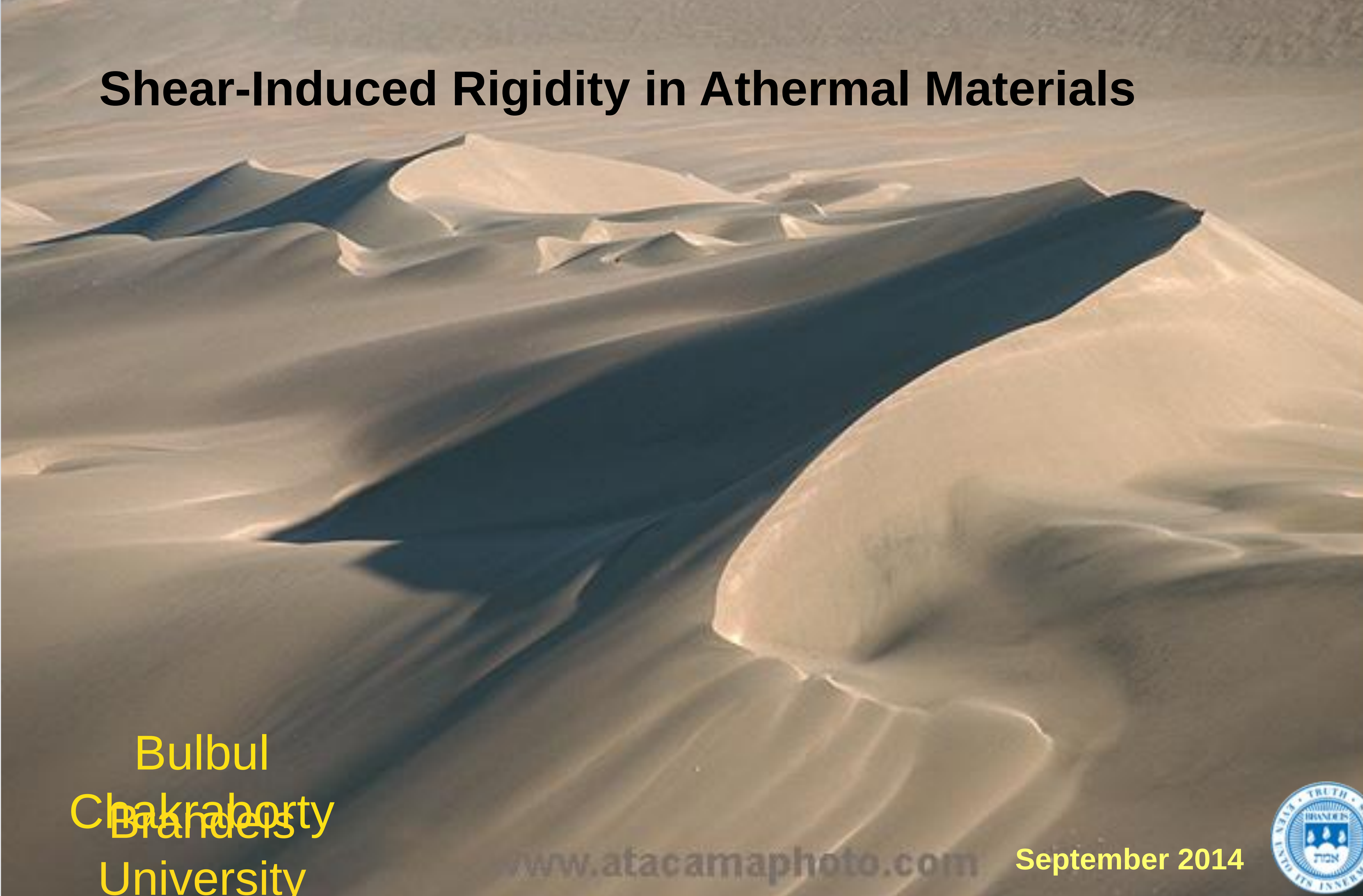
Shear-Induced Rigidity in Athermal Materials



Bulbul Chakraborty
Brandeis University

Sumantra Sarkar, Dapeng Bi
Bob Behringer, Jie Zhang, Jie Ren, Joshua Dijksman

September 2014

Shear-Jamming & Shear-Thickening

- Dry grains at low densities undergo a transition from unjammed to fragile to shear-jammed solely through changing the shape of a 2D box (no change in density)

- Athermal (non-Brownian) suspensions undergo a discontinuous shear-thickening transition (DST) as shear rate is increased.

- Experiments and/or simulations show that the driving leads to a proliferation of frictional contacts. The qualitative reason is that constant volume frustrates dilatancy

- Simulations of DST show that the transition does not occur if grains are frictionless

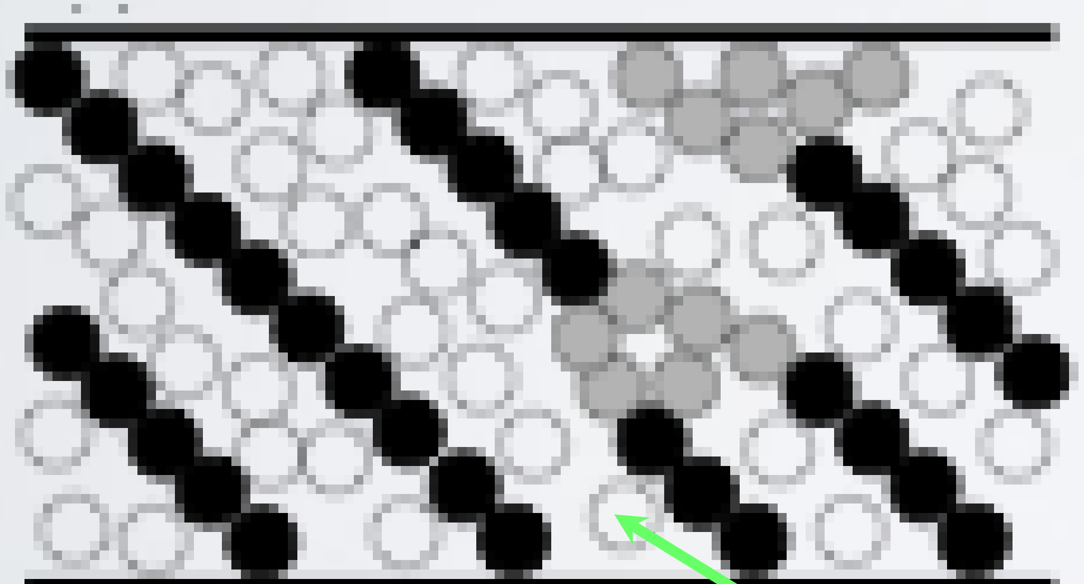
- What type of solids are these ? How do we understand the transition ?

Shear-Jamming

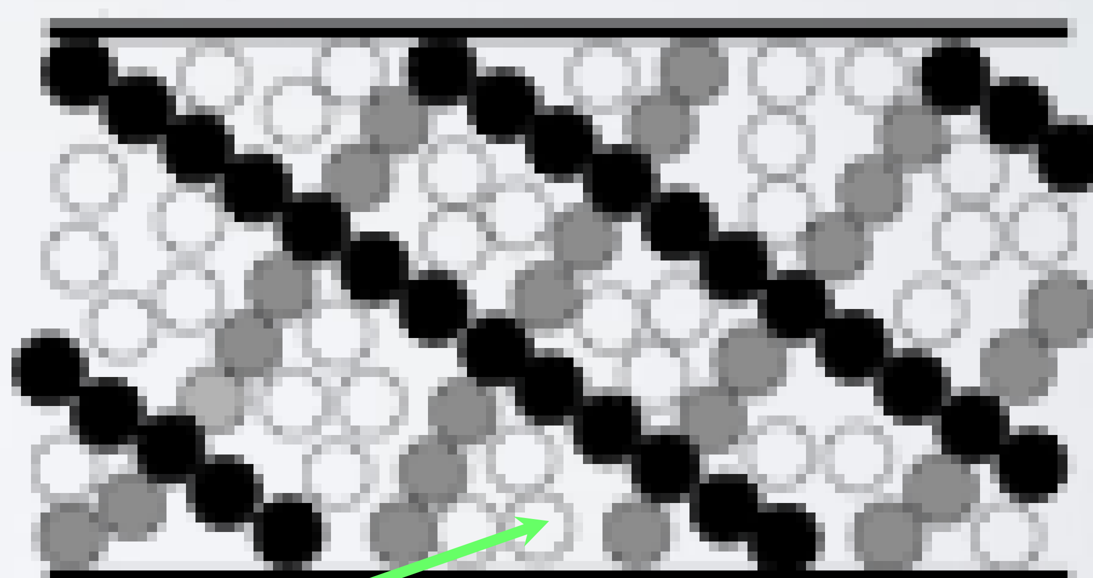
"Jamming apparently occurs because the particles form "force chains" along the compressional direction. Even for spherical particles the lubrication films cannot prevent contacts; once these arise, an array or network of force chains can support the shear stress indefinitely. By this criterion, the material is a solid. In this Letter, we propose some simple models of jammed systems like this, whose solidity stems directly from the applied stress itself".

Cates et al: Phys Rev. Lett 81, 1841 (1998)

1D Force Network



2D Force Network



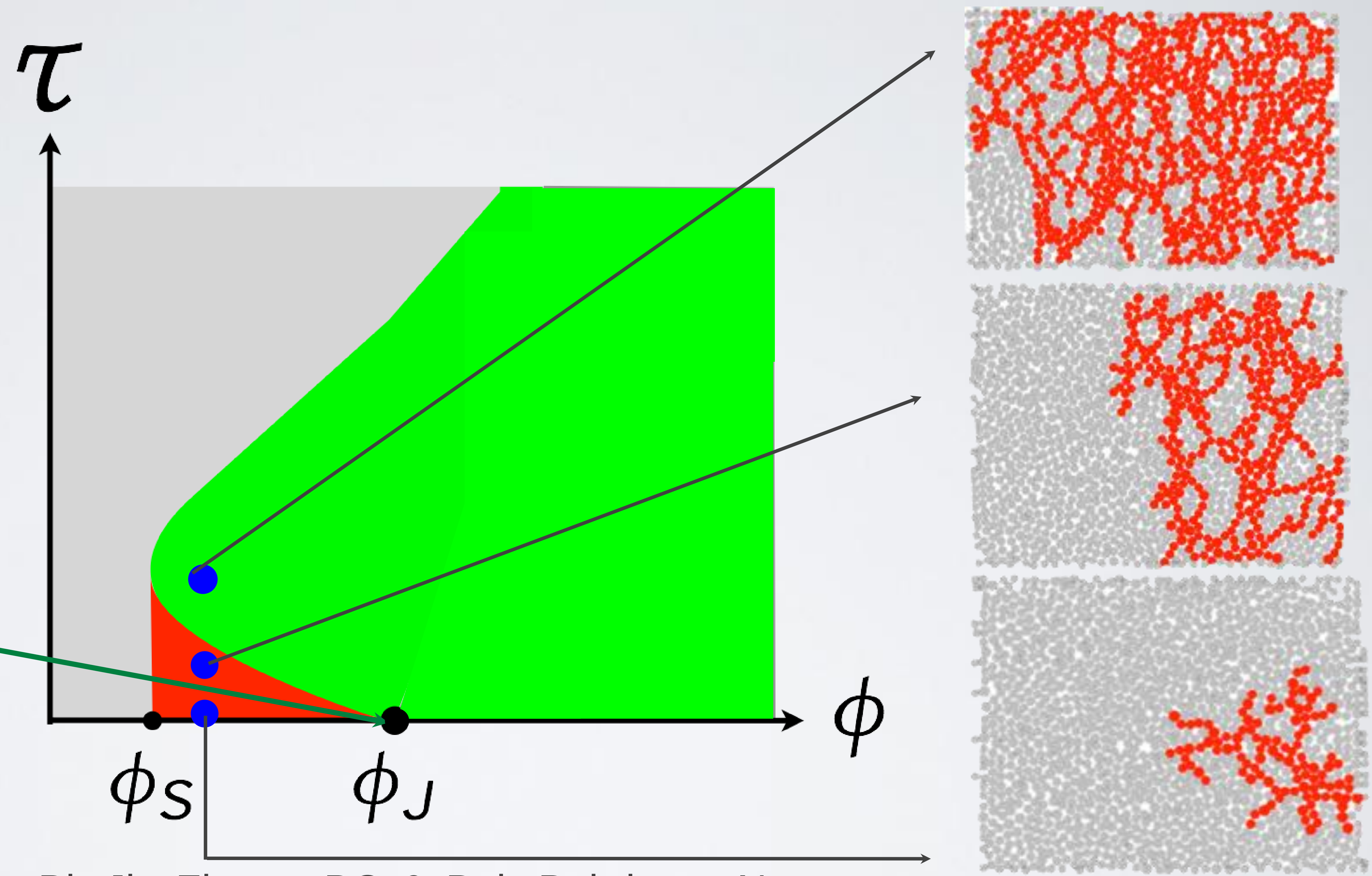
"Spectator Particles"

