# Memory effects in avalanche dynamics: a key to the statistical properties of earthquakes

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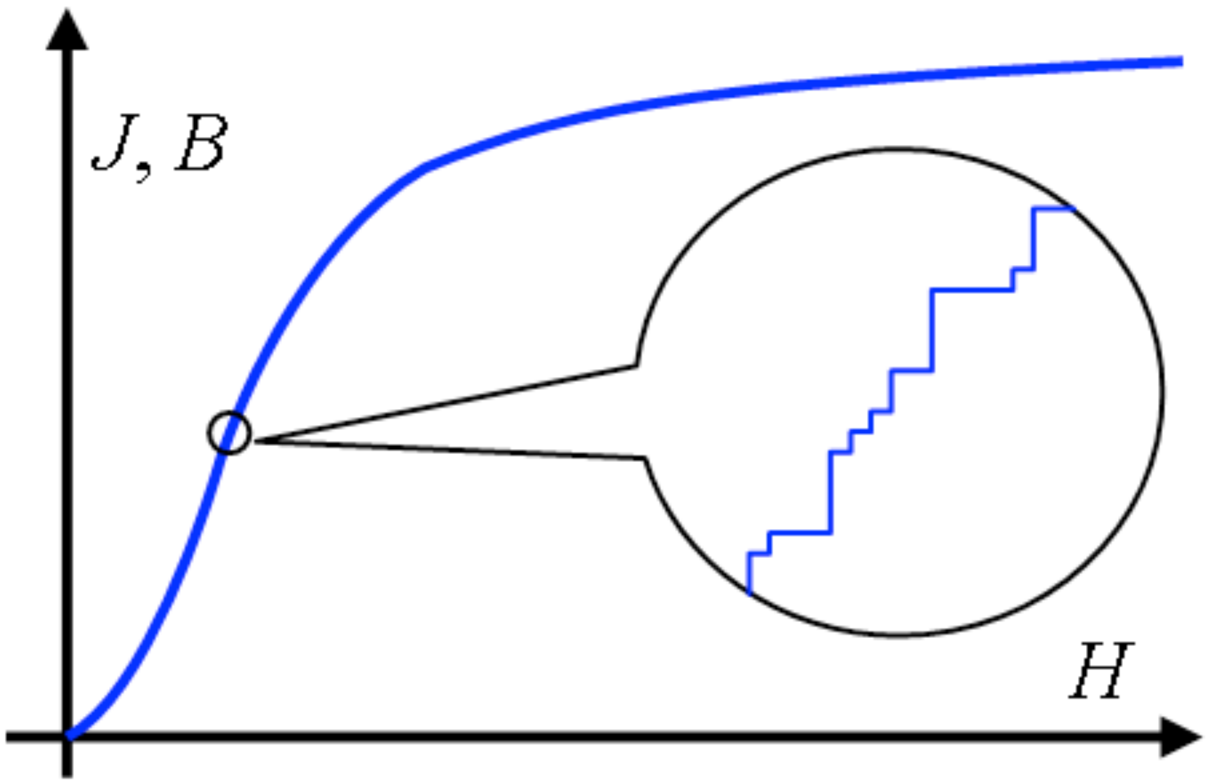
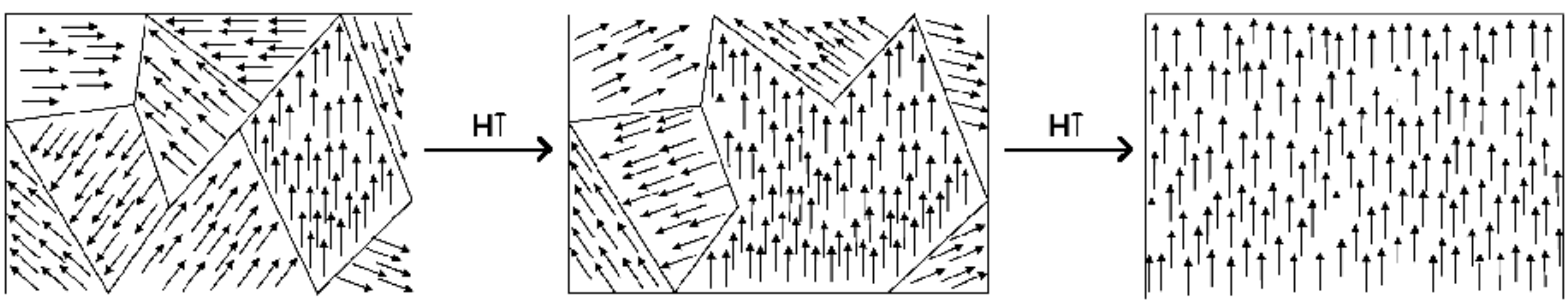
**Phys. Rev. Lett. (2014)**

# Barkhausen noise

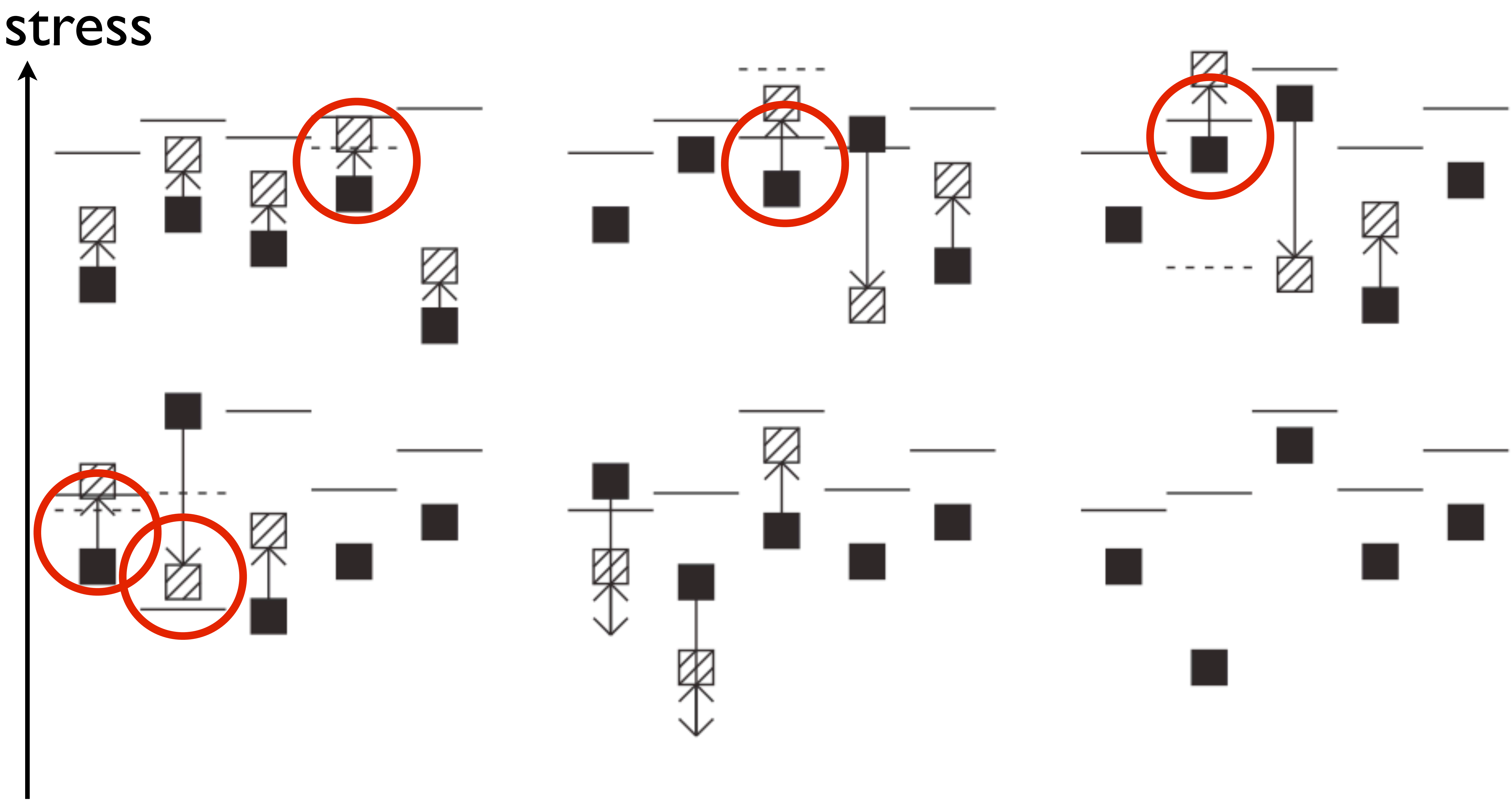



H increases slowly while the magnetization J displays little jumps: the avalanches




Interpretation: Domain walls dynamics

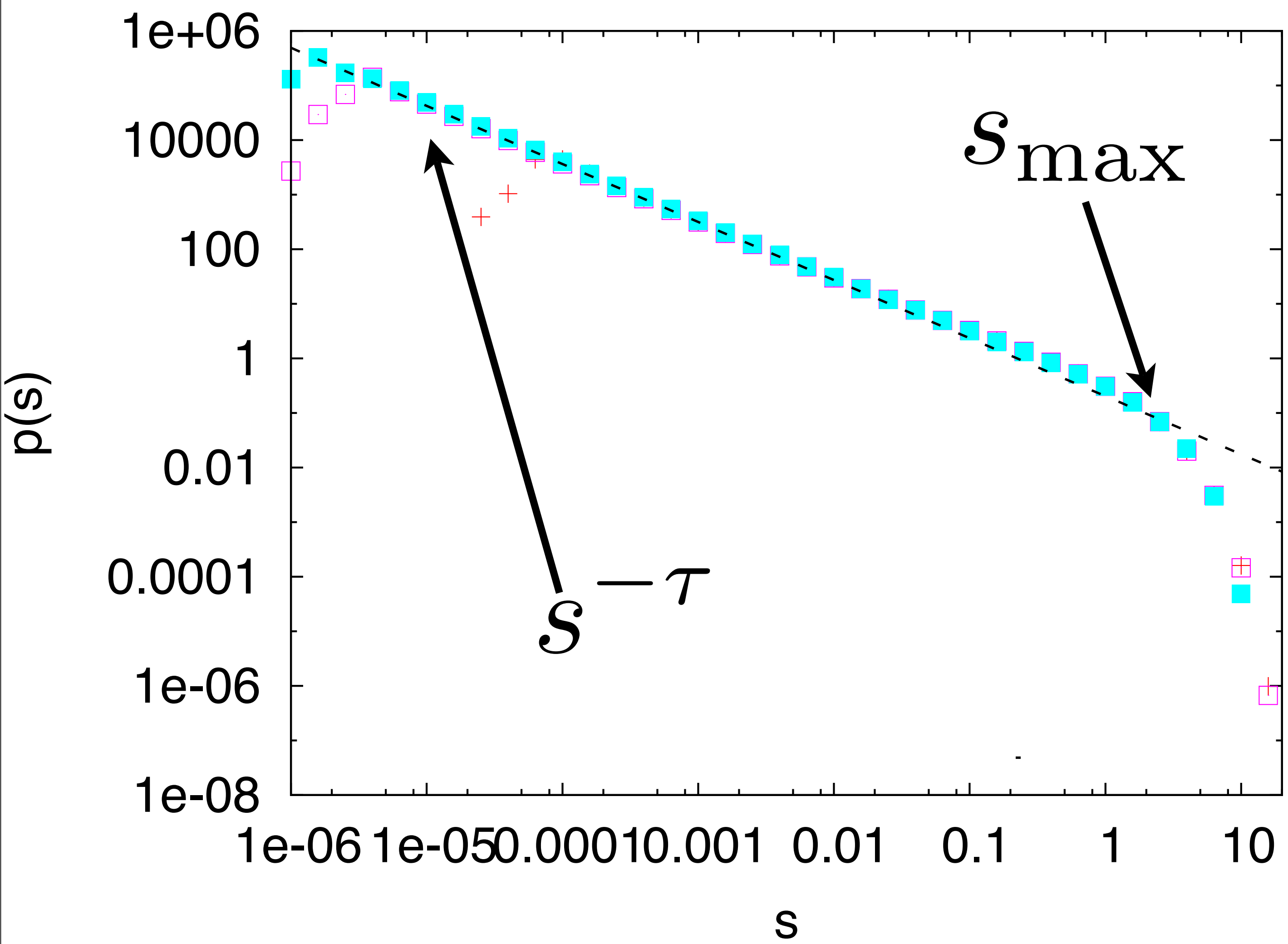
# Avalanche dynamics




Avalanche magnitude: S=5 (number of active sites)

# Avalanche models: main ingredients and success

- Energy injected and dissipated: out-of-equilibrium
- Existence of random thresholds
- Extended system with strong interactions



