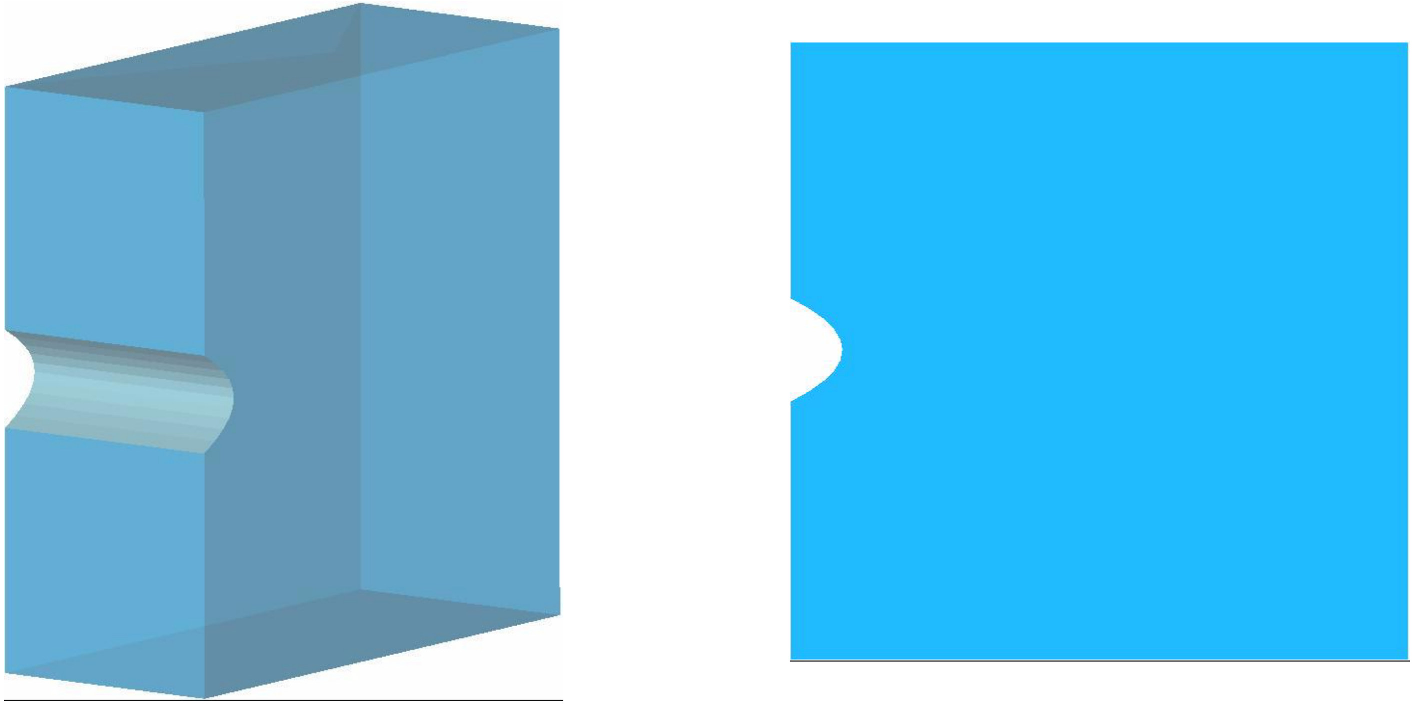
Ever more singular:
How *crack front* geometry determines *crack front* dynamics

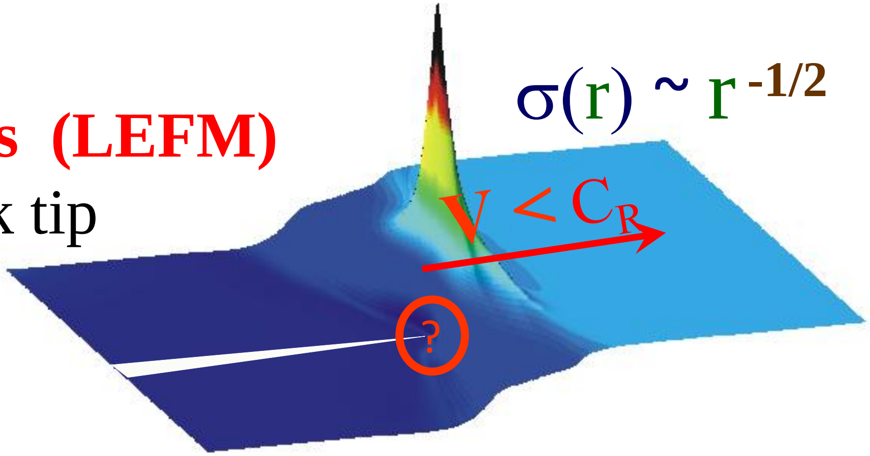


Itamar Kolvin, Gil Cohen and Jay Fineberg

The Racah Institute of physics, the Hebrew University of Jerusalem, Israel

Thanks to the **E**uropean **R**esearch **C**ouncil
and the **I**srael **S**cience **F**oundation
for funding this stuff…

Fracture mechanics in ~1 min



Linear Elastic Fracture Mechanics (LEFM)

Singularity of the stress at the crack tip

Speed limit **= c_R**

Equation of motion ⇔ Energy balance:

Energy flux into the crack tip = dissipation

$$G = \Gamma$$

LEFM: ignore what happens within the singular region…

(fine for this talk…)

A.Livne, E. Bouchbinder and J.Fineberg, PRL **101**, 264301 (2008).
E. Bouchbinder, A. Livne and J. Fineberg, PRL **101**, 264302 (2008).
E. Bouchbinder, A. Livne and J. Fineberg, JMPS **57**, 1568-1577 (2009).

Slowing things down:

Understanding dynamic fracture through brittle gels

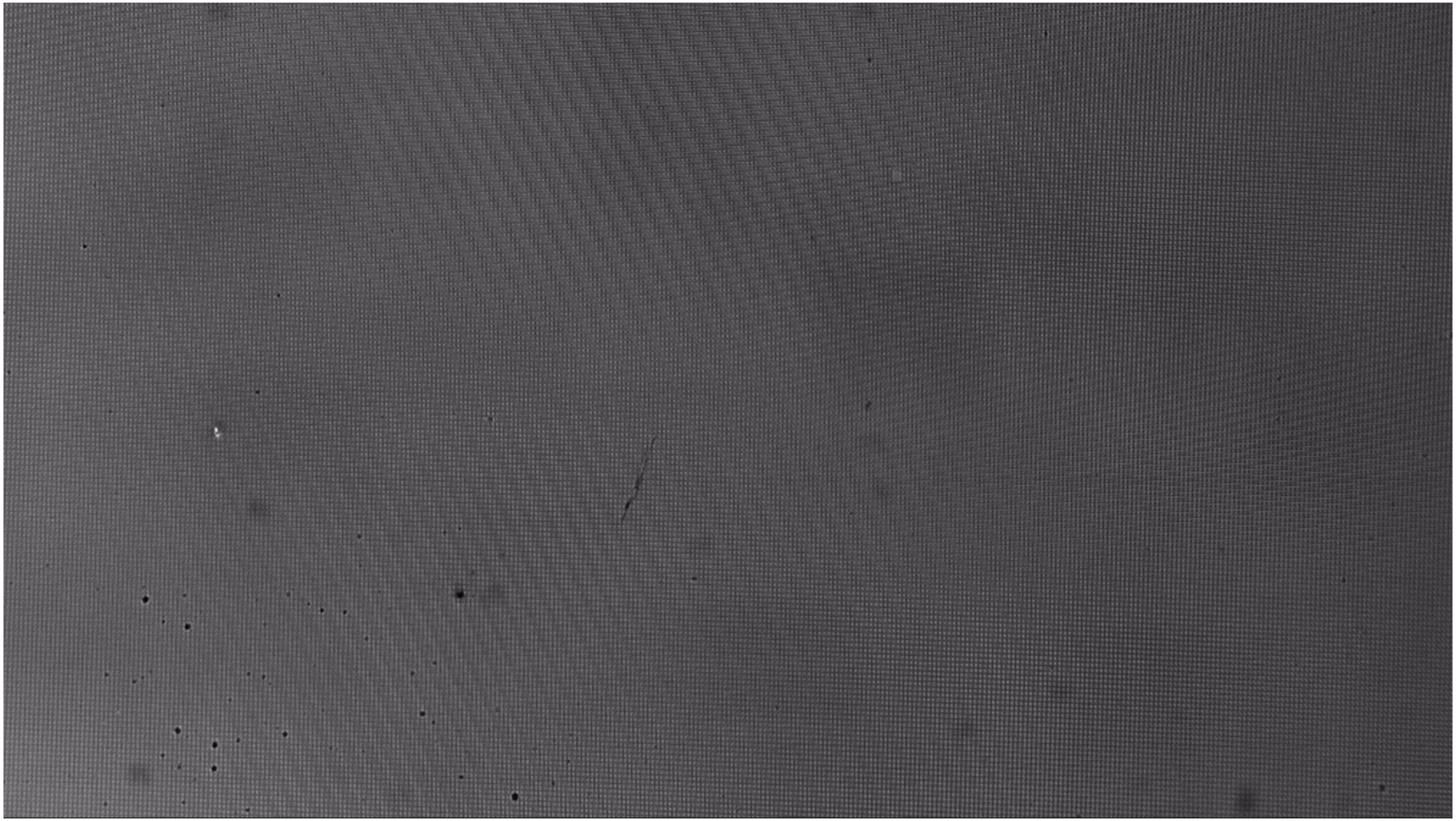
Fracture of polyacrylamide ➔ dynamic fracture in ***slow motion*** by reducing sound velocities by **2-3 orders of magnitude**

Material	**Young's Modulus (kPa)**	**Poisson ratio**	**C_R (m/s)**
Gel X% acrylamide Y% bis-acrylamide	100-1000	0.5	**5-14**
PMMA	3,900,000	0.35	**930**
Soda-Lime glass	70,000,000	0.22	**3340**

Change in the gel's composition → Change in elastic constants

Young's modulus E=**100**-560 kPa
Fracture energy Γ=**13**-60 J/m^2

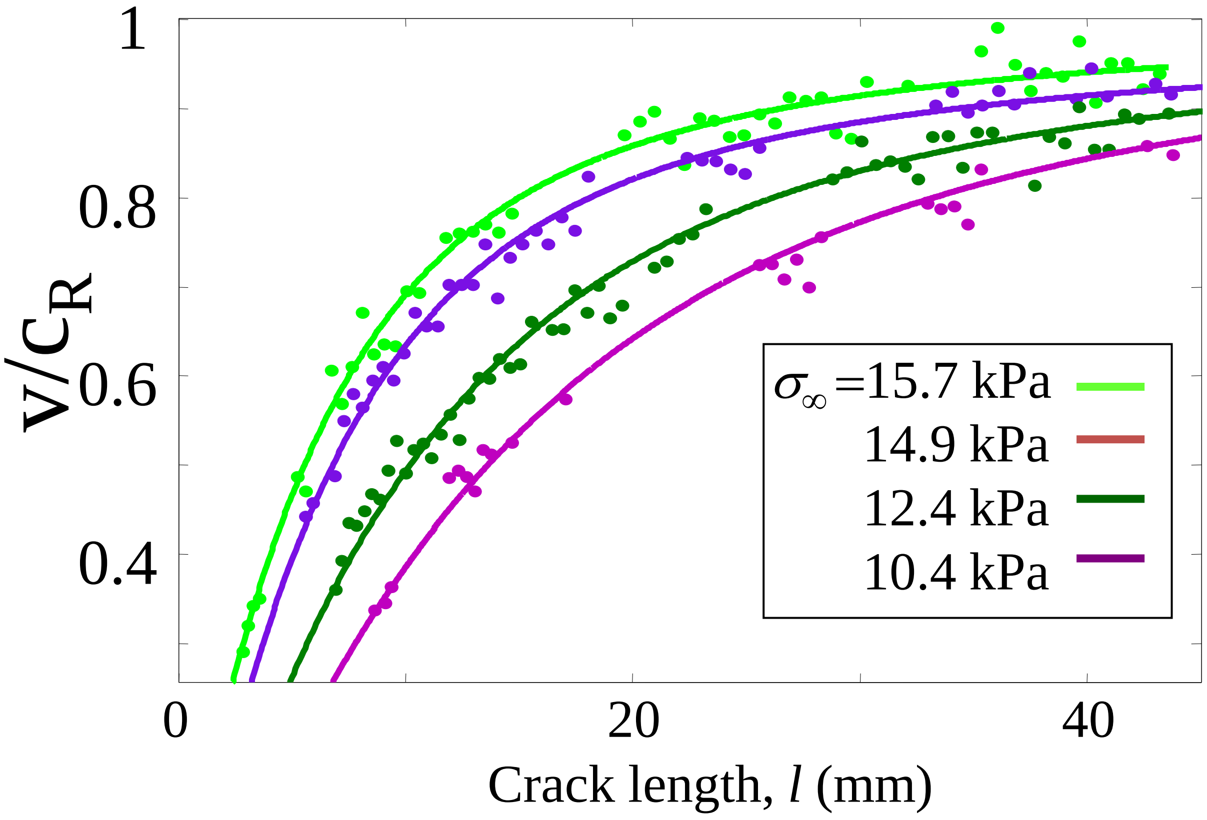
A simple crack moving at $0.6C_R \sim 3m/s$



Gels really probe dynamic fracture: e.g. checking the equation of motion: $G=\Gamma$

In an ***infinite medium*** and constant stress, σ_∞: $\Gamma = G(l,v) \approx \frac{1-\nu^2}{E}\frac{8l}{\pi}\sigma_\infty^2\left(1-v/c_R\right)$

L.B. Freund, Dynamic Fracture Mechanics (1990)



The equation of motion $G=\Gamma$ works perfectly:

Gels are perfectly representative of brittle materials!