Shear-induced Solidification in a Model Granular System

- No Thermal Fluctuations
- Purely Repulsive, contact Interactions, friction
- States controlled by driving at the boundaries
- Structure emerges that supports further shearing: a solid

Shear-Jamming Experiments
(Quasistatic Forward + Cyclic Shear)

Isostatic Point of frictionless grains: given a geometry, all forces can be determined.



Max Bi, Jie Zhang, BC & Bob Behringer Nature (2011)

Conditions of Static, Mechanical Equilibrium
(In DST, need to include the velocity-dependent forces)

This complicates the isostatic story

Local force & torque balance satisfied for every grain

Number of equations = N d (d+1)/2

Friction law on each contact $f_t \leq \mu f_N$

For infinite friction coefficient, isostatic number is d + 1 (there are d unknowns at every contact

Positivity of all forces $f_N \geq 0$

Imposed stresses determine sum of stresses over all grains

For arbitrary friction coefficients, need to know how many contacts are fully mobilized, and forces have to satisfy the inequality constraint

Imposing the conditions through gauge potentials (2D)

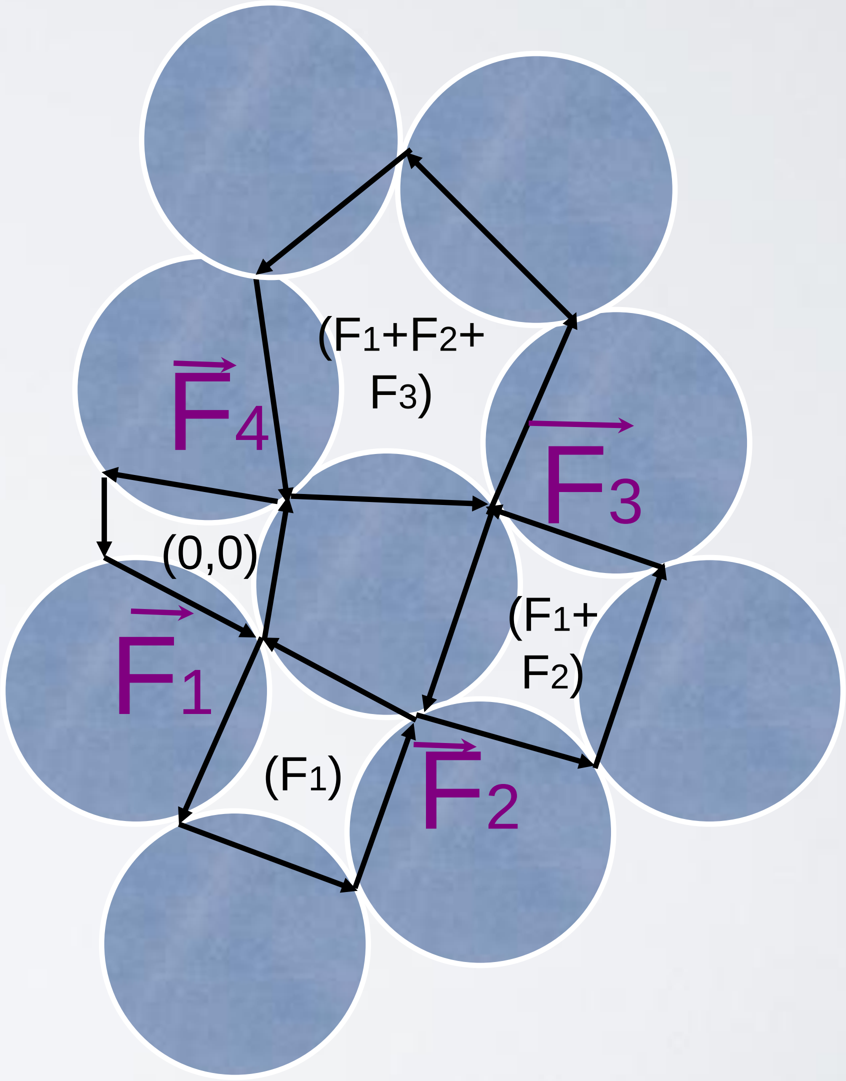
Ball & Blumenfeld (2002), Henkes, Bi, & BC (2007---), DeGuili (2011--)

- Vector fields enforce force balance constraint
- Additional scalar field enforces torque balance
- There is a relation between the two

Looking at the vector fields

We refer to them as heights: like a vector height field familiar in the context of groundstates of some frustrated magnet

Here the fields are continuous



Imposing the conditions through gauge potentials (2D)

Ball & Blumenfeld (2002), Henkes, Bi, & BC (2007---), DeGuili (2011--)

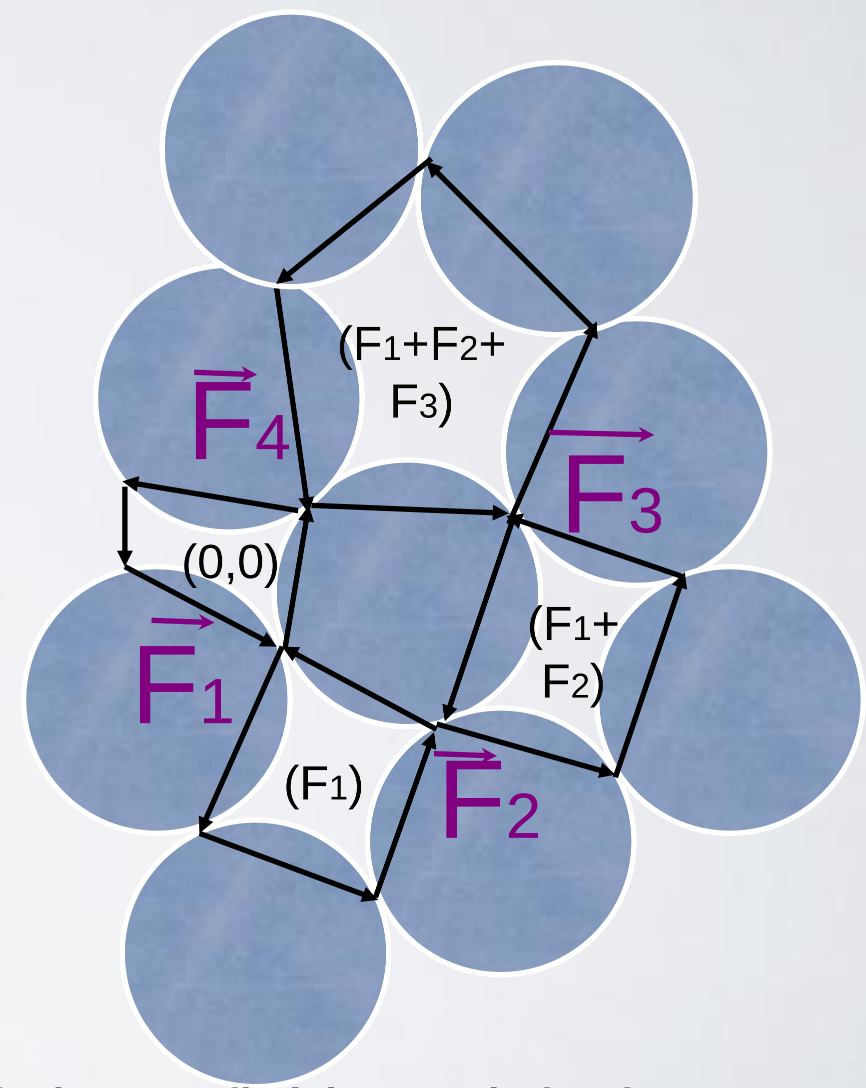
loops enclosing voids

$$\boldsymbol{f}_g^c = \boldsymbol{\rho}^{\ell'} - \boldsymbol{\rho}^{\ell},$$

$$\boldsymbol{r}^c \times \boldsymbol{f}_g^c = \varphi^{\ell'} - \varphi^{\ell} + \boldsymbol{r}^{\ell'} \times \boldsymbol{\rho}^{\ell'} - \boldsymbol{r}^{\ell} \times \boldsymbol{\rho}^{\ell}$$

