Avalanches in Turbulent Confined Plasmas

P.H. Diamond

CMTFO, CASS, Dept. of Physics, UCSD
WCI Center for Fusion Theory, NFRI

KITP Avalanches Program; 2014

Outline:

- A very brief primer on tokamak turbulence and transport
- Avalanches in turbulent transport
- Zonal flows and the secondary pattern selection problem
- ExB staircase and avalanches

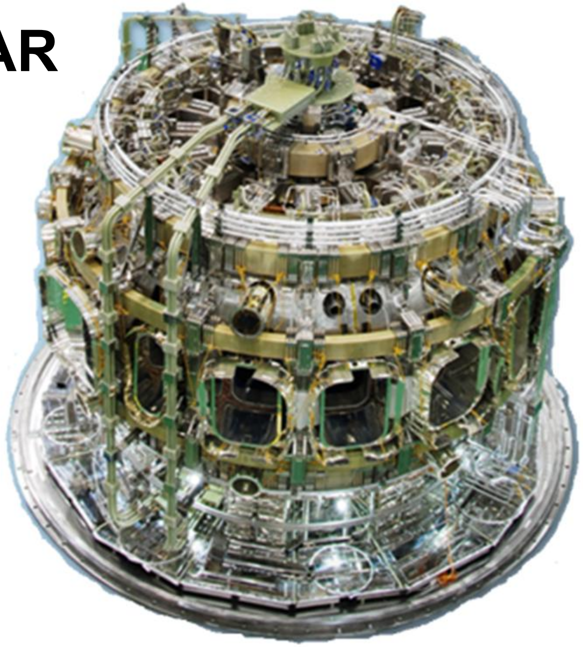
  Staircase as a heat flux jam
- Discussion

What is a Tokamak?

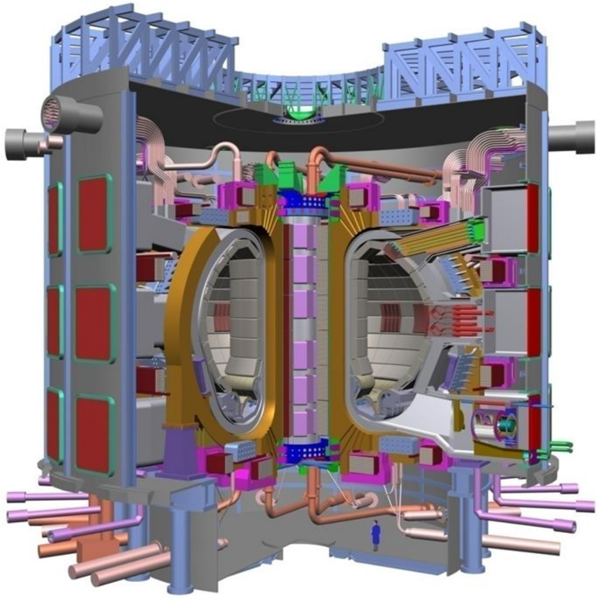
N.B. No advertising intended…

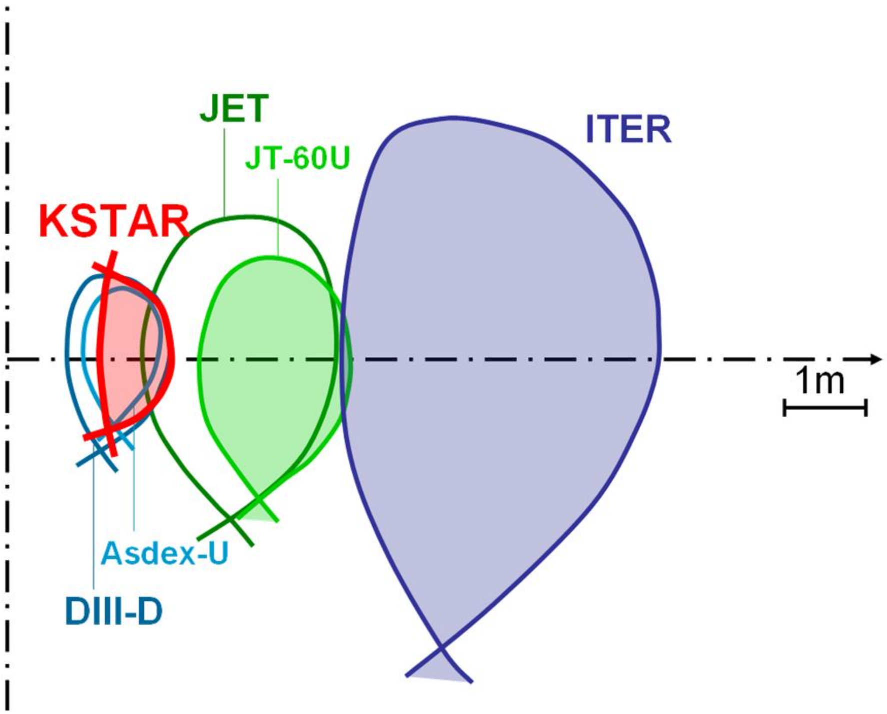
Tokamak: the most intensively studied magnetic confinement device

KSTAR



ITER





PARAMETERS	ITER	KSTAR
Major radius	6.2m	1.8m
Minor radius	2.0m	0.5m
Plasma volume	830m^3	17.8m^3
Plasma current	15MA	2.0MA
Toroidal field	5.3T	3.5T
Plasma fuel	H, D-T	H, D-D
Superconductor	Nb_3Sn, NbTi	Nb_3Sn, NbTi

Basic of Magnetic Fusion

What is required for ignition?

- Fuel: D, T
- Amount/density n
- Ignition temperature T
- Energy confinement time τ_E

- Energy content
- Confinement

Confinement time τ_E
set by turbulent transport

$$\text{Fusion power} \sim n^2T^2(\sim \beta^2 B^4) \geq \text{Loss power} \sim \frac{nT}{\tau_E}$$

$$n \cdot T \cdot \tau_E \geq 3 \times 10^{28}\ \text{m}^{-3}\text{Ks}$$

Lawson criterion for D-T fusion

⇒ Good confinement
required for ignition!

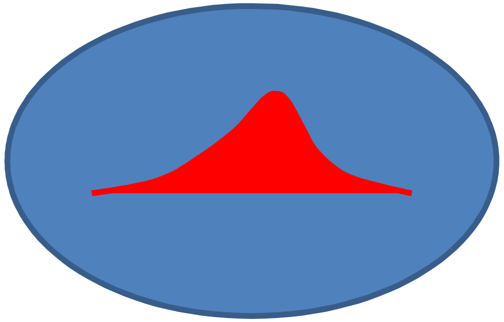
$\beta = P_{Th}/P_{B^2}$
Limited by stability

Tokamak Turbulence and Transport

→ How do plasmas form a profile?

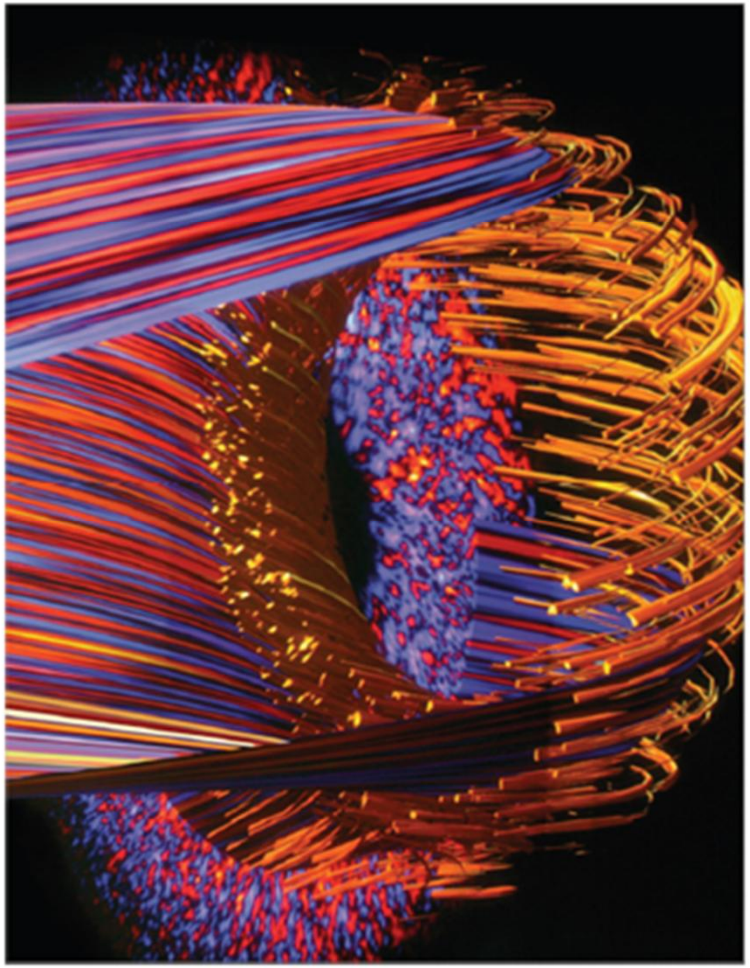
→ What limits gradients?

Primer on Turbulence in Tokamaks I

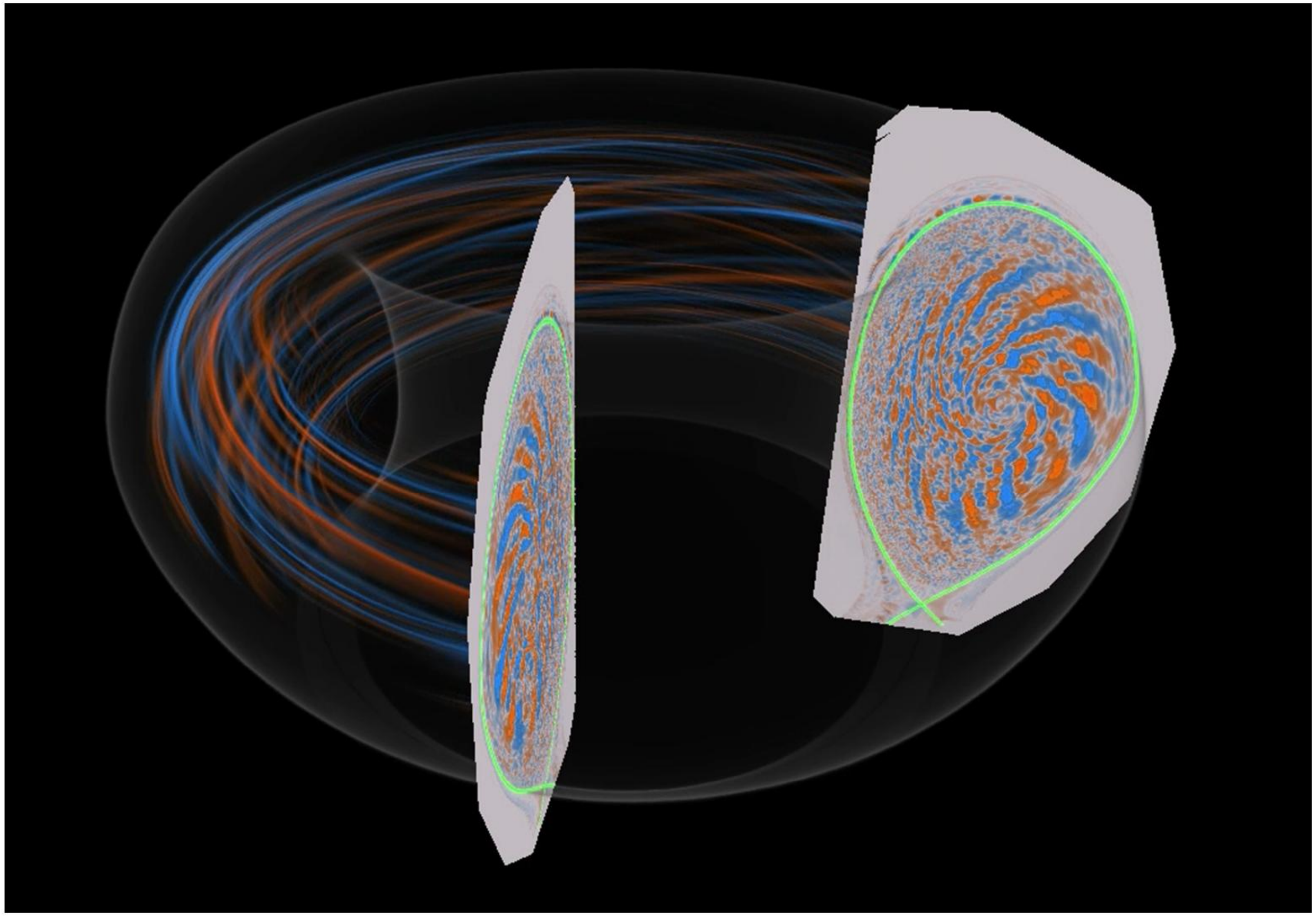


- Strongly magnetized
  - Quasi 2D cells
  - Localized by $\vec{k} \cdot \vec{B} = 0$ (resonance)
- $\vec{V}_\perp = +\frac{c}{B}\,\vec{E} \times \hat{z}$
- $\nabla T_e, \nabla T_i, \nabla n$ driven
- Akin to thermal Rossby wave, with: g → magnetic curvature
- Resembles to wave turbulence, not high Re Navier-Stokes turbulence
- Re ill defined, $K \;\leq 1$

Primer on Turbulence in Tokamaks II

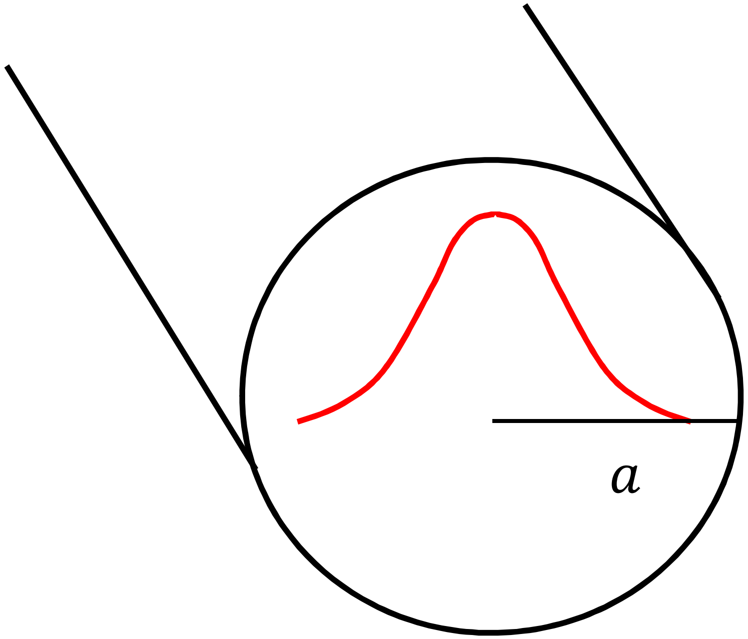


[Klasky, ORNL; Ethier, Wang, PPPL]



S. Ku et al, EPS/ICPP 2012

Primer on Turbulence in Tokamaks III



- $\nabla T, \nabla n$, etc. driver
- Quasi-2D, elongated cells aligned with B_0
- Characteristic scale ~ few ρ_i
- Characteristic velocity $v_d \sim \rho_* c_s$

2 scales:

$\rho \equiv$ gyro-radius

$a \equiv$ cross-section

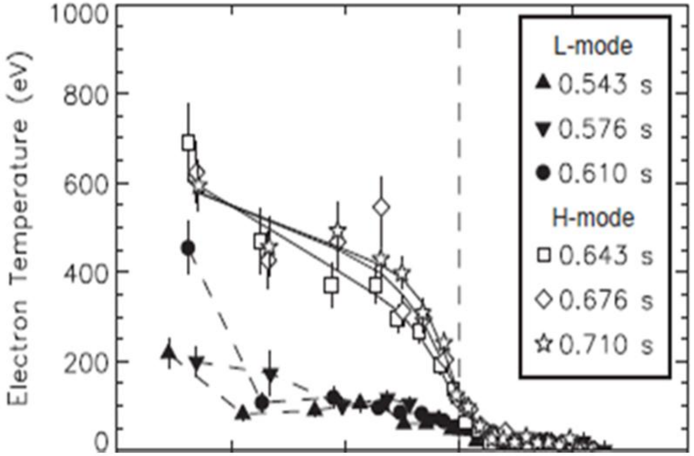
$\rho_* \equiv \rho / a$ ➔ key ratio

- Transport scaling: $D \sim \rho v_d \sim \rho_* D_B \sim D_{GB}$
- i.e. Bigger is better! ➔ sets profile scale via heat balance
- Reality: $D \sim \rho_*^{\alpha} D_B$, $\alpha < 1$ ➔ why??