T. Goldman, A. Livne, J. Fineberg, Phys. Rev. Lett. **104**, 1144301(2010).

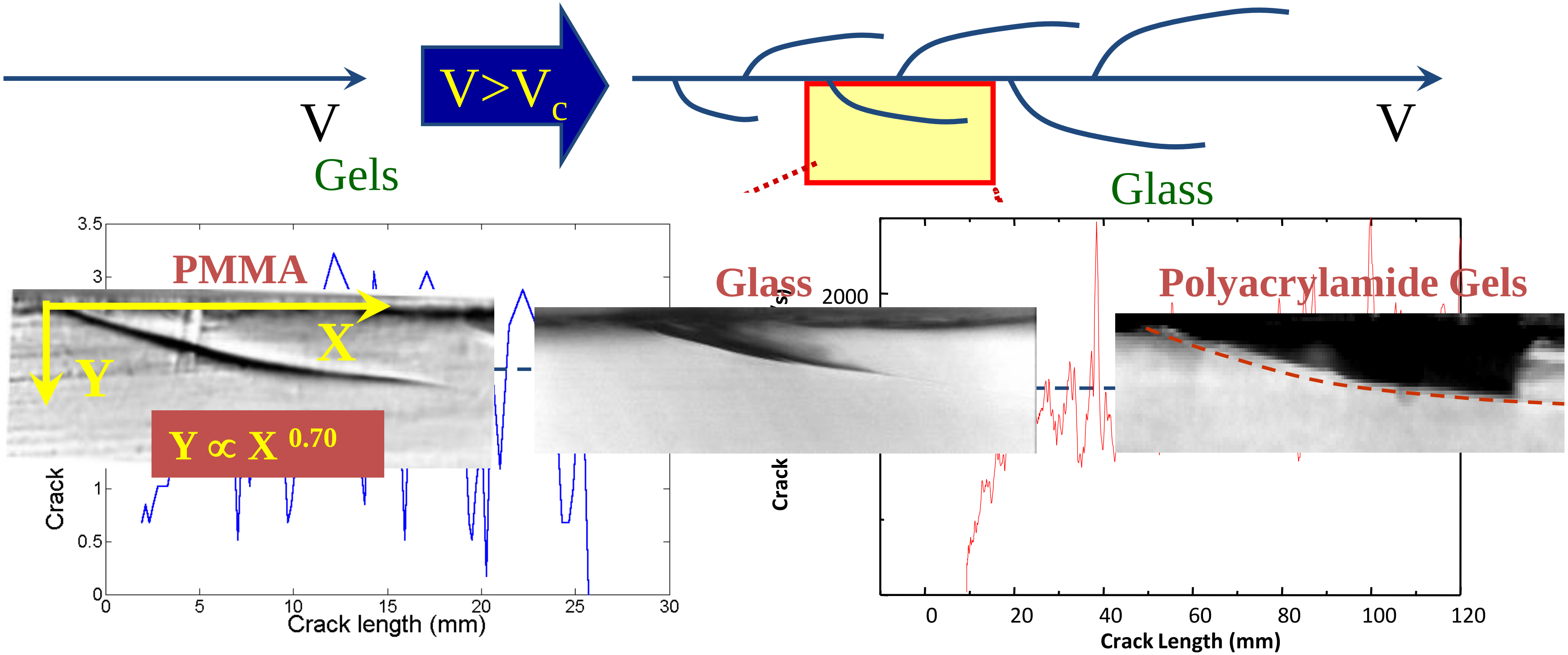
Excellent agreement with Fracture Mechanics for a simple crack

Gels are a convenient testing ground for fracture mechanics

What happens when cracks stop being simple?

The Micro-branching instability

- At a critical velocity a single crack **may** become *unstable* to frustrated micro-branches
- In gels, Micro-branches have the ***same functional form*** as in other brittle materials

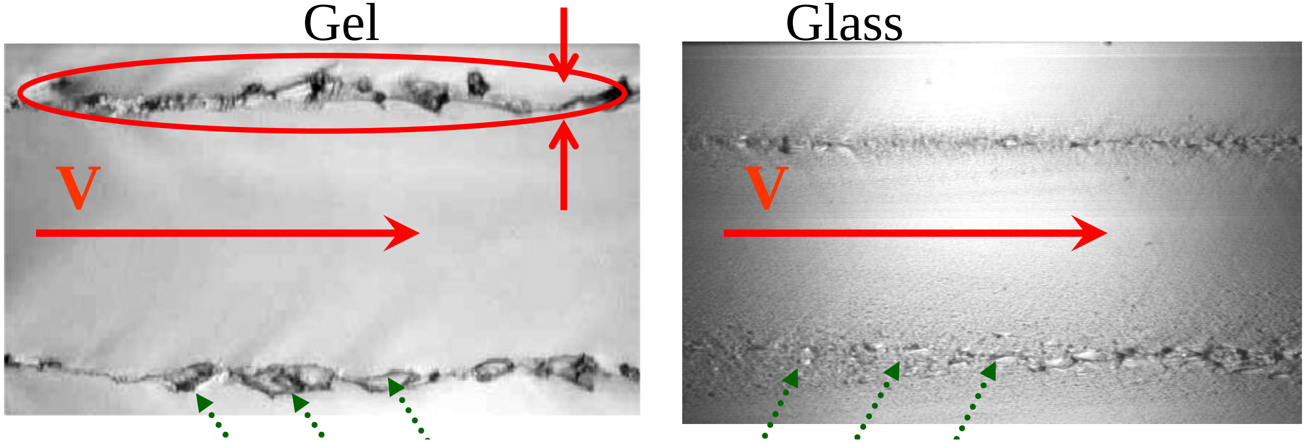


A. Livne, G. Cohen and J. Fineberg, Phys. Rev. Lett. **94,** 224301 (2005)

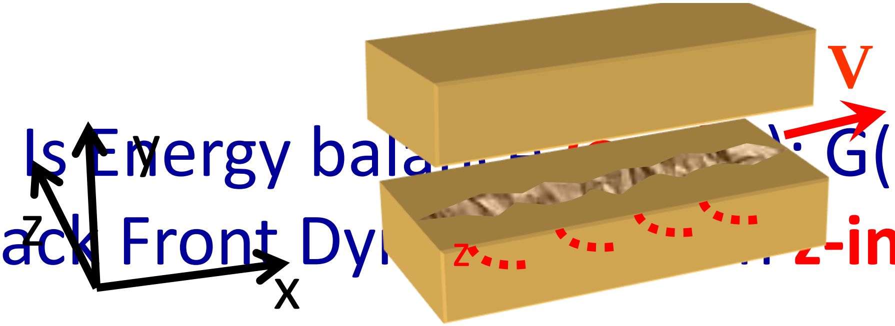
Micro-branches within a ***Crack Front***:

Micro-branches are ***Energy Sink***s that are:

- **Localized** within the crack front (z direction)
- **Align** in **chains** along the propagation (x) direction.
- **Bi-stable** within a crack front



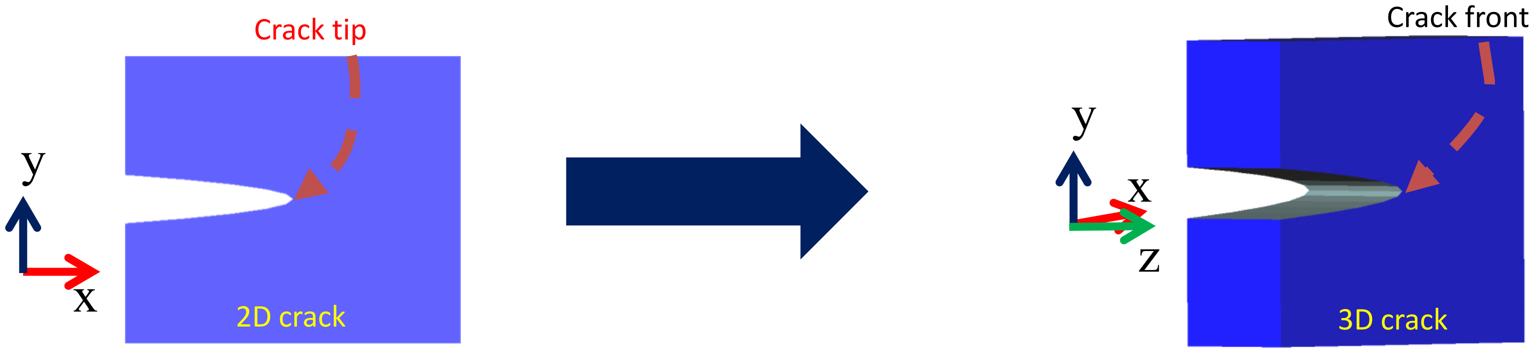
Micro-branches (fracture surface view)



Is Energy bala … : $G(v,z)=\Gamma(z)$

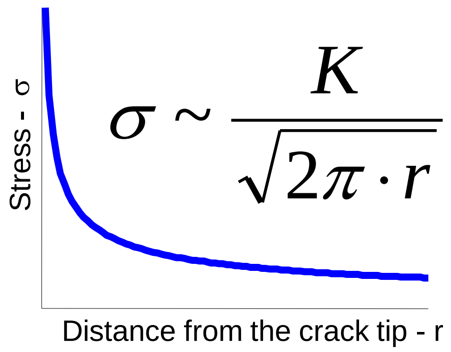
What are Crack Front Dy … **z-invariance** is **broken**??

Crack fronts: A transition from 2D to 3D understanding of fracture



The conventional view

- Materials fail at a crack tip because stresses become singular



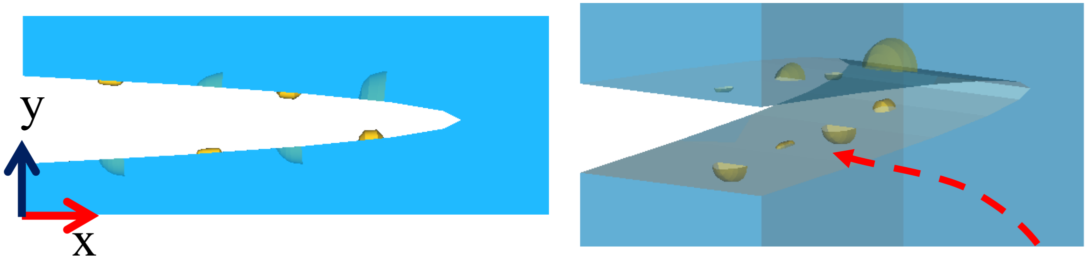
$$\sigma \sim \frac{K}{\sqrt{2\pi \cdot r}}$$

- Crack tip equation of motion:

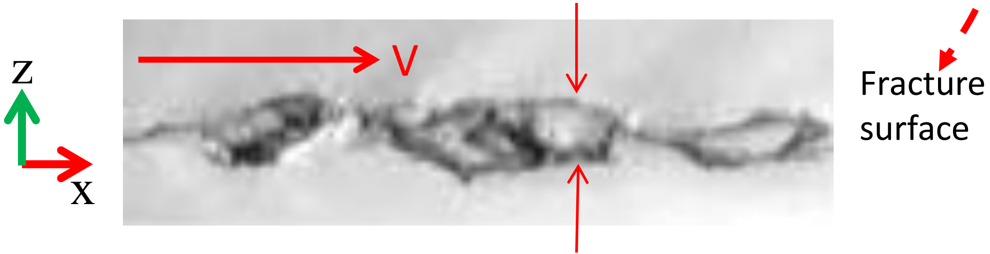
Elastic energy available for fracture $\mathbf{G} = \Gamma$ **Energy needed for fracture**

Challenges to the 2D view – front instabilities

The micro-branching instability –



localized out-of-plane structures



Other front instabilities

Front waves (Ramanathan & Fisher ‘97, Sharon *et al.* ‘01)

Stepped surfaces (Sommer ‘69, Tanaka *et al.* ‘98,00’, Baumberger *et al.* ’08,’13)

Measuring rapid crack fronts Dynamics

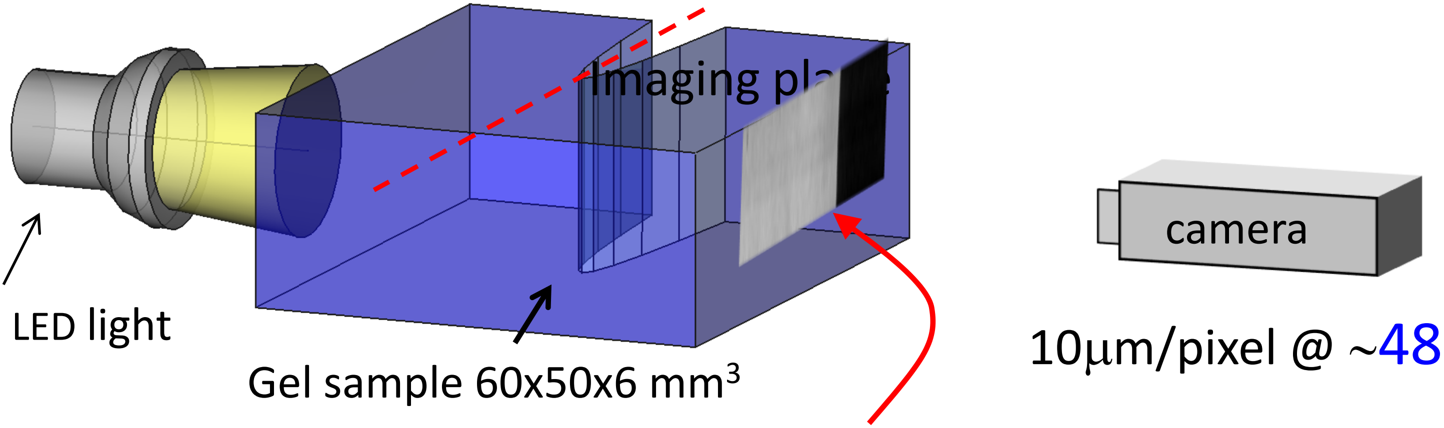
Problem #1: Cracks are fast (~3 km/sec in glass)

Solution: Use gels (~3 m/sec in polyacrylamide)

$$c \sim \sqrt{\frac{E}{\rho}} \qquad \begin{aligned} E &\sim 90\ kPa \\ c &\sim 5\ m/s \end{aligned}$$

Problem #2: How to image a crack front?

