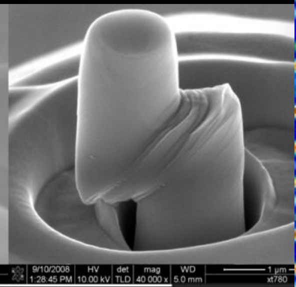
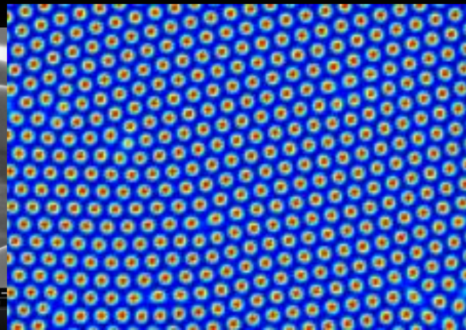
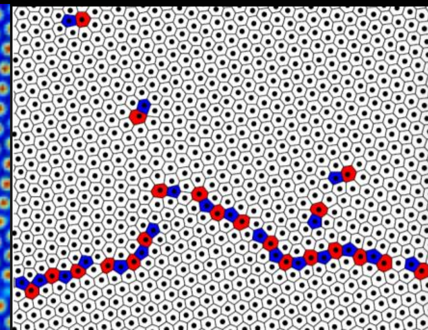

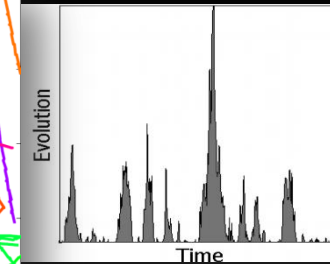

Statistics of intermittent fluctuations in dislocation-mediated plasticity

Luiza Angheluta

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University of Oslo, Norway

PGP

complex

The Research Council of Norway

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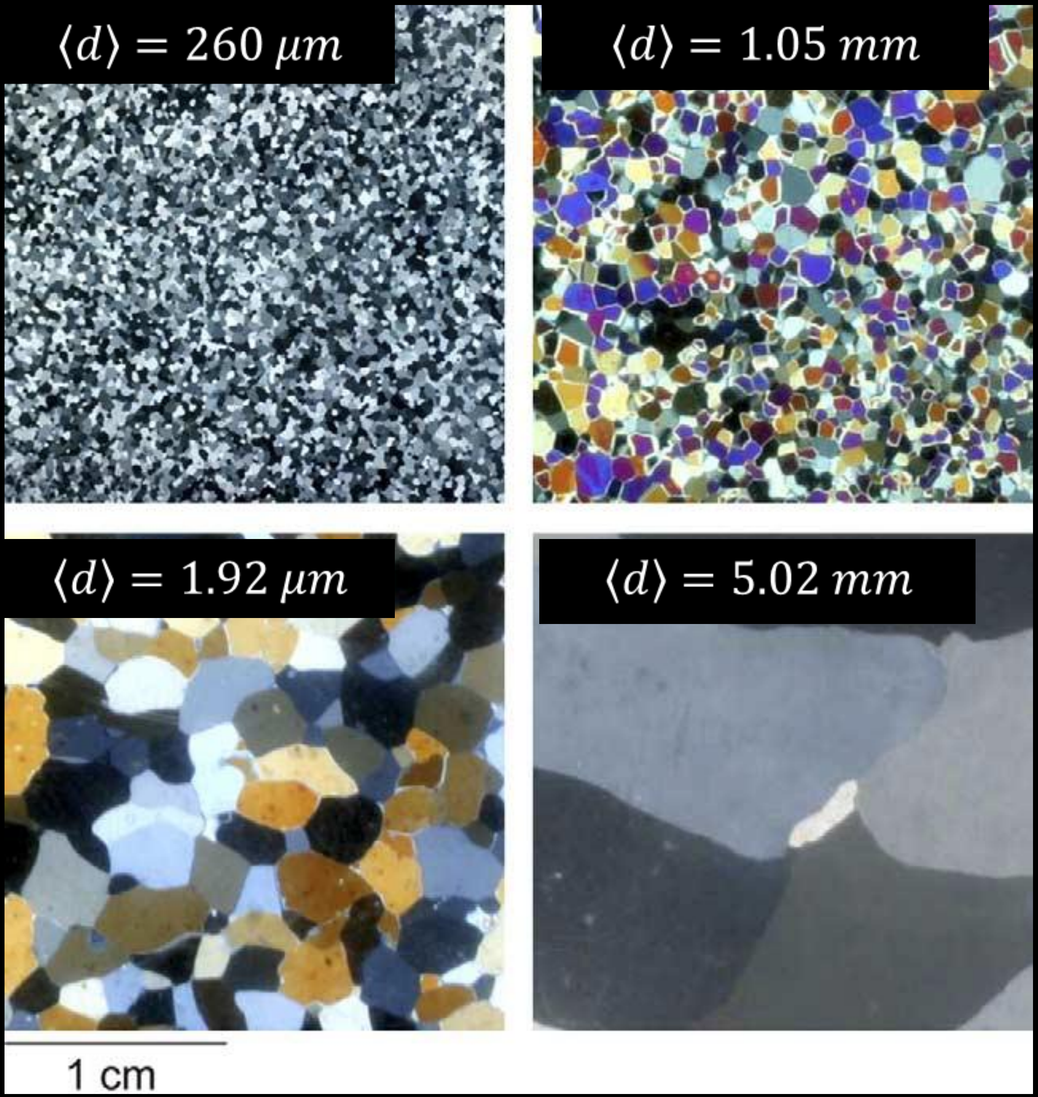
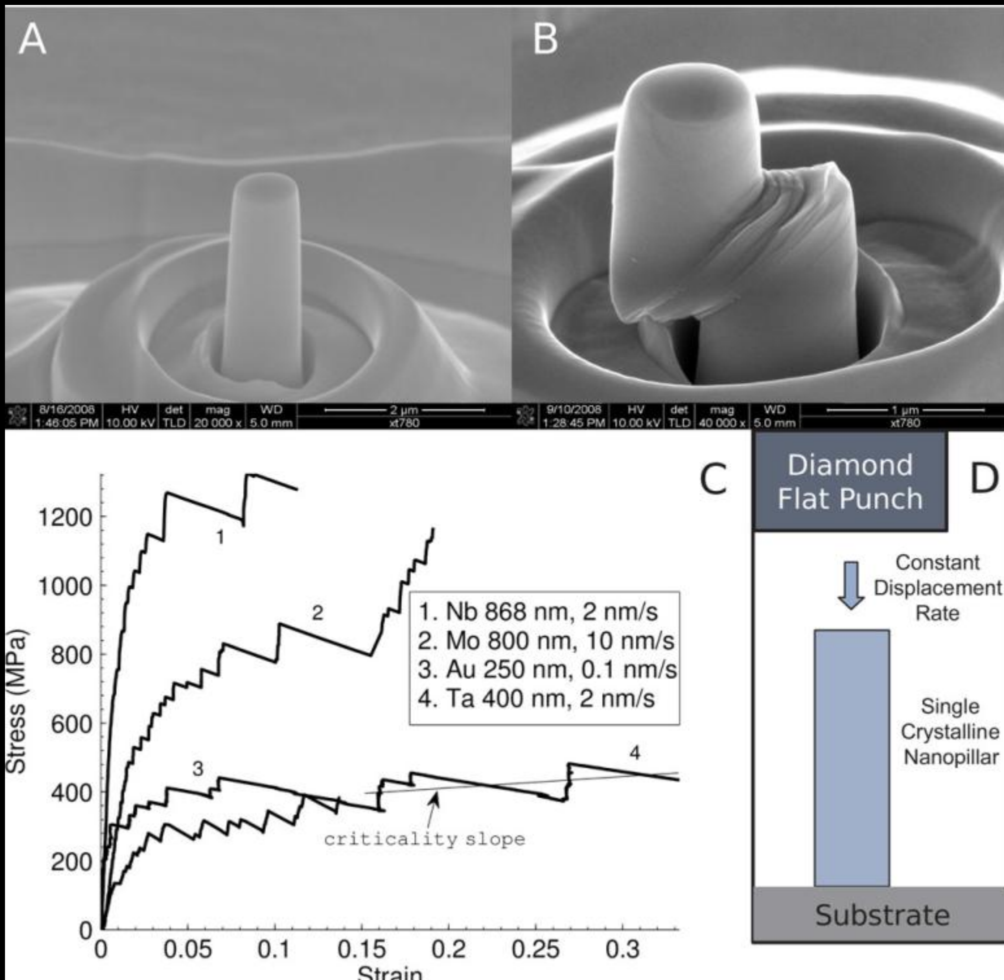
- Polycrystalline materials exhibit complex plastic deformation behavior due to the nontrivial response of single crystals
- Contrary to the smooth, macroscopic plastic flow, the small-scale deformation of single crystals is highly intermittent in time and heterogeneous in space

