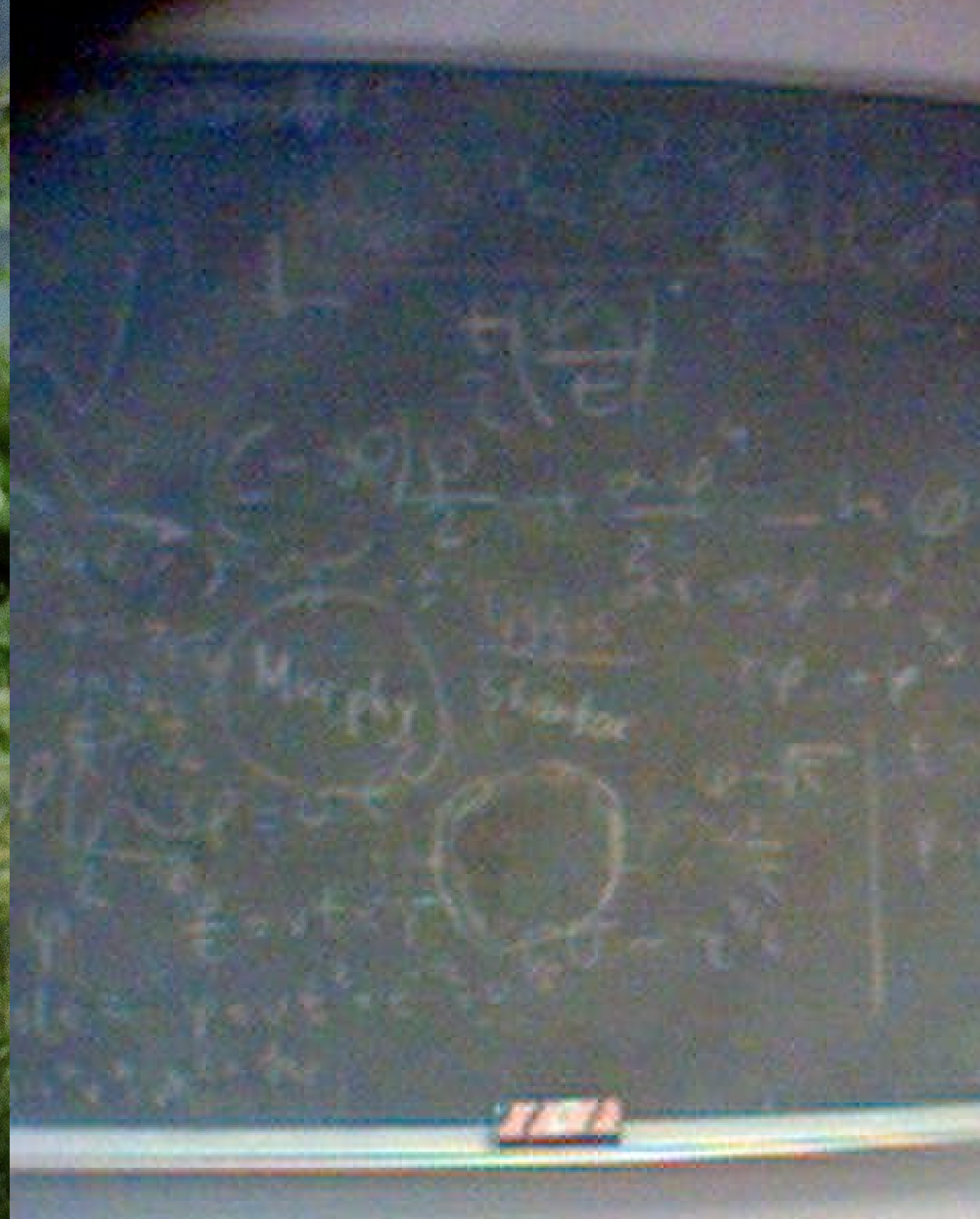


Murphy

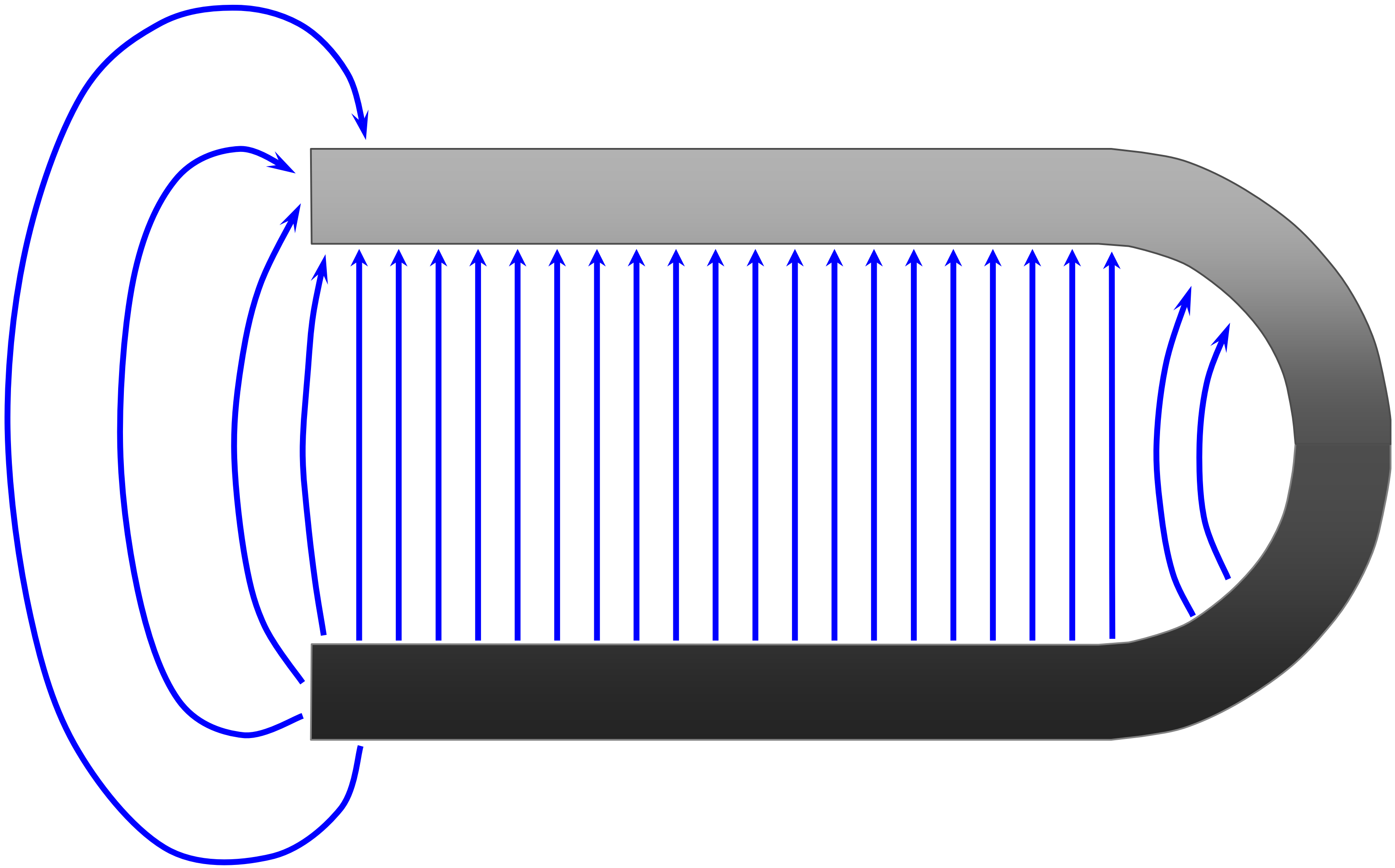
Current Voltage Curves in the regime of strong pinning and the Coulomb's Law of Friction

with Alexander Thomann, Gianni Blatter

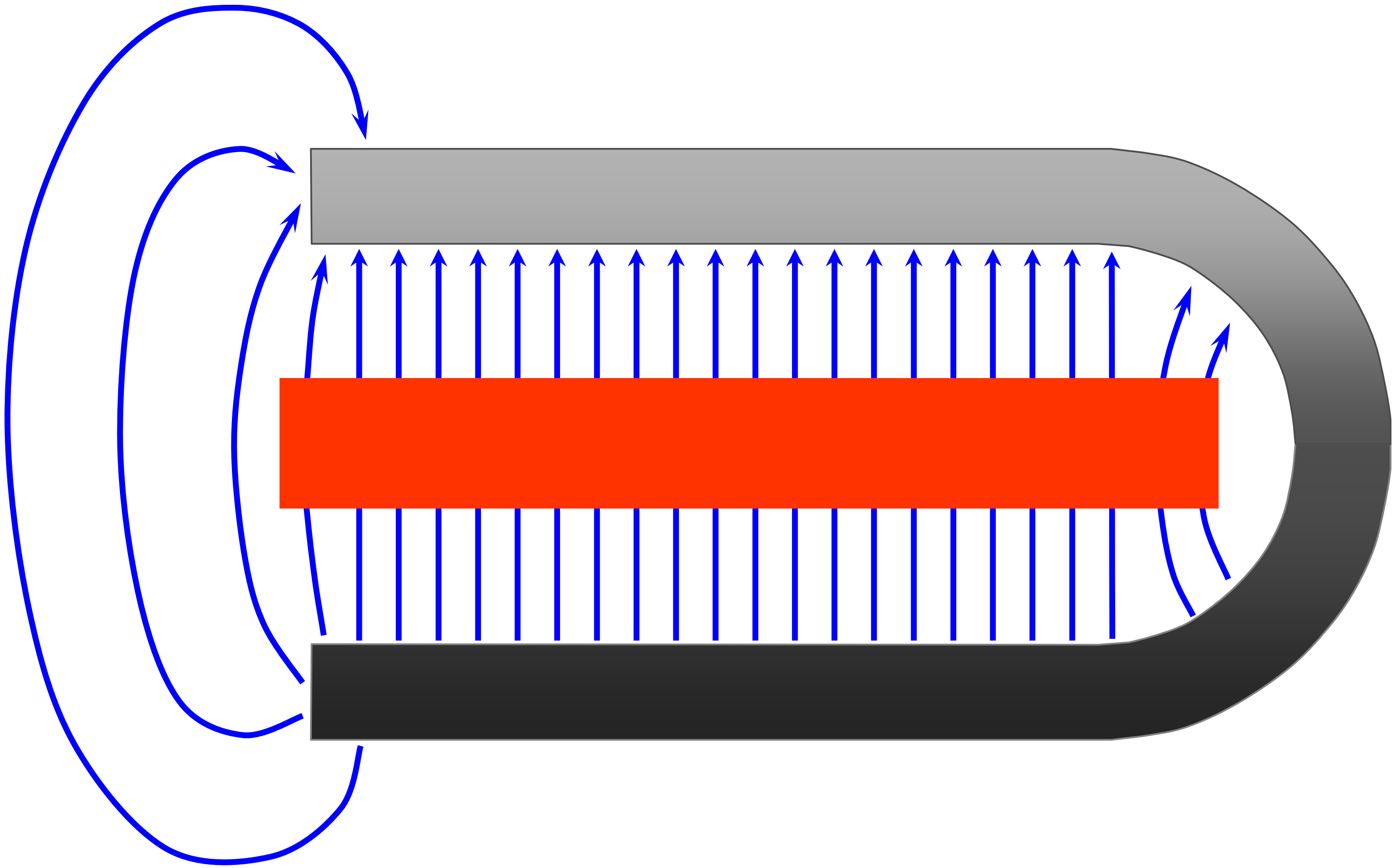
ETH Zurich, Switzerland

Phys. Rev. Lett. **108**, 217001 (2012)