scalar potential

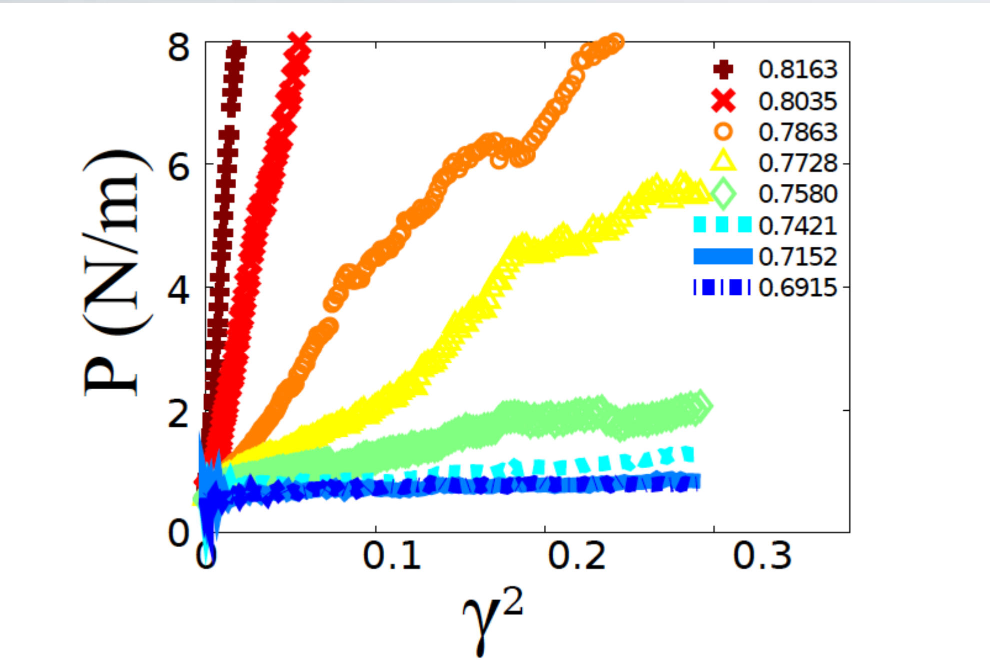
height vector



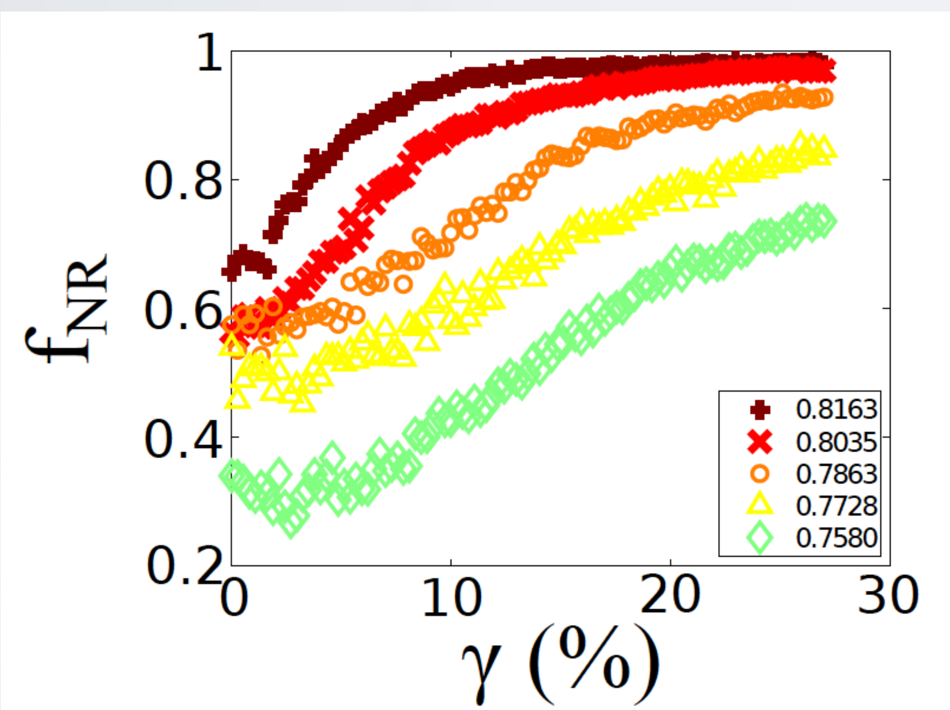
Gauge potentials: irrelevant additive constant. Any set of these fields satisfy force and torque balance. There are constraints relating the two potentials.

