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Quantum creep and localization in disordered quantum rotors

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Collaboration on disordered quantum rotor model

Pierre Le Doussal (LPTENS, Paris)

Kay J. Wiese (LPTENS, Paris)

The FRG world

Outline

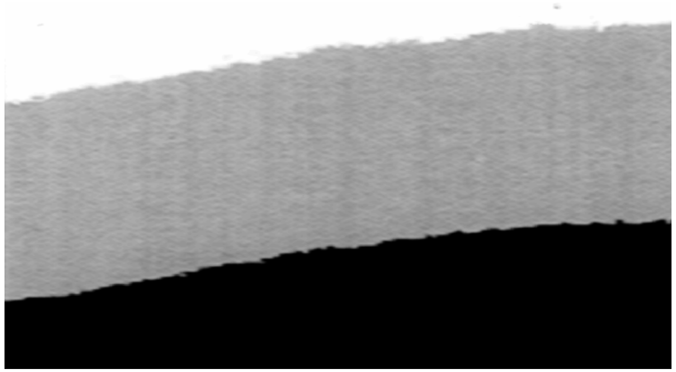
- Disordered elastic systems
- Classical and quantum creep

- Disordered quantum rotors
- Functional renormalization group (FRG) apporach to random field systems
- Localization of spin-waves and activated dynamics
- Summary and open questions

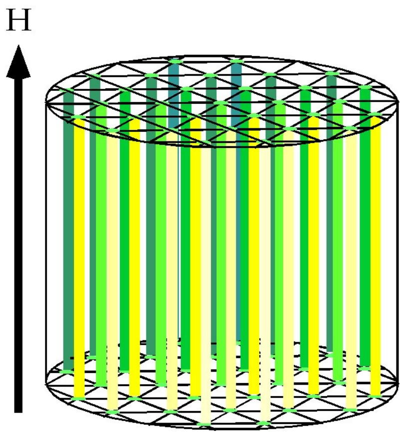
Disordered elastic systems

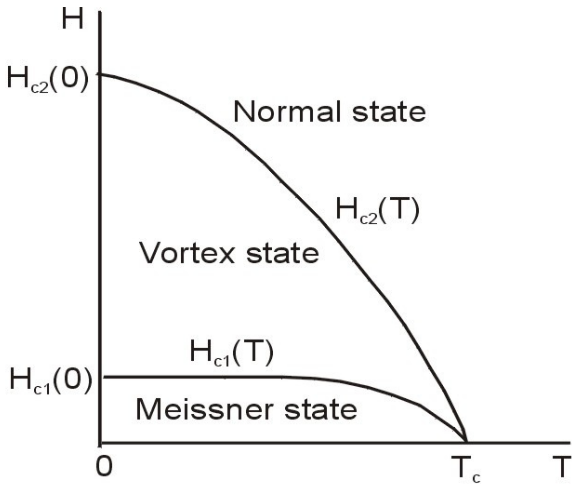
Domain wall in a disordered Ising ferromagnet (cobalt film)

S. Lemerle, et al 1998



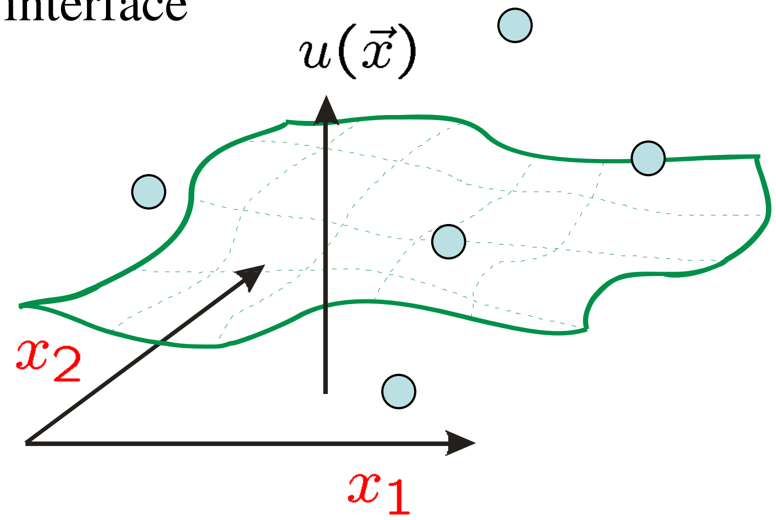
Vortices in type-II superconductor





A. A. Abrikosov, 1957

interface



Hamiltonian

$$\mathcal{H} = \int d^d x \left[\frac{c}{2}(\nabla u(x))^2 + V(x,u)\right]$$

Random potential:

$$\overline{V(x,u)V(x',u')} = R(u-u')\delta^d(x-x')$$

Roughness exponent:

$$C(x-x') = \overline{(u(x)-u(x'))^2} \sim |x-x'|^{2\zeta}$$

Overdamped dynamics:

