Single spin dynamics vs. magnetization conserving dynamics in disordered systems

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in colaboration with

Helmut G. Katzgraber, Vladimir Dobrosavljevic, Gergely T. Zimanyi

Outline

- Short introduction to spin glasses: from order to disorder
- Avalanches with single flip dynamics: EA and SK model
- Avalanches with magnetization conserving dynamics: Coulomb-glass model
- Discussion and work in progress: single flip vs magn. cons. dynamics

Disordered systems

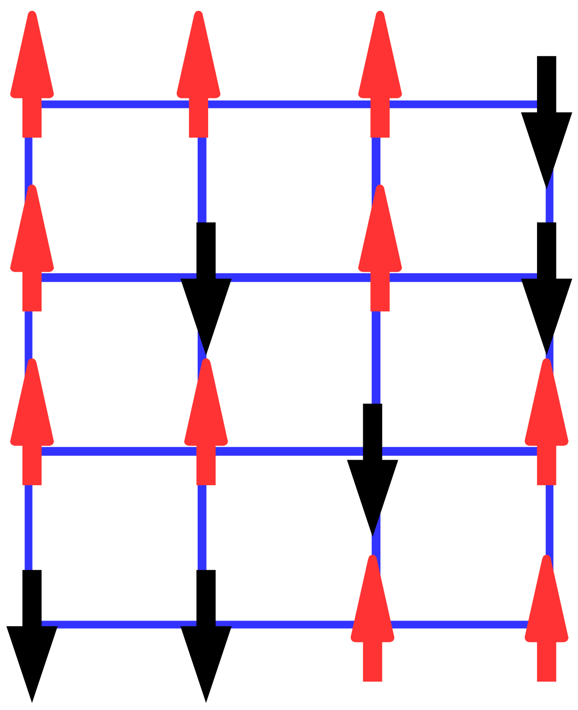
Ideally ordered:

Disordered systems

Real life disordered:

From order to disorder: From the Ising to the spin-glass model

Ising Model



$$T > T_c$$

From order to disorder: From the Ising to the spin-glass model

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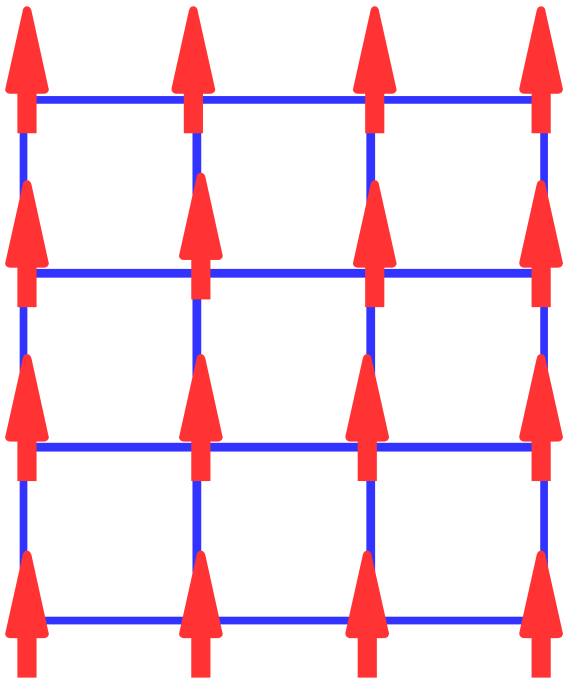

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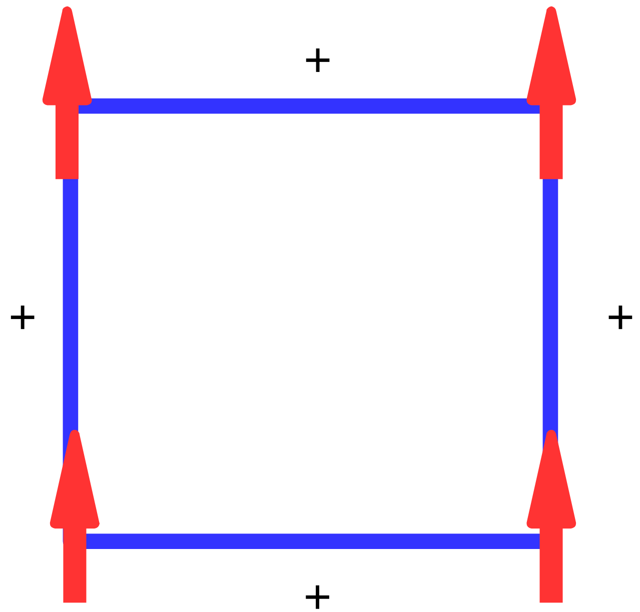

