Avalanches and complex spatial-temporal patterns: branching process approaches to seismic occurrence

Dina Dargo

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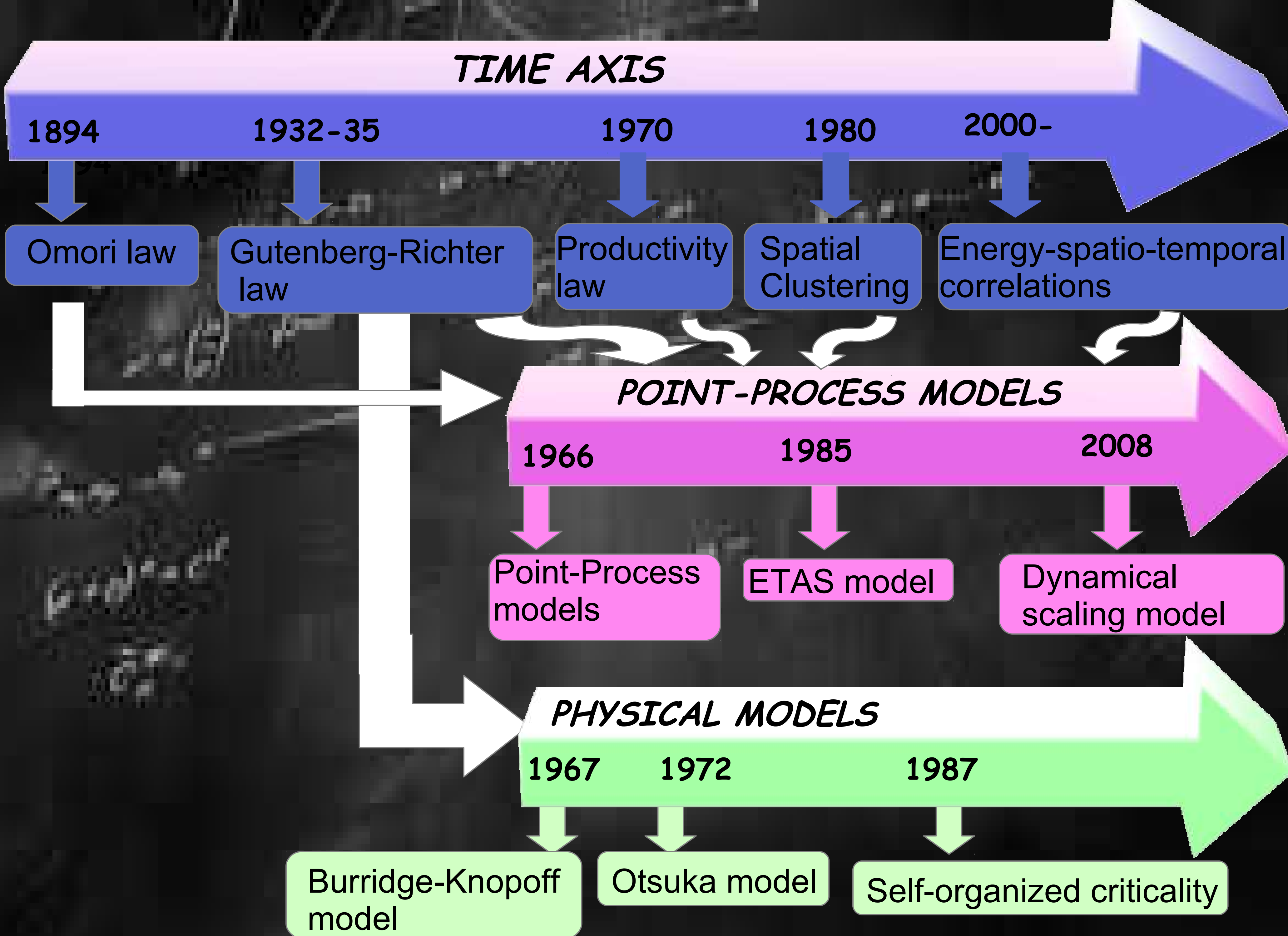
Lucilla de Arcangelis, Cataldo Godano



TIME AXIS
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1970
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Spatial Clustering
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ETAS model
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Self-organized criticality

TIME AXIS

1894

Omori law

OMORI LAW

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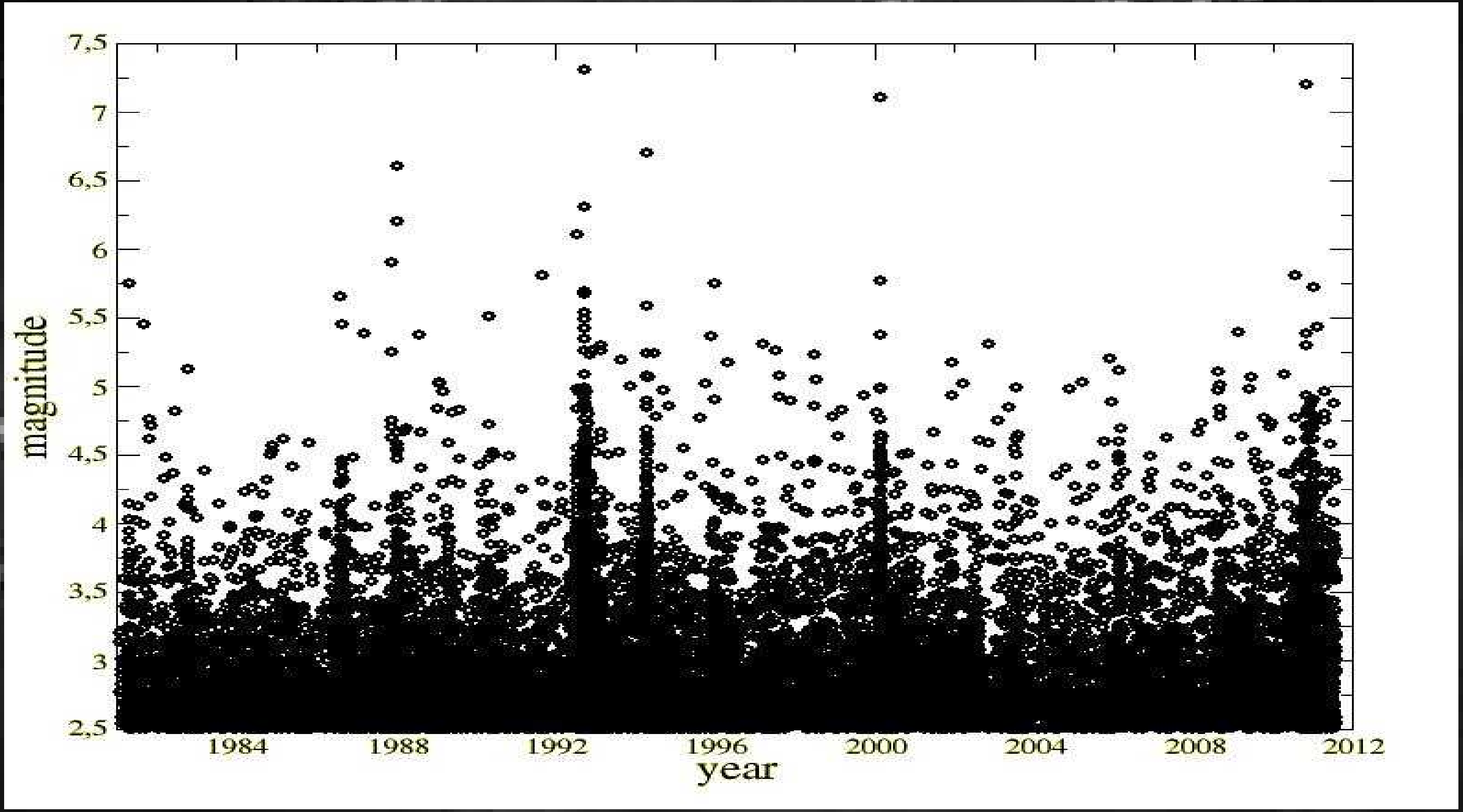
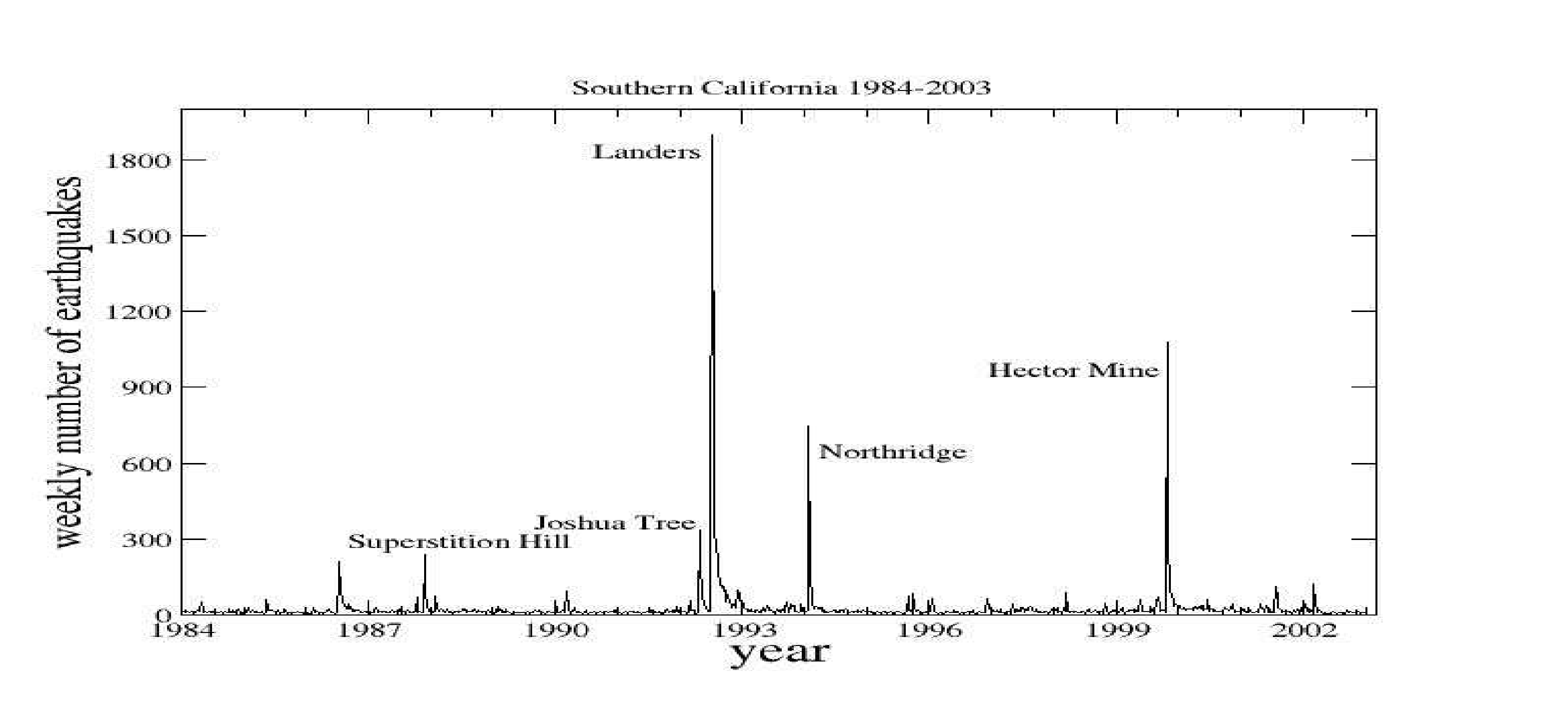


OMORI LAW

Aftershock sequences



The majority of events in seismic catalogs are aftershocks!

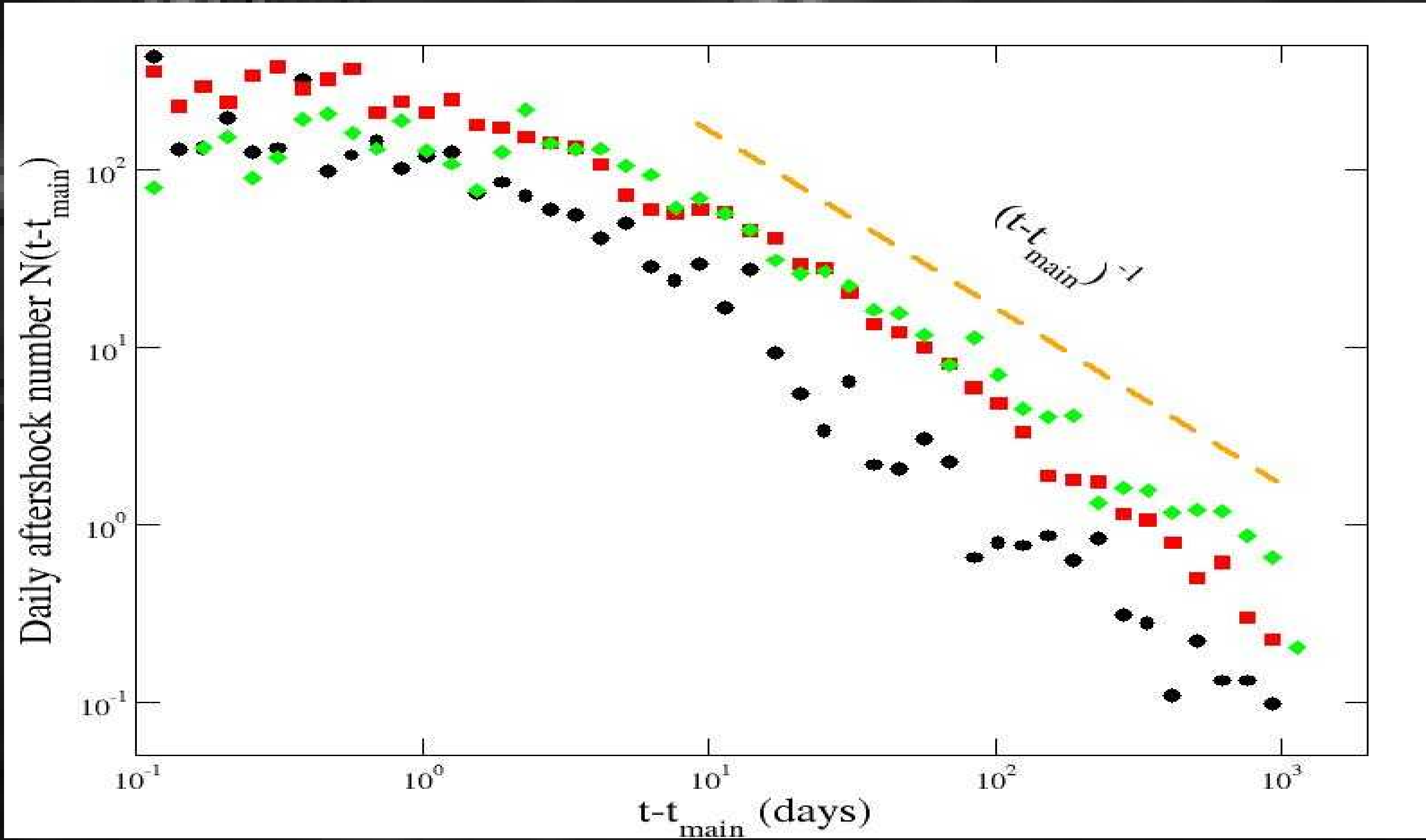
OMORI LAW

Fusakichi Omori published his work on the aftershocks of earthquakes, in which he stated that aftershock frequency decreases by roughly the reciprocal of time after the main shock.

$$n(t) = \frac{k}{t}$$

$$n(t) = \frac{k}{(t+c)^p}$$

The modified version of Omori's law, now commonly used, was proposed by Utsu in 1961, With typical values of p [0.75:1.5].