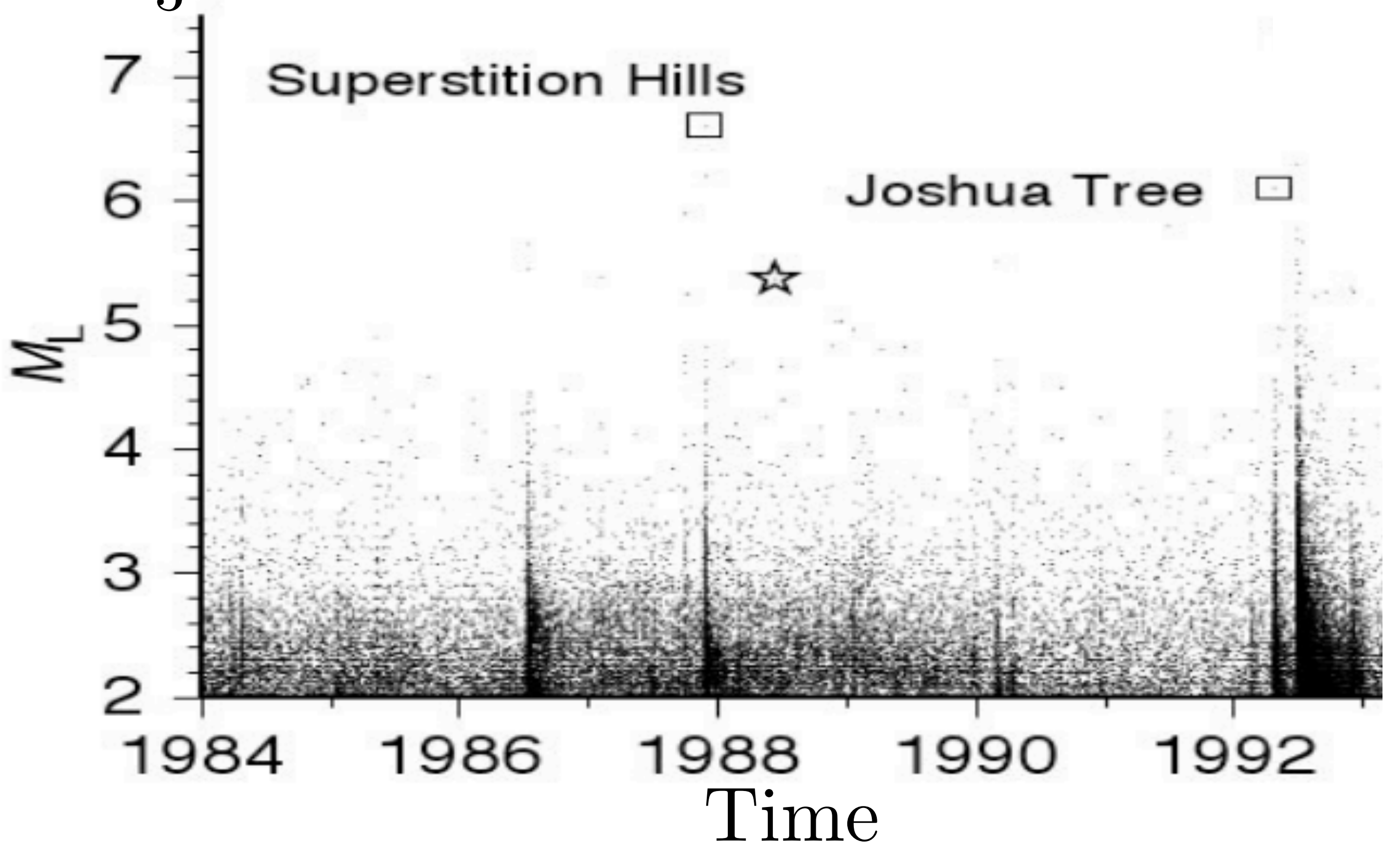

- free scale statistics: universality
- cut-off controlled by the dissipation

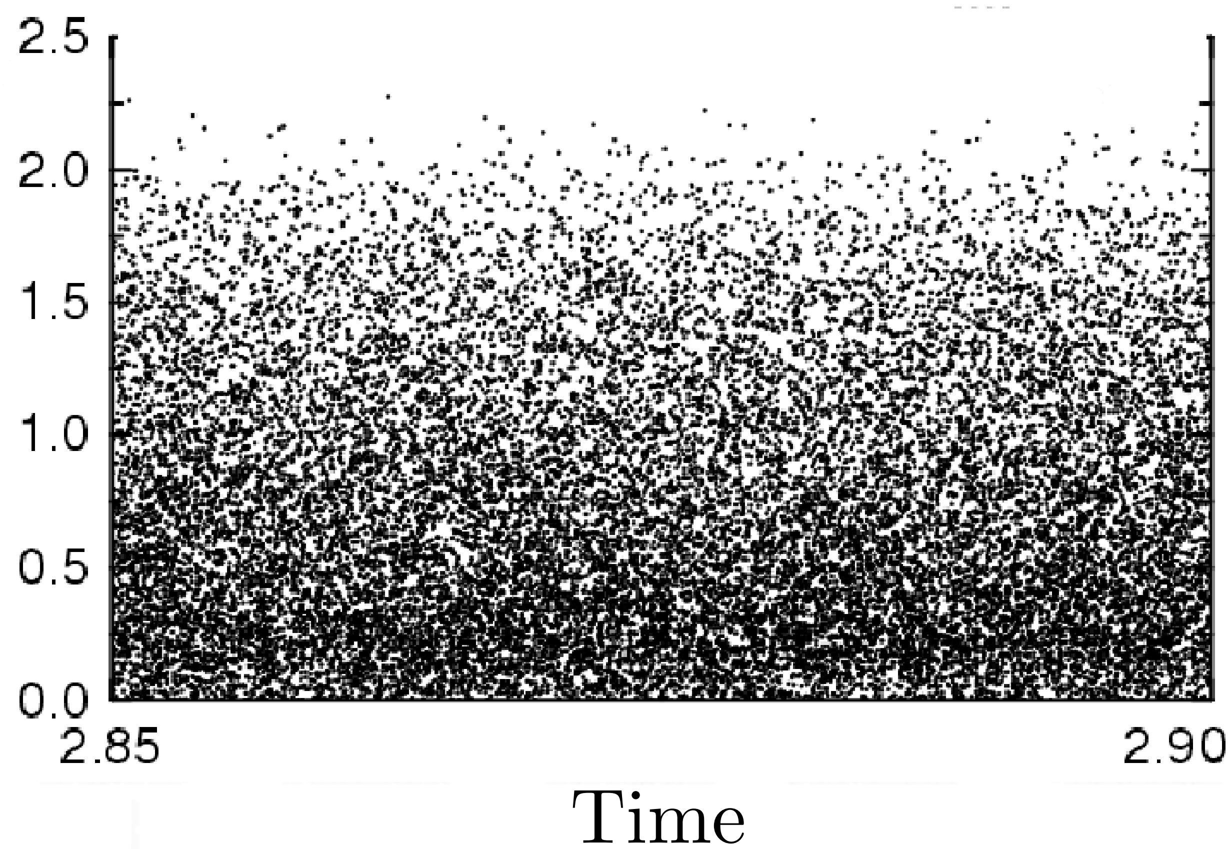
# Limits: Gutenberg Richter law, aftershocks ...

- Gutenberg Richter exponent $\tau_{GR} \simeq 1.7$
- Correlated Aftershocks

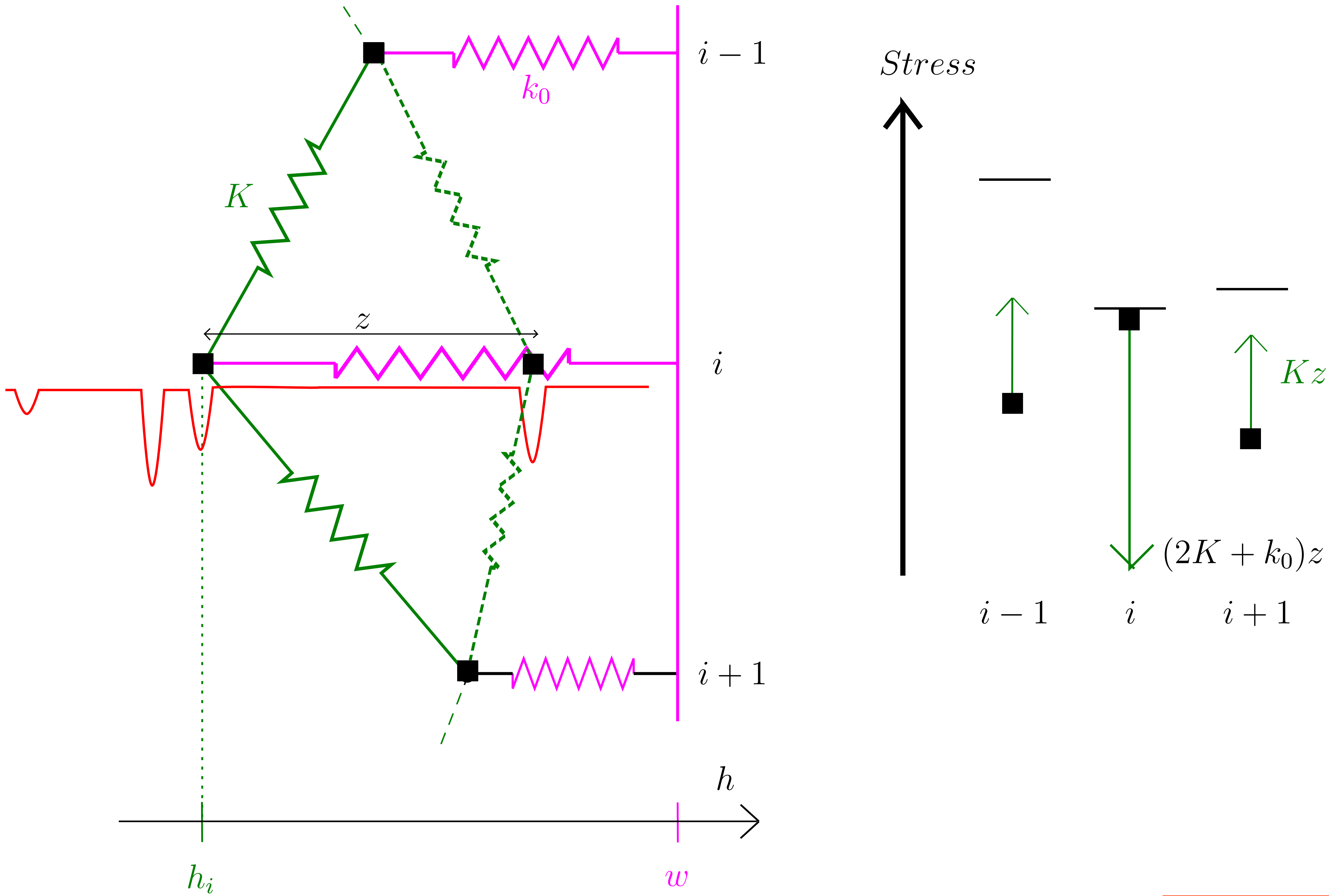
- Magnitude Exponent $\tau \leq 1.5$
- Uncorrelated avalanches

$M = \frac{2}{3} \log S$



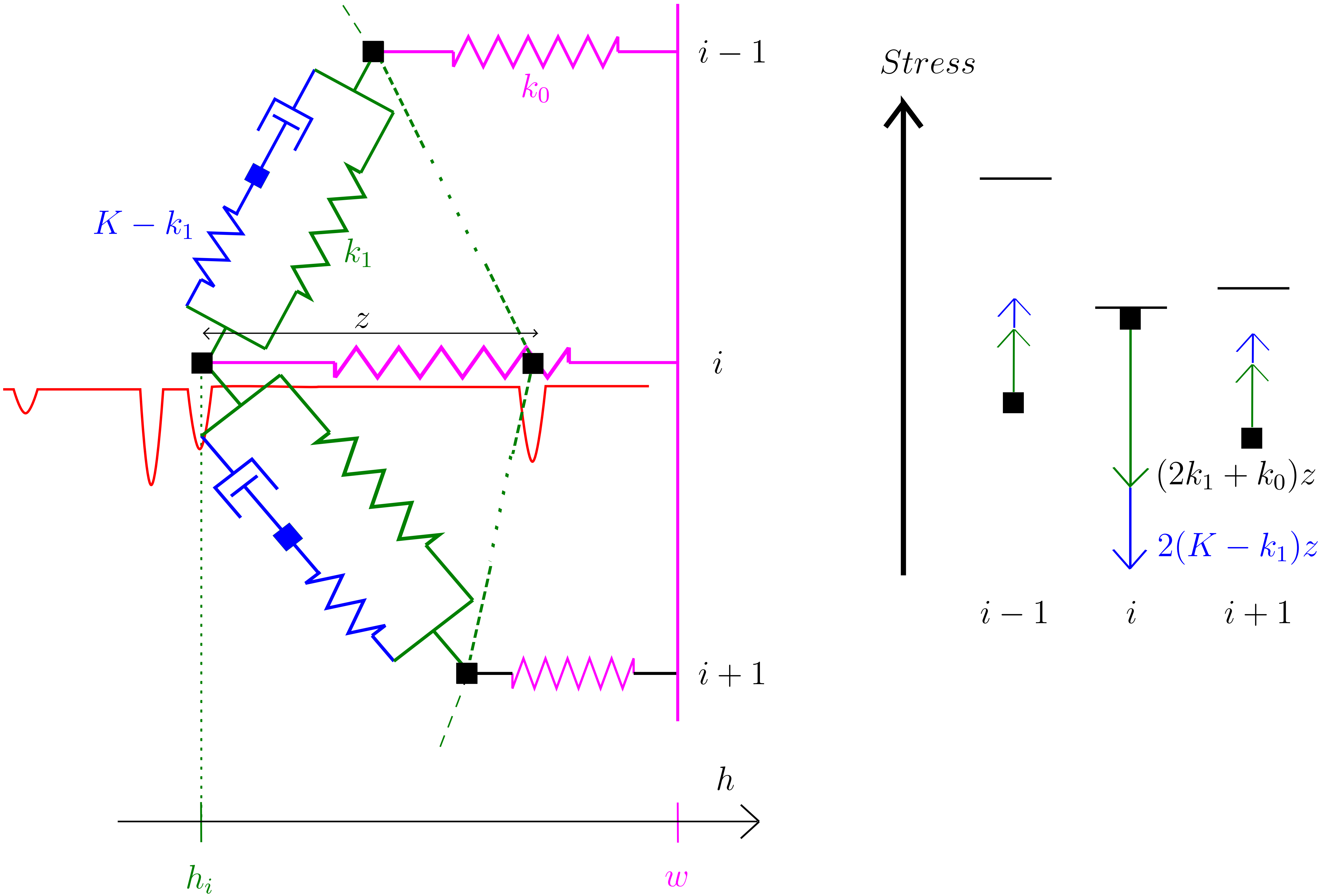



# Elastic Solid driven on a disordered substrate




$$\partial_t h_i(t) = \underbrace{k_0(w-h_i) + K(h_{i+1}+h_{i-1}-2h_i)}_{\sigma(h_i)} + \sigma^{th}(h_i)$$