Solution: Look ***through*** the gel



Collimated LED light

Gel sample 60x50x6 mm^3

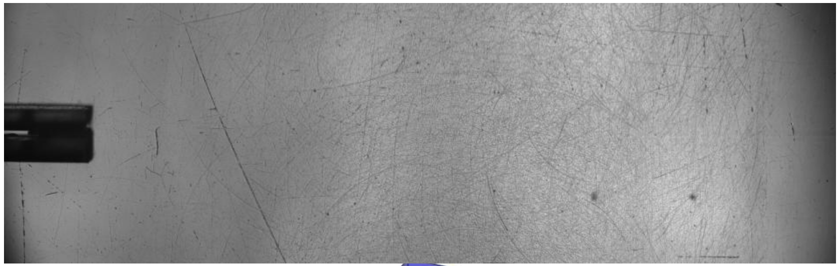
10μm/pixel @ ~48000 fps

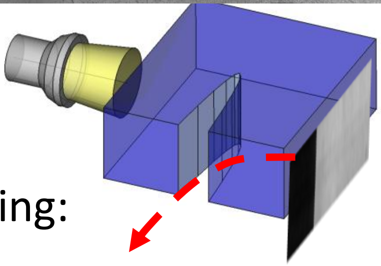
The **front** becomes a moving shadow across the image

Simple crack

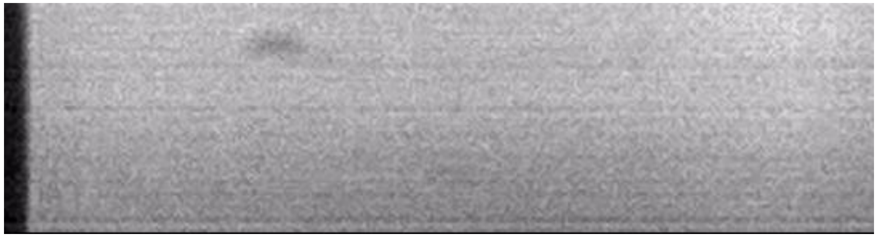
Micro-branching crack

Crack tip imaging:

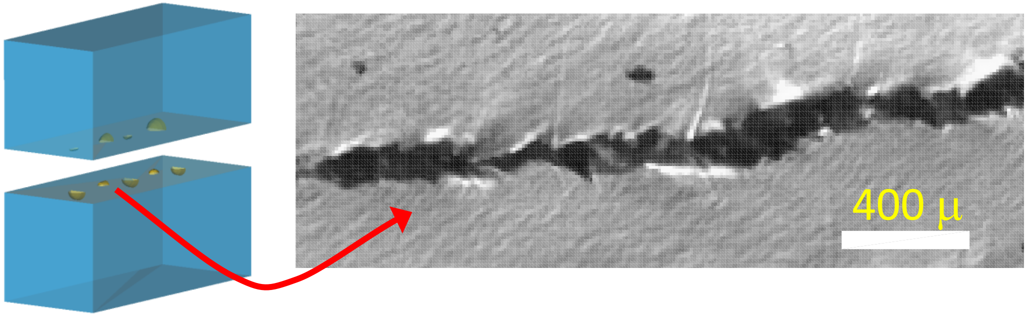




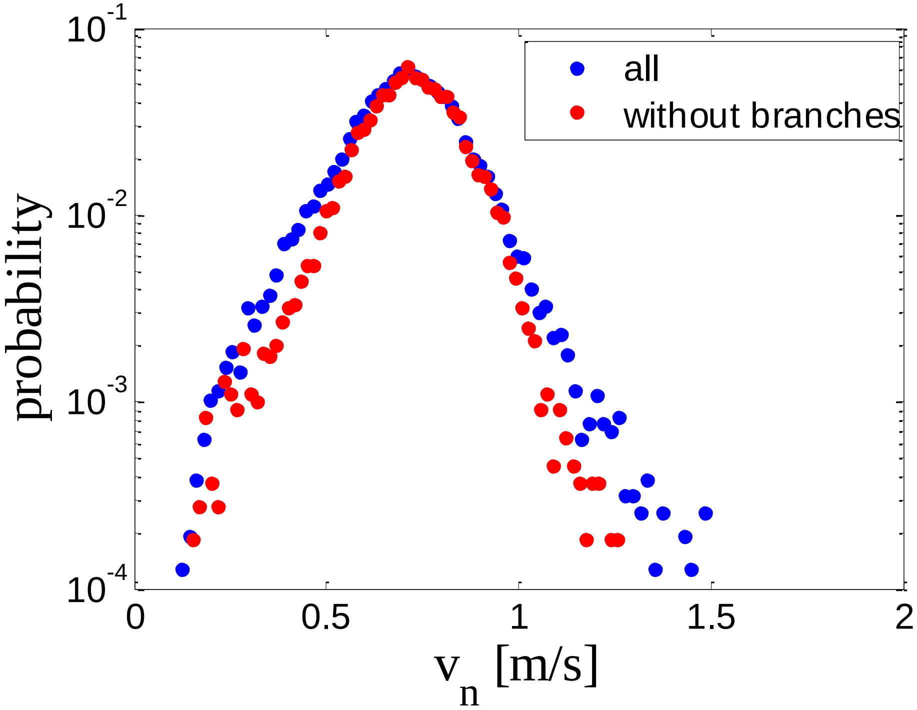
Front imaging:



The fracture surface *post-mortem*:



Once cracks stop being geometrically simple, their ***dynamics*** become pretty ***complex***



Velocity fluctuations along the front

~ Exponential tails

↕

Long-range correlations

Can we understand the

Front shapes and dynamics

Velocity fluctuations

Do “simple” **2D** Fracture Mechanics work in an **intrinsically 3D world**???

The dramatis personae: Enter v(z), $K(z)$, $\Gamma(z)$

Crack tip singularity
$$\boxed{\sigma \sim \frac{K}{\sqrt{2\pi \cdot r}}}$$

Elastic energy available for fracture

$$\mathbf{G(\,v\,,\,}K\mathbf{\,)} = \Gamma(\mathbf{v})$$

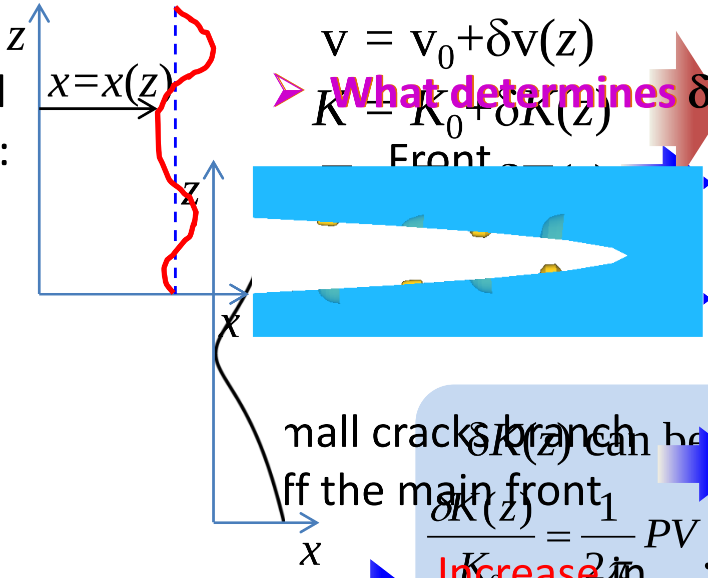
Crack velocity

Energy needed for fracture

Crack tip eq. of motion: $v = v(\,K\,,\,\Gamma\,)$

Transition to front dynamics: Assume the eq of motion is locally valid $v(z) = v(K(z),\Gamma(z))$

Perturb around a straight front:



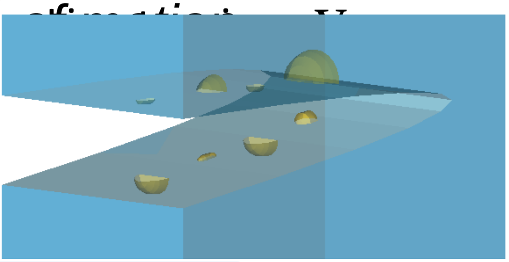
$v = v_0+\delta v(z)$

$K = K_0+\delta K(z)$

Front

- What determines δK(z) in microbranching?

Front eq. of motion:
$$\frac{\delta v(z)}{V} \sim \frac{\delta K(z)}{K_0} - \frac{\delta\Gamma(z)}{\Gamma_0}$$



$\delta K(z)$ can be **computed:**

$$\frac{\delta K(z)}{K_0} = \frac{1}{2\pi} PV \int \frac{x(z')-x(z)}{(z'-z)^2} dz'$$

mall cracks branch
ff the main front

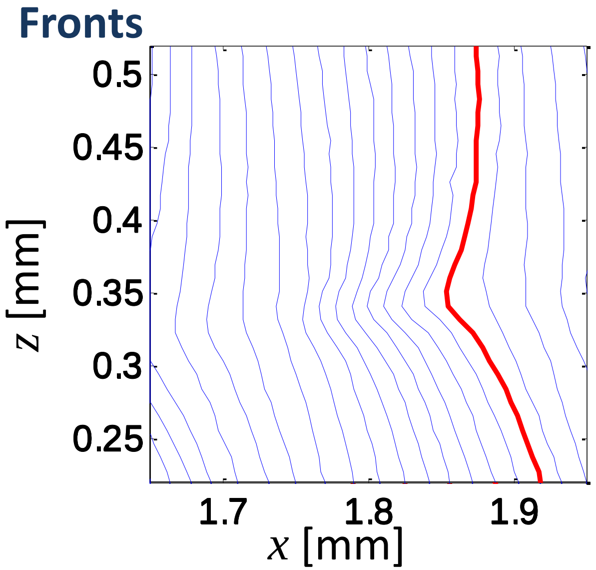
More surface is created

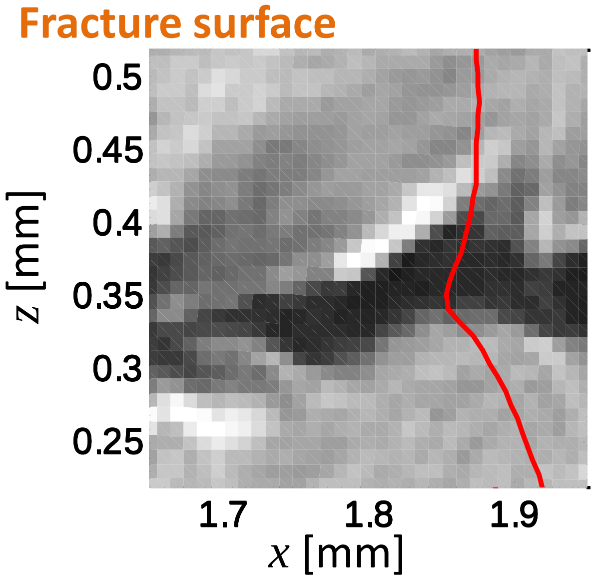
Increase in Fracture Energy

Pinning of the front

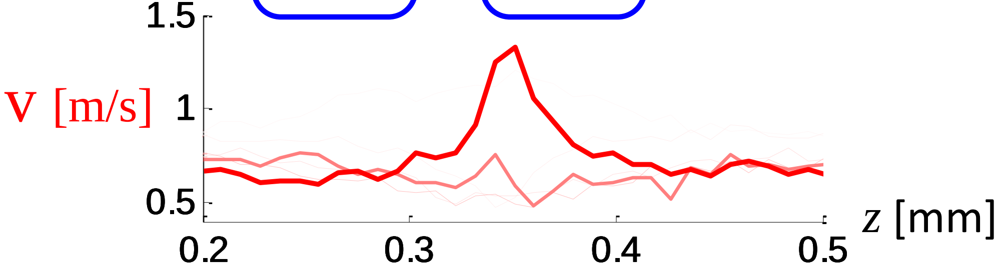
*Rice, '85. First order in $x'(z)$, neglecting dynamic effects

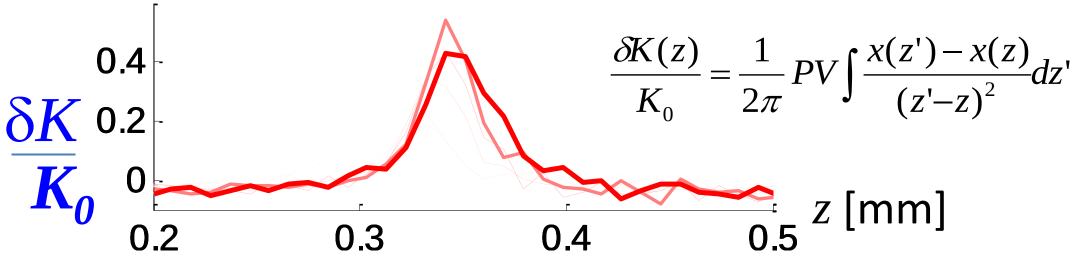
The life and times of a microbranching event





$$\frac{\delta v(z)}{v_0} \sim \boxed{\frac{\delta K(z)}{K_0}} - \boxed{\frac{\delta \Gamma(z)}{\Gamma_0}}$$





$$\frac{\delta K(z)}{K_0} = \frac{1}{2\pi} PV \int \frac{x(z') - x(z)}{(z' - z)^2} dz'$$

- Micro-branch **initiation increases** $\delta\Gamma(z) > 0$
- The front is locally stretched as micro-branches progress due to the **inhomogeneity of $\Gamma(z)$**
- **Upon micro-branch arrest, $\delta\Gamma(z) = 0$** while **$\delta K(z) > 0$** ➔ fronts are **locally accelerated**

The crack front is a “slingshot”, cocked by micro-branch initiation

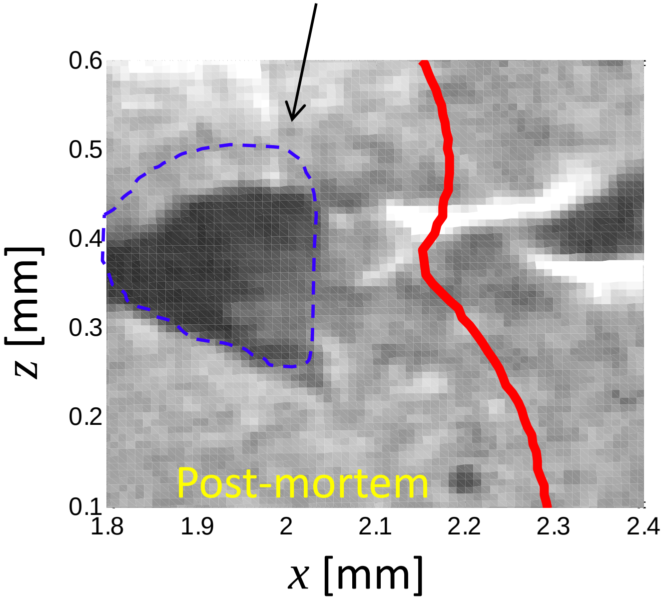
What determines the magnitude of velocity at release?