TIME AXIS

1894

Omori law

1932-35

Gutenberg-Richter law

Power law in the size distribution

In 1932 Wadati published a paper entitled "On the frequency distribution of earthquakes". In the paper he proposed a power law distribution for the eneergy and used Japanese data to estimate the exponent.
The paper received poor attention because the title was vague.

In 1935 in the first paper on the instrumental magnitude Richter proposed a fast decay of the shocks number for large magnitude.

In 1941 Guttenberg-Richter proposed an exponential distribution.

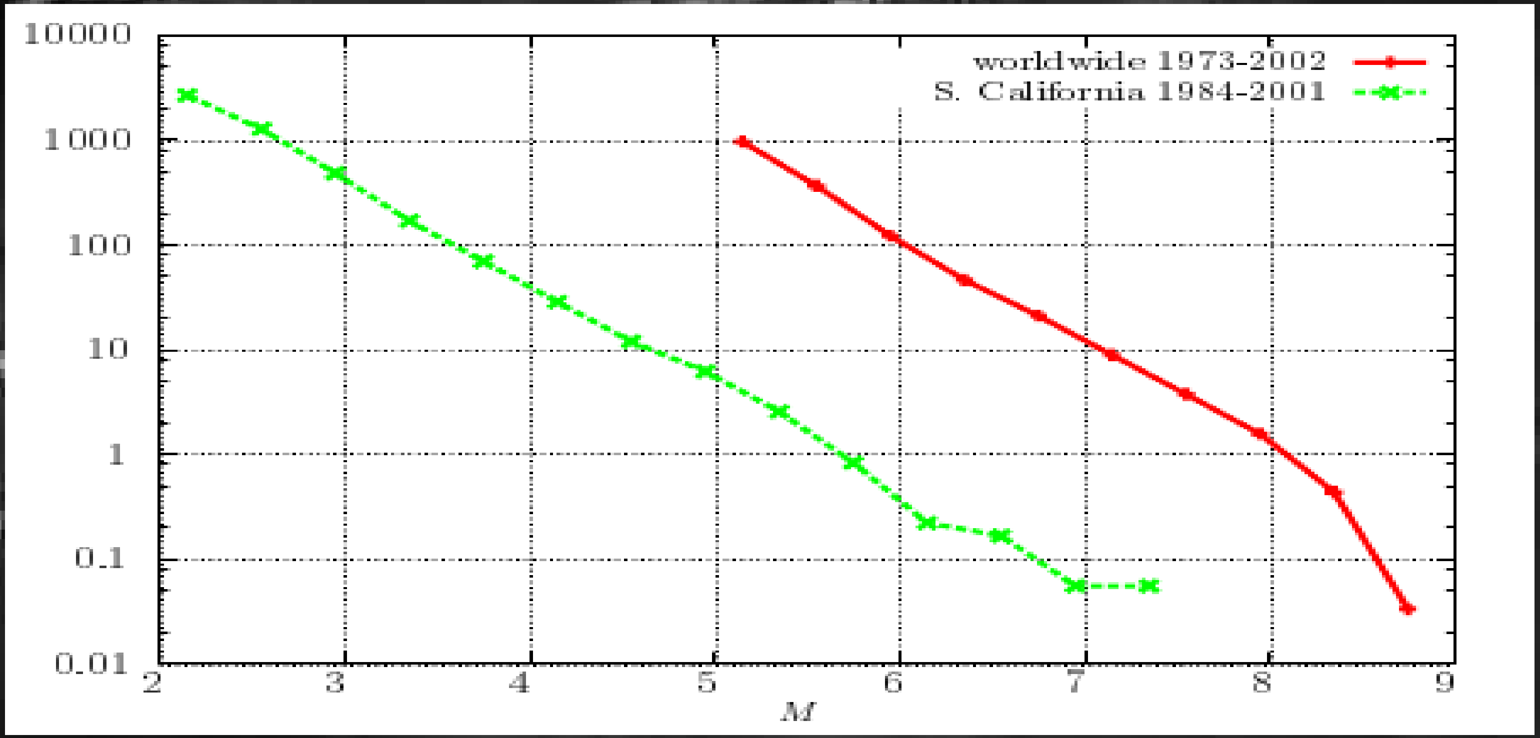
$$N(m) \sim 10^{-bm} \quad m \sim 3/2 \log_{10}(E) \Rightarrow N \sim E^{-1-2/3b} \quad b \simeq 1$$

$$E \propto S$$

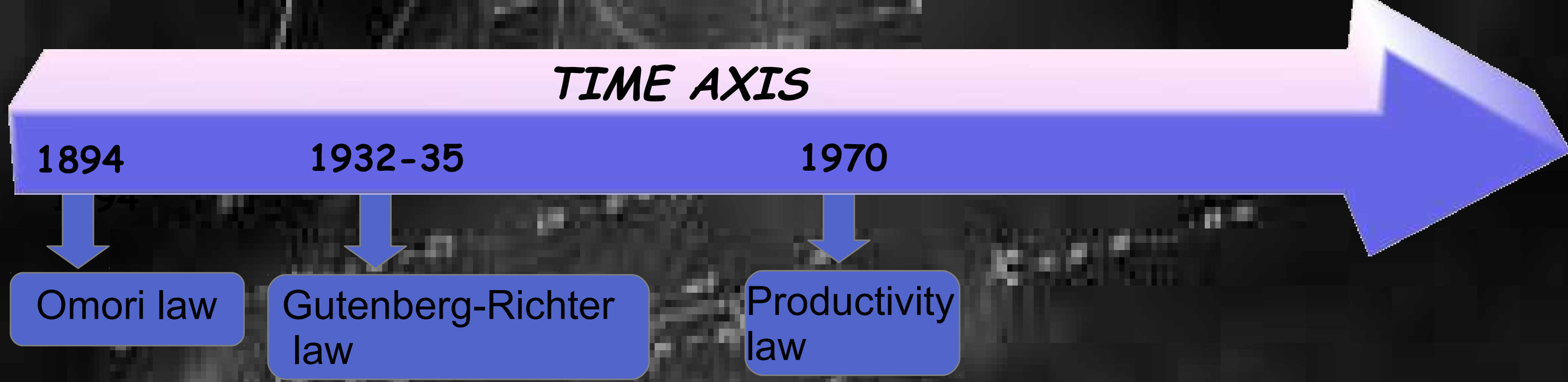
→ Power law distribution of erathquake sizes

Power law in the size distribution

Number of earthquakes per year worldwide and in Southern California



GUTENBERG-RICHTER LAW: $P(m) \cong 10^{-b\,m}$
POWER LAW for Size distribution $P(S) \cong S^{-1-b2/3}$
Using $m = (2/3)\log_{10}(S)$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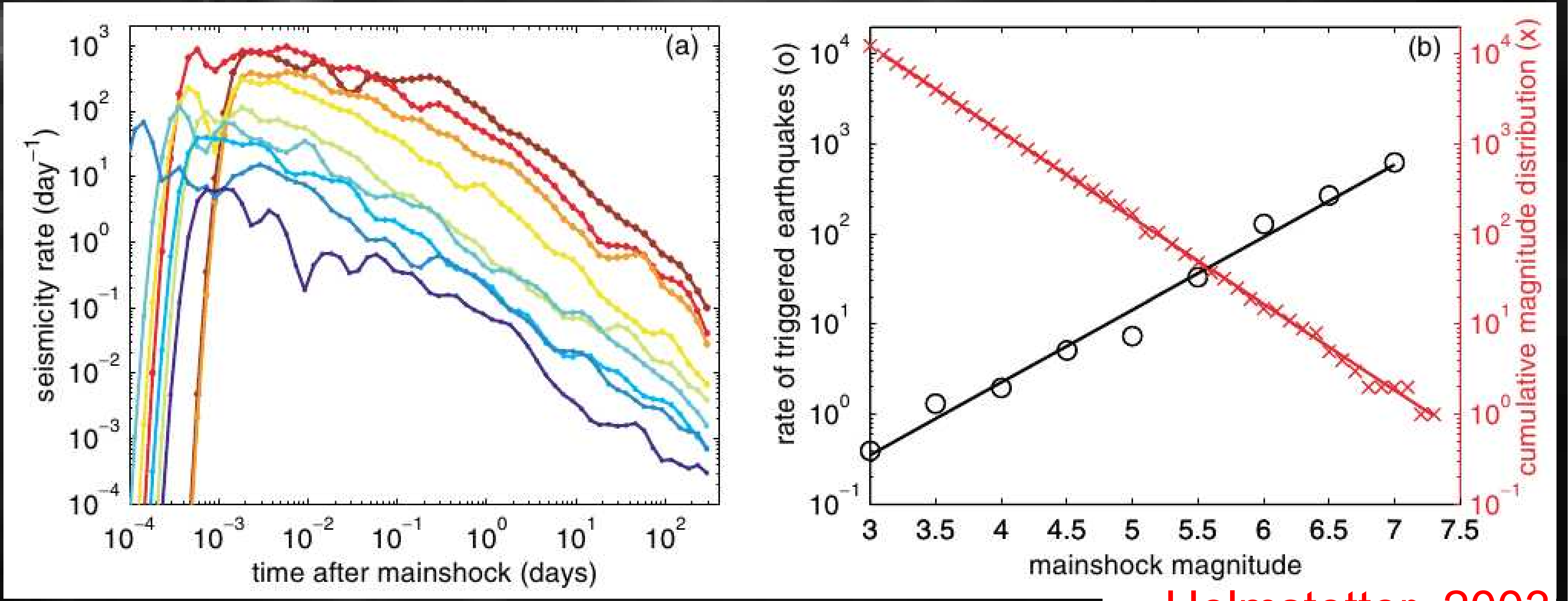
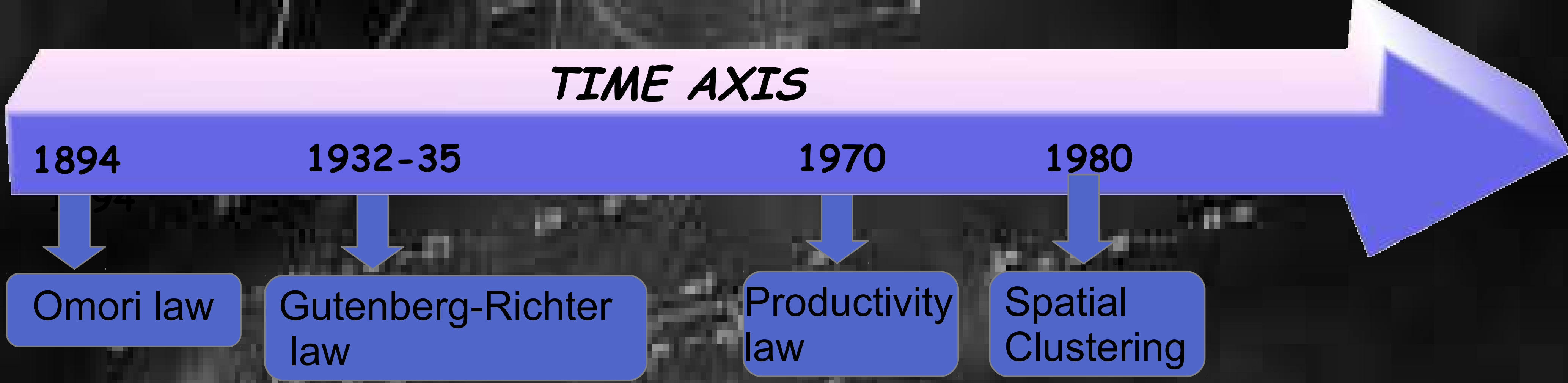
Productivity law

In 1970 Utsu observed that the number of aftershocks is exponential with the mainshock magnitude

$$N_{aftershocks} \sim 10^{\alpha m} \quad m \sim 3/2 \log_{10}(E) \Rightarrow N_{aftershocks} \sim E^{1+2/3\alpha} \quad \alpha \simeq 1$$



Helmstetter, 2003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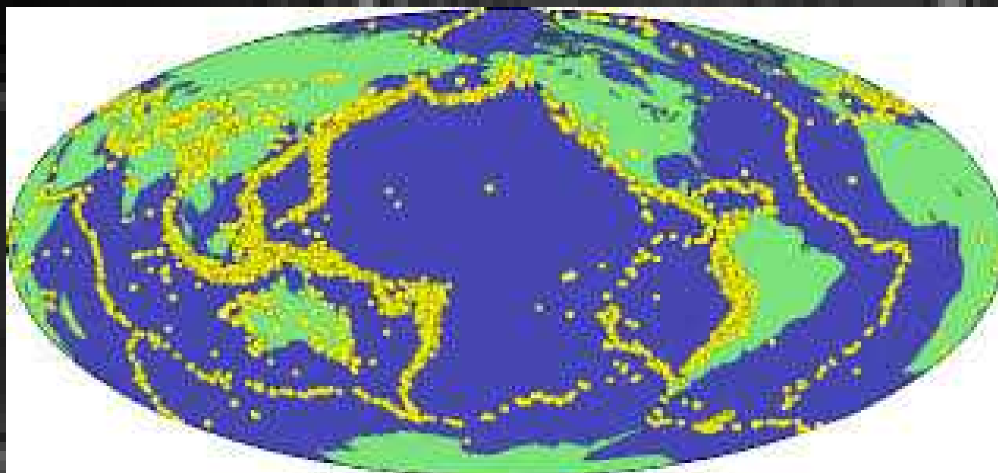
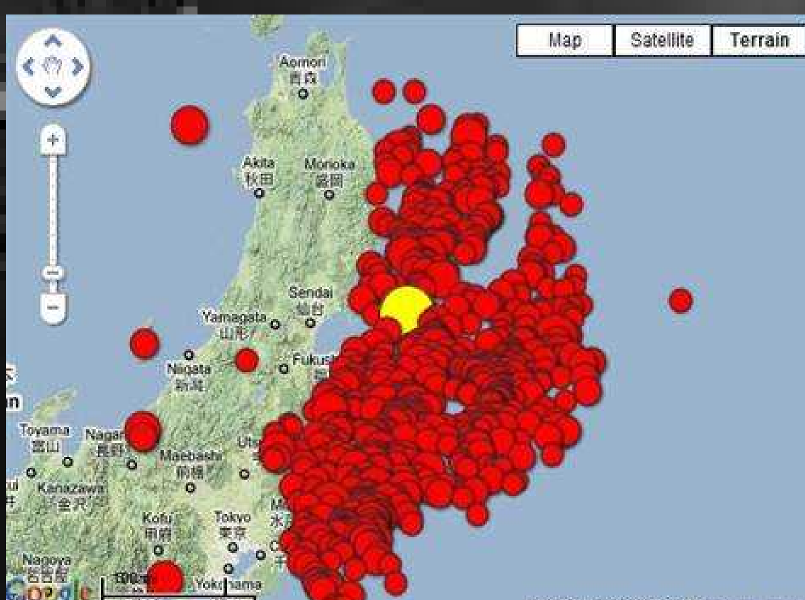
Spatial Clustering