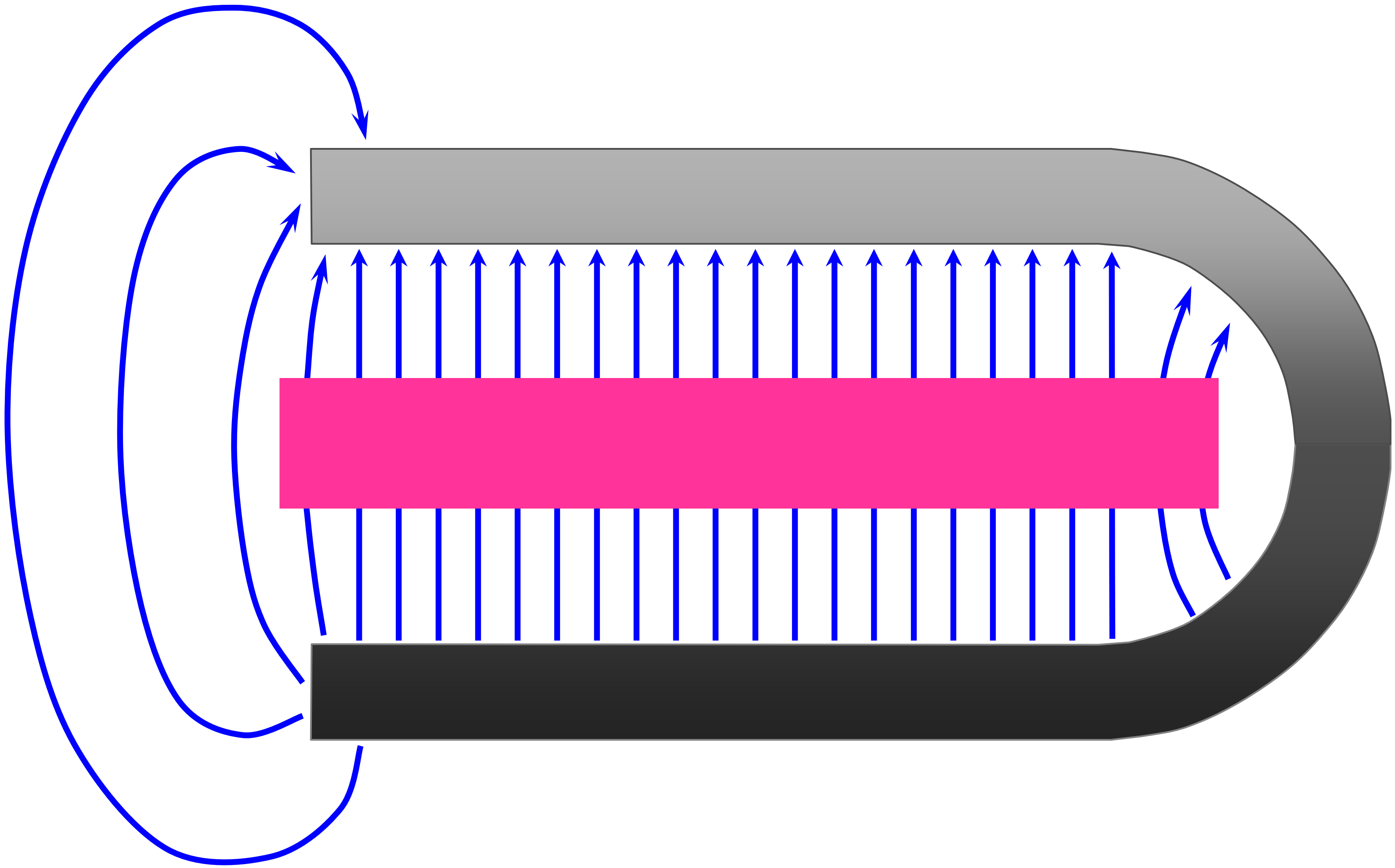
Permanent magnet with magnetic field lines



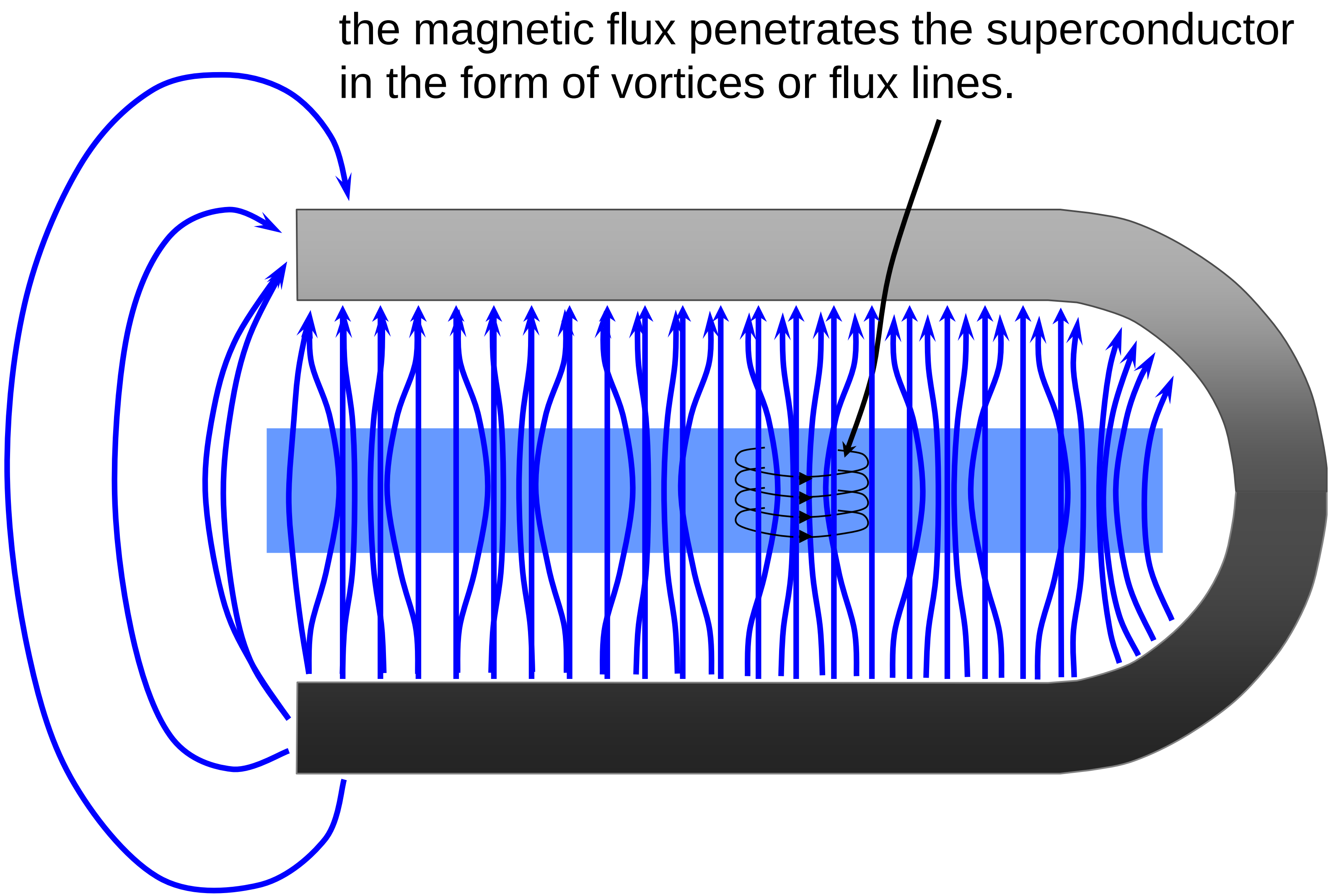
Inserting a normal metal ......



and cooling it down ......

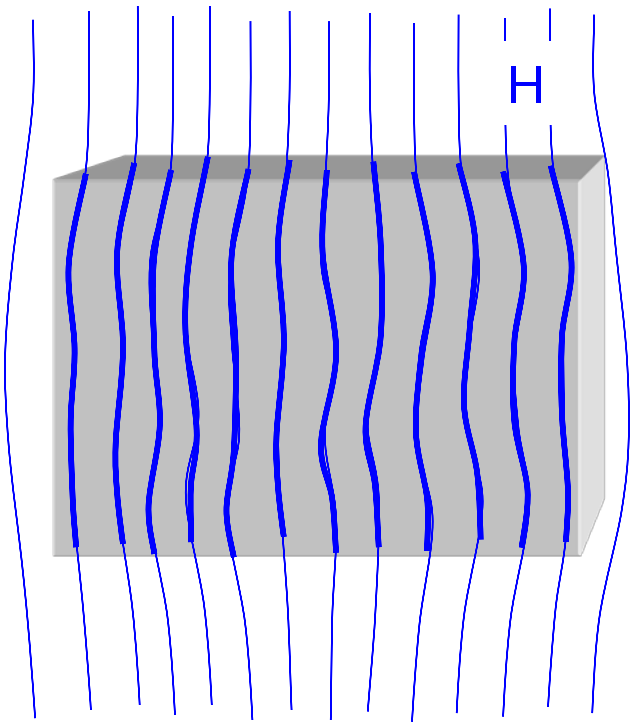


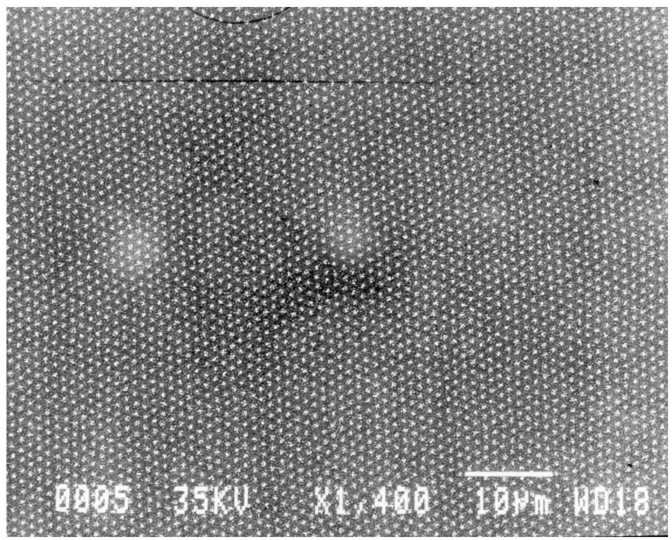
..... into the superconducting state,



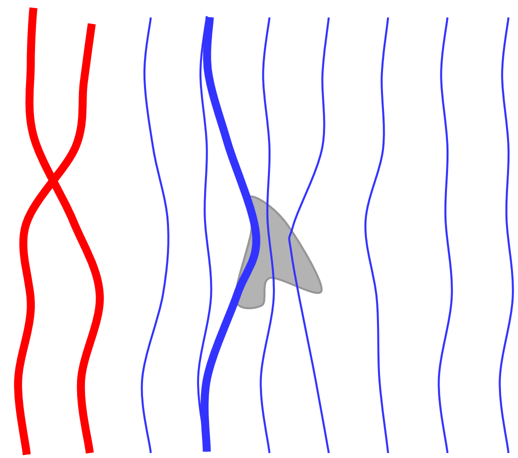
Physics of Vortex Matter

In a type II superconductor the magnetic field penetrates through the superfluid via creation of topological defect lines: vortices.

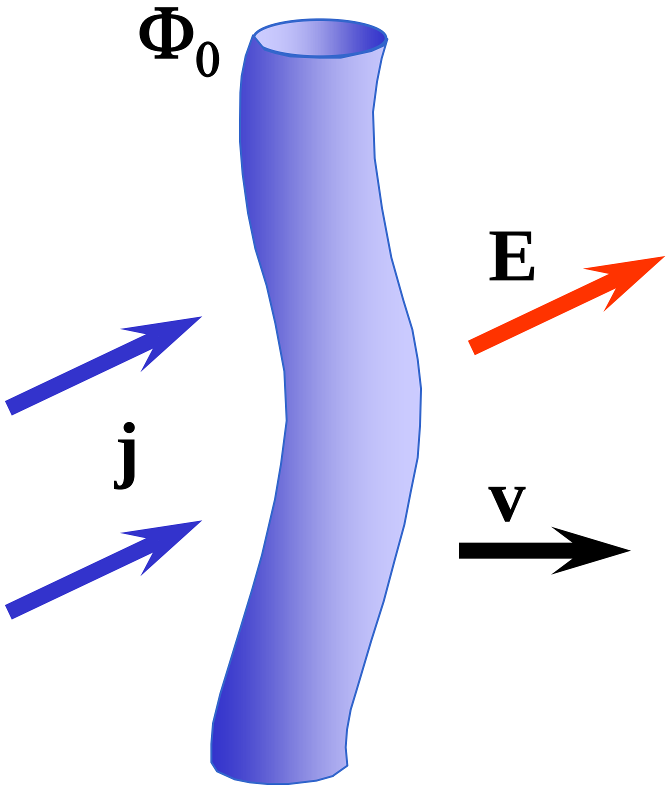




weak pinning by defects



Vortex dynamics



Vortex equation of motion

$$\eta \mathbf{v} = \frac{\Phi_0}{c}\, \mathbf{j} \times \mathbf{n}$$

dissipative force

Lorentz force

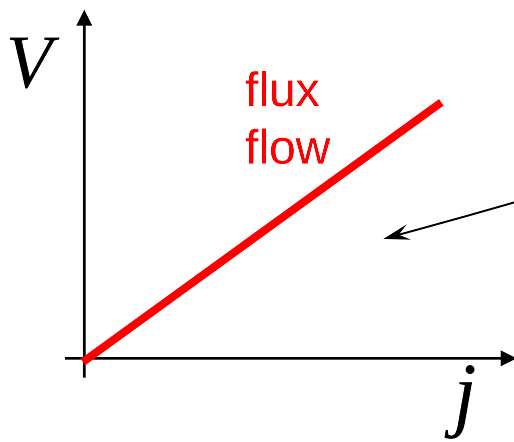
External current produces the Lorentz force which leads to the vortex motion,

$$v = \frac{\Phi_0}{c}\frac{1}{\eta}\, j,$$

and the dissipative motion produces the flux flow resistivity,

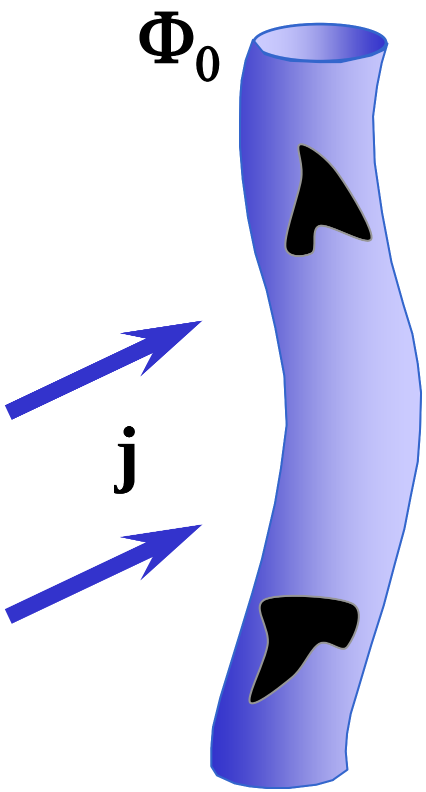
$$\mathbf{E} = \frac{1}{c}\mathbf{B} \times \mathbf{v}$$

$$\Rightarrow \quad \rho_{\mathrm{ff}} \simeq \rho_n \frac{B}{H_{c_2}}.$$



Thus the superconductor loses its most valuable property of **dissipation free** current transport.