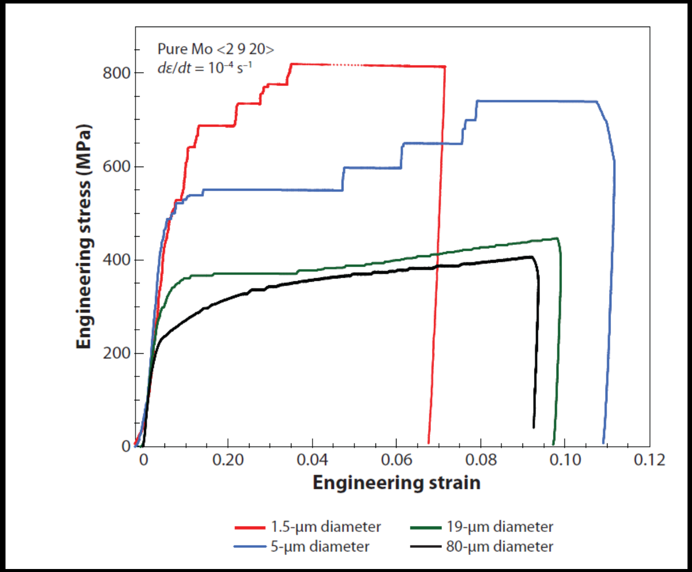


Richeton et al., Acta Materialia 53 (2005)

Intermittency in small-scale plasticity (micron and submicron scales)

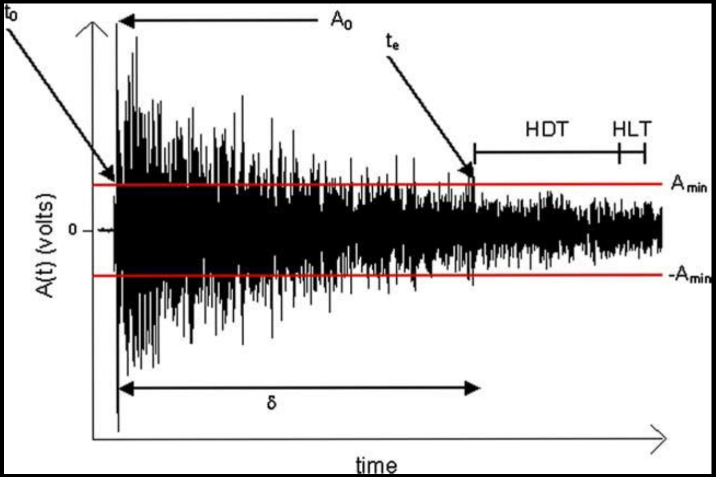


- Smaller is stronger
  $\sigma_f \sim d^{-n}$
- Bursty deformation
  $P(s) \sim s^{-\tau}$



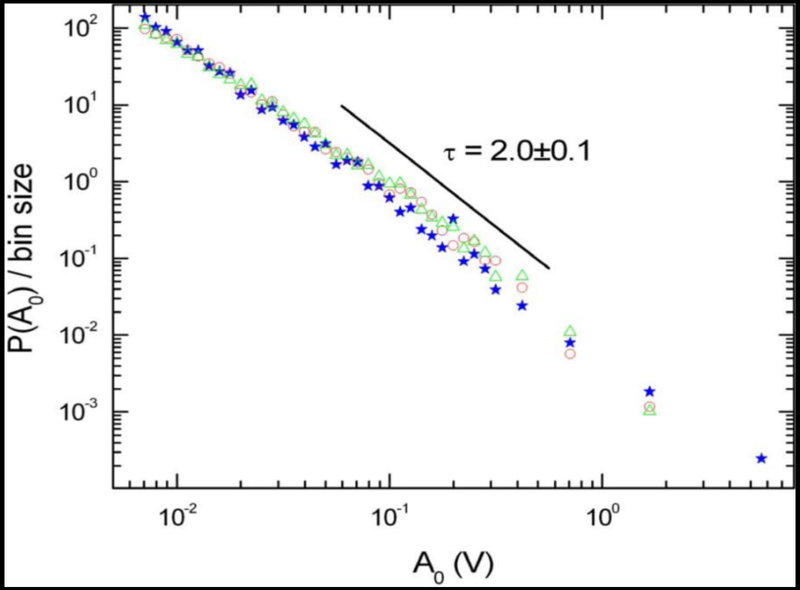
Uchic et al., Annu. Rev. Mater. Res. (2009); Friedman et al., PRL (2012);

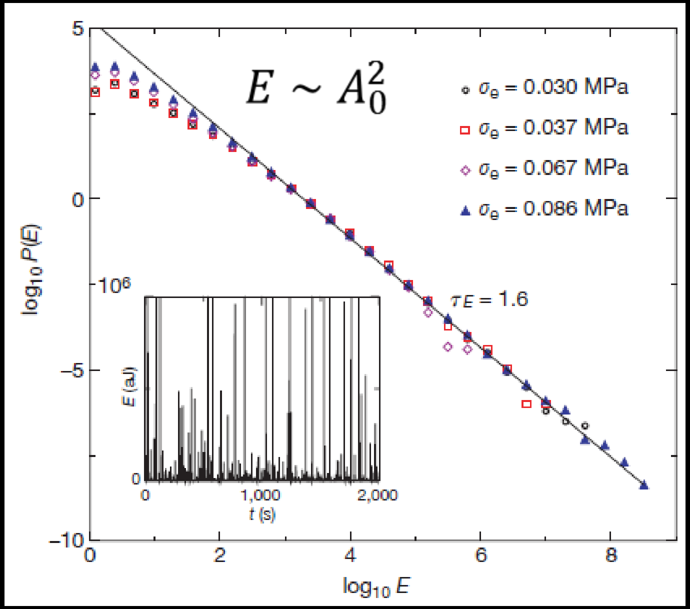
Acoustic emission measurements



Ansatz: moving dislocation ~ **sound wave**

$$A_0 \sim \dot{\epsilon}_{max} \sim b\rho_d v_{\max}$$





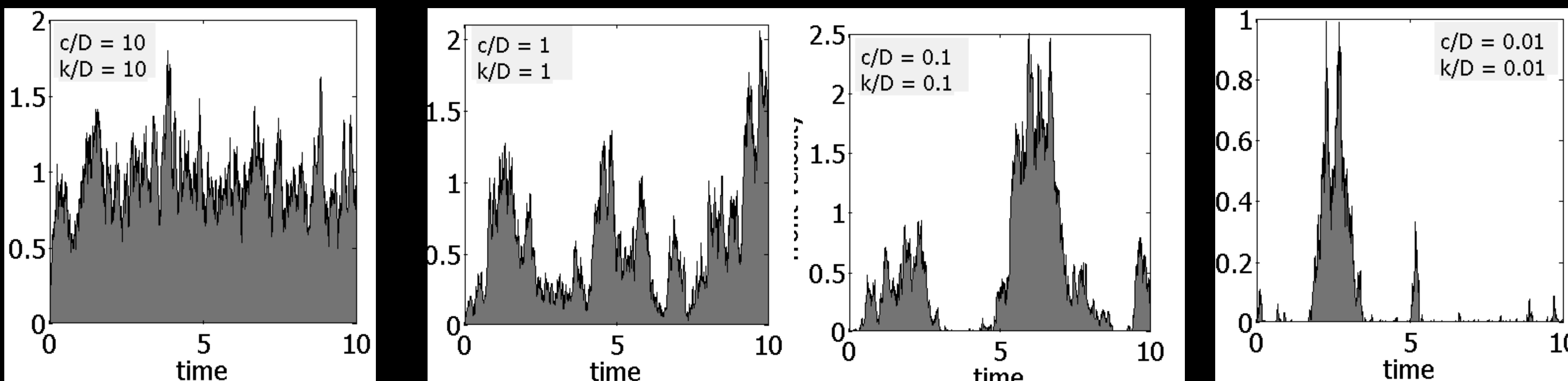
Q: Can these exponents be calculated within the mean field theory of depinning?

Weiss and Grasso *J.* Phys. Chem. B *(*1997), Richeton et al. Acta Materialia (2005), Richeton, et al. Mat. Sci.Eng . (*2006)*

Fluctuations of global strain rate $\dot{\epsilon} \sim b\rho_d v$

Described by the mean field interface depinning model

$$\frac{dv}{dt} = -kv + c + \sqrt{v}\xi(t), \langle \xi(t)\xi(t') \rangle = 2\sigma\delta(t - t')$$

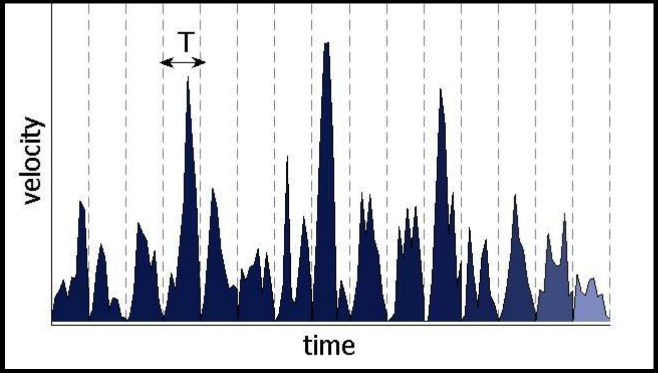


As the driving rate approaches critical point, the dynamics becomes intermittent.