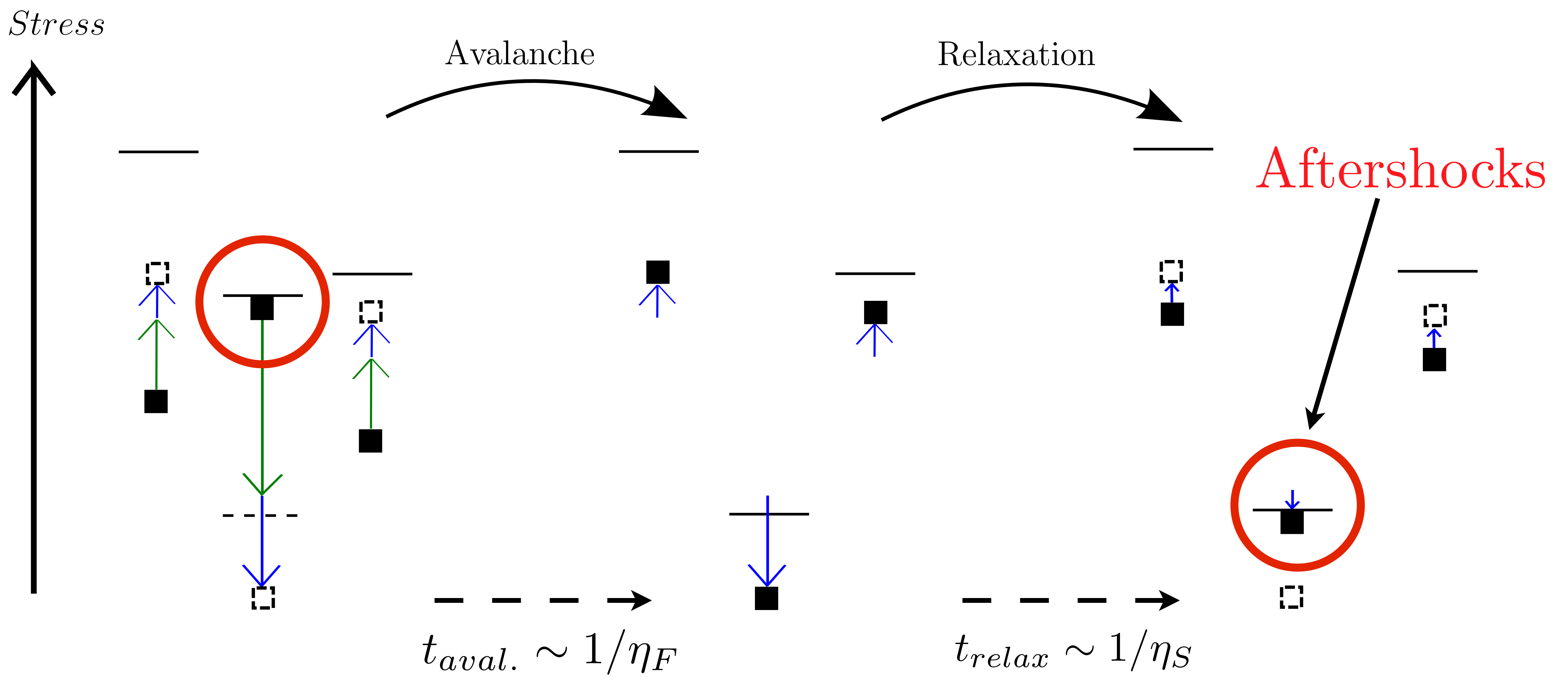
# Visco-Elastic Solid driven on a disordered substrate




$$\eta_F \partial_t h_i(t) = k_0(w-h_i) + k_1 \Delta h_i + (K-k_1)(\Delta h_i - u_i) + \sigma^{th}(h_i)$$

$$\eta_S \partial_t u_i(t) = (K-k_1)(\Delta h_i - u_i)$$

# Visco-Elastic Solid and Aftershocks



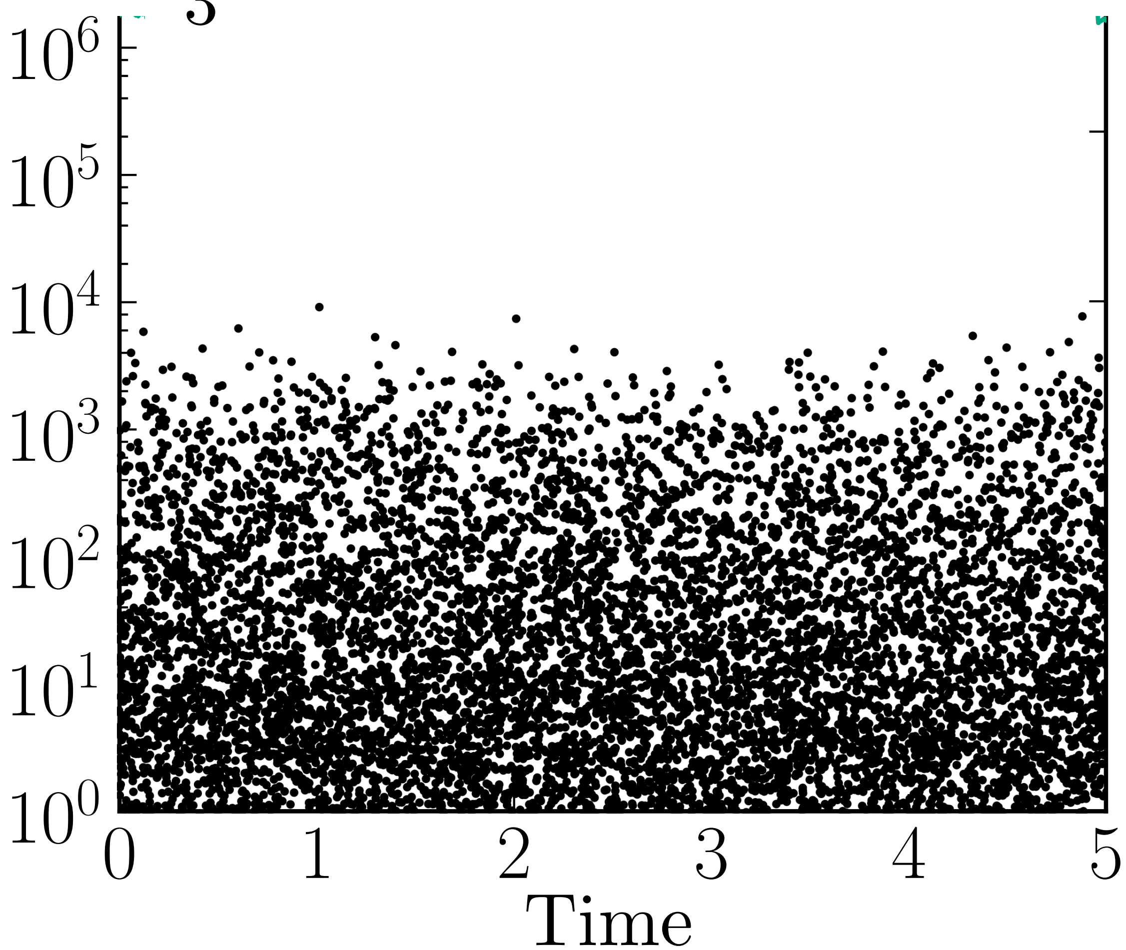

$$\eta_F \partial_t h_i(t) = k_0(w - h_i) + k_1 \Delta h_i + (K - k_1)(\Delta h_i - u_i) + \sigma^{th}(h_i)$$