$$S_i = -1, 1$$

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} S_i S_j$$

From order to disorder: From the Ising to the spin-glass model

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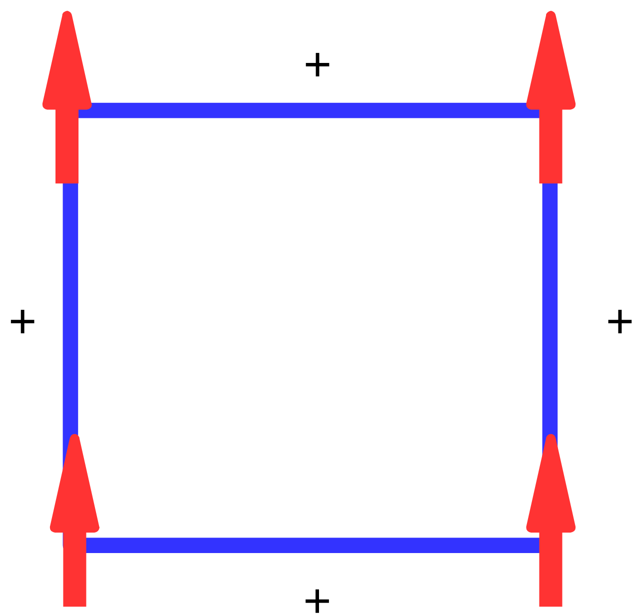

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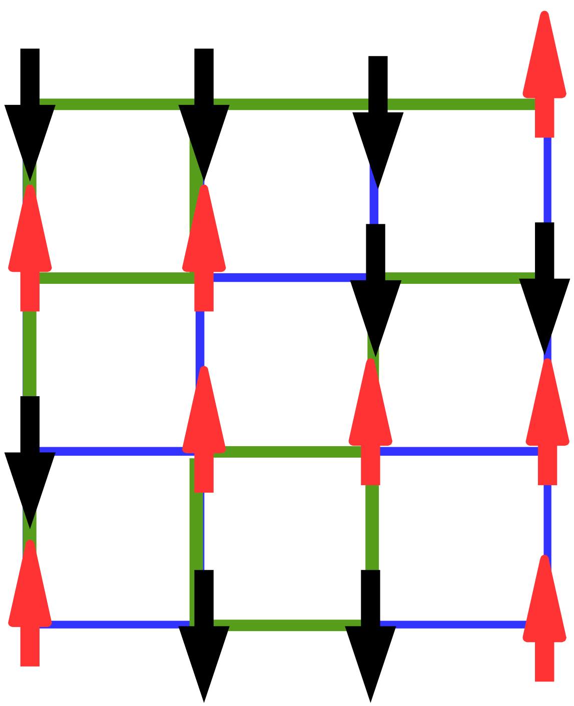
Spin Glass



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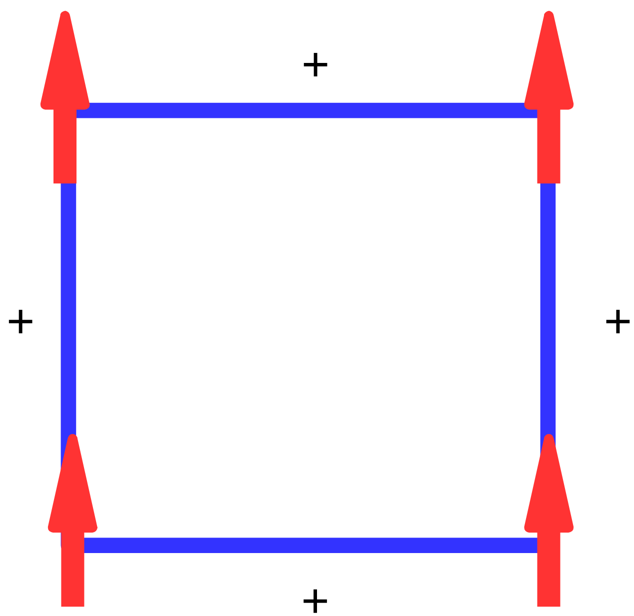