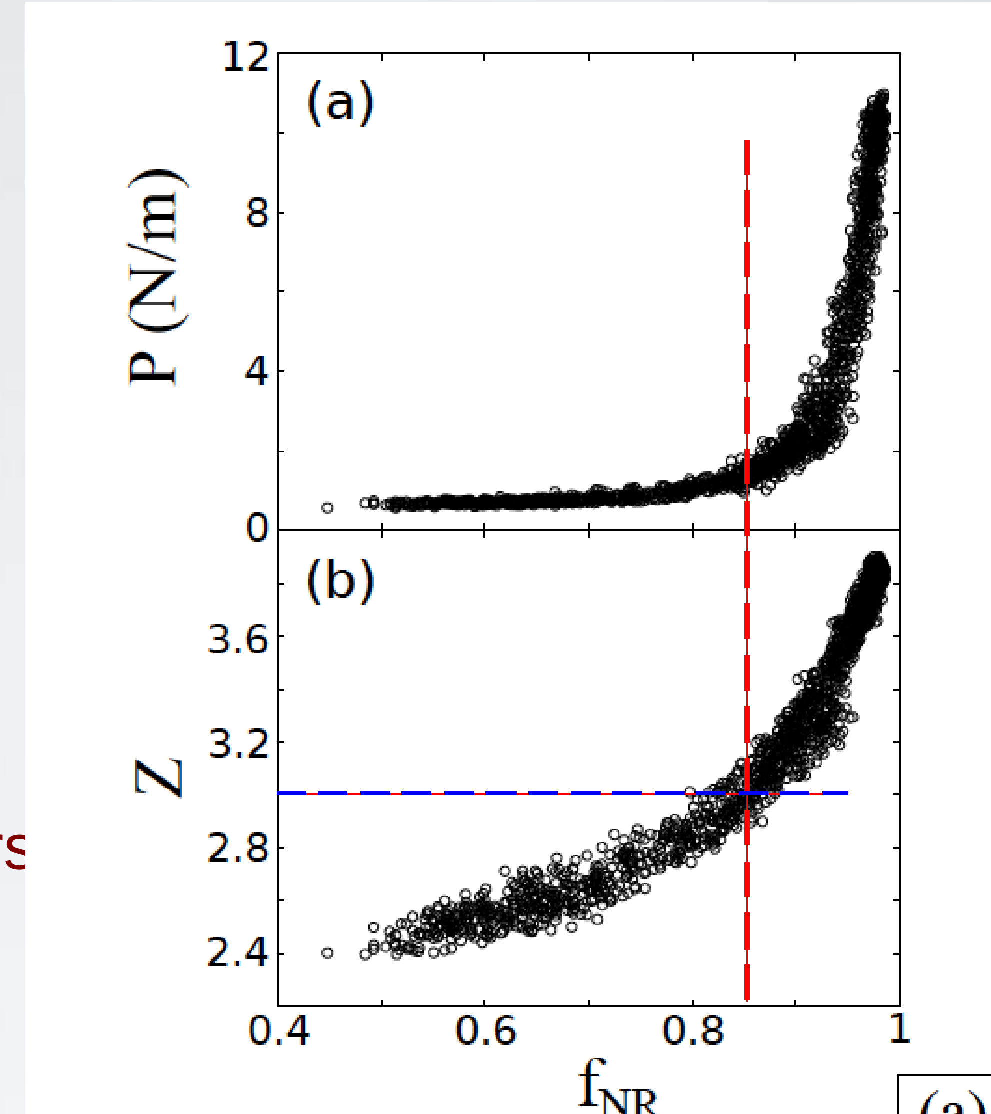
A Tour of Shear Jamming Experiments

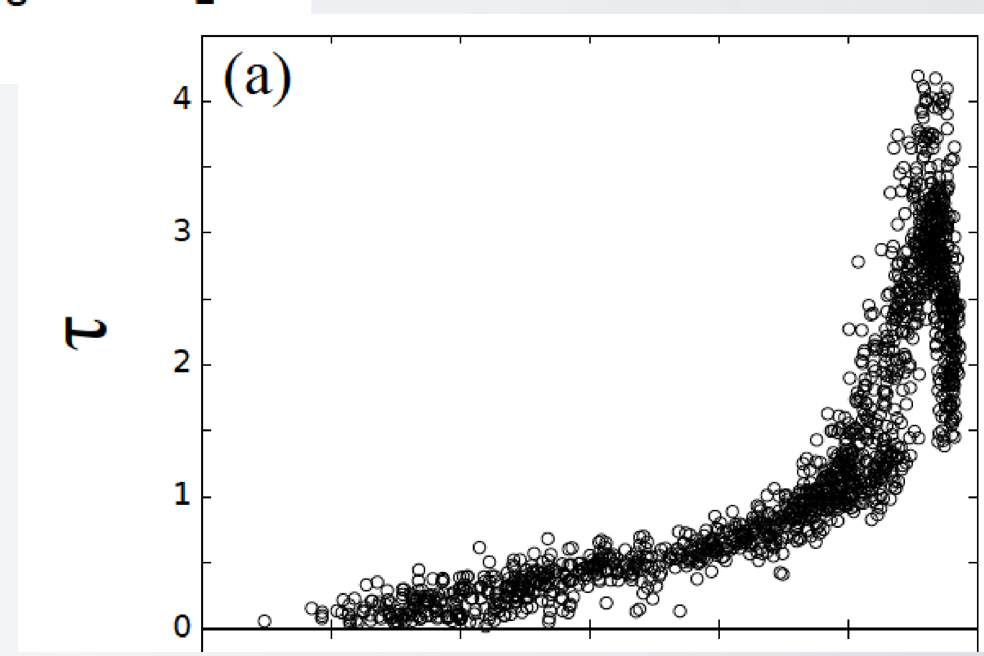
Reynolds Pressure



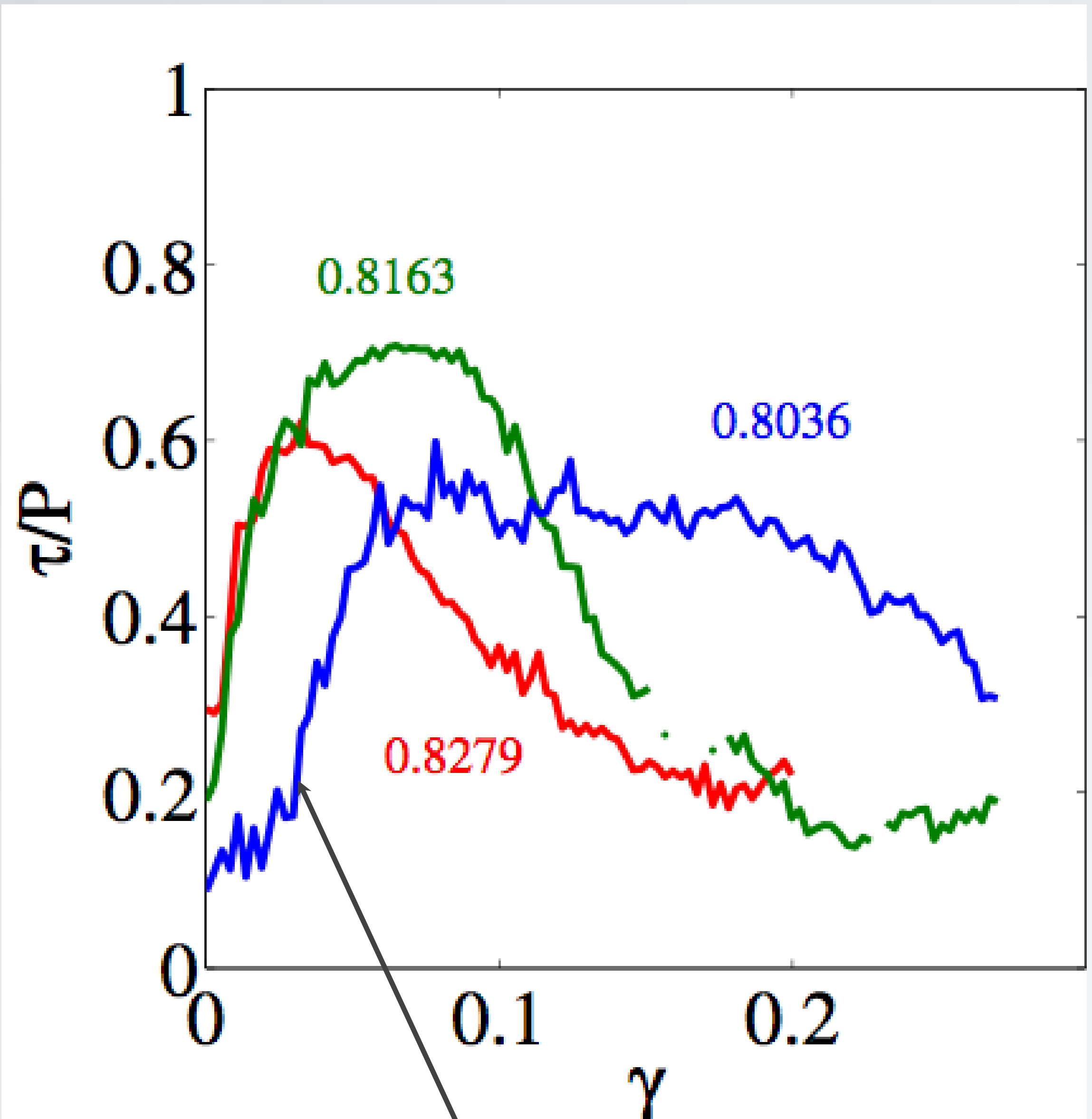
Non-rattler Fraction (Non-Spectators

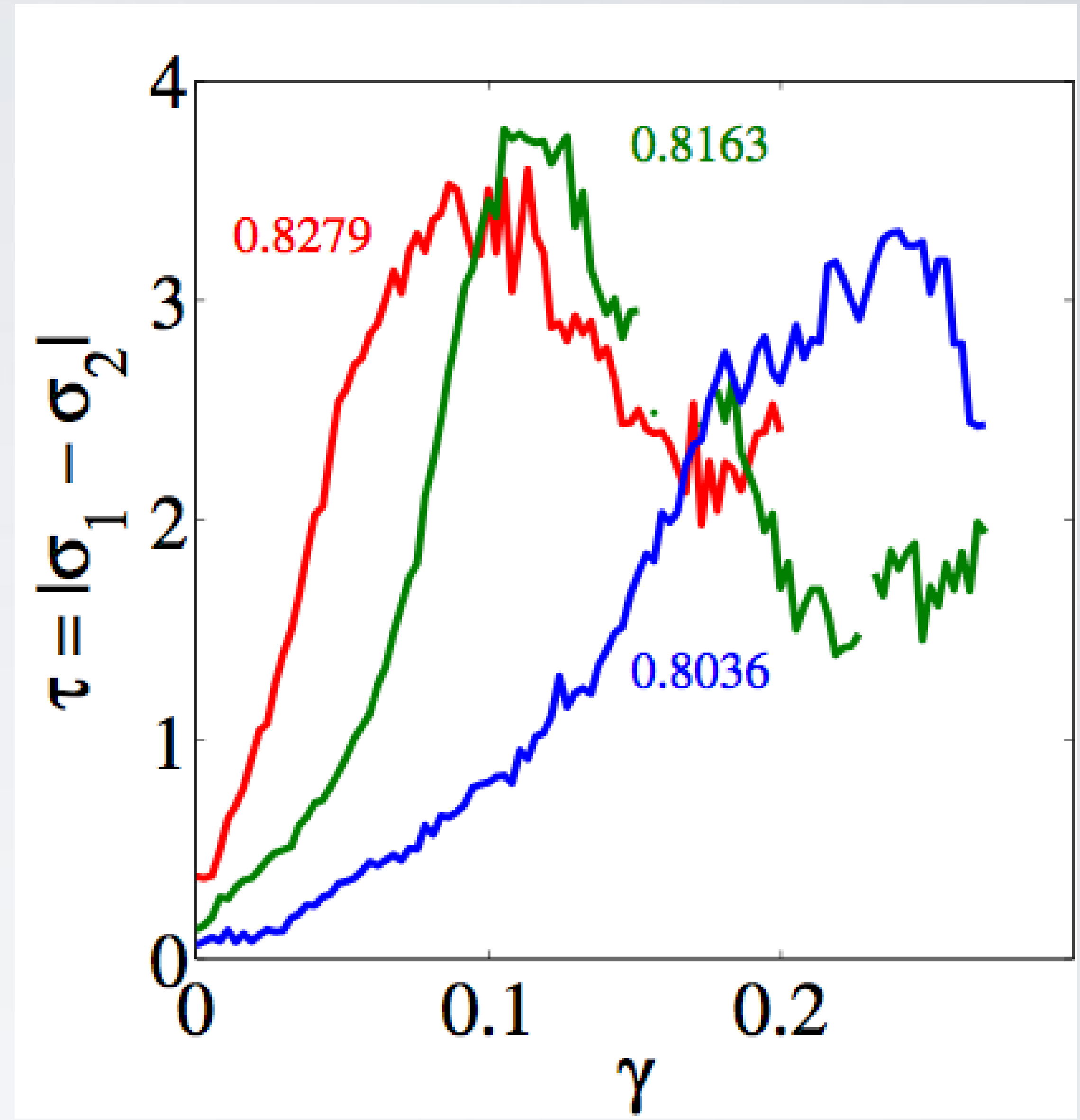






Shear Stress vs Strain

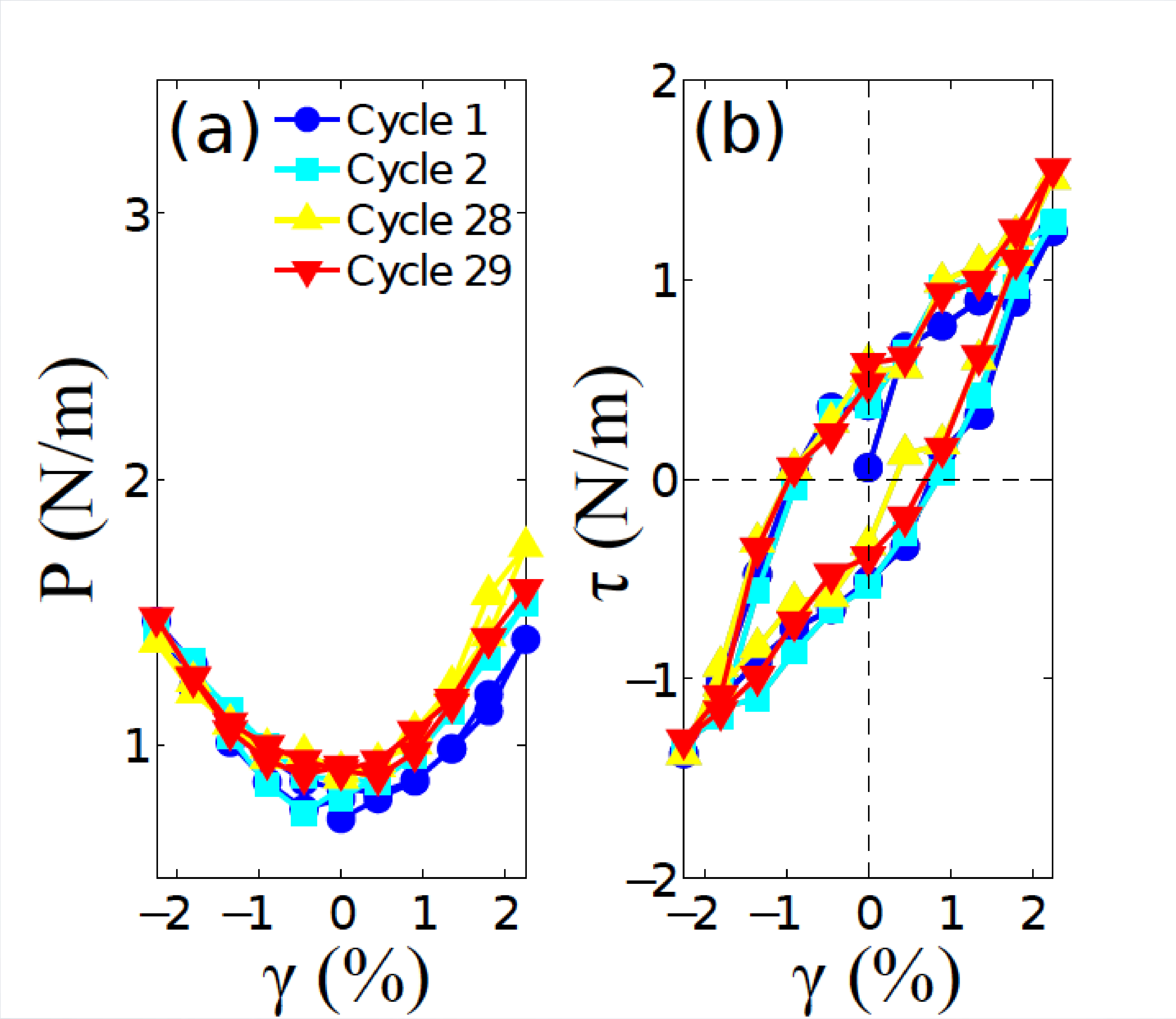




Looks deceptively like an elastic regime but is not

Curves for different packing fractions can be scaled: the strain at which the stress peaks, goes to zero as the packing fraction approaches that of the isotropic state.

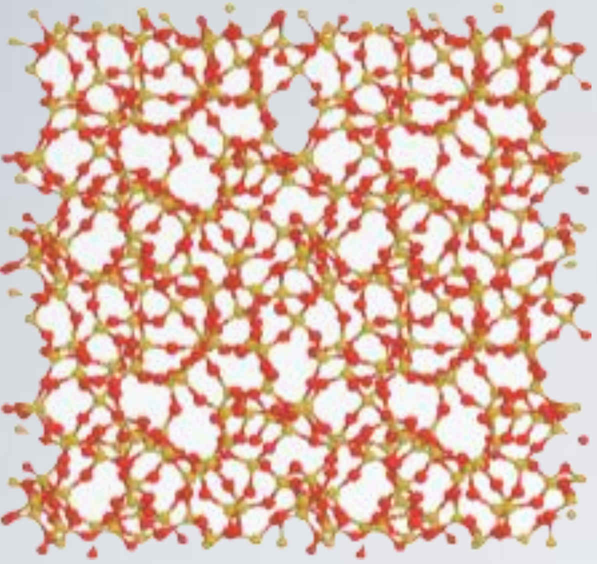
Cyclic Shear

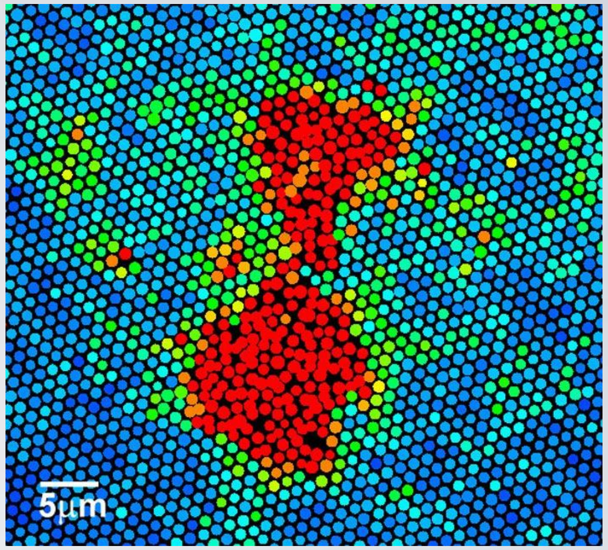


Packing Fraction = 0.825

Small strains: below peak of shear stress curve

Rigidity of amorphous solids





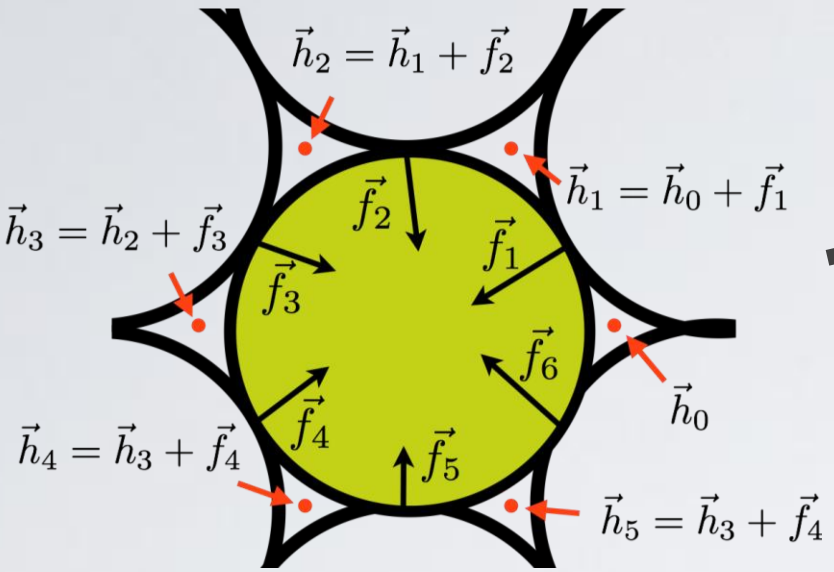
- Permanent density modulations distinguish solids from liquids: a necessary condition for shear rigidity.