Depinning critical point $c \to 0, k \to 0$

B. Alessandro et al. J.Appl.Phys. (1990)

Extreme value statistics of avalanches



$$\max_{0\leq t\leq T} v(t)\,, with\ v(0) = v(T) = \varepsilon\ and\ T\ fixed$$

Cumulative distribution of the maximum velocity

$$C\left(\max_{0\leq t\leq T} v(t) \leq v_m | T\right) = \frac{\int_{v(0)=\varepsilon}^{v(T)=\varepsilon} \mathcal{D}v(t) P(\{v(t)\}) \prod_{0\leq t\leq T} \Theta(v_m - v(t))}{\int_{v(0)=\varepsilon}^{v(T)=\varepsilon} \mathcal{D}v(t) P(\{v(t)\})}$$

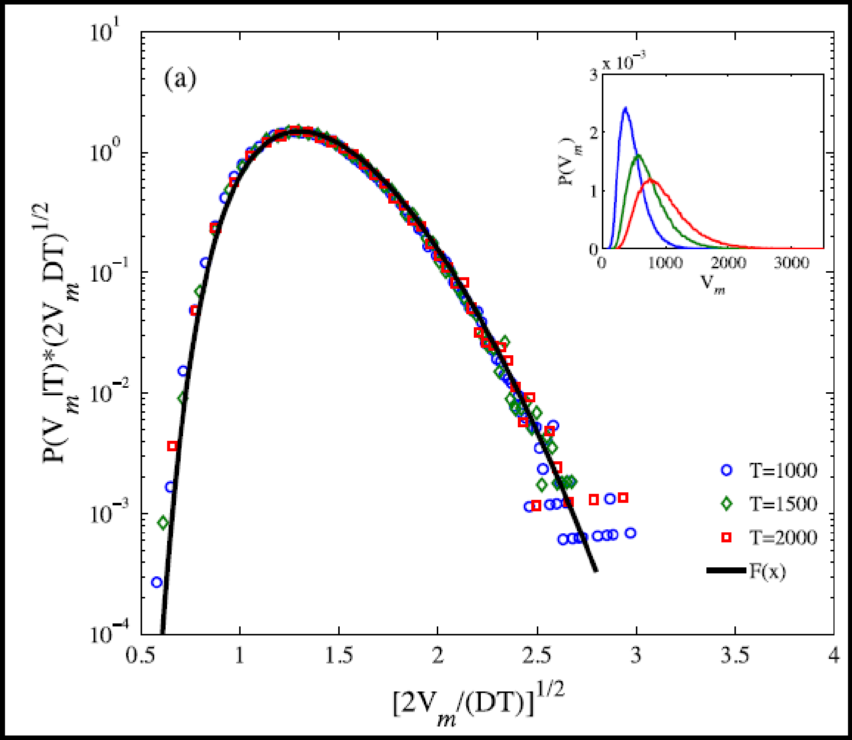
For a discrete, uncorrelated, and identically distributed velocity signal

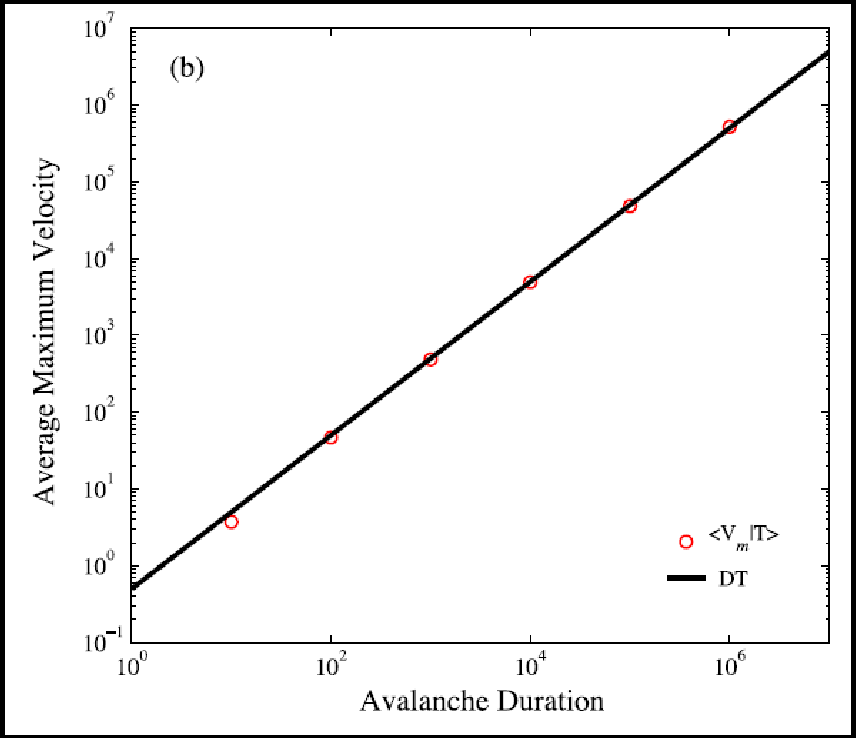
$$C_N\left(\max_{n=1,\cdots,N} v_n \leq v_m\right) = \left(\int_0^{v_m} dv\, p(v)\right)^N$$

Avalanche velocity is strongly correlated, and non-identically distributed due to boundary conditions

M. LeBlanc**, L. Angheluta**, K. Dahmen, N. Goldenfeld, PRL (2012)

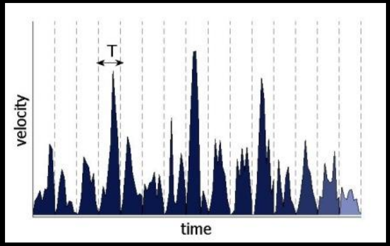
Extreme statistics of avalanches





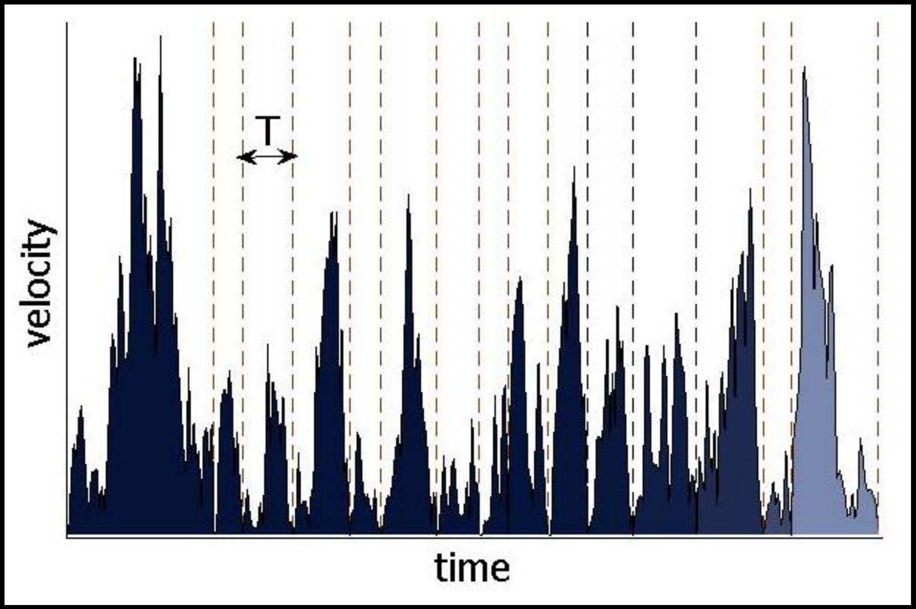
Universal scaling form

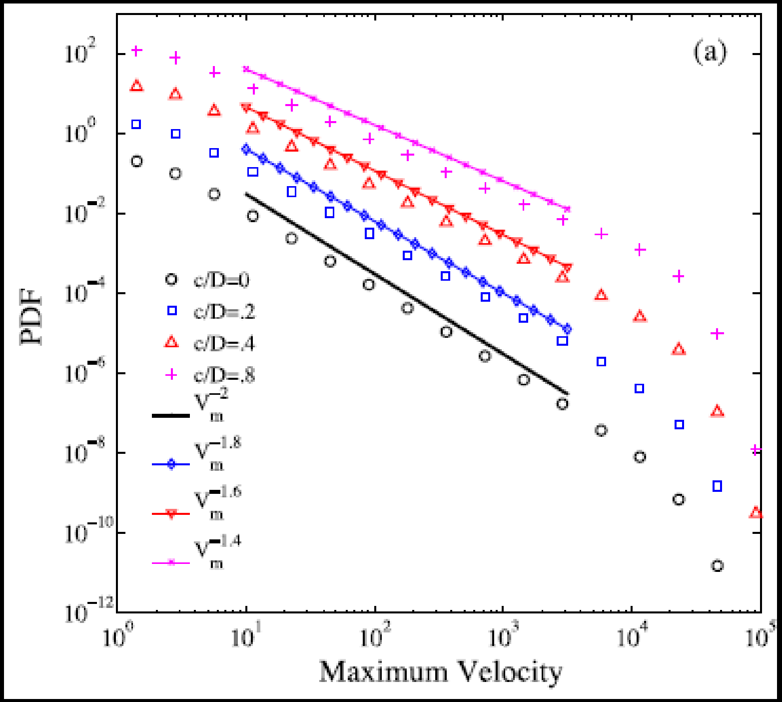
$$P(v_m|T) = \frac{1}{\sqrt{2\sigma T v_m}} F\left(\sqrt{\frac{2v_m}{\sigma T}}\right)$$



M. LeBlanc, **L. Angheluta**, K. Dahmen, N. Goldenfeld, PRL (2012), PRE (2013)

Extreme statistics of plastic bursts

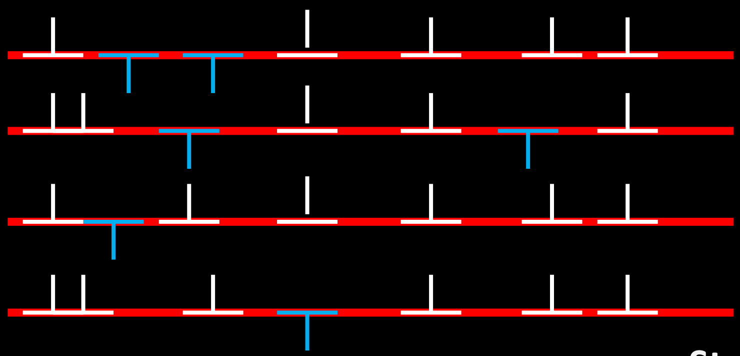




- $v_m = \max_{0 \le t \le T} v(t)$, *where T is arbitrary*
- $P(v_m) = \int dT\, D(T) P(v_m|T)$, $\;P(\mathrm{v_m}) = v_m{}^{-2+c} G\left(\frac{k\, v_m}{2\sigma}\right)$
- Exponent depends on how far away the system is from the critical point.