What determines the moment of release?

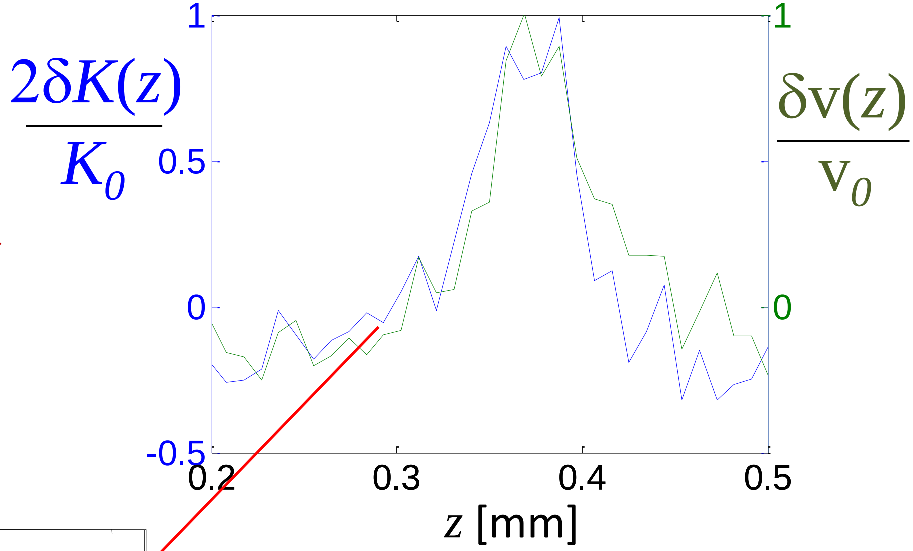
The velocity at the moment of complete release
(when $\delta\Gamma \sim 0$)

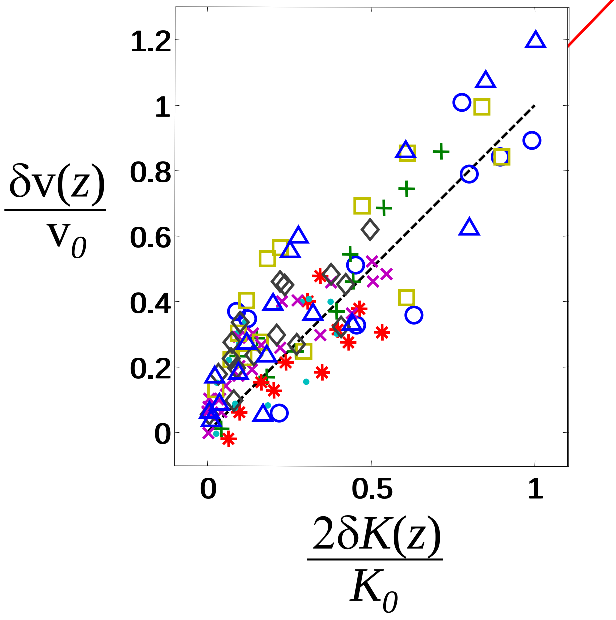
$$\frac{\delta K(z)}{K_0} = \frac{1}{2\pi} PV\int \frac{x(z')-x(z)}{(z'-z)^2} dz'$$

Immediately after a micro-branch **dies** $\delta\Gamma \sim 0$



…δv and δK *measured independently* become directly correlated





Accumulated data from nine different instances of microbranch death and release

The same data in a "y=x" plot

Geometry + **Fracture Mechanics** determines **dynamics**

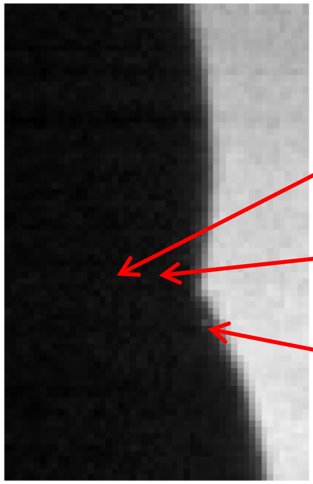
$$\frac{\delta v(z)}{v_0} \sim \frac{2\delta K(z)}{K_0}$$

What makes the stress grow so large?

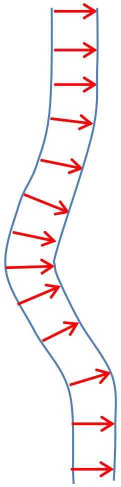
⇔ What determines the moment of release?

Let's analyze the movie front-by-front:

Front dynamics of one big microbranching event



- At first the front is locally retarded because of **micro-branch nucleation + growth** (decrease in velocity = increase in fracture energy)
- The curved front **collapses** into a cusp-like shape
- **Cusp formation** is immediately followed by **release**

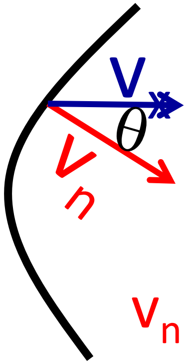


A finite-time singularity scenario

1. Fronts propagate in the **normal** direction ➔ Develop **cusps** ⇔ **shocks** in curvature
2. Diverging curvature means **diverging δK** ➔ Micro-branches **yield** to diverging stress ➔ fronts are **released** from pinning

How to test these assumptions?

Normal propagation + curvature = Burgers equation



$v_n \sim$ constant

$\partial_t x = v_x(z) = v_n \cos\theta$

$\sim v_n\cdot(1-\theta^2/2)$

$\sim v_n\cdot(1-(\partial x/\partial z)^2/2)$

taking $\partial/\partial z$ + using the slope:

$u(z) \equiv \partial x/\partial z$

$$\partial_t u + v_n\cdot u(z)\cdot\partial_z u = 0$$

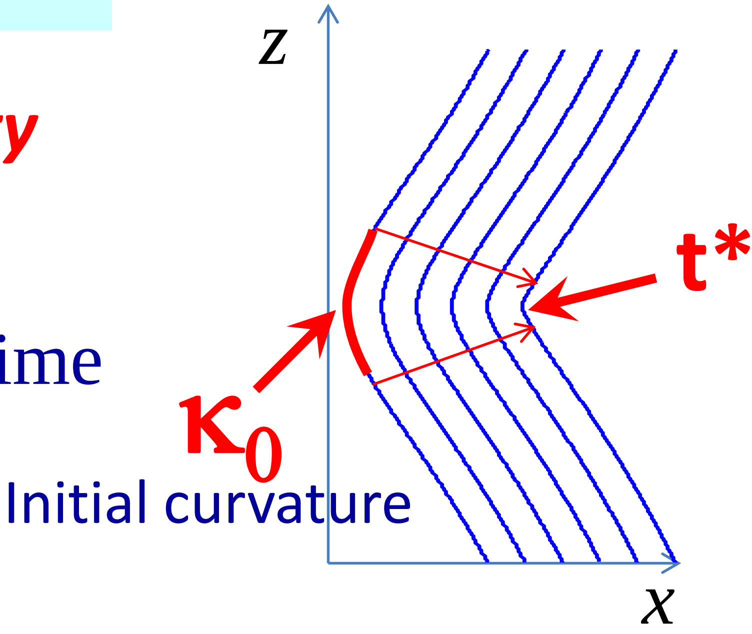
Burger's equation for the **slope** u(z)

Burger's Eqn ⇔ ***Finite time singularity***

$$\frac{\partial^2 x}{\partial^2 z} \sim \frac{1}{t^* - t}$$

with $t^* = 1/(\kappa_0 \cdot v_n)$

CUSP formation time

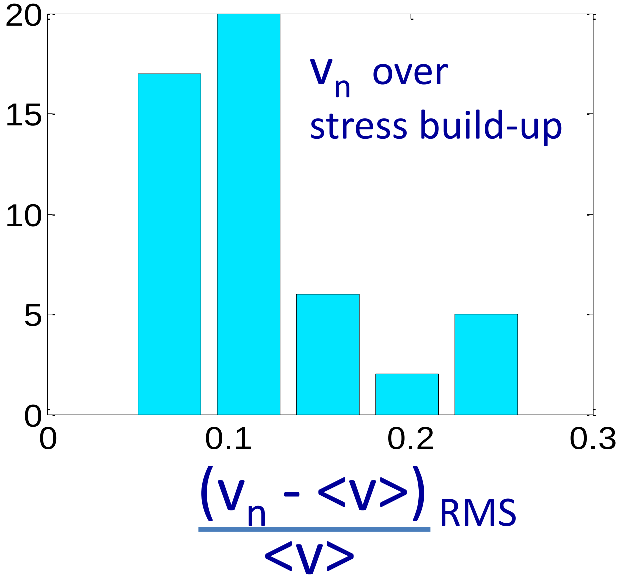


Assumptions for Burger's equation:

- $v_n \sim$ ***constant***
- initial ***curvature*** of the front (**due to $\delta\Gamma>0$**)

No explicit fracture mechanics input required ⇔ only geometry!

How **constant *is*** the V_n during the stress buildup?



$$\frac{(v_n - <v>)_{RMS}}{<v>}$$

v_n varies by ~10% over stress build-up

v_n statistics are not far off from the assumptions!

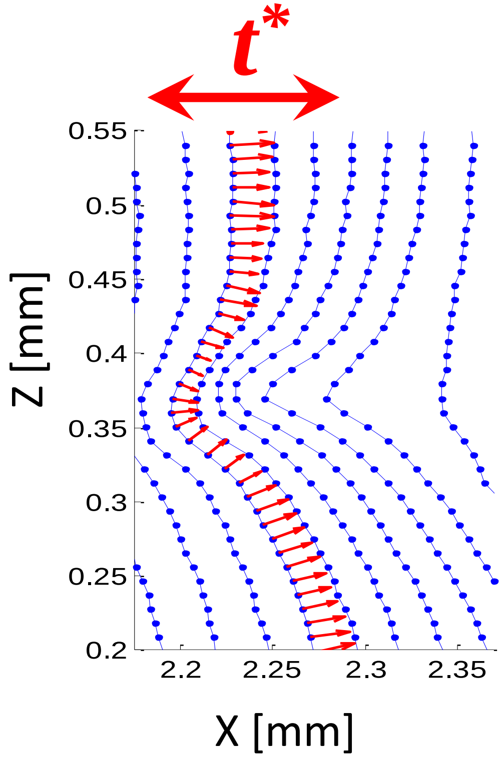
Is ***cusp formation*** at all related to ***micro-branch dynamics***?