Spatial clustering

The earthquakes are clustered in space along hierarchical fault structures
[Kagan-Knopoff 1980]

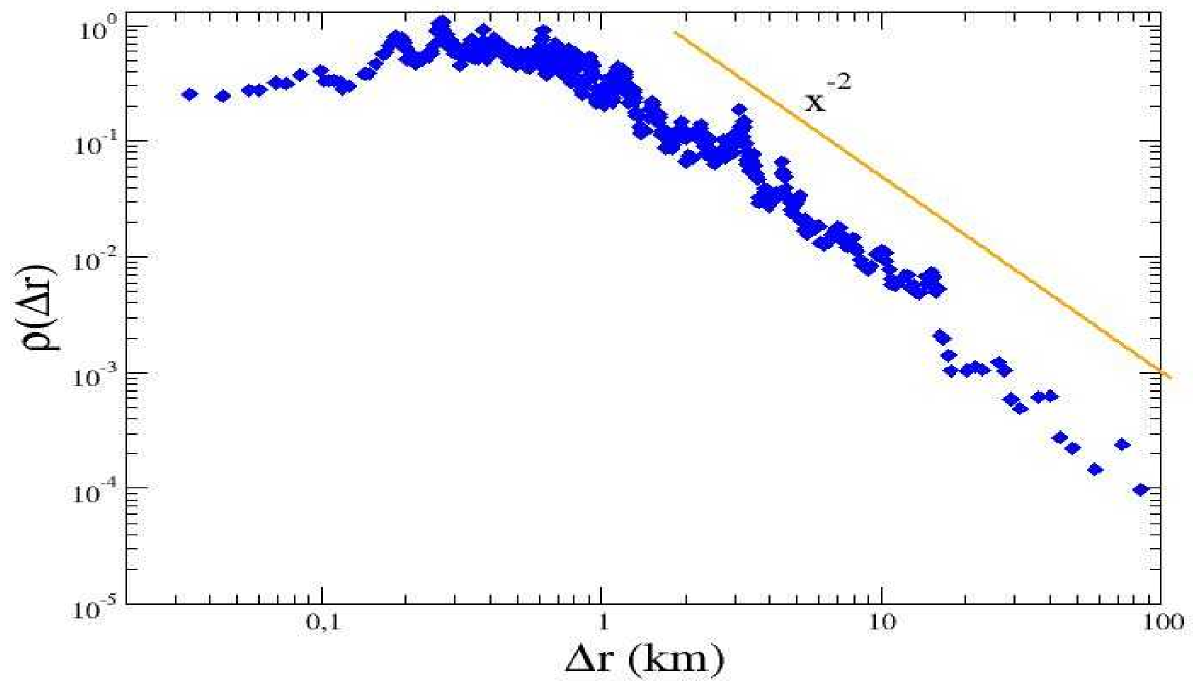




Aftershocks are preferentially localized close in spaxe to the mainshock

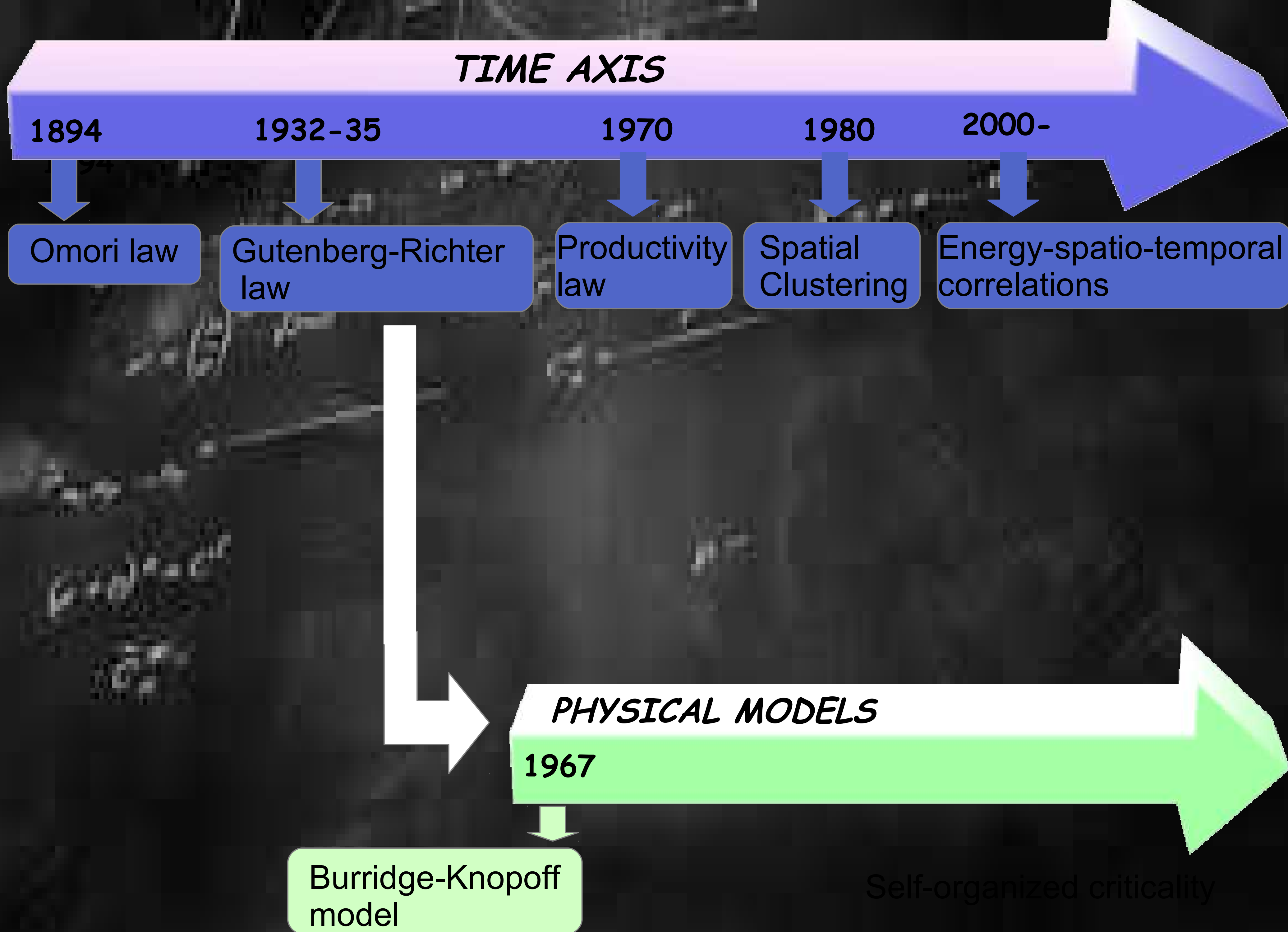
Spatial clustering

Aftershock linear density



TIME AXIS

- 1894 — Omori law
- 1932-35 — Gutenberg-Richter law
- 1970 — Productivity law
- 1980 — Spatial Clustering
- 2000- — Energy-spatio-temporal correlations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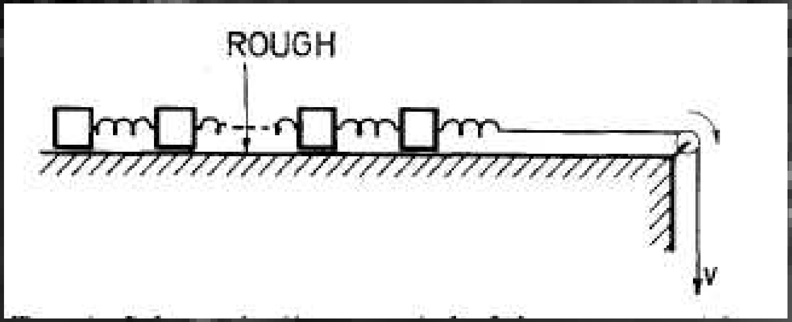
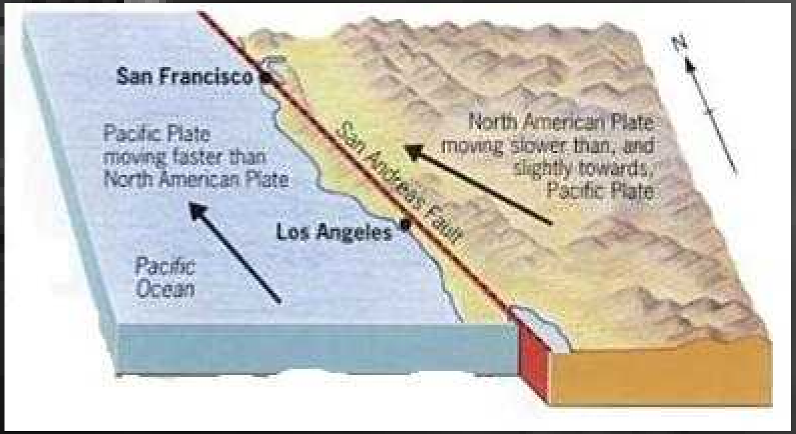
Self-organized criticality

Burridge-Knopoff model

A seismic fault is described as an elastic string everywhere in contact with a frictional surface which retards the motion.





It is a simple but quite realistic description of a real sesimic fault that is an elastic medium under shear in contact with a rough surface.

Burridge-Knopoff model

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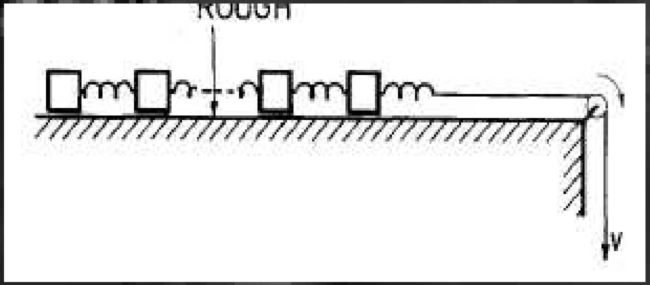