Pinning



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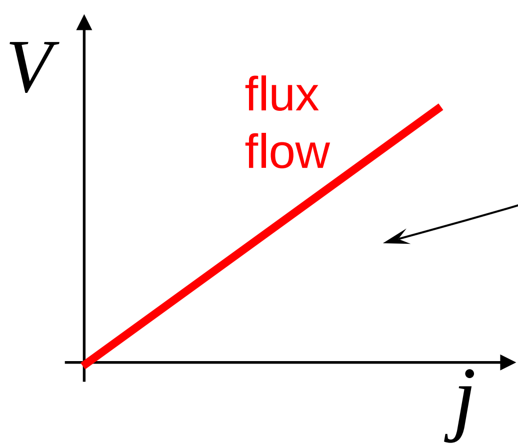

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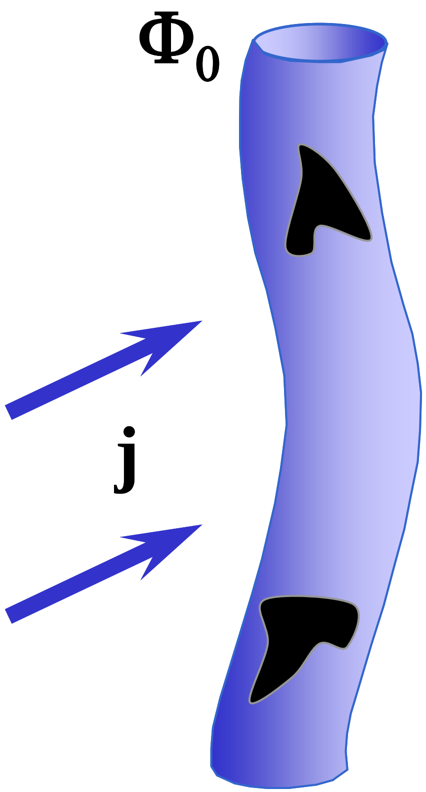

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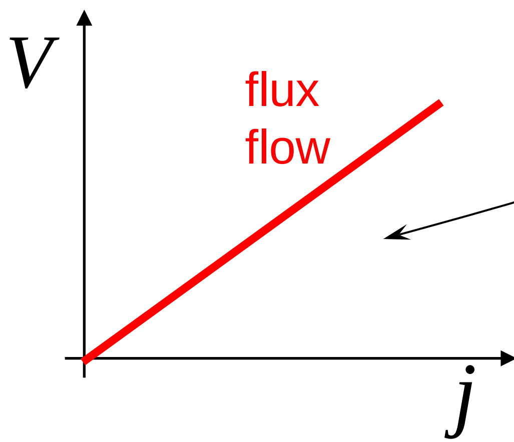

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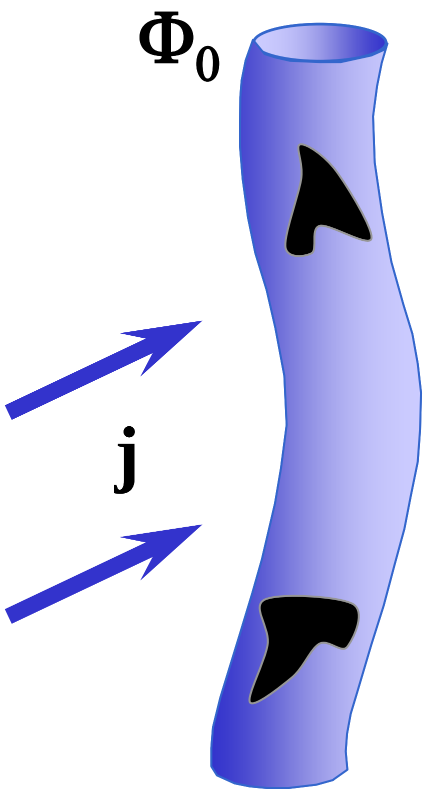

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pinning force

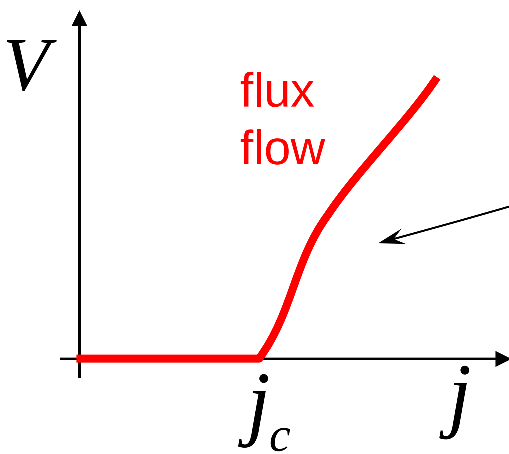
Lorentz force

The Lorentz force is balanced by the pinning force and velocity

$$\mathrm{v} = 0$$

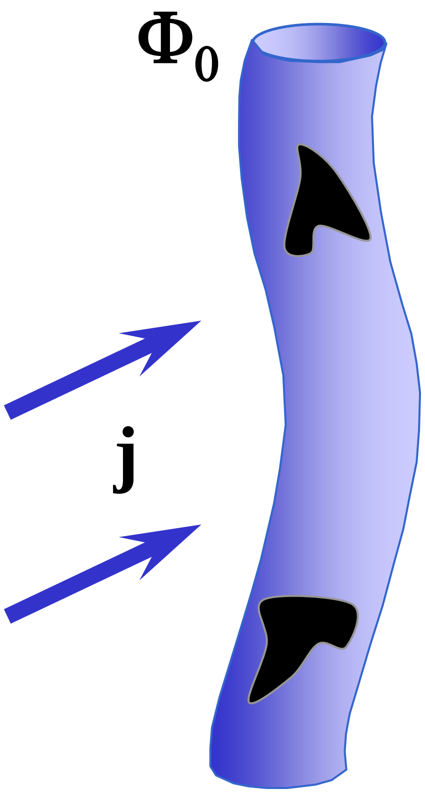
below the critical current density

$$j < j_c.$$



Thus the superconductor loses its most valuable property of **dissipation free** current transport.

Pinning



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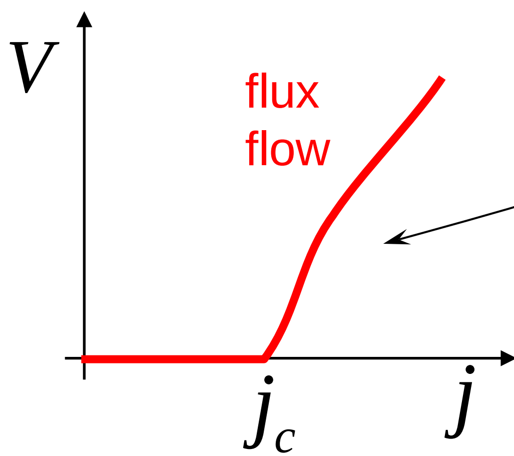
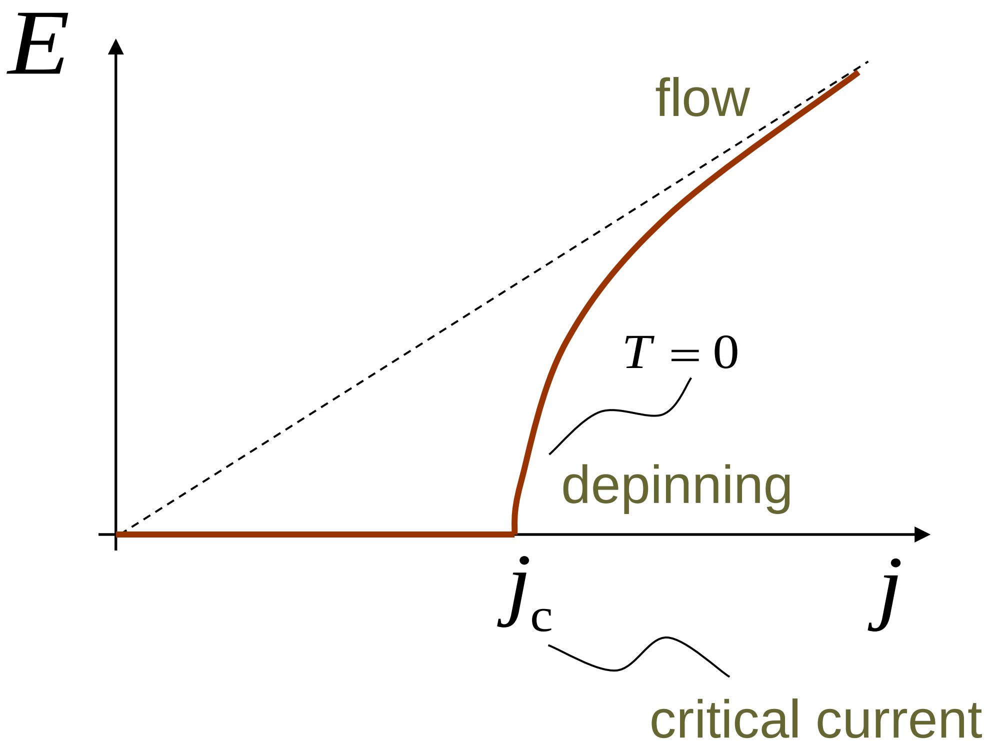


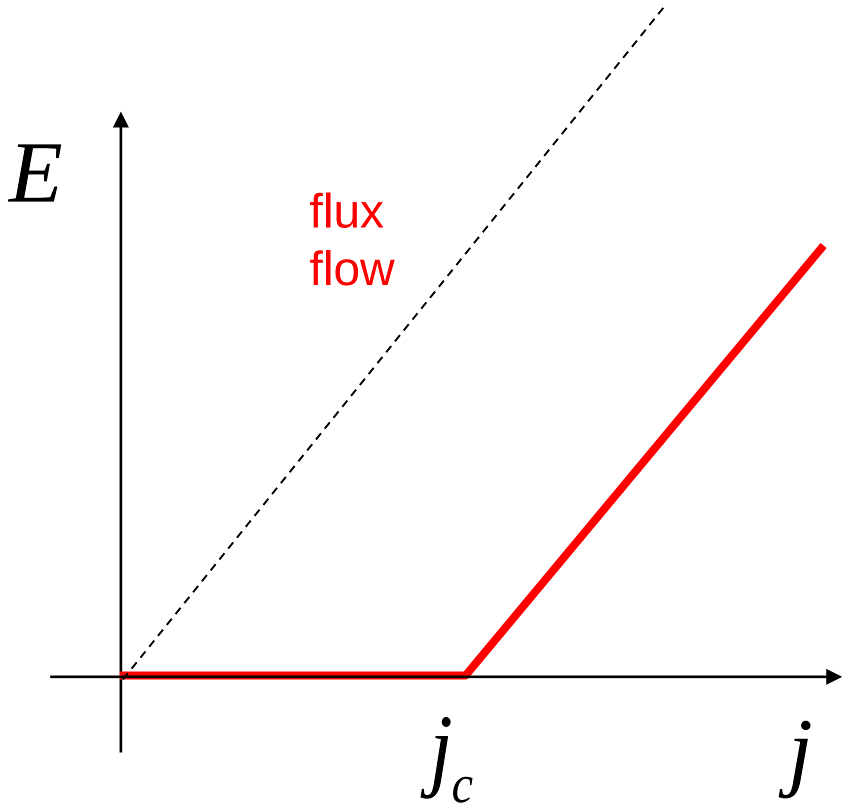
Thus the superconductor recovers the property of **dissipation free** current transport.

Typical current voltage curve



For strong pinning

$$\boxed{E = \rho_{\text{ff}}\,(j - j_c)}$$



Coulomb law of dry friction

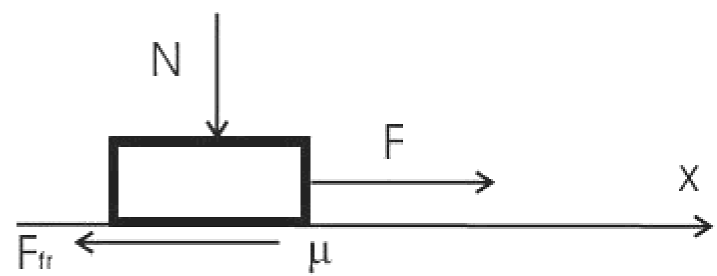
Basics of friction

Fact or Friction

500,000 BC: Friction of Rubbing Sticks / Flint Caused Heat = ***FIRE!***

3,500 BC: Rolling Friction was less than Sliding Friction = ***WHEEL!***

1495 AD: Basic Precepts and Laws by Di Vinci = ***Modern Measurements of Friction!***



It is estimated, that from 1/3 to 1/2 of the total energy produced in the world is consumed by friction.