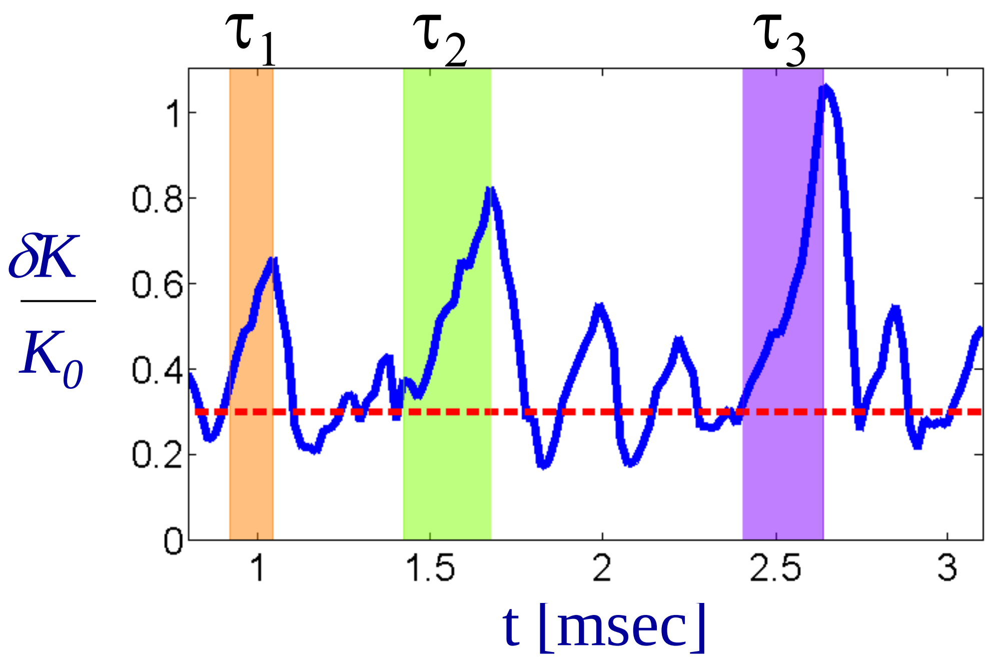
$t^* = 1/(\kappa_0 \cdot v_n)$

CUSP formation time

Front "***stretching***" time, τ

τ ~ micro-branch ***lifetime***

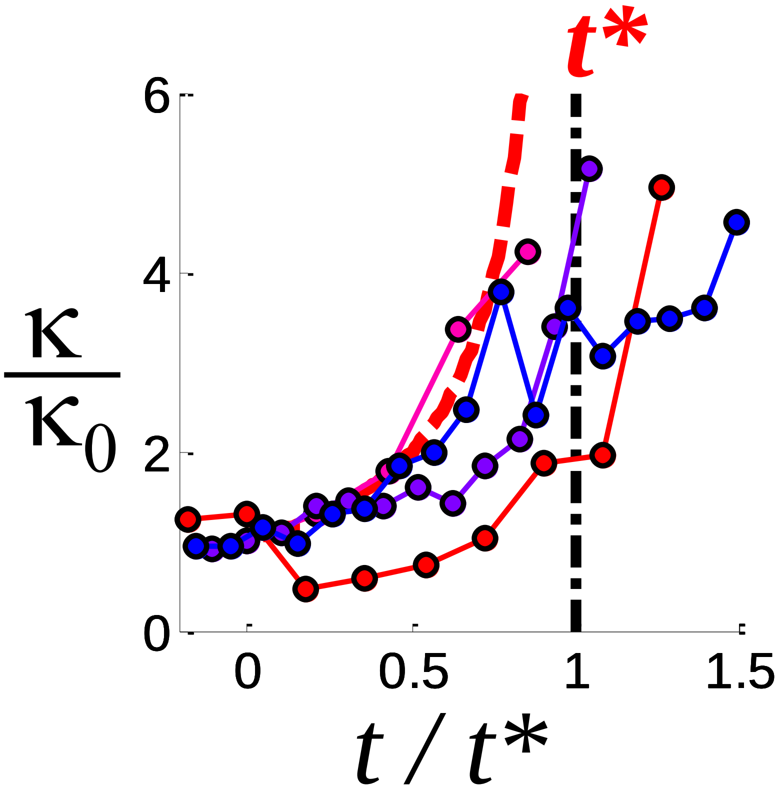


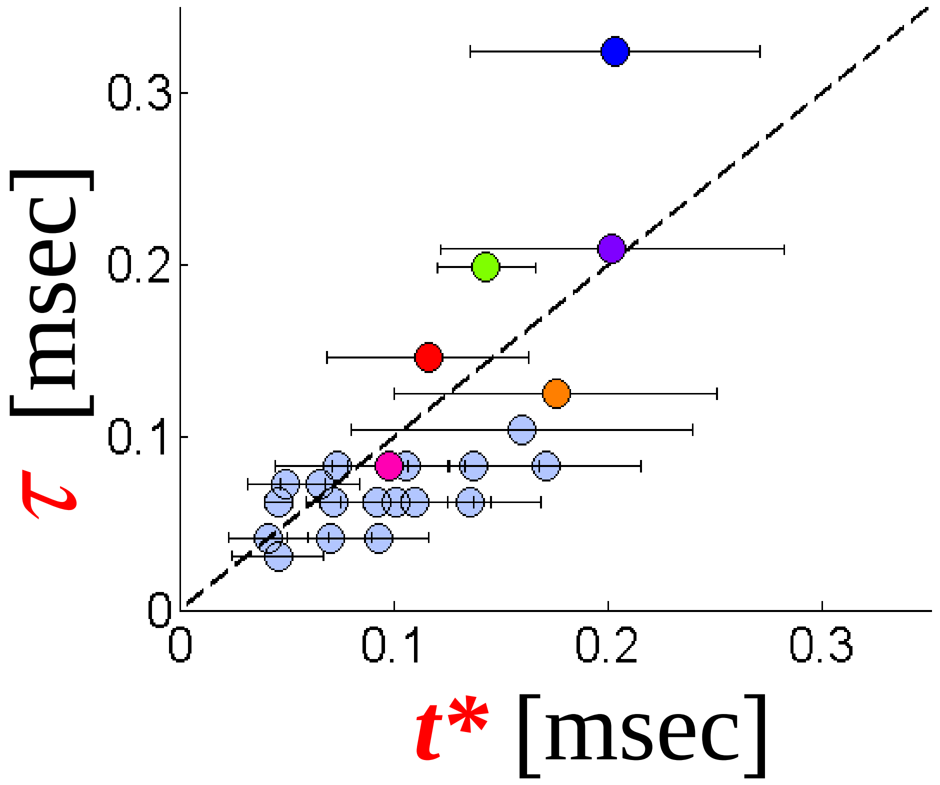


Geometry ??? *Dynamics of cracks*

Are t^* and τ related?

Are *t** and *τ* related? *Yes!*

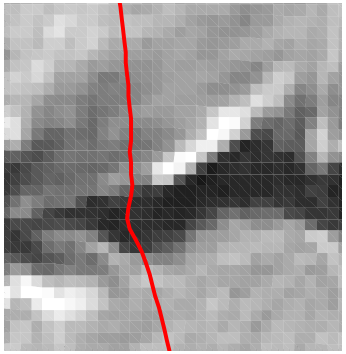




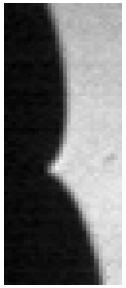
Cusp formation t^* ~ Micro-branch lifetimes τ !

Summary: Front geometry drives Front dynamics

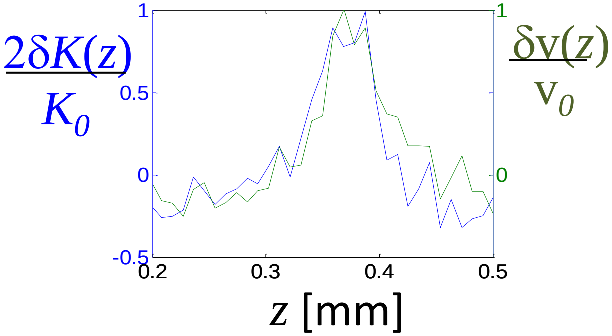
Micro-branching provides insight into crack front dynamics



- When a micro-branch is nucleated the front *curves* due to *increased* fracture energy



- Crack front curvature spontaneously generates a *cusp*
- The formation of the cusp ⬄ singular “line tension” δK
- Front **velocities** at release are determined by front **geometry + Fracture mechanics**



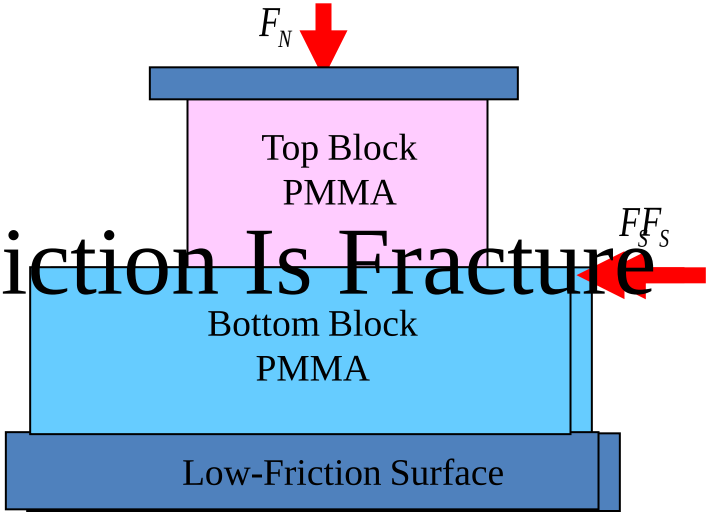
- ***Singular*** line tension ➔ cusp **collapse** ⬄ **Micro-branch Death**

Thank you!

Friction is Fracture:
Fracture Processes Drive Frictional Motion

Ilya Svetlizky & Jay Fineberg

The Racah Institute of Physics
The Hebrew University of Jerusalem



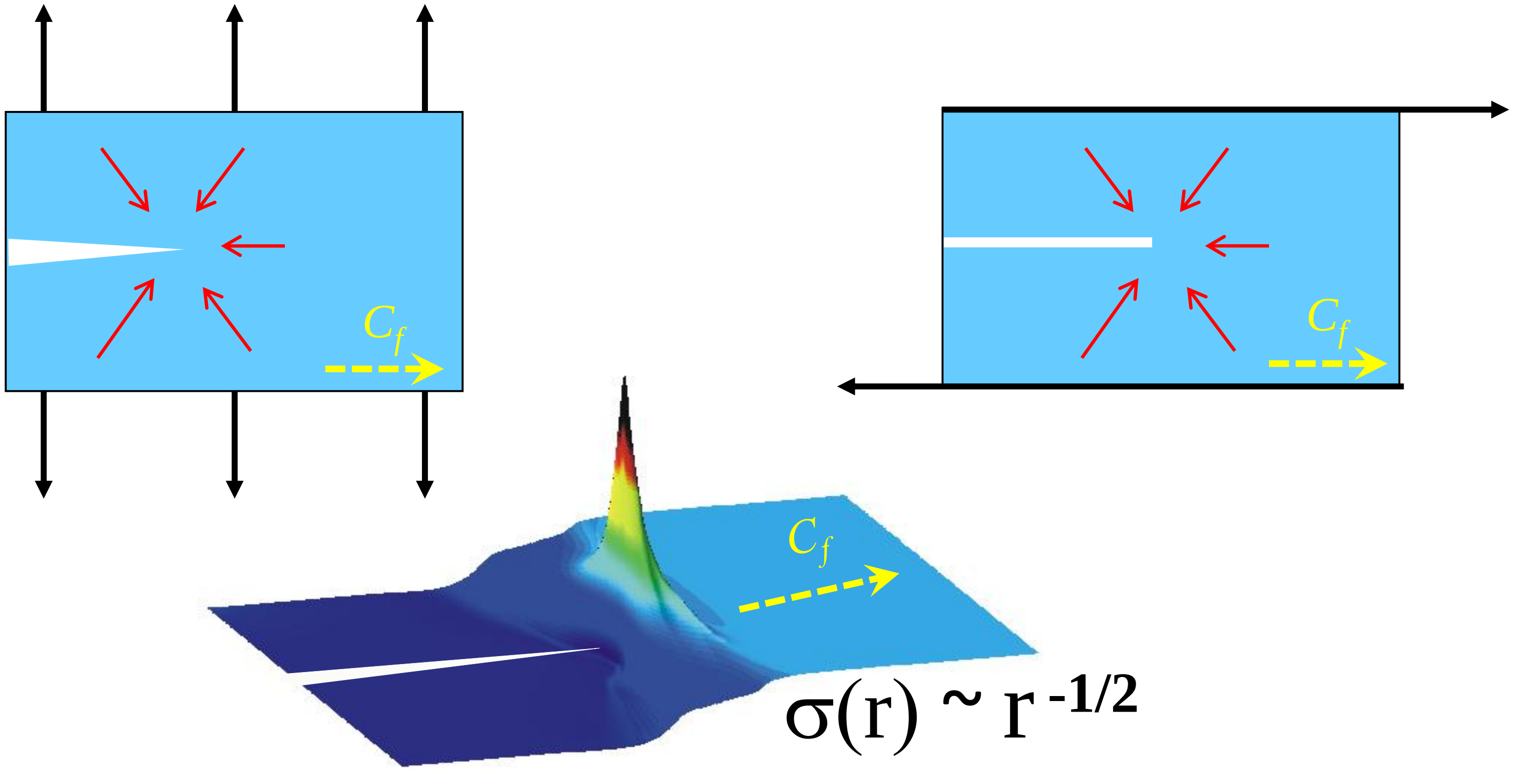
Friction Is Fracture

Review: Fracture

Linear Elastic Fracture Mechanics (LEFM)

Tension (σ_{yy})

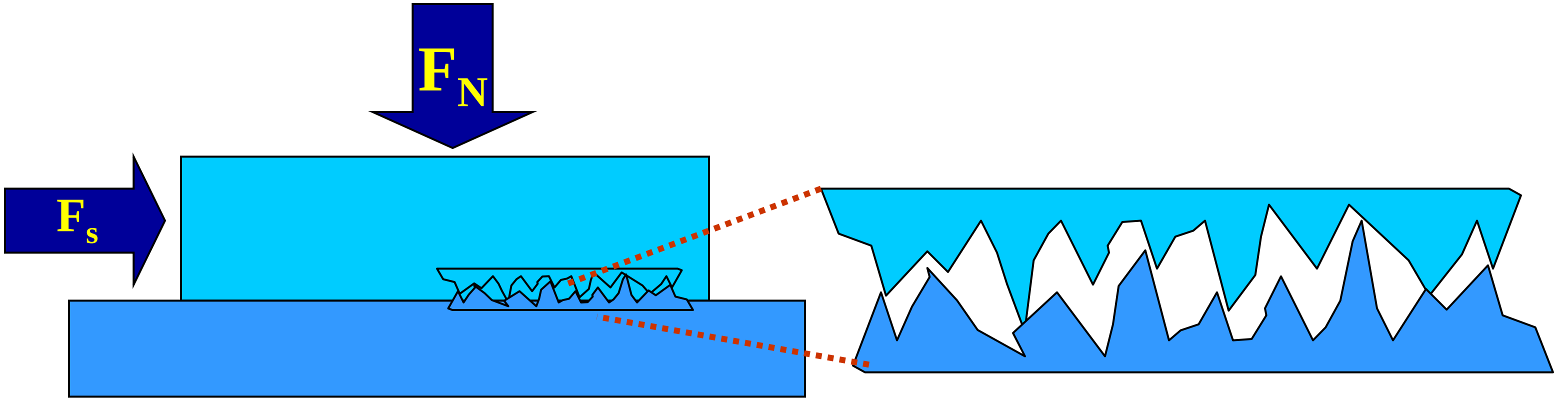
Shear (σ_{xy})



$\sigma(r) \sim r^{-1/2}$

- Linear elasticity → singularity of the stress at the crack tip
- Energy balance → Dissipation = Energy flux into the crack tip
- Important velocity scale C_R ,Rayleigh wave speed (1255m/s for PMMA)

Frictional Interface



- Net contact area = ***A*** << Nominal contact area

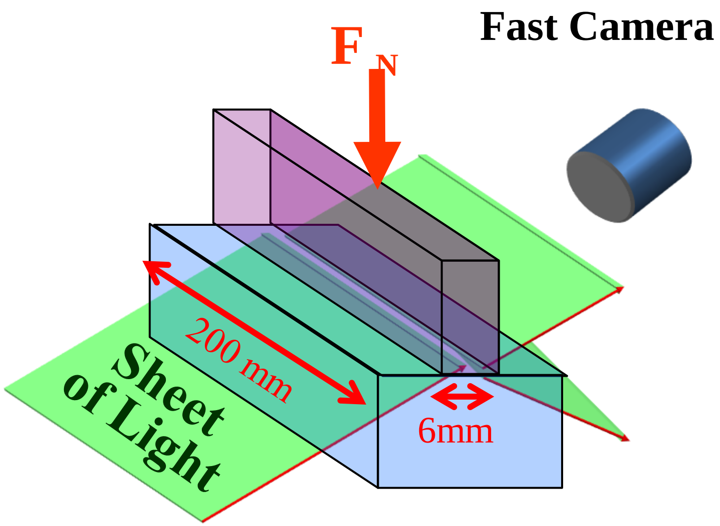
- At the transition from stick to slip contacts are being broken and reduce *A*.