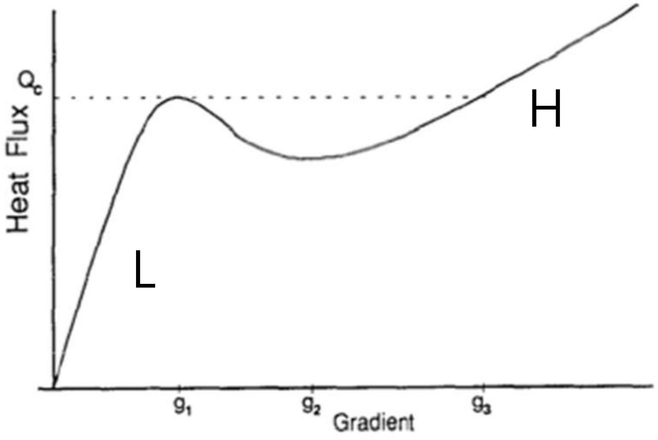
L→H Transition → Transport Barrier Formation

- A Remarkable Phenomenon: Plasma Spontaneously Self-Organizes to Improved Confinement
  - L→H Transition – jam forms at edge

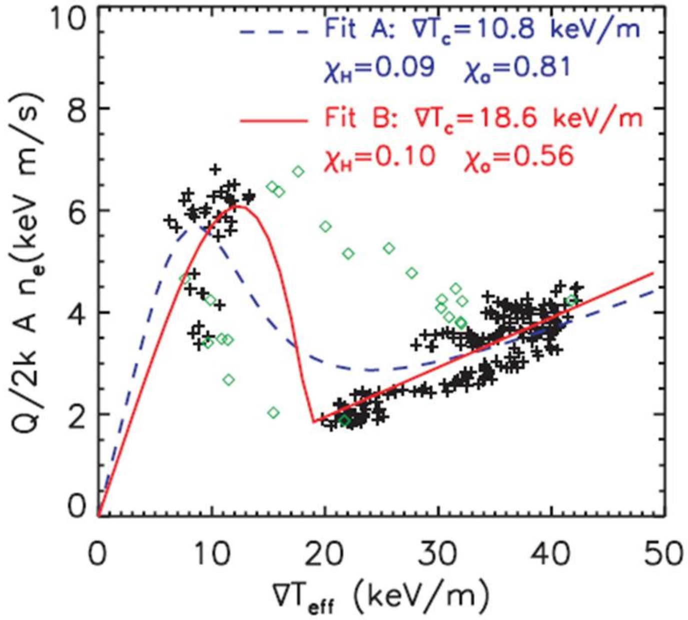
A. Hubbard et. al. 2002



J.W. Huges et al., PSFC/JA-05-35



-∇T



  - Transport bifurcation, 'phase transition' ⇒ P_{thresh}, hysteresis, etc.
  - Characterized by reduction of transport, turbulence in localized edge layer
  - Likely related to V_{ExB} shear suppression of turbulent transport in edge layer

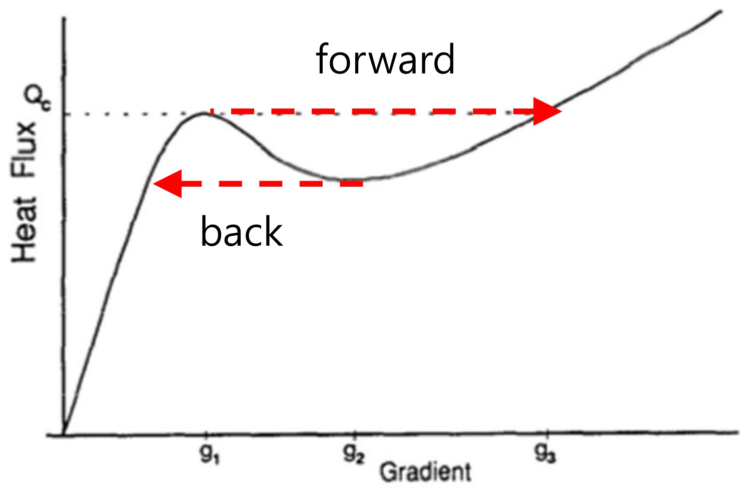
→ Coupling of Transport Bifurcation to turbulence, $\langle v_E \rangle'$ suppression

→ Non-linear Fick’s Law, extension

$$Q = -\frac{\chi_T}{1+\alpha {v_E'}^2}\nabla T - \chi_{neo}\nabla T$$

Shearing feedback

$$v_E' = -\frac{\partial}{\partial r}\left(\frac{c}{eB}\frac{\nabla p}{n_0}\right) \quad p = n_0 T$$



Heat flux S-curve induced by profile-dependent shearing feedback

Profile Bifurcation

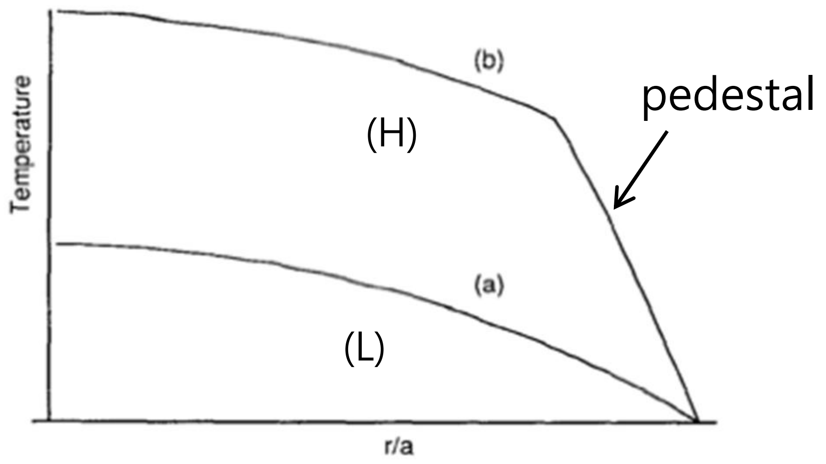


FIG. 2. Temperature profiles near the power threshold (arbitrary units): (a) $Q(a) = 0.99Q_c$; (b) $Q(a) = 1.01Q_c$

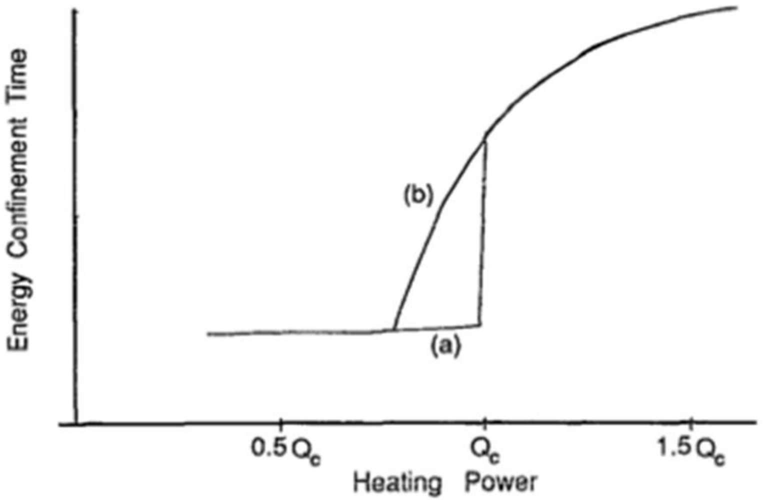
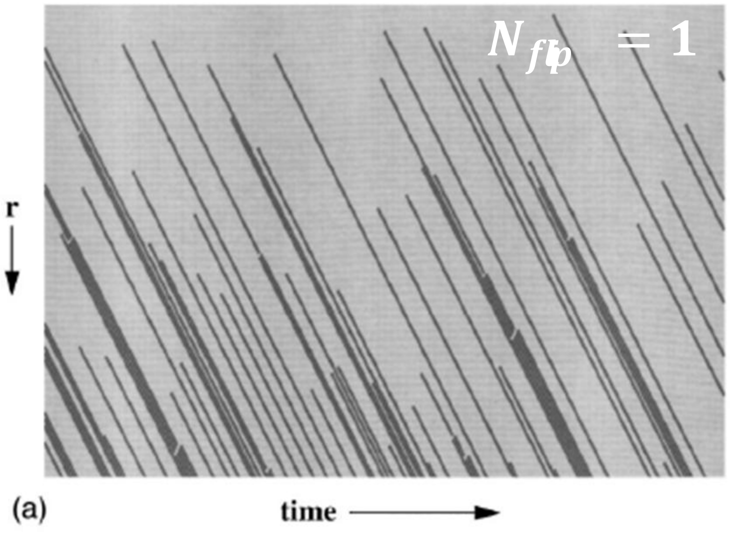


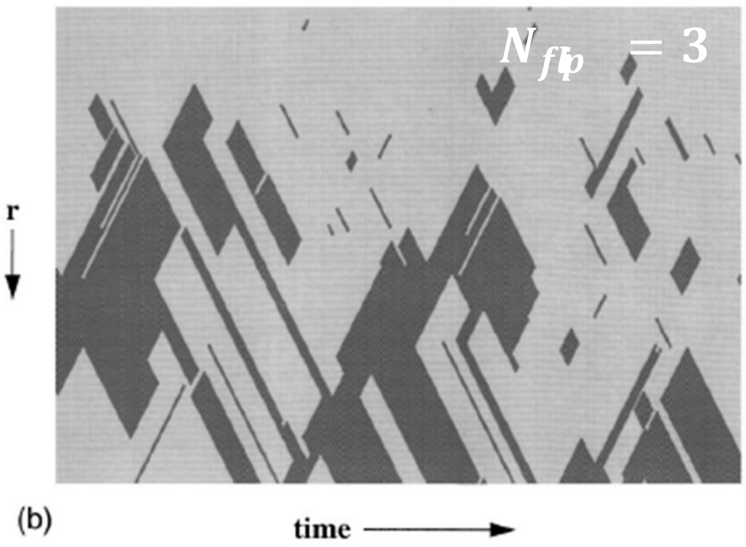
FIG. 4. Power hysteresis in the energy confinement time (arbitrary units): (a) increasing power; (b) decreasing power.

Avalanches in Turbulent Transport

Basic Phenomenology of CA Models – and Transport

- See: P.D. and Hahm, PoP'95; Newman, et al. PoP'96
- Avalanches happen:



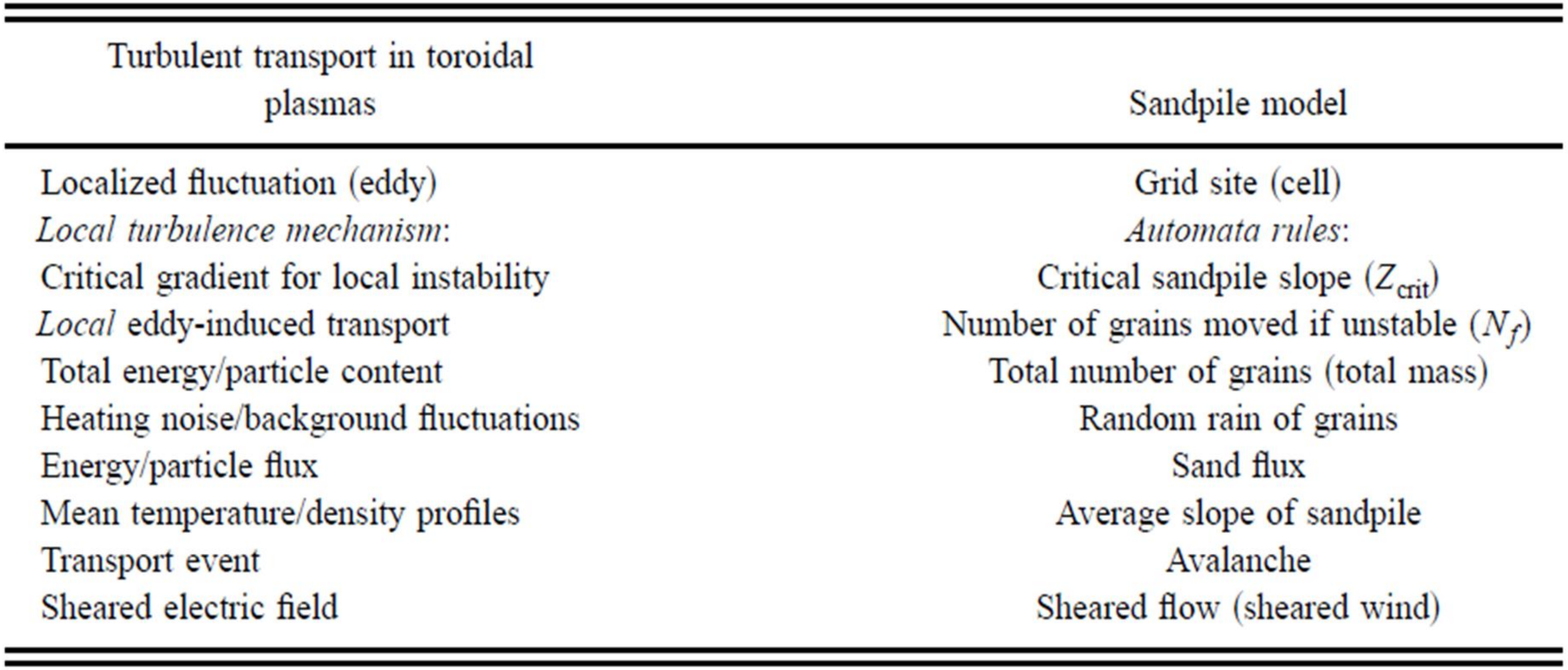


➔ broad spectrum of inward, outward propagating avalanches evident

- What is an avalanche?
  - sequence of correlated toppling or eddy over-turning events
  - akin to fall of dominos
  - typically: $\Delta_c < l_{aval} < L_p$ → meso-scale

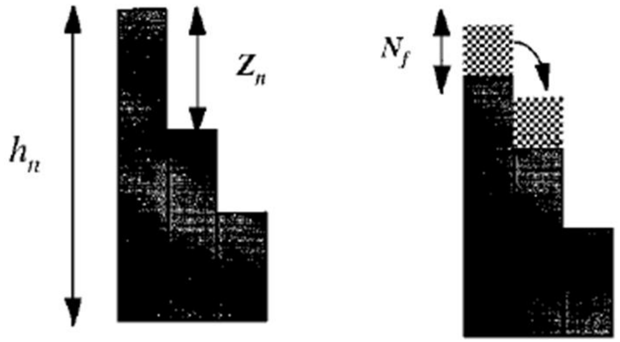
- Cells “pinned” by magnetic geometry
- Remarkable Similarity:

TABLE I. Analogies between the sandpile transport model and a turbulent transport model.



Turbulent transport in toroidal plasmas	Sandpile model
Localized fluctuation (eddy)	Grid site (cell)
Local turbulence mechanism:	*Automata rules*:
Critical gradient for local instability	Critical sandpile slope (Z_{crit})
Local eddy-induced transport	Number of grains moved if unstable (N_f)
Total energy/particle content	Total number of grains (total mass)
Heating noise/background fluctuations	Random rain of grains
Energy/particle flux	Sand flux
Mean temperature/density profiles	Average slope of sandpile
Transport event	Avalanche
Sheared electric field	Sheared flow (sheared wind)

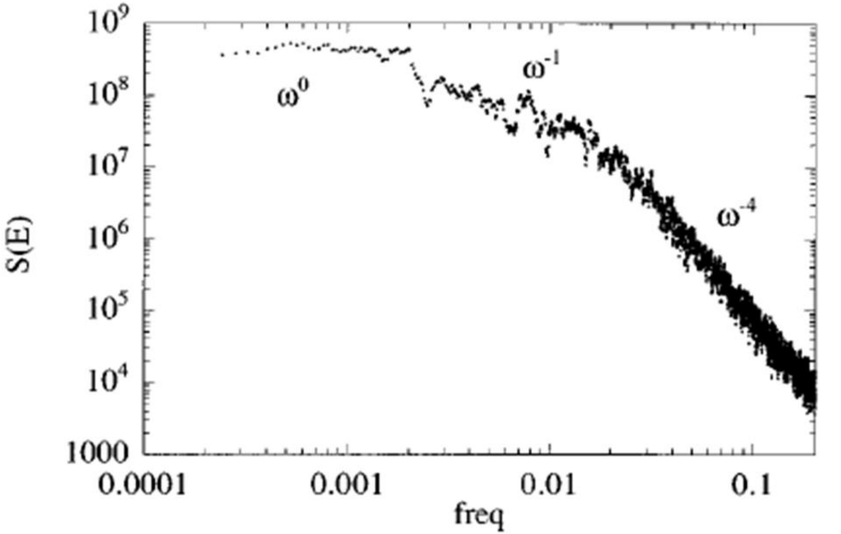
Automaton toppling
↔ Cell/eddy overturning



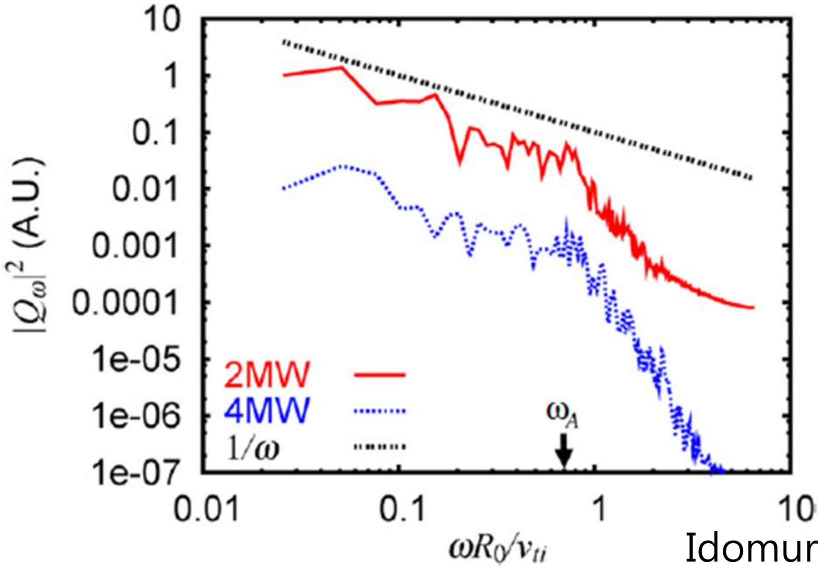
A cartoon representation of the simple cellular automata rules used to model the sandpile.