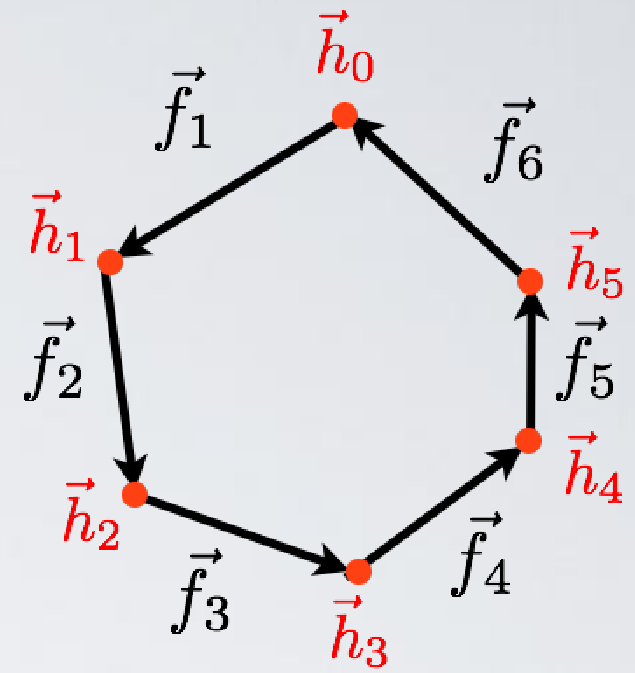
- Broken translational symmetry (not obvious because structure is not crystalline) Positional correlations : not destroyed by small thermal fluctuations, average survives

- Traditionally: energy or entropy gain leads to solidification

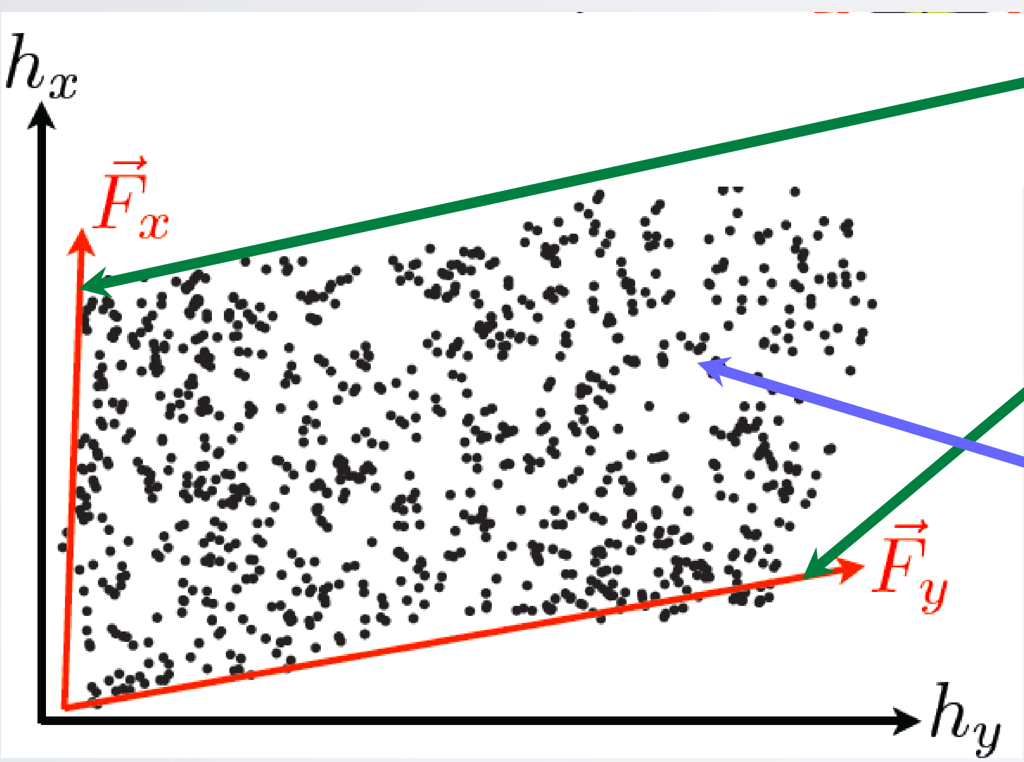
- The granular story has to be one of CONSTRAINTS: there are no entropic (thermal) or energetic preferences for any positional patterns.

Looking at the vector, height fields only





Height difference across the boundaries: determined by boundary stresses

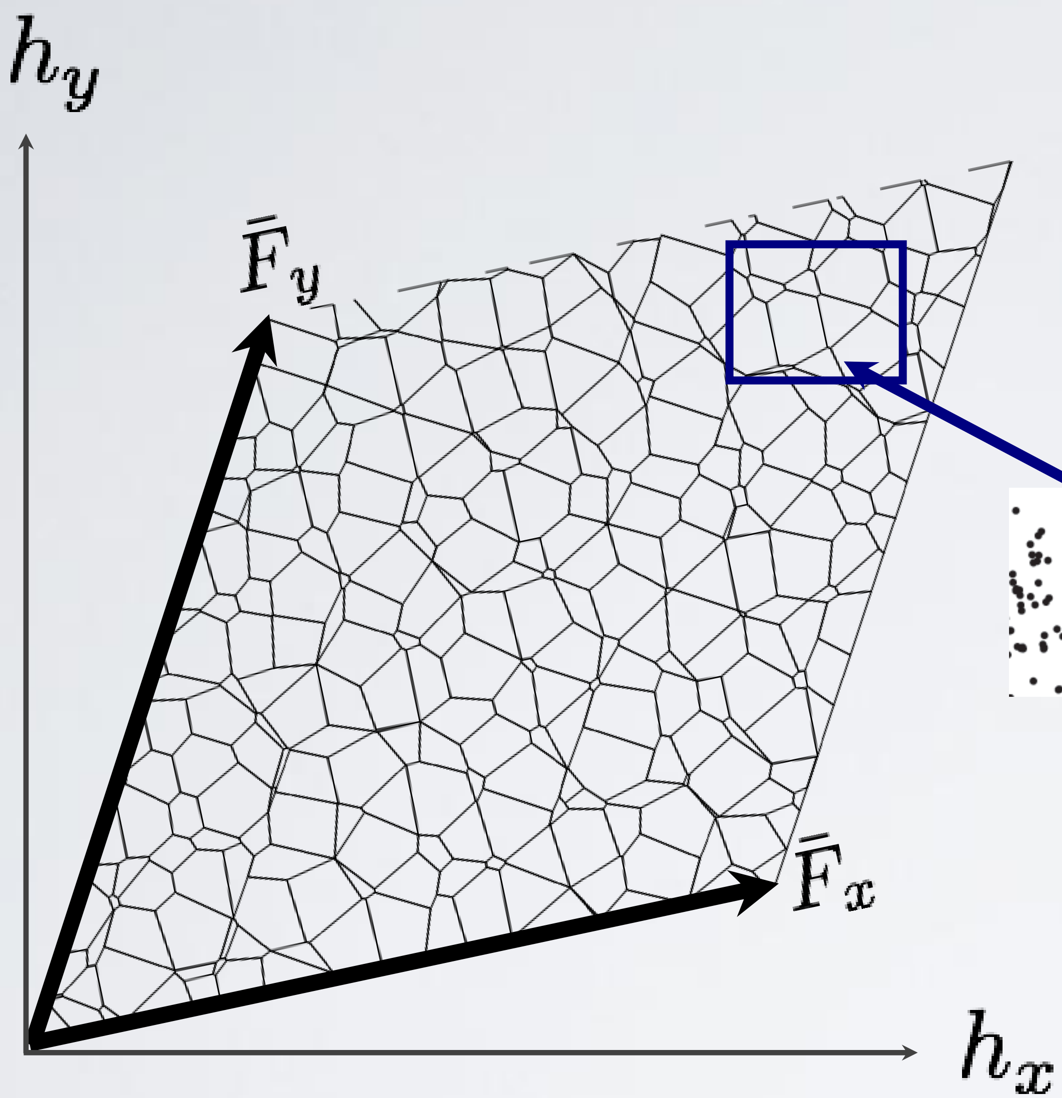


Points: height vectors at a void starting from some arbitrary origin

Reciprocal space: a tiling of polygons

for systems where forces are all repulsive, we have one sheet of tiling

Torque Balance | Friction law on each contact



- IF these introduce correlations, then maybe ...
- Example: Polygons have to be convex for frictionless grains

$$\langle \rho(\vec{h}) \rangle \neq const$$

- Change bounding box: points cannot rearrange freely
- Rigidity
- Changing bounding box is the analog of straining a spring network
- Spring constant is not fixed but depends on configuration

$$\text{Changing } \vec{F}_x, \vec{F}_y \longrightarrow \text{distorting } \langle \rho(\vec{h}) \rangle$$