Leonardo da Vinci (1452-1519)
Guillaume Amontons (1699)
Leonhard Euler (1750)
Charles-Augustin de Coulomb (1785)

Laws of dry friction

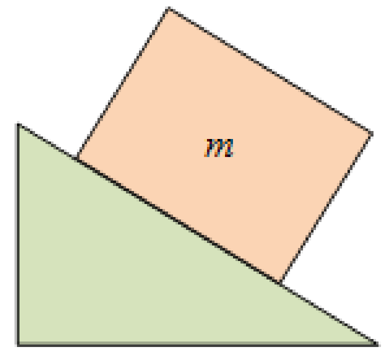
The elementary properties of sliding (kinetic) friction were discovered by experiment in the 15th to 18th centuries and were expressed as three empirical laws:

Amontons' First Law: The force of friction is directly proportional to the applied load.

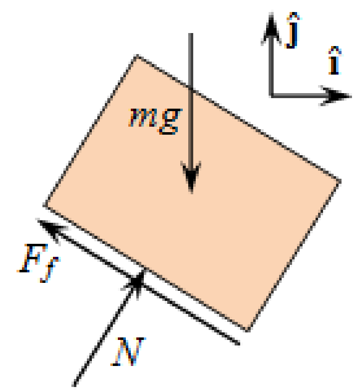
Amontons' Second Law: The force of friction is independent of the apparent area of contact.

Coulomb's Law of Friction: Kinetic friction is independent of the sliding velocity.

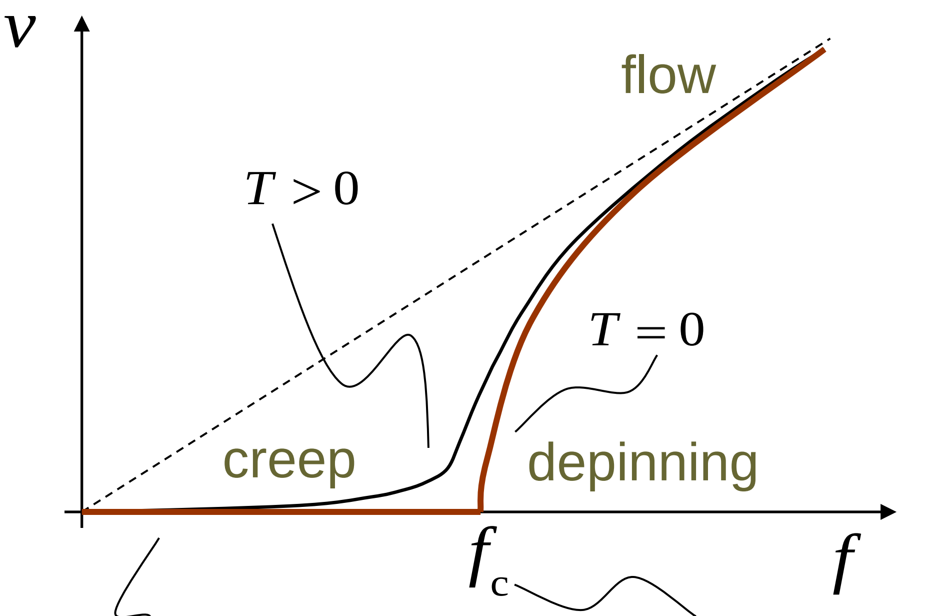
A block on a ramp



Free body diagram of just the block



Consequences of disorder

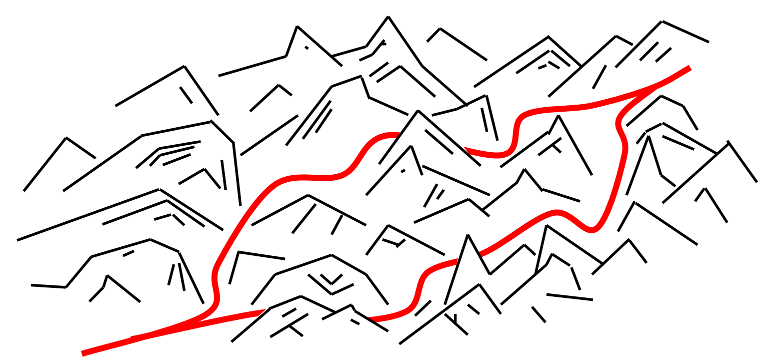


non-linear response at vanishing drive is the signature of a super-conducting phase

Dynamics

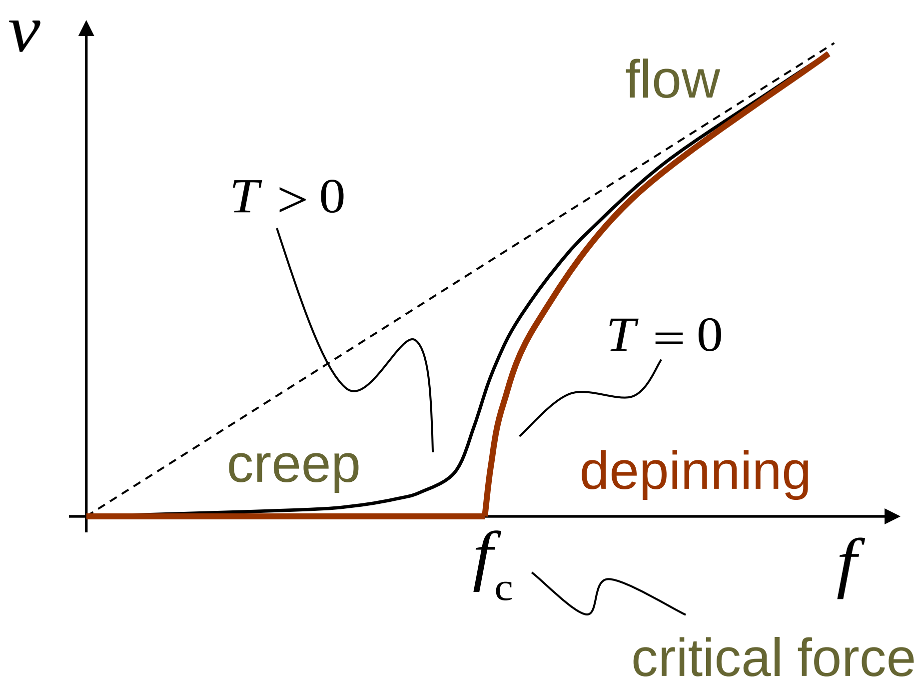
Pinning - critical force
Creep at finite T

Well understood within the framework of weak collective pinning theory



vortex in a random potential

Weak collective pinning (wcp) & creep



small scales: pinning

random summation of pinning forces of competing defects: fluctuations in the defect density produce a non-zero critical force

$$f_c \sim \left\langle f_{\text{pin}}^2 \right\rangle^{1/2} \sim \frac{1}{V_c}\left[f_p^2 \; n_p (\xi/a_0)^2 V_c \right]^{1/2}$$

Larkin volume
elastic energy ~ pinning energy

$$\left\langle E_{\text{pin}}^2 \right\rangle^{1/2} \sim E_{\text{elastic}} \rightarrow V_c$$

large scales: creep

$$v \sim v_0 \exp\left[-\frac{U_c}{\mathrm{T}} \left(\frac{f_c}{f} \right)^{\mu} \right]$$

in the 3-dimensional case

$$\boxed{j_c \sim j_0 (a_0 \xi^2 n_p)^2 \frac{\xi^2}{\lambda^2} \left(\frac{f_p}{\varepsilon_0 \xi / a_0} \right)^4}$$

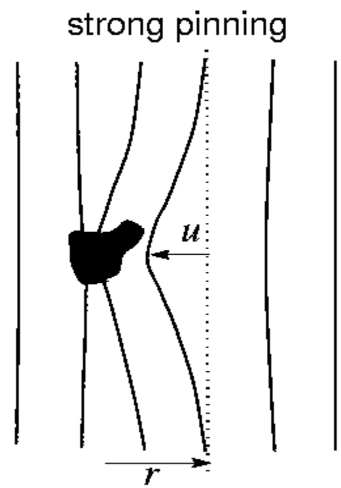
Strong pinning (Labusch (1969), Larkin-Ovchinnikov)

To calculate the mean pinning force:

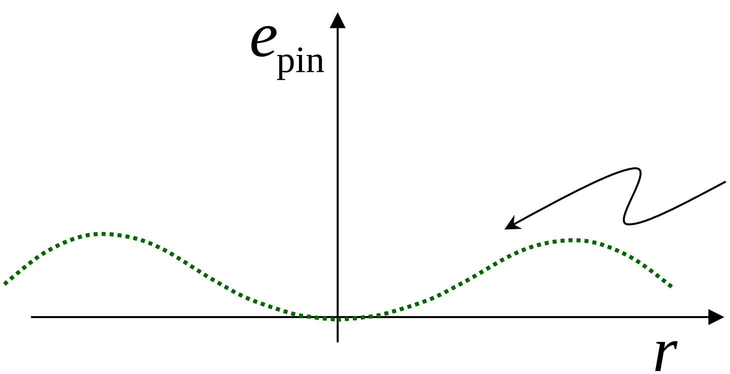
1. Find the force of interaction of the vortex lattice with defect
2. Average the force over randomly positioned defects

At low impurity concentration different defects do not interfere and in linear approximation in n_{p}

$$j_c \times B / c = F_{\mathrm{pin}} = n_{\mathrm{p}} \langle f_{\mathrm{pin}} \rangle$$

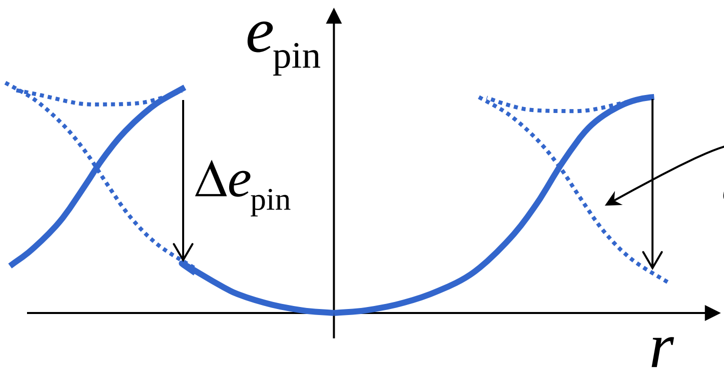


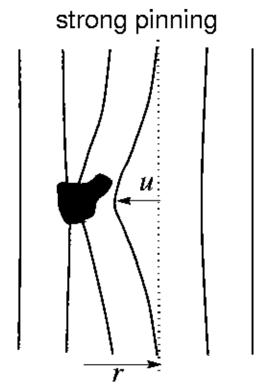
$$f_{\mathrm{pin}} = -\nabla_r e_{\mathrm{pin}}(r)$$

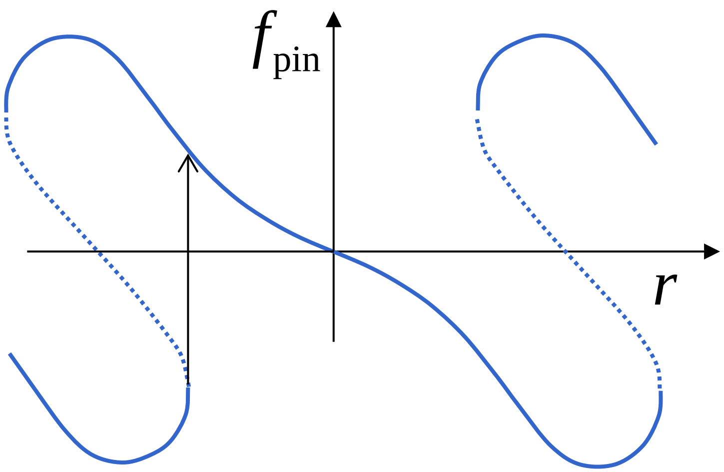


When pinning is weak, $e_{\mathrm{pin}}(r)$ is a single valued function. Then the lattice is smoothly dragged across the pinning landscape and $\langle f_{\mathrm{pin}} \rangle = 0$ ⟹ no critical current.

Then pinning is due to **fluctuations** in the pinning potential, Strong pins induce plastic deformations in the vortex lattice and the energy landscape $e_{\mathrm{pin}}(r)$ becomes a multi-valued function in the displacement r.