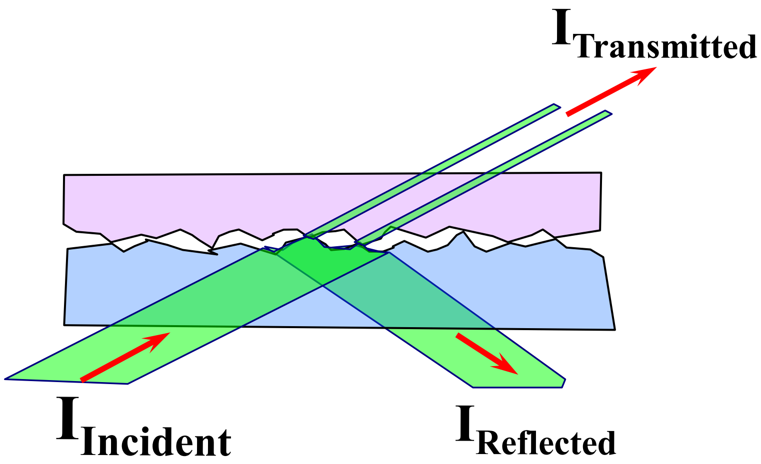
We'll show that:

Rupture of contacts described by classic Fracture Mechanics

F. Philip Bowden and David Tabor (1950)

Real Contact Area *A* Visualization





$I_{transmitted} \propto A$

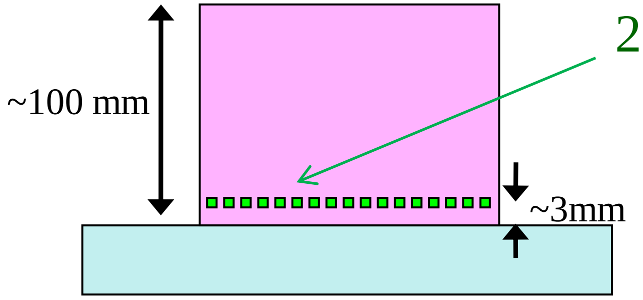
Frame Rate ~ 580,000 Frames/sec

Resolution: 1280 Pixels / 200mm

$$A(x,t)= I(x,t) = \int I(x,y,t)\, dy$$

S. M. Rubinstein, G. Cohen, and J. F., Nature 430, 1005-1009 (2004)

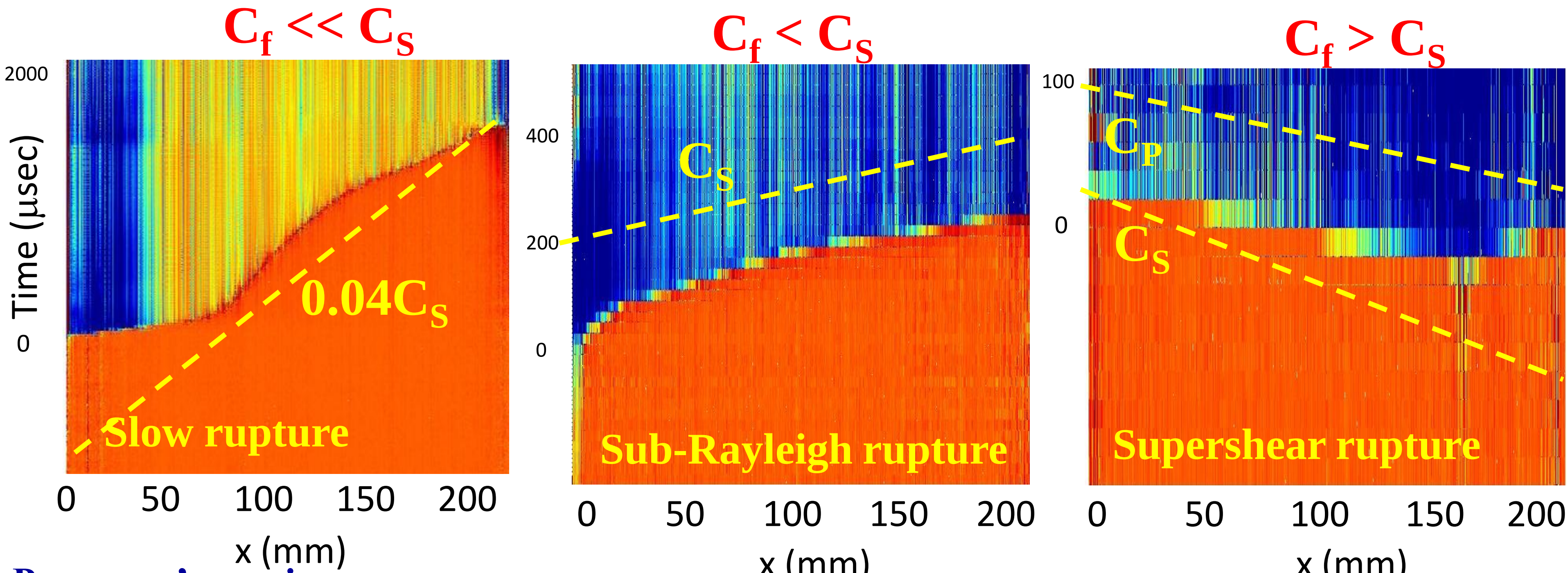
Strain measurements



2D strain tensor measured at 20 locations

(~1 μV signal at 1 MHz rate)

What types of rupture events occur upon slip initiation?



Poor man’s movie:

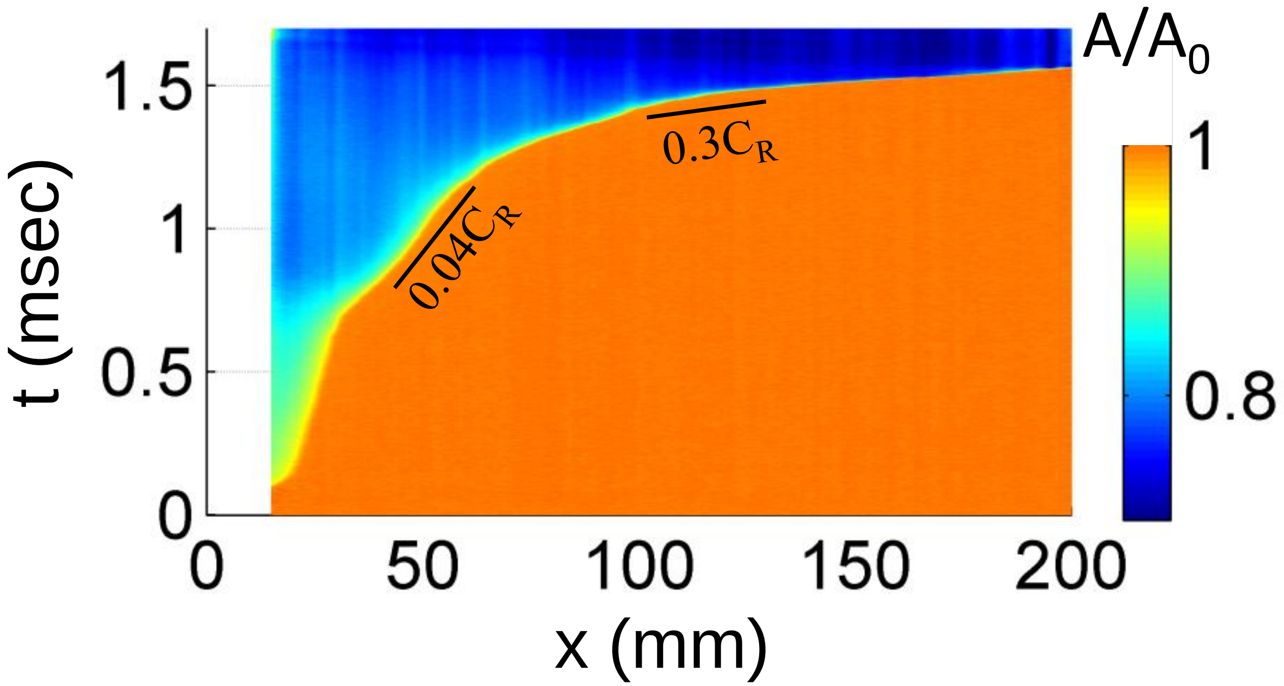
Horizontal lines are $A(x)$ over the entire interface separated in time by 2µs

Different ruptures modes:

are **determined** by **local pre-stresses** near the interface!

Here we’ll concentrate on ruptures where $C_f < C_S$

“Slow” Rupture Fronts



Each t is a snapshot of the real area of contact across the entire interface (X-t plot)

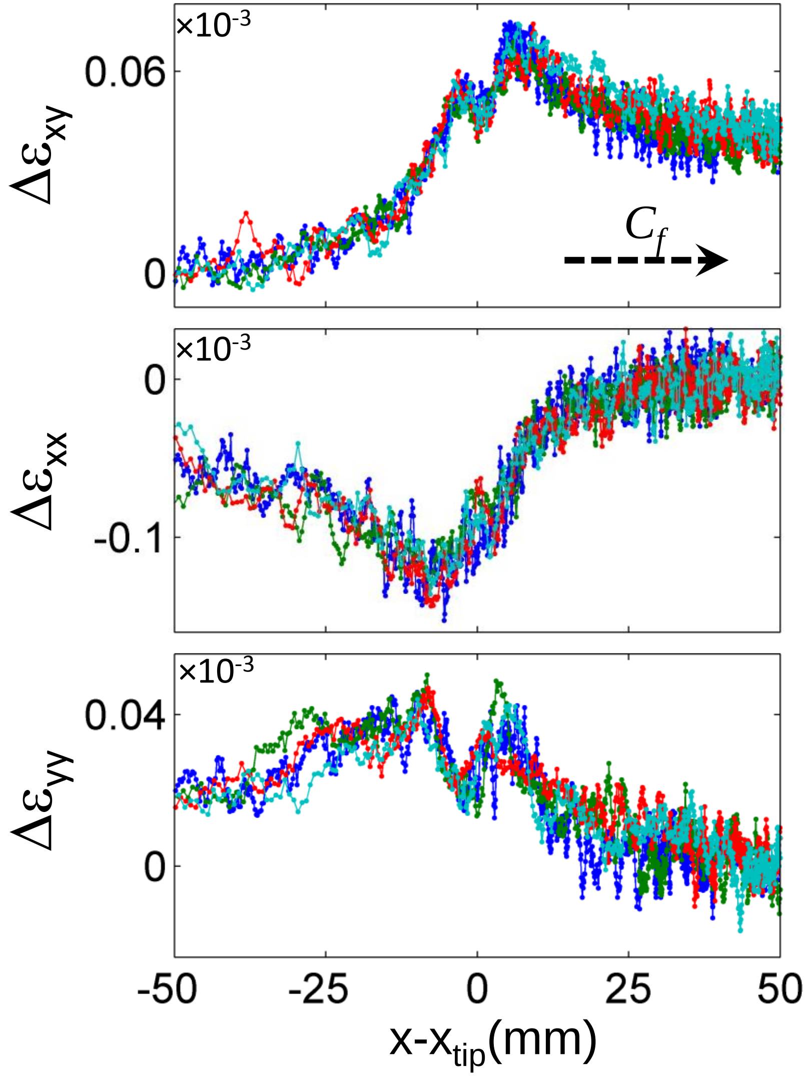
Block detachment is mediated by propagating crack-like front

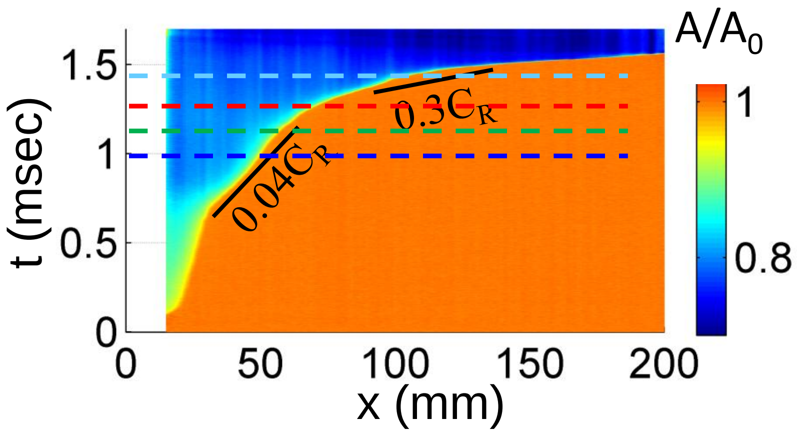
⇩

Friction ⬄ Dynamic Fracture Problem

Characterizing “Slow” Ruptures

$0.04C_R<C_f<0.3C_R$





x_{tip} - Rupture tip location

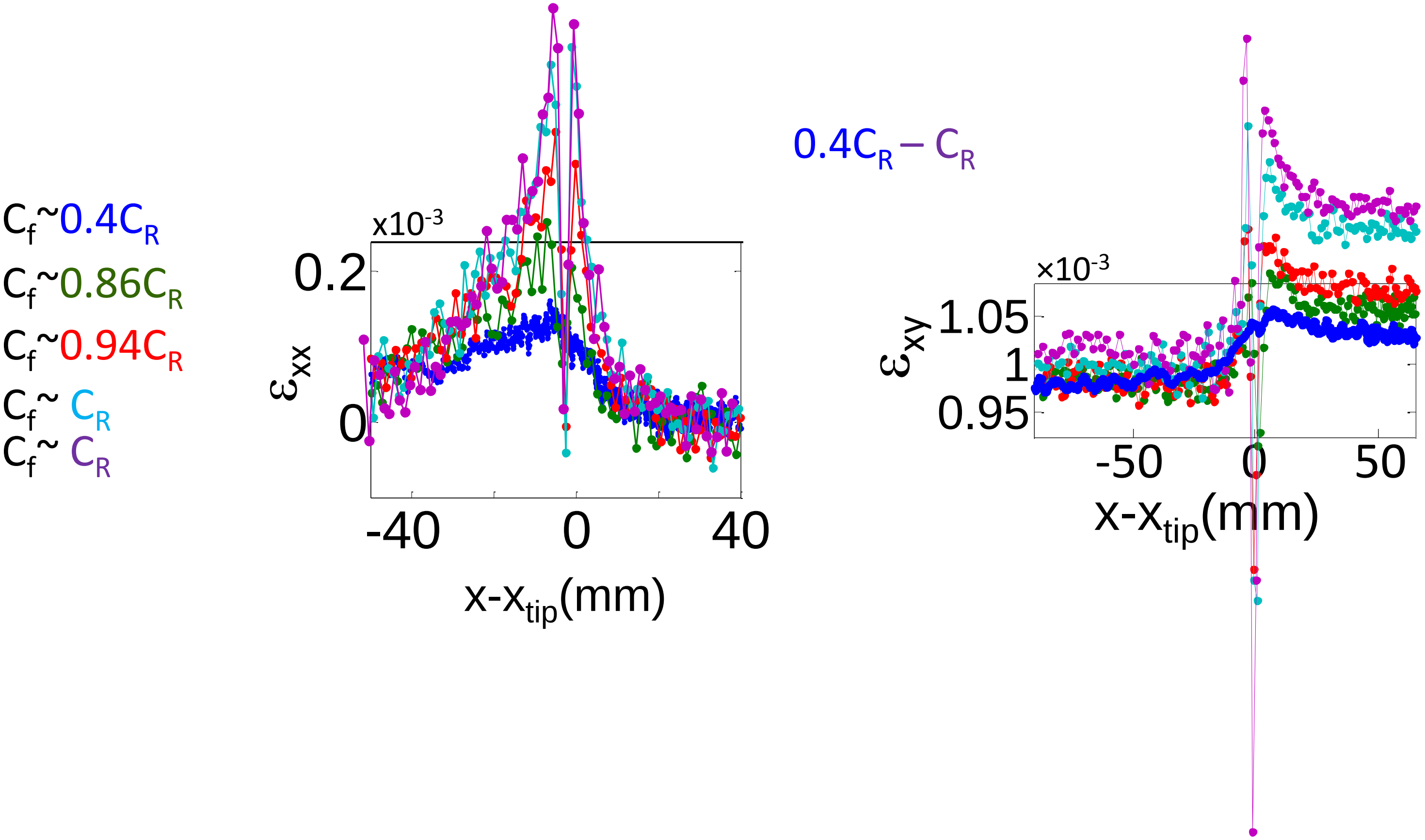
ε_{ij} - Strain tensor

$C_f<0.3C_R$:

Spatial profiles of strain collapse to a single functional form.