Test of persistent pattern in height space

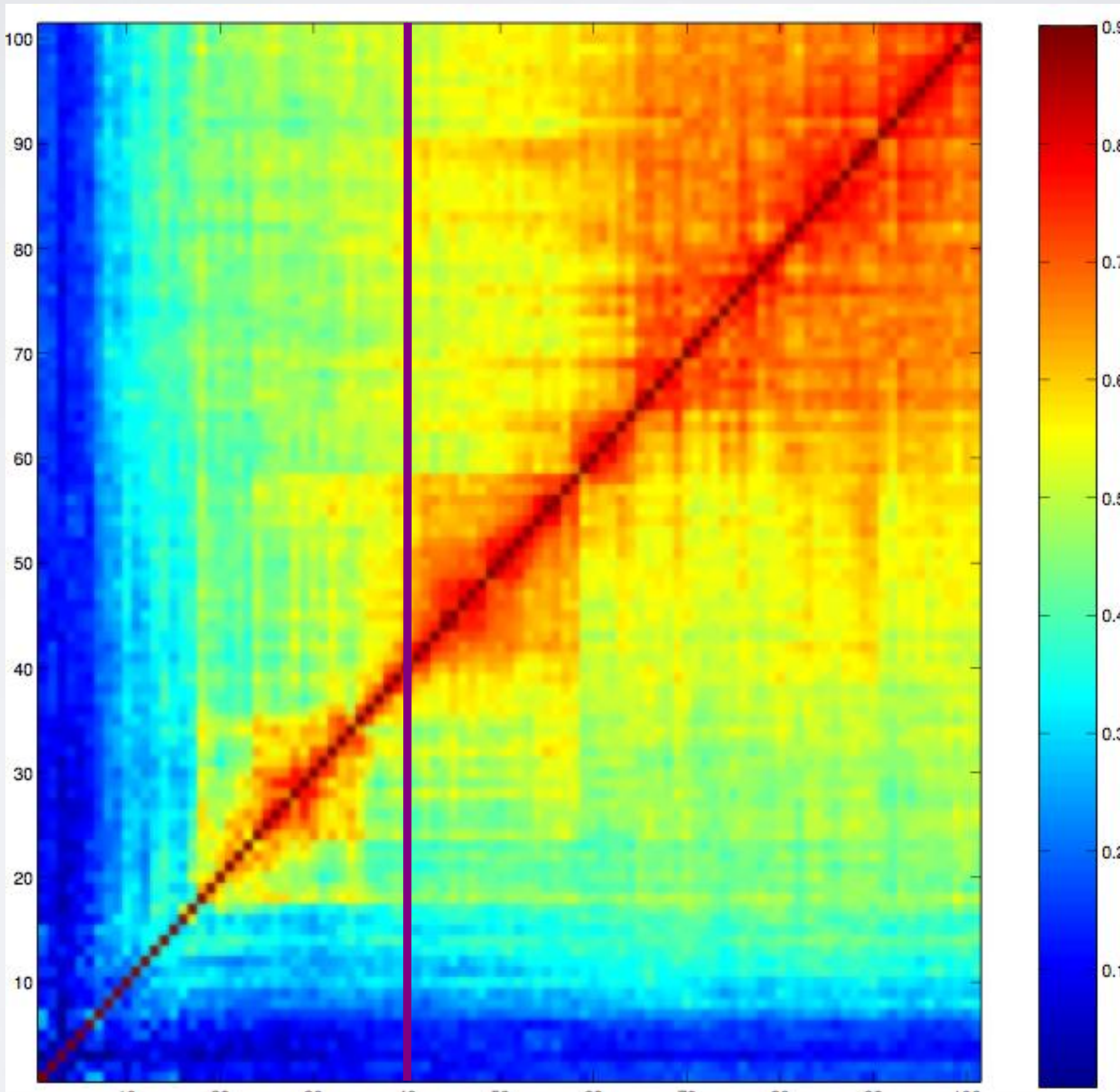
Overlap between two configurations

Grid stretched affinely with bounding box

$$d^{\alpha,\beta} = \sum_{m,n} \rho^{\alpha}_{m,n} \rho^{\beta}_{m,n}$$

If height pattern evolves affinely, large overlap
The stress-generated pattern can sustain further loading

Persistence of patterns

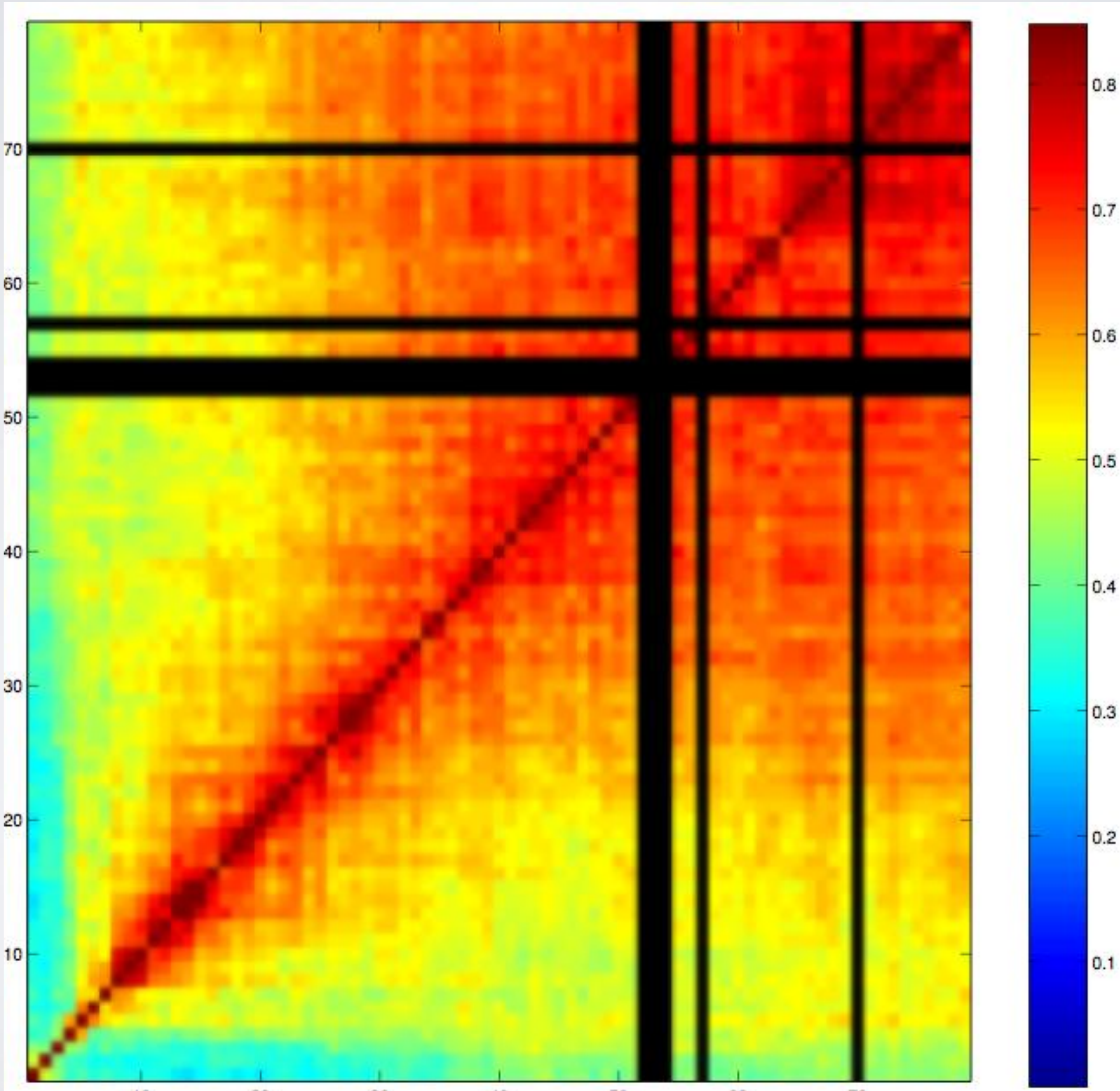


Strain Step

Strain Step

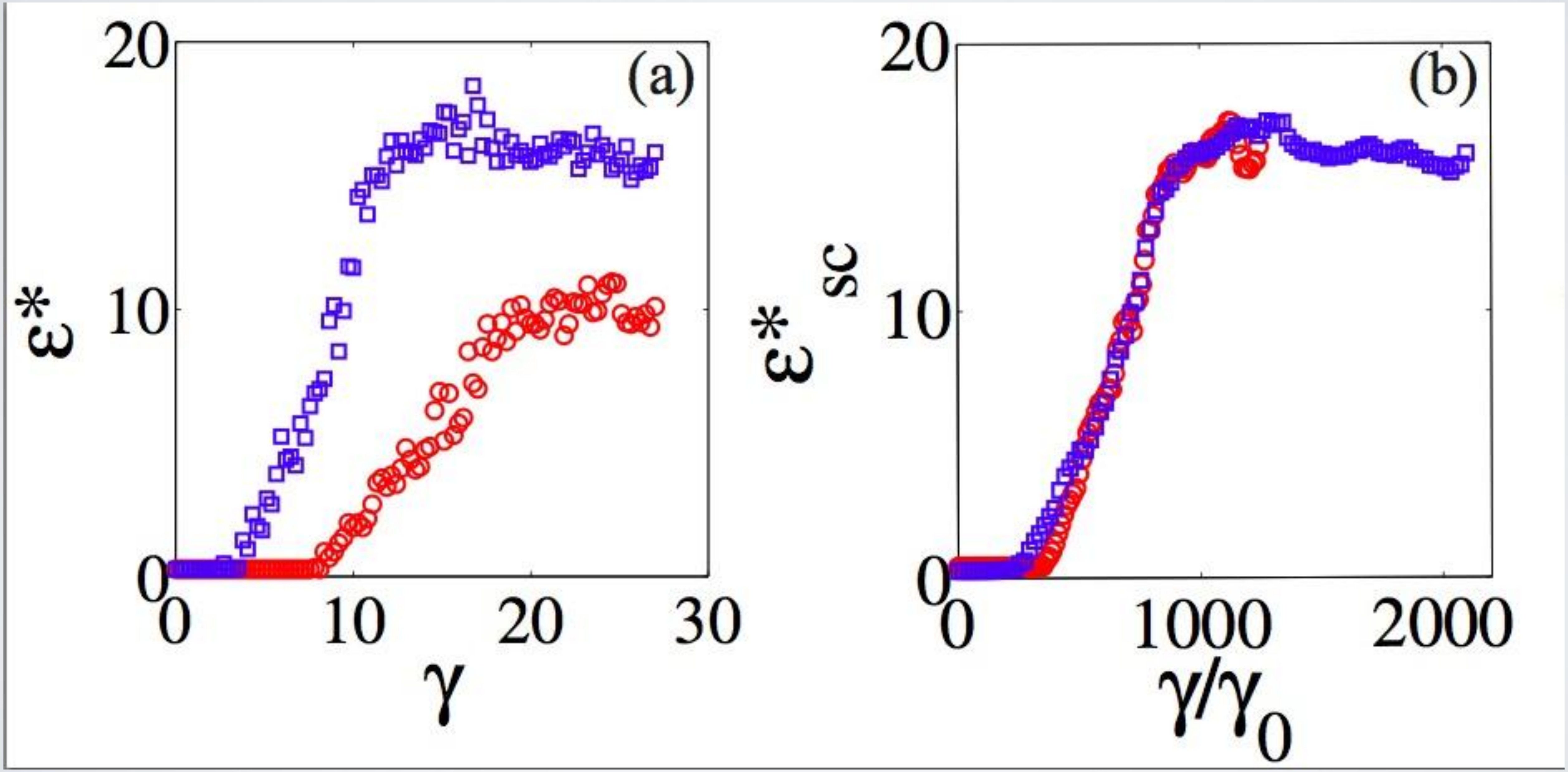
Sarkar et al PRL (2013)