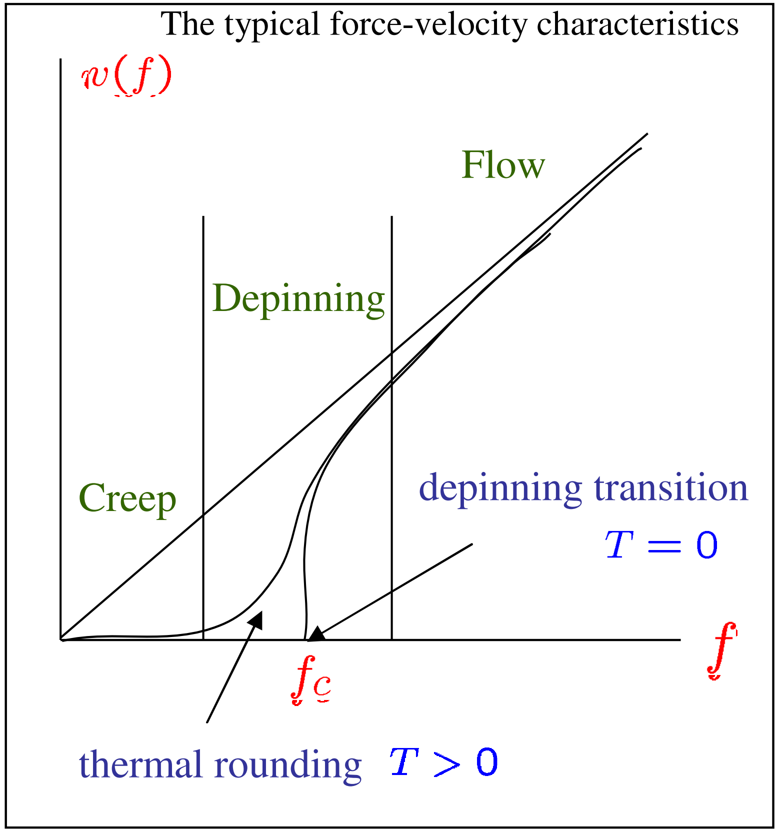
friction elastic force pinning force thermal noise driving force

$$\eta\partial_t u_{xt} = c\nabla^2 u_{xt} + F(x,u_{xt}) + \xi(x,t) + f$$

Pinning force correlator ($\Delta(u) = -R''(u)$) : $\overline{F(x,u)F(x',u')} = \Delta(u-u')\delta^d(x-x')$

Thermal noise: $\langle\xi(x,t)\xi(x',t')\rangle = 2\eta T\delta^d(x-x')\delta(t-t')$

Depinning transition and creep



Depinning transition ($f > f_c$, $T = 0$)

velocity: $v \propto (f - f_c)^{\beta}$

correlation length: $\xi \propto (f - f_c)^{-\nu}$

dynamic exponent: $t \sim x^z$

$$\nu = \frac{1}{2 - \zeta_{\rm dep}}, \quad \beta = \nu(z - \zeta_{\rm dep})$$

D.S. Fisher, 1986

T. Nattermann, S. Stepanow, et al 1992

P. Chauve, P. Le Doussal, K. Wiese, 2001

Classical creep ($f \ll f_c$, $T > 0$)

velocity: $v \propto \exp\left(-\frac{U_c}{T}\left(\frac{f}{f_c}\right)^{-\mu}\right)$

$$\mu = (d + 2\zeta)/(2 - \zeta)$$

M.V. Feigelman, V.B. Geshkenbein, A.I. Larkin, V.M. Vinokur, Phys. Rev. Lett. 63, 2303 (1989)

M.P.A. Fisher, Phys. Rev. Lett. 62, 1415 (1989)

Quantum creep ($f \ll f_c$, $T = 0$, $\hbar > 0$)

G. Blatter, V.B. Geshkenbein Phys. Rev. Lett. 66, 3297 (1991)

Y. Yeshurun et al, Rev. Mod. Phys. 68, 911 (1996)

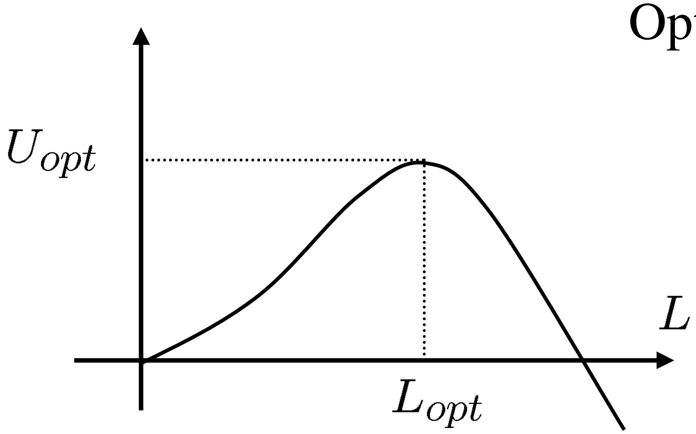
velocity: $v \propto \exp\left(-\frac{U_c}{\hbar}\left(\frac{f}{f_c}\right)^{-\mu'}\right)$???