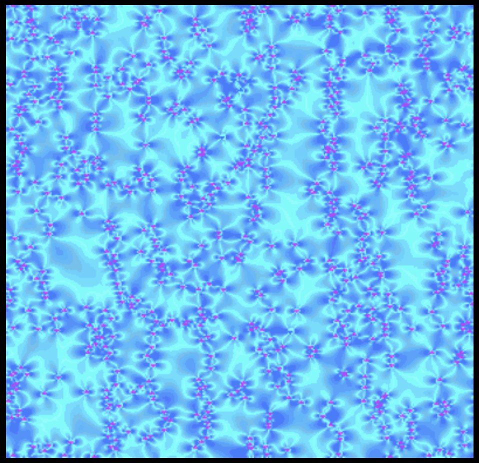
M. LeBlanc, **L. Angheluta**, K. Dahmen, N. Goldenfeld, PRL (2012), PRE (2013)

From global to local properties: Collective dislocation dynamics

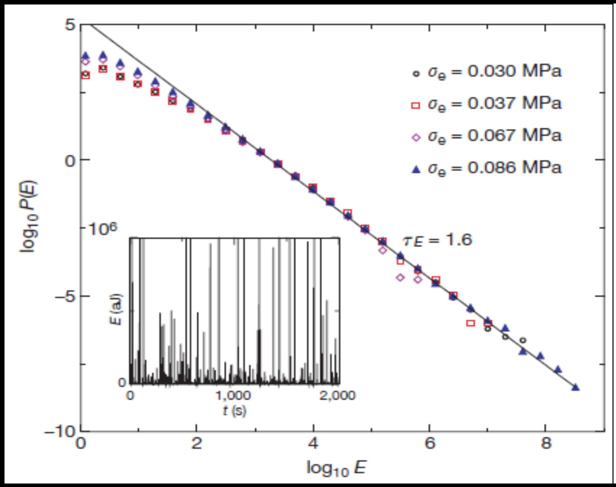


$$\sigma_{xy}(\vec{r}) = b\mu \frac{x(x^2 - y^2)}{2\pi(1-\nu)(x^2+y^2)^2}$$

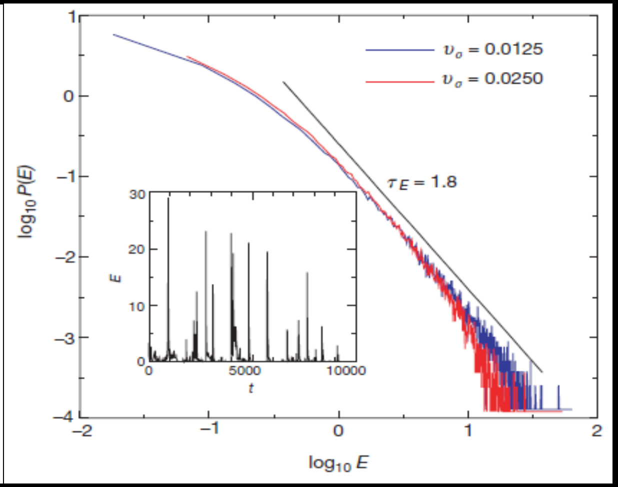
$$\frac{\eta}{b_i}\frac{dx_i}{dt} = b_i \sum_{j \neq i} \sigma_{xy}\left(\vec{r_j} - \vec{r_i}\right) - b_i \sigma_e\,, \qquad i = 1 \cdots N$$



Statistics of acoustic energy bursts in ice single crystals



Statistics of energy bursts in dislocations dynamics



M. C. Miguel et. al, Nature (2001)

Q: How is the local stress or dislocation velocity distributed due to the long-range interactions?

Mapping to Dyson’s model (1962)

Consider a population of edge dislocations of the same Burgers vectors along an isolated slip line

$$\frac{dx_i}{dt} = \sum_{j\neq i} \frac{1}{x_i - x_j} - \kappa x_i + \xi_i(t), \qquad i = 1 \cdots N$$

$$\langle \xi_i(t) \xi_j(t') \rangle = 2\sigma \delta_{i,j} \delta(t - t')$$

$$\sigma = \frac{2\pi(1-\nu)}{\mu b^3} \mathrm{k_B T}$$

Strength of the thermal energy relative to the mutual interaction energy

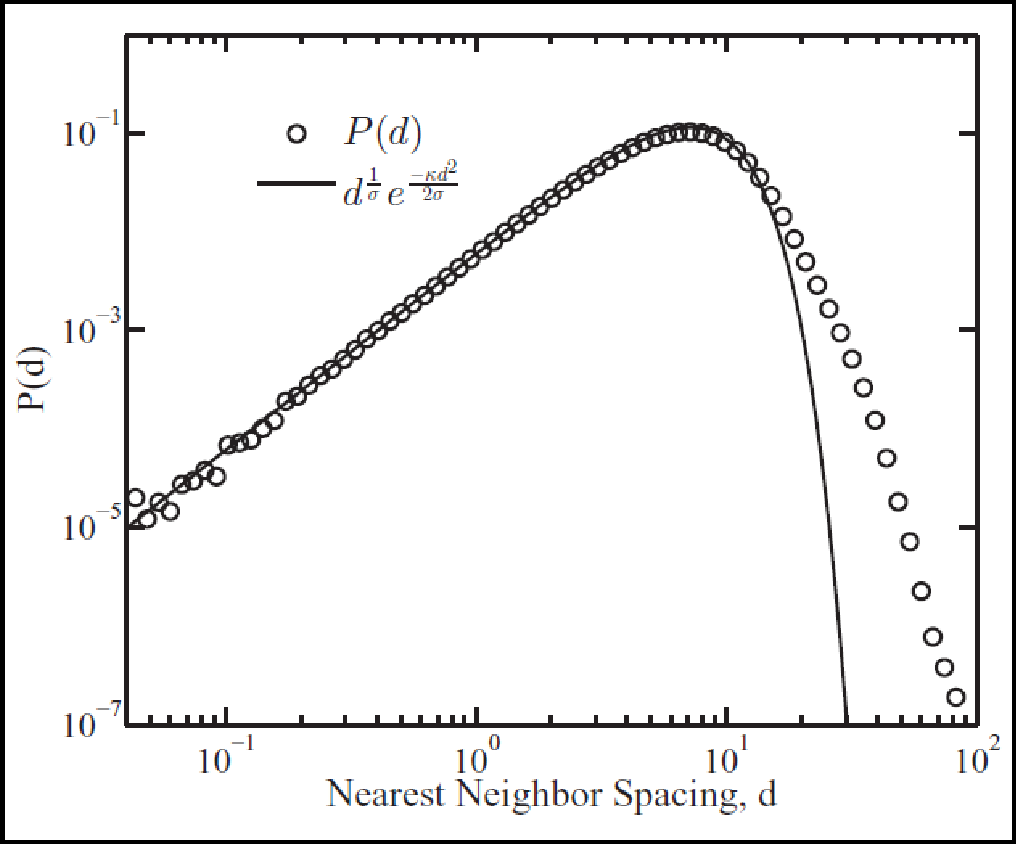
Jafarpour, **L. Angheluta**, N. Goldenfeld, PRE (2013); F. Dyson, J. Math. Phys. **3**, 1191 (1962)

Configurational probability distribution

$$\rho(x_1, \cdots, x_N) \sim \prod_{i<j} |x_i - x_j|^{1/\sigma} exp(-\frac{\kappa}{2\sigma}\Sigma_i x_i^2)$$