As a result the averaging produces a non-zero pinning force determined by the jump Δe_{pin} connecting different metastable states.

$$\langle f_{\mathrm{pin}} \rangle = -\int_0^{a_0} dx \frac{\partial_x e_{\mathrm{pin}}(x)}{a_0} = \frac{\Delta e_{\mathrm{pin}}}{a_0}$$

$$j_c = \frac{c}{B} \langle f_{\mathrm{pin}} \rangle$$

The weak- to strong pinning crossover is given by the **Labusch**'s criterion,

$$\partial_x f_{\mathrm{p}} = \left[\int \frac{d^3k}{(2\pi)^3} G_{xx}^{\mathrm{elas}}(k) \right]^{-1} = \bar{C}.$$

Disorder average - force against drag

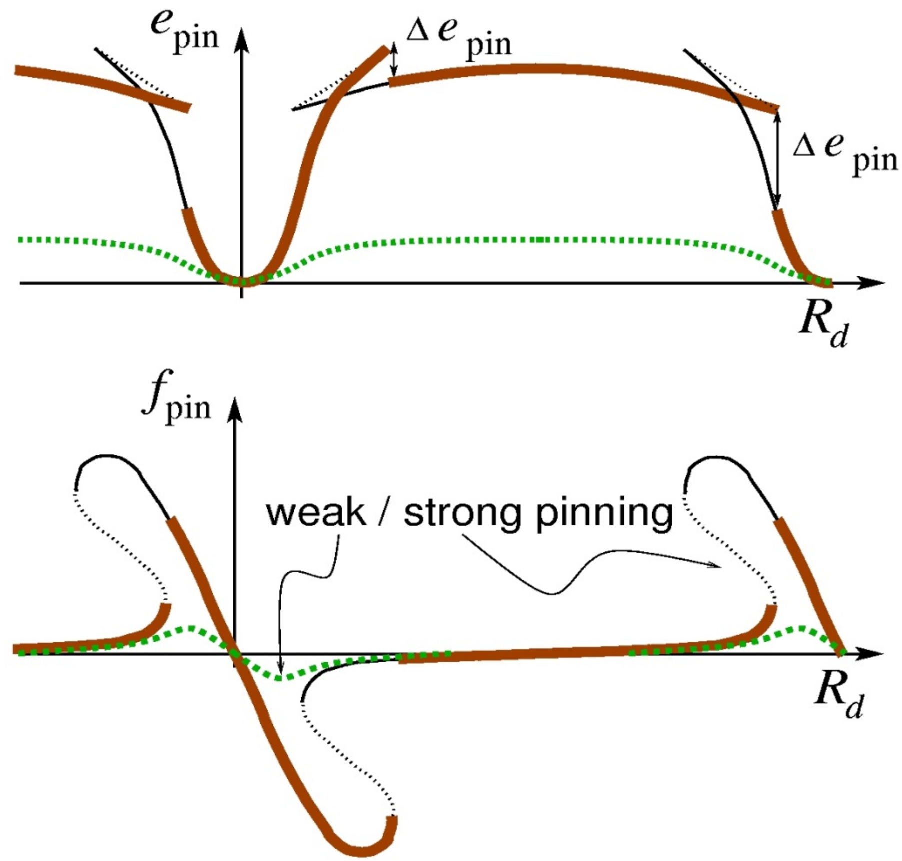
- drag the vortex lattice over the pinning landscape
- the averaging has to account for the preparation
- add up all pinning forces (producing a maximal negative force)

$$\left\langle f_{\text{pin}} \right\rangle = -\int_0^{L_x} dx \int_0^{L_y} dy \, \frac{\partial_x e_{\text{pin}}(x,y)}{L_x L_y}$$

$$= -\int_0^{a_0} dy \, \frac{\Delta e_{\text{pin}}(y)}{a_0^2}$$

the pinning force can be expressed through the jumps in the pinning energy

reminds of first order phase transitions (spinodals, jumps)



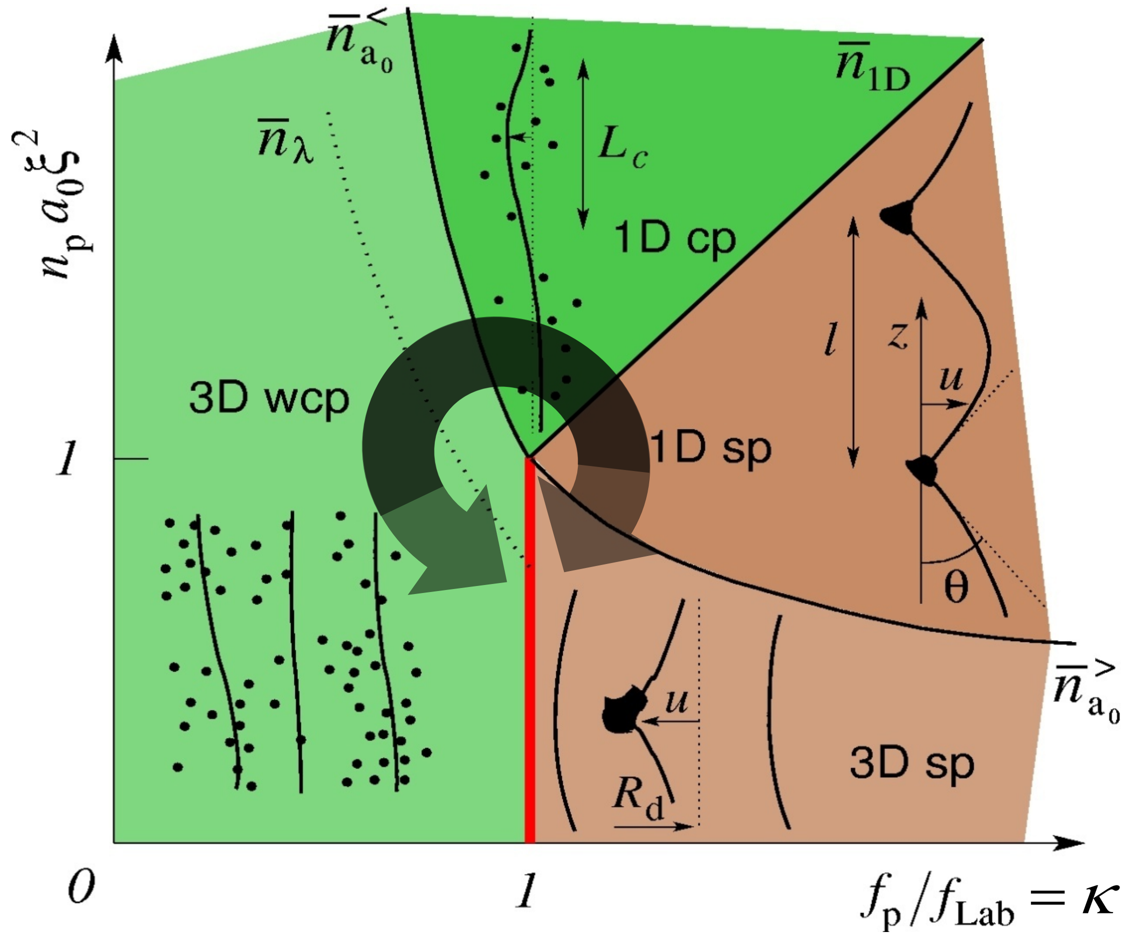
sp

$$\left\langle f_{\text{pin}} \right\rangle \neq 0$$

wp

$$\left\langle f_{\text{pin}} \right\rangle = 0$$

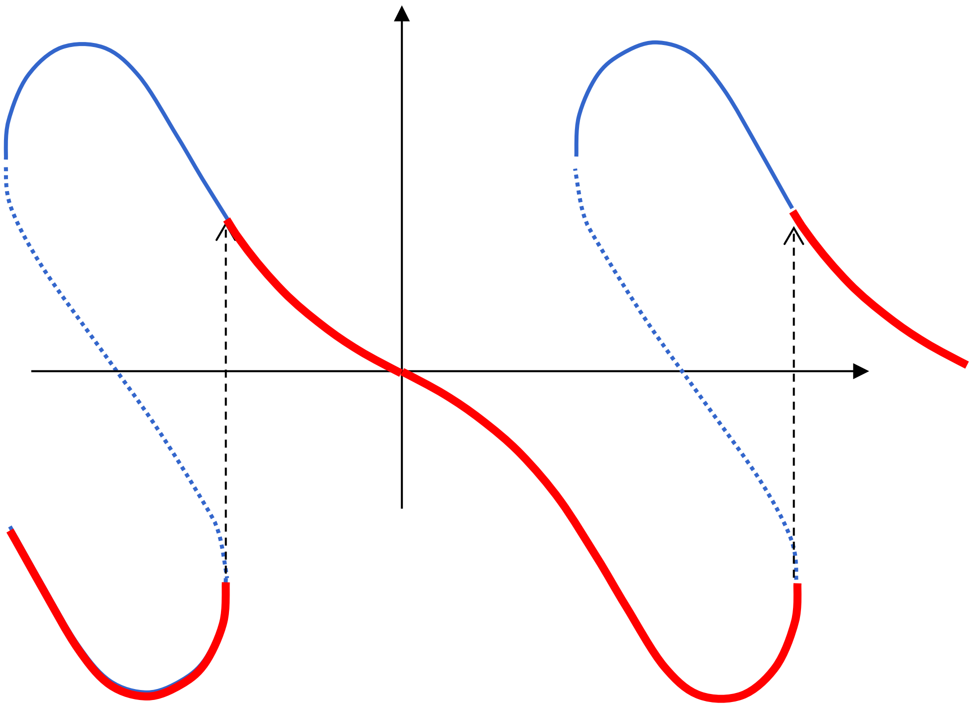
Pinning diagram

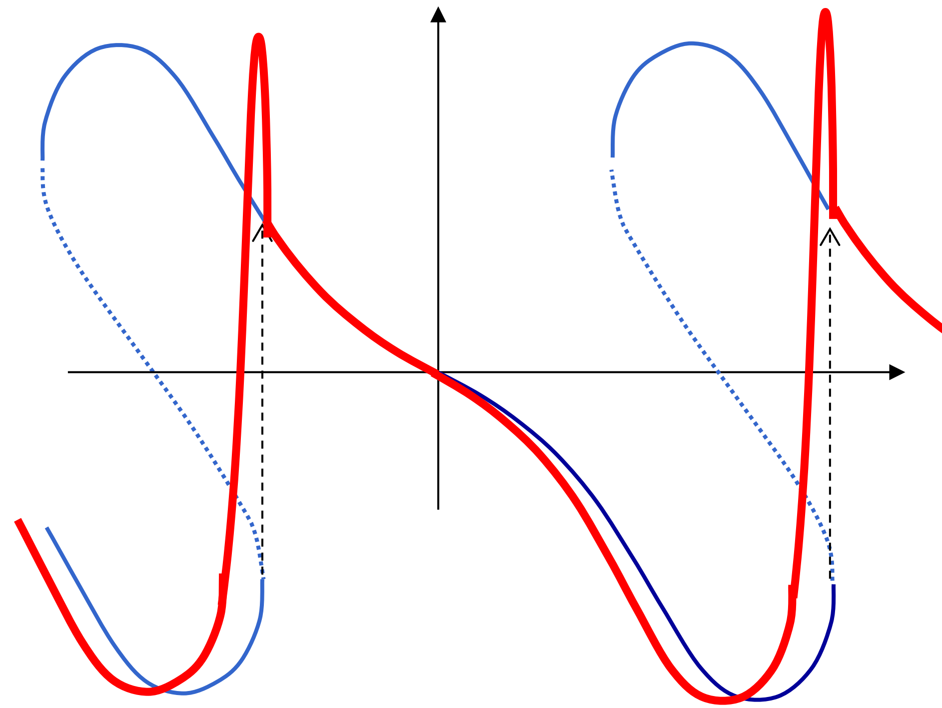


Strong pinning - dynamics

Statics - jumps

The dynamical solution can not jump





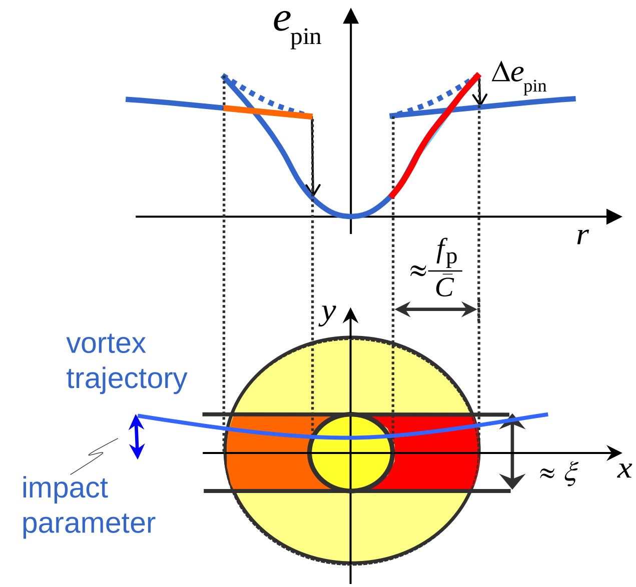
Dynamic Aspects of Strong Pinning (Thomann, Geshkenbein, Blatter)

Goal: Solve force balance equation

$$\eta \mathrm{v} = F_L - \langle F_{\mathrm{pin}}(r,t,\mathrm{v}) \rangle$$

viscous force density (Bardeen Stephen η), mean vortex velocity v

external (Lorentz) force density $F_L \sim jB/c$



Calculate v dependent mean pinning force $\langle F_{\mathrm{pin}}(r,t,\mathrm{v}) \rangle$ assuming

- dilute limit (single-pin physics)

$\rightarrow$ average over disorder and time

implicit equation

vortex displacement

free flow

Greens function

$$u(t) = (f/\eta)\, t + \underbrace{\int_{-\infty}^{t} dt' G(t-t') f[u(t')]}_{\text{pinning correction}}$$