Persistence of patterns

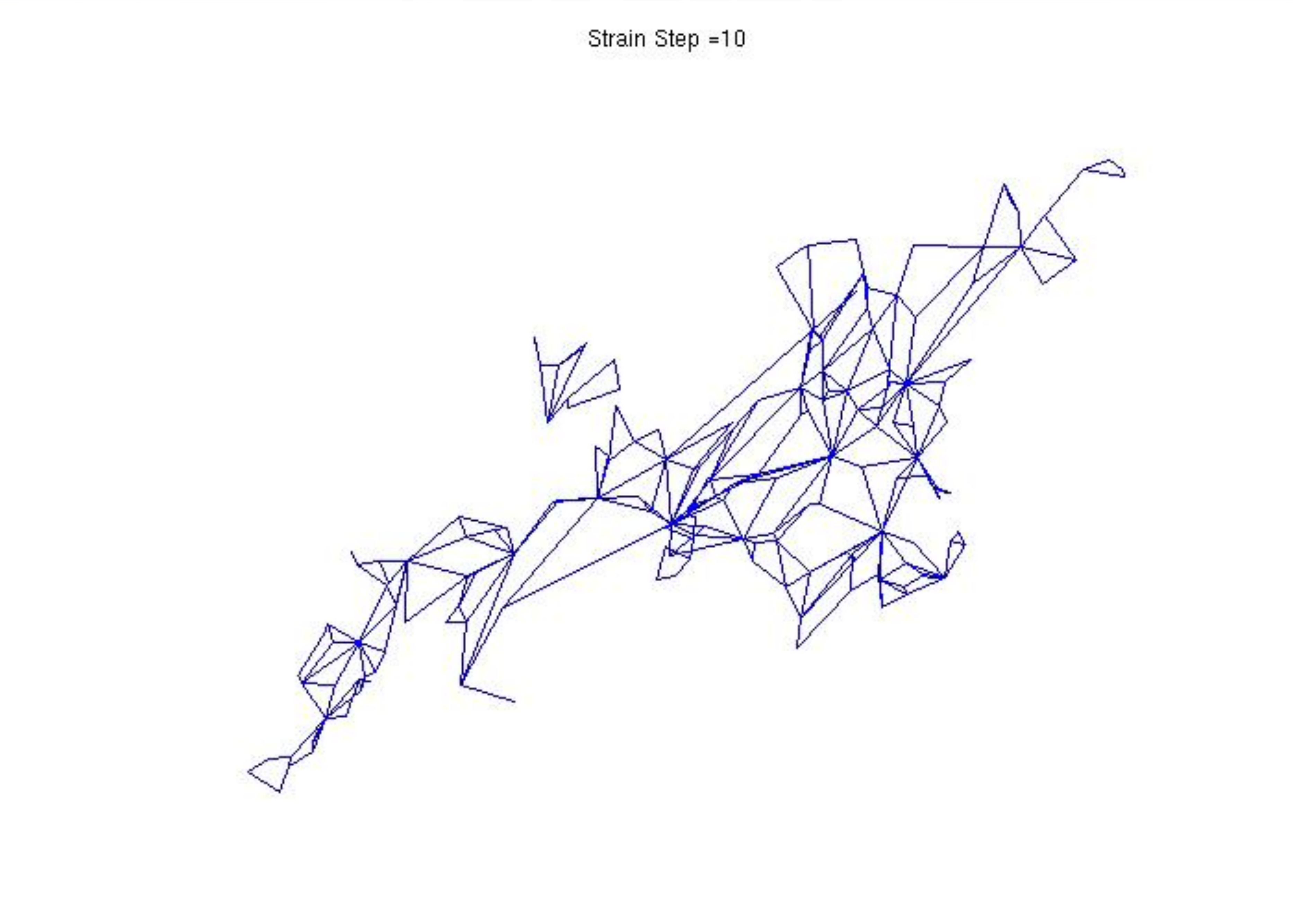


ar et al PRL (2013)