$$\eta_S \partial_t u_i(t) = (K - k_1)(\Delta h_i - u_i)$$

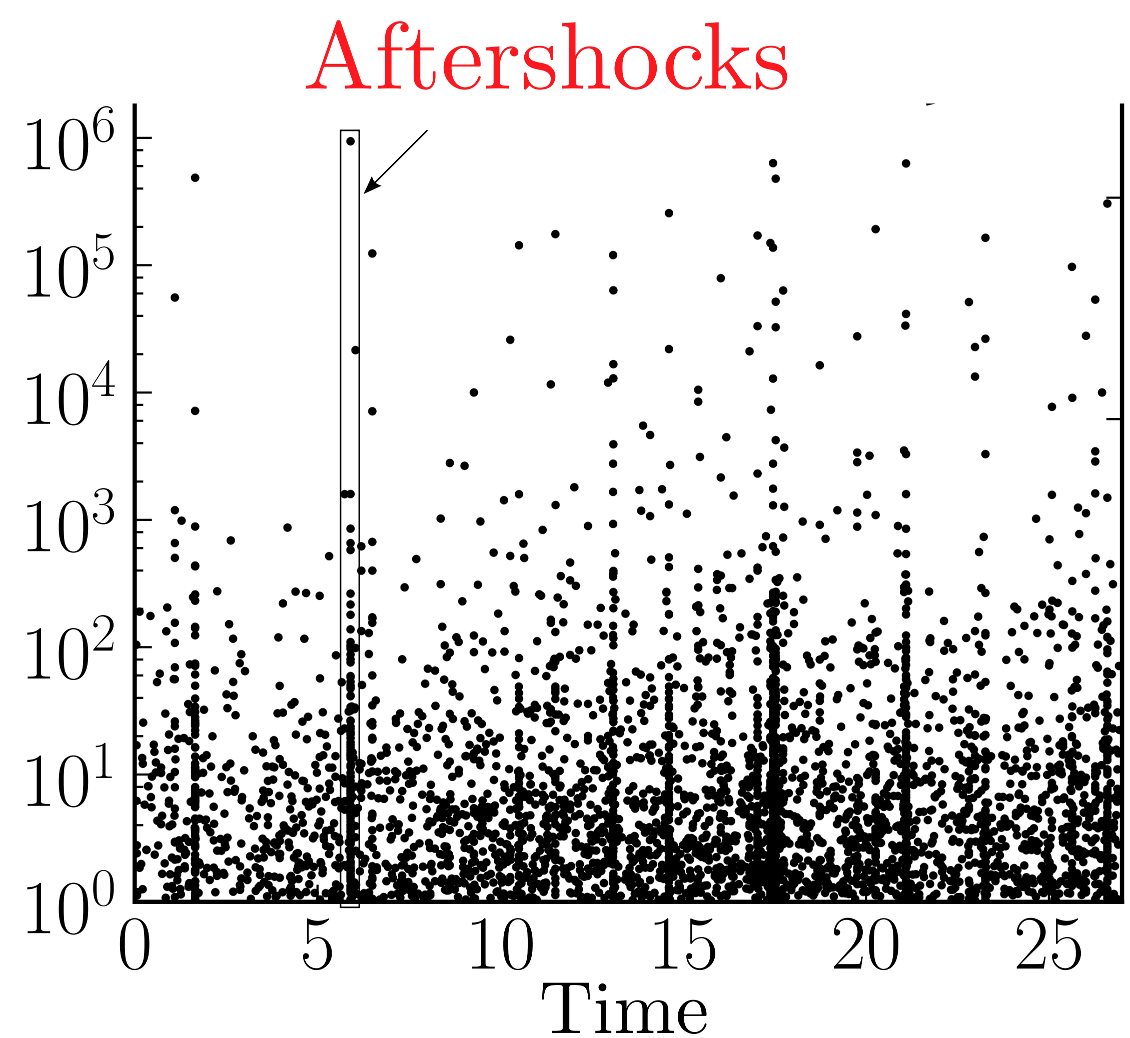
# Aftershocks Dynamics

## 2D Elastic Solid

$$M = \frac{2}{3} \log S$$

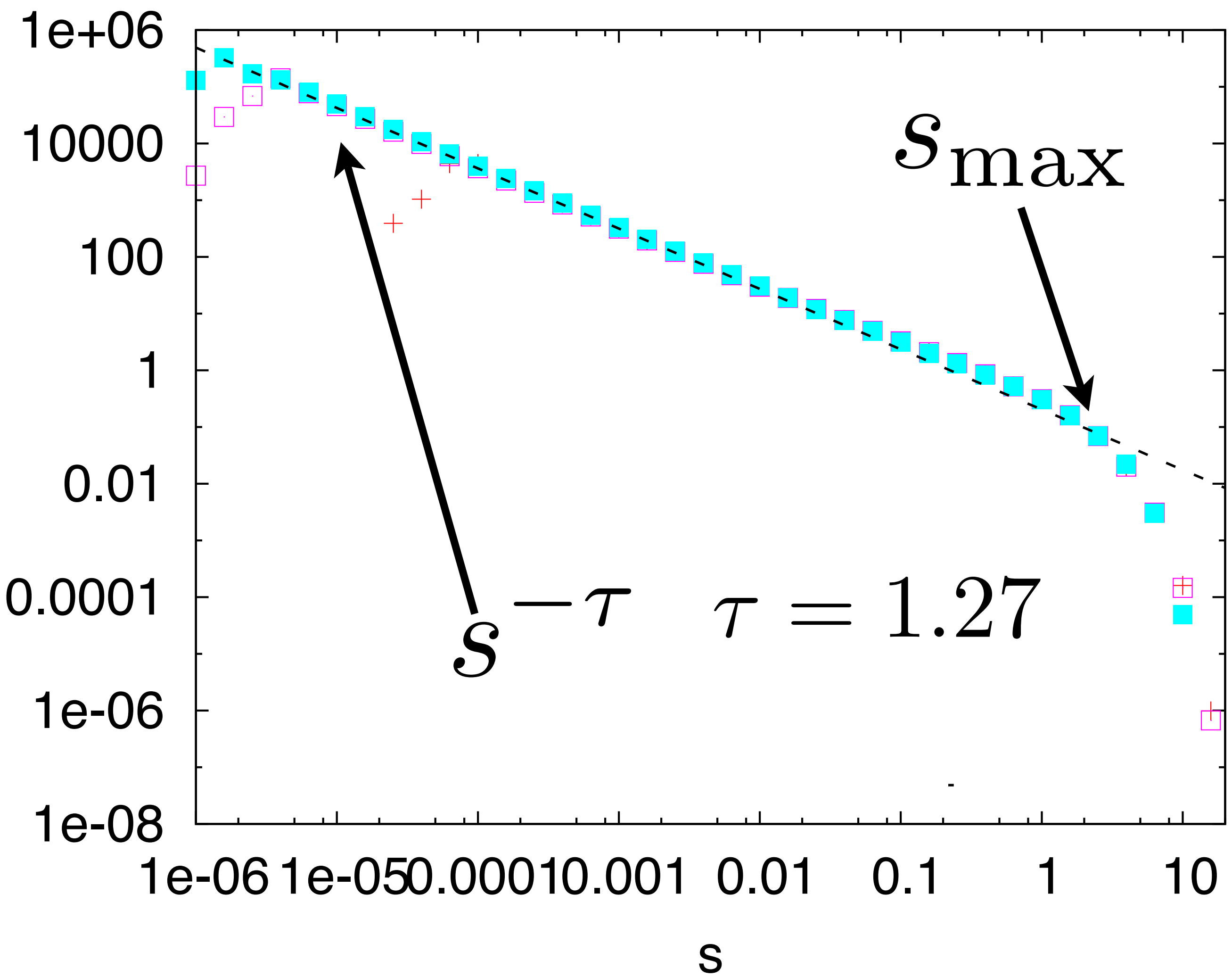



## 2D Visco-Elastic Solid

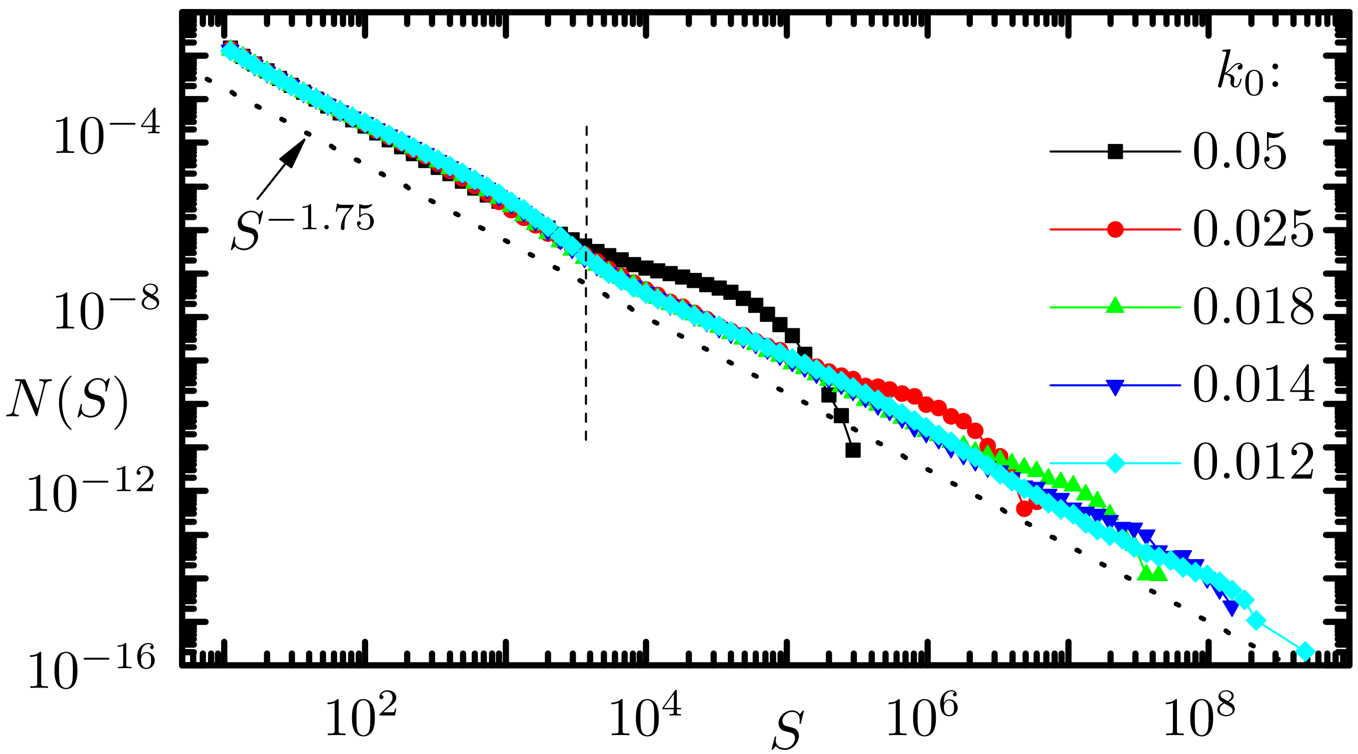


# Gutenberg Richter Law

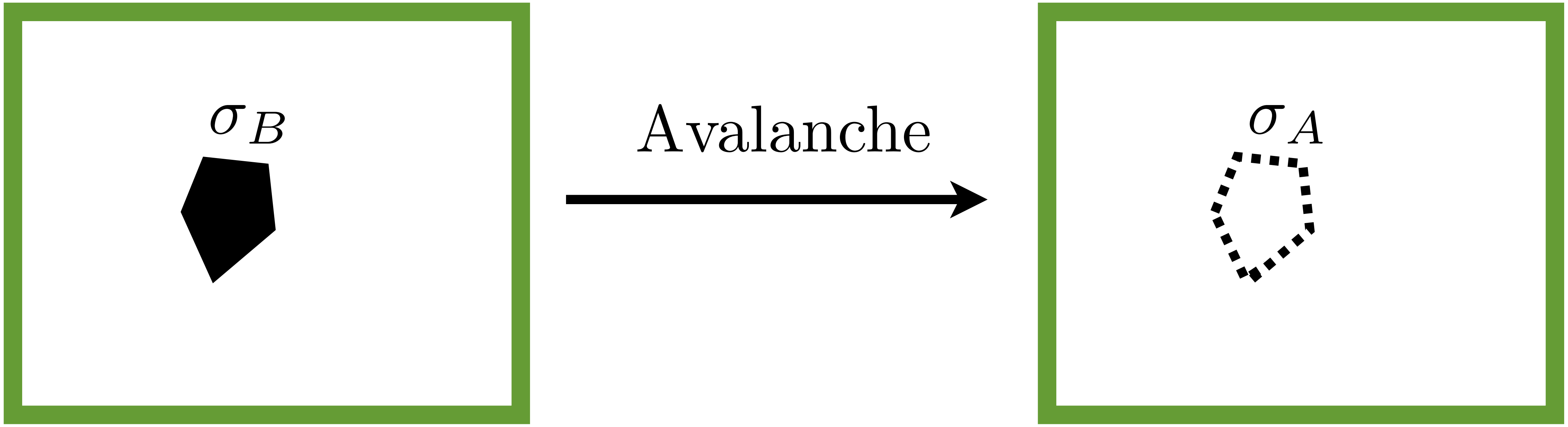
## 2D Elastic Solid

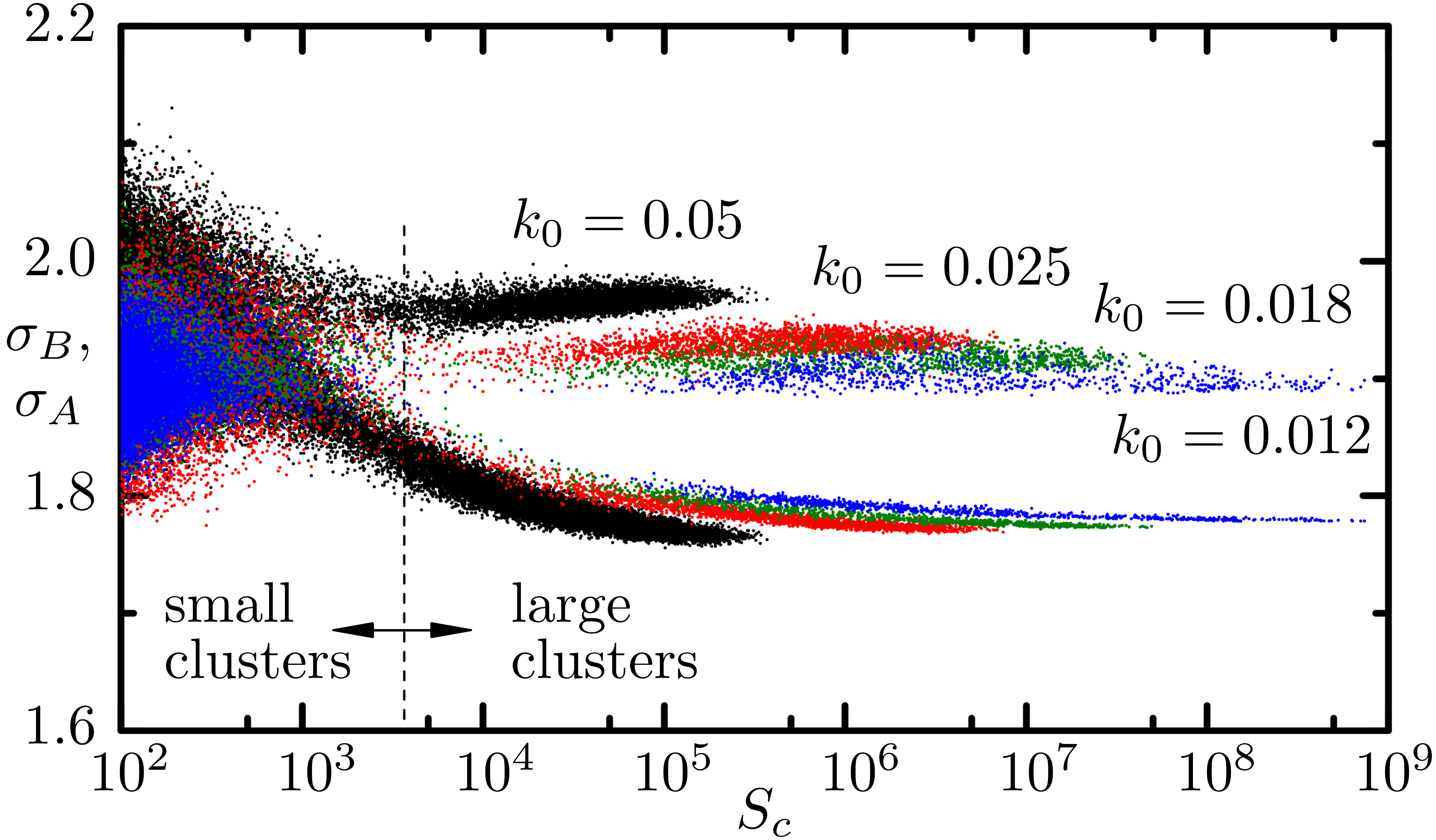



## 2D Visco Elastic Solid

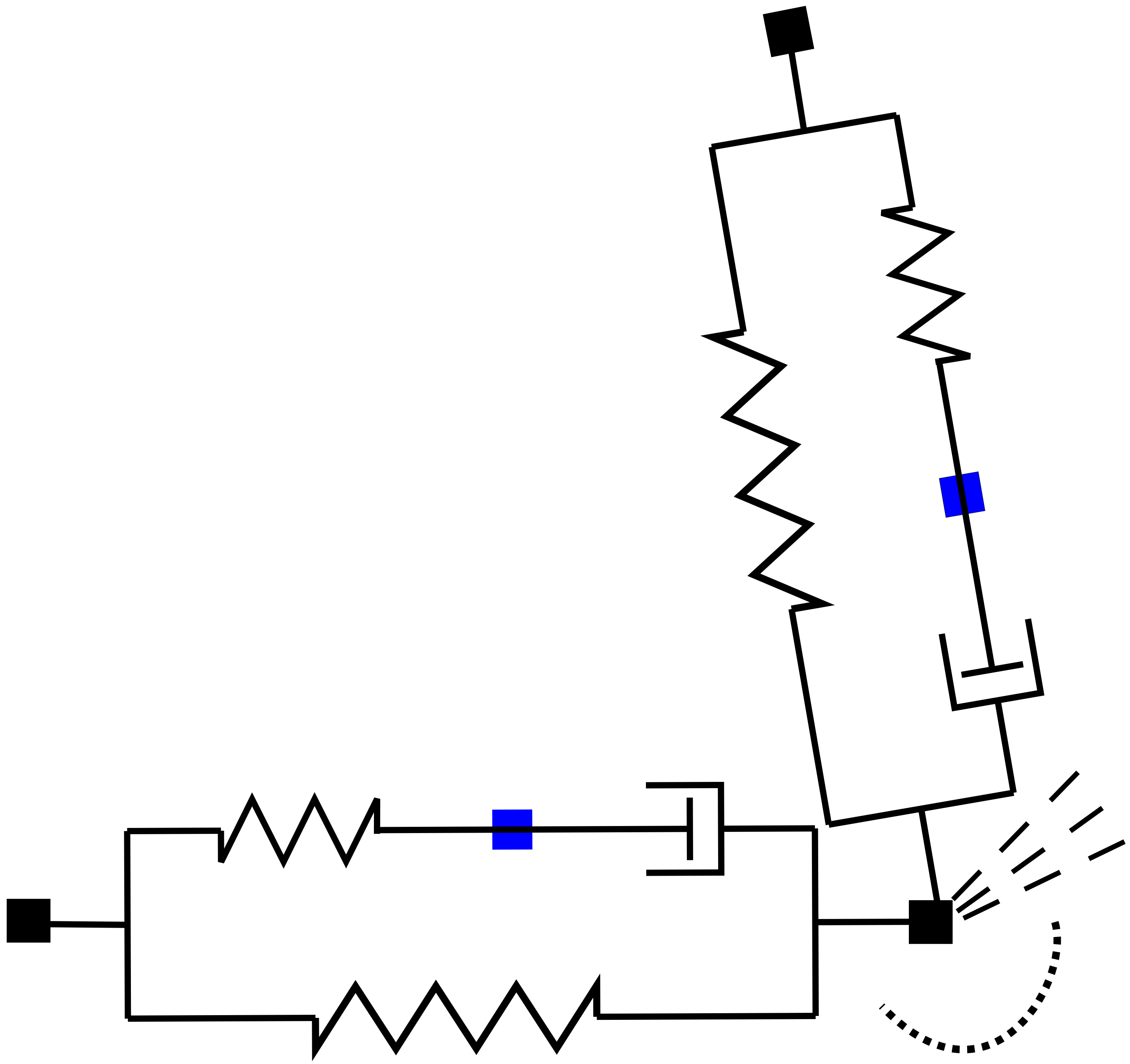


# Stress Dissipation



# Mean Field Solution: fully connected model



$$\Delta h_i \rightarrow \overline{h} - h_i$$

$$\Delta u_i \rightarrow \overline{u} - u_i$$

# Mean Field Solution: fully connected model

$$\partial_t h_i(t) = \underbrace{k_0(w - h_i) + K(\overline{h} - h_i)}_{\sigma_i} + \sigma^{th}(h_i)$$