Scaling arguments

The typical displacement of the interface $\delta u(L) \sim r_f \left(\frac{L}{L_c}\right)^{\zeta}$

The typical fluctuation of the energy $E(L) \sim U_c \left(\frac{L}{L_c}\right)^{d-2+2\zeta}$ $U_c \sim c L_c^{d-2} r_f^2$

Effective barrier $\sim E(L) - f L^d \delta u(L)$



Optimal barrier

$$L_{opt} \sim L_c \left(\frac{f}{f_c}\right)^{-\frac{1}{2-\zeta}}$$

$$U_{opt} \sim U_c \left(\frac{f}{f_c}\right)^{-\mu}$$

$$f_c \sim c r_f / L_c^2$$

$$v \propto \exp\left(-U(f)/T\right) \qquad \mu = (d+2\zeta)/(2-\zeta)$$

Functional renormlaization group approach to classical creep

P. Chauve, T. Giamarchi, P. Le Doussal, PRB 62, 6241 (2000)

Arrhenius law (typical events) vs. rare events

$$v \propto \exp\left(-U_{opt}(f)/T\right) \qquad v \propto \exp\left(-[U_{opt}(f)/T]^{\alpha}\right)$$

Quasi-classical Langevin equation approach to quantum creep

D.A. Gorokhov, D.S. Fisher, G. Blatter, PRB 66, 214203 (2002)

Dynamics:

inertia

$$\eta \partial_t u_{xt} + \rho \partial_t^2 u_{xt} = c\nabla^2 u_{xt} + F(x, u_{xt}) + \xi(x,t) + f$$

Colored noise:

$$\langle \xi(x,t)\xi(x',t')\rangle = \delta^d(x-x')\kappa(t-t')$$

The effective noise correlator :

$$\kappa(\omega) = \eta\hbar \coth\left[\frac{\hbar\omega}{2T}\right]$$

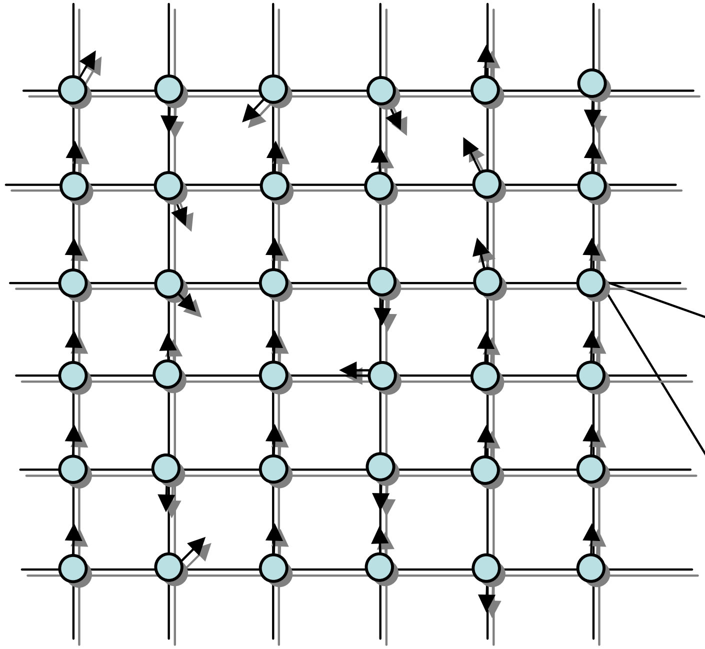
The main conclusion from FRG is the relevance of inertia and rare events

$$v \propto \exp\left(-\left[\frac{S_\rho}{\hbar}\left(\frac{f_c}{f}\right)^{(d+2\zeta-1)/(2-\zeta)}\right]^2\right)$$

Disordered quantum rotors

Model

O(N) quantum rotors on a *d*-dimensional hyper-cubic lattice



$\hat{\mathbf{n}}_i$ - orientation of the rotor on site *i* ($\hat{\mathbf{n}}_i^2 = 1$)