Are avalanches a consequence of the toy CA model? NO!

- Avalanches observed, studied in flux driven simulations
  - First: Carreras , et. al. PoP'96 → resistive interchanges
  - GK: GYSELA, GT5D, XGC1p …



Newman PoP96



Idomura NF09

- Comment:
  - flux tube and δf simulations and those which artificially constrain ∇P, will not capture (full) avalanche dynamics
  - avalanching not captured in quasi-linear models

Transport: Local or Non-local?

- 40 years of fusion plasma modeling
  - local, diffusive transport

$$Q = -n\chi(r)\nabla T$$

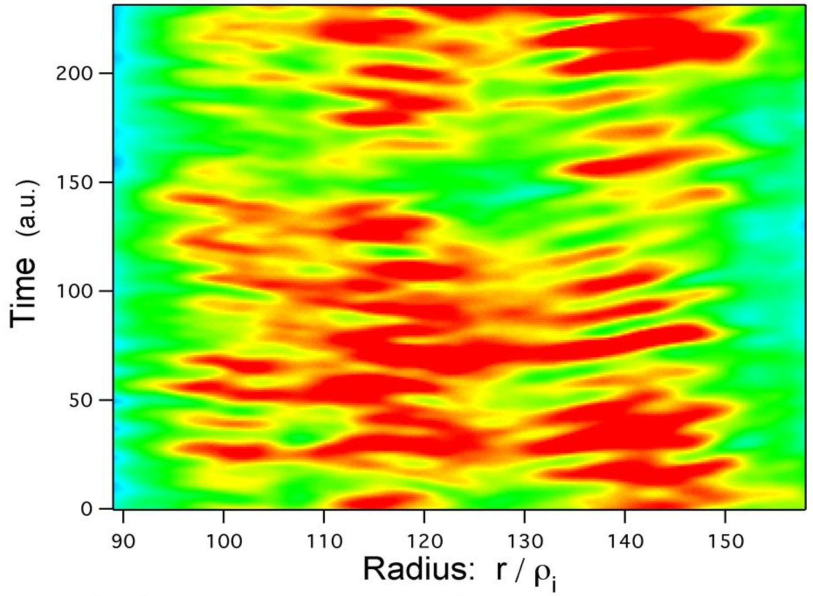
- 1995 → increasing evidence for:
  - transport by avalanches as in sand pile/SOCs
  - turbulence propagation and invasion fronts
  - non-locality of transport

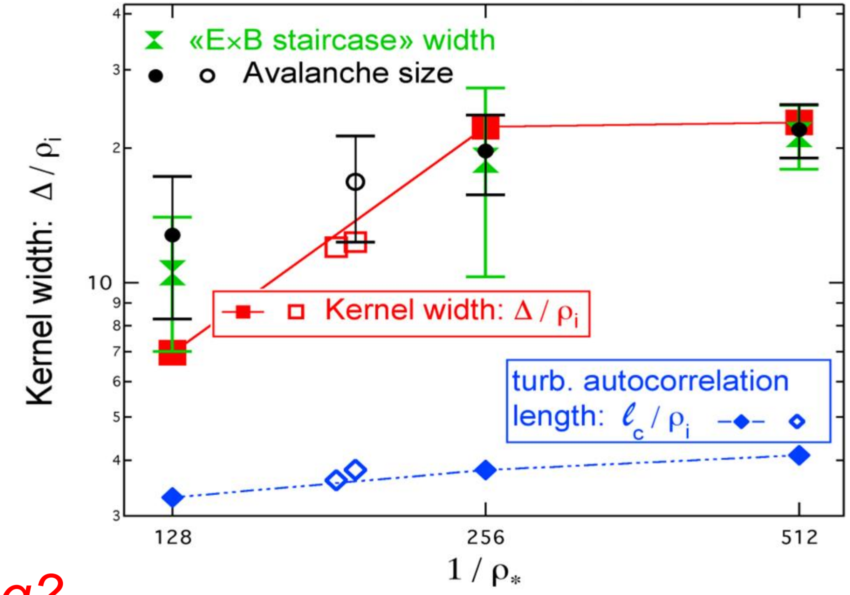
$$Q = -\int \kappa(r,r')\nabla T(r')dr'$$

- Physics:
  - Levy flights, SOC, turbulence fronts…
- Fusion:
  - gyro-Bohm breaking

    (ITER: significant ρ_* extension)

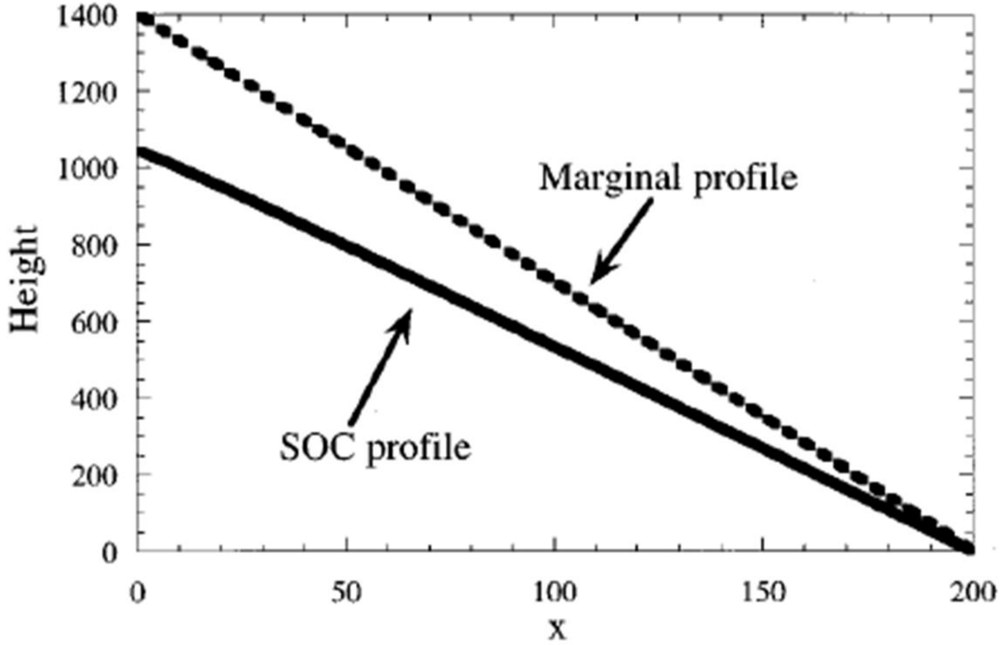
→ fundamentals of turbulent transport modeling?





Guilhem Dif-Pradalier et al. PRL 2009

What Do Profiles Look Like?



Newman PoP96

- SOC profile ≠ linearly marginal profile
- For moderate drive, SOC occupation profile < marginal profile
- N.B. Important
  - Observe SOC profile approaches marginal profile near boundary
  - Flip intensity largest near boundary → losses
  - As deposition increases, edge gradient steepens

➔ with bi-stable flux, transport bifurcation naturally initiated first, at boundary

Heat avalanche dynamics model (Continuum)

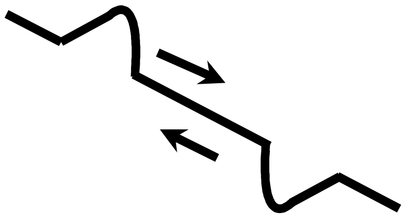
Hwa+Kardar ’92, P.D. + Hahm ’95, Carreras, et al. ’96, ... GK simulation, ... Dif-Pradalier ’10

- δT :deviation from marginal profile → conserved order parameter

- Heat Balance Eq.: $\partial_t \delta T + \partial_x Q[\delta T] = 0$ → up to source and noise

- Heat Flux $Q[\delta T]$? → utilize symmetry argument, ala’ Ginzburg-Landau

  - Usual: → joint reflectional symmetry (Hwa+Kardar’92, Diamond+Hahm ’95)



$$\delta T \leftrightarrow -\delta T$$

$$x \leftrightarrow -x$$

$$Q = Q_0(\delta T)$$

$$= \frac{\lambda}{2}\delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T$$

hyperdiffusion

lowest order → Burgers equation

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

- External Shear

No shear

shear

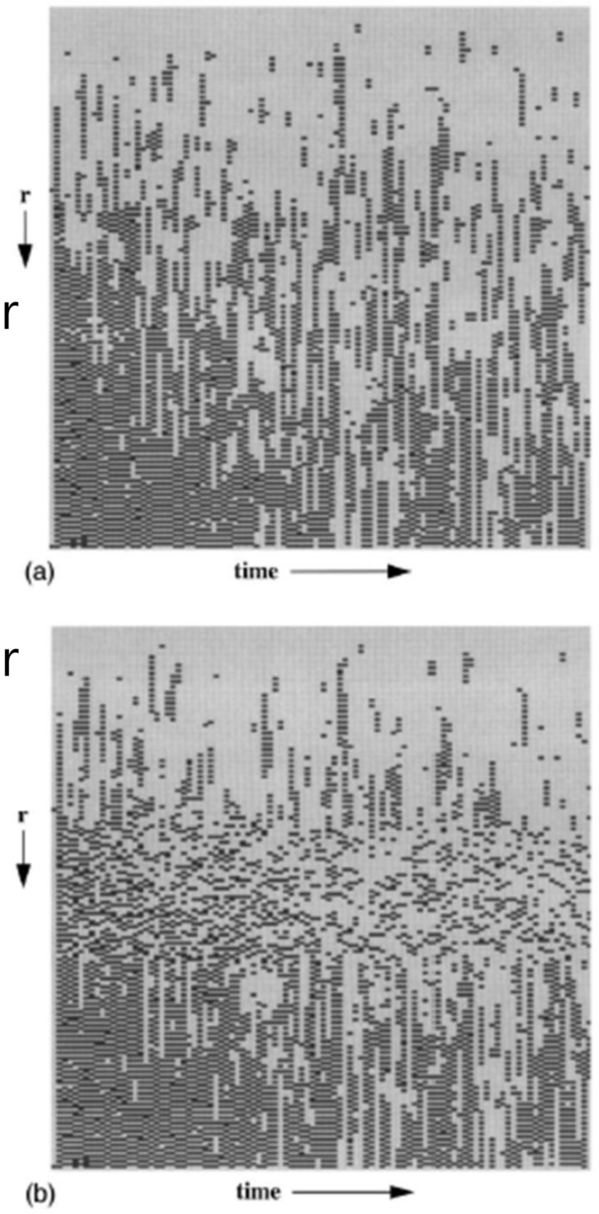


FIG. 11. Time evolution of the overturning sites (like Fig. 4). The avalanches do not appear continous in time because only every 50th time step is shown. (a) The shear-free case shows avalanches of all lengths over the entire radius. (b) The case with sheared flow shows the coherent avalanches being decorrelated in the shear zone in the middle of the pile.

How is transport suppressed?

➔ shear decorrelation!

Back to sandpile model:

2D pile +

sheared flow of

grains

Shearing flow
decorrelates
Toppling sequence

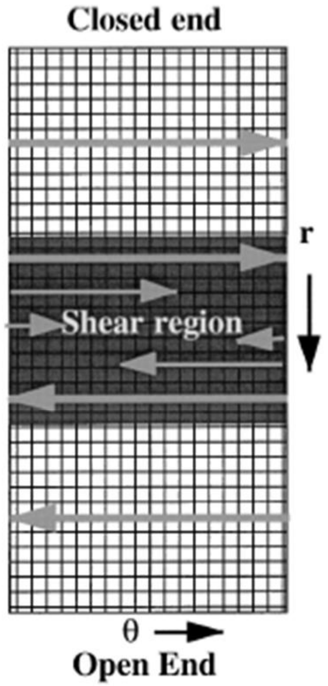


FIG. 10. A cartoon of the sandpile with a shear flow zone. The whole pile is flowing to the right at the top and to the left at the bottom connected by a variable sized region of sheared flow.

Avalanche coherence destroyed by shear flow

- Implications:

Spectrum of Avalanches

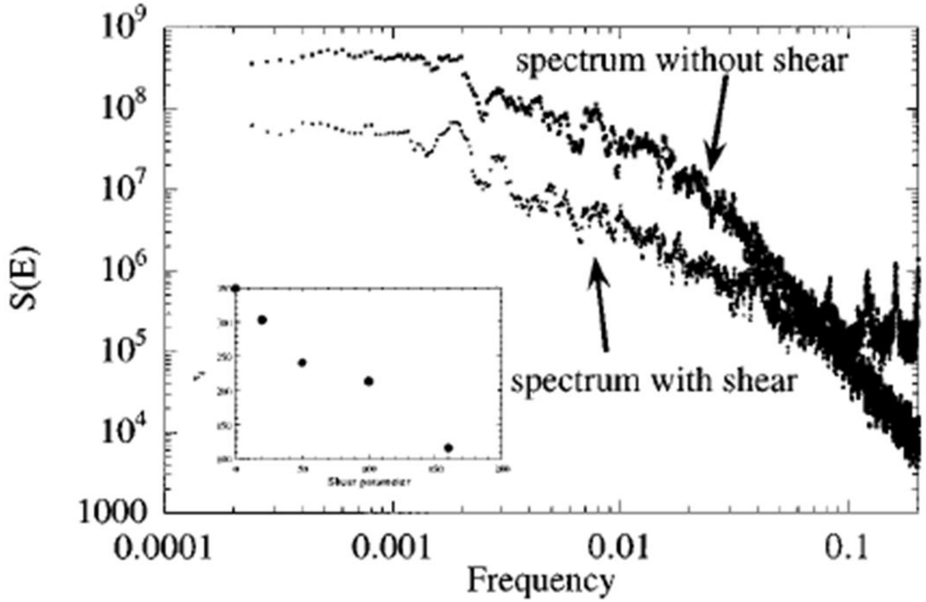
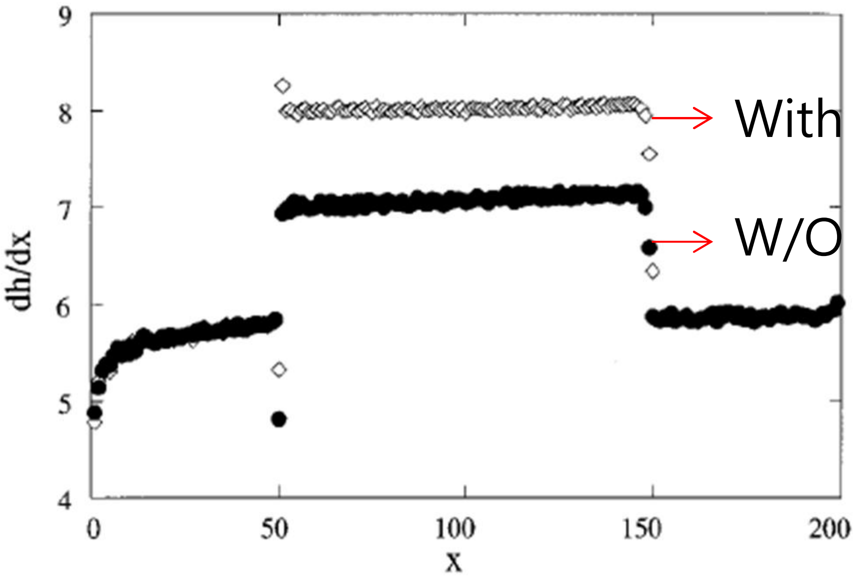


FIG. 12. (a) Frequency spectra with and without a shear flow region. This shows a marked decrease in the low-frequency power (with shear) and a commensurate increase in high-frequency power. (b) The insert shows the decorrelation time ($\tau_d = 1/\varpi$) as a function of the shear parameter (the product of the shearing rate and the size of the shear zone).