Does this collapse continue for even **higher** front velocities?

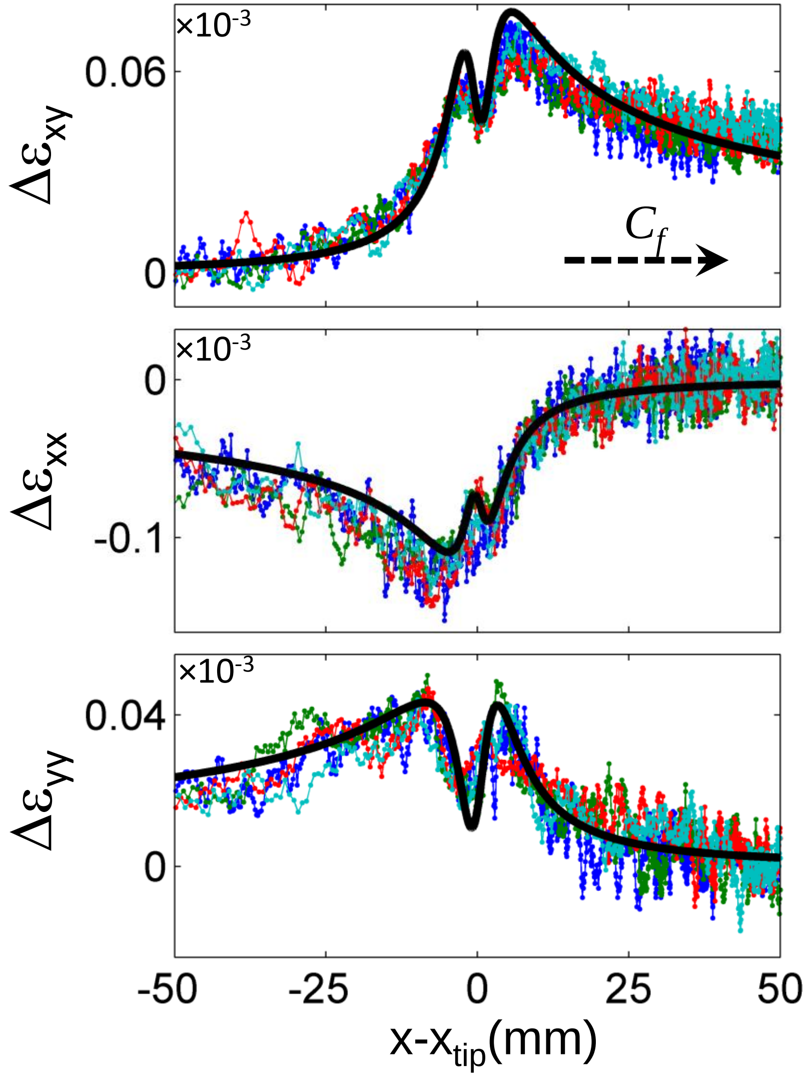
No data collapse at high velocities!

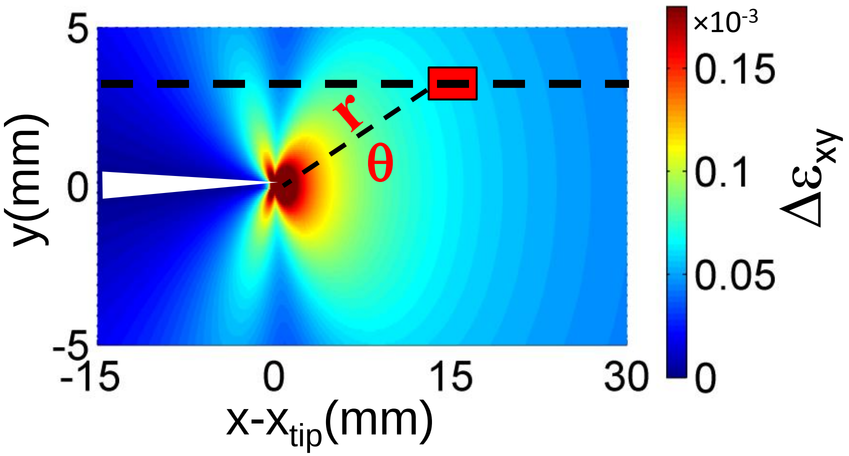


So, how can we explain this mess?! → LEFM

Comparing Strain Measurements To LEFM

$0.04C_R<C_f<0.3C_R$

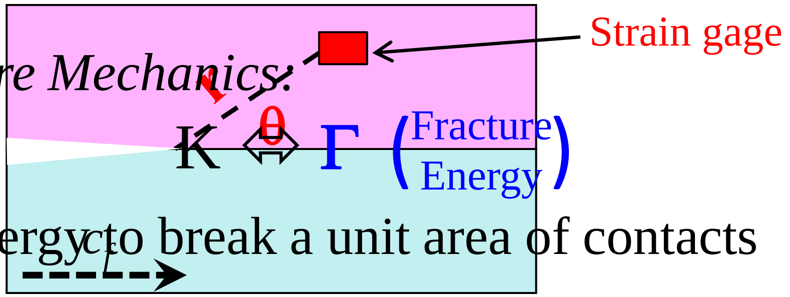




$$\Delta\varepsilon_{i,j} = \frac{K}{r^{1/2}} \Sigma'_{i,j}(\theta, c_f)$$

One free parameter K fits ***all*** of the data well

Fracture Mechanics:



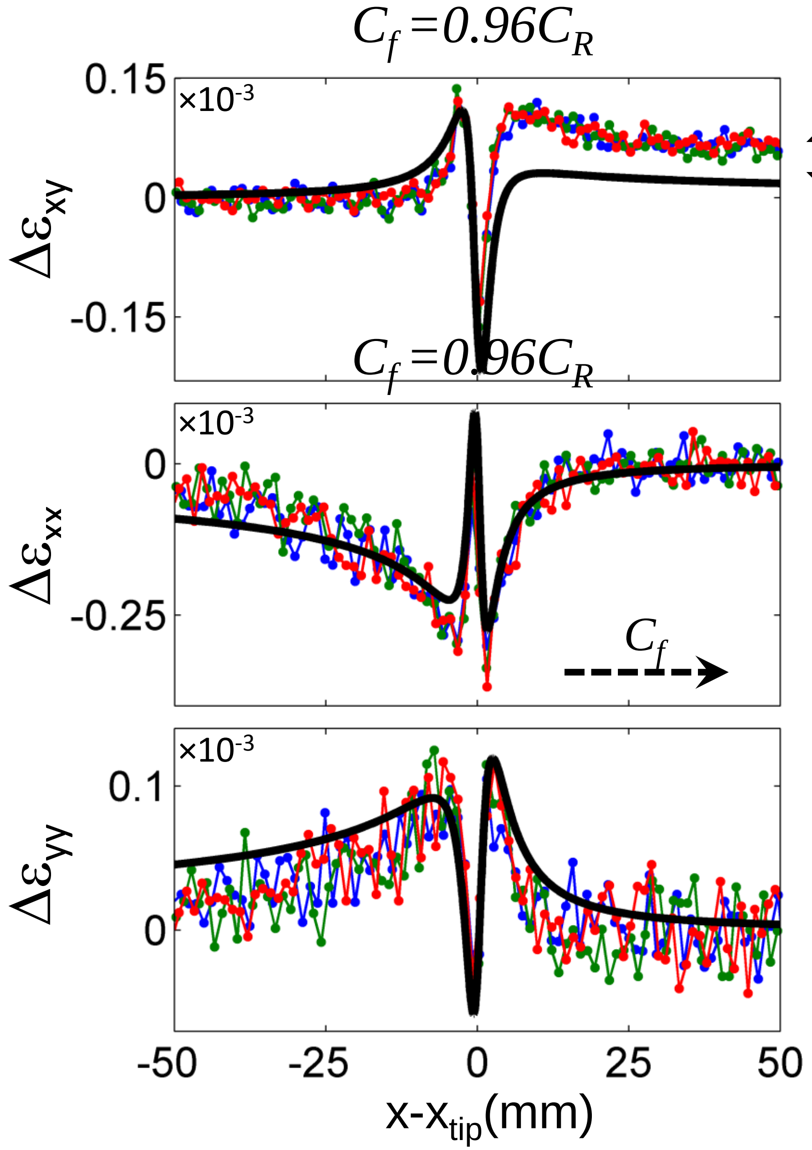
K ⇔ Γ (Fracture Energy)

Γ= energy to break a unit area of contacts

$\Gamma \sim 1\ \mathrm{J/m^2}$

Comparing Strain Measurements To LEFM

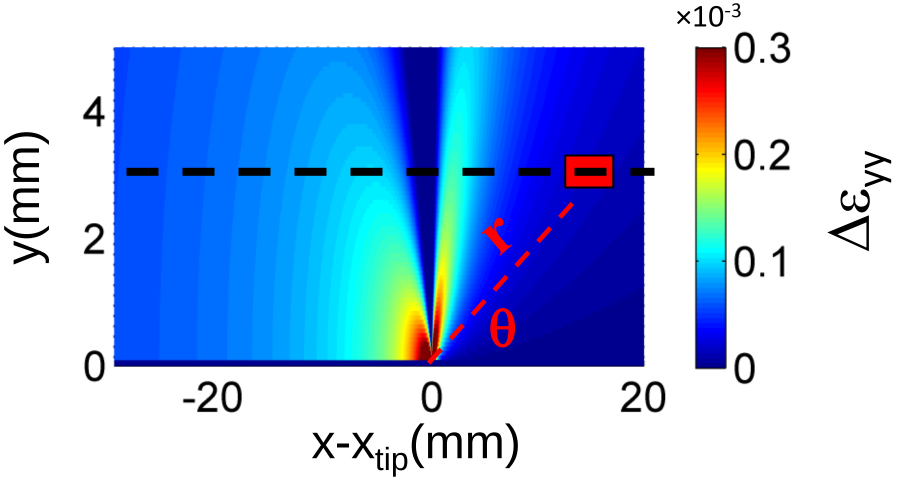
No free parameter - using same Γ (Fracture energy)



Failure at $x\text{-}x_{tip} > 0$

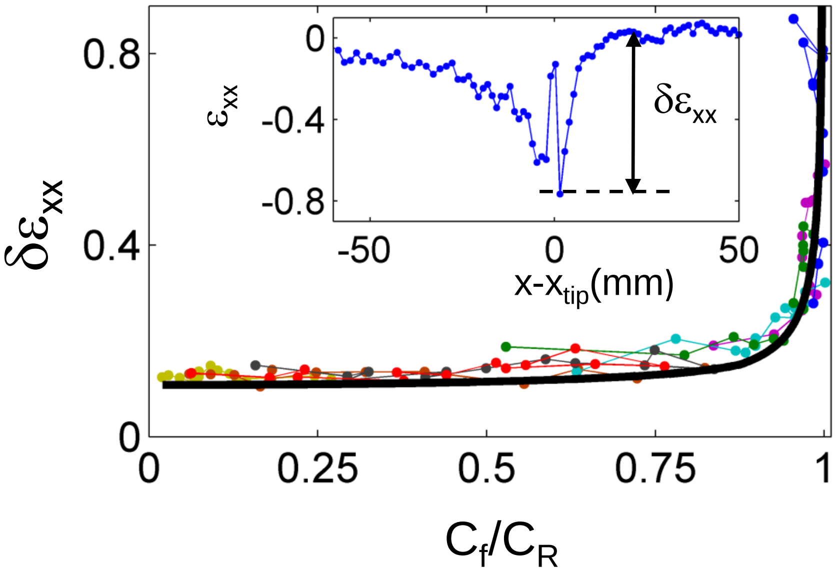
ment at high velocities

$$\Delta\varepsilon_{i,j} = \frac{K}{r^{1/2}} \Sigma'_{i,j}(\theta, c_f)$$



Comparing LEFM to Measurements at All Velocities:

One free parameter **Γ** (Fracture energy)