$\hat{L}_{i\mu\nu} = \hat{n}_{i\mu}\hat{p}_{i\nu} - \hat{n}_{i\nu}\hat{p}_{i\mu}$ - the angular momentum operator

$$[\hat{n}_{i\mu}, \hat{p}_{j\nu}] = i\hbar\delta_{ij}\delta_{\mu\nu}$$

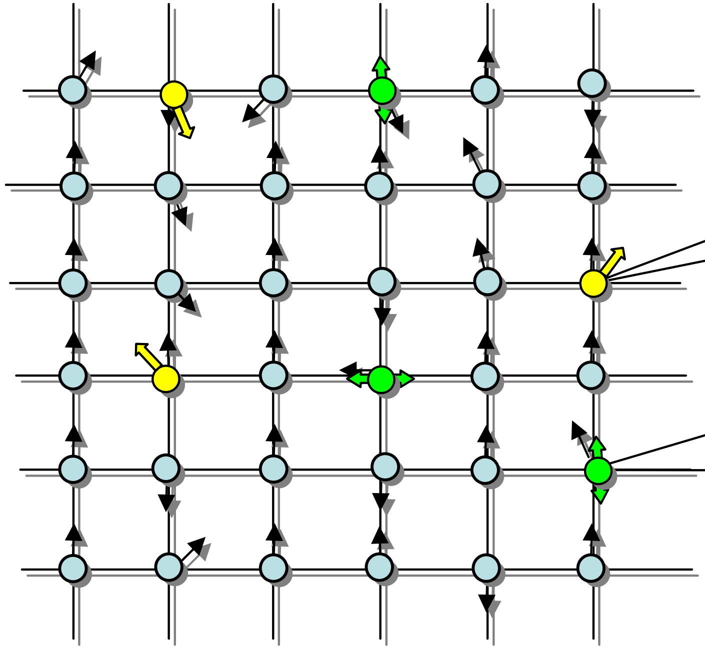
$\frac{1}{2I}\hat{\mathbf{L}}_i^2$ - kinetic energy

Hamiltonian of interacting quantum rotors

$$\mathcal{H}_0 = \frac{1}{2I}\sum_i \hat{\mathbf{L}}_i^2 - J\sum_{\langle i,j\rangle} \hat{\mathbf{n}}_i\hat{\mathbf{n}}_j$$

Model

O(N) quantum rotors with random fields and/or random anisotropy



Quenched random field $\mathbf{h}_i$

linear coupling $-\mathbf{h}_i \cdot \hat{\mathbf{n}}_i$

Quenched random anisotropy $\mathbf{d}_i$

quadratic coupling $-(\mathbf{d}_i \cdot \hat{\mathbf{n}}_i)^2$

Hamiltonian of interacting quantum rotors

$$\mathcal{H} = \frac{1}{2I} \sum_i \hat{\mathbf{L}}_i^2 - J \sum_{\langle i,j \rangle} \hat{\mathbf{n}}_i \hat{\mathbf{n}}_j - \sum_i \mathbf{h}_i \cdot \hat{\mathbf{n}}_i - \sum_i (\mathbf{d}_i \cdot \hat{\mathbf{n}}_i)^2$$

$O(N)$ quantum-mechanical nonlinear sigma model

S. Chakravarty, B. I. Halperin, D. R. Nelson, PRB 39, 2344 (1989)

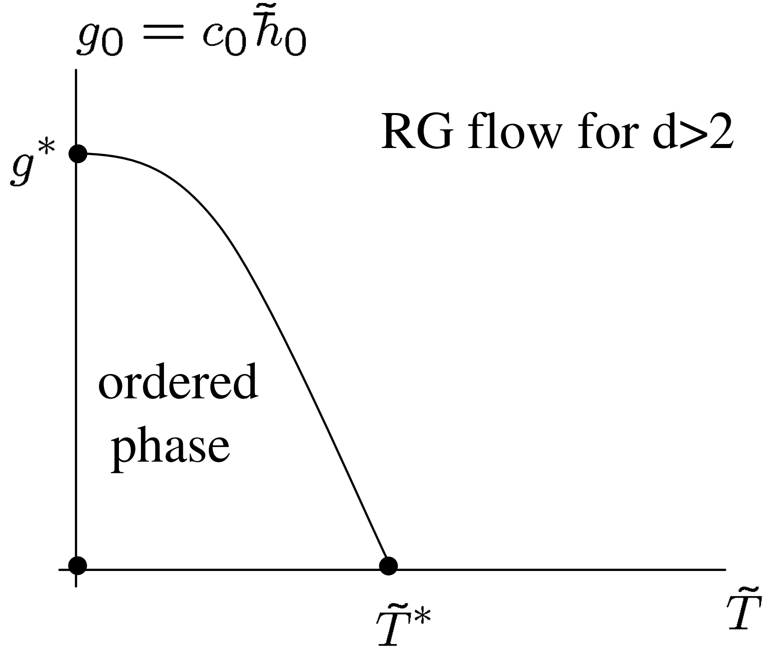
The imaginary time action $$\mathcal{Z} = \int \mathcal{D}\mathbf{n}\, \delta(|\mathbf{n}| - 1) e^{-\mathcal{S}[\mathbf{n}]/\hbar}$$