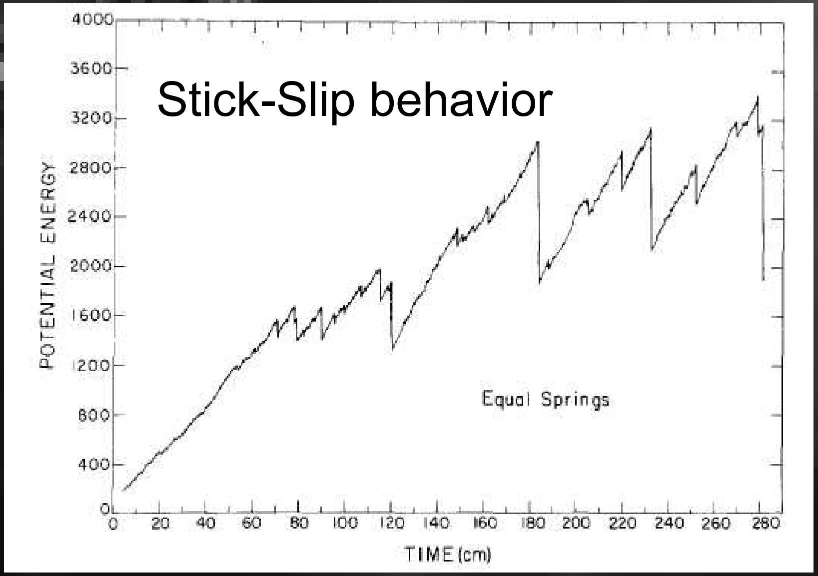

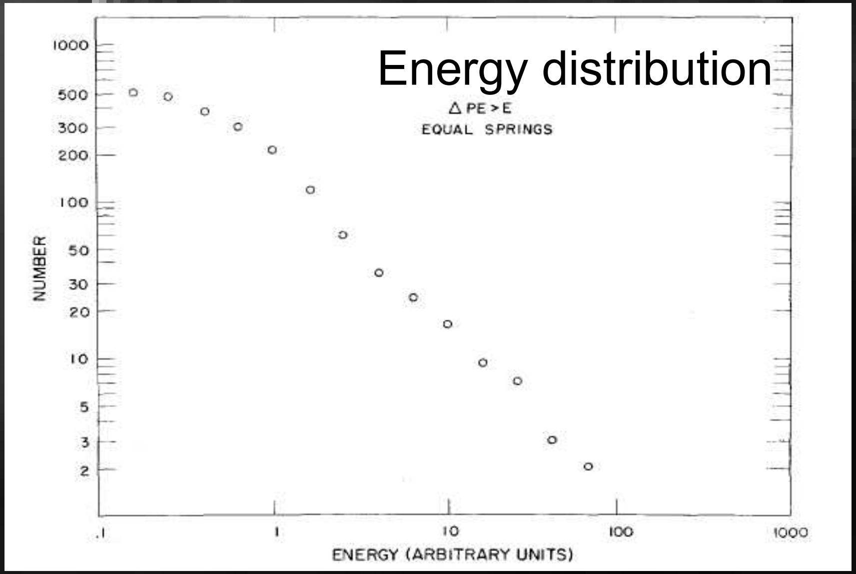

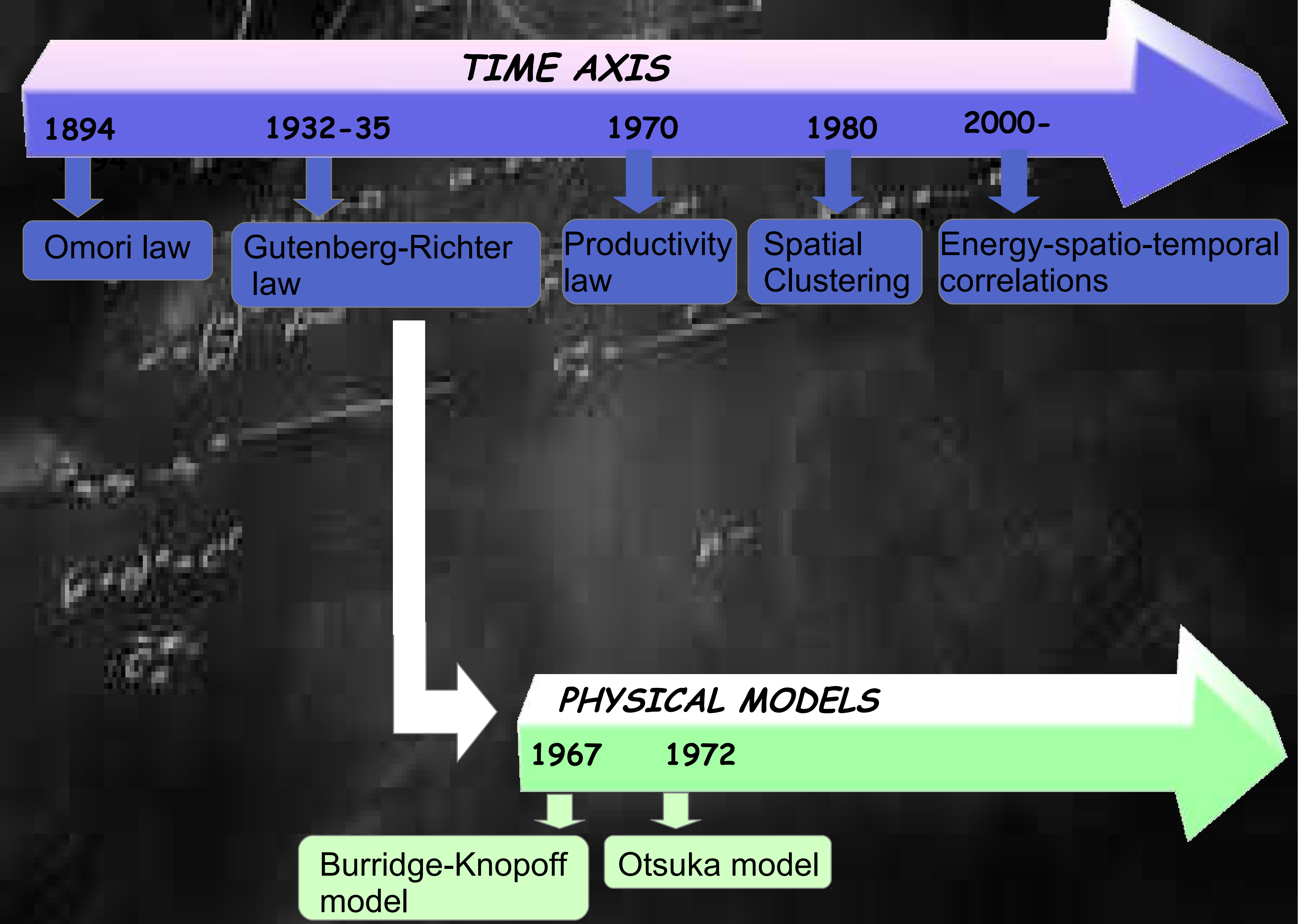
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Otsuka (chain-reaction) model

It is probably the first time that the concept of “avalanche” is proposed in the seismological context.
It is just a chain-reaction model.

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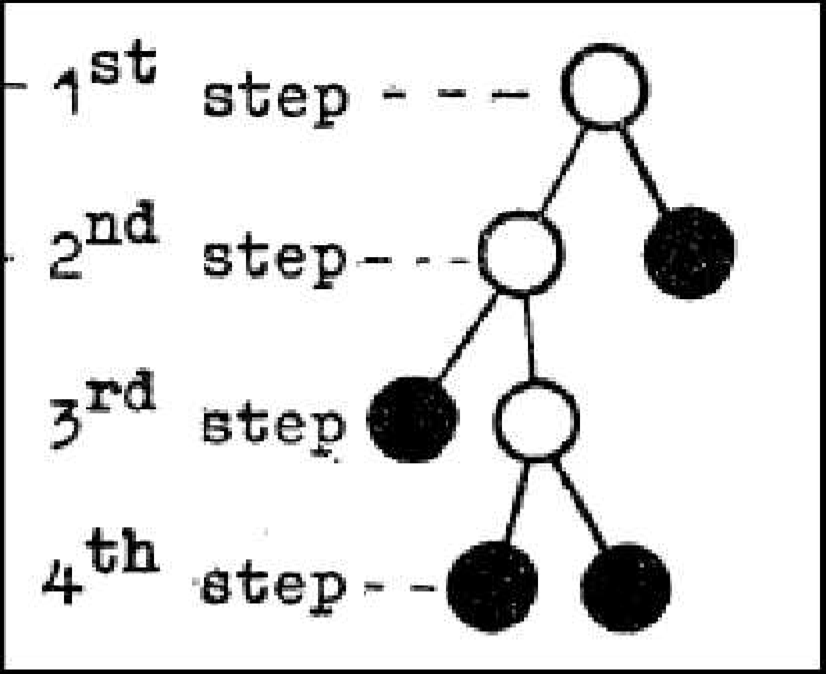

Otsuka, *Zisin* 1971

Otsuka model

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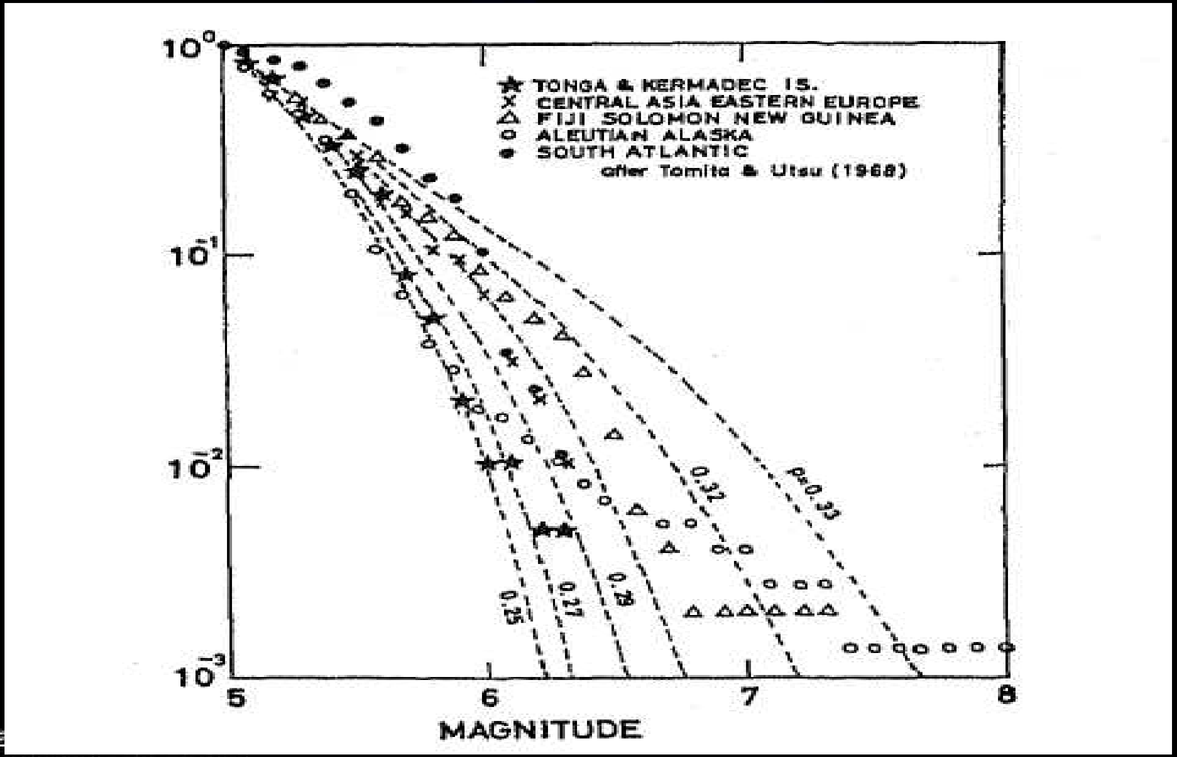
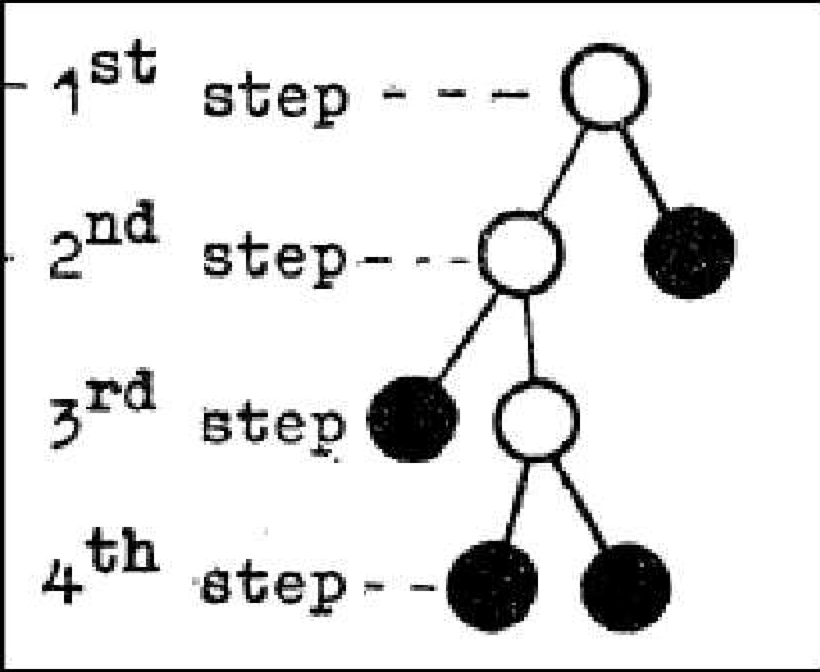
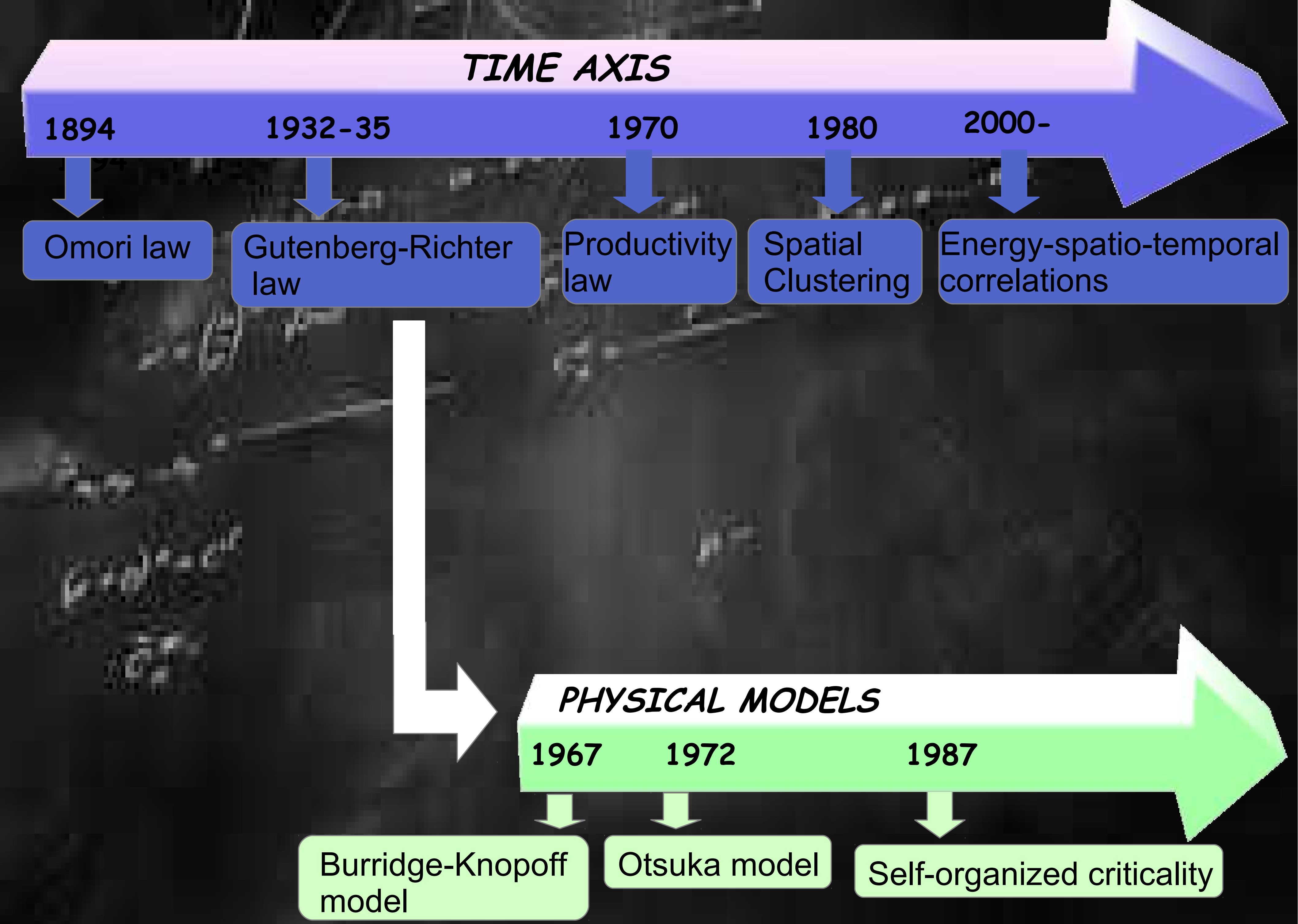