With shear

W/O shear

FIG. 14. The slopes of a sandpile with a shear region in the middle, including all the shear effects (diamonds) and just the transport decorrelation and the linear effect (circles).

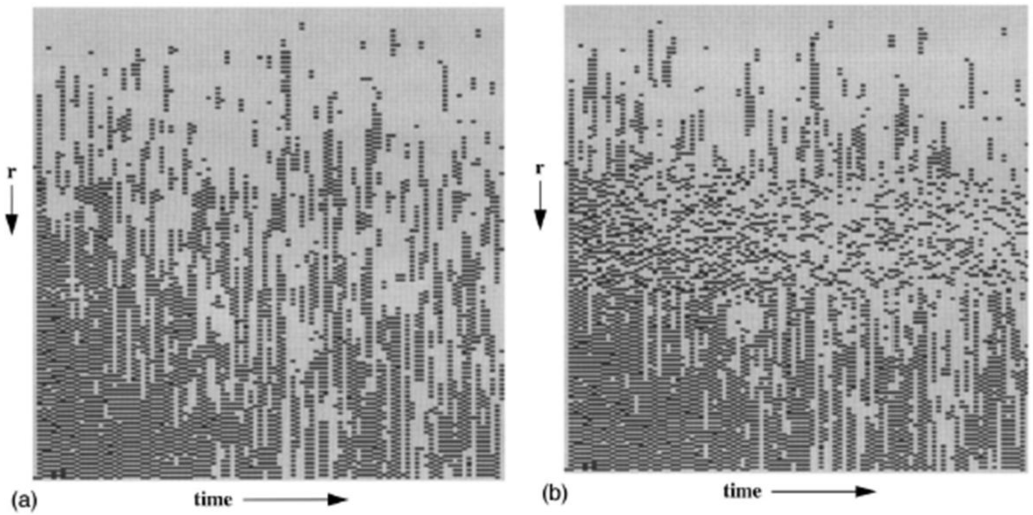


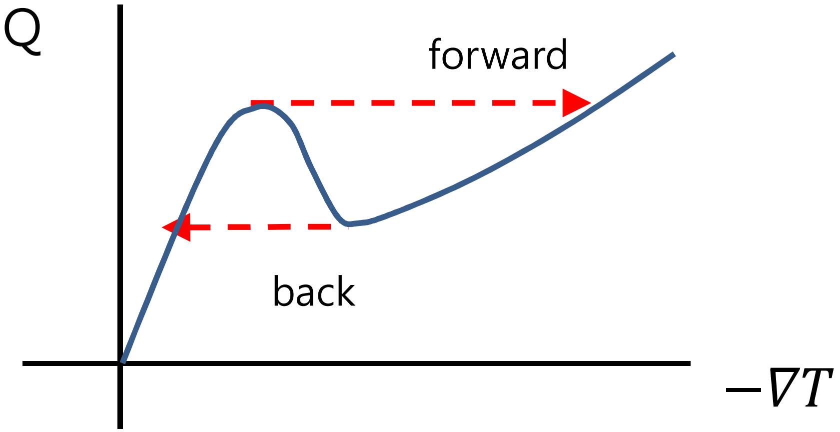
FIG. 11. Time evolution of the overturning sites (like Fig. 4). The avalanches do not appear continous in time because only every 50th time step is shown. (a) The shear-free case shows avalanches of all lengths over the entire radius. (b) The case with sheared flow shows the coherent avalanches being decorrelated in the shear zone in the middle of the pile.

Concept of a Transport Bifurcation
i.e. how generate the sheared flow??

N.B. Edge sheared flow / transport barrier ➔ L→H transition

→ First Theoretical Formulation of L→H Transition as an

- Transport Bifurcation
- $\langle E_r \rangle'$ Bifurcation
- $Q = -\frac{\chi}{1+\alpha\langle V_E\rangle'^2}\nabla T - \chi_0 \nabla T$



➔ Appearance of S-curve in a Physical Model of L→H Transition

➔ Formulation of Criticality Condition (Threshold) for Transport Bifurcation

→ Theoretical Ideas on Hysteresis, ELMs, Pedestal Width, ...

L→H Transition

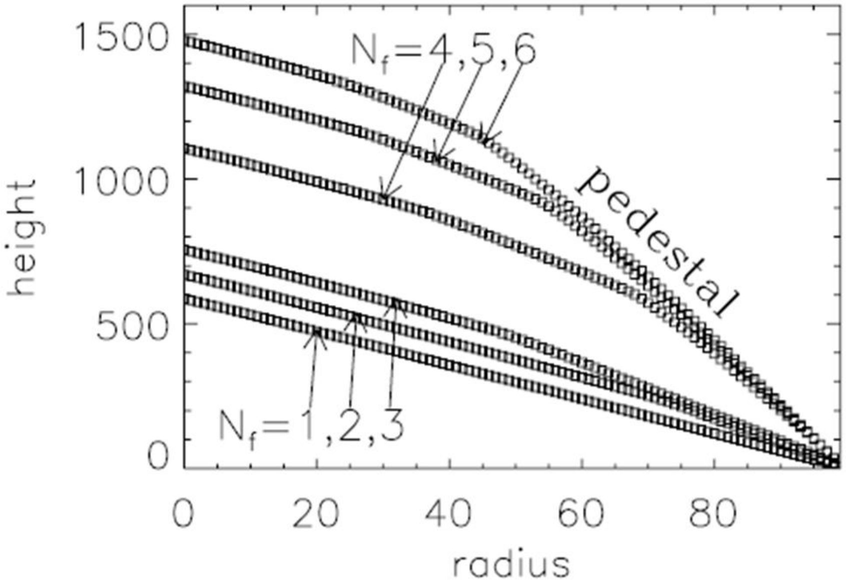
- Now try bi-stable toppling rule, i.e. if $Z_i - Z_{i+1}$ large enough

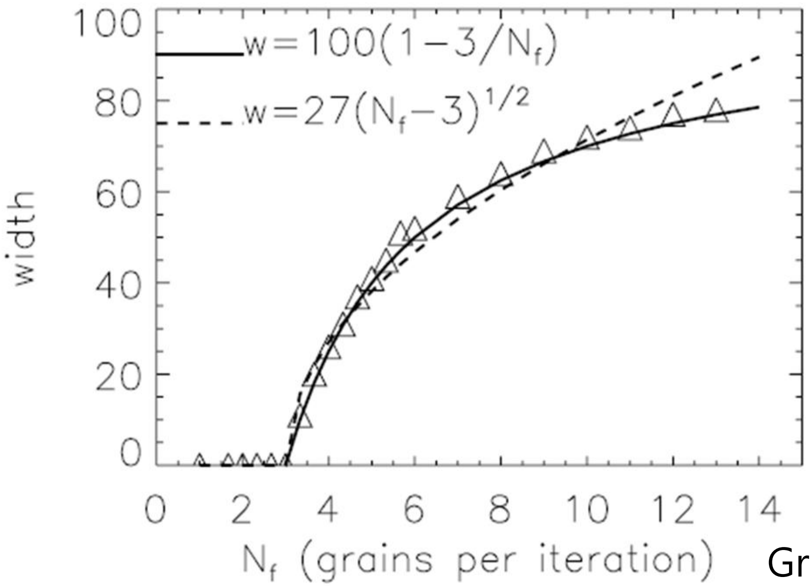
  ➔ reduced or no toppling

- Obvious motivation is $Q = -\frac{\chi \nabla P}{1+\alpha {V'}_E^2}$ and $V_E \approx \frac{c}{eB}\frac{\nabla P}{n}$

- Hard gradient limit imposed

- Transitions happen, pedestal forms!

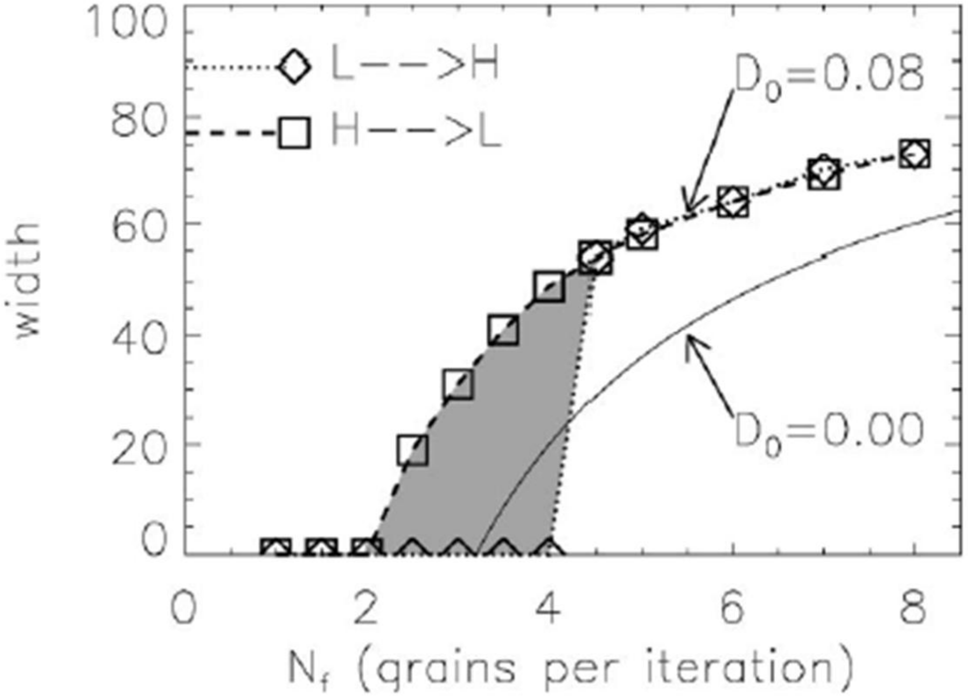




Gruzinov PRL2002

Note

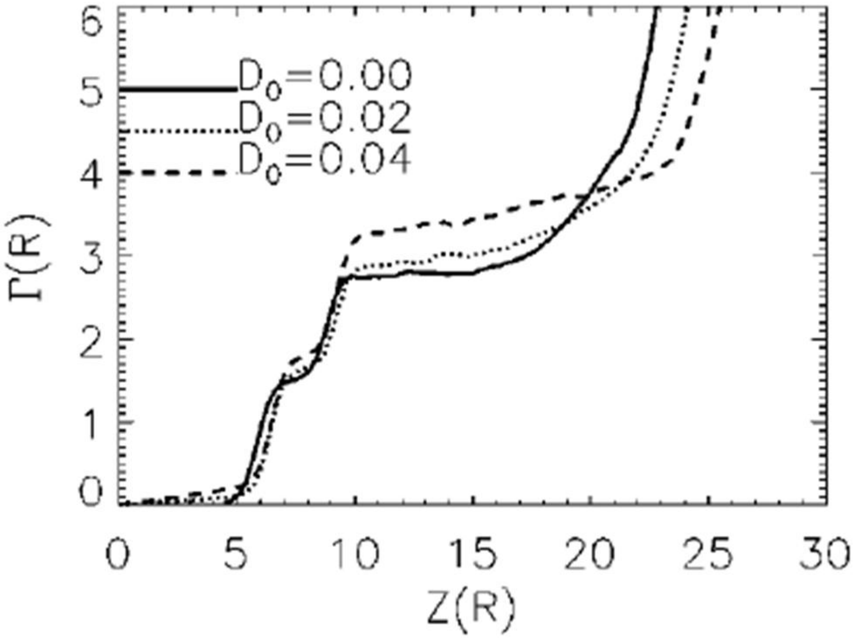
- Critical deposition level required to form pedestal (“power threshold”)
- Pedestal expands inward with increasing input after transition triggered
- Now, including ambient diffusion (i.e. neoclassical)
  - N_F threshold evident
  - Asymmetry in L→H and H→L depositions

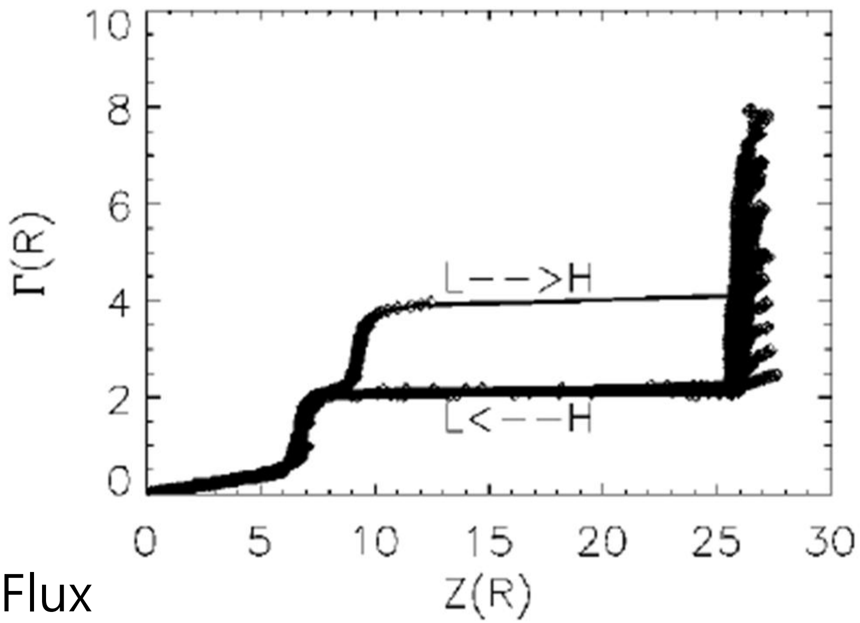


Gruzinov PoP2003

Hysteresis Happens!

- Hysteresis loop in mean flux-gradient relation appears for $D_0 \neq 0$
- Hysteresis is consequence of different transport mechanisms at work in "L" and "H" phases
- Diffusion 'smoothes' pedestal profiles, allowing filling limited ultimately by large events





$\Gamma(R)$=Flux

$Z(R)$=Mean Slope

Gruzinov PoP2003

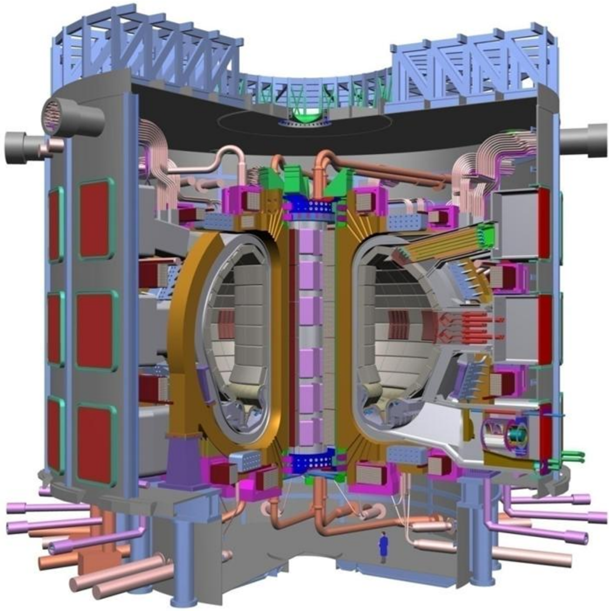
Zonal Flows and
the Secondary Pattern
Selection Problem

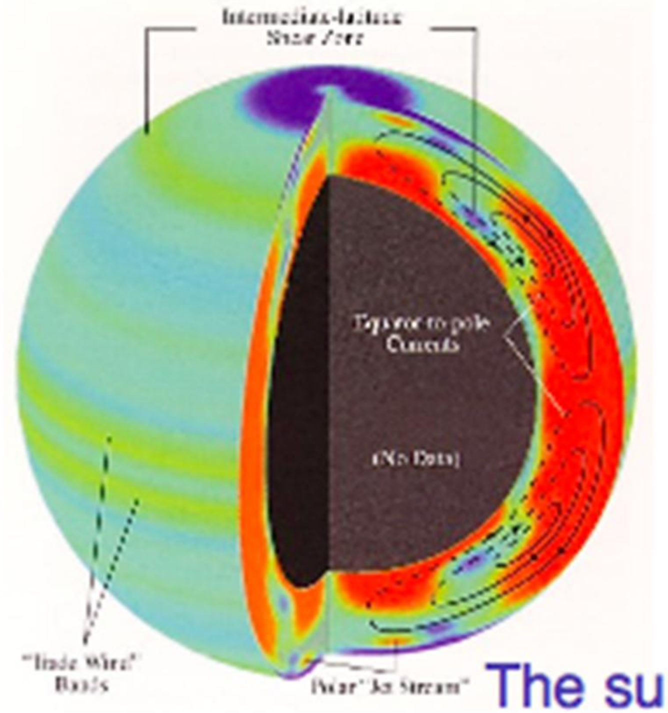
Preamble I

- Zonal Flows Ubiquitous for:
  - ~ 2D fluids / plasmas $R_0 < 1$

    Rotation $\vec{\Omega}$, Magnetization $\vec{B}_0$, Stratification

  Ex: MFE devices, giant planets, stars…





The sun

Preamble II

- What is a Zonal Flow?
  - $n = 0$ potential mode; $m = 0$ (ZFZF), with possible sideband (GAM)
  - toroidally, poloidally symmetric *ExB* shear flow
- Why are Z.F.'s important?
  - Zonal flows are secondary (nonlinearly driven):
    - modes of minimal inertia (Hasegawa et. al.; Sagdeev, et. al. '78)
    - modes of minimal damping (Rosenbluth, Hinton '98)
    - drive zero transport ($n = 0$)
  - natural predators to feed off and retain energy released by gradient-driven microturbulence

Zonal Flows I

- Fundamental Idea:
  - Potential vorticity transport + 1 direction of translation symmetry → Zonal flow in magnetized plasma / QG fluid
  - Kelvin's theorem is ultimate foundation

- G.C. ambipolarity breaking → polarization charge flux → Reynolds force
  - Polarization charge ➡ $-\boxed{\rho^2}\nabla^2\phi = \boxed{n_{i,GC}(\phi) - n_e(\phi)}$

    polarization length scale *ion GC* *electron density*

  - so $\Gamma_{i,GC} \neq \Gamma_e$ ➡ $\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi} \right\rangle \neq 0$ ⟷ 'PV transport'

    polarization flux → What sets cross-phase?