The imaginary time action

$$\mathcal{S}\,[\mathbf{n}] = \frac{\rho_0}{2} \int_0^{\hbar/T} d\tau \int d^d x \left[\frac{1}{c_0^2} (\partial_\tau \mathbf{n}(\tau, x))^2 + (\nabla \mathbf{n}(\tau, x))^2 \right]$$

$\rho_0 = b^{2-d} J$ - bare spin-stiffness

$c_0 = b\sqrt{J/I}$ - bare spin-wave velocity



$g^* = 2(d-1)/(N-2)$

$\tilde{T}^* = (d-2)/(N-2)$

The imaginary time action for the disordered system

$$\mathcal{S}\left[\mathbf{n}\right] = \frac{\rho_0}{2}\int_{\tau,x}\left[\frac{1}{c_0^2}\left(\partial_\tau \mathbf{n}(\tau,x)\right)^2 + \left(\nabla \mathbf{n}(\tau,x)\right)^2\right] - \int_{\tau,x}\sum_{\mu=1}^{\infty}\sum_{i_1\ldots i_\mu} h^{(\mu)}_{i_1\ldots i_\mu}(x) n_{i_1}(\tau,x)\ldots n_{i_\mu}(\tau,x)$$

μ rank anisotropies with zero mean and correlators

D. S. Fisher, PRB 31, 7233 (1985)

$$\overline{h^{(\mu)}_{i_1\ldots i_\mu}(x) h^{(\nu)}_{j_1\ldots j_\nu}(x')} = \delta^{\mu\nu}\delta_{i_1 j_1}\ldots\delta_{i_\mu j_\nu} r^{(\mu)}\delta(x-x')$$

The replicated action ($R(z) = \sum_\mu r^{(\mu)} z^\mu$)

$$\mathcal{S}_n\left[\{\mathbf{n}\}\right] = \frac{\rho_0}{2}\sum_{a=1}^{n}\int_{\tau,x}\left[\frac{1}{c_0^2}\left(\partial_\tau \mathbf{n}_a(\tau,x)\right)^2 + \left(\nabla \mathbf{n}_a(\tau,x)\right)^2\right] - \frac{1}{2\hbar}\sum_{a,b=1}^{n}\int_{\tau,\tau' x} R(\mathbf{n}_a(\tau,x)\cdot\mathbf{n}_b(\tau',x))$$

For d<4 there is no true long-range order

All $r^{(\mu)}$ are relevant operators

We need functional renormalization group!

Functional renormalization group

A. Andreanov, AAF, arXiv:1403.5529

The reduced renormalized quantities:

$$\tilde{R}_\ell(\phi) = K_d R_\ell(\cos\phi)\rho_\ell^{-2}\Lambda_\ell^{d-4} \qquad \tilde{T}_\ell = K_d T_\ell \rho_\ell^{-1}\Lambda_\ell^{d-2} \qquad \tilde{\hbar}_\ell = K_d \hbar \rho_\ell^{-1}\Lambda_\ell^{d-1}$$

random field $2\pi-$periodic, random anisotropy $\pi-$periodic

The flow equations ($\varepsilon = 4 - d$):

$$\partial_\ell \tilde{R}_\ell(\phi) = \varepsilon \tilde{R}_\ell(\phi) + \tilde{R}''_\ell(\phi)[\Gamma_\ell - \tilde{R}''_\ell(0)] + \frac{1}{2}[\tilde{R}''_\ell(\phi)]^2 + (N-2)\left(\frac{\tilde{R}'_\ell(\phi)^2}{2\sin^2\phi} + \left[\frac{\tilde{R}'_\ell(\phi)}{\tan\phi} + 2\tilde{R}_\ell(\phi)\right][\Gamma_\ell - \tilde{R}''_\ell(0)]\right)$$

$$\partial_\ell \ln \tilde{T}_\ell = 2 - d - (N-2)\tilde{R}''_\ell(0)$$

$$\partial_\ell \ln \tilde{\hbar}_\ell = 1 - d - (N-2)\tilde{R}''_\ell(0)$$

$$\partial_\ell \ln c_\ell = -\frac{1}{6}\left[(N+1)\tilde{R}^{(4)}_\ell(0) + (N-2)\tilde{R}''_\ell(0)\right]$$

The boundary layer width (the strengh of thermal and quantum fluctuation)

$$\Gamma_\ell = \frac{1}{2}c_\ell \tilde{\hbar}_\ell \coth\left[\frac{c_\ell \tilde{\hbar}_\ell}{2\tilde{T}_\ell}\right] = \begin{cases} \tilde{T}_\ell & \text{if } \tilde{\hbar}_\ell \to 0 \\ \frac{1}{2}c_\ell \tilde{\hbar}_\ell & \text{if } \tilde{T}_\ell \to 0 \end{cases}$$