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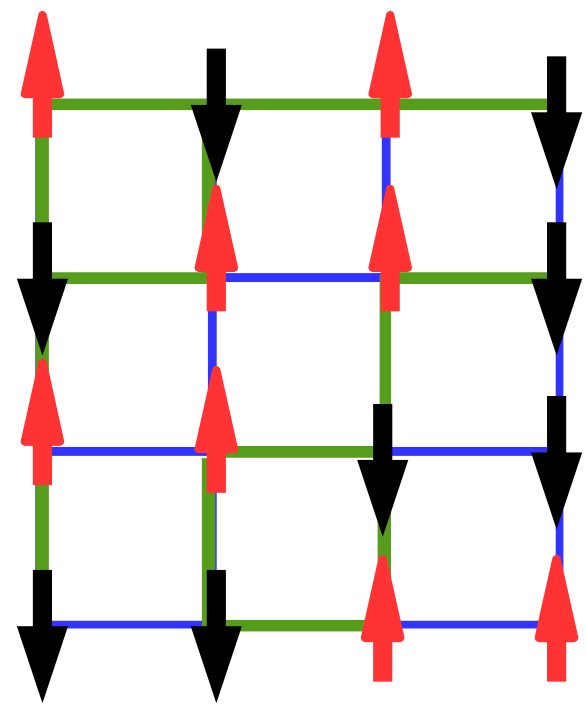

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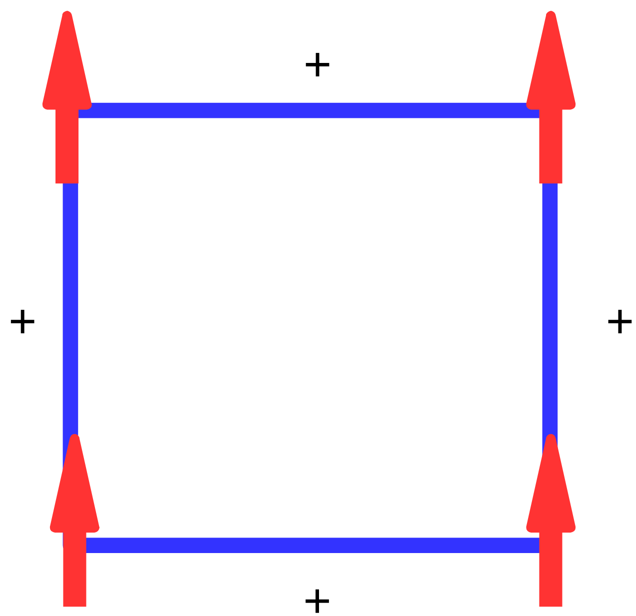

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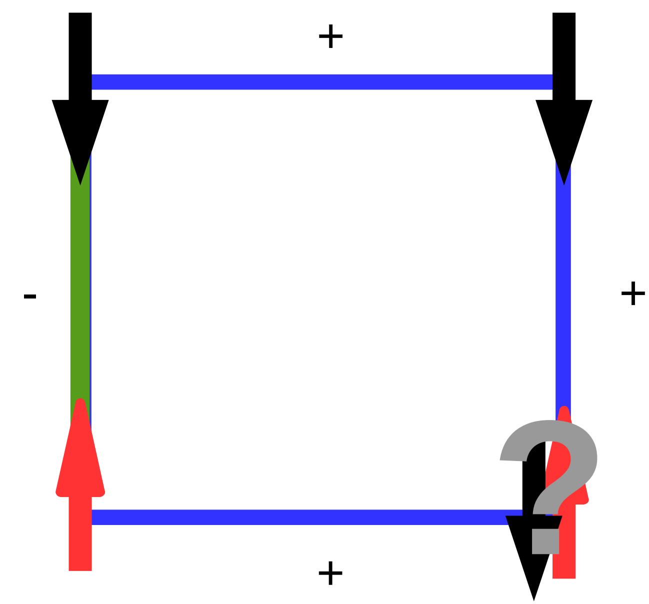

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From order to disorder: From the Ising to the spin-glass model