Probability of the nearest neighbor

$$\rho(d) \sim d^{\frac{1}{\sigma}} e^{-\kappa d^2/2\sigma}$$



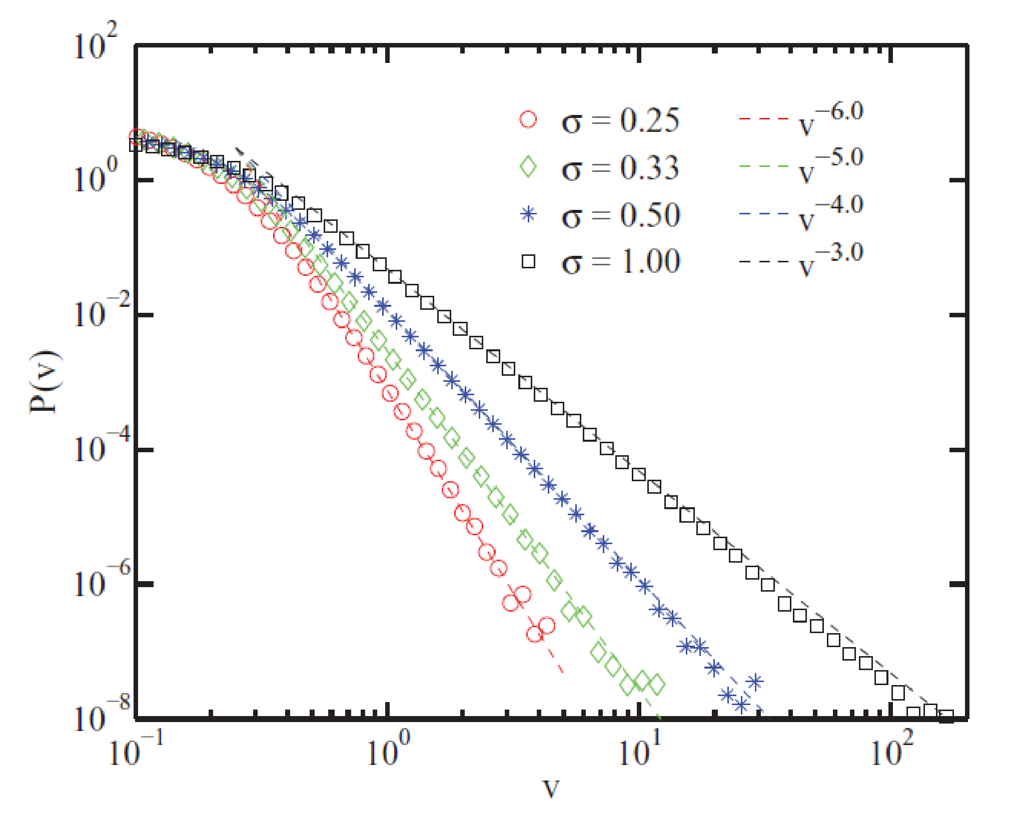
Jafarpour**, L. Angheluta**, N. Goldenfeld, PRE (2013)

Velocity probability distribution

$$P(v_1, \cdots, v_N) = \sum_{\{x_i\}} \rho(x_1, \cdots, x_N) \left| \frac{dx_i}{dv_j} \right|$$

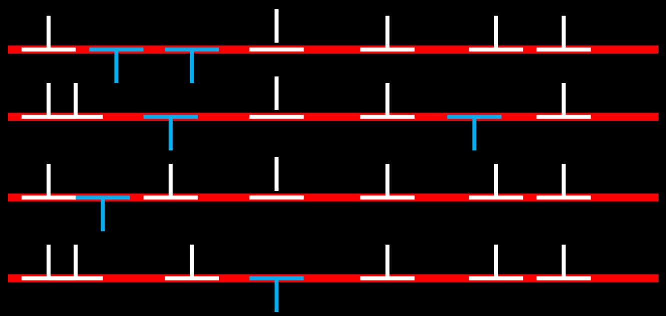
Probability of velocity induced by the nearest neighbor
(asymptotic limit of the PDF's tail)

$$P(v) \sim v^{-2-1/\sigma}$$



Jafarpour**, L. Angheluta**, N. Goldenfeld, PRE (2013)

Dyson's model of a neutral system in 2D



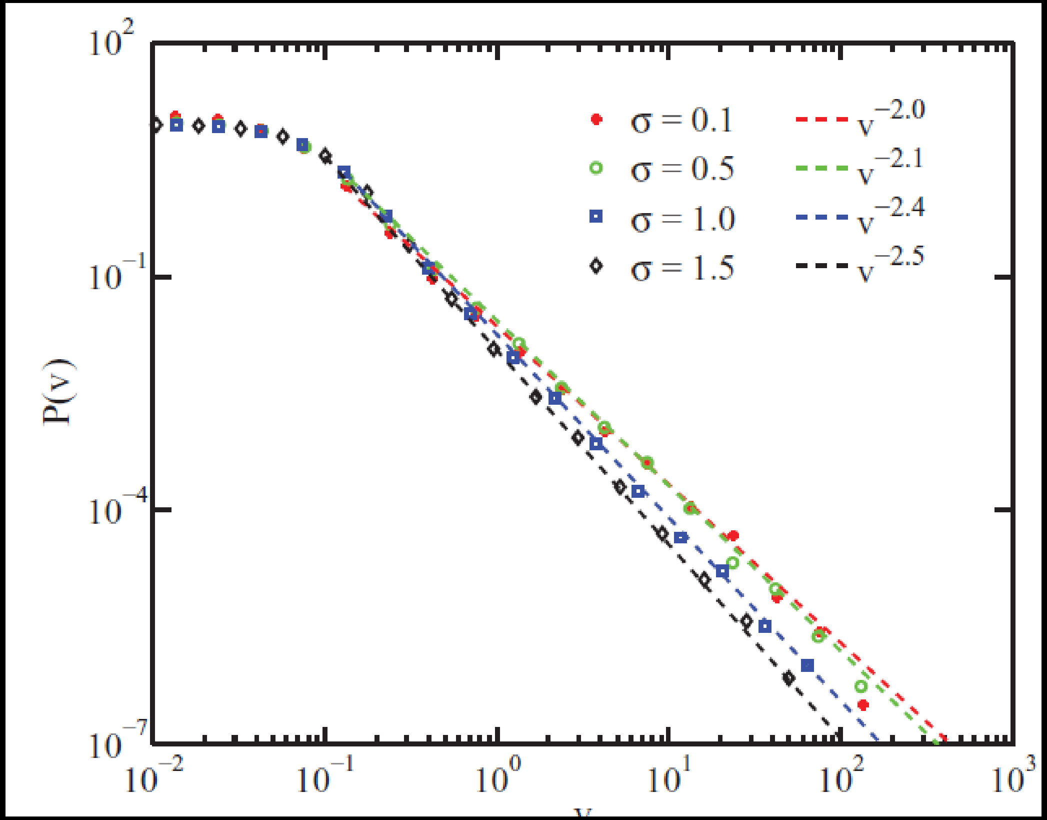
Pairing transition

$\sigma < 1$ Dislocations of opposite Burgers vectors tends to pair up

$$P(v) \sim v^{-2}$$

$\sigma > 1$ Dislocation pairs dissociate

$$P(v) \sim v^{-2.5}$$



F. Jafarpour, **L. Angheluta**, N. Goldenfeld, PRE (2013)

What is the effect of dislocation density fluctuations on the intermittency of global strain rate?

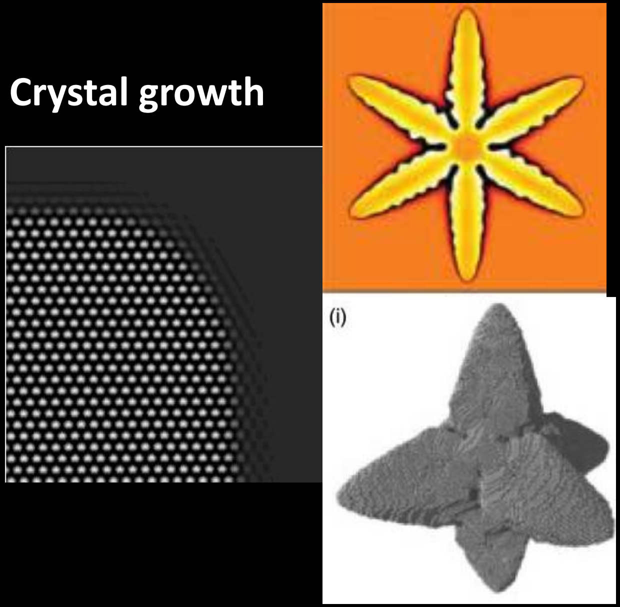
- Most numerical studies consider fixed dislocation density or ad-hoc rules of dislocation reactions & nucleations
- Instead we use an order parameter model (phase field crystal) where dislocations are emergent structural defects in the crystalline order parameter

Phase Field Crystal

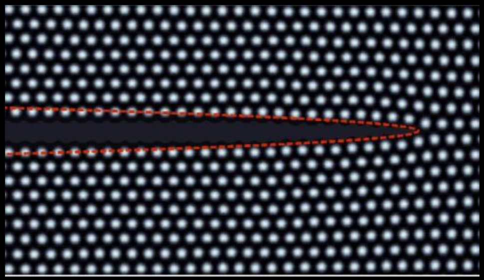
$$\frac{\partial^2 \rho}{\partial t^2} + \beta \frac{\partial \rho}{\partial t} = \alpha^2 \nabla^2 [(1 + \nabla^2)^2 \rho + \mathrm{r}\rho + \rho^3]$$

- Atomistic-scale elastic and plastic deformations on a diffusive timescale
- Efficient modelling of microstructures, grain boundary dynamics, defects, phase transitions

