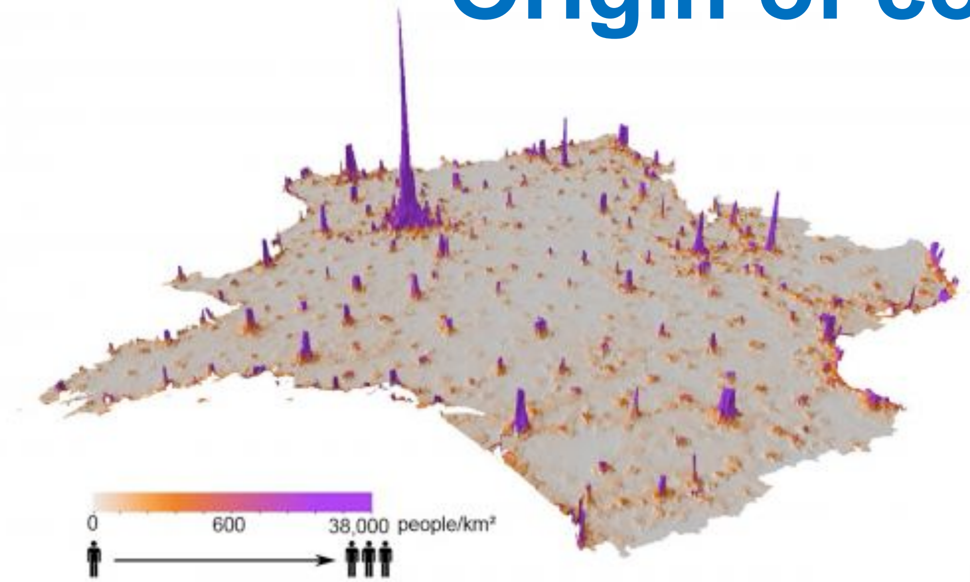


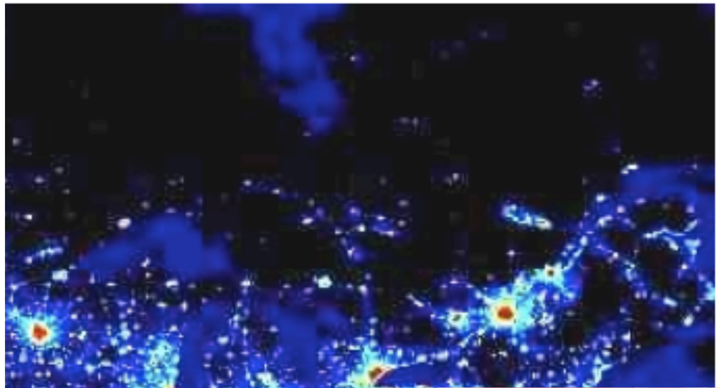
Spatial energy distribution

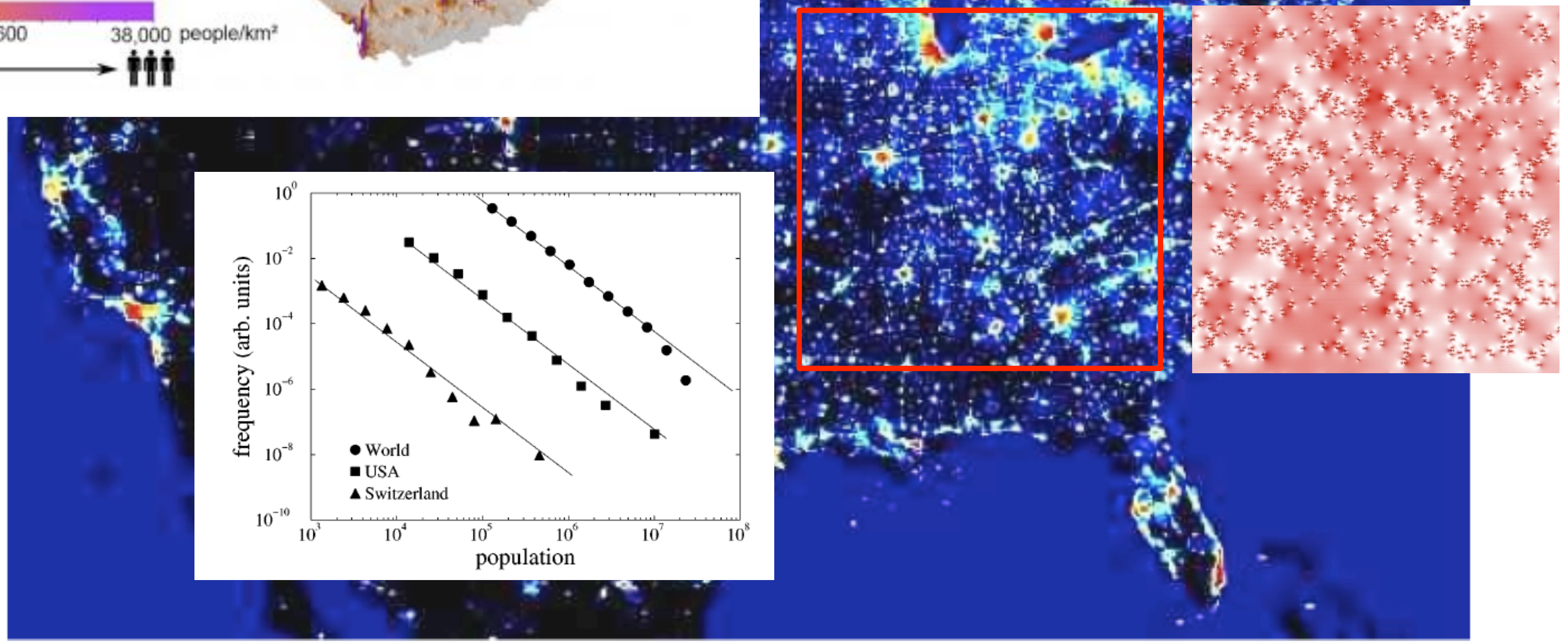




Origin of correlations?





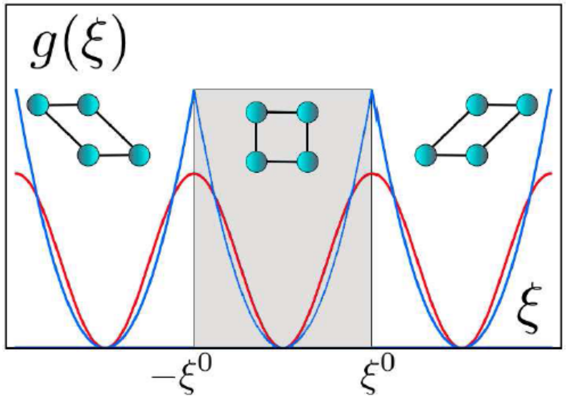


Equivalent automaton model

$$\nu \dot{u} = -\partial \Phi(u) / \partial u$$

$$\nu \to 0$$

$$\partial \Phi / \partial u = 0$$



piece-wise quadratic potential

$$g(\xi) = \frac{1}{2}(\xi - d)^2$$

integer valued field: plastic strain

$$d(i, j)$$

Elimination of elastic fields

$$\hat{\xi}(\mathbf{q}) = (s_y^+(\mathbf{q})s_y^-(\mathbf{q})\hat{d}(\mathbf{q}) + \hat{H}(\mathbf{q}))/\hat{\lambda}(\mathbf{q})$$

quenched disorder

$$\hat{H}(\mathbf{q}) = s_x^-(\mathbf{q})s_x^+(\mathbf{q})\hat{h}_1(\mathbf{q}) + s_y^-(\mathbf{q})s_y^+(\mathbf{q})\hat{h}_2(\mathbf{q})$$

elasticity

$$s_a^{\mp}(\mathbf{q}) = \pm(1 - \cos(q_a) \pm i\sin(q_a))$$

$$\hat{\lambda}(\mathbf{q}) = 2K(\cos(q_x) - 1) + s_y^-(\mathbf{q})s_y^+(\mathbf{q})$$