The FRG flow for d<4

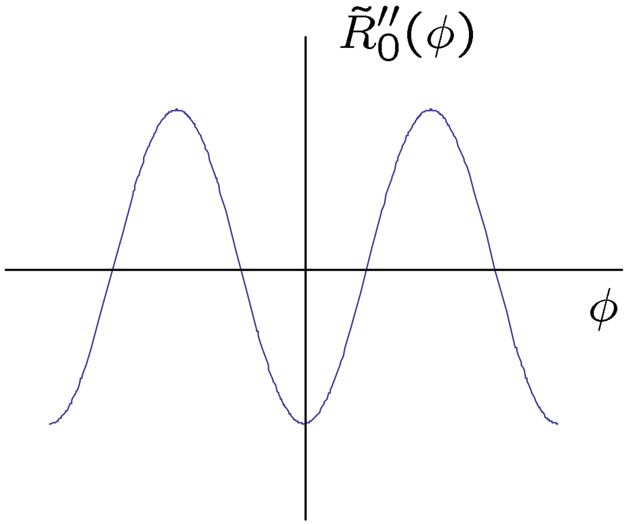
The FRG flow neglecting thermal and quantum fluctuations, i.e. for $\Gamma_\ell = 0$

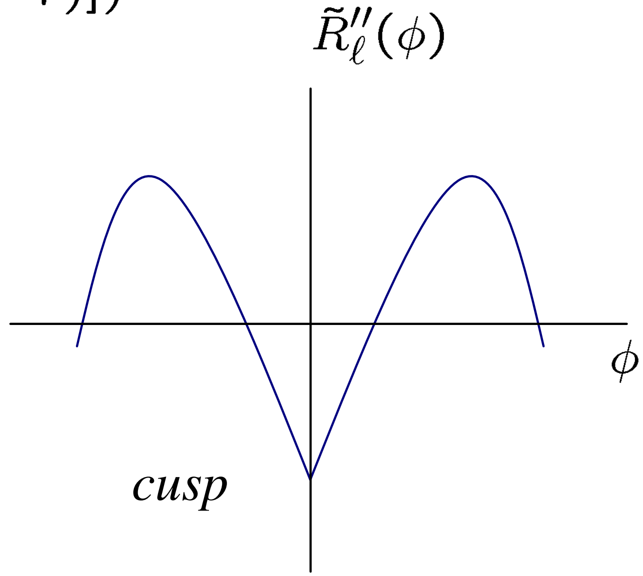
The bare disorder correlator

$$\tilde{R}_0(\phi) = \gamma \cos^2 \phi \qquad \text{Random anisotropy (RA)}$$

The running disorder correlator becomes non-analytic beyond the Larkin scale

$$\ell_c \approx \frac{1}{\varepsilon} \ln(1 + 3\varepsilon/[8\gamma(N+7)])$$





Stable fixed point for $2 \leq N < N_c$ $\qquad N_c = 2.835$ (RF) $\qquad N_c = 9.441$ (RA)

In statics:

D.E. Feldman, PRL 88, 177202 (2002)

P. Le Doussal, K.J. Wiese, PRL 96, 197202 (2006)

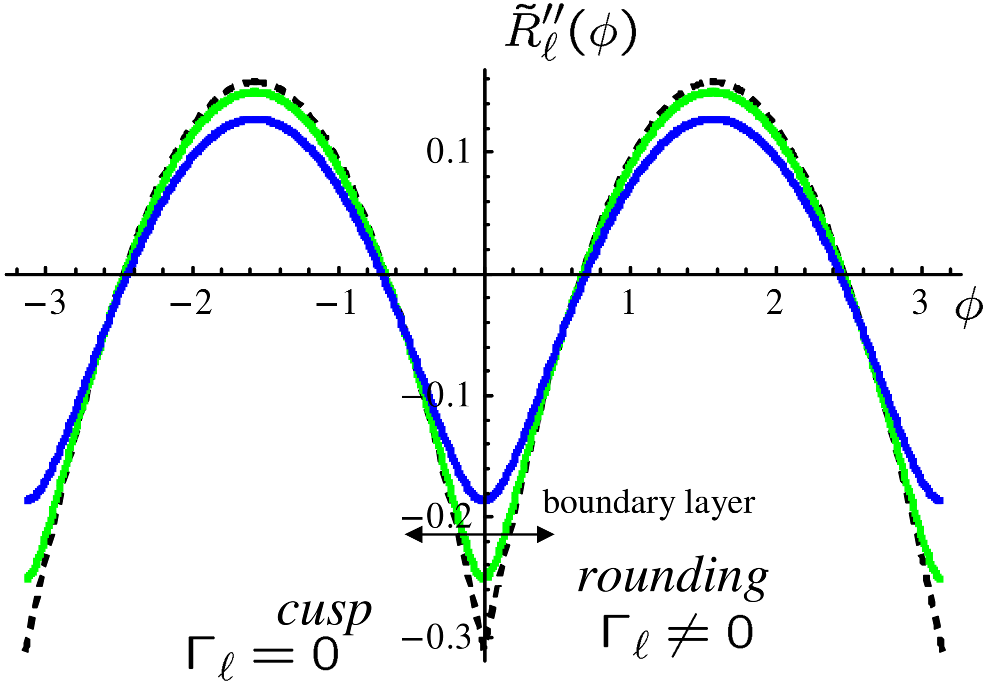
M. Tissier, G. Tarjus, PRB 74, 214419 (2006)