$a \simeq a_0$ effective periodicity

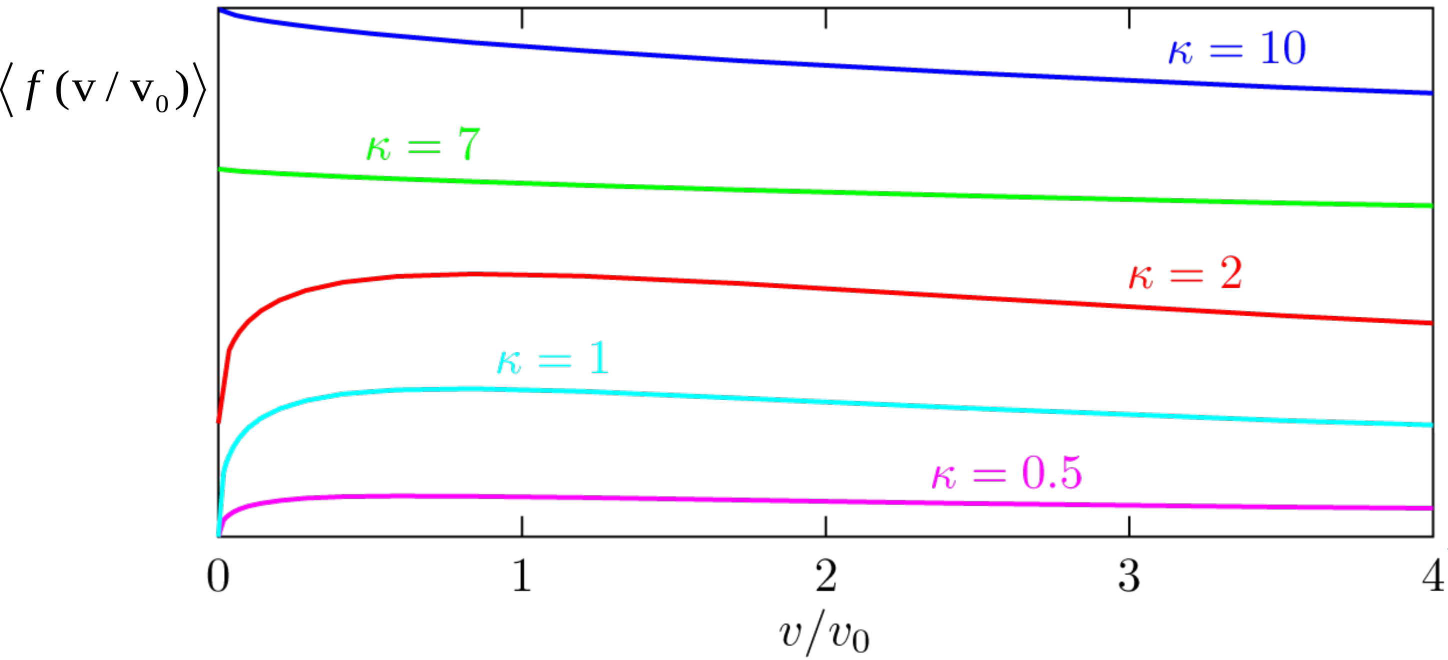
$$\langle F_{\mathrm{pin}}(\mathrm{v}) \rangle = n_p \left\langle \int \frac{dx}{a} f[u(x)] \right\rangle_b$$

Numerics

For any $f(u)$ one can solve integral equation

$$u(t) = (f/\eta)\,t + \int_{-\infty}^{t} dt'\,G(t-t')\,f[u(t')]$$

For Lorentzian pin $\quad e_{\mathrm{p,L}}(u) = -\dfrac{e_0}{1+u^2/\xi^2}$



large-v behavior $\sim 1/\sqrt{\mathrm{v}}$

small-v corrections $\sim \pm\sqrt{\mathrm{v}}$

A. I. Larkin & Yu. N. Ovchinnikov

$\mathrm{v}_0 = \xi\lambda c_{66}/\eta a_0^3$ $\qquad \kappa = f_{\mathrm{p}}/f_{\mathrm{Lab}}$

I – V curves

It is important, that the average pinning force from the single defect $\left\langle \int \frac{dx}{a} f[u(x), \mathrm{v}/\mathrm{v}_0] \right\rangle_b$

has typical velocity scale $\mathrm{v}_0 = \xi\lambda c_{66} / \eta a_0^3$

independent on the defects density n_p

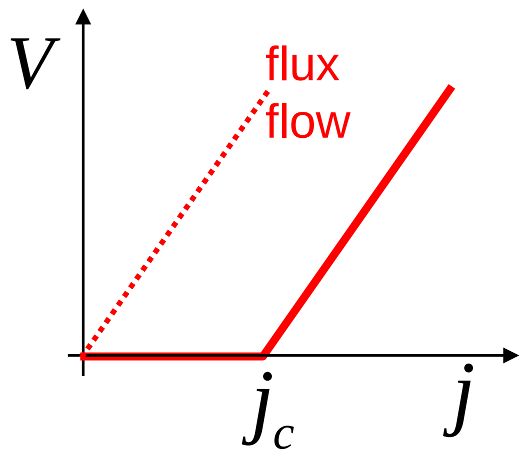
But the force balance equation $\eta \mathrm{v} = F_L - n_p \left\langle \int \frac{dx}{a} f[u(x), \mathrm{v}/\mathrm{v}_0] \right\rangle_b$

$\Rightarrow F_c \propto n_p$ And typical flow velocity above the critical current

$$\mathrm{v}_c \sim F_c / \eta \propto n_p \ll \mathrm{v}_0 \qquad \mathrm{v}_c / \mathrm{v}_0 \sim \kappa n_p a_0 \xi^2 \ll 1$$

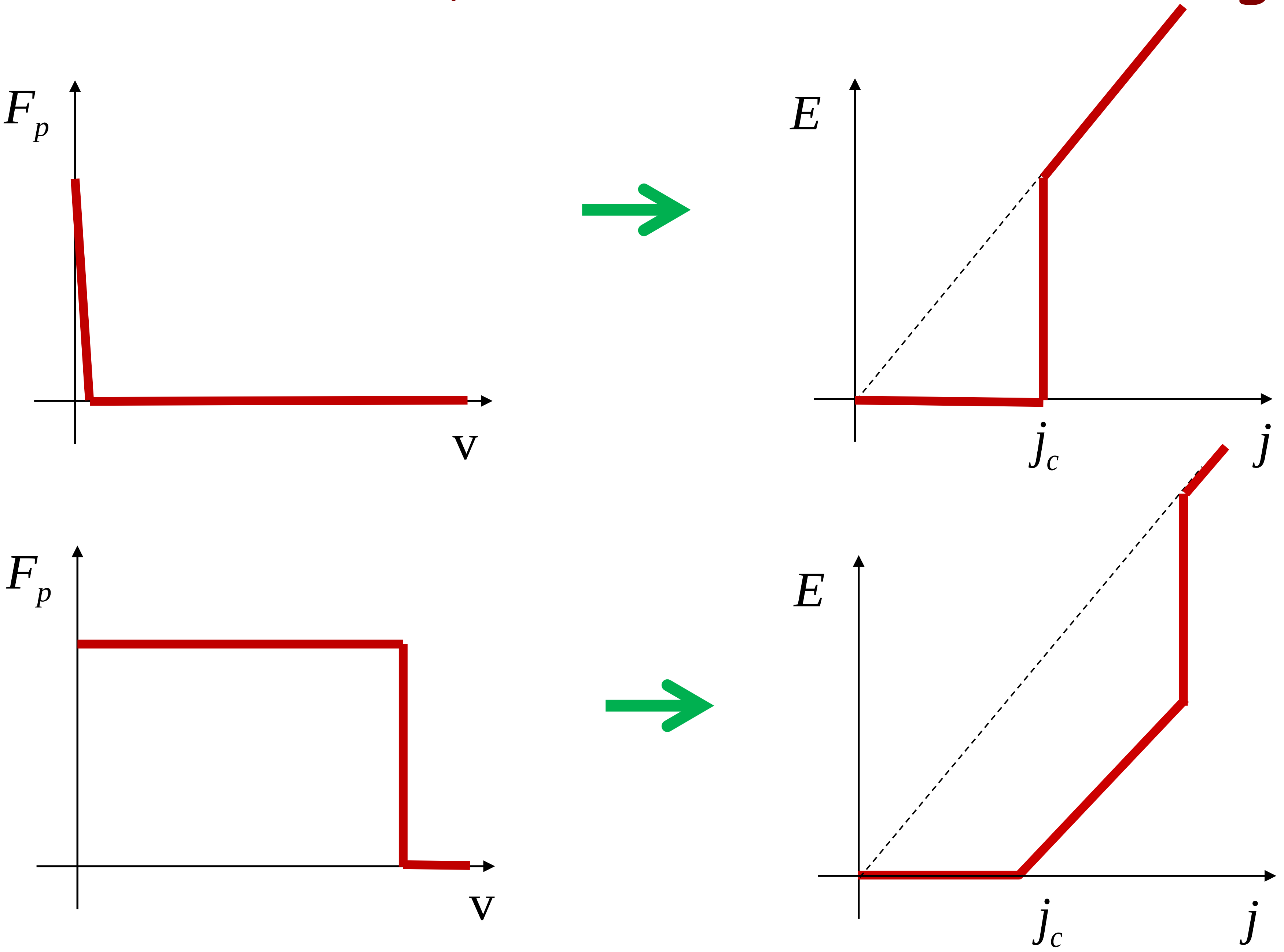
Thus we expect nearly linear force-velocity curve shifted to critical force:

$$\boxed{\eta \mathrm{v} = F_L - F_c}$$

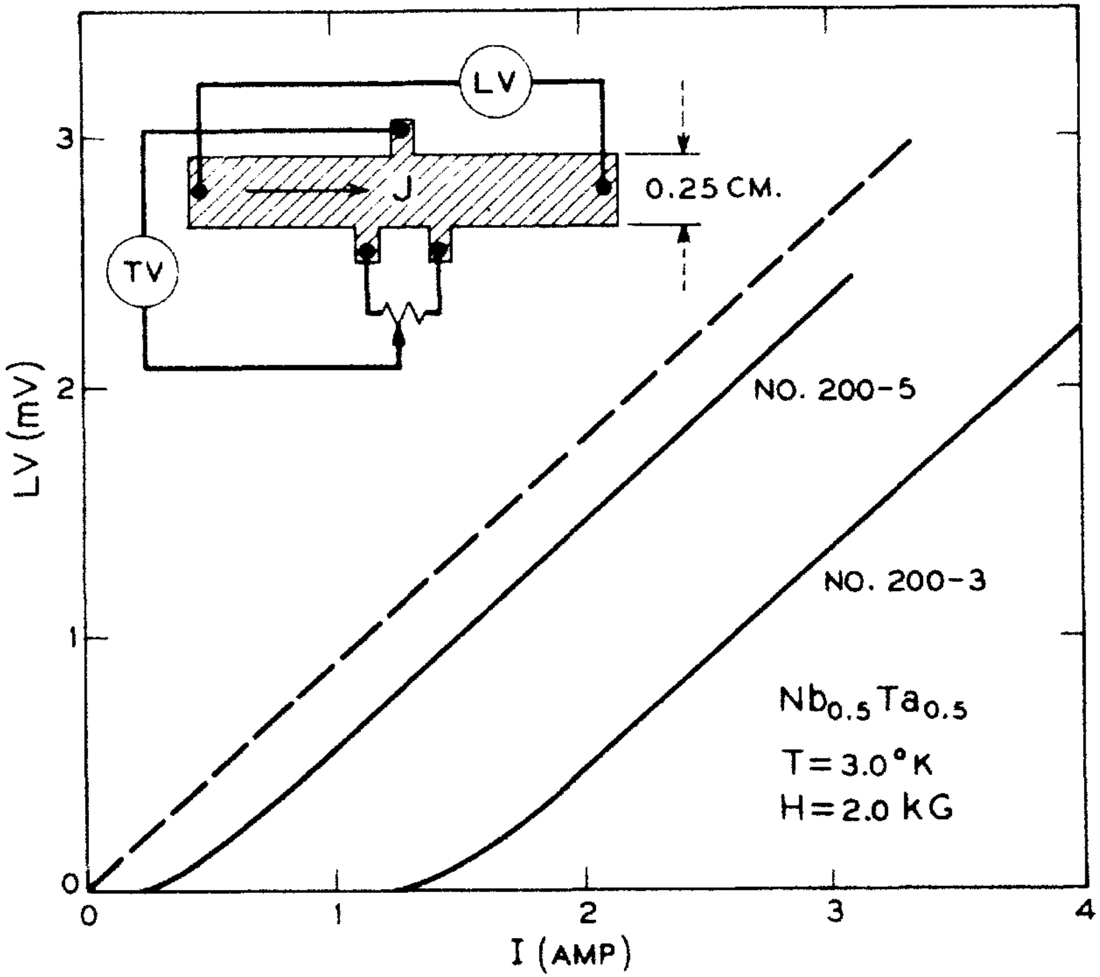


Coulomb law of dry friction

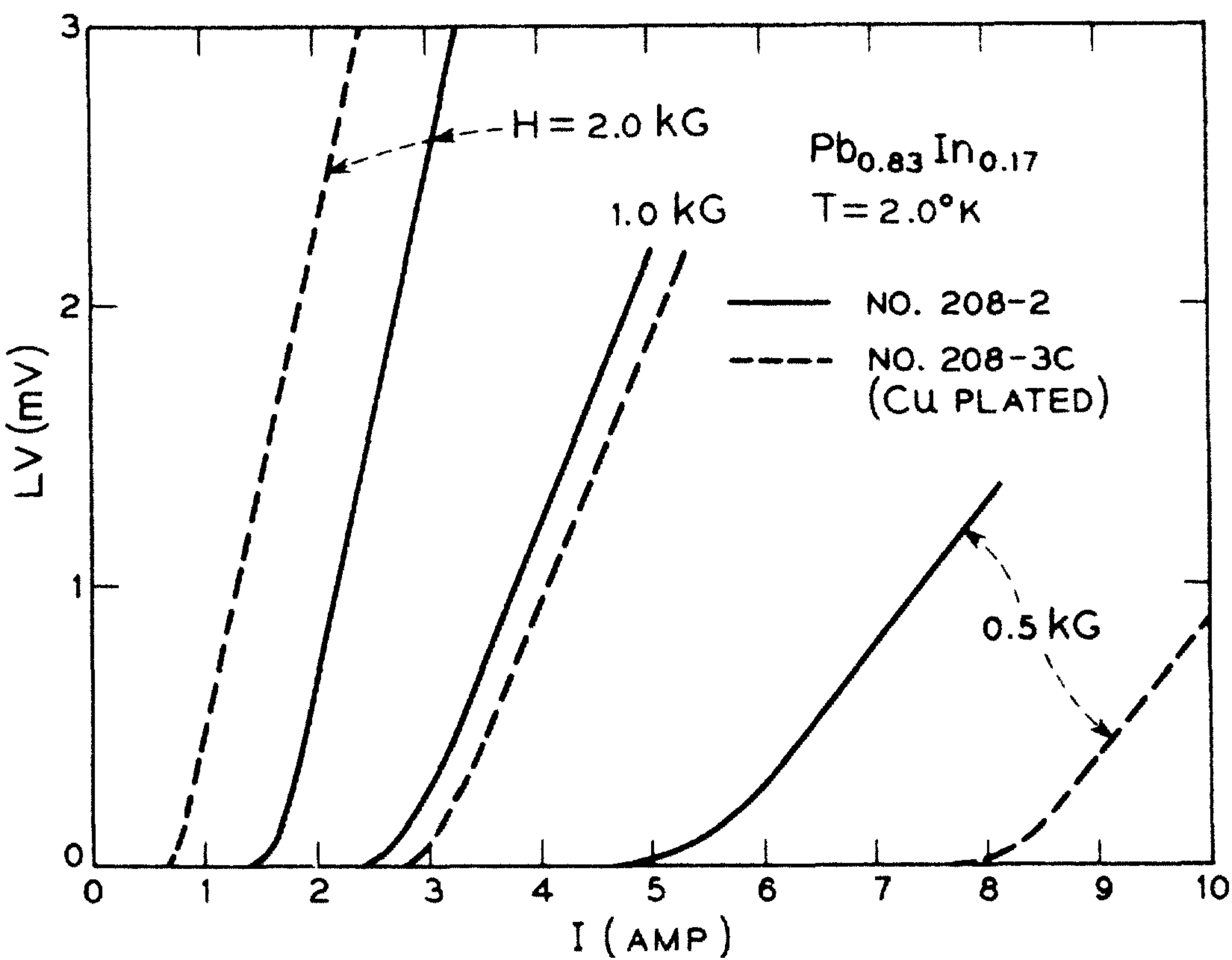
Force velocity   Current Voltage



Experiment



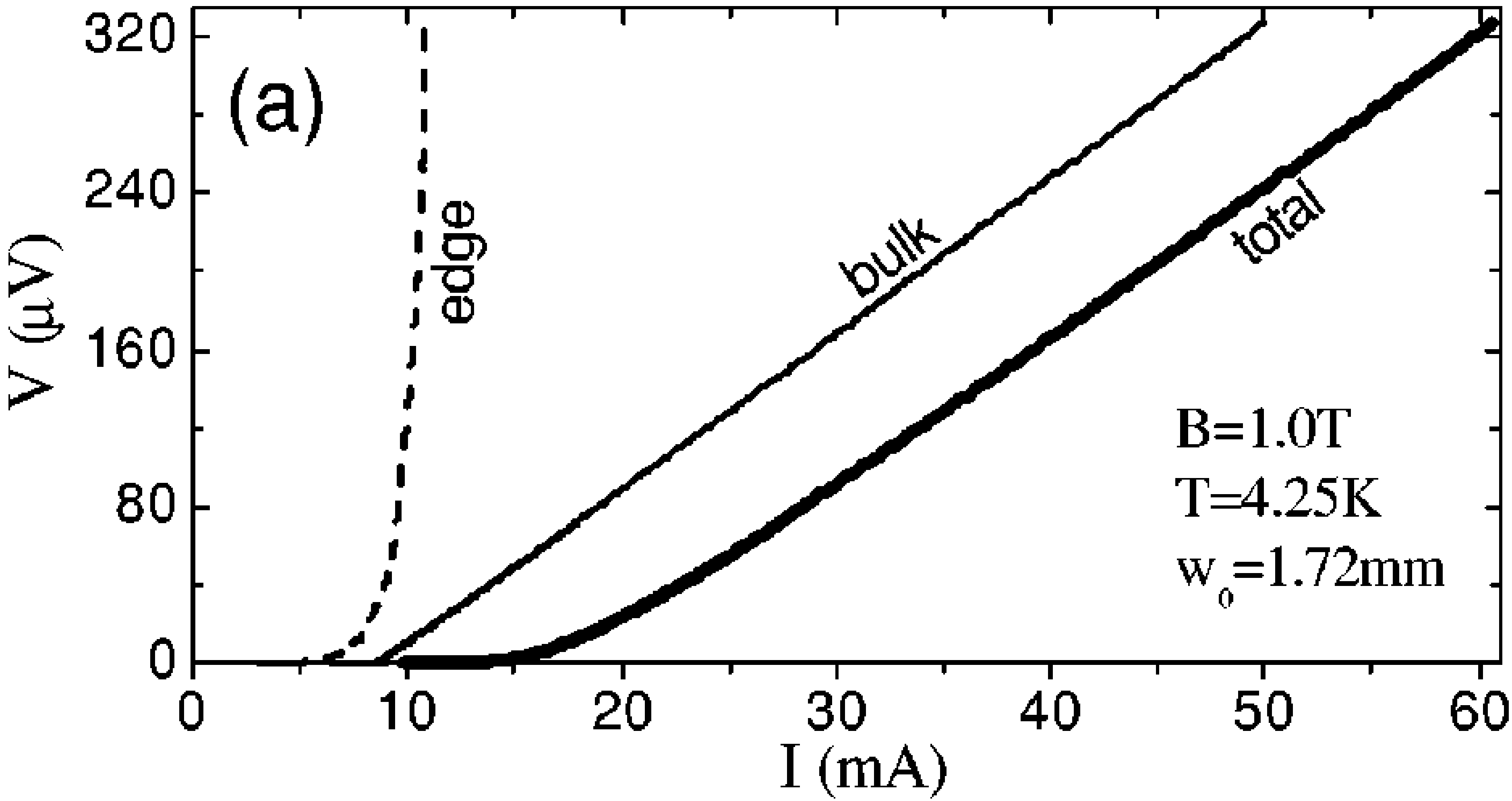
from A. R. Strnad, C. F. Hempstead, and Y. B.Kim, PRL **13, 794 (1964)**



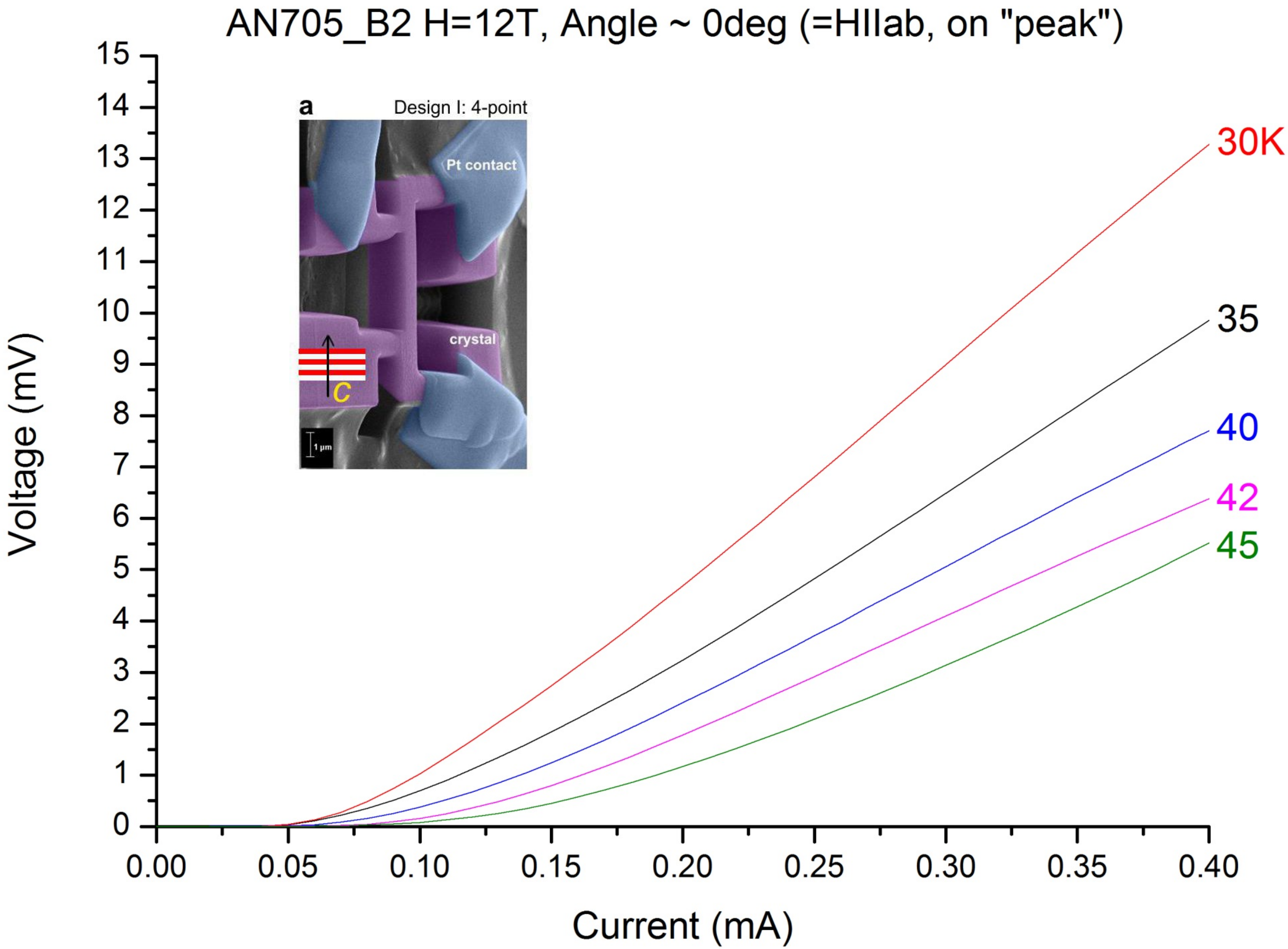
H = 2.0 kG
1.0 kG
0.5 kG
$Pb_{0.83}In_{0.17}$
T = 2.0°K
NO. 208-2
NO. 208-3C
(Cu PLATED)
LV (mV)
I (AMP)
0
1
2
3
4
5
6
7
8
9
10

Z. L. Xiao, E. Y. Andrei, Y. Paltiel, E. Zeldov, P. Shuk, and M. Greenblatt,
Phys. Rev. B **65**, 094511 (2002).

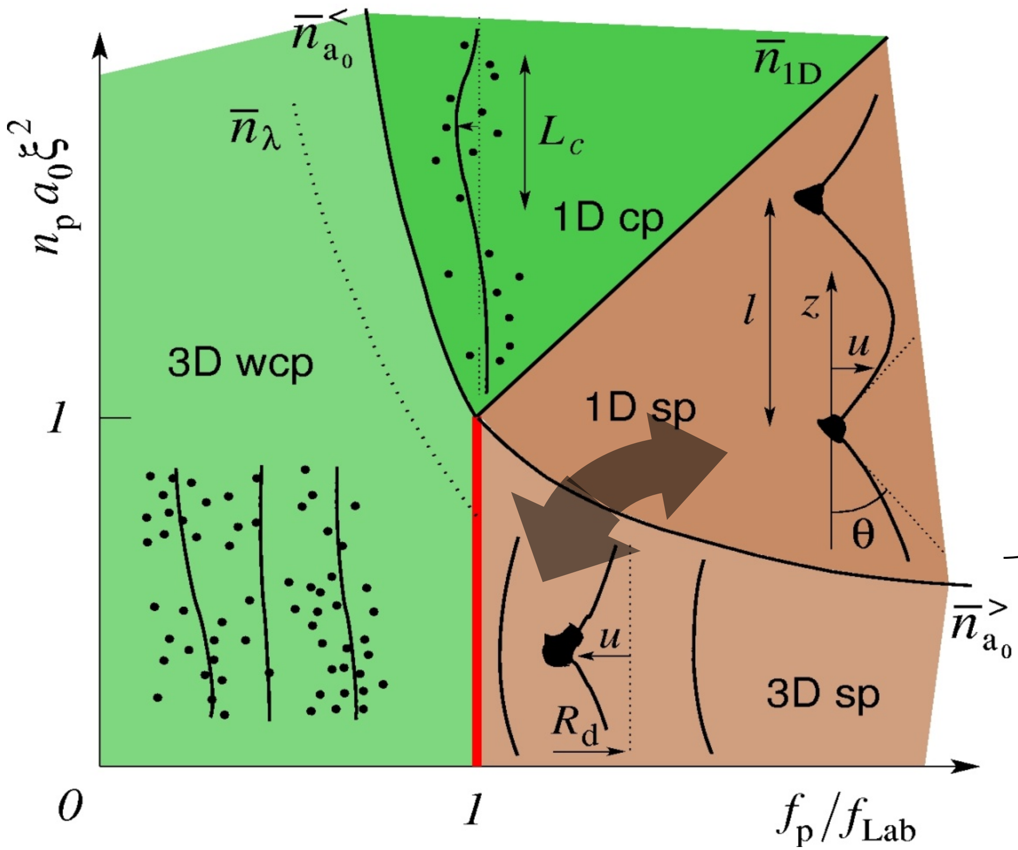
Edge and bulk transport in the mixed state of a type-II superconductor