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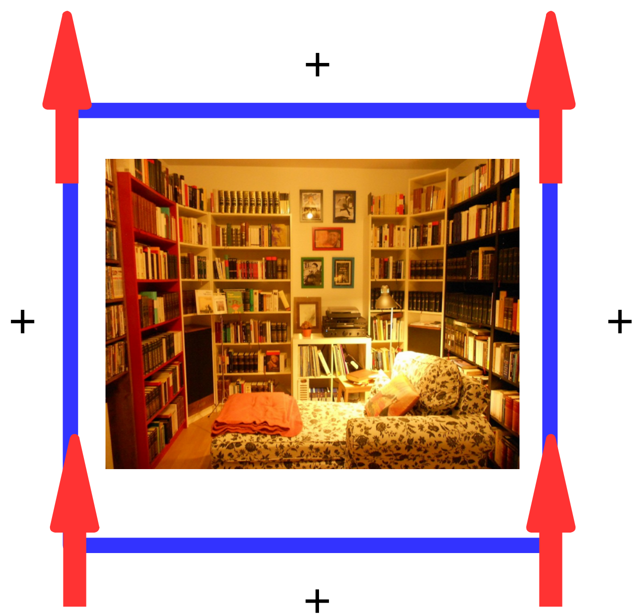

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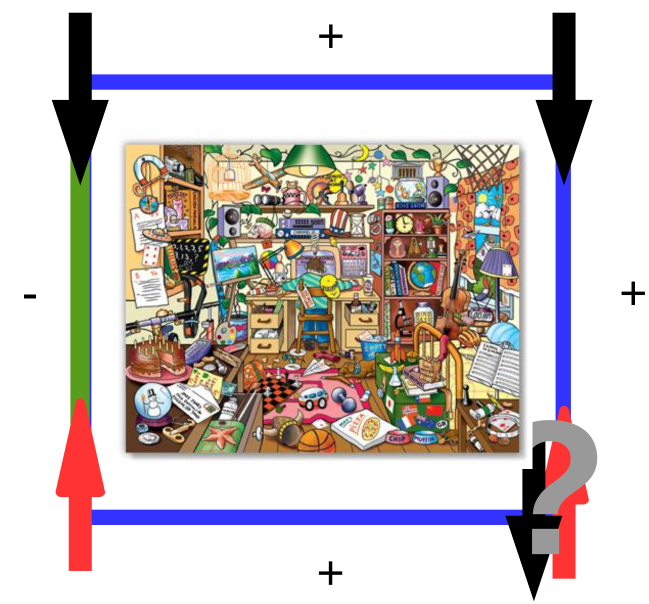

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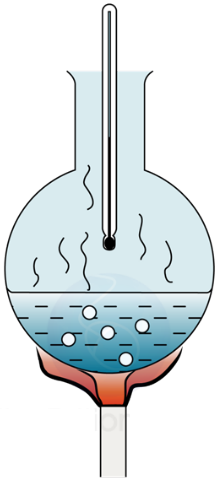
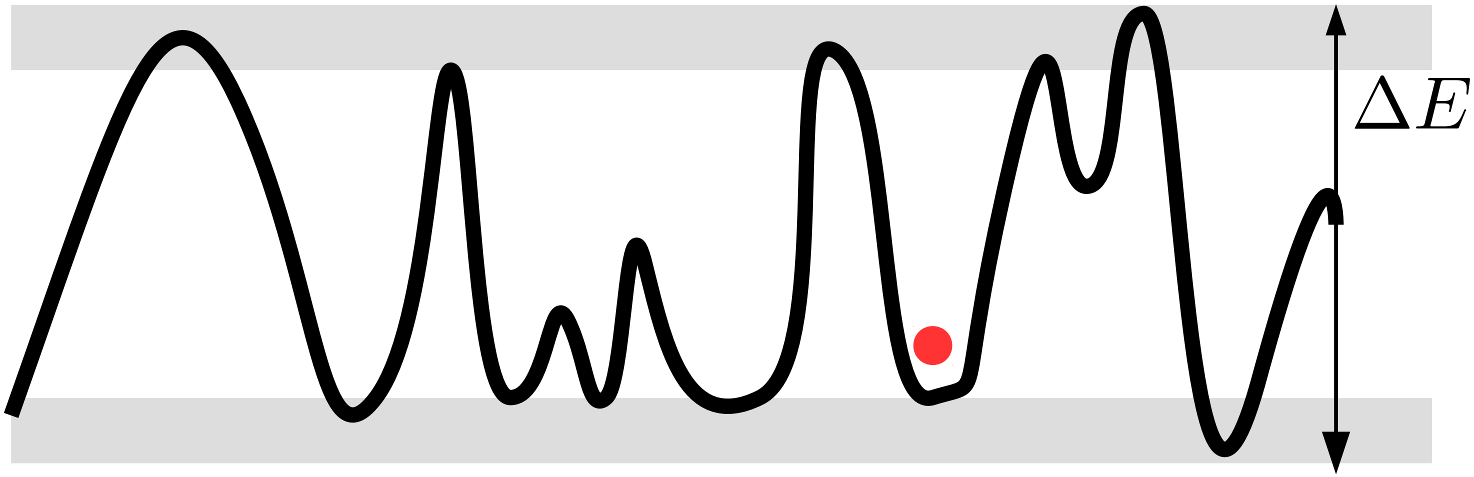
Characteristics of glassy systems