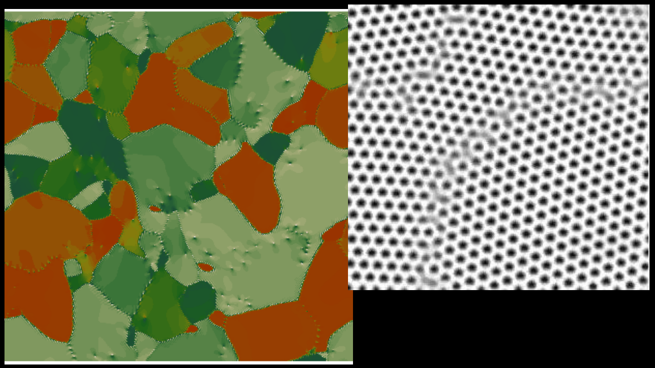
Crystal growth



Deformation and fracture



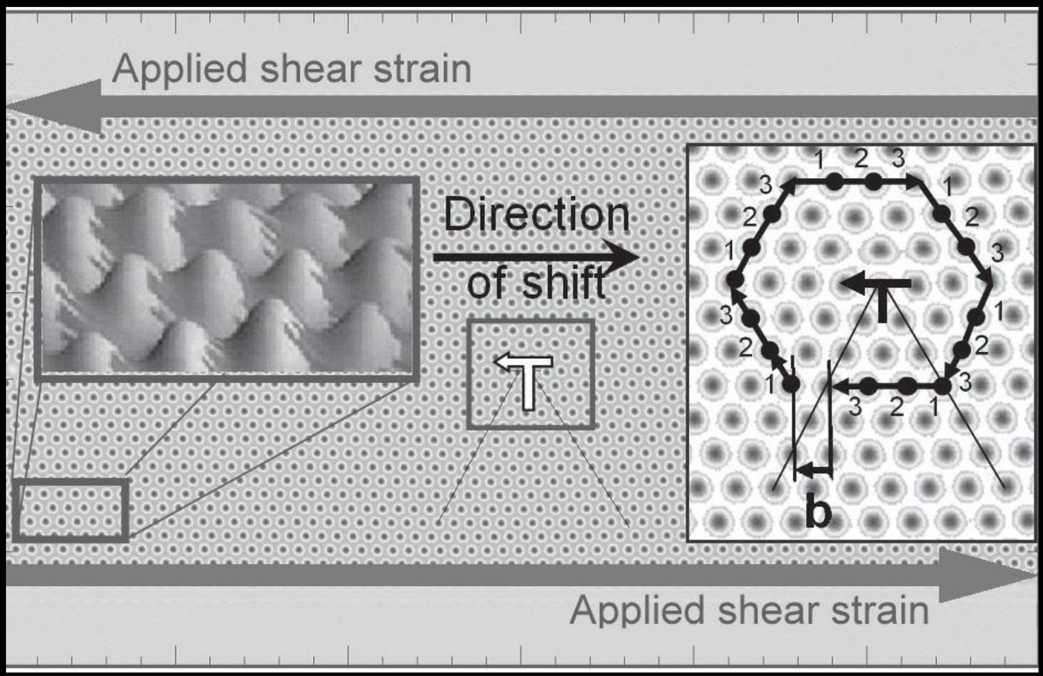
Grain boundaries

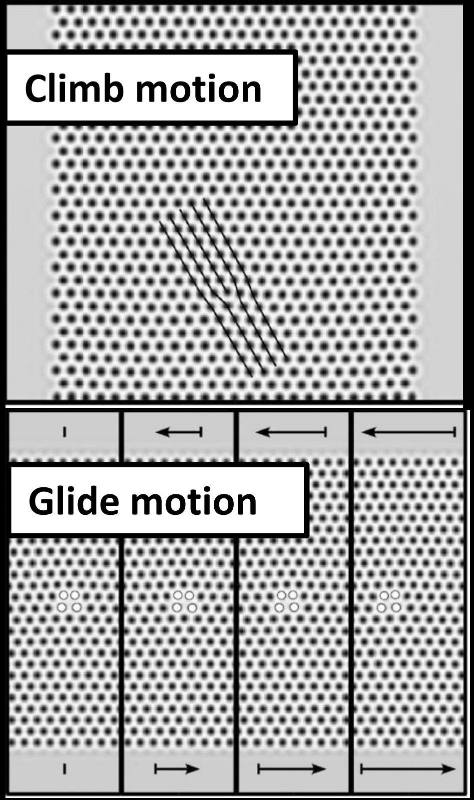


K.R: Elder and M.Grant , PRE (2004), Emmerich et al. Adv. Phys. (2012)

Plastic deformation in Phase Field Crystal

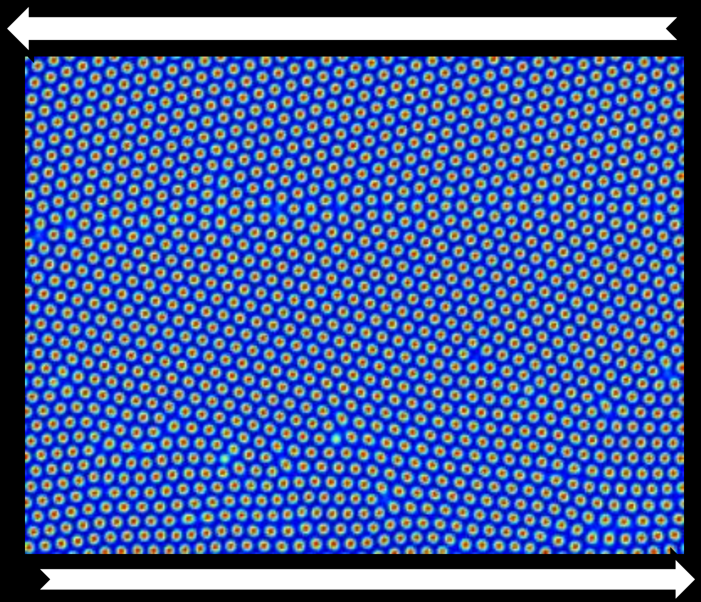
- Crystal defects generated by **thermal** nucleations or when the local **shear stress** exceeds a critical value (**No adhoc rules of nucleation or annihilation**)
- Long-range elastic interactions