$\sigma^{th}(h_i) = 1$ no aftershocks

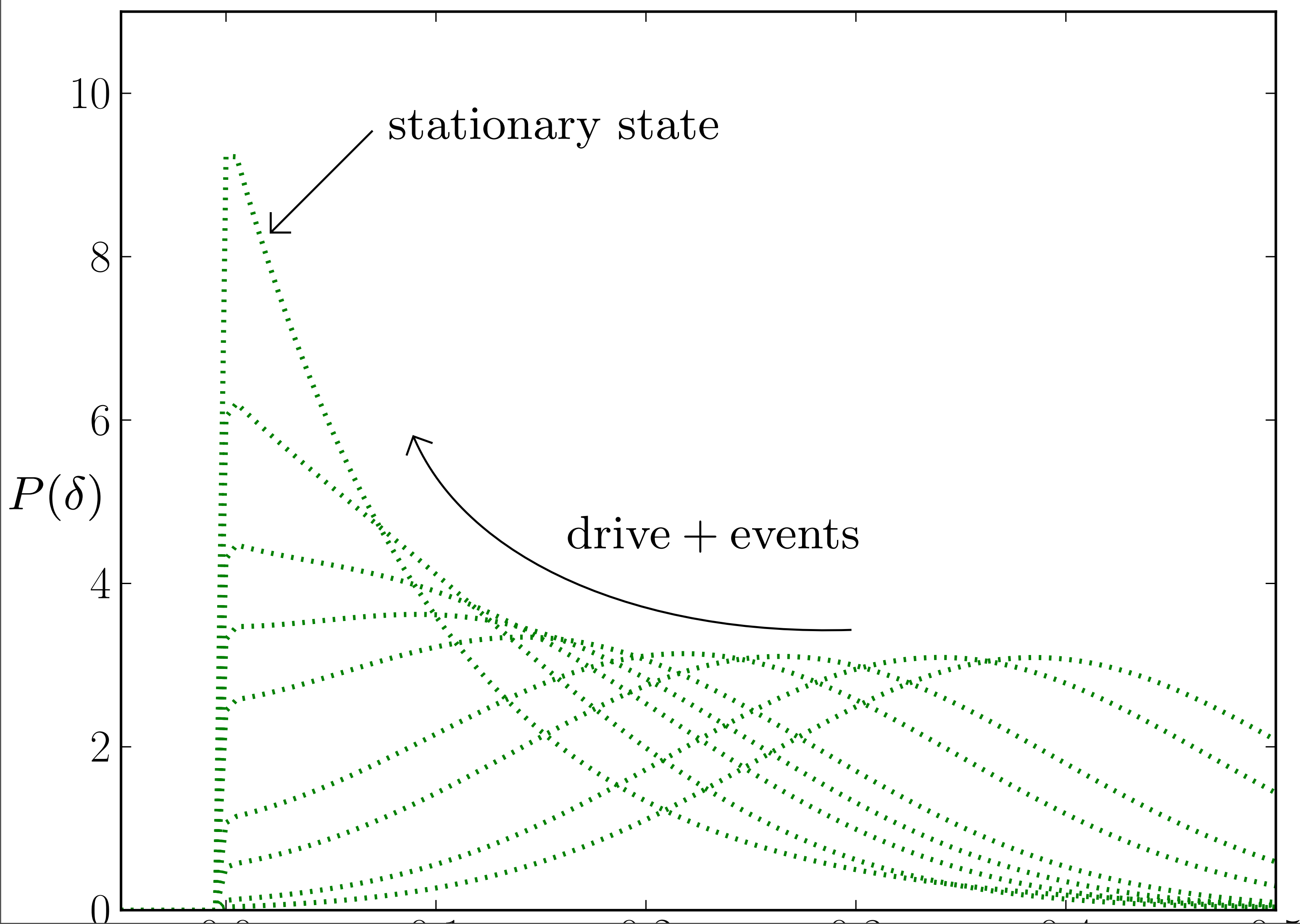
$\delta_i = 1 - \sigma_i$ distance to instability

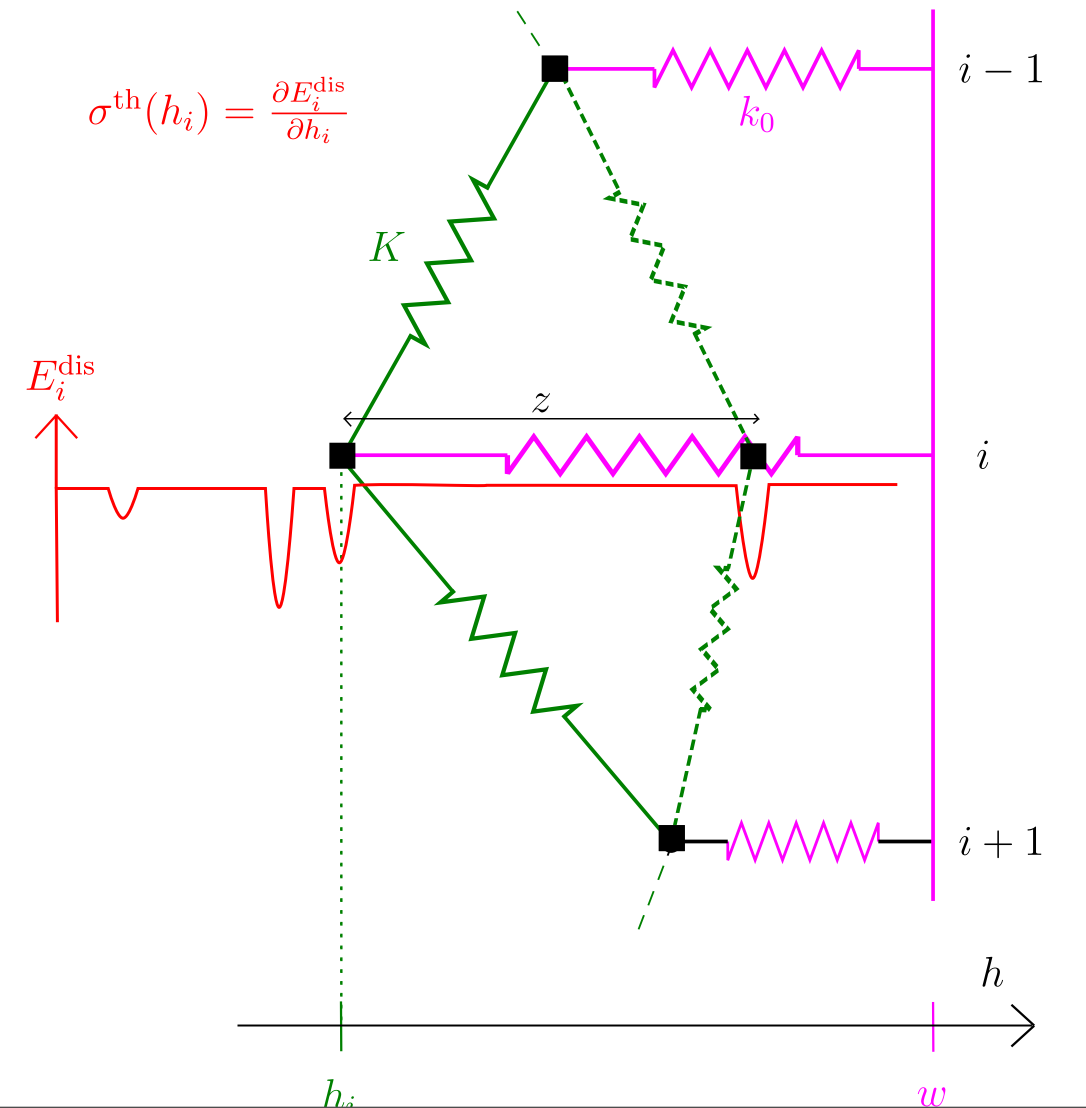
If $\delta_i = 0$

- $\delta_i^{\text{new}} = (K + k_0)z$ stress drop Random distance $g(z)$
- $\delta_j^{\text{new}} = \delta_j - \frac{Kz}{N}$ stress redistribution

$$\partial_t h_i(t) = \underbrace{k_0(w - h_i) + K(\overline{h} - h_i)}_{\sigma(h_i)} + \sigma^{th}(h_i)$$

The central object is $P_w(\delta)$



$$\delta_i = 1 - \big(k_0(w - h_i) + K(\bar{h} - h_i)\big)$$

when $w \to w + dw$

- Stress Shift: $P_{w+dw}(\delta) = P_w(\delta + k_0 dw)$

$$\delta_i = 1 - \left(k_0(w - h_i) + K(\bar{h} - h_i)\right)$$

when $w \to w + dw$

- Stress Shift: $P_{w+dw}(\delta) = P_w(\delta) + k_0 dw \, P'_w(\delta) + \dots$

$$\delta_i = 1 - \left(k_0(w - h_i) + K(\bar{h} - h_i)\right)$$

when $w \to w + dw$

- Stress Shift: $P_{w+dw}(\delta) = P_w(\delta) + k_0 dw\, P'_w(\delta) + \dots$
- Stress Drop: $\underbrace{P_w(0) k_0 dw}_{\text{unstable sites}} \underbrace{g(\frac{\delta}{K + k0})/(K + k0)}_{\text{stress drop}}$