- Complex energy landscapes
- Slow dynamics, time scales diverge: $\propto e^{\Delta E/T}$



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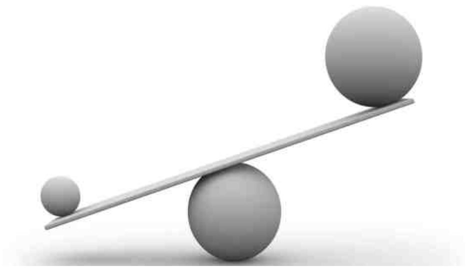
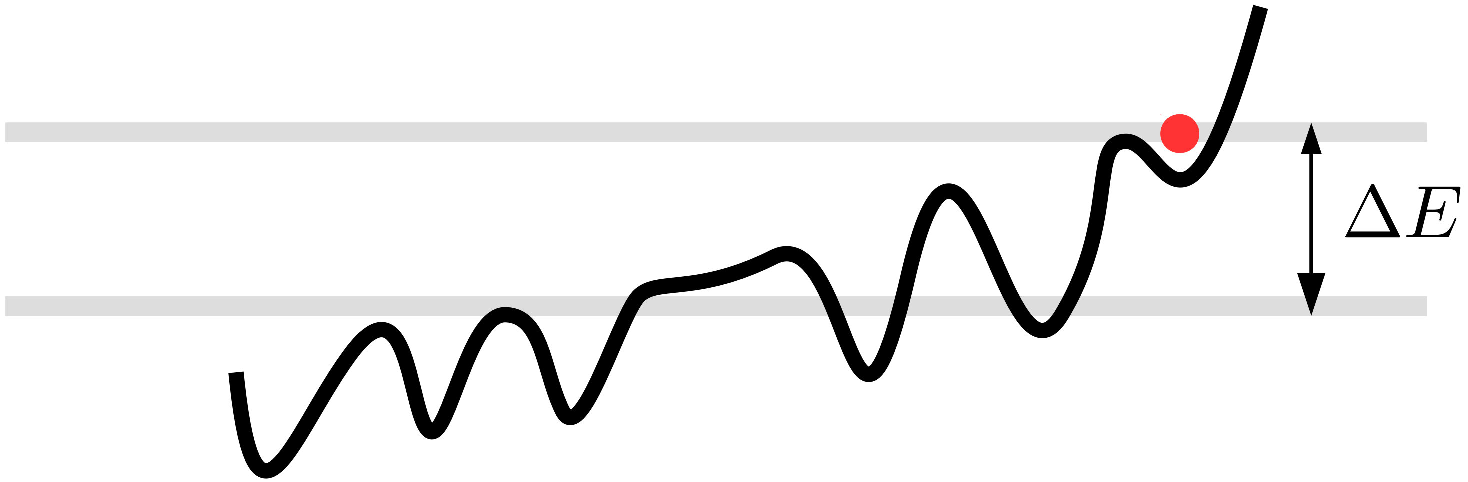


Out of equilibrium

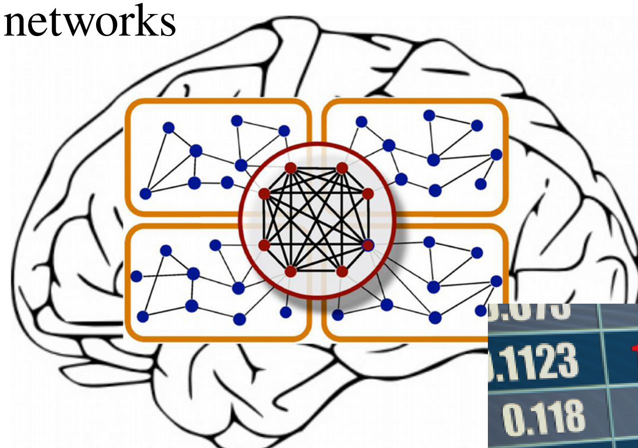
- Complex energy landscapes
- External force, e.g. tilt landscape