Otsuka, *Zisin* 1971

Seismic fault viewed as patches that may fail and trigger other patches to fail with some probability and so on
Extensions by Vere-Jones 1976, Kagan 1982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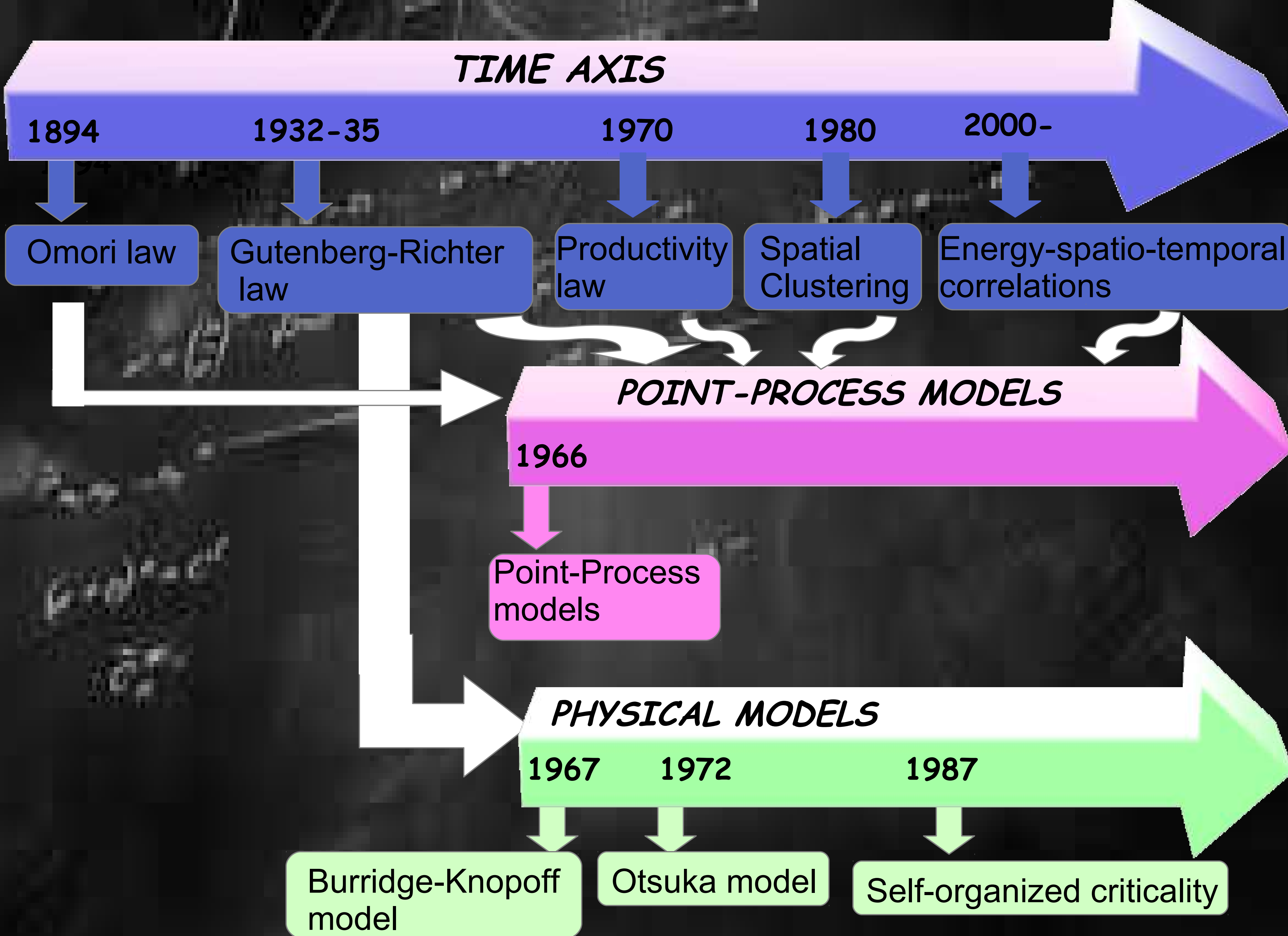
Self-organized critical models

The sand pile model paradox
Bak -Tang-Wiesenfeld

A common explanation for power laws in the size distribution
(see also Self-organized branching processes Zapperi et al. 1995)

Temporal corellations are absent in the sand-pile model
Many extensions have been proposed to include temporal correlations....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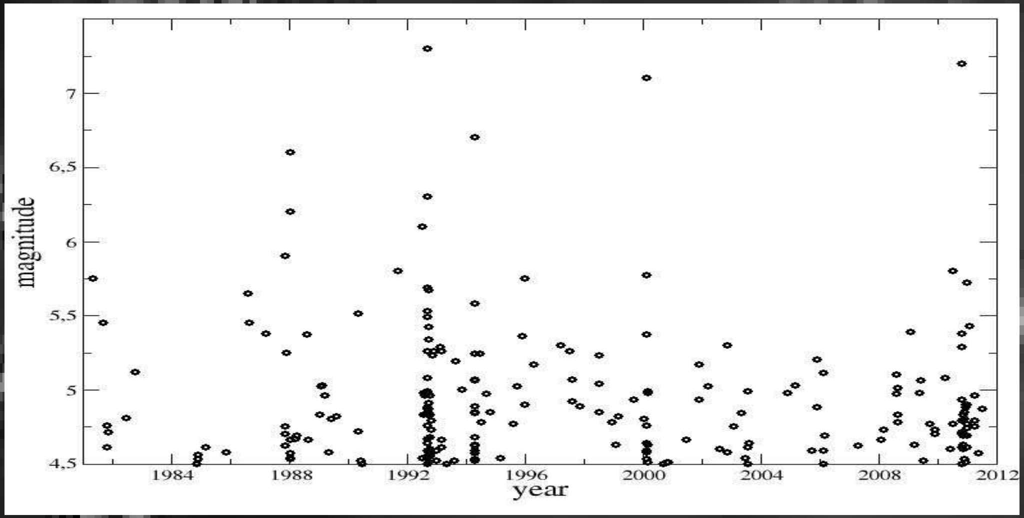
Point-Process approach

Looking at the recording of a seismic station, earthquakes usually appear as isolated pulses. This is due to the fact that the duration of an earthquake is much smaller than the average temporal distance between events.
For istance, for m=2 earthquake typical duration is 0.1 sec whereas typical temporal distances are larger than 1 minute.



Temporal clustering was clearly evident already with data available at the beginning of the 60's

TRIGGER POINT-PROCESS (Vere-Jones)
Cluster centres Poisson distributed in time would be regarded as ``ancestors'', and the cluster members as the first generation ``offsprings''. Clearly, the scenario can be iterated any number of times, and we can talk of n-th ordering clustering processes corresponding to the n-th generation offsprings.

Triggering Point-Process

Seismic rate: number of events for unit of time

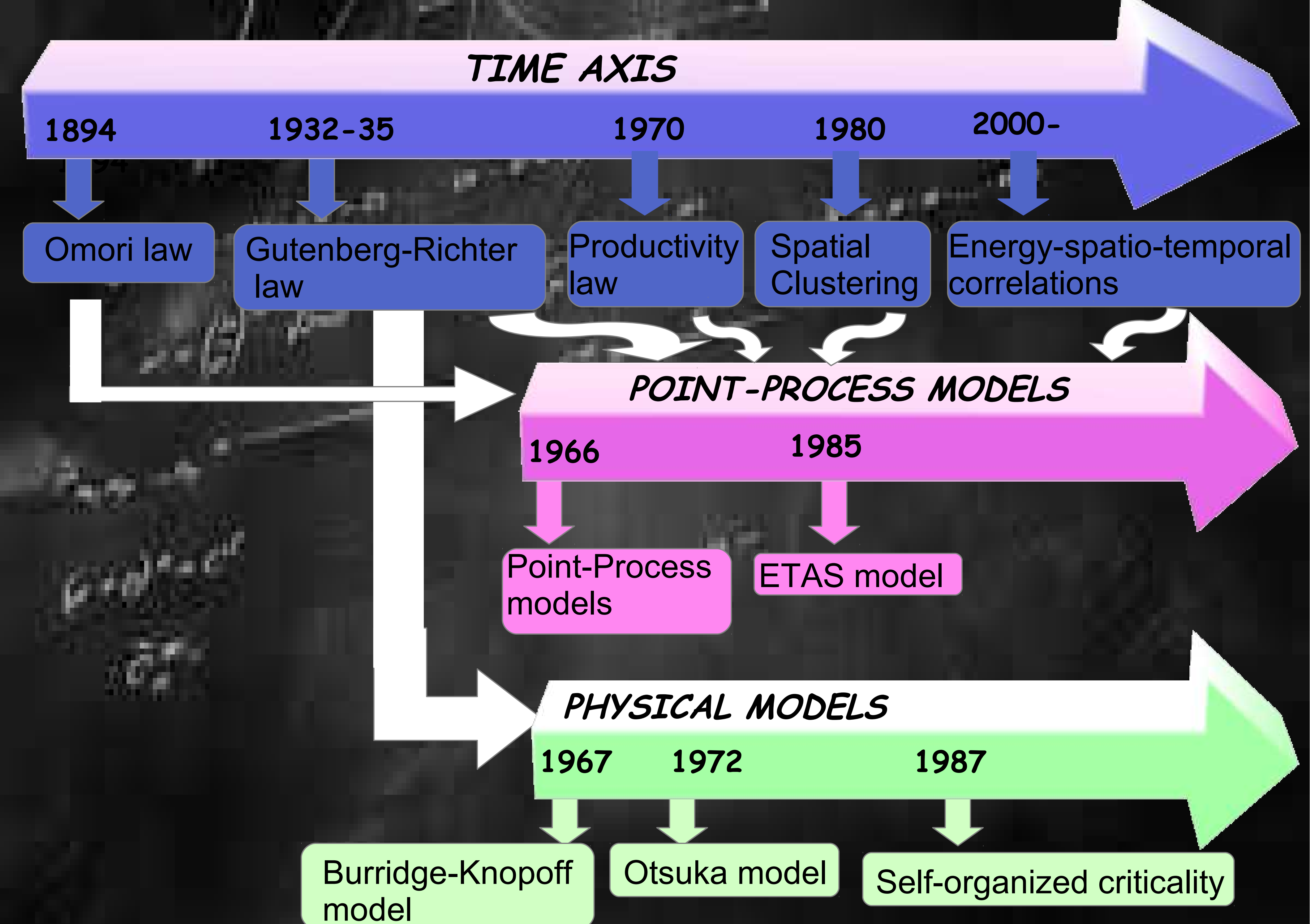
$$\lambda\left(t,\left|\left[t_i\right]\right.\right)=\mu+G\left(t-t_i\right)$$

λ is the fundamental quantity in seismic forecasting since it is proportional to the probability to have a future earthquake on the basis of the previous historical information

Generalization including also time and space

$$\lambda\left(E,r,t,\left|\left\{E_i,r_i,t_i\right\}\right.\right)=\mu\left(E,r\right)+G\left(E,r,t,\left|\left\{E_i,r_i,t_i\right\}\right.\right)$$

Macroscopic approach: an earthquake is viewed as just one point in a 5-dimensional space. All the details (the duration, the spatial extension.....) are neglected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Epidemic time aftershock sequence model

$$\lambda\left(E,r,t,\left|\left\{E_i,r_i,t_i\right\}\right.\right)=\mu\left(E,r\right)+G\left(E,r,t,\left|\left\{E_i,r_i,t_i\right\}\right.\right)$$

Empirical laws (Omori law, GR law, Productivity law and spatial clustering) are implemented to model the kernel G

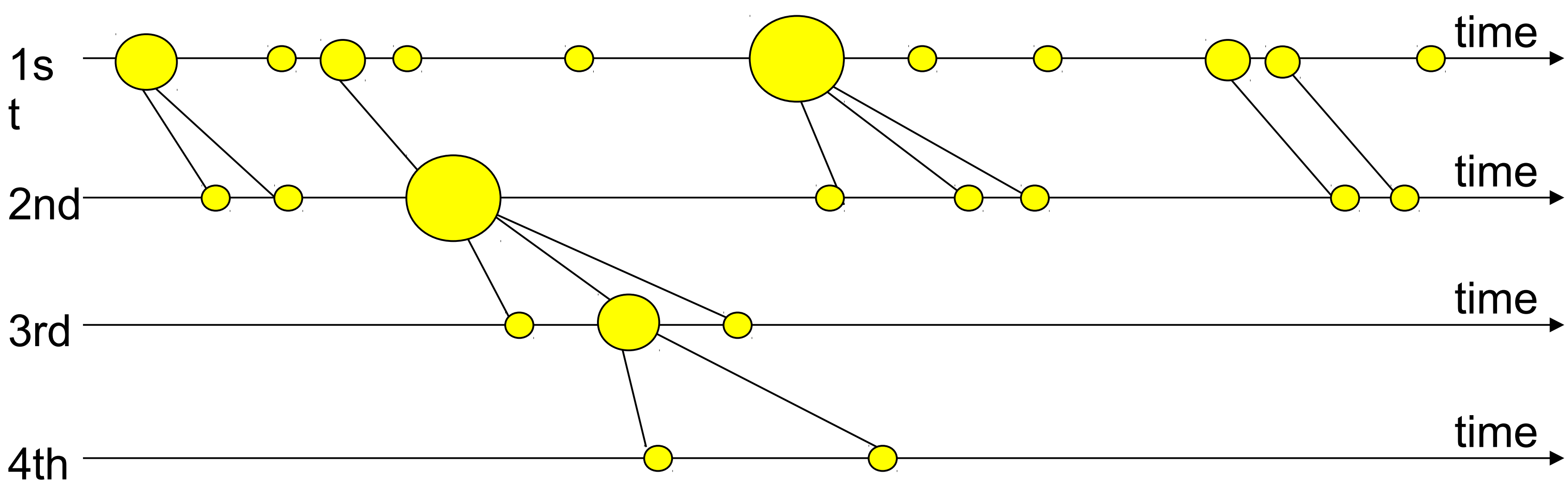
$$G\propto\sum E^{-\beta}E_i^{\beta'}\left(t-t_i\right)^{-p}\left(\frac{r-r_i}{E_i^{\gamma}}+d_z\right)^{-\delta}$$

The sum extends to all previous earthquakes

Epidemic time aftershock sequence (ETAS) model

$$\lambda\left(E,r,t,\left|\left[H_i\right]\right.\right)=\mu+\sum E^{-\beta}E_i^{\beta'}\left(t-t_i\right)^{-p}\left(\frac{r-r_i}{E_i^{\gamma}}+d_z\right)^{-\delta}$$