$$\delta_i = 1 - \left(k_0(w - h_i) + K(\bar{h} - h_i)\right)$$

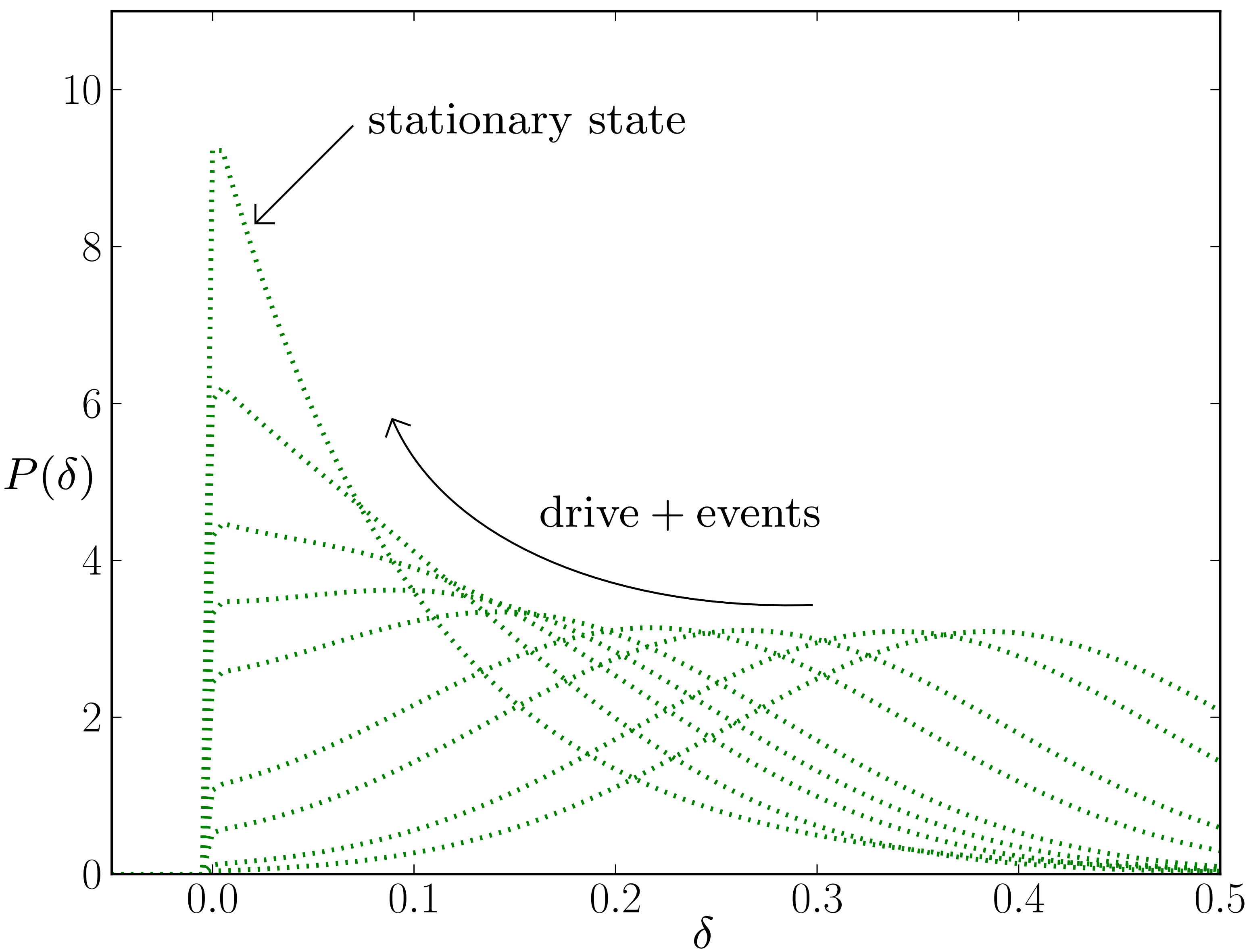
when $w \to w + dw$

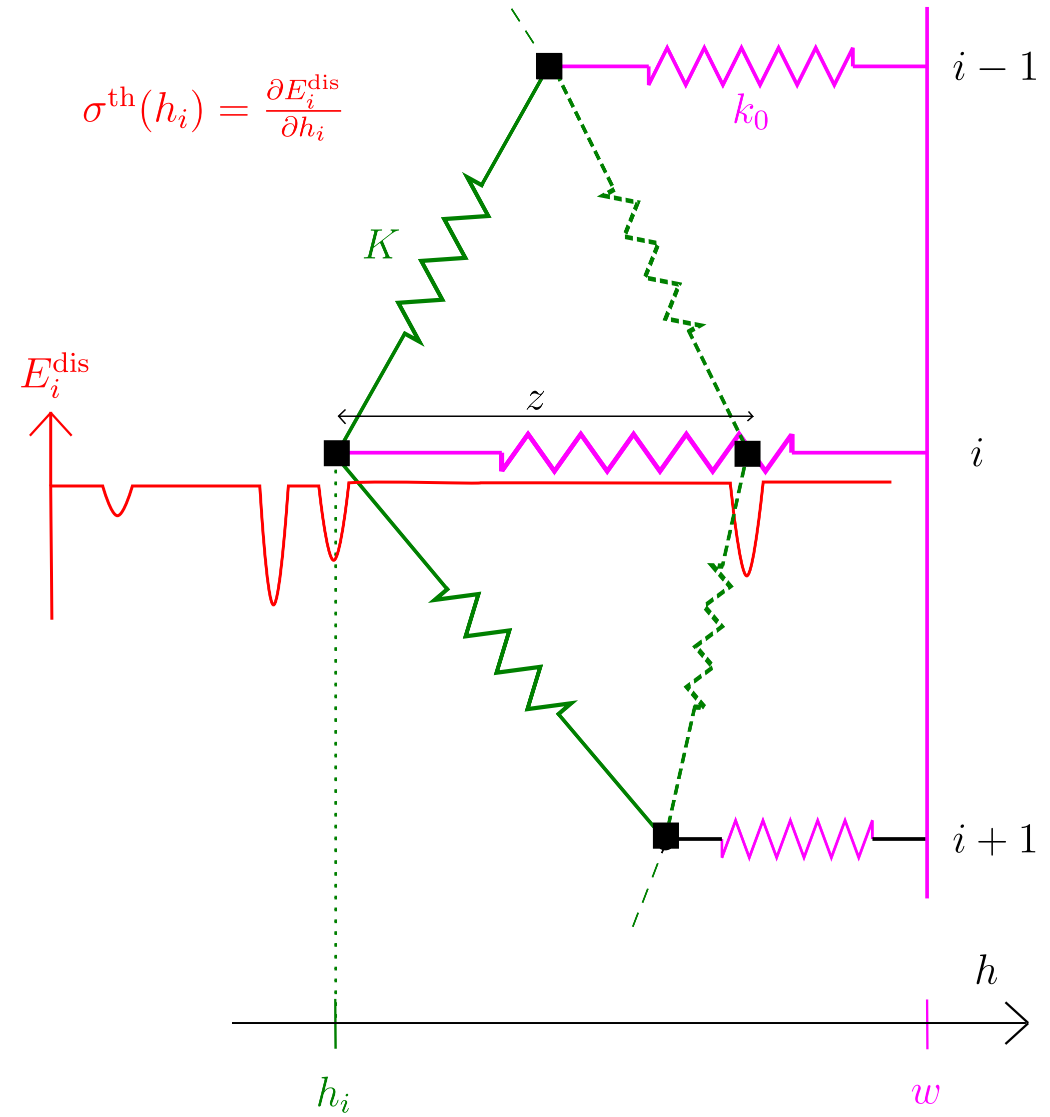
- Stress Shift: $P_{w+dw}(\delta) = P_w(\delta) + k_0 dw\, P'_w(\delta) + \dots$
- Stress Drop: $\underbrace{P_w(0) k_0 dw}_{\text{unstable sites}} \underbrace{g(\frac{\delta}{K + k0})/(K + k0)}_{\text{stress drop}}$

$$\frac{P_{1\text{step}}(\delta) - P_w(\delta)}{kdw} = P'_w(\delta) + P(0)\frac{1}{K + k0} g(\delta/(K + k_0))$$

# Fixed Point solution

$$\frac{P_{1\text{step}}(\delta) - P_w(\delta)}{kdw} = P'_w(\delta) + P(0)\frac{1}{K+k0}g(\delta/(K+k_0))$$



# Fixed Point solution

$$P'_w(\delta) = -\frac{P(0)}{K+k_0}\, g(\delta/(K+k_0))$$

$$P(0) = \frac{1}{K+k0}\overline{z} \quad \text{normalization}$$

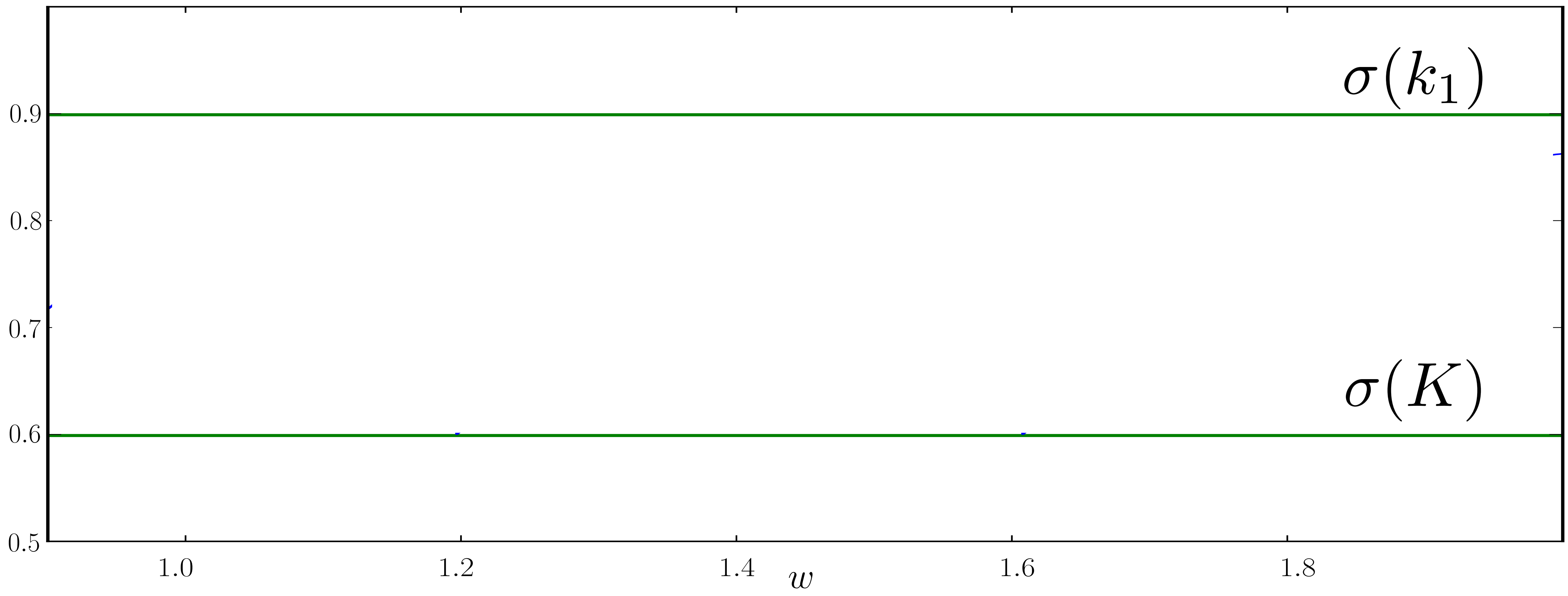
$$P(\delta) = \frac{1 - G(\delta/(K+k0))}{(K+k0)\overline{z}}$$

$$g(z) = e^{-z} \;\rightarrow\; P(\delta) = \frac{1}{K+k_0}\, e^{-\frac{\delta}{K+k_0}}$$

The total stress $\sigma = 1 - \overline{\delta} = 1 - (K + k_0)$

# Elastic Solid driven on a disordered substrate

## stress decreases with solid rigidity



# Elastic Solid driven on a disordered substrate

For $N$ blocks $\delta_1 < \delta_2 < \ldots < \delta_N$ are iid drawn from $P(\delta)$

$$\int_0^{\delta_1} d\delta\, P(\delta) = \frac{1}{N} \;\rightarrow\; \delta_1 \approx \frac{1}{NP(0)} = \frac{K + k0}{N}$$

Similarly, $\delta_2 \approx 2\frac{K+k_0}{N}, \delta_3 \approx 3\frac{K+k_0}{N}, \ldots$

If $\delta_1$ becomes unstable, it gives a random kick to all the sites $\simeq K/N$

# Elastic Solid driven on a disordered substrate

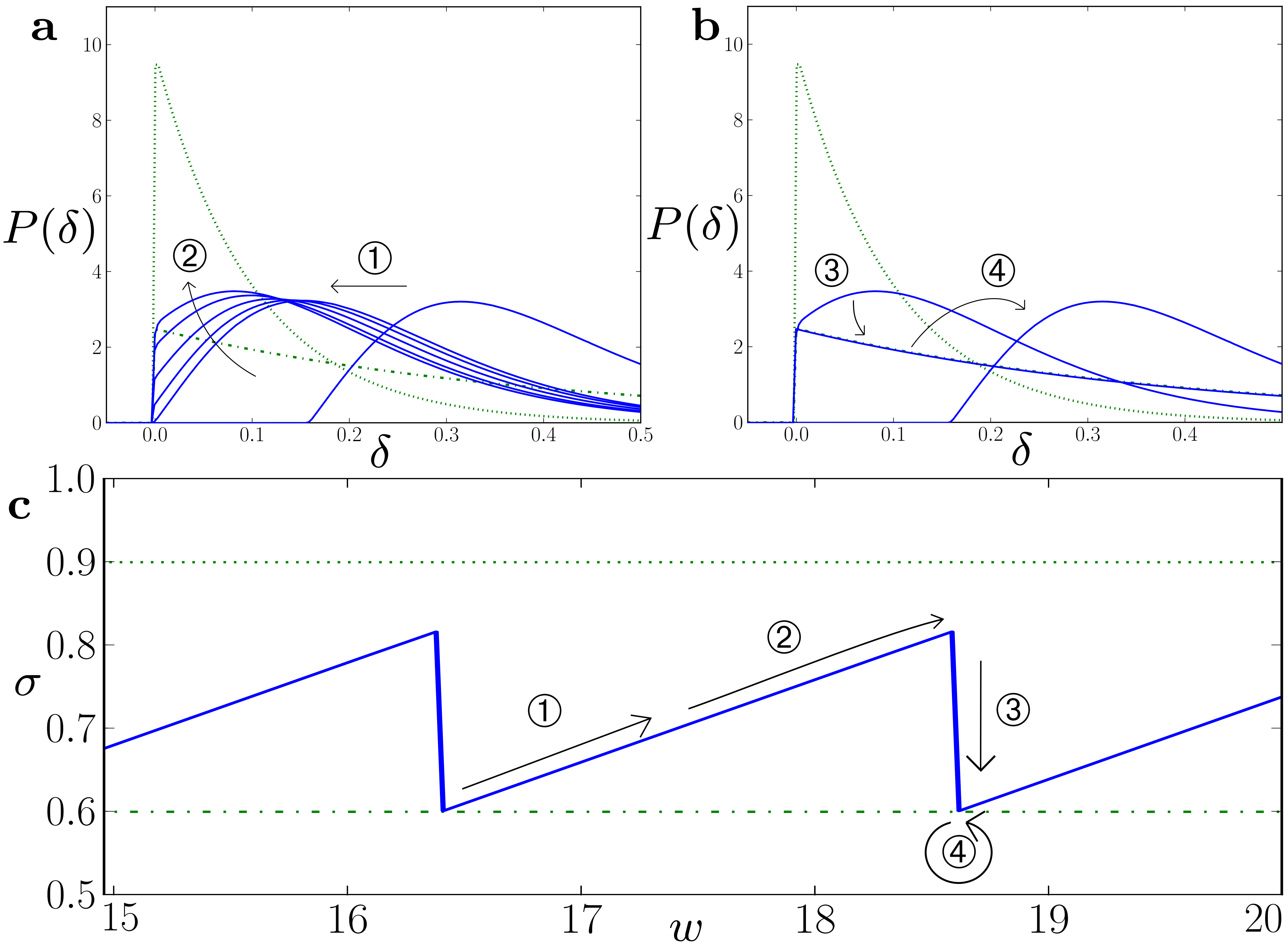
For N Sites:

- $\delta_1 \sim (K + k_0)/N \; \delta_2 \sim 2(K + k_0)/N; \delta_3 \sim 3(K + k_0)/N \ldots$
- The stress redistribution gives an increment $\sim K/N$