Applications

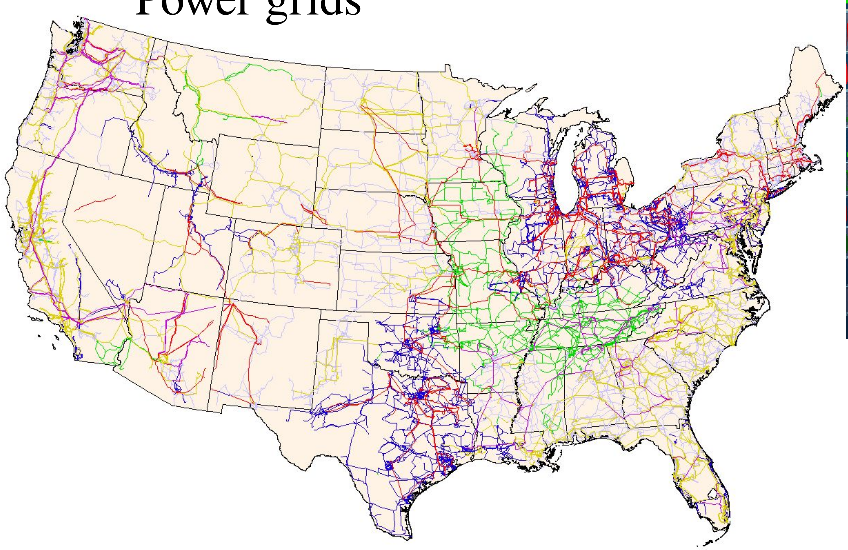
Traffic

Neural networks



Stock market

Power grids



Traveling salesman problem

Avalanches and self-organized criticality

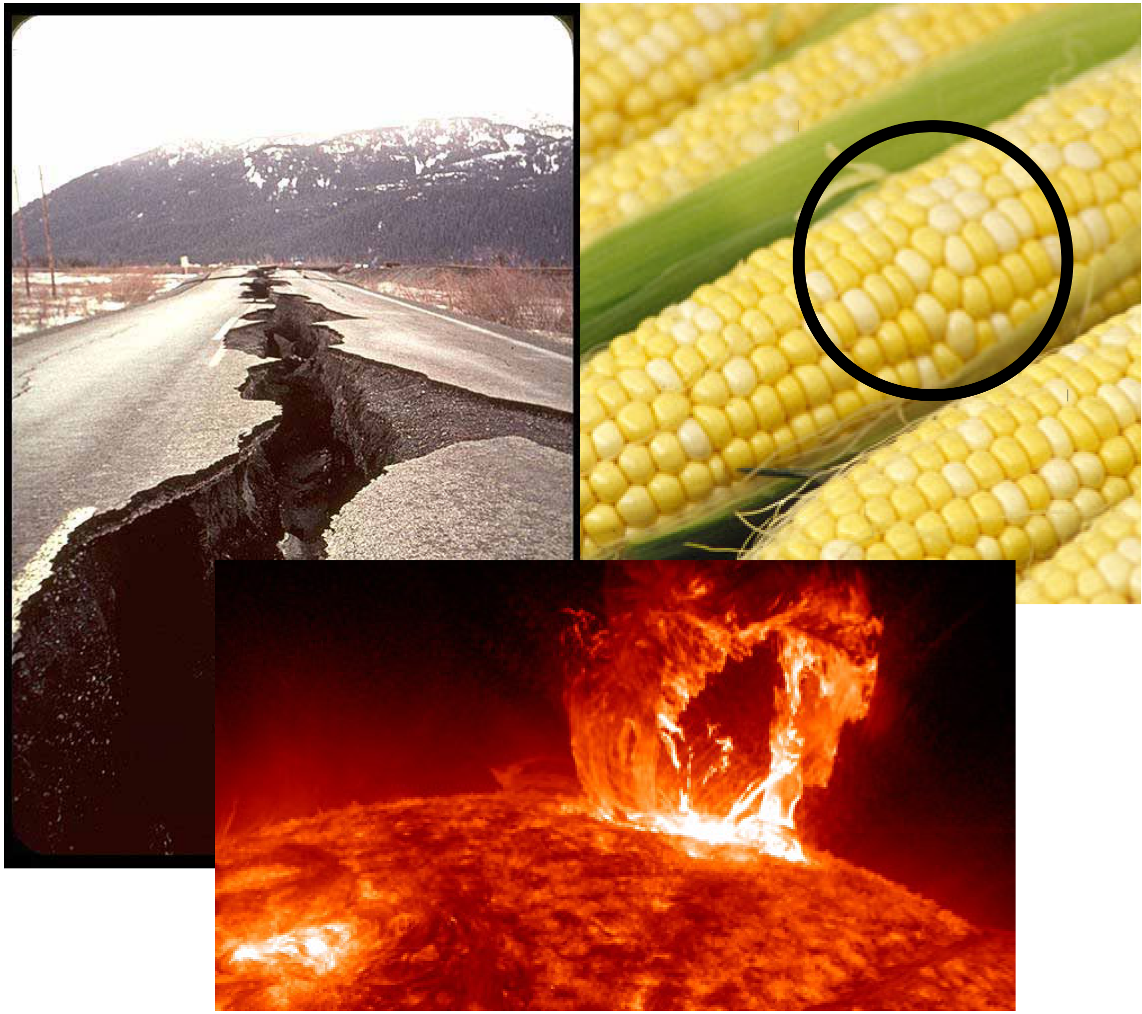
Self-organized criticality (SOC): *property of dissipative systems that drive themselves into a scale-invariant state*