The thermal and quantum fluctuations flow to zero $\Gamma_\ell \to 0$

The large scale behavior is controlled by a zero temperature quasi-classical fixed point

$$\tilde{T}_\ell = \tilde{T}_0 e^{-\theta(\ell-\ell_c)} \qquad \tilde{\hbar}_\ell = \tilde{\hbar}_0 e^{-\theta_{\hbar}(\ell-\ell_c)} \qquad \theta = \theta_{\hbar} - 1 = d - 2 + (N-2)\tilde{R}^{*\prime\prime}(0)$$

Example: 3D *O(3)* RA model:



dashed line – fixed point $\tilde{R}^{*\prime\prime}(\phi)$

green line – $\Gamma_\ell = 0.05$

blue line – $\Gamma_\ell = 0.1$

$$\tilde{R}''_\ell(0) \approx \tilde{R}^{*\prime\prime}(0)$$

$$\tilde{R}^{(4)}_\ell(0) \approx 6\Omega/[\Gamma_\ell(N+1)]$$

$$\Omega = \frac{1}{2}\tilde{R}^{*\prime\prime}(0)[\tilde{R}^{*\prime\prime}(0)(N-2) - \varepsilon]$$

For d<4 the fixed point is stable and describes a quasi-long-range-ordered (QLRO) phase

The connected and disconnected correlators

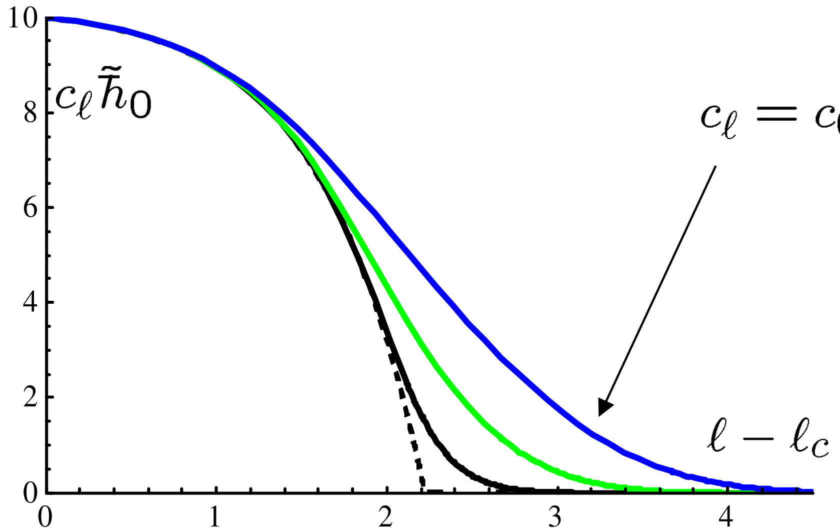
$$G_{\text{con}}(x) \sim 1/x^{d-2+\eta} \qquad \eta = -\tilde{R}^{*\prime\prime}(0)$$

$$G_{\text{dis}}(x) \sim 1/x^{d-4+\bar{\eta}} \qquad \bar{\eta} = \varepsilon - (N-1)\tilde{R}^{*\prime\prime}(0)$$

The spectrum of excitations remains gapless in the whole quantum QLRO phase

The spin wave velocity flow:

$$\partial_\ell \ln c_\ell = -\frac{\Omega}{\Gamma_l}$$



$$c_\ell = c_0 e^{-\frac{\Omega}{\tilde{T}_0 \theta}[e^{\theta(\ell-\ell_c)}-1]}$$

$$L = \Lambda_0^{-1} e^\ell$$

The localization length at zero temperature

$$\ell_{\text{loc}} - \ell_c = \frac{1}{\theta_\hbar} \ln\left[1 + \frac{c_0 \tilde{\hbar}_0 \theta_\hbar}{2\Omega}\right]$$

The residual quantum tunneling

We assumed that Γ_ℓ is determined exclusively by the low frequency part of the spectrum!