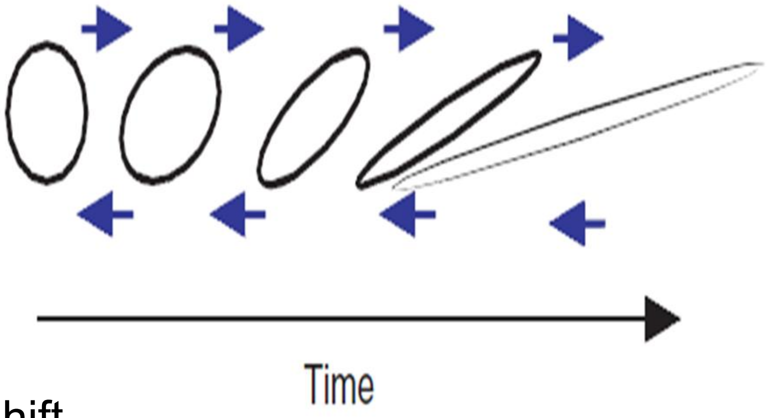
  - If 1 direction of symmetry (or near symmetry):

$$-\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi} \right\rangle = -\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E} \right\rangle$$ (Taylor, 1915)

$-\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E} \right\rangle$ ➡ Reynolds force ➡ Flow

Shearing I

- Coherent shearing: (Kelvin, G.I. Taylor, Dupree’66, BDT‘90)
  - radial scattering + $\langle V_E \rangle' \rightarrow$ hybrid decorrelation
  - $k_r^2 D_\perp \rightarrow (k_\theta^2 \langle V_E \rangle'^2 D_\perp / 3)^{1/3} = 1/\tau_c$
  - shaping, flux compression: Hahm, Burrell ’94



Response shift
and dispersion

- Other shearing effects (linear):
  - spatial resonance dispersion: $\omega - k_\| v_\| \Rightarrow \omega - k_\| v_\| - k_\theta \langle V_E \rangle' (r - r_0)$
  - differential response rotation → especially for kinetic curvature effects

→ N.B. Caveat: Modes can adjust to weaken effect of external shear

(Carreras, et. al. ‘92; Scott ‘92)

Shearing II

- Zonal Shears: Wave kinetics (Zakharov et. al.; P.D. et. al. '98, et. seq.)

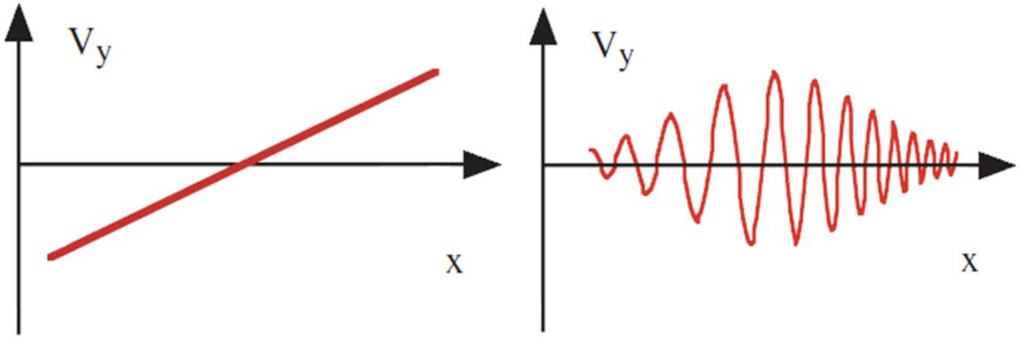
  Coherent interaction approach (L. Chen et. al.)

- $dk_r / dt = -\partial(\omega + k_\theta V_E) / \partial r \, ; \; V_E = \langle V_E \rangle + \widetilde{V}_E$

  Mean shearing $: k_r = k_r^{(0)} - k_\theta V_E' \tau$

  Zonal Random shearing $: \left\langle \delta k_r^2 \right\rangle = D_k \tau$

  $$D_k = \sum_q k_\theta^2 \left| \widetilde{V}_{E,q}' \right|^2 \tau_{k,q}$$



- Wave ray chaos (not shear RPA) underlies $D_k \rightarrow$ induced diffusion
- Induces wave packet dispersion
- Applicable to ZFs and GAMs

- Mean Field Wave Kinetics

$$\frac{\partial N}{\partial t} + (\vec{V}_{gr} + \vec{V}) \cdot \nabla N - \frac{\partial}{\partial r}(\omega + k_\theta V_E) \cdot \frac{\partial N}{\partial \vec{k}} = \gamma_{\vec{k}} N - C\{N\}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial t}\langle N \rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r} \langle N \rangle = \gamma_{\vec{k}} \langle N \rangle - \langle C\{N\} \rangle}$$

Zonal shearing

Shearing III

- Energetics: Books Balance for Reynolds Stress-Driven Flows!
- Fluctuation Energy Evolution – Z.F. shearing

$$\int d\vec{k}\,\omega\left(\frac{\partial}{\partial t}\langle N\rangle-\frac{\partial}{\partial k_r}D_k\frac{\partial}{\partial k_r}\langle N\rangle\right)\Rightarrow\frac{\partial}{\partial t}\langle\varepsilon\rangle=-\int d\vec{k}\,V_{gr}(\vec{k})D_{\vec{k}}\frac{\partial}{\partial k_r}\langle N\rangle \qquad V_{gr}=\frac{-2k_rk_\theta V_*\rho_s^2}{\left(1+k_\perp^2\rho_s^2\right)^2}$$

Point: For $d\langle\Omega\rangle/dk_r<0$, Z.F. shearing damps wave energy

- Fate of the Energy: Reynolds work on Zonal Flow

Modulational Instability

$$\partial_t\delta V_\theta+\partial\left(\delta\left\langle\tilde{V}_r\tilde{V}_\theta\right\rangle\right)/\partial r=-\gamma\delta V_\theta$$

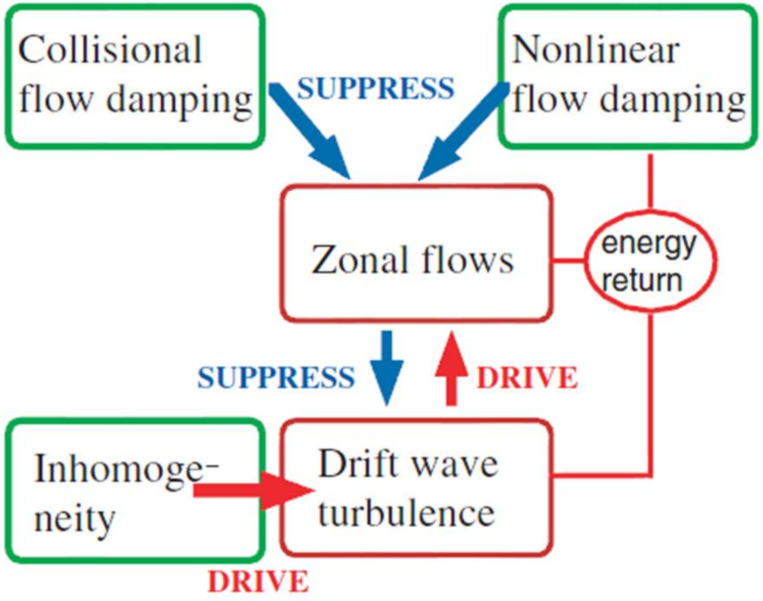
$$\delta\left\langle\tilde{V}_r\tilde{V}_\theta\right\rangle\sim\frac{k_rk_\theta\delta\Omega}{(1+k_\perp^2\rho_s^2)^2}$$

N.B.: Wave decorrelation essential:
Equivalent to PV transport
(c.f. Gurcan et. al. 2010)

- Bottom Line:
  - Z.F. growth due to shearing of waves
  - "Reynolds work" and "flow shearing" as relabeling → books balance
  - Z.F. damping emerges as critical; MNR '97

Feedback Loops I

- Closing the loop of shearing and Reynolds work


Collisional flow damping

SUPPRESS

Nonlinear flow damping

Zonal flows

energy return

SUPPRESS

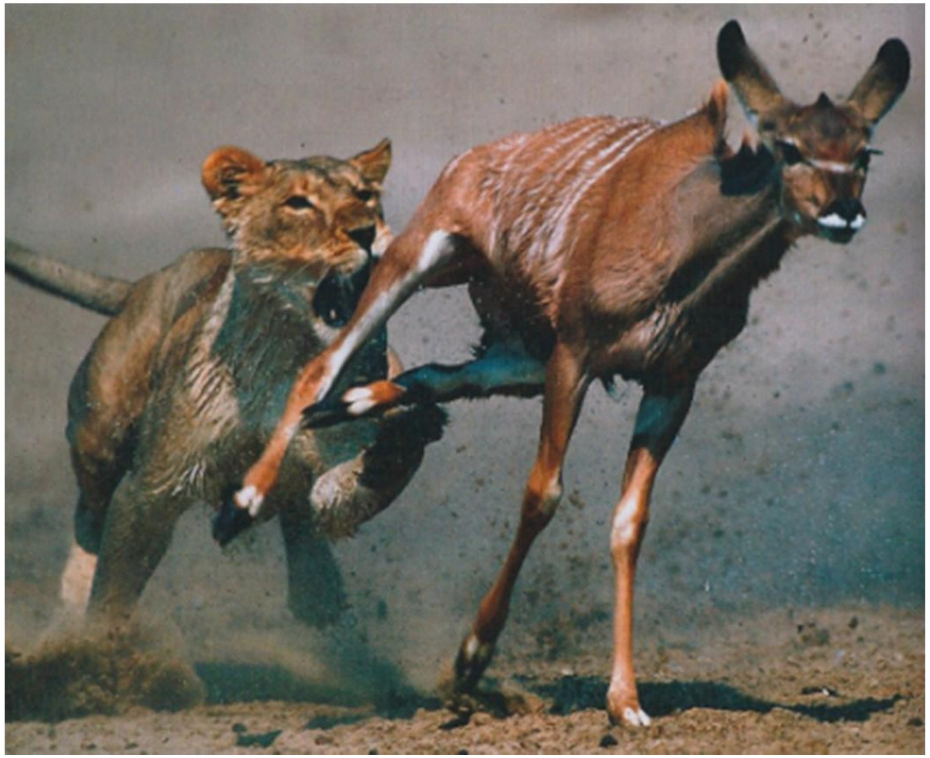
DRIVE

Inhomogeneity

Drift wave turbulence

DRIVE

- Spectral ‘Predator-Prey’ equations



Prey → Drift waves, <*N*>

$$\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle = \gamma_k\langle N\rangle - \frac{\Delta\omega_k}{N_0}\langle N\rangle^2$$

Predator → Zonal flow, $|\phi_q|^2$

$$\frac{\partial}{\partial t}|\phi_q|^2 = \Gamma_q\left[\frac{\partial\langle N\rangle}{\partial k_r}\right]|\phi_q|^2 - \gamma_d|\phi_q|^2 - \gamma_{NL}[|\phi_q|^2]|\phi_q|^2$$

Feedback Loops II

- Recovering the ‘dual cascade’:
  - Prey → <N> ~ <Ω> ⇒ induced diffusion to high k_r ⇒ Analogous → forward potential enstrophy cascade; PV transport
  - Predator → $|\phi_q|^2 \sim \langle V_{E,\theta}^2 \rangle$
    - ⇒ growth of *n=0, m=0* Z.F. by turbulent Reynolds work
    - ⇒ Analogous → inverse energy cascade

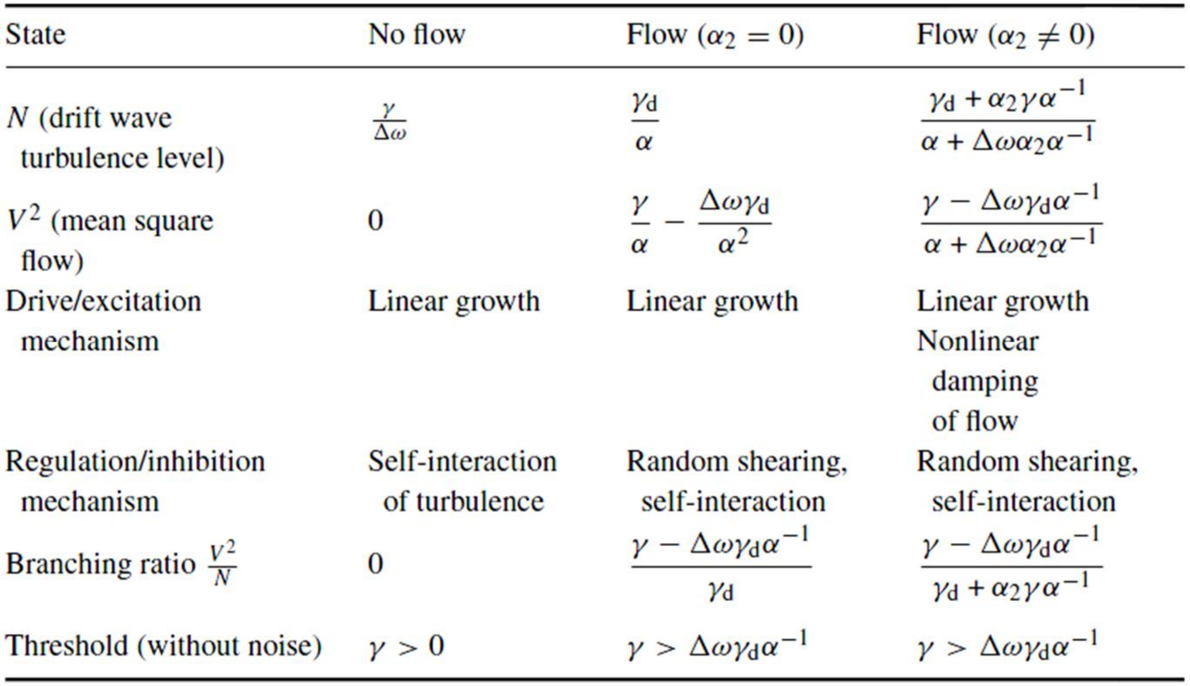
- Mean Field Predator-Prey Model

(P.D. et. al. ’94, DI^{2}H ‘05)

$$\frac{\partial}{\partial t} N = \gamma N - \alpha V^2 N - \Delta\omega N^2$$

$$\frac{\partial}{\partial t} V^2 = \alpha N V^2 - \gamma_d V^2 - \gamma_{NL}(V^2) V^2$$

System Status



State	No flow	Flow ($\alpha_2 = 0$)	Flow ($\alpha_2 \neq 0$)
N (drift wave turbulence level)	$\frac{\gamma}{\Delta\omega}$	$\frac{\gamma_\mathrm{d}}{\alpha}$	$\frac{\gamma_\mathrm{d} + \alpha_2\gamma\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$
V^2 (mean square flow)	0	$\frac{\gamma}{\alpha} - \frac{\Delta\omega\gamma_\mathrm{d}}{\alpha^2}$	$\frac{\gamma - \Delta\omega\gamma_\mathrm{d}\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$
Drive/excitation mechanism	Linear growth	Linear growth	Linear growth Nonlinear damping of flow
Regulation/inhibition mechanism	Self-interaction of turbulence	Random shearing, self-interaction	Random shearing, self-interaction
Branching ratio $\frac{V^2}{N}$	0	$\frac{\gamma - \Delta\omega\gamma_\mathrm{d}\alpha^{-1}}{\gamma_\mathrm{d}}$	$\frac{\gamma - \Delta\omega\gamma_\mathrm{d}\alpha^{-1}}{\gamma_\mathrm{d} + \alpha_2\gamma\alpha^{-1}}$
Threshold (without noise)	$\gamma > 0$	$\gamma > \Delta\omega\gamma_\mathrm{d}\alpha^{-1}$	$\gamma > \Delta\omega\gamma_\mathrm{d}\alpha^{-1}$

A Central Question: Secondary Pattern Selection

- Two secondary structures suggested
  - Zonal flow → quasi-coherent, regulates transport via shearing
  - Avalanche → stochastic, induces extended transport events
- Nature of co-existence?