Thresholds

$$\Delta\xi = \xi - (\xi_0 + \xi_h)$$

$$\hat{\xi}_0(\mathbf{q}) = t\delta(\mathbf{q})$$

$$\hat{\xi}_h(\mathbf{q}) = \hat{H}(\mathbf{q})/\hat{\lambda}(\mathbf{q})$$

$$-\xi^0 - \xi(h,t) < \Delta\xi(i,j) < \xi^0 - \xi(h,t)$$

$$\xi(h,t) = [\hat{\xi}_h]^{-1}_{\mathbf{q}} + t$$

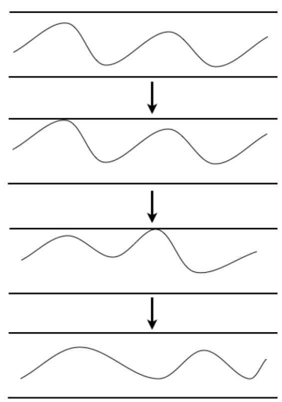
Automaton

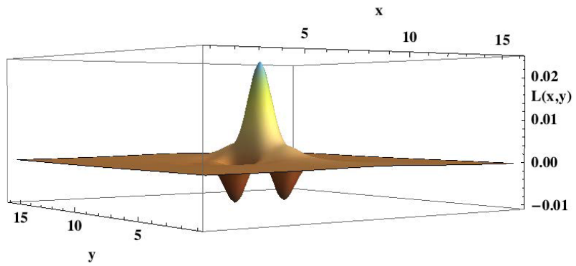
$$d \rightarrow d + M(\Delta\xi)$$

$$M(\Delta\xi) = \begin{cases} +1, \text{ if } \Delta\xi > \xi^{0} - \xi(h,t), \\ -1, \text{ if } \Delta\xi < -\xi^{0} - \xi(h,t) \\ \quad 0 \text{ otherwise.} \end{cases}$$

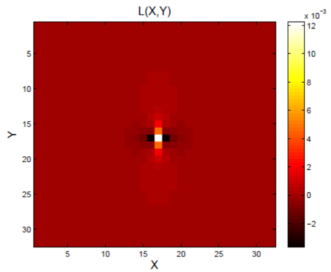
$$\hat{\Delta}\xi \rightarrow \hat{\Delta}\xi - \hat{L}(\mathbf{q})\hat{M}(\Delta\xi)$$

$$\hat{L}(\mathbf{q}) = \frac{\sin(q_y/2)^2}{\sin(q_y/2)^2 + K\sin(q_x/2)^2}$$



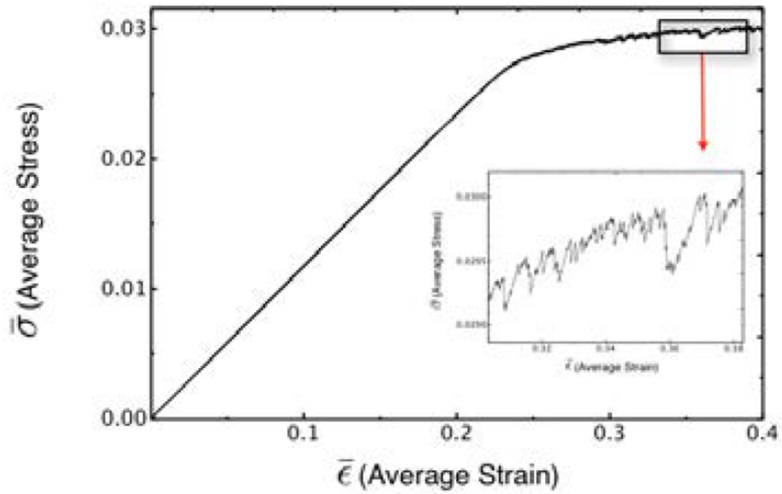


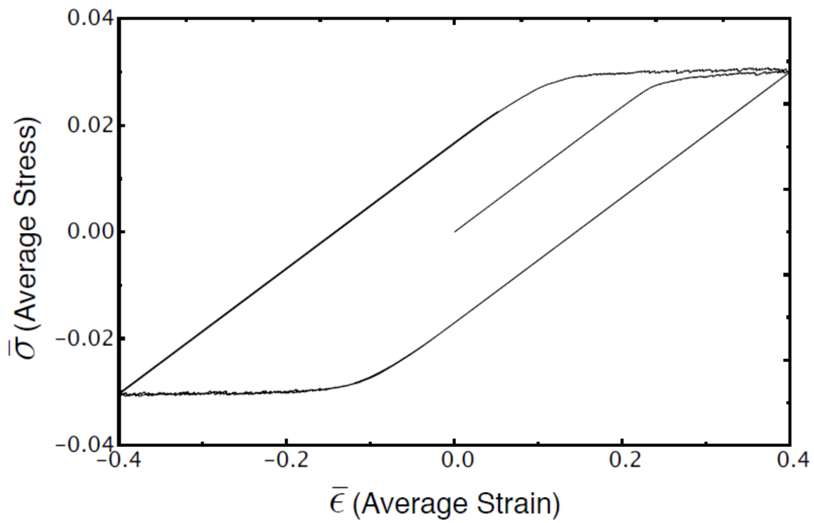
(a)