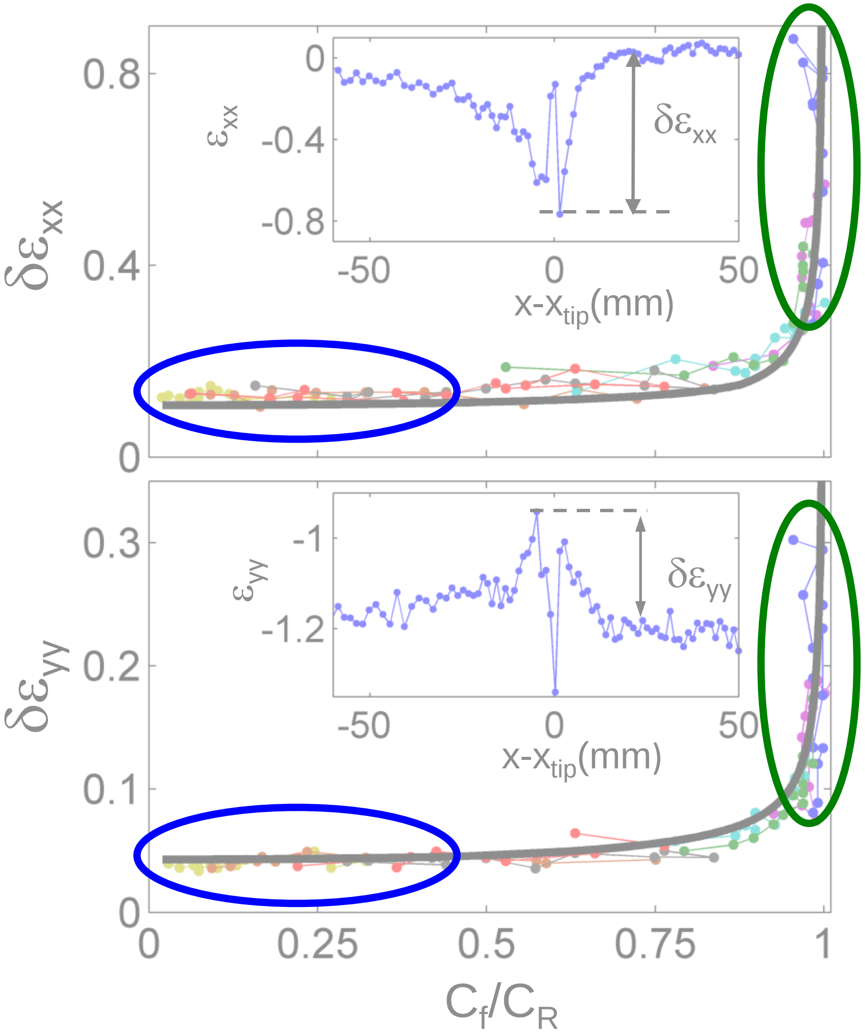


Great quantitative agreement for all velocities

Comparing LEFM to Measurements at All Velocities:

One free parameter **Γ** (Fracture energy)

Problem with stress drop prediction

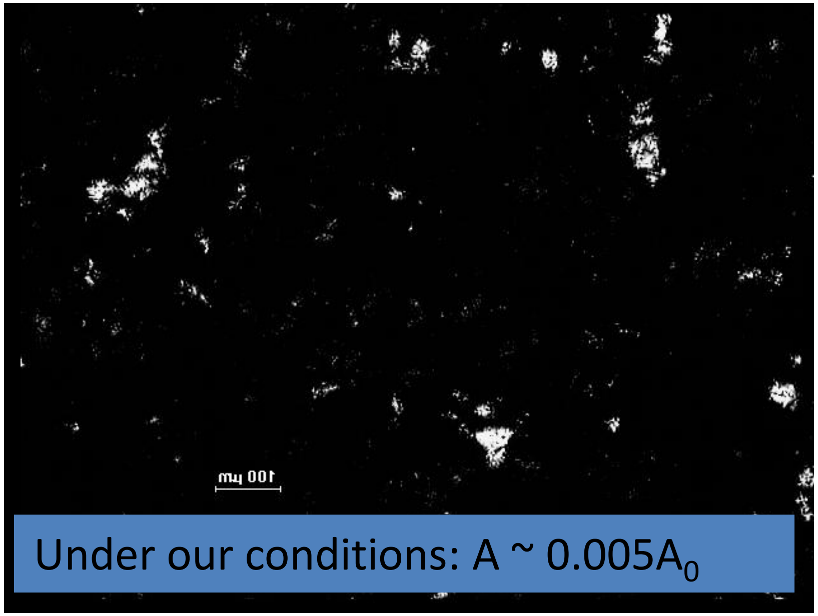


Great quantitative agreement
for all velocities

Does the value of Γ ~1J/m^2 make sense?

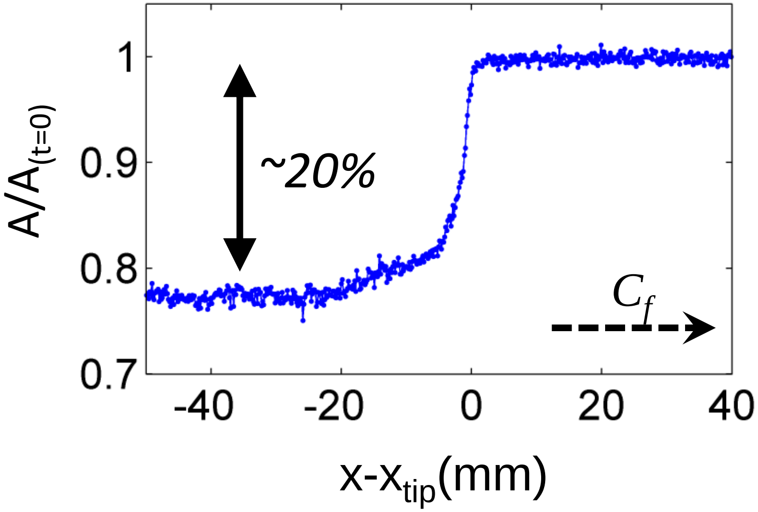
Yes! When interface sparseness is taken into account $\Gamma \Leftrightarrow \Gamma_{\text{bulk}}$

Real area of contact - PMMA



Under our conditions: $A \sim 0.005A_0$

J.H. Dieterich, B.D. Kilgore Tectonophysics 256 (1996) 219-239



$$\Gamma_{\text{bulk}} = \Gamma \cdot A_0/\Delta A = 1 / (0.2 \times 0.005) \sim 1000 \text{ J/m}^2$$

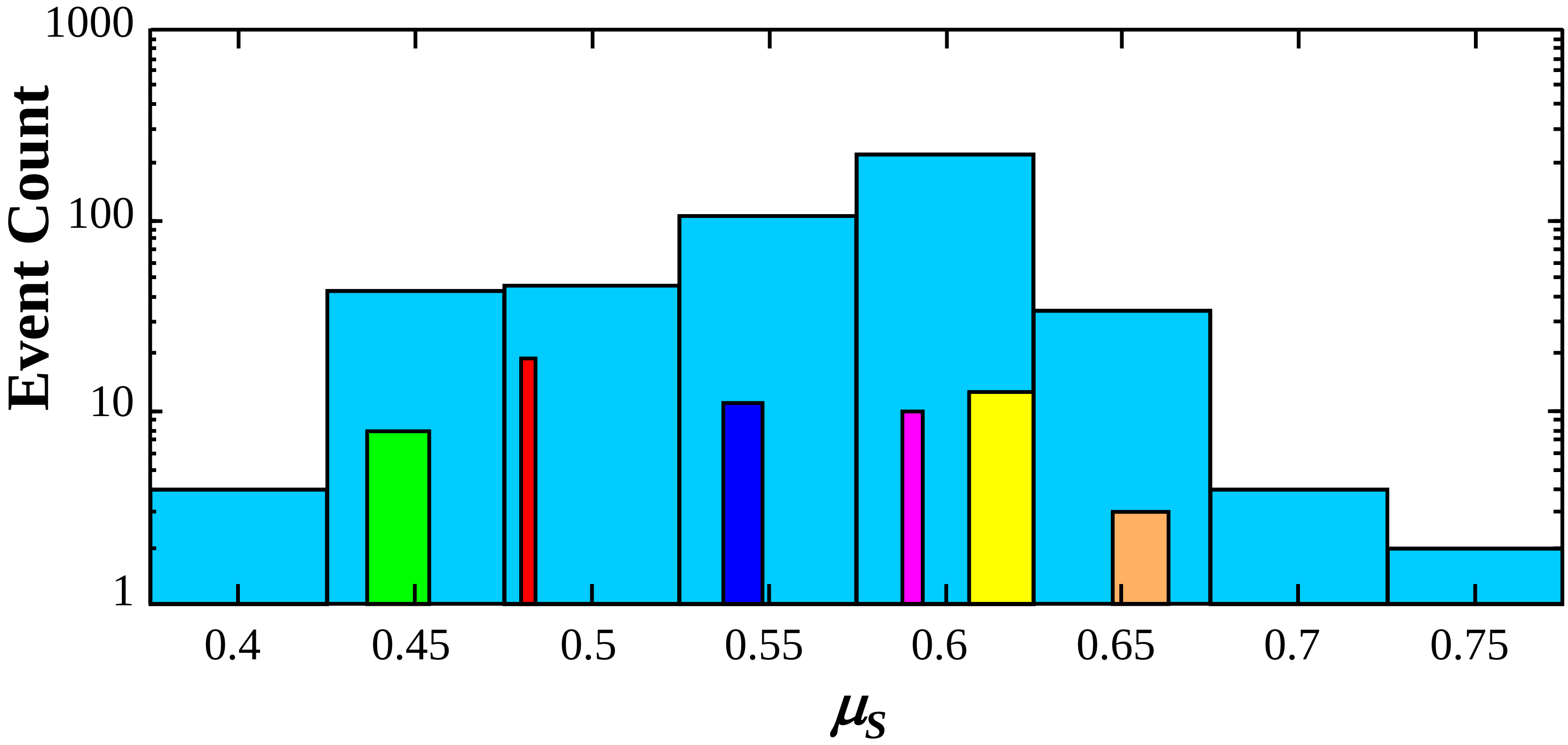
➔ Γ_{bulk} ~ the measured bulk fracture energy for PMMA!

Well… What about **friction** (we are talking about friction)?

How is this compatible with a ***characteristic***
static friction coefficient?

It’s actually not….

In general:



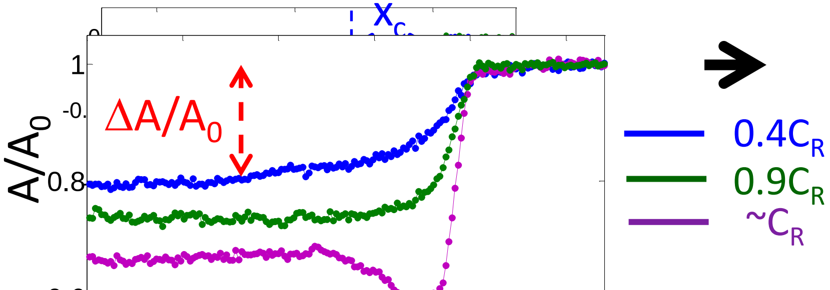
μ_S can vary by ~ a factor of 2 – for the same materials under the same ambient conditions!

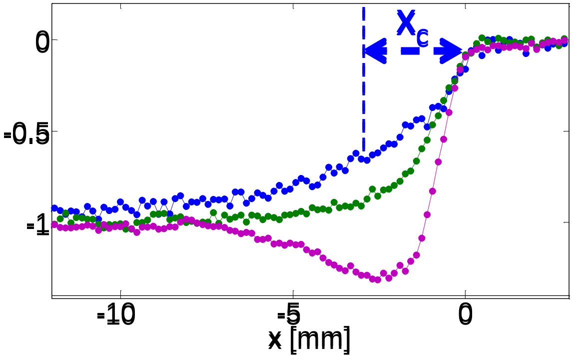
O. Ben-David and J. Fineberg, Phys. Rev. Lett. **106**, 254301 (2011).

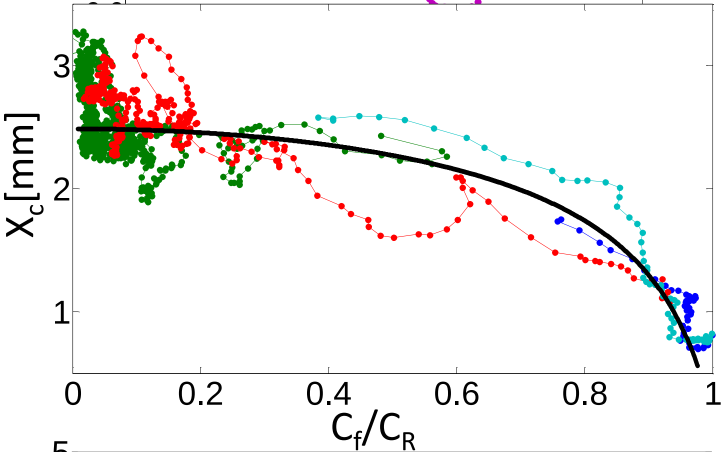
Dissipation ⇔ $\Delta A(x,t)$ at the tip of a rupture front

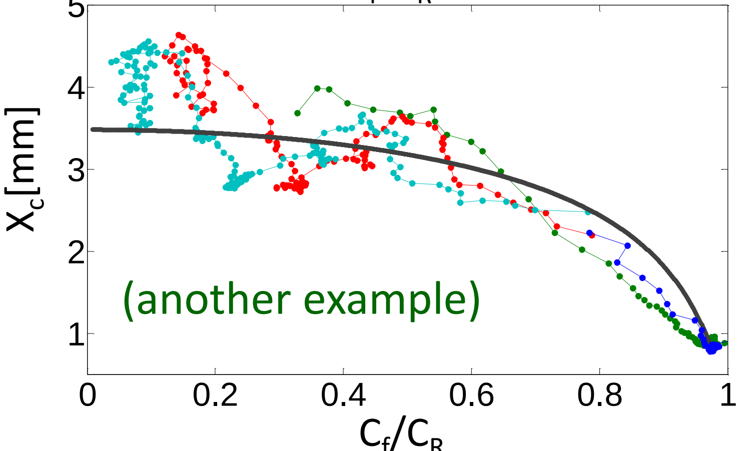
Characterizing the dissipation scale, X_c

A(x) characterizes the dissipation at each x, c_f









X_c ***contracts*** as $c_f \rightarrow c_R$!

X_c contracts due to ***relativistic effects*** at high front velocities

$$X_c = X_{c0}/f(c_f)$$

$1/\, f(c_f)$ ~ Lorentz contraction of length scales

(for anti-plane $f(c_f)$ = $(1-(c_f/c_s)^2)^{-1/2}$)

J. R Rice (1980)
M.Ohnaka & T.Yamashita JGR (1989)
Y.Bar Sinai, Efim A.Brener, E.Bouchbinder GRL (2012)

Summary

Friction is (really) Fracture

- Singular fields at the rupture tip ⇔ Classic Shear Fracture
- Measured fracture energy (Γ~1J/m^2) ⇔ ~bulk fracture energy
- Cohesive zone size contracts according to “Lorentz Contraction”

Questions:

- As $C_f \rightarrow C_R$, classic solution fails to describe $\varepsilon_{xy}(x-x_{tip}>0)$
- Rupture Nucleation

Thank you!