Philip Moll, *et al.*, SmFeAs(O,F)



Pinning diagram



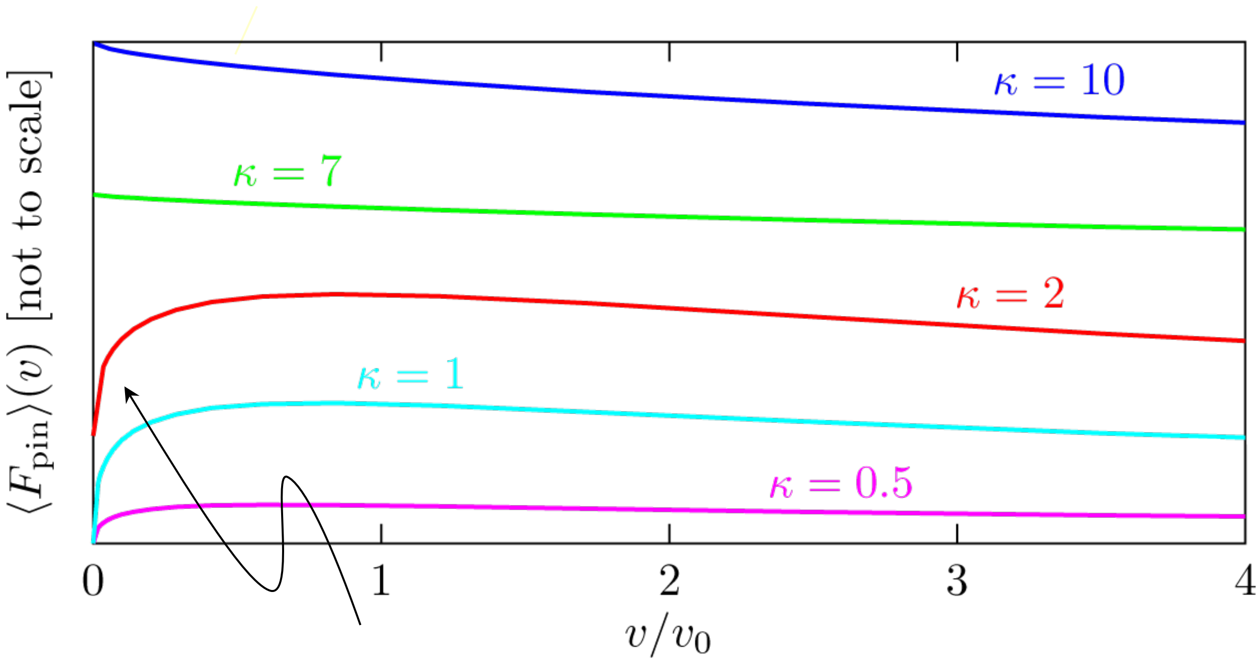
$$\mathrm{v}_c / \mathrm{v}_0 \sim \kappa n_p a_0 \xi^2 \sim 1$$

Exceptions to Coulomb's Laws

Third Law: Friction is not always independent of velocity.
If we exclude very low speeds and very high speeds, the friction coefficient is constant and independent of sliding velocity.
But at very high speeds, the friction coefficient generally has a slightly negative slope; that is, the friction coefficient decreases gradually as the speed increases.

At very low speeds, the friction coefficient generally increases gradually with a decrease in sliding velocity.

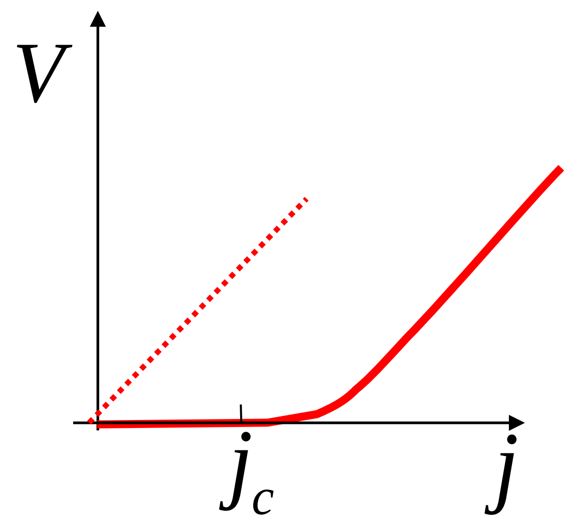
Corrections to linear behavior



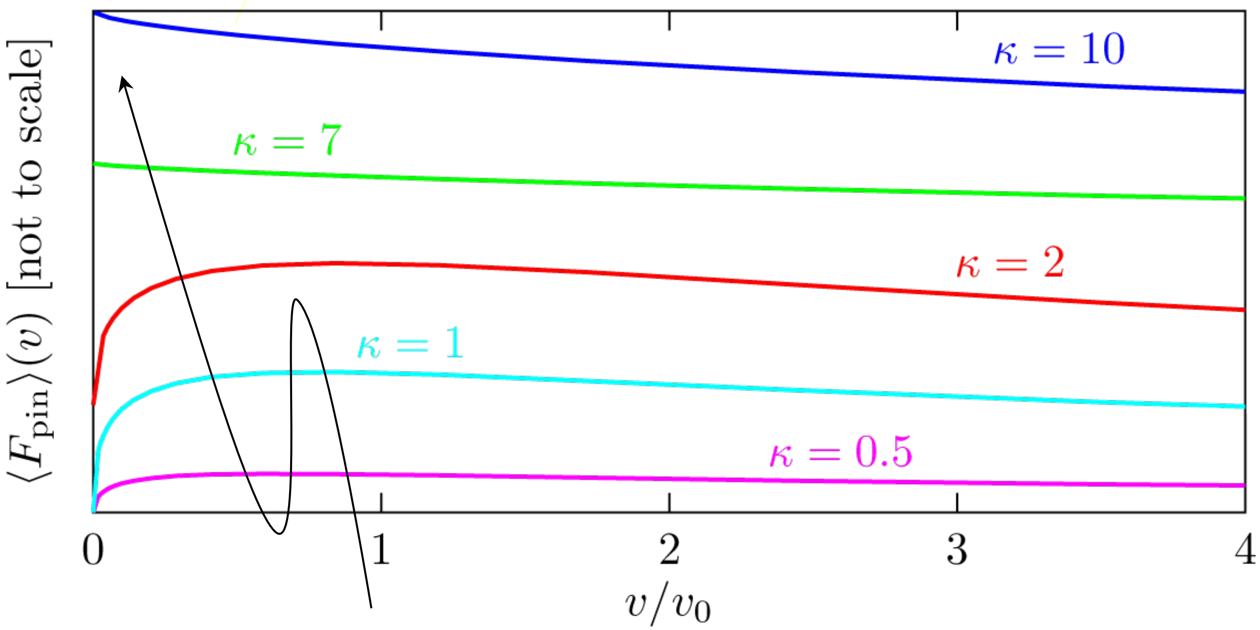
For relative weak pins at low velocities $\langle F_{\mathrm{pin}}(\mathrm{v})\rangle = F_c(0) + \mathrm{const}\sqrt{\mathrm{v}}$ for $\mathrm{v} \to 0$

The force balance equation $\eta \mathrm{v} = F_L - F_c - \mathrm{const}\sqrt{\mathrm{v}}$

And close to the critical force $\mathrm{v} \propto \left(F_L - F_c\right)^2$



Strong pins - hysteresis

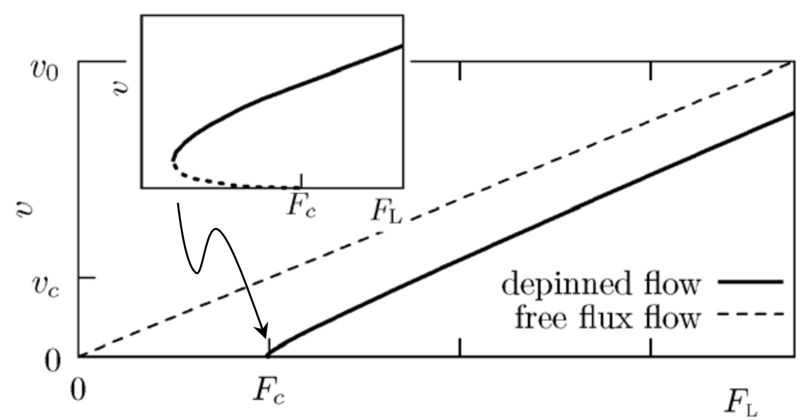


For strong centers sign of corrections changes $\langle F_{\rm pin}(\mathrm{v})\rangle = F_c(0) - \mathrm{const}\sqrt{\mathrm{v}}$ for $\mathrm{v} \to 0$

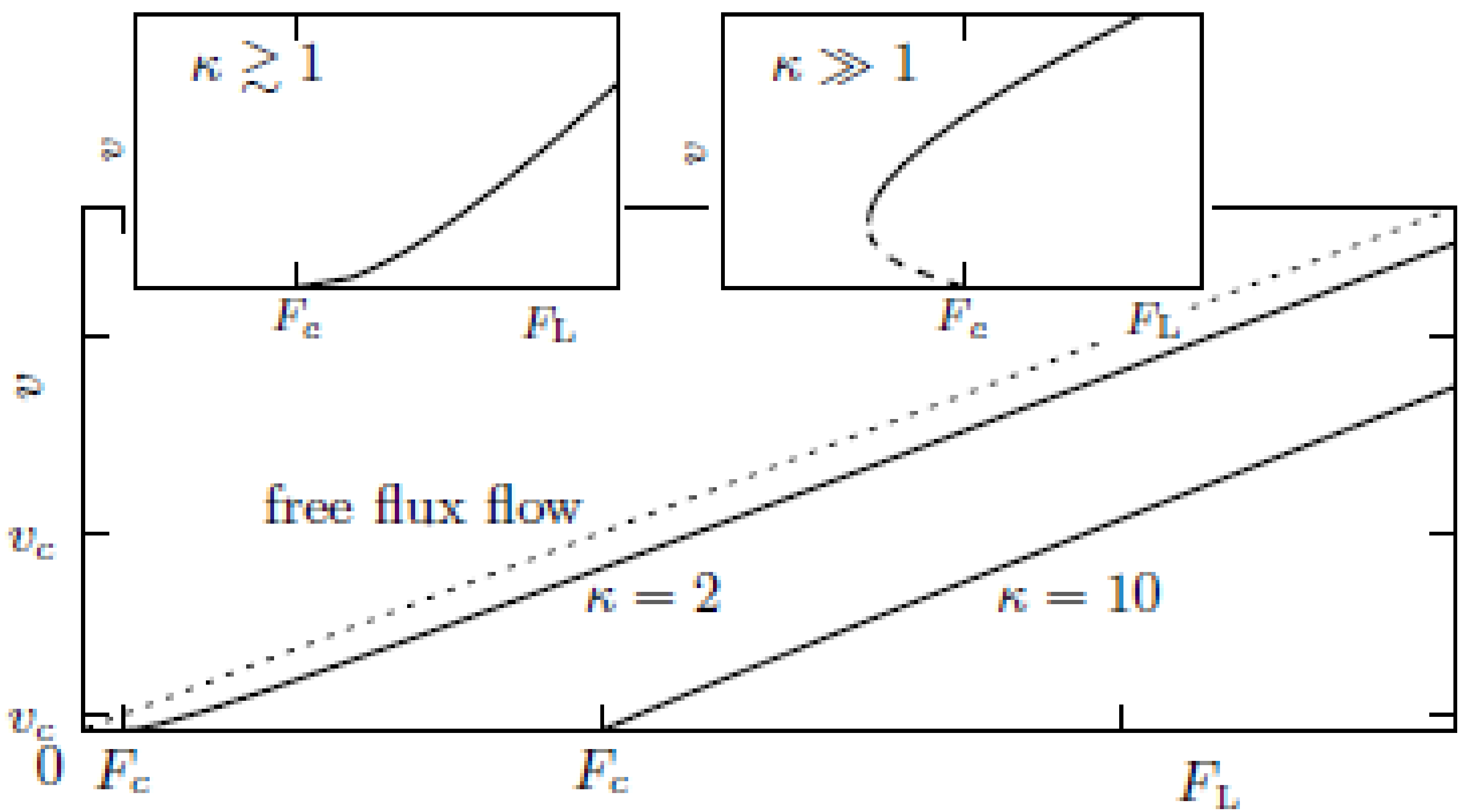
Bi-stable force–velocity relation
with **jumps:** $\sqrt{v}$ vs. ηv

The force balance equation $\eta \mathrm{v} = F_L - F_c + \mathrm{const}\sqrt{\mathrm{v}}$

And close to the critical force $\mathrm{v} \propto (F_c - F_L)^2, \quad F_L < F_c$



Evolution of the current voltage curves with the pinning strength



Correlations

These parabolic corrections become visible (e.g. size of the jumps) very close to the critical force:

$$\eta \mathrm{v} \simeq F_L - F_c \propto n_p^2$$

For such small velocities correlations between the pinning centers should be taken into account.

Our expansion parameter is density of the pins n_p

Since we don't know critical force with such precision it is difficult to say how much this nonlinear behavior will survive.