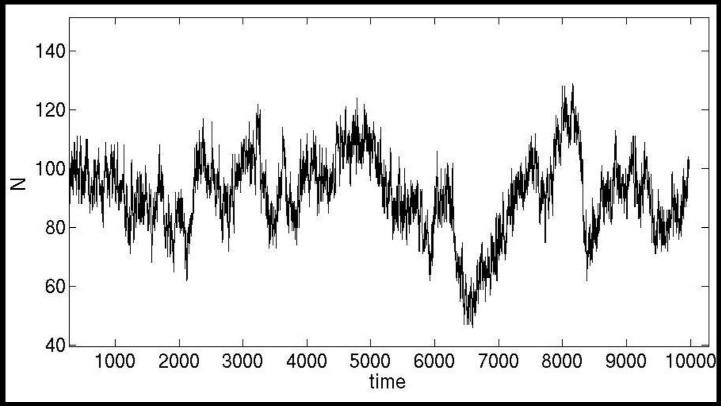


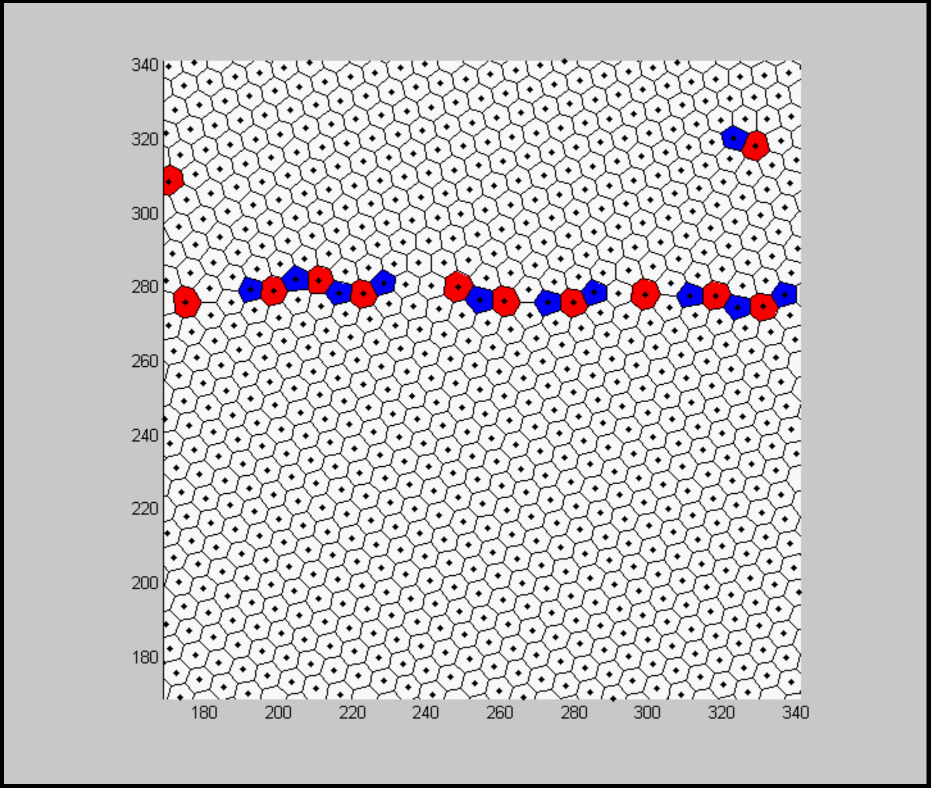


J. Berry et al. PRE (2006), P. Stefanovic et al. PRL (2006)

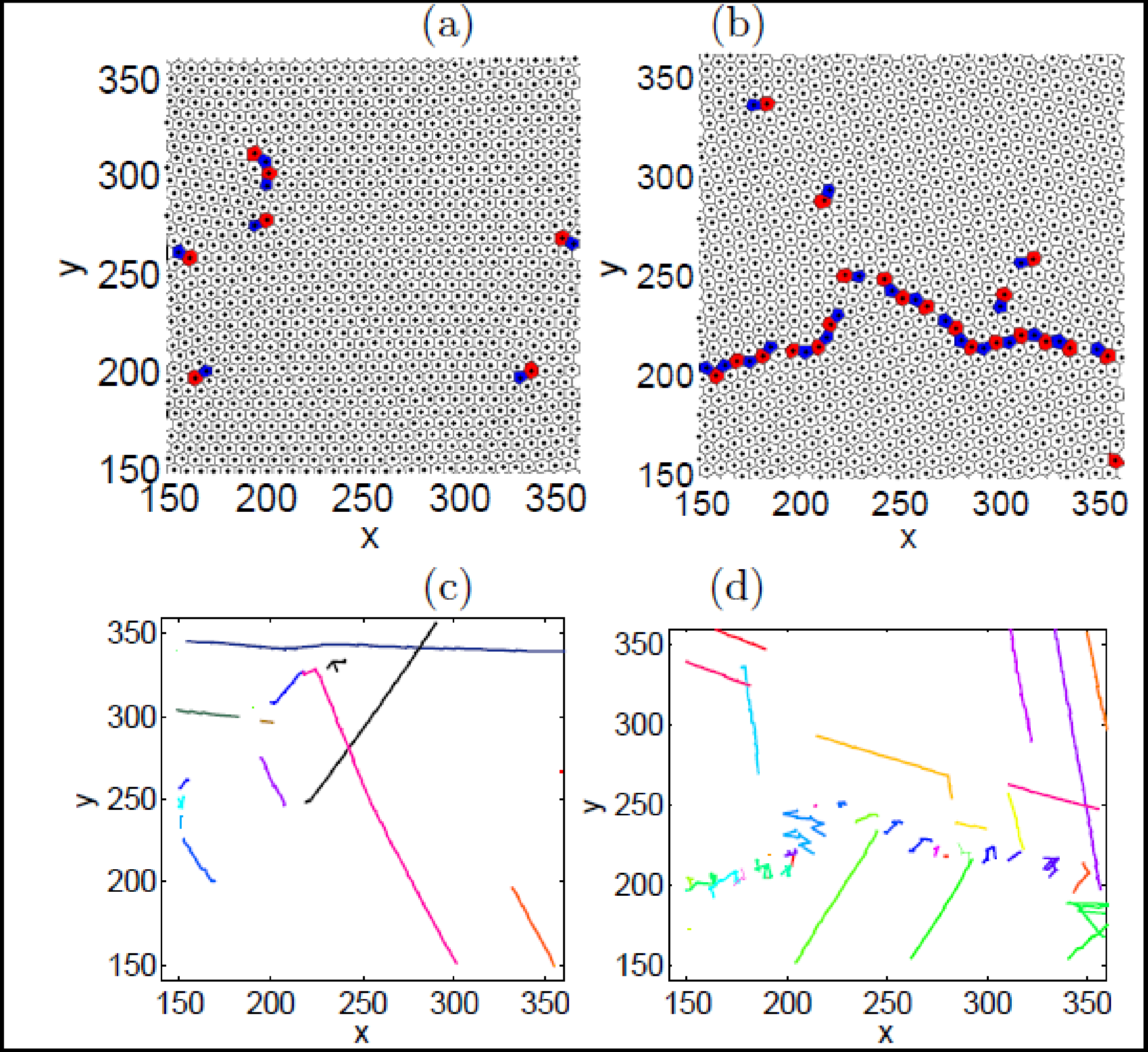
Plastic deformations under constant shear rate







Slow versus fast moving dislocations



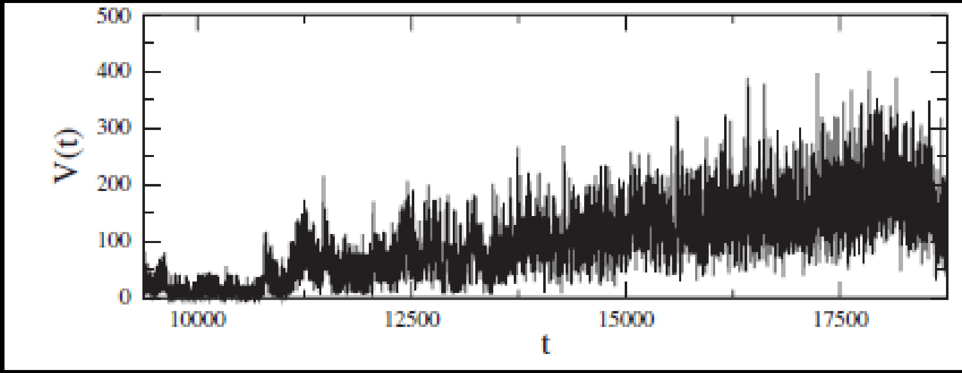
Avalanche statistics of collective velocity

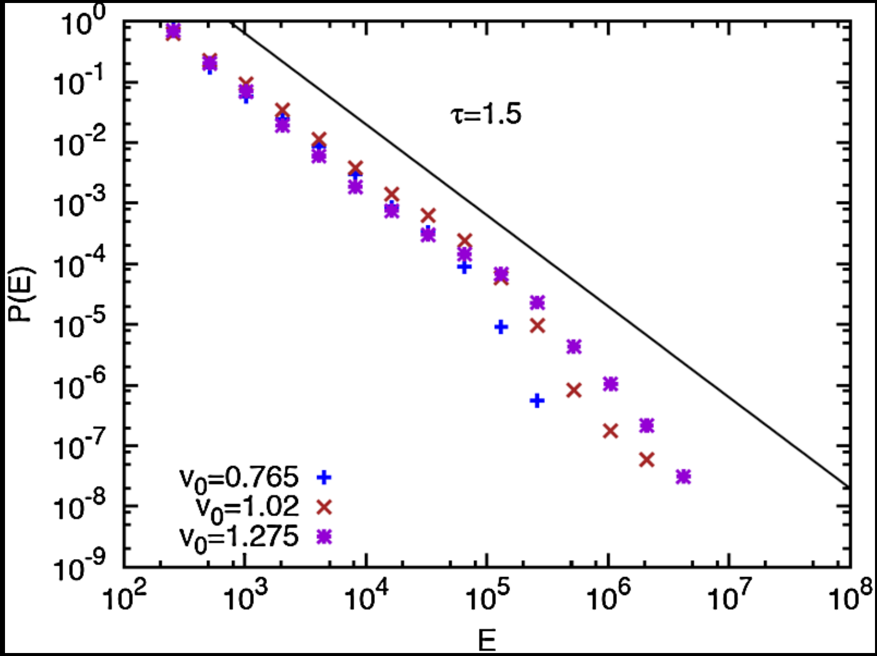
Collective dislocation velocity ~ global strain rate