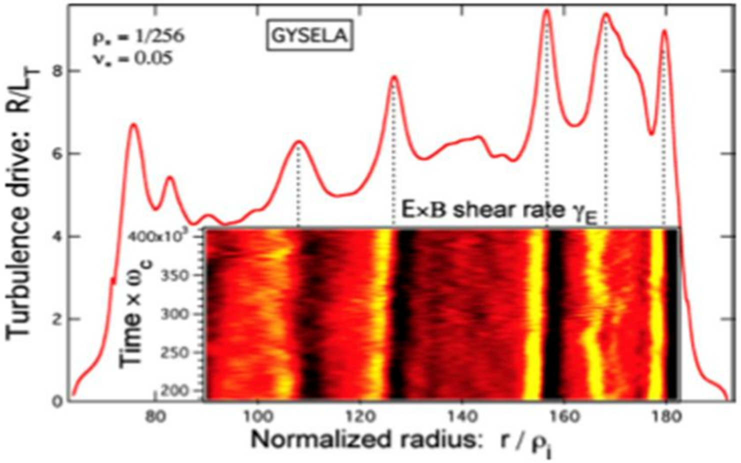
Staircases and Traffic Jams

Single Barrier → Lattice of Shear Layers

→ Jam Patterns

Highlights

Observation of ExB staircases



→ Failure of conventional theory

(emergence of particular scale???)

Model extension from Burgers to telegraph

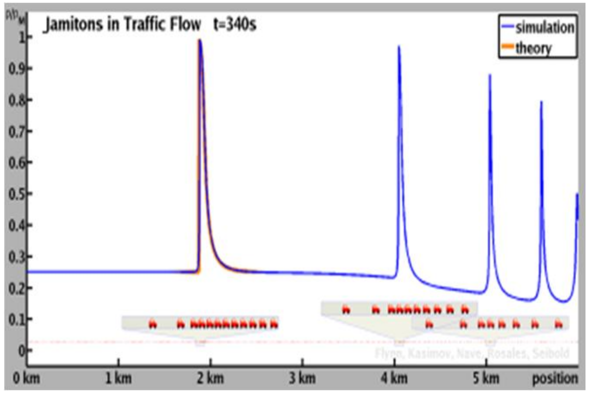
$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

$$\Rightarrow \boxed{\tau \partial_t^2 \delta T} + \partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

finite response time → like drivers' response time in traffic

Analysis of telegraph eqn. predicts heat flux jam

- scale of jam comparable to staircase step

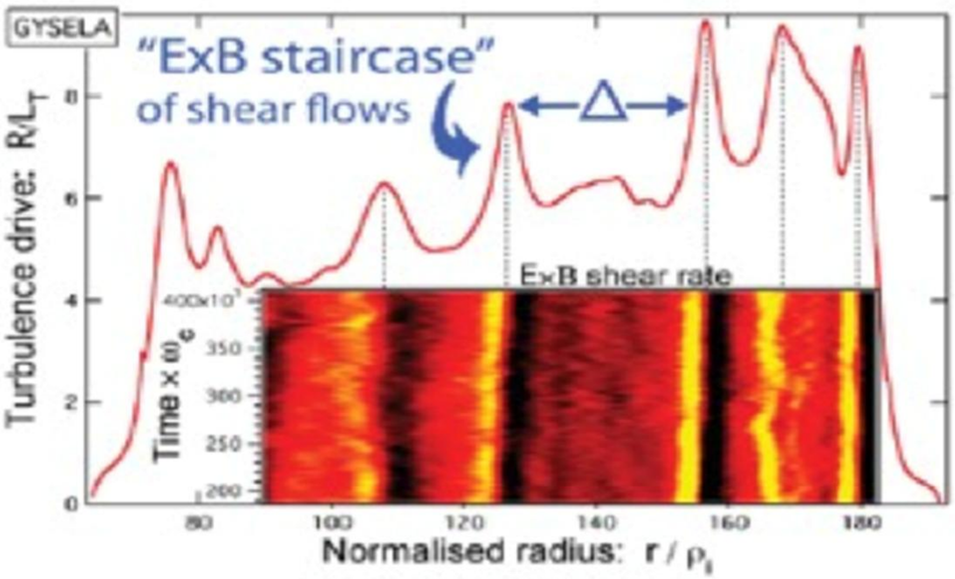


Motivation: ExB staircase formation (1)

- ExB flows often observed to self-organize in magnetized plasmas

  eg.) mean sheared flows, zonal flows, ...

- `ExB staircase' is observed to form (G. Dif-Pradalier, P.D. et al. Phys. Rev. E. '10)



[from Dunkerton et al. 2008]

- flux driven, full f simulation

- Quasi-regular pattern of shear layers and profile corrugations

- Region of the extent $\Delta \gg \Delta_c$ interspersed by temp. corrugation/ExB jets

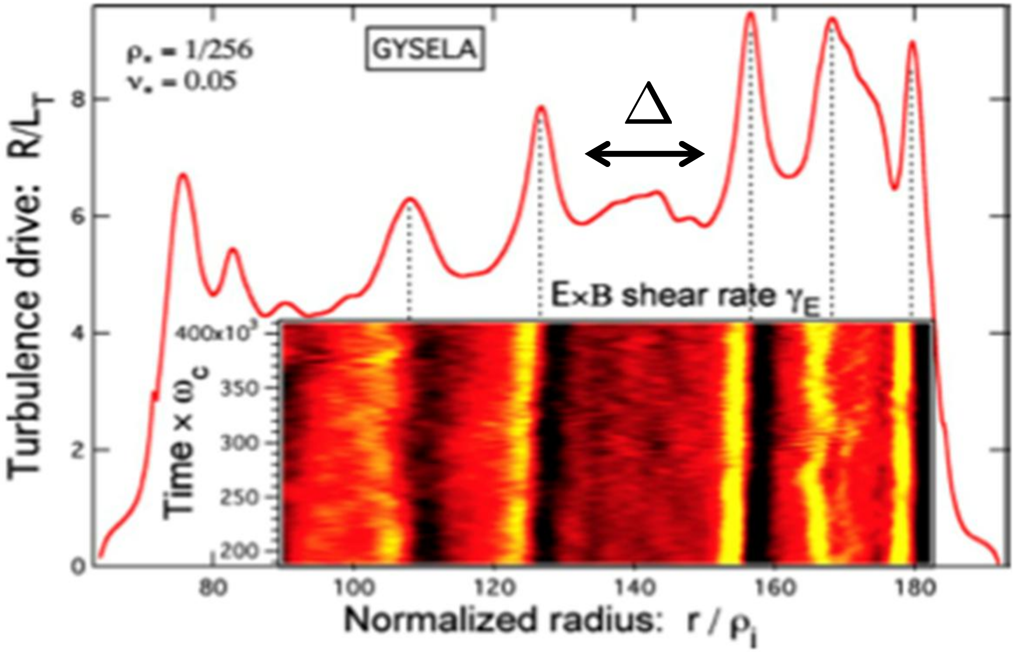
  → ExB staircases

- so-named after the analogy to PV staircases and atmospheric jets

- Step spacing → avalanche outer-scale

ExB Staircase (2)

- Important feature: co-existence of shear flows and avalanches



- Seem mutually exclusive ?!?

  → strong ExB shear prohibits transport

  → avalanches smooth out corrugations

- Can co-exist by separating regions into:

  1. avalanches of the size $\Delta \gg \Delta_c$

  2. localized strong corrugations + jets

- How understand the formation of ExB staircase???

  - What is process of self-organization linking avalanche scale to ExB step scale?

    i.e. how explain the emergence of the step scale ???

Staircases, cont'd

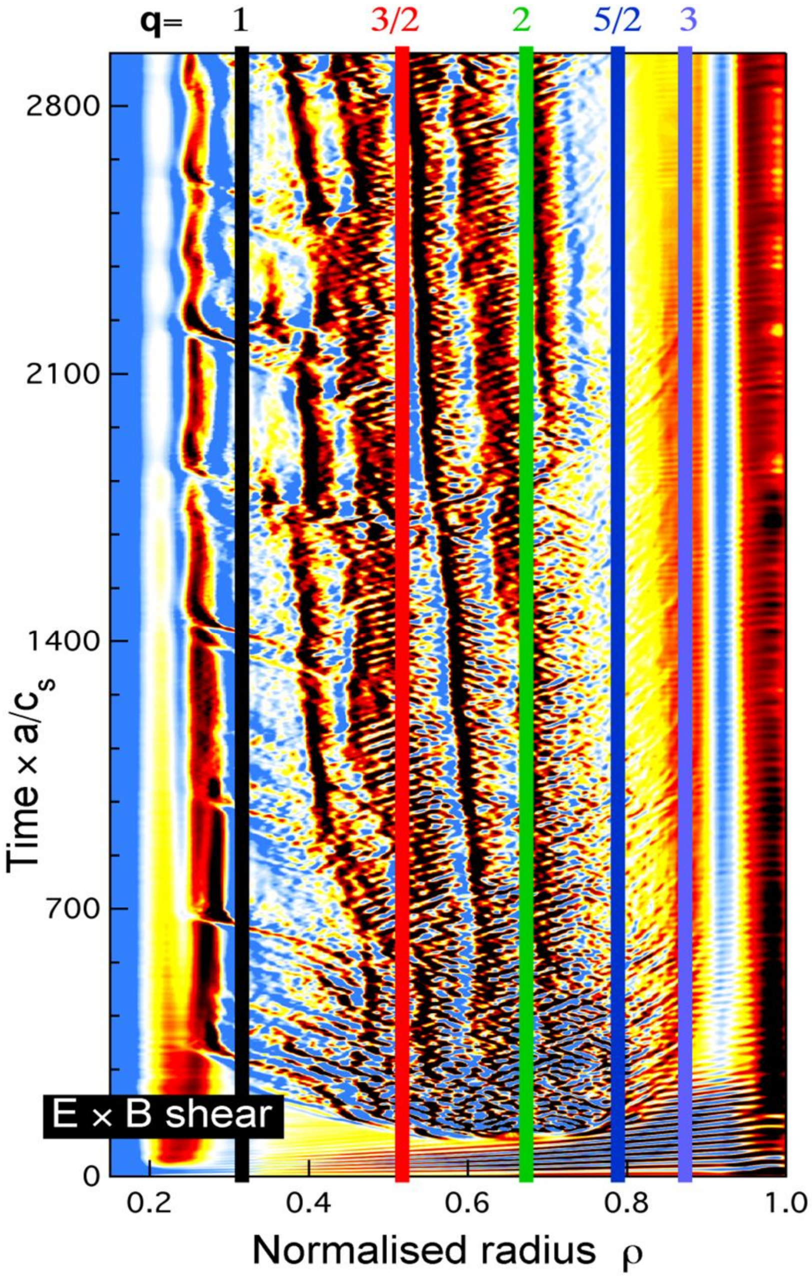
- The point:
  - fit: $Q = -\int dr' \kappa(r,r') \nabla T(r')$ $\kappa(r,r') \sim \frac{S^2}{(r-r')^2 + \Delta^2}$ → some range in exponent
  - $\Delta >> \Delta_c$ i.e. $\Delta \sim$ Avalanche scale >> $\Delta_c \sim$ correlation scale
  - Staircase 'steps' separated by Δ ! → stochastic avalanches produce quasi-regular flow pattern!?

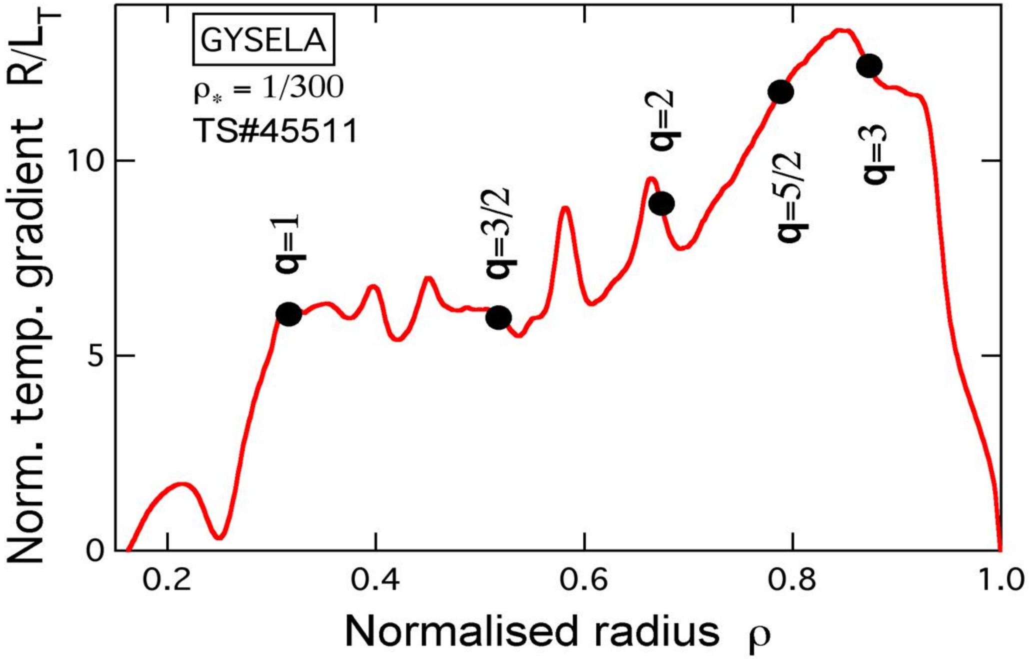
    N.B.

    - The notion of a staircase is not new – especially in systems with natural periodicity (i.e. NL wave breaking…)
    - What IS new is the connection to stochastic avalanches, independent of geometry
  - What is process of self-organization linking avalanche scale to zonal pattern step?

    i.e. How extend predator-prey feedback model to encompass both avalanche and zonal flow staircase? Self-consistency is crucial!