Can be viewed as a branching process



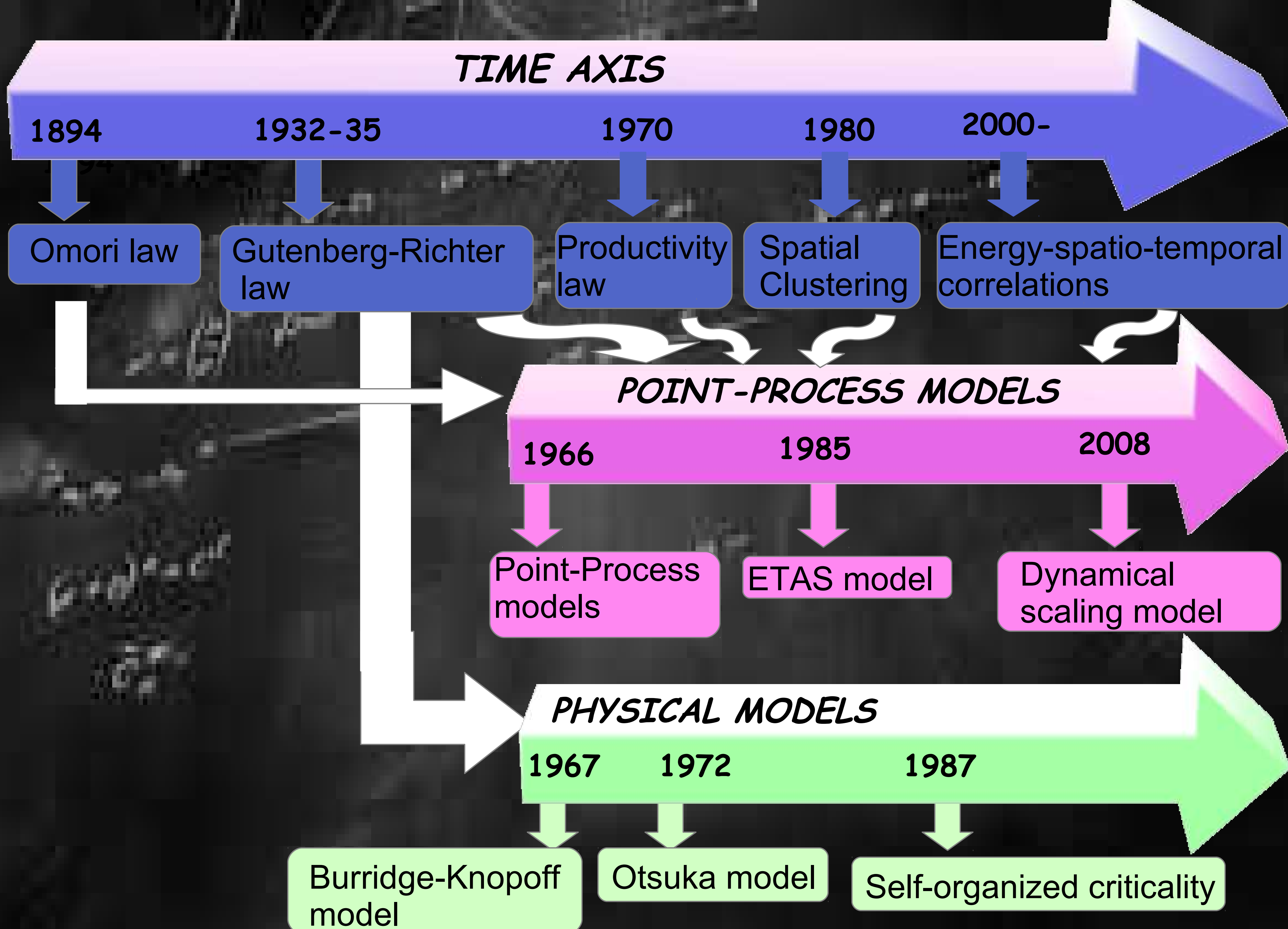
Totally different approach with respect to the Otsuka model

Epidemic time aftershock sequence (ETAS) model

Probably the most efficient tool actually available for seismic forecasting.

Notwistanding its simiplicity many non trivial patterns arise because of the many body interaction.

http://www.corssa.org/articles/themev/zhuang_et_al_c/zhuang_et_al_c.pdf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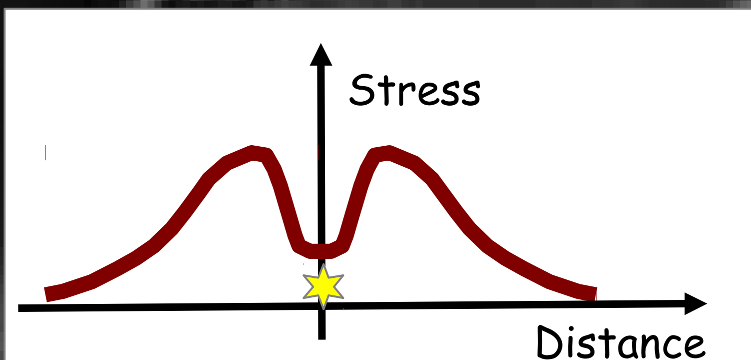
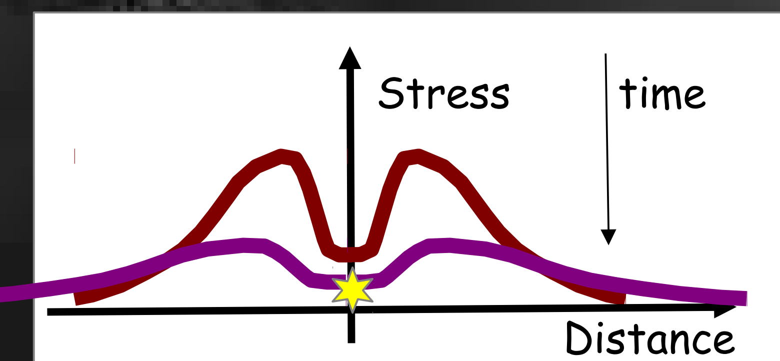
Triggering Point-Process from a physical perspective

The term $G(E,r,t|E_i,r_i,t_i)$ describes interaction between two earthquakes

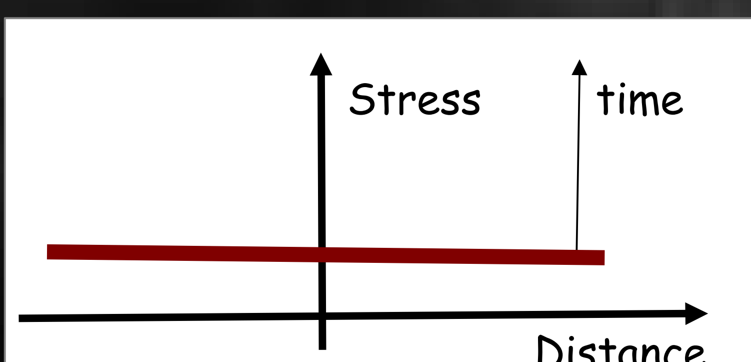


An earthquake modifies the stress filed in the surrounding area rising the probability of subsequent shocks



Some relaxation mechanism is present so that stress released evolves in time

$$\sigma_i(r-r_i,t-t_i)$$



Uniform stress increases at costant rate because of tectonic drive

$$\sigma_B(r,t)$$

Triggering Point-Process from a physical perspective

Earth Crust Elasticity

$$\sigma_{TOT}(r,t)=\sigma_B(r,t)+\sum\sigma_i(r-r_i,t-t_i)$$

Seismic rate and stress relationship (Beeler and Lockner [2003])

$$\lambda(r,t)\propto\dot{\sigma}_{TOT}(r,t) \quad \text{if} \quad \dot{\sigma}_{TOT}\ll 1$$

$$\lambda(r,t)\sim\exp\left[\sigma_{TOT}(r,t)\right] \quad \text{if} \quad \dot{\sigma}_{TOT}\gg 1$$

After mainshock relaxation shear rate evolves slowly

$$\lambda(r,t)=\mu(r)+\sum\lambda_i(r-r_i,t-t_i)$$

Triggering Point-Process from a physical perspective

Explitely including the energy

$$\lambda(E,r,t) = \mu(E,r) + \sum G(E, r - r_i, t - t_i | E_i)$$

The sum extends over all previuos earthquakes and G becomes a two-point correlation function, i.e. The probability that at time t an event is triggered by a previous one occured at time t_i.

The correlation function in the ETAS model

FACTORIZATION

$$G(E, t-t_i, r-r_i | E_i) = P(E)\,Q(E_i)\,T(t-t_i)\,R(r-r_i)$$

EMPIRICAL LAWS FOR P,Q,T,R

$P(E) \cong E^{-1-b2/3}$ Gutenberg-Richter law

$Q(E) \cong E^{-1-\alpha 2/3}$ Productivity law

$T(t) \cong (t+c)^{-p}$ Omori law

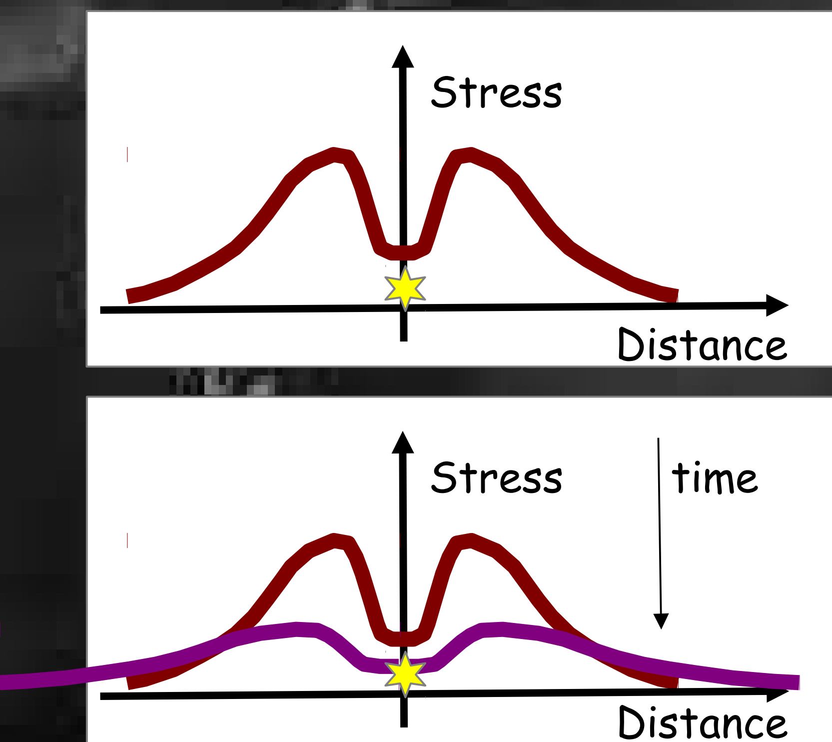
$R(x) \cong x^{-\delta}$ spatial clustering

The correlation function G within a scaling approach

$$G\left(E, t-t_i, r-r_i \middle| E_i\right)$$

without he hypothesis of FACTORIZATION

Relevant time and temporal scales in the process



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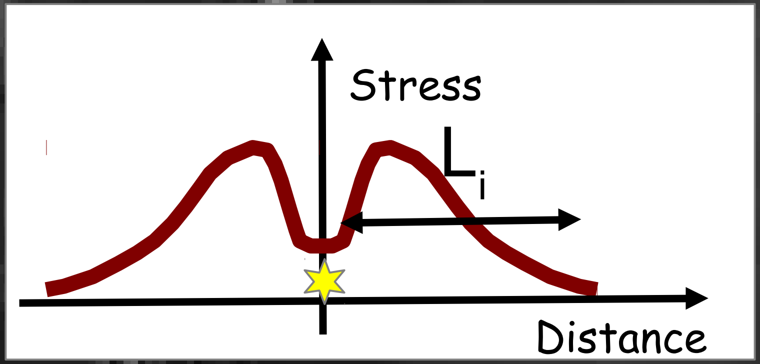

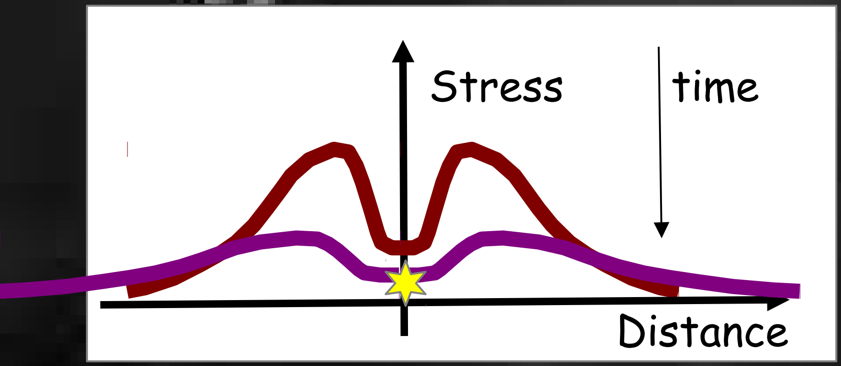

$$L_i \propto E_i^{\gamma} \propto 10^{0.5 m_i}$$

The correlation function G within a scaling approach

$$G\left(E, t-t_i, r-r_i \middle| E_i\right)$$

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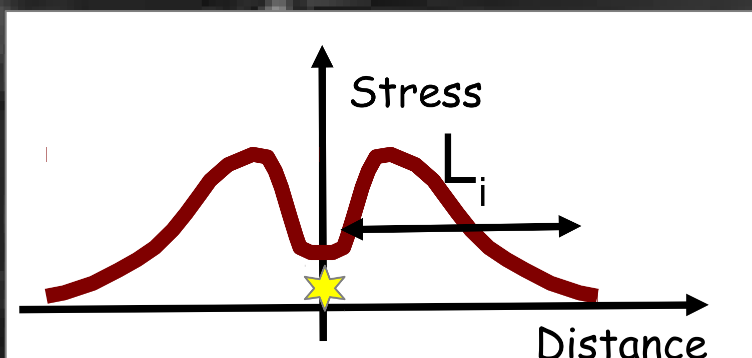

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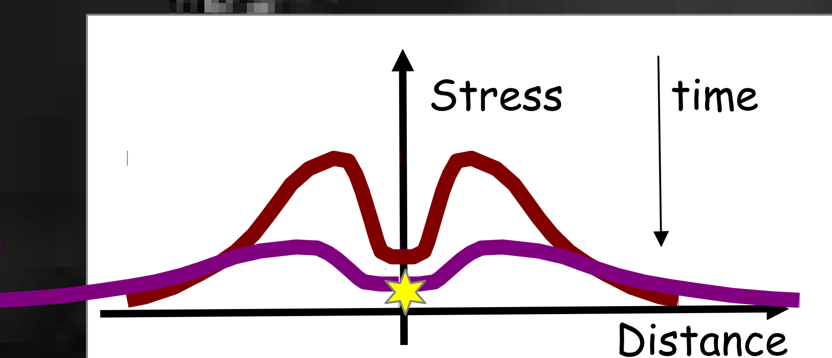


τ characteristic relaxation time

The correlation function G within a scaling approach

Rewriting G in terms of all scales in the process

$$G\left(E,t-t_i,r-r_i \middle| E_i\right)=F\left(L,\delta t,\delta r,L_i,\tau\right)$$

Where $L \propto E^{\gamma} \propto 10^{0.5m}$
is the typical length
of the triggered event

The correlation function G within a scaling approach

Rewriting G in terms of all scales in the process

$$G\left(E, t-t_i, r-r_i \middle| E_i\right) = F\left(L, \delta t, \delta r, L_i, \tau\right)$$

Where $L \propto E^{\xi} \propto 10^{0.5m}$
is the typical length
of the triggered event