$$V(t) = \sum_i |v_i|$$

Energy released during a slip avalanche of duration T

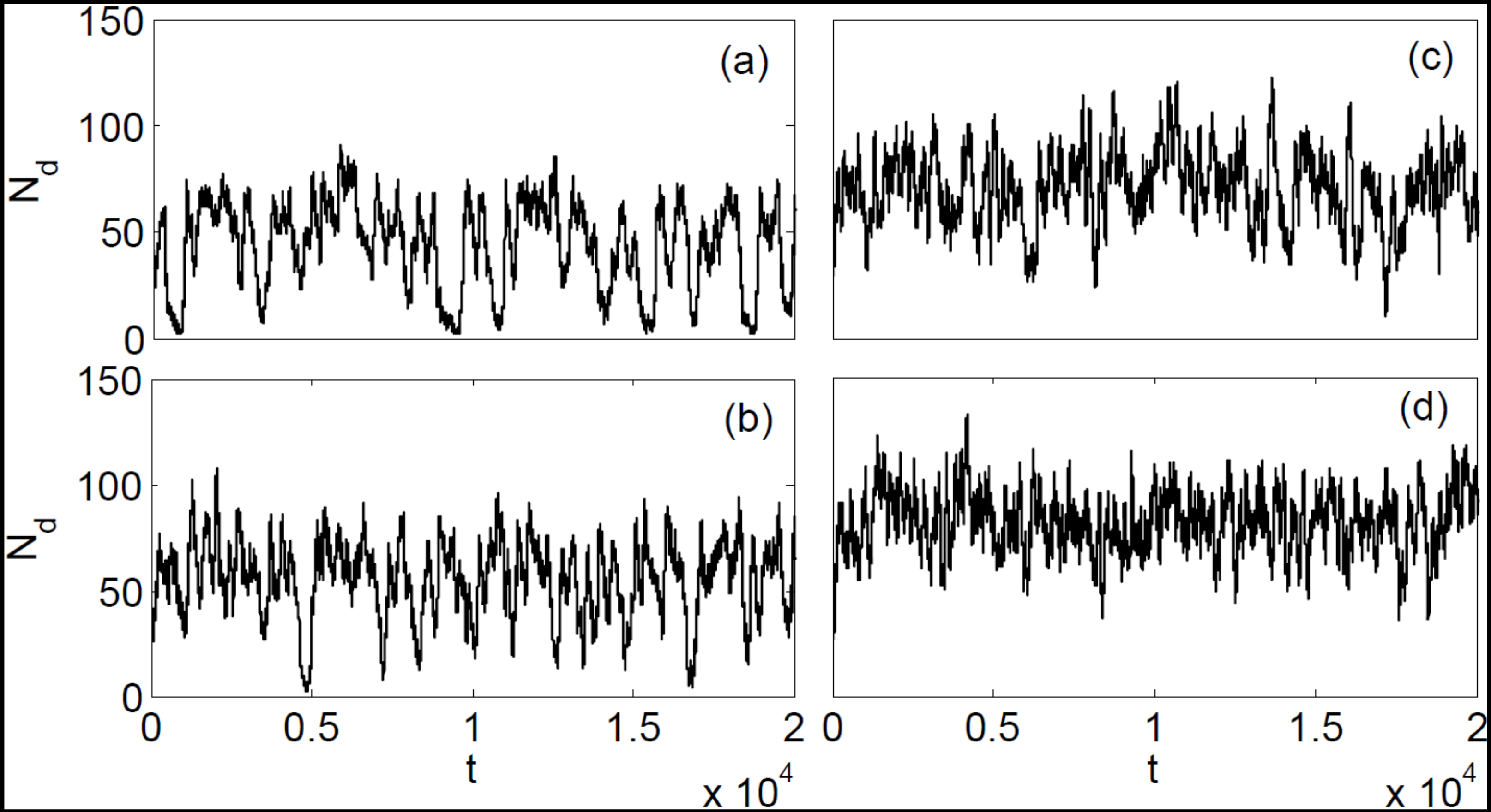
$$\mathrm{E} = \int_0^T dt\, V^2(t)$$





P. Y. Chan et. al, PRL (2010)

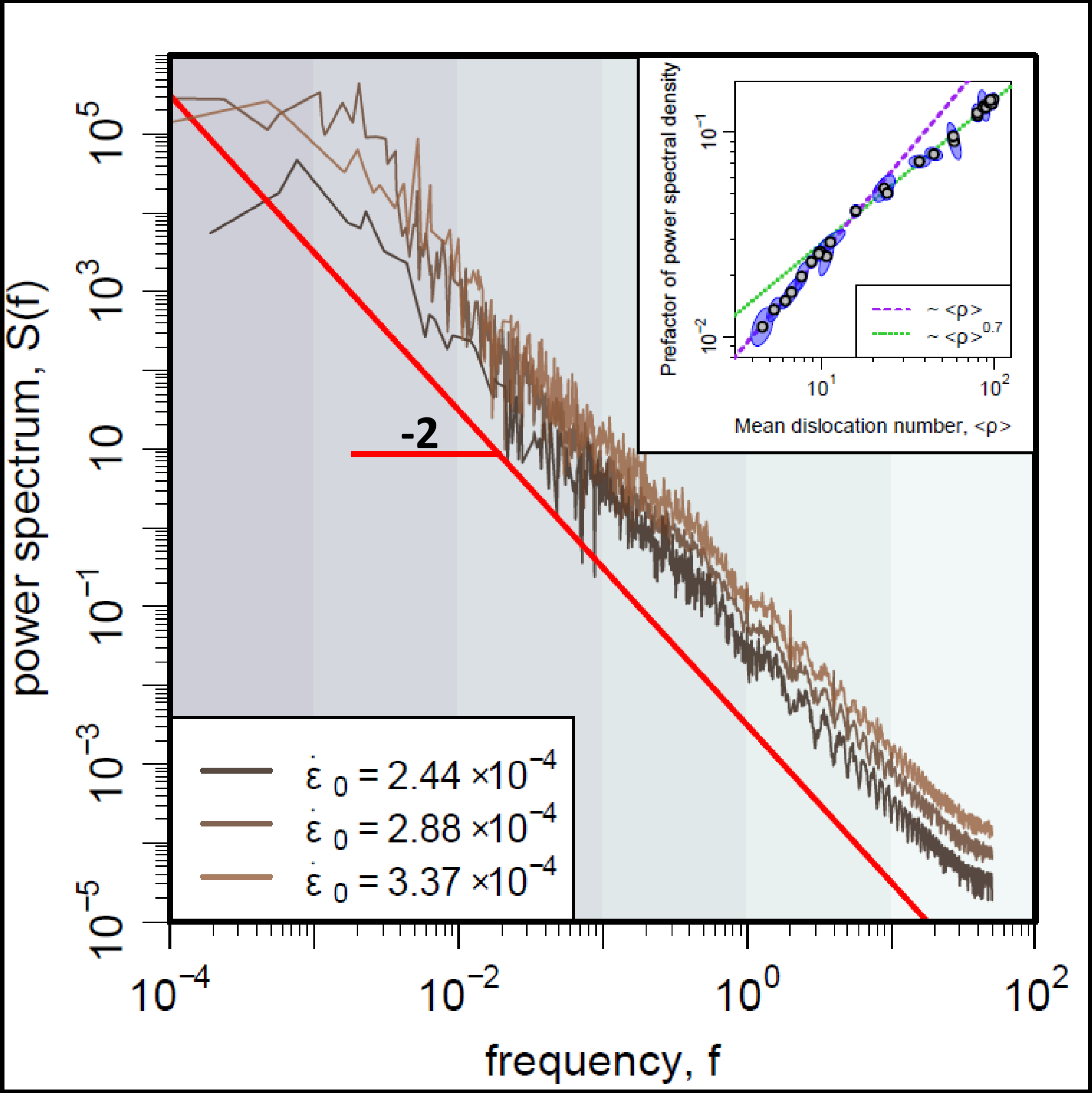
Dislocation density fluctuations



Applied strain rate increases from (a) to (d)

J. Tarp, **L. Angheluta**, J. Mathisen, N. Goldenfeld, submitted

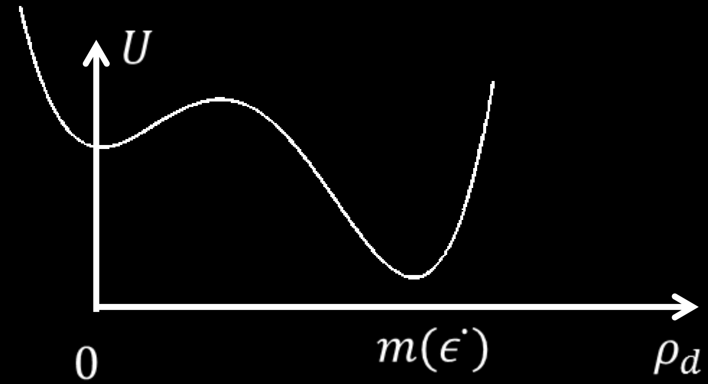
Power spectrum density of N_d

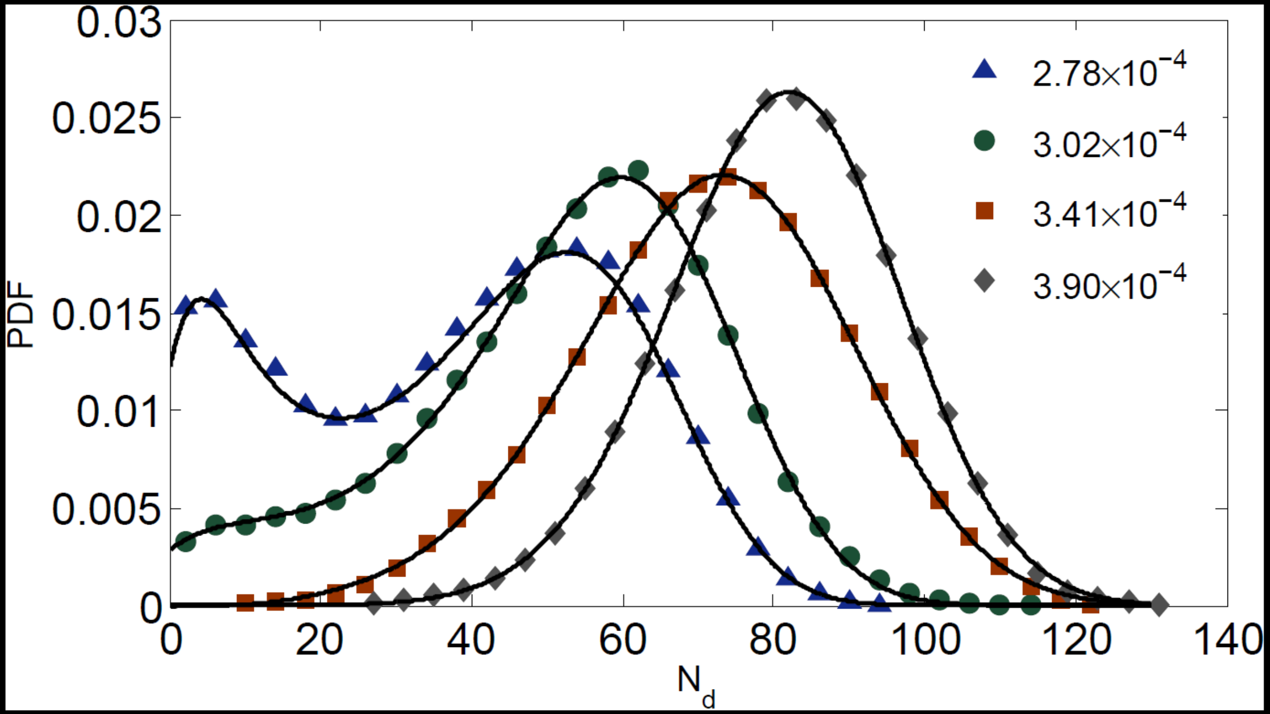


J. Tarp, **L. Angheluta**, J. Mathisen, N. Goldenfeld, submitted

Non Gaussian statistics captured by stochastic dynamics

$$\dot{\rho_d} = -\frac{d}{d\rho_d}U(\rho_d, m) + \left(1 + \frac{\rho_d}{m}\right)\delta\dot{\epsilon}(t),$$
$$\langle\delta\dot{\epsilon}(t)\delta\dot{\epsilon}(t')\rangle = 2D\delta(t - t')$$





J. Tarp, **L. Angheluta**, J. Mathisen, N. Goldenfeld, submitted

Summary

- AE statistics of plastic bursts predicted within the mean field theory of depinning by extreme value statistics
- Logarithmic interactions lead to generic power-law distributions of local quantities
- Dislocation density is a highly fluctuating quantity:
  - power spectrum is similar to that of the global strain rate
  - non-Gaussian distributed fluctuations and bimodal at low drivings.