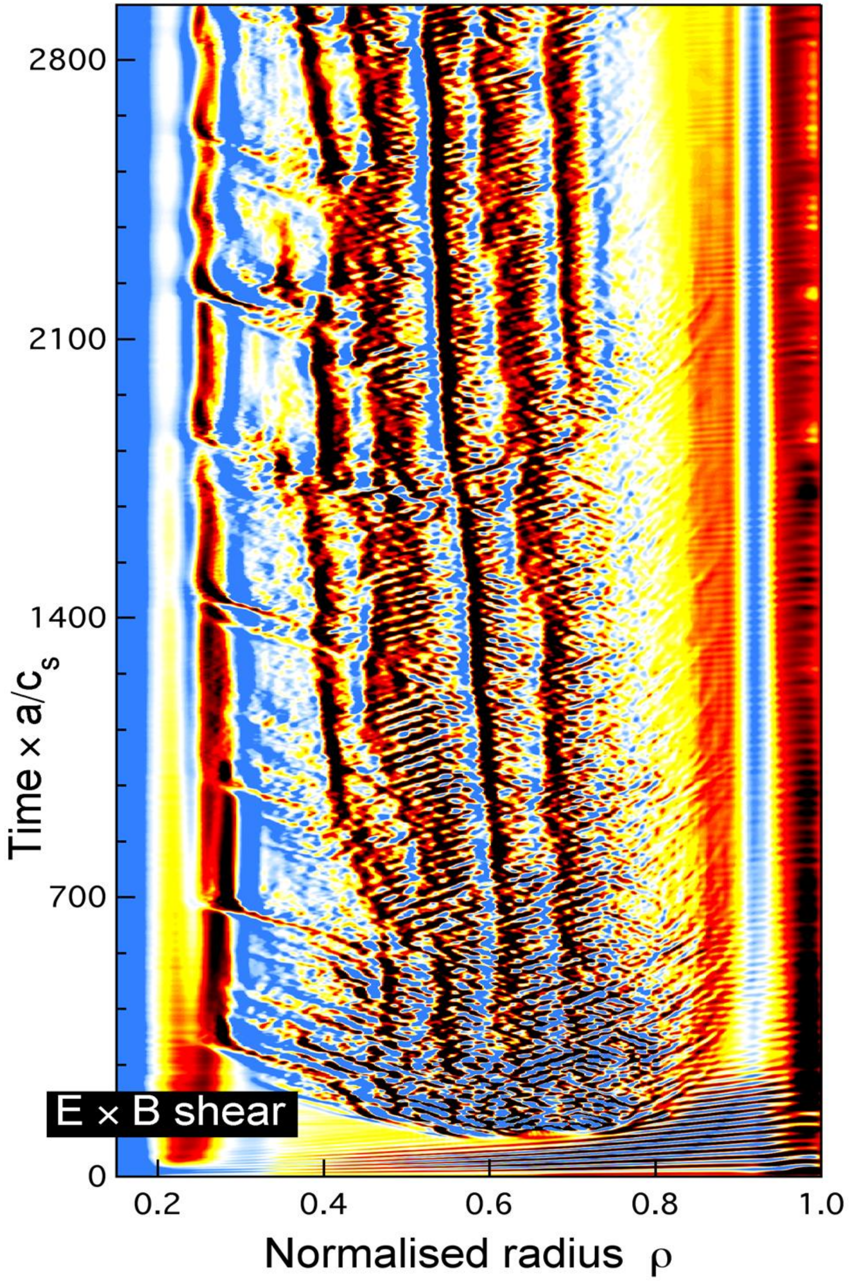
Corrugation points and rational surfaces – no relation!





Step location not tied to magnetic geometry structure in a simple way

Staircases build up from the edge



→ staircases may not be related to zonal flow eigenfunctions

→ How describe generation mechanism??

(GYSELA simulation)

Towards a model

- How do we understand quasi-regular pattern of ExB staircase, generated from stochastic heat avalanche???

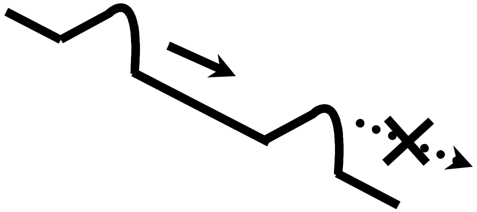
- An idea: jam of heat avalanche

corrugated profile ↔ ExB staircase

→ corrugation of profile occurs by 'jam' of heat avalanche flux

* → time delay between $Q[\delta T]$ and δT is crucial element

like drivers' response time in traffic



→ accumulation of heat increment
→ stationary corrugated profile

- How do we actually model heat avalanche 'jam'??? → origin in dynamics?

Traffic jam dynamics: ‘jamiton’

- A model for Traffic jam dynamics → Whitham

$$\rho_t + (\rho v)_x = 0$$

$$v_t + v v_x = -\frac{1}{\tau}\left\{ v - V(\rho) + \frac{\nu}{\rho}\rho_x \right\}$$

→ Instability occurs when $\tau > \nu/(\rho_0^2 V_0'^2)$

$$D_{eff} = \nu - \tau\rho_0^2 V_0'^2 < 0$$ → clustering instability

→ Indicative of jam formation

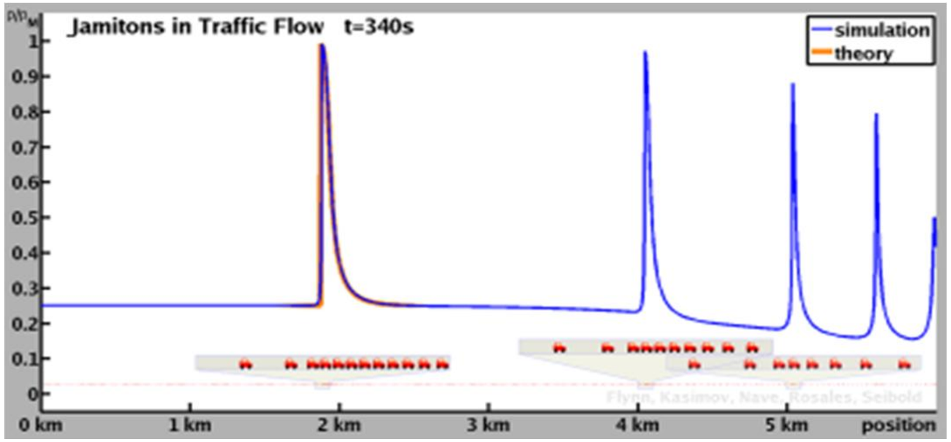
ρ → car density

v → traffic flow velocity

$V(\rho) - \frac{\nu}{\rho}\rho_x$ → an equilibrium traffic flow

τ → driver’s response time

- Simulation of traffic jam formation



http://math.mit.edu/projects/traffic/

→ Jamitons (Flynn, et.al., ’08)

n.b. I.V.P. → decay study

Heat avalanche dynamics model (`the usual')

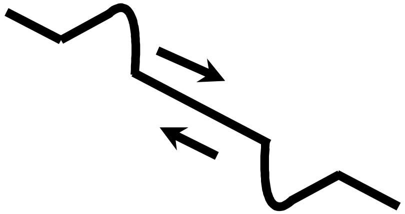
Hwa+Kardar ’92, P.D. + Hahm ’95, Carreras, et al. ’96, ... GK simulation, ... Dif-Pradalier ’10

- δT :deviation from marginal profile → conserved order parameter

- Heat Balance Eq.: $\partial_t \delta T + \partial_x Q[\delta T] = 0$ → up to source and noise

- Heat Flux $Q[\delta T]$? → utilize symmetry argument, ala’ Ginzburg-Landau

  - Usual: → joint reflectional symmetry (Hwa+Kardar’92, Diamond+Hahm ’95)



$$\delta T \leftrightarrow -\delta T$$

$$x \leftrightarrow -x$$

$$Q = Q_0(\delta T)$$
$$= \frac{\lambda}{2}\delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T$$

→ hyperdiffusion

lowest order → Burgers equation

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

An extension of the heat avalanche dynamics

- An extension: a finite time of relaxation of Q toward SOC flux state

$$\partial_t Q = -\frac{1}{\tau}(Q - Q_0(\delta T)) \qquad Q_0[\delta T] = \frac{\lambda}{2}\delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T$$

(Guyot-Krumhansl)

→ In principle $\tau(\delta T, Q_0)$ ⟷ large near criticality (∼ critical slowing down)

i.e. enforces time delay between δT and heat flux

- Dynamics of heat avalanche:

$$\underline{\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T} - \chi_4 \partial_x^4 \delta T - \tau \partial_t^2 \delta T$$

$\downarrow$

→ Burgers
(P.D. + T.S.H. ’95)

New: finite response time

→ Telegraph equation

n.b. model for heat evolution

diffusion → Burgers → Telegraph

Relaxation time: the idea

- What is 'τ ' physically? → Learn from traffic jam dynamics
- A useful analogy:

heat avalanche dynamics	traffic flow dynamics
temp. deviation from marginal profile	local car density
heat flux	traffic flow
mean SOC flux (ala joint relflection symmetry)	equilibrium, steady traffic flow
➡ heat flux relaxation time	driver's response time

- driver's response can induce traffic jam

- jam in avalanche → profile corrugation → staircase?!?

- Key: instantaneous flux vs. mean flux

Time delay: microscopic foundation?

- Relaxation by plasma turbulence = mixing of phase space density

$$\frac{df}{dt} = 0 \Rightarrow \partial_t \langle \delta f(1) \delta f(2) \rangle + \frac{1}{\tau_{mix}} \langle \delta f(1) \delta f(2) \rangle = -\langle \tilde{v}_r \delta f \rangle \langle f \rangle'$$

phase space density correlation =
'phasetrophy'

turbulent mixing
↓
i.e. PV mixing time sets delay

production
due gradient relaxation

- Energy moment leads to heat flux evolution equation (Gurcan '13)

$$\partial_t Q = -\frac{1}{\tau_{mix}}(Q - Q_0) \qquad Q_0 = -\chi_{turb} \nabla T$$

→ Heat flux relaxes toward the mean value, in the mixing time

The delay time is a natural consequence of phase space density mixing. The delay time is typically in the order of mixing time.

Heat flux dynamics: when important?

- Heat flux evolution: $\partial_t Q = -\frac{1}{\tau_{mix}}(Q - Q_0)$ → time delay, when important?

Conventional Transport Analysis

$\tau_{mix} \ll$ time scale of interest

→ Heat flux relaxes to the mean value immediately

$$Q = Q_0$$

→ Profile evolves via the mean flux

$$\partial_t T + \partial_x Q_0 = 0$$

then

diff. $\partial_t T = \chi \partial_x^2 T$

Burgers $\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$

New approach for transport analysis

→ mixing time can be long, so

$\tau_{mix} \sim$ time scale of interest
mesoscale

→ Heat evo. and Profile evo. must be treated self-consistently

$$\begin{cases} \partial_t Q = -\frac{1}{\tau}(Q - Q_0) \\ \partial_t \delta T + \partial_x Q[\delta T] = 0 \end{cases}$$

then telegraph equation:

$$\boxed{\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T - \tau \partial_t^2 \delta T}$$

Brief summary on model extension

	Heat Flux	Profile evo.	
Usual:	$Q = Q_0[\delta T]$	$\partial_t T = \chi \partial_x^2 T$ $\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$	Diffusion Burgers
Extended:	$\partial_t Q = -\frac{1}{\tau}(Q - Q_0)$	$\tau \partial_t^2 \delta T + \partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$	telegraph

finite response time

- Physical idea: analogy to traffic dynamics, drivers' response time

- Microscopic foundation: mixing of phase space density

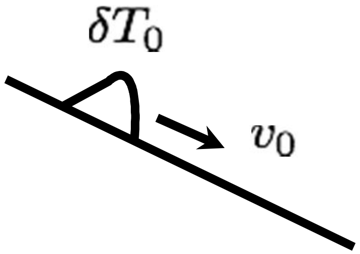
- Finite response time → Heat dynamics described by telegraph eqn.

  → Wavy feature, speed determined by $\sqrt{\chi_2/\tau}$

- Connects avalanche dynamics to elasticity in/of turbulence

Analysis of heat avalanche dynamics via telegraph

- How do heat avalanches jam?



- Consider an initial avalanche,
  with amplitude δT_0,
  propagating at the speed $v_0 = \lambda\delta T_0$

→ turbulence model dependent

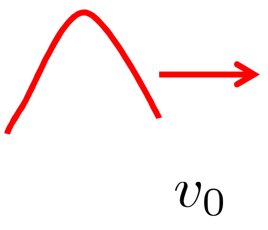
- Dynamics:

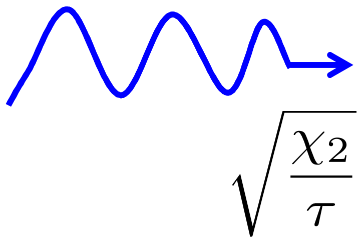
$$\partial_t\widetilde{\delta T} + v_0\partial_x\widetilde{\delta T} = \chi_2\partial_x^2\widetilde{\delta T} - \chi_4\partial_x^4\widetilde{\delta T} - \tau\partial_t^2\widetilde{\delta T}$$

pulse

'Heat flux wave': $\sqrt{\frac{\chi_2}{\tau}}$
telegraph → wavy feature

two characteristic propagation speeds





→ In short response time (usual) heat flux wave propagates faster

→ In long response time, heat flux wave becomes slower and pulse starts overtaking.
What happens???

Analysis of heat avalanche jam dynamics

- In large tau limit, what happens? → Heat flux jams!!

- Recall plasma response time akin to driver's response time in traffic dynamics

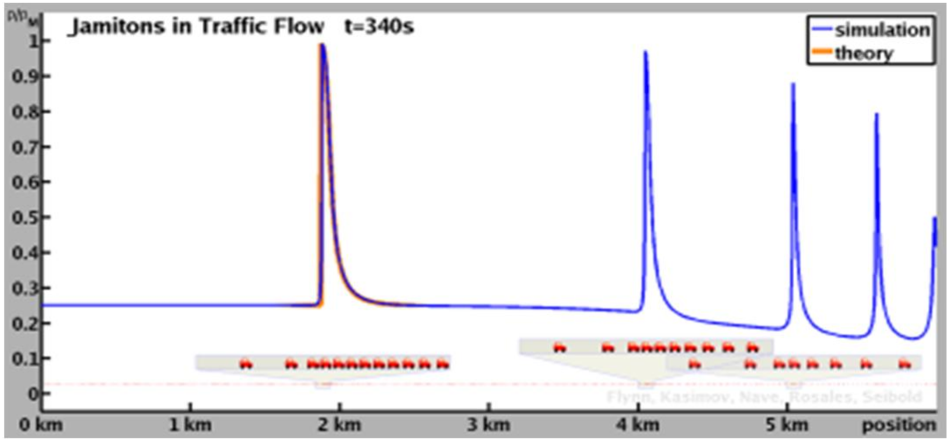
- negative heat conduction instability occurs (as in clustering instability in traffic jam dynamics)

$$\partial_t \widetilde{\delta T} + v_0 \partial_x \widetilde{\delta T} = \chi_2 \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T} - \tau \partial_t^2 \widetilde{\delta T}$$

$$\rightarrow \underline{(\chi_2 - v_0^2 \tau)} \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T}$$

<0 when overtaking

→ clustering instability



Flynn, Kasimov, Nave, Rosales, Seibold

n.b. akin to negative viscosity instability of ZF in DW turbulence

instead ZF as secondary mode in the gas of primary DW

➔ Heat flux 'jamiton' as secondary mode in the gas of primary avalanches

Analysis of heat avalanche jam dynamics

- Growth rate of the jamiton instability

$$\gamma = -\frac{1}{2\tau} + \frac{1}{2\tau}\sqrt{\frac{r+1}{2} - 2\tau\chi_2 k^2\left(1+\frac{\chi_4 k^2}{\chi_2}\right)} \qquad r = \sqrt{\left\{4\tau\chi_2 k^2\left(1+\frac{\chi_4 k^2}{\chi_2}\right) - 1\right\}^2 + 16v_0^2 k^2\tau^2}$$

- Threshold for instability

$$\tau > \frac{\chi_2}{v_0^2}\left(1+\frac{\chi_4 k^2}{\chi_2}\right)$$

→ critical minimal delay time

n.b. $1/\tau = 1/\tau[\mathcal{E}]$

→ clustering instability strongest near criticality

- Scale for maximum growth

$$k^2 \cong \frac{\chi_2}{\chi_4}\sqrt{\frac{\chi_4 v_0^2}{4\chi_2^3}}$$

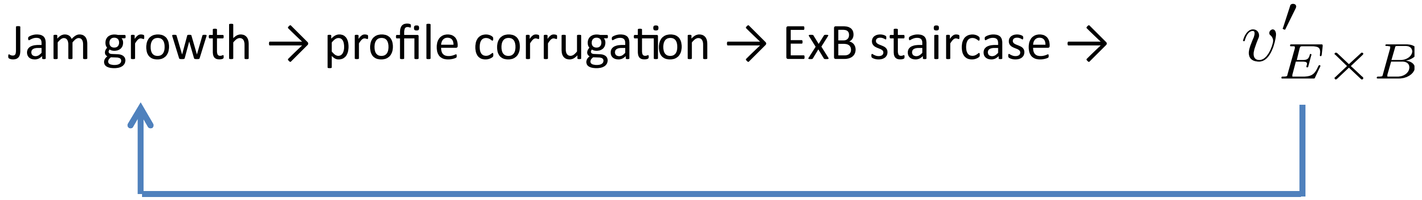
from

$$\frac{\partial\gamma}{\partial k^2} = 0 \quad\Rightarrow\quad 8\tau\frac{\chi_4^2}{\chi_2}k^6 + 4\tau\chi_4 k^4 + 2\frac{\chi_4}{\chi_2}k^2 + 1 - \frac{v_0^2\tau}{\chi_2} = 0$$

→ staircase size, $\Delta_{stair}^2(\delta T)$, δT from saturation: consider shearing

Scaling of characteristic jam scale

- Saturation: Shearing strength to suppress clustering instability



Jam growth → profile corrugation → ExB staircase → $v'_{E\times B}$

→ estimate, only

→ saturated amplitude: $\frac{\delta T}{T_i} \sim \frac{1}{v_{thi}\rho_i}\sqrt{\frac{\chi_4}{\tau}}$