SCALE INVARIANCE ASSUMPTION

Introducing a scaling factor a

$$F\left(L, \delta t, \delta r, L_i, \tau\right) \simeq a^{-2-\gamma} F\left(\frac{L}{a}, \frac{\delta t}{a^{\gamma}}, \frac{\delta r}{a}, \frac{L_i}{a}, \frac{\tau}{a^{\gamma}}\right)$$

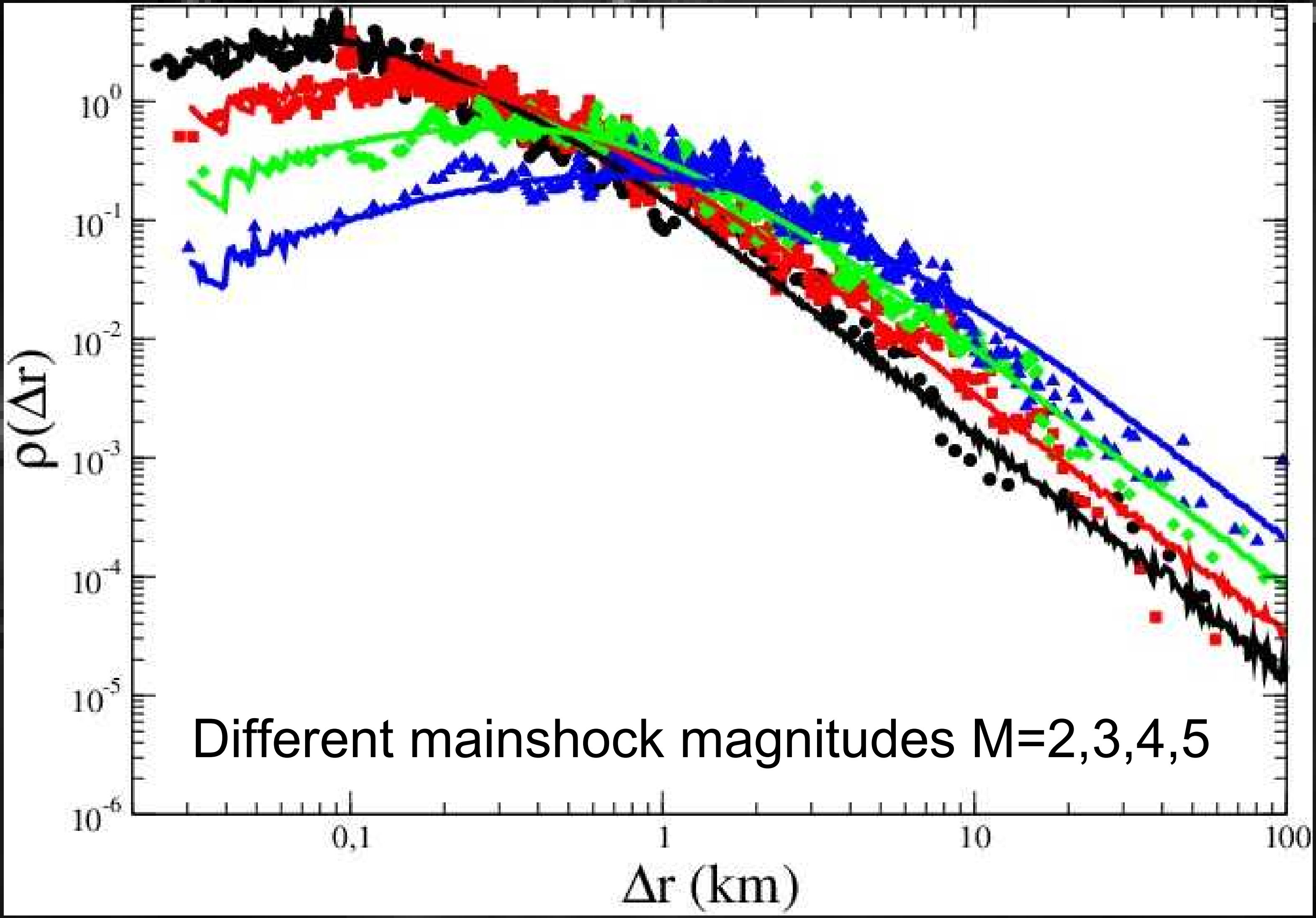
Where γ is a scaling exponent

The correlation function G within a scaling approach

Setting $a=L_i$

$$F\left(L,\delta t,\delta r,L_i,\tau\right)\simeq L_i^{-2-\gamma}H\left(\frac{L}{L_i},\frac{\delta t}{L_i^{\gamma}},\frac{\delta r}{L_i},\frac{\tau}{L_i^{\gamma}}\right)$$

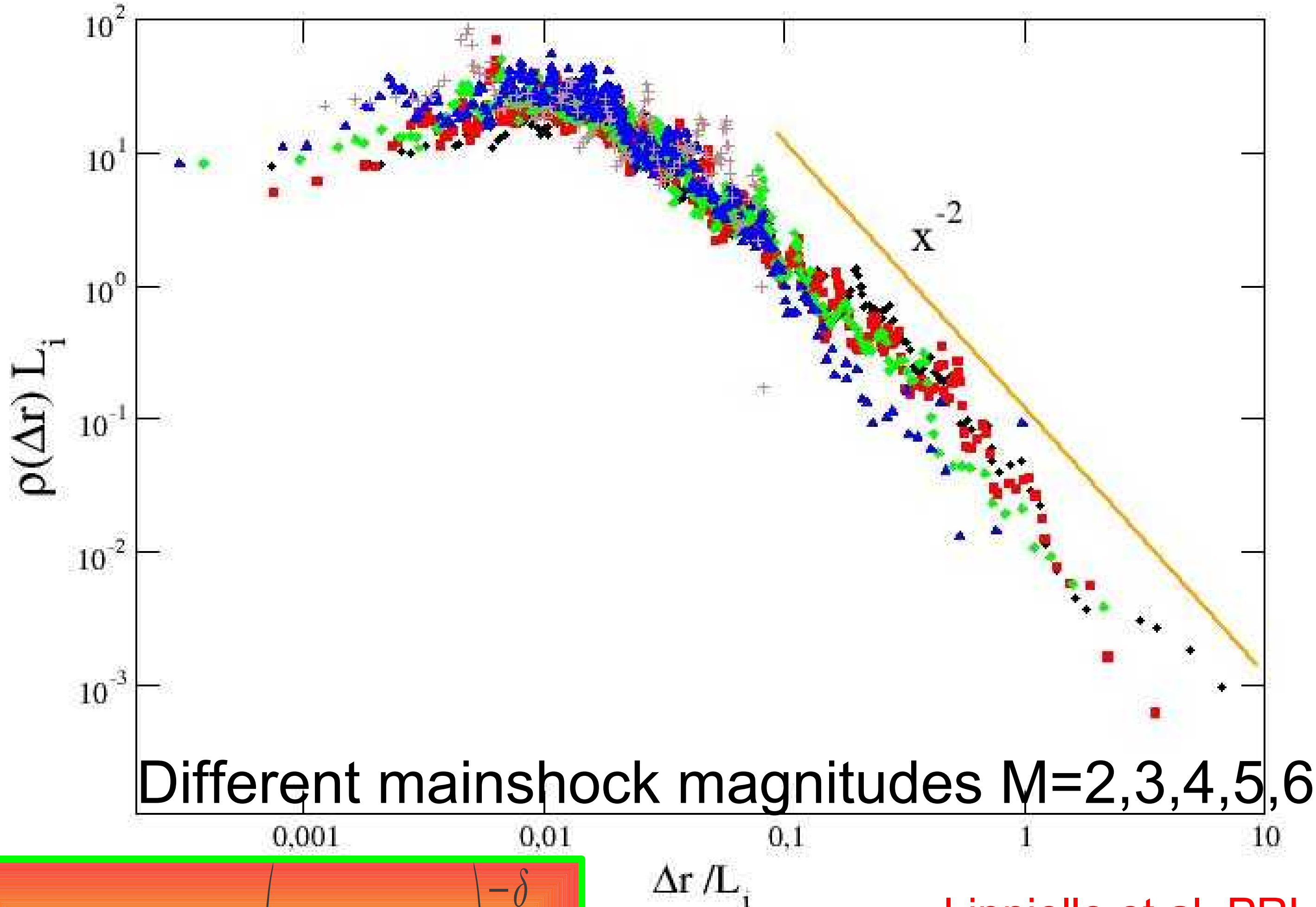
EMPIRICAL VERIFICATIONS OF THE SCALING HYPOTHESIS



Aftershock spatial density

Lippiello et al, PRL 2009

AFTERSHOCK SPATIAL DENSITY DISTRIBUTION

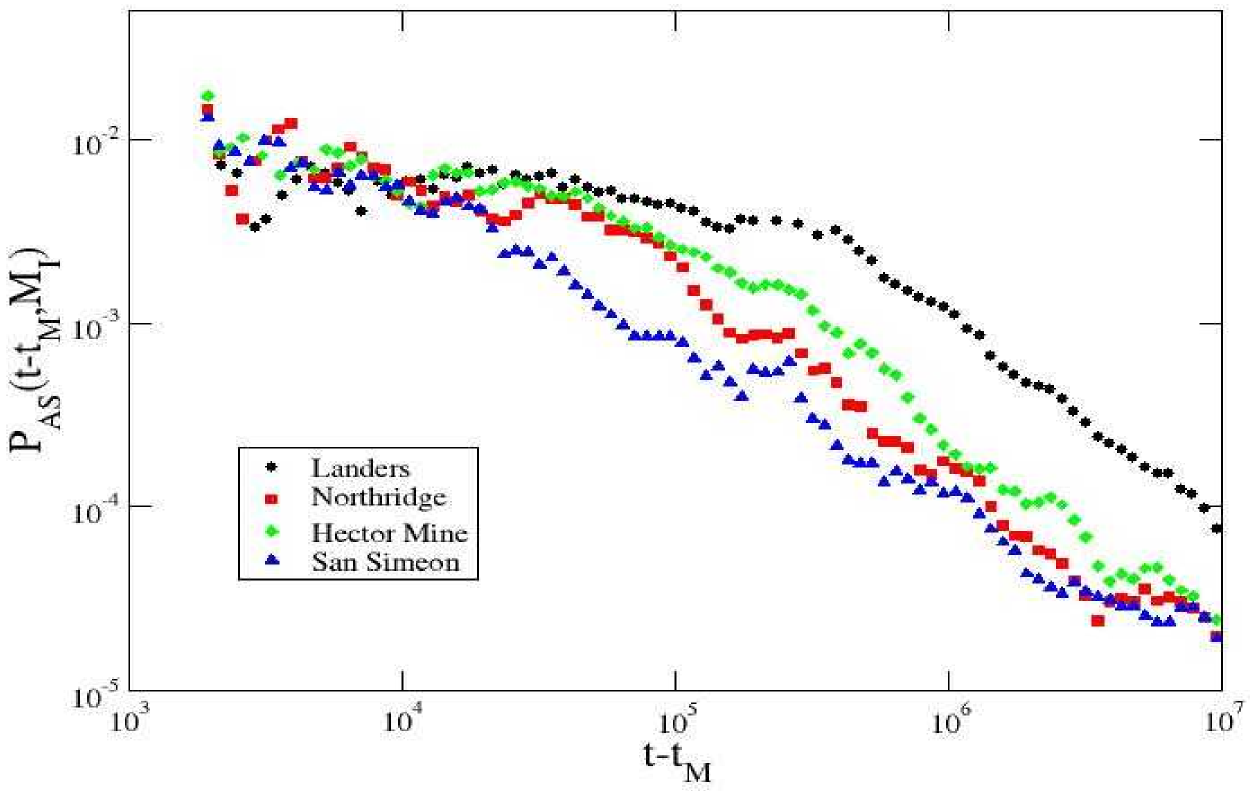


Lippiello et al, PRL 2009

$$R\left(r-r_i\right)=\left(\frac{r-r_i}{L_i}+d_z\right)^{-\delta}$$

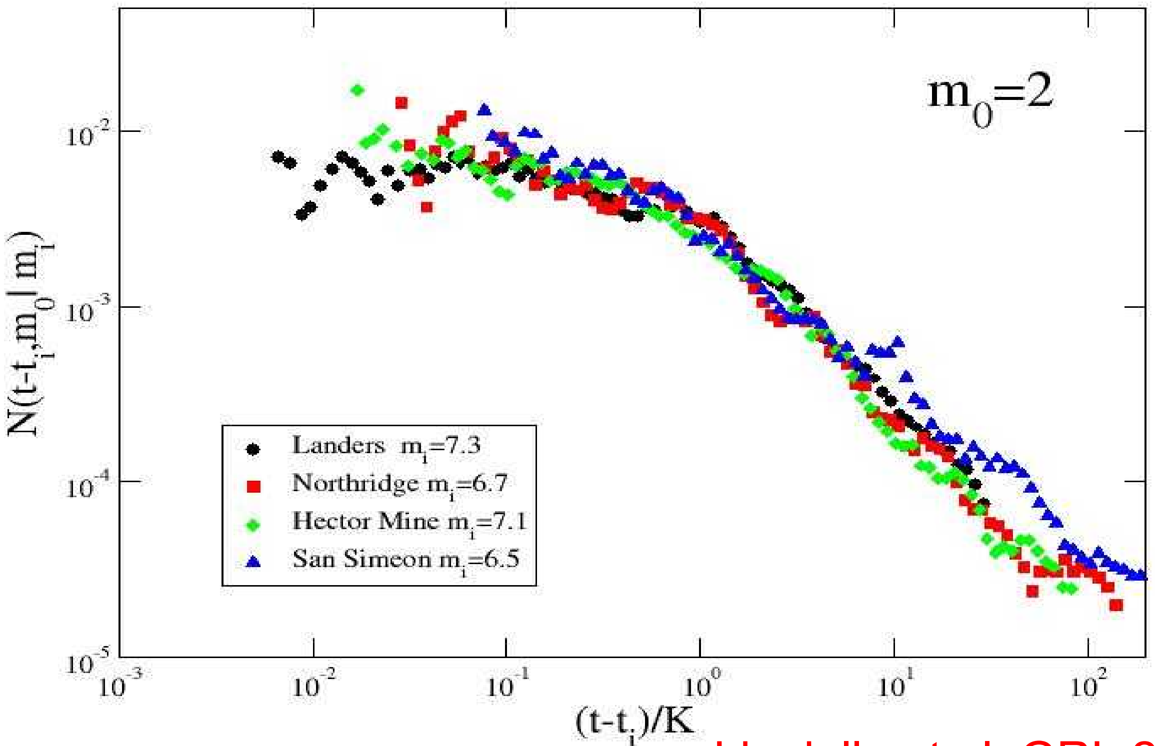
SPACE-ENERGY SCALING

Omori decay in experimental catalogs



Different colors are different mainshock magnitudes

Omori decay in experimental catalogs



Lippiello et al, GRL 2007

Different colors are different mainshock magnitudes

$$T\left(t-t_i\right)=\left(\frac{t-t_i}{L_i^{\gamma}}+c\right)^{-p}$$

TIME-ENERGY SCALING

With $\gamma=2$

DYNAMICAL SCALING IN SEISMICITY

$$G = M_i^{-2} F\left(\frac{M}{M_i}, \frac{(t-t_i)}{M_i}, \frac{(r-r_i)}{M_i^{1/2}}\right)$$

For convenience let us introduce the quantity $M = L^2 = 10^{-m}$

Magnitude Distribution

$$\rho(M) = \int d\,\Delta t_i \int d\,\Delta r_i\, G(L, \Delta t_i, \Delta r_i | M_i) = F_1\left(\frac{M}{M_i}\right)$$

setting $F_1(x) = (x+d)^{-1}$

$$\rho(M) = \left(\frac{M}{M_i} + d\right)^{-1}$$

The costant d is necessary for normalization

DYNAMICAL SCALING IN SEISMICITY

Remembering that $M_i = 10^{m_i}$

$$\rho(M) \propto 10^{-m} 10^{m_i}$$

Gutenberg-Richter law is recovered and also productivity law with the condition $\alpha = b$.
GR law and productivity are not independent laws.