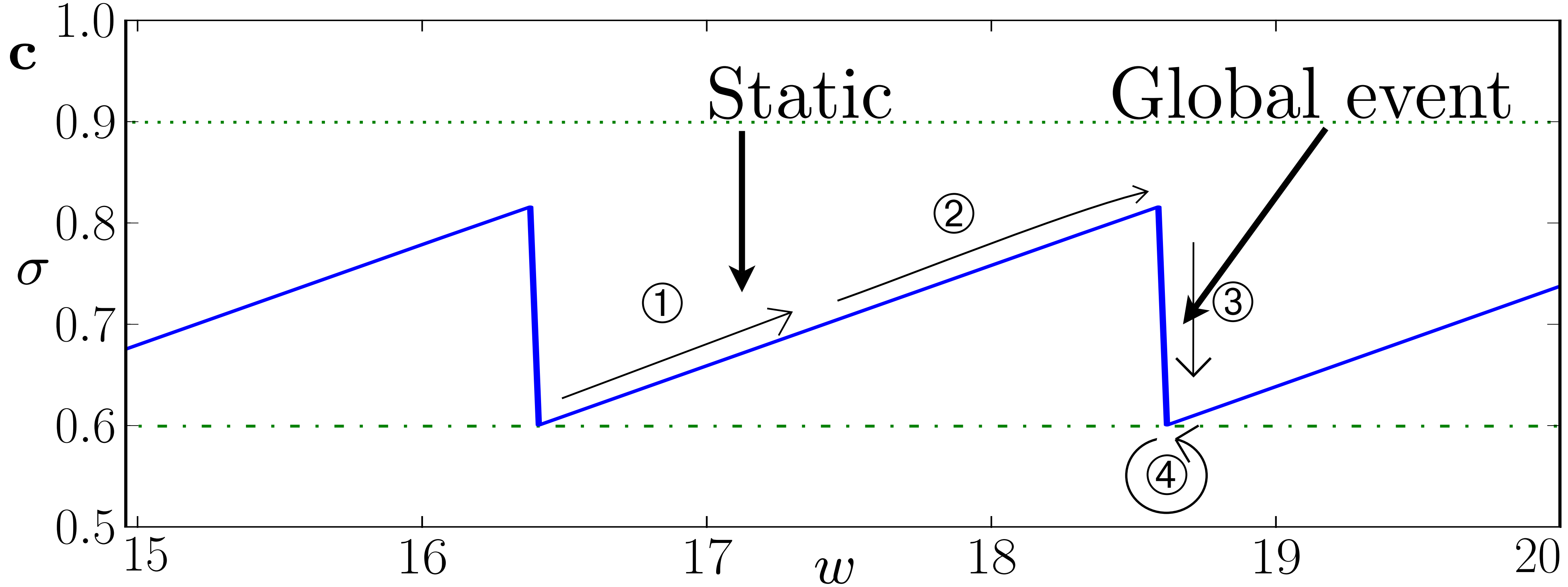
An avalanche of size S corresponds to a Brownian excursion with drift

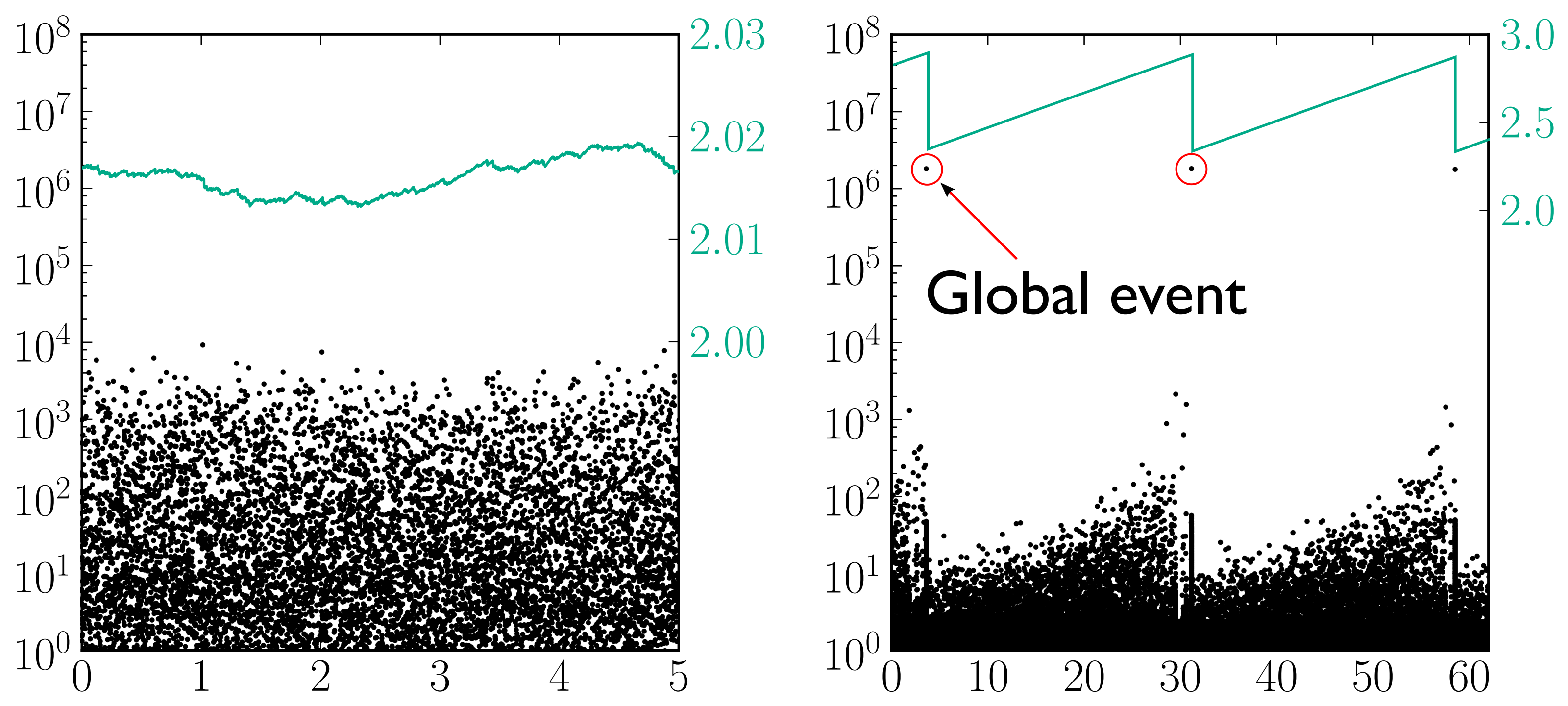
- $P(S) \sim S^{-3/2}$
- $S_{\max} \sim k_0^{-2}$