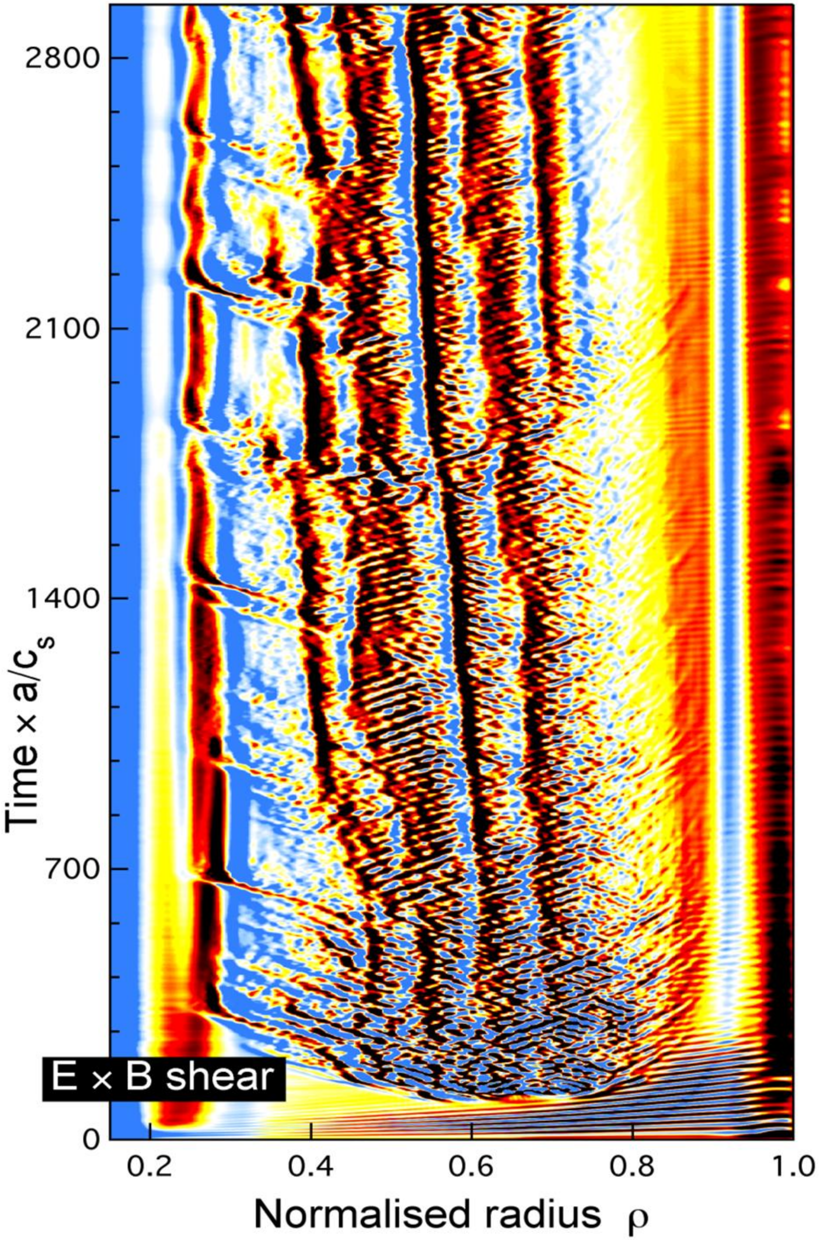
- Characteristic scale

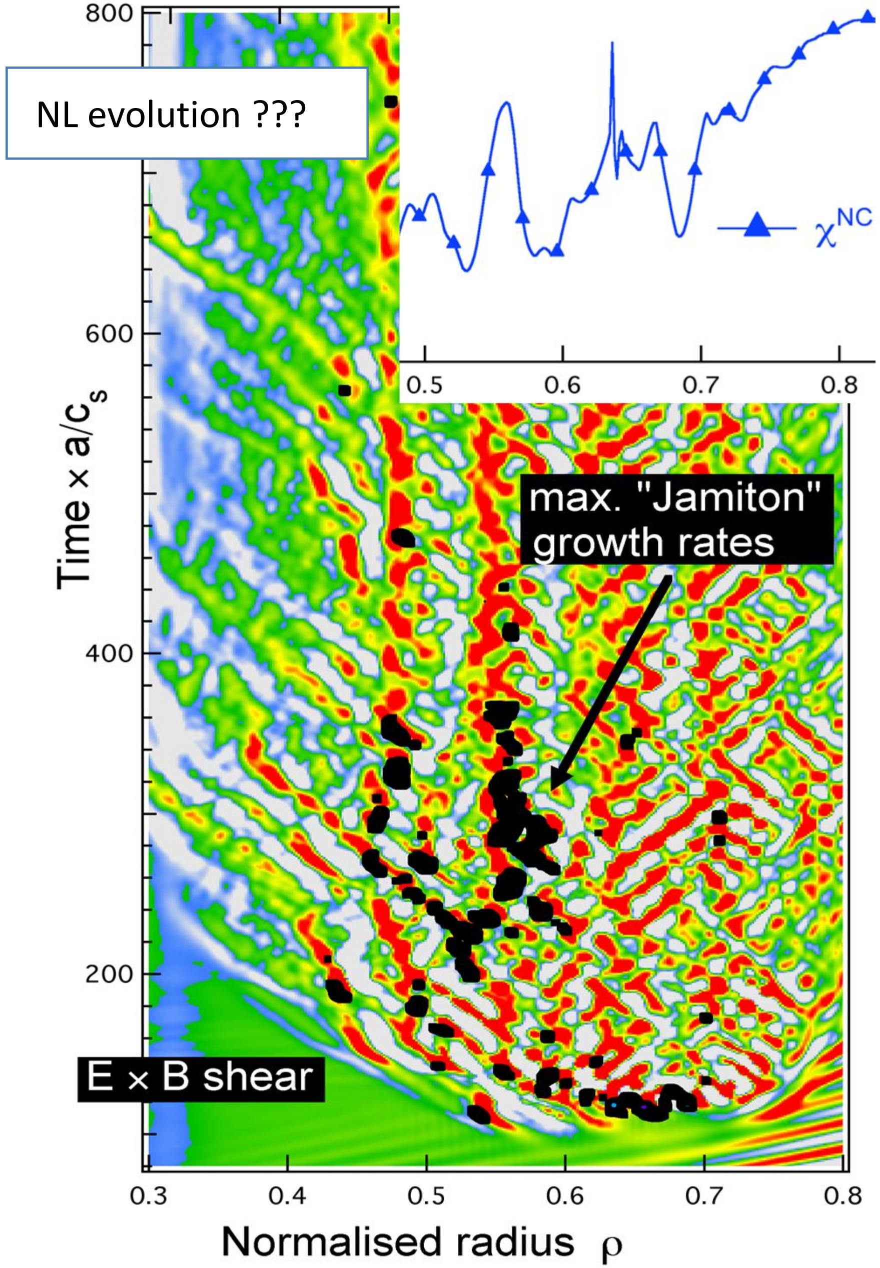
$$\Delta^2 \sim k^{-2}(\delta T) \sim \frac{2v_{thi}}{\lambda T_i}\rho_i\sqrt{\chi_2\tau} \qquad \chi_2 \sim \chi_{neo}$$

- Geometric mean of ρ_i and $\sqrt{\chi_2\tau}$: ambient diffusion length in 1 relaxation time

- ‘standard’ parameters: $\Delta \sim 10\Delta_c$

Jam growth qualitatively consistent with staircase formation





NL evolution ???

outer radius:
large chi
→ smear out instability
or
→ heat flux waves propagate faster
→ harder to overtake, jam

good agreement in early stage

Dif-Pradalier ’13 caveat: based on model with compressional waves

Summary

- A model for ExB staircase formation

  - Heat avalanche jam → profile corrugation → ExB staircase

  - model developed based on analogy to traffic dynamics → telegraph eqn.

- Analysis of heat flux jam dynamics

  - Negative conduction instability as onset of jam formation

  - Growth rate, threshold, scale for maximal growth

  - Qualitative estimate: scale for maximal growth $\Delta \sim 10\Delta_c$

    → comparable to staircase step size

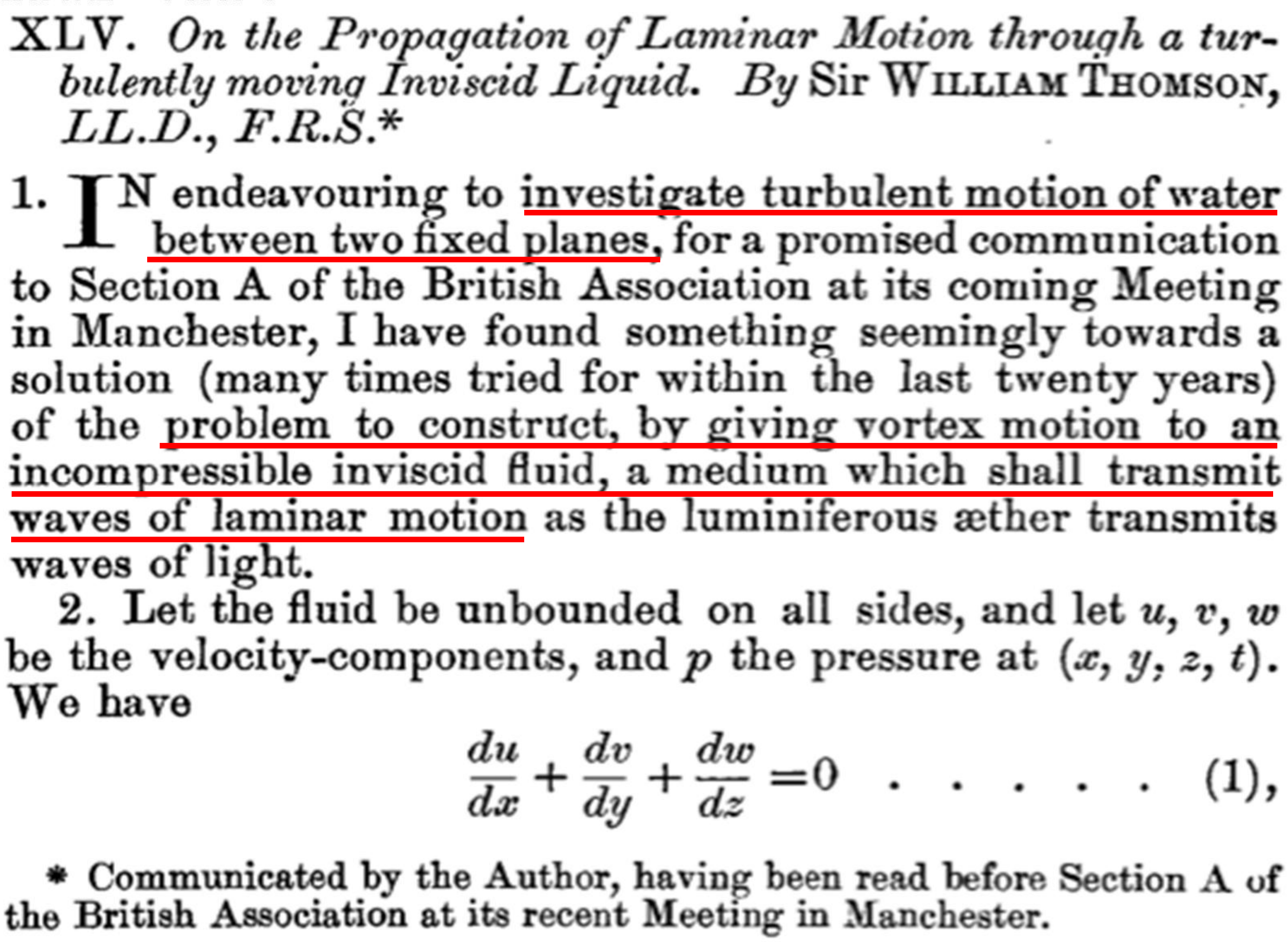
Ongoing Work

- This analysis ↔ set in context of heat transport
- Implications for momentum transport? ➔
  - consider system of flow, wave population, wave momentum flux
  - time delay set by decay of wave population correlation due ray stochastization → elasticity
  - flux limited PV transport allows closure of system

Aside: FYI – Historical Note

- –→ Collective Dynamics of Turbulent Eddy
- – 'Aether' I – First Quasi-Particle Model of Transport?!

– Kelvin, 1887



XLV. *On the Propagation of Laminar Motion through a turbulently moving Inviscid Liquid. By* Sir WILLIAM THOMSON, *LL.D., F.R.S.**

1. IN endeavouring to investigate turbulent motion of water between two fixed planes, for a promised communication to Section A of the British Association at its coming Meeting in Manchester, I have found something seemingly towards a solution (many times tried for within the last twenty years) of the problem to construct, by giving vortex motion to an incompressible inviscid fluid, a medium which shall transmit waves of laminar motion as the luminiferous æther transmits waves of light.

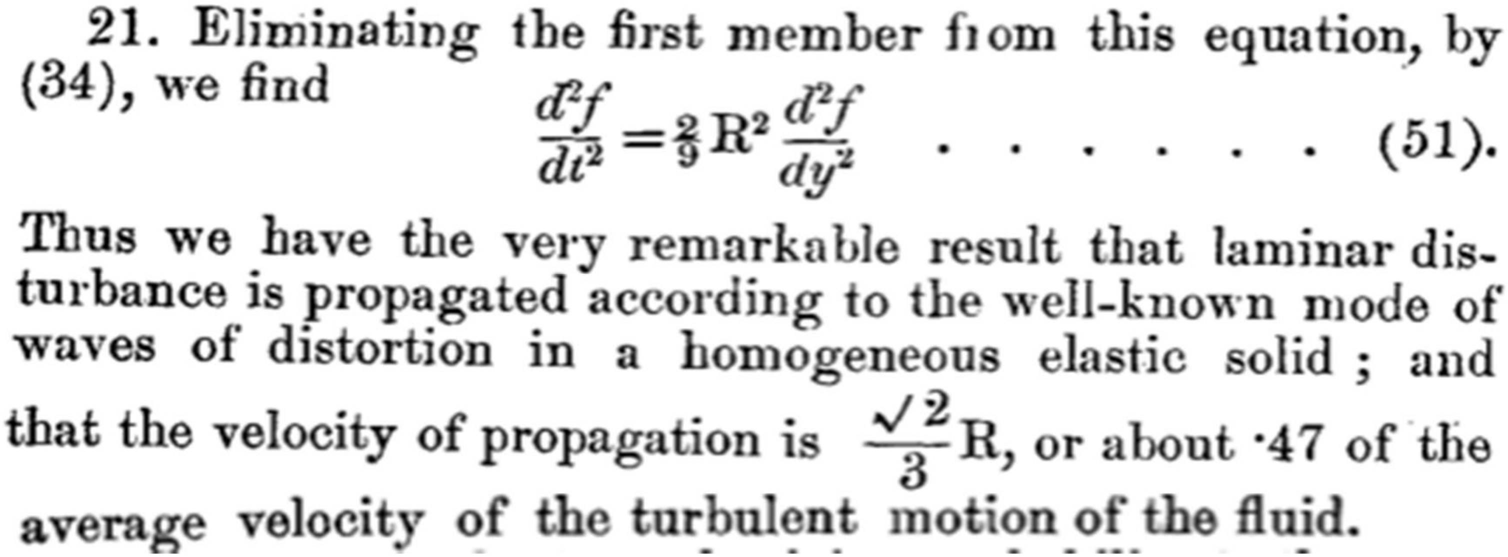
2. Let the fluid be unbounded on all sides, and let u, v, w be the velocity-components, and p the pressure at (x, y, z, t). We have

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0 \quad . \quad . \quad . \quad . \quad . \quad (1),$$

---

* Communicated by the Author, having been read before Section A of the British Association at its recent Meeting in Manchester.

$R^2 \sim \langle \tilde{v}^2 \rangle$



21. Eliminating the first member from this equation, by (34), we find

$$\frac{d^2f}{dt^2}=\tfrac{2}{9}\mathrm{R}^2\frac{d^2f}{dy^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51).$$

Thus we have the very remarkable result that laminar disturbance is propagated according to the well-known mode of waves of distortion in a homogeneous elastic solid ; and that the velocity of propagation is $\frac{\surd 2}{3}\mathrm{R}$, or about ·47 of the average velocity of the turbulent motion of the fluid.

- → time delay between Reynolds stress and wave shear introduced
- → converts diffusion equation to wave equation
- → describes wave in ensemble of vortex quasi-particles
- – c.f. “Worlds of Flow”, O. Darrigol

Fig. 1.