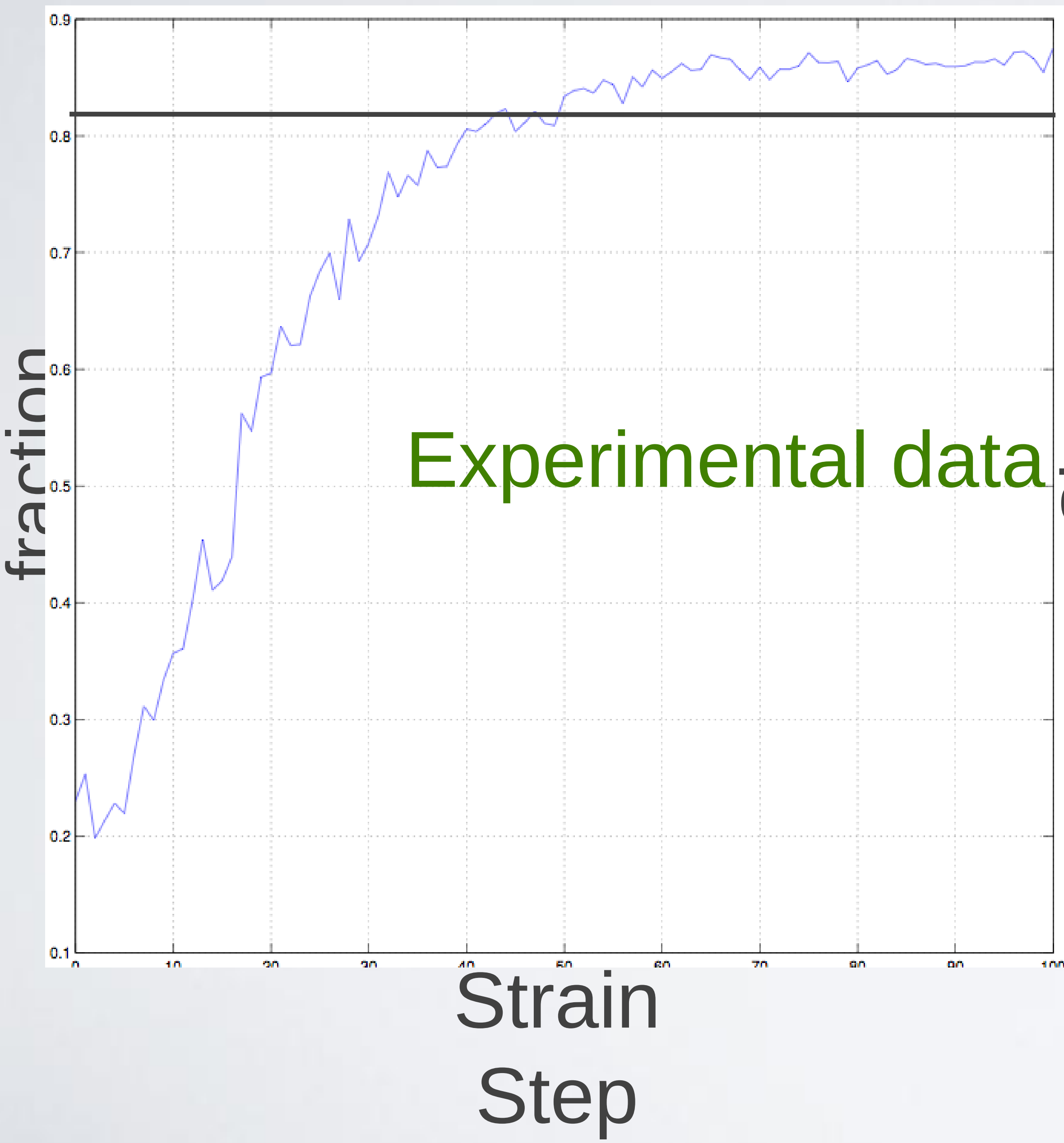
Measure of rigidity

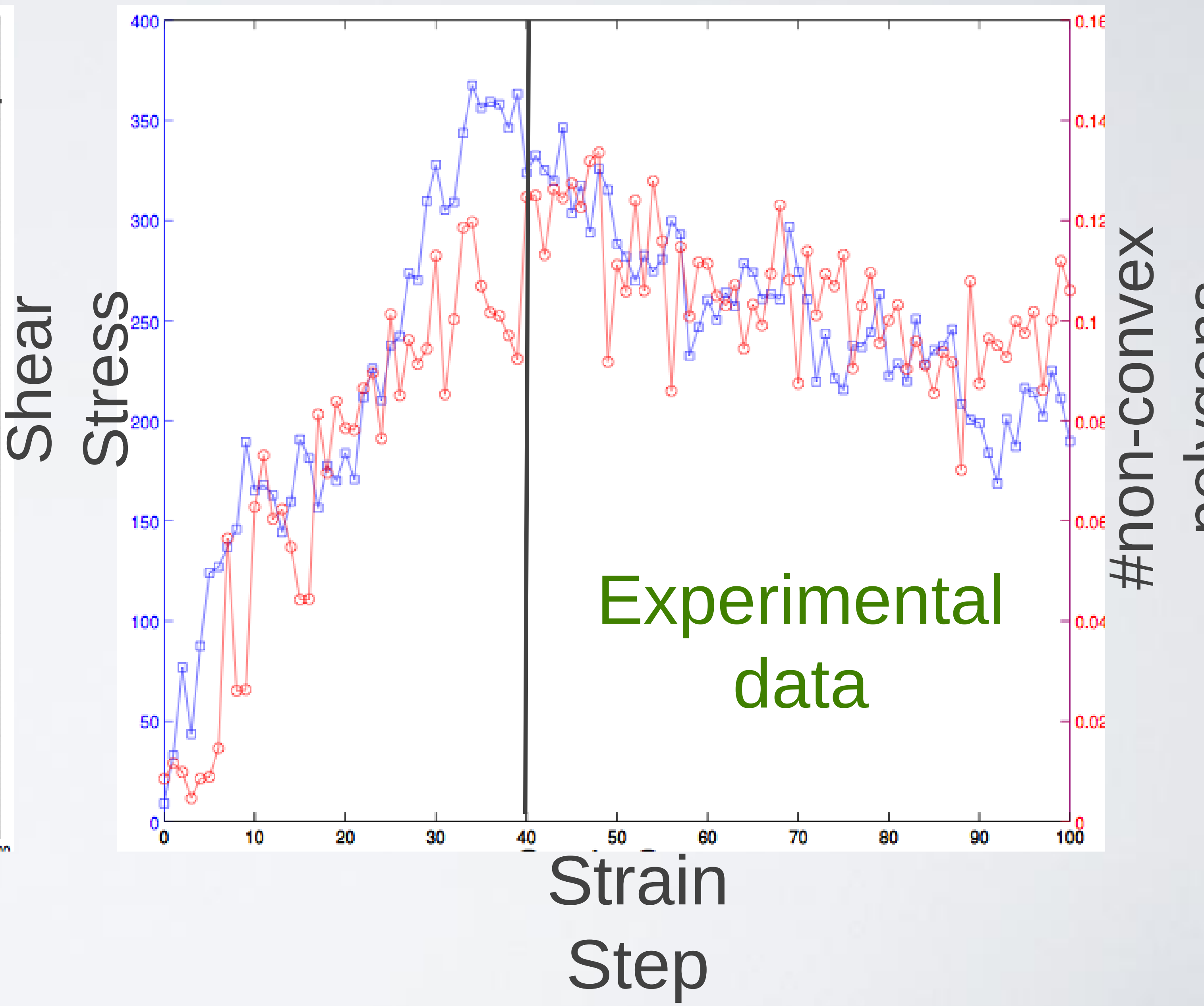


Dual to Spring Networks

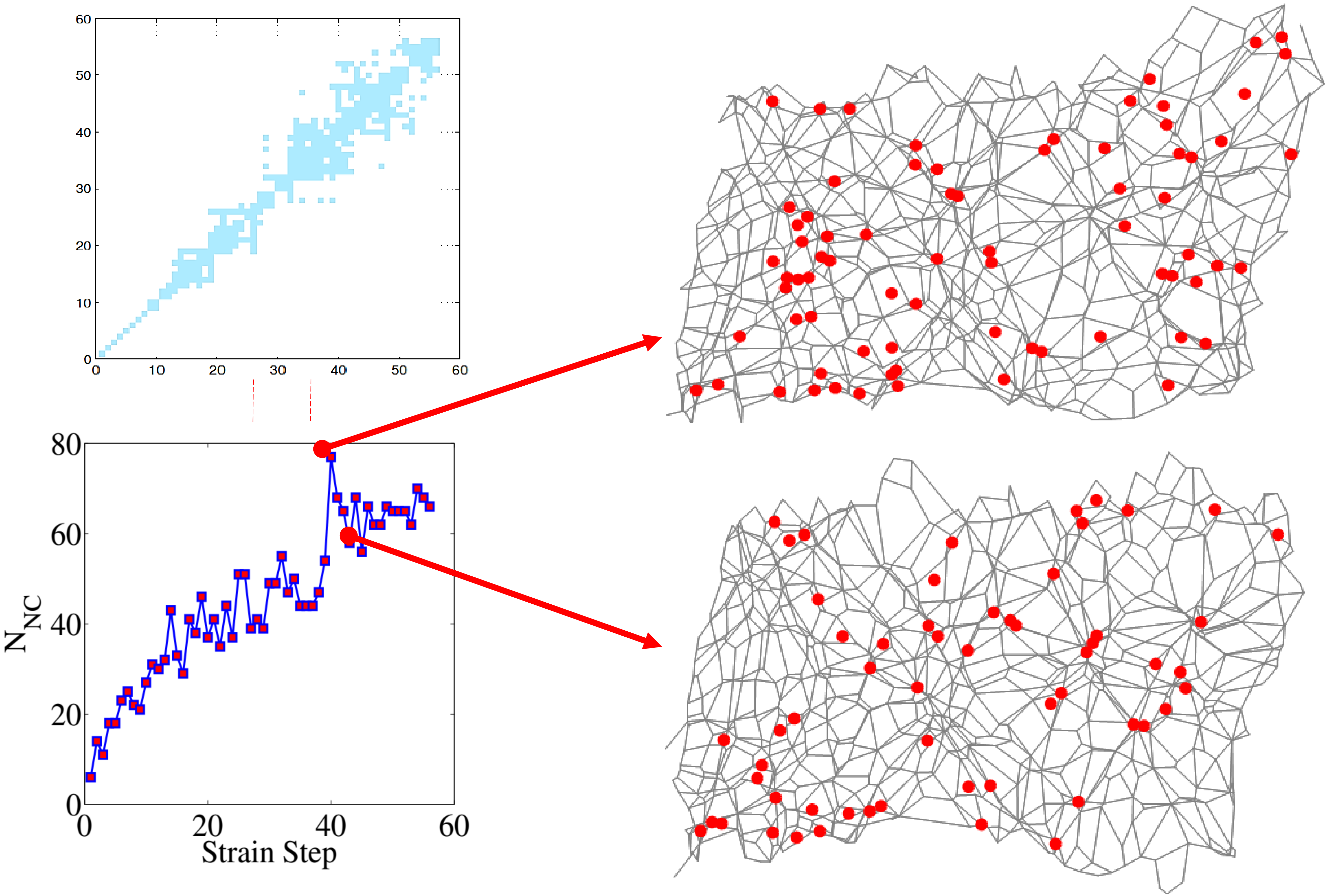


Non-convex polygons
Forward shear



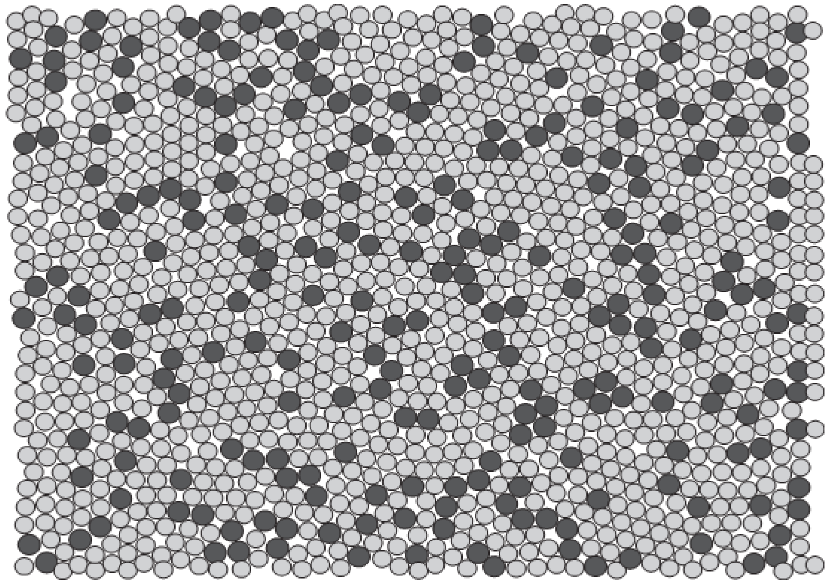


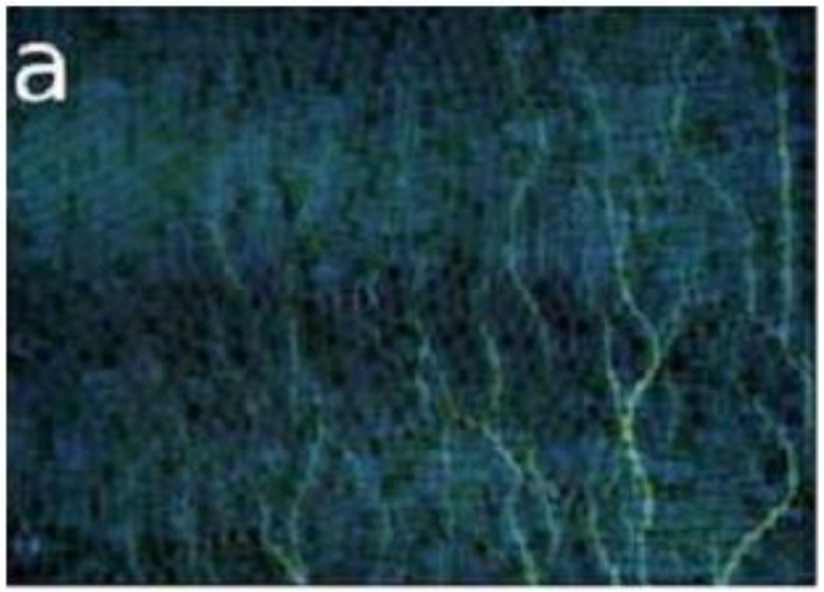
Failures: mini avalanches



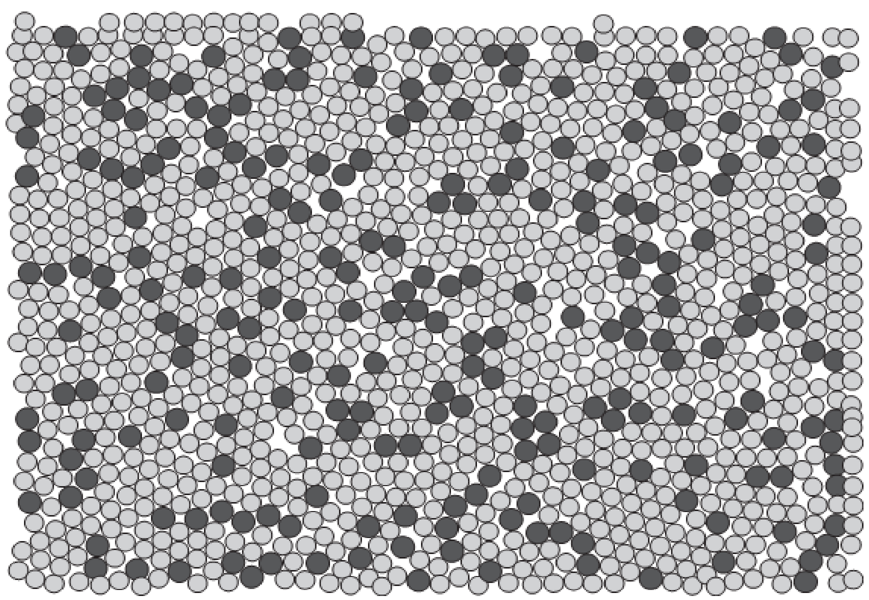
Separation of Structure and Stress

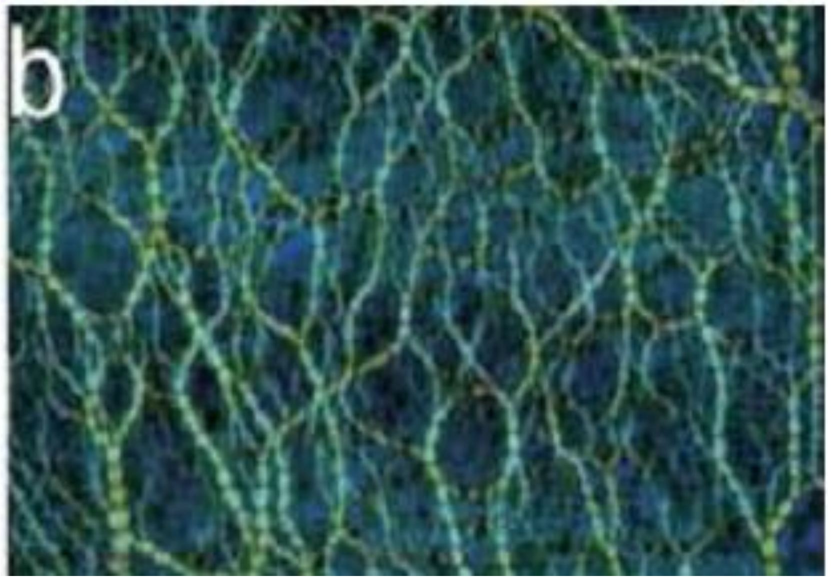
Real Space | Real space with forces | Reciprocal Space

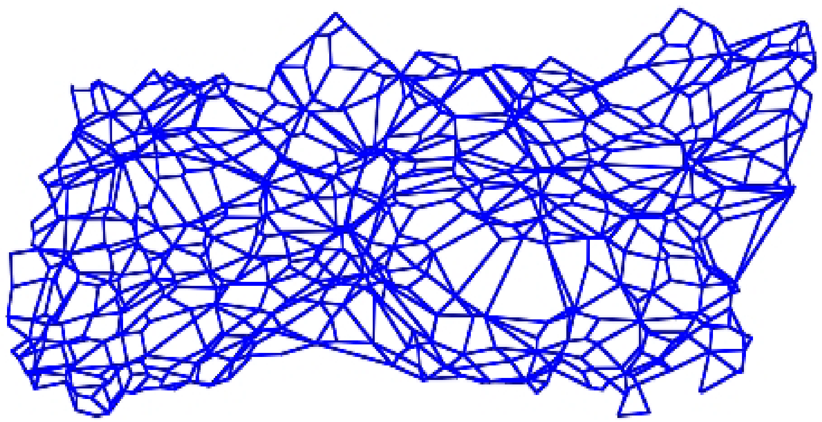




Unjammed







Jammed

Zhang et.al., Soft Matter, 2010

Sarkar et.al., PRL, 2013

“Frozen” structure, Large changes in force network