# Visco-Elastic Solid driven on a disordered substrate

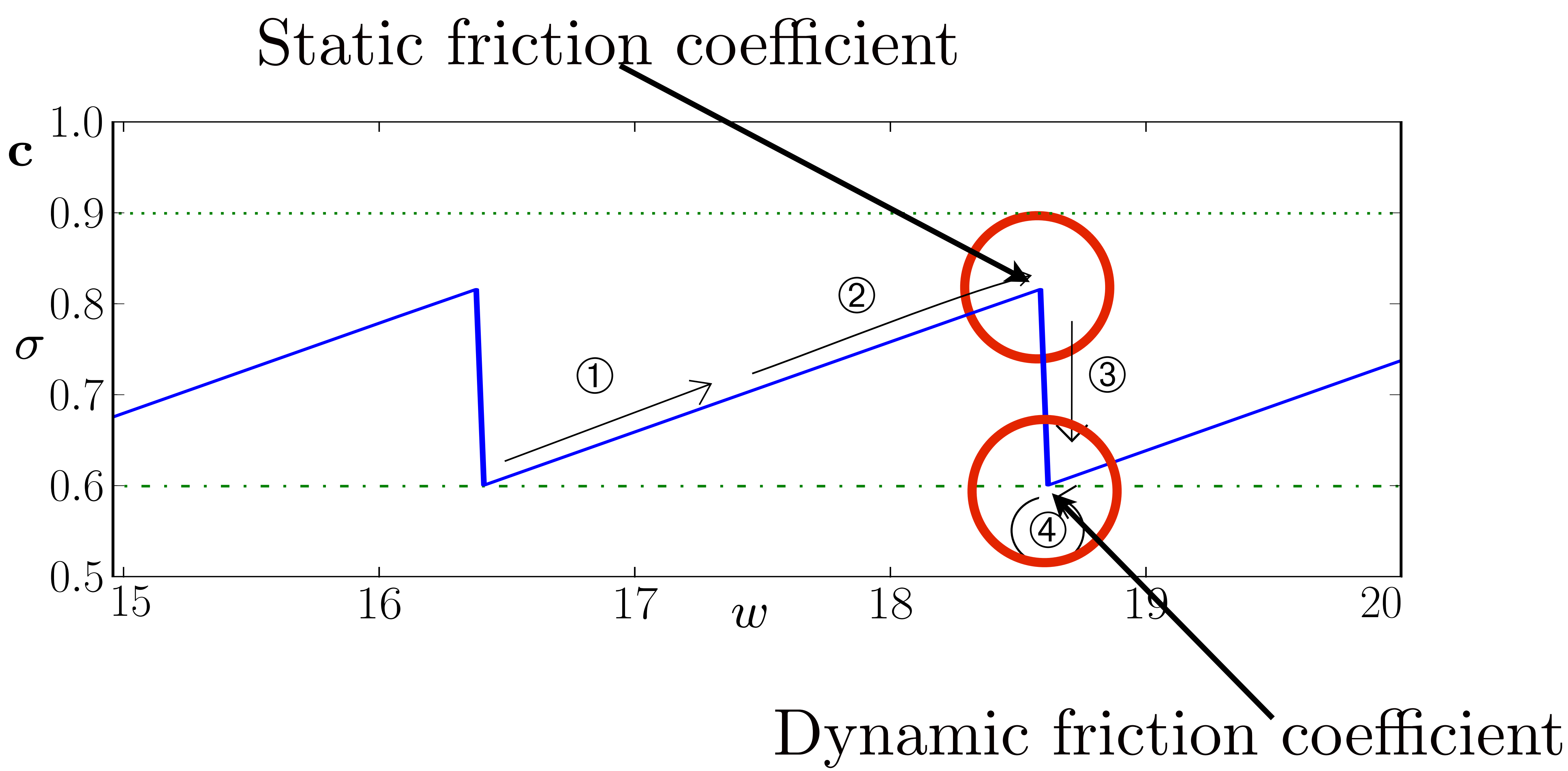


# Visco-Elastic Solid driven on a disordered substrate



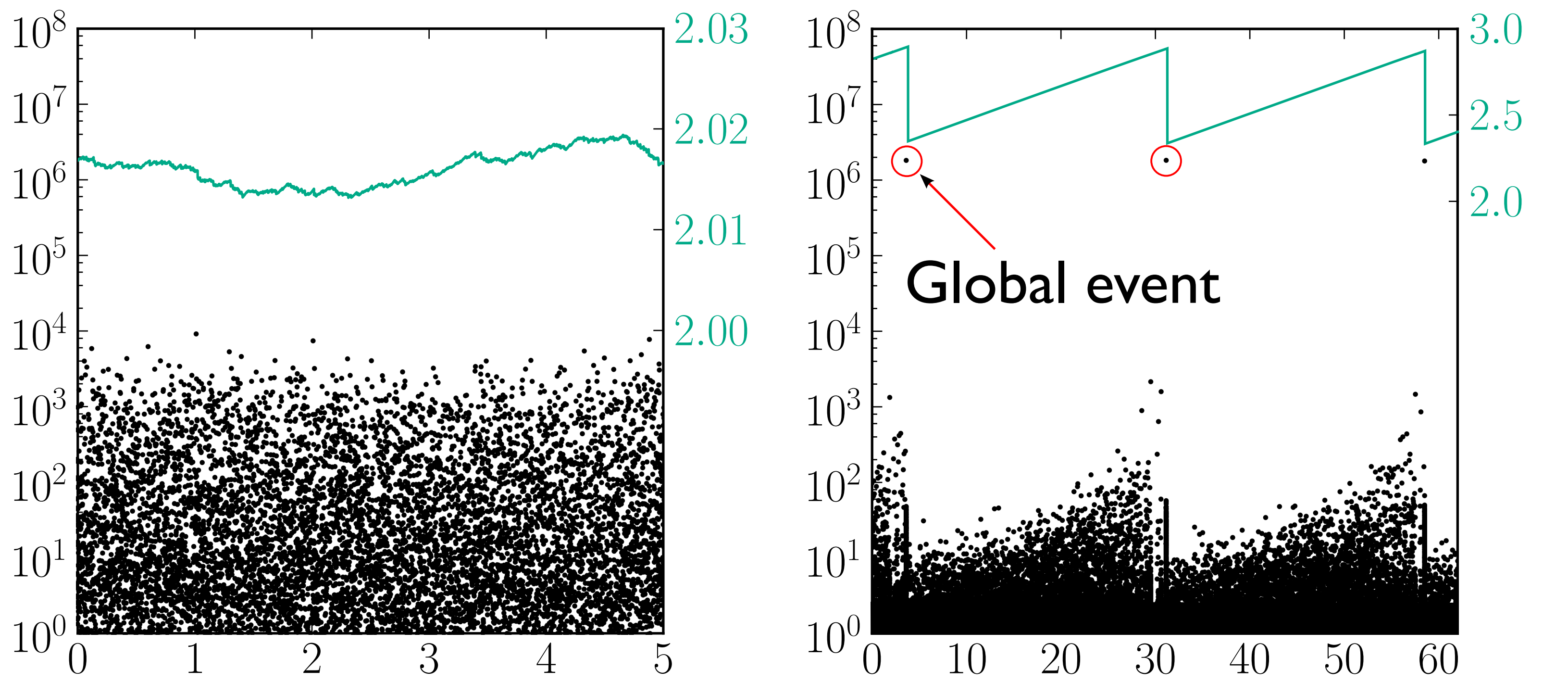



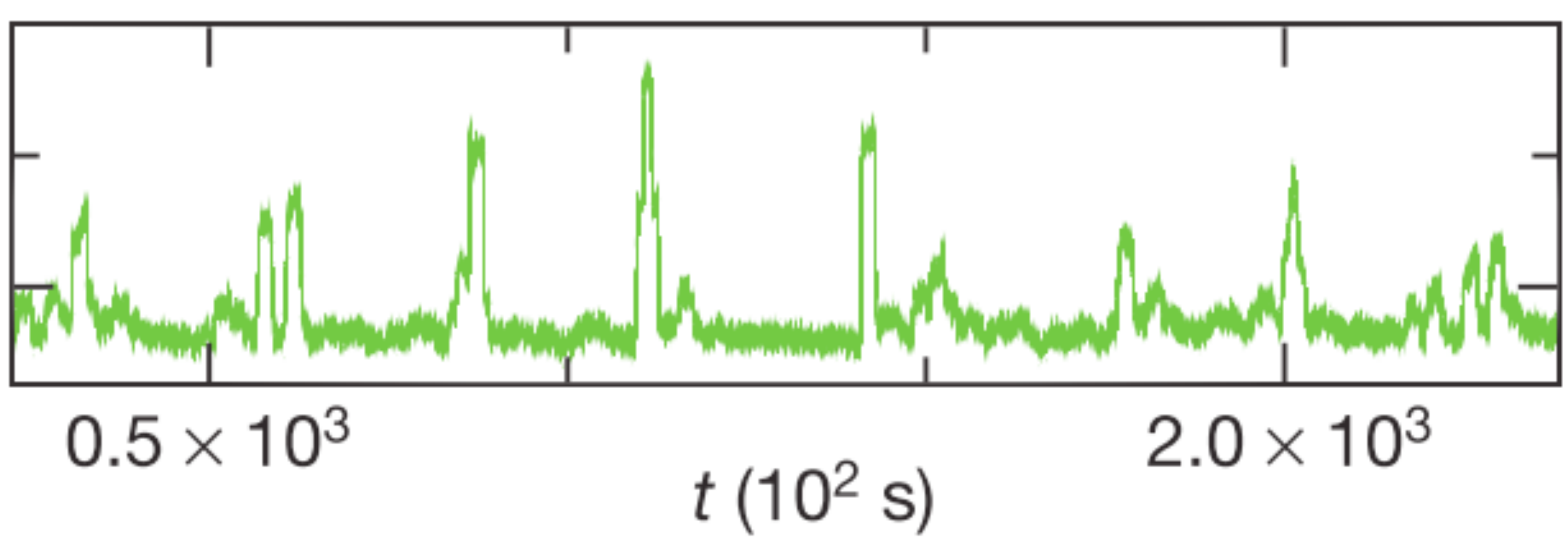
# Visco-Elastic Solid driven on a disordered substrate




## Stick Slip Instability

# Seismic Cycle and avalanche oscillator

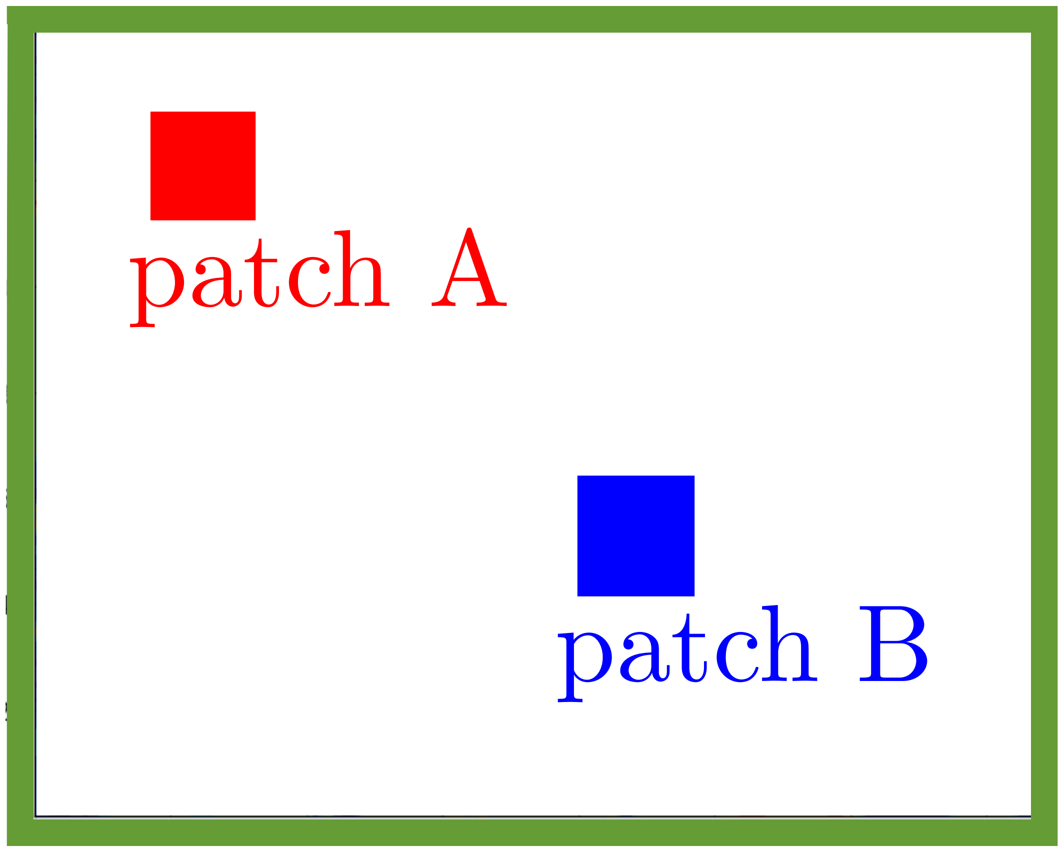


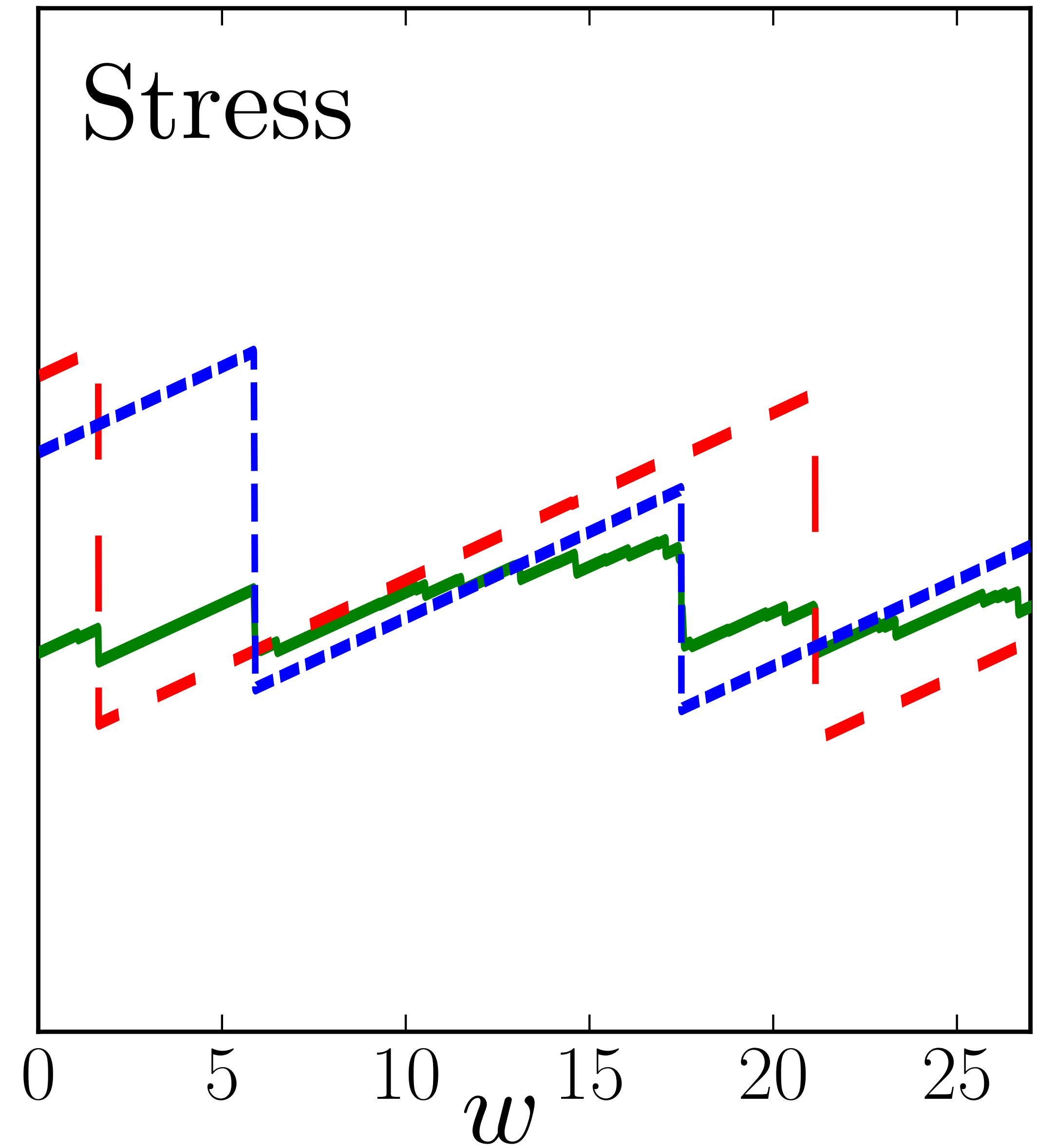




Micro-crystals compression

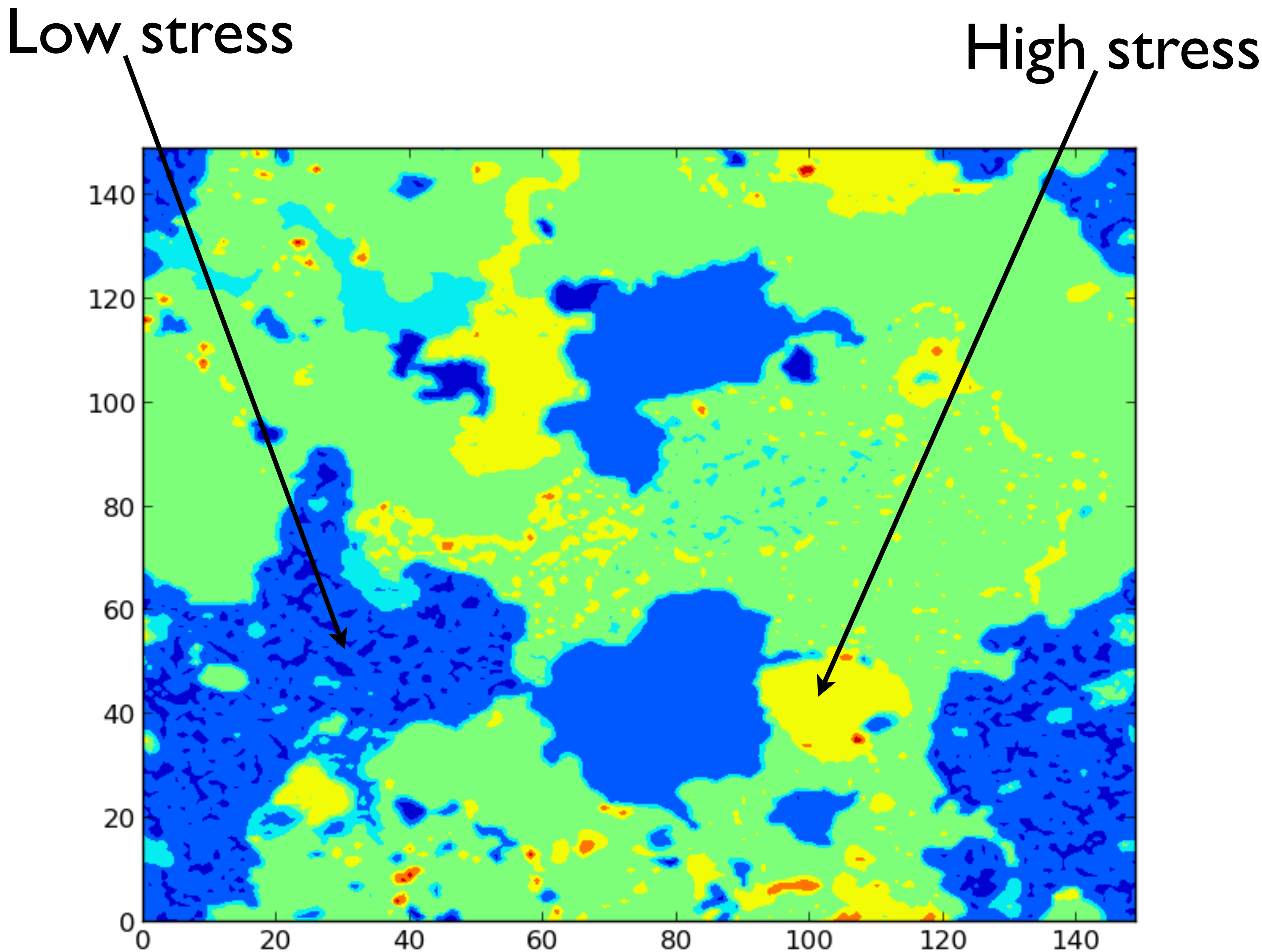

S. Papanikolaou et al., Nature (2012)

# Back to 2D: local oscillations



# Back to 2D: stress map



# Conclusions

- Visco-elastic interfaces have a rich dynamics with main shock and aftershocks
- In D=2 the results are consistent with GR exponent, displays spatial correlations and periodic behavior
- In mean field we understand the periodicity as a stick slip behavior of a system with a large number of degree of freedom