We have ignored the costant d that however plays an important role introducing non trivial correlations.

DYNAMICAL SCALING IN SEISMICITY

OMORI law

$$\rho(\Delta t_i) = \int dM \int d\,\Delta r_i\, G(M, \Delta t_i, \Delta r_i | M_i) = F_2\left(\frac{\Delta t_i}{M_i}\right)$$

$$\text{setting} \quad F_2(x) = (x+c)^{-1}$$

$$\rho(\Delta t_i) = \left(\frac{\Delta t_i}{M_i} + c\right)^{-1}$$

The costant c is necessary for normalization

Omori law is recovered and also productivity law.
The scaling analysis reveals that also Omori behavior and Productivity law are not independent.

DYNAMICAL SCALING IN SEISMICITY

OMORI law

Remembering that $M_i = 10^{m_i}$

$$\rho(\Delta t_i) \simeq M_i (\Delta t_i)^{-1} \propto 10^{m_i} (\Delta t_i)^{-1}$$

Omori law is recovered and also productivity law.
The scaling analysis reveals that also Omori behavior and Productivity law are not independent.

A COMMON ORIGIN FOR POWER LAWS IN SEISMIC OCCURRENCE

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LETTERS

Common dependence on stress for the two fundamental laws of statistical seismology

Clément Narteau[1], Svetlana Byrdina[1,2], Peter Shebalin[1,3] & Danijel Schorlemmer[4]

Numerical simulations

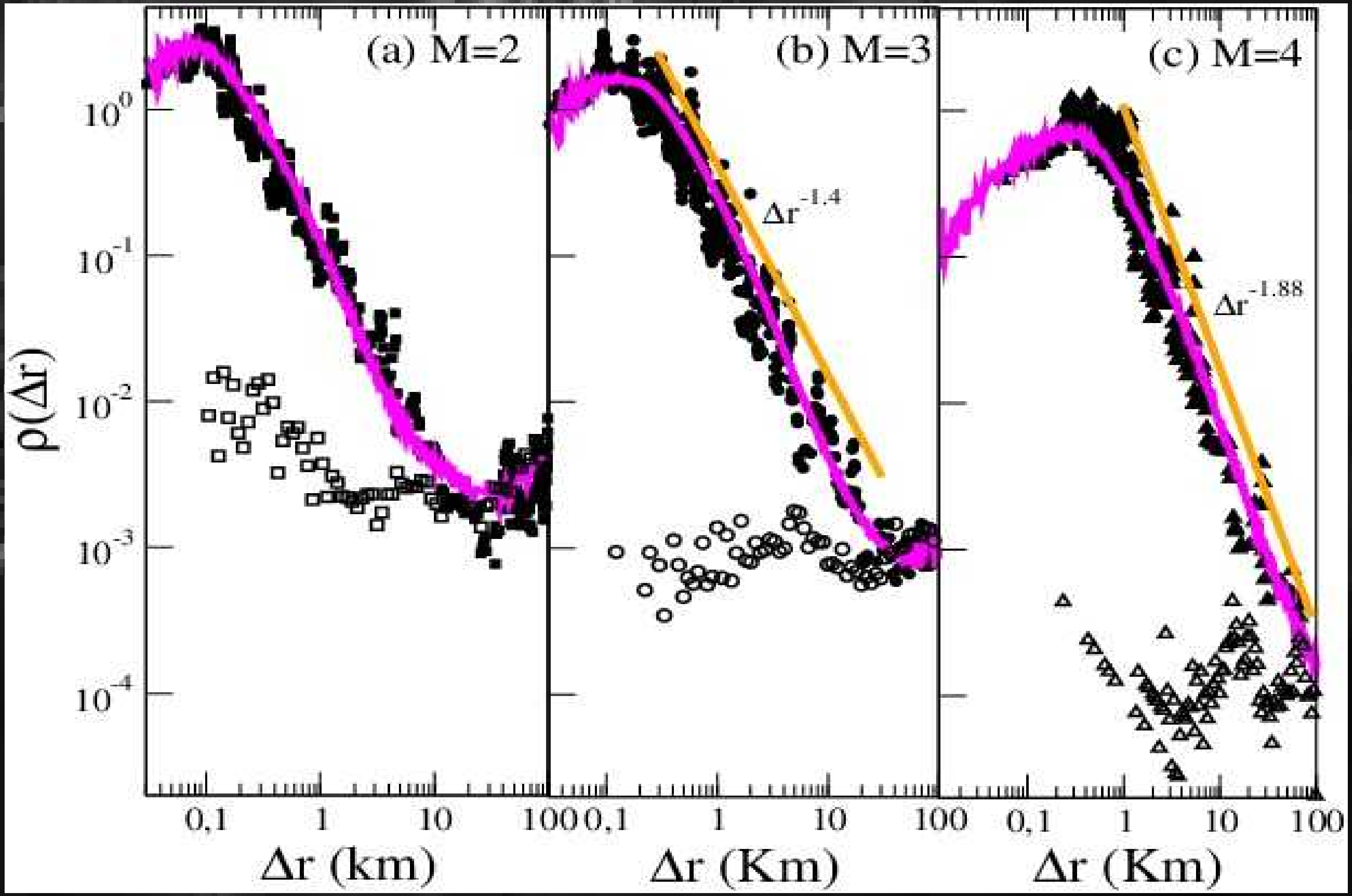
$$\lambda(E,r,t)=\mu(E,r)+\sum G(E,r-r_i,t-t_i|E_i)$$

$$G \propto h\left(\frac{t-t_i}{c\,10^{b(m_i-m)}}, \frac{r-r_i}{(t-t_i)^{1/\gamma}}\right)$$

$$G \propto (t-t_i)^{1/\gamma}\left(\frac{t-t_i}{c\,10^{b(m_i-m)}}+1\right)^{-1}\left(\frac{r-r_i}{(t-t_i)^{1/\gamma}}+1\right)^{-\mu}$$

Linear Density distribution

Lippiello et al, PRL 2009



Magnitude correlations

Magnitude Distribution

$$\rho(M) = \left(\frac{M}{M_i} + d\right)^{-1}$$

The costant d is necessary for normalization

The importance of magnitude correlations for seismic forecasting

The presence of correlations between subsequent earthquake magnitudes provides a first answer to the question concerning the existence of premonitoring indications on the subsequent earthquake magnitude. Magnitude correlations, indeed, imply that earthquake occurrence modify physical properties in such a way to influence the subsequent earthquake magnitude. The understanding of these modifications represent a possible tool to predict features of the next seismic event.
In the opposite scenario, magnitudes are not affected by preexisting phyiscal properties and therefore, magnitudes are totally uncorrelated.

Experimetal evidences: Lippiello et al. 2007,....,2012

Linear Density distribution and stress diffusion

The agreement between experimental and numerical results supports the validity of the scaling relation

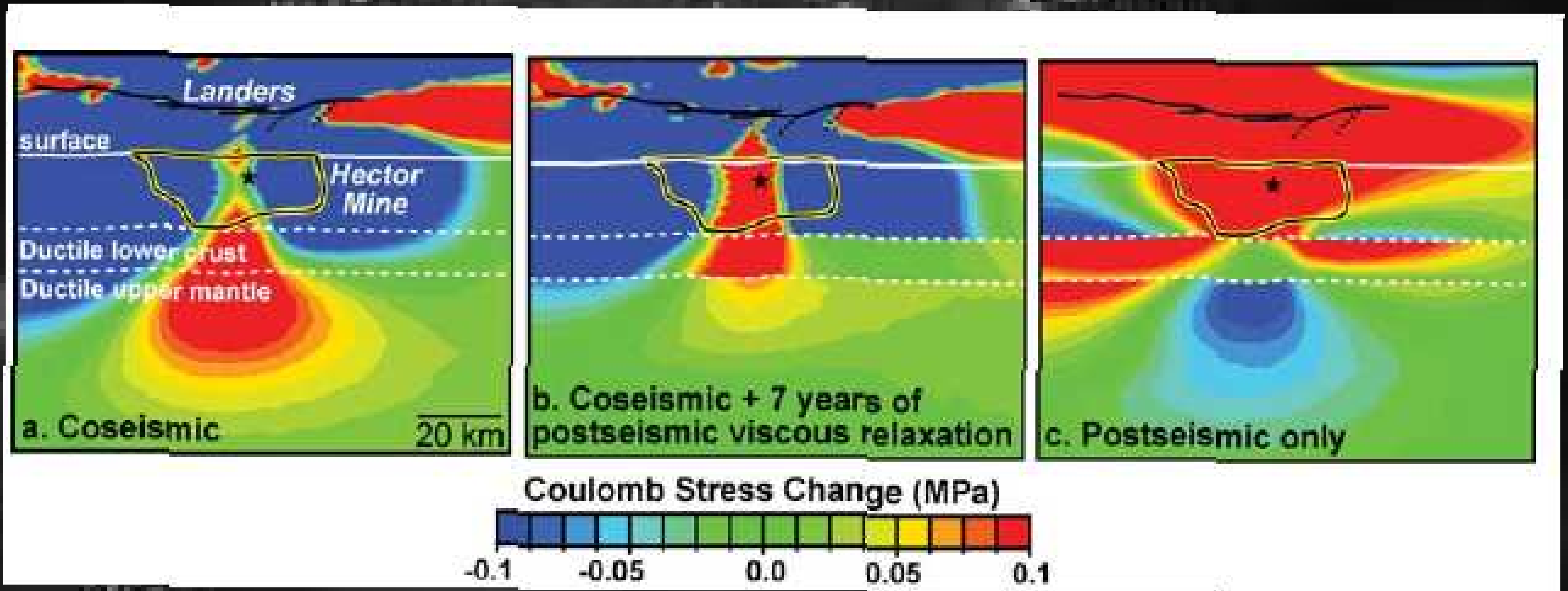
$$\Delta r \propto \Delta t^{H} \quad \text{with} \quad H = 1/\gamma \approx 0.5$$

which implies that the evolution in time of stress is consistent with a diffusion equation.

Static stress diffusion has been proposed has one of the main mechanisms responsible of aftershock triggering

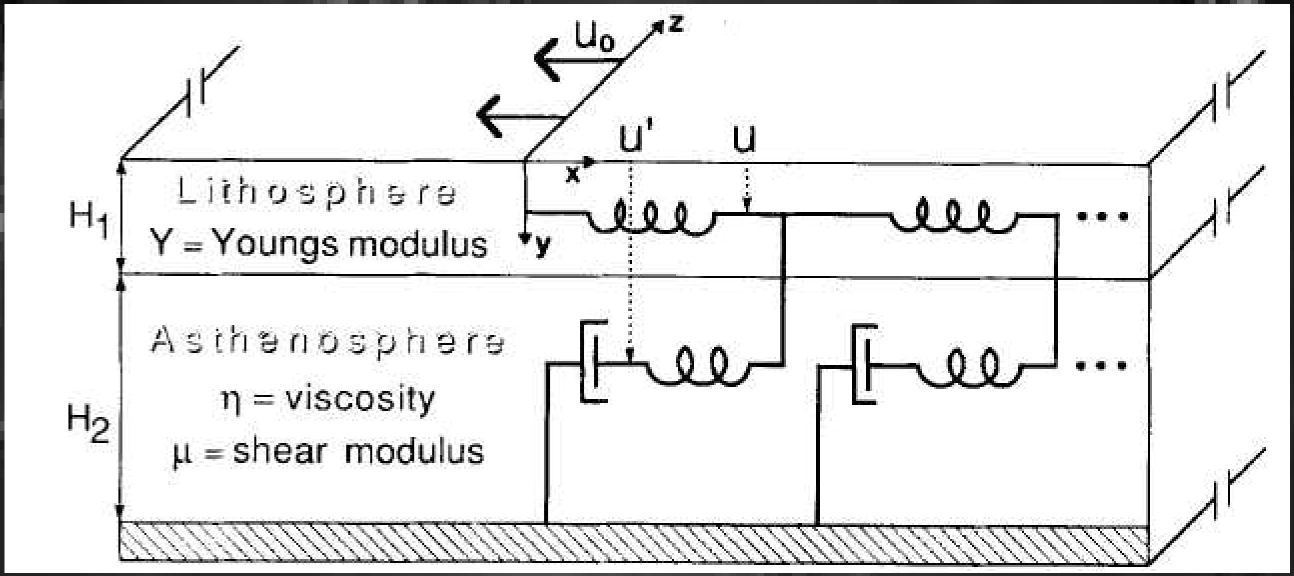
The very good fit of numerical simulations support this conclusion: Numerical results are obtained with H=0.47

Stress diffusion



FREED, Nature 2002

Stress diffusion



Stress transfer between faults through viscous relaxation may be a general cause of earthquake clustering. For example, Lynch et al. (2003) suggest that seismicity on a northern and southern San Andreas–type fault system can become coupled by the transfer of stress through lower crustal flow. Chèry et al. (2001) appeals to a similar stress transfer process to explain a sequence of three M > 8 earthquakes that occurred in Mongolia during a 52-year period despite great distances (400 km) that separate the events. And in a global review of the relative distance and time delay separating pairs of earthquakes, Marsan & Bean (2003) found that seismic activity diffuses away from an earthquake as the delay time increases following its occurrence, which they attributed to viscous diffusion of stress in the upper mantle.

OPEN QUESTIONS

- Is it possible to recover the observed scaling invariance in physical models for seismic occurrence?

- Is it the same scaling invariance observed in other physical processes?

- Is it possible to develop a theory to explain the origin of scaling invariance?

- Is it possible to develop a branching model that combines the microscopic chain-reaction model to the triggering models?