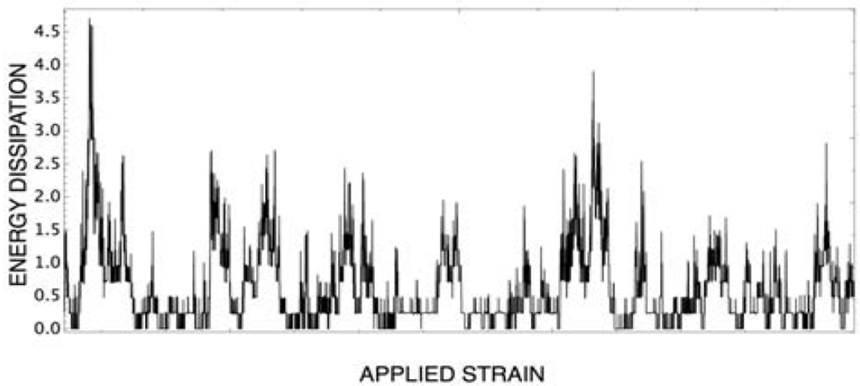


(b)

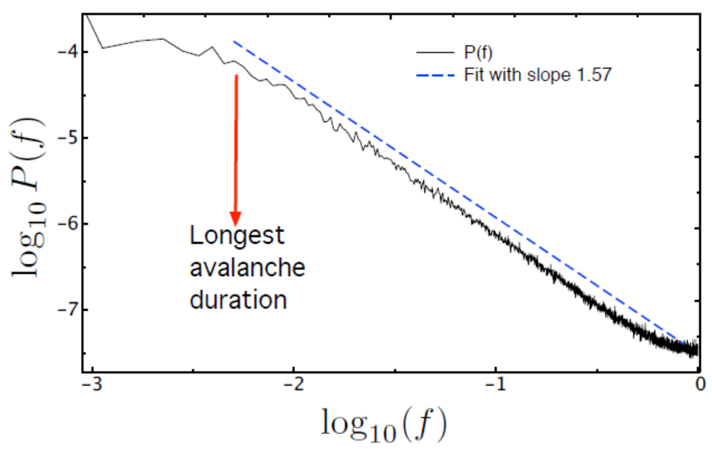
Macroscopic response

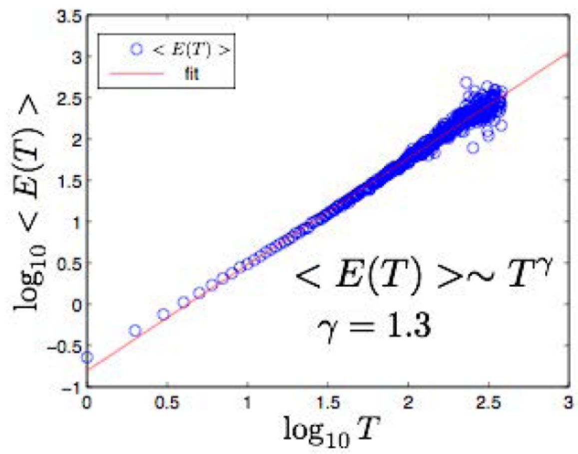


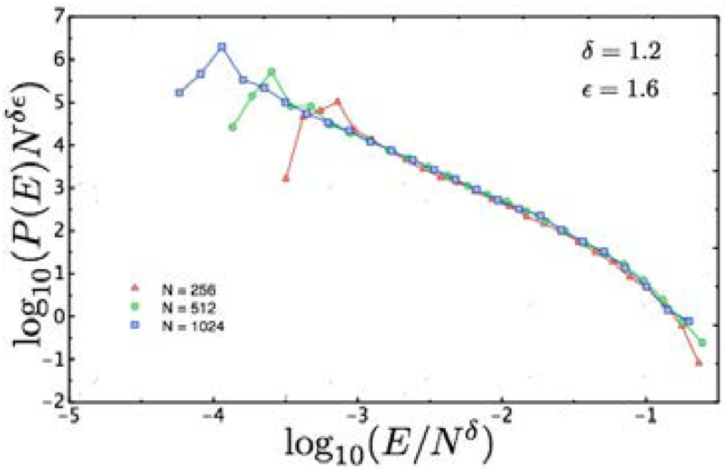




Scaling relations in automaton model



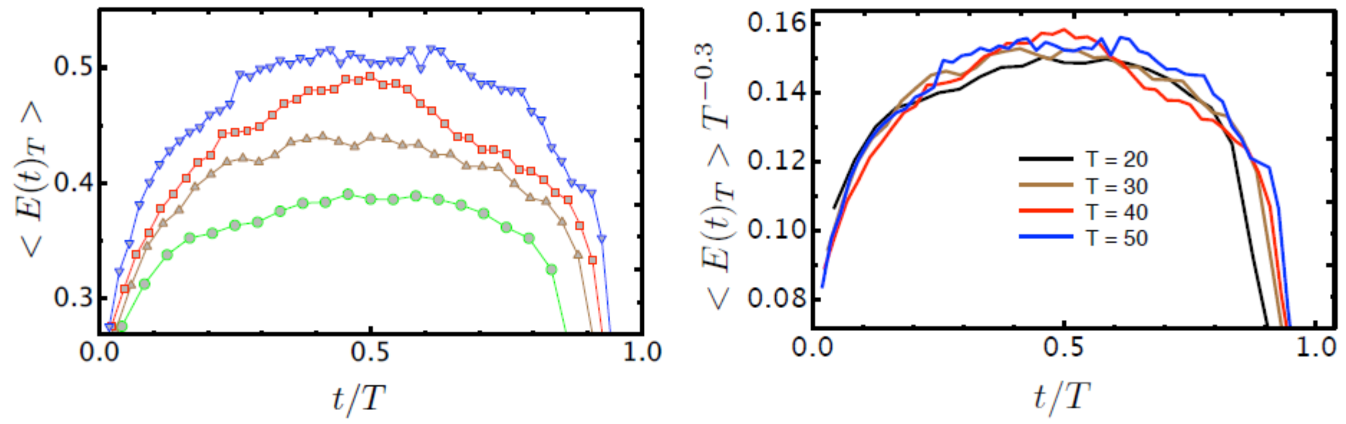




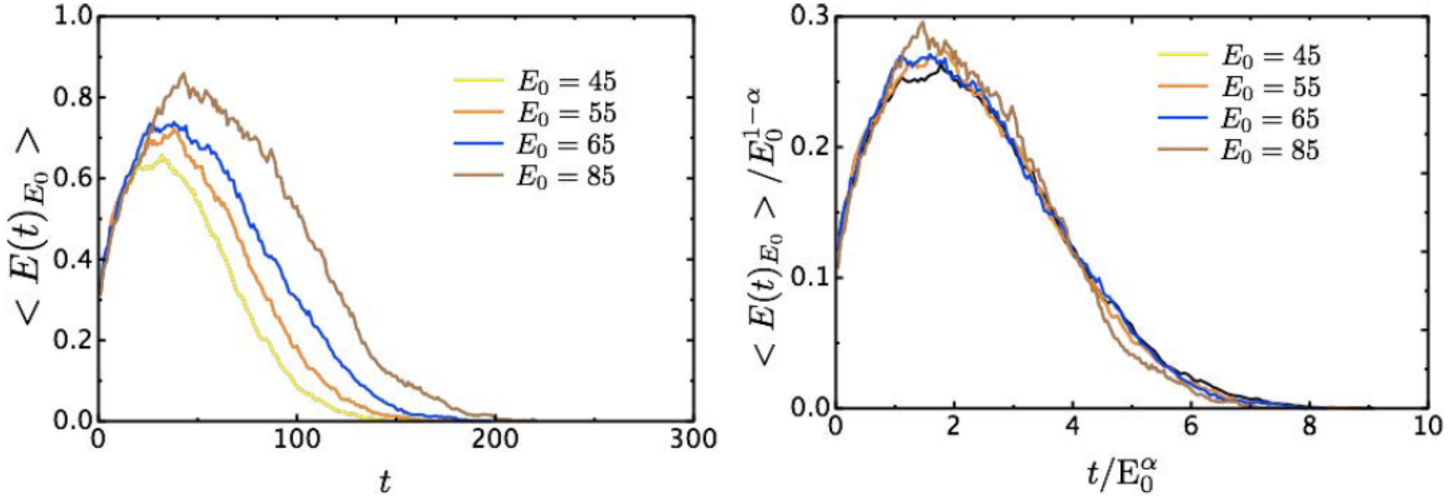
$$P(E) = E^{-\epsilon}\varphi(E/E_c)$$

$$E_c \sim N^{\delta}$$

Scaling collapse of avalanche shapes



$$< E(t)_T >= T^{\gamma-1} e(t/T)$$



$$< E(t)_{E_0} >= E_0^{1-\alpha} \tilde{e}(t/E_0^{\alpha})$$

Conclusions

- Model of crystal plasticity allowing one to study cooperative effects in dislocation dynamics *without additional assumptions* such as nucleation rules and kinetic relations. The model shows an excellent agreement with experimental observations of fluctuations during plastic yielding in HCP crystals.

- In the limit of slow driving continuous dynamics can be reduced to an integer automaton. Despite partial linearization implied by the use of piece-wise quadratic potential and the replacement of fast stages of dynamics by a sequence of jumps, the automaton model exhibits the *same critical behavior* as the original ODE based model.

- Our numerical study suggests that the automaton model has some form of *Abelian property* which makes it amenable in principle to rigorous mathematical treatment.

References:

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