The generalized effective action

$$\mathcal{S}_n^{(0)} = \frac{\rho_0}{2} \sum_{a=1}^{n} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^d q}{(2\pi)^d} \left[D(\omega) + q^2\right] |\mathbf{n}_a(\omega, q)|^2$$

The flow of the spectrum

$$\partial_\ell \tilde{D}_\ell(\omega) = 2\tilde{D}_\ell(\omega) + \frac{2\Omega}{\Gamma_\ell} \frac{\tilde{D}_\ell(\omega)}{1 + \tilde{D}_\ell(\omega)} \qquad \tilde{D}_\ell(\omega) = \Lambda_\ell^{-2} D_\ell(\omega)$$

The zero-temperature boundary layer width is

$$\Gamma_\ell = \frac{\tilde{\hbar}_\ell}{\Lambda_\ell} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{1 + \tilde{D}_\ell(\omega)}$$

Contribution to the spin-wave velocity due to renormalization of the high frequency part of the spectrum

$$c(L) \sim \exp\left[-\frac{1+\theta}{2\theta} \left(\frac{L}{L_{\mathrm{loc}}}\right)^{2(\theta+1)}\right]$$

Activated critical dynamics in spin systems with random fields

The critical temperature is a function of the random field strength Δ and vanishes at the quantum critical point $T_c(\Delta^*) = 0$

The relaxation time at the classical critical point $\tau \sim e^{C\xi^\theta/T}$ ~~$\tau \sim \xi^z$~~

phenomenological RG ⟶ $\nu(d-\theta) = 2-\alpha$ violation of hyperscaling

D. S. Fisher, PRL 56, 416 (1986)

The relaxation time at the quantum critical point $\tau \sim e^{C\xi^\Psi}$

F. Anfuso, A. Rosch, EPJ 69, 465 (2007)

Ψ is unknown

"Evidence for the activated dynamic scaling cannot be obtained from the ε*-expansion (or from other perturbative approaches such as a* $2+\varepsilon$ *-expansion). Even for the classical transition, the only available evidence comes from numerical calculations and agreement with the phenomenology seen in experiments."*

T. Senthil, PRB 57, 8375 (1998)

Order-disorder transition for d>4

For d>4 the FRG fixed point is singly unstable and describes the transition for $N > N_c$

In the classical regime $\xi \sim |T - T_c|^{-\nu}$

$$\nu = \frac{1}{|\varepsilon|}$$

At the quantum critical point $\xi \sim |\Delta - \Delta^*|^{-\nu}$

The connected and disconnected correlators at the transition

$$G_{\rm con}(x) \sim 1/x^{d-2+\eta} \qquad \eta = -\tilde{R}^{*\prime\prime}(0)$$

$$G_{\rm dis}(x) \sim 1/x^{d-4+\bar{\eta}} \qquad \bar{\eta} = \varepsilon - (N-1)\tilde{R}^{*\prime\prime}(0)$$

The relaxation time at the classical critical point $\tau \sim e^{C\xi^\theta/T}$

$$C = \Omega \rho_0 \Lambda_0^{\theta-d+2}/K_d\theta \qquad \theta = 2 + \eta - \bar{\eta}$$

The relaxation time at the quantum critical point $\tau \sim e^{C'\xi^\Psi/\hbar^2}$

$$C' = 2C^2\theta/(1+\theta) \qquad \boxed{\Psi = 2(\theta+1)}$$

Summary

The behavior of disordered quantum rotors is controlled by a quasi-classical zero temperature (*i.e.* infinite randomness) FP with an infinite dynamic critical exponent.

Below the lower critical dimension d=4 the system has a quantum QLRO phase with a power-law decay of correlations. At zero temperature the spin-wave excitations are localized at a finite length scale. For T>0 the spin-waves propagate via thermal activation over the energy barriers which diverge in the thermodynamic dynamic limit so that the system never thermally equilibrates.

Above the lower critical dimension the system of quantum rotors undergoes an order-disorder phase transition with activated dynamics which is strongly suppressed in the vicinity of the quantum critical point.

Open questions

- consistency of the boundary layer picture in higher loop orders ?
- effect of magnetic field, delocalization in analogy with depinning transition?
- Langevin dynamics, aging ?

Many-body localization

In isolated interacting quantum systems, failure of equilibration may lead to a so-called many-body localization.

D.M. Basko, I.L. Aleiner, B.L. Altshuler, Annals of Physics 321, 1126 (2006)

thermal phase

eigenstate thermalization hypothesis (ETH)

transition

many-body localized phase

ETH violation

D.A. Huse, R. Nandkishore, V. Oganesyan, A. Pal, S.L.Sondhi, PRB 88, 014206 (2013)

Many-body localized phases can exhibit *symmetry breaking* in dimensions normally forbidden and order can arise even in the absence of a bulk gap.

One-dimensional system may be described by an *infinite-randomness fixed point* with an infinite dynamic critical exponent

R. Vosk, E. Altman, PRL 110, 067204 (2013)

A. Pal, D. A. Huse, PRB 82, 174411 (2010)

Is there analog in higher dimensions?