- Slow driving or energy input
- Fast relaxation events (avalanches, earthquakes,...)
- Power-law distribution of the response with an exponential cutoff that scales with the system size
- No tuning parameter

Avalanches and self-organized criticality

SOC found in different natural systems

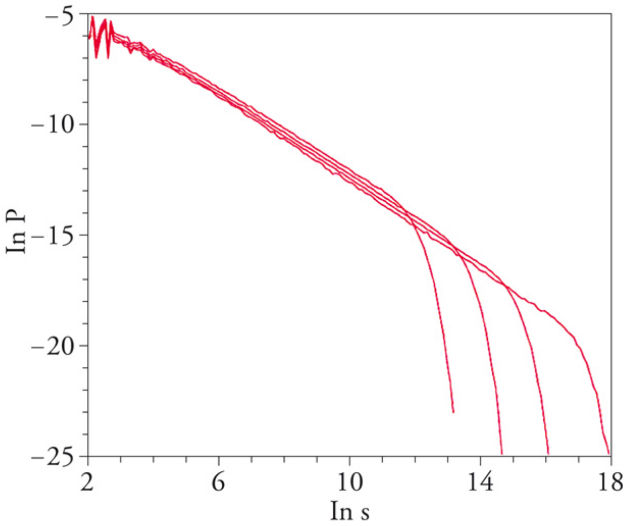
- Earthquakes
- Solar flares
- Dislocations flow
- etc.

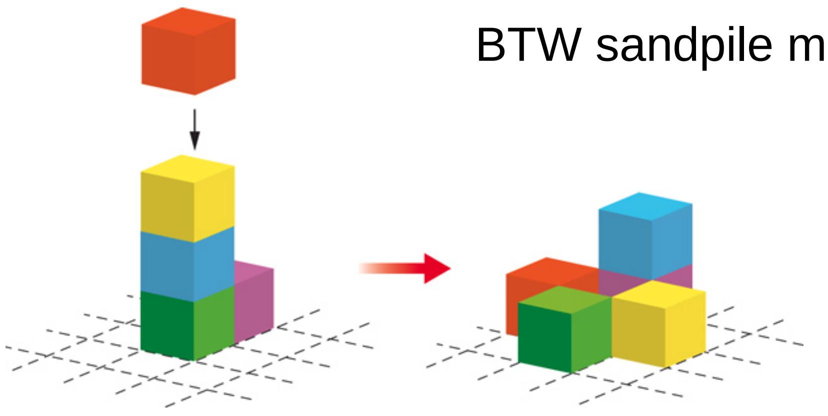


Avalanches and SOC

To understand better the emergence of SOC

- Use models as simple as possible showing SOC
- So far mostly celular automata models





BTW sandpile model

Alessandro Vespignani, Enciclopedia della Scienza e della Tecnica (2007)

Avalanches and self-organized criticality in spin models

What about spin models?

Avalanches and self-organized criticality in spin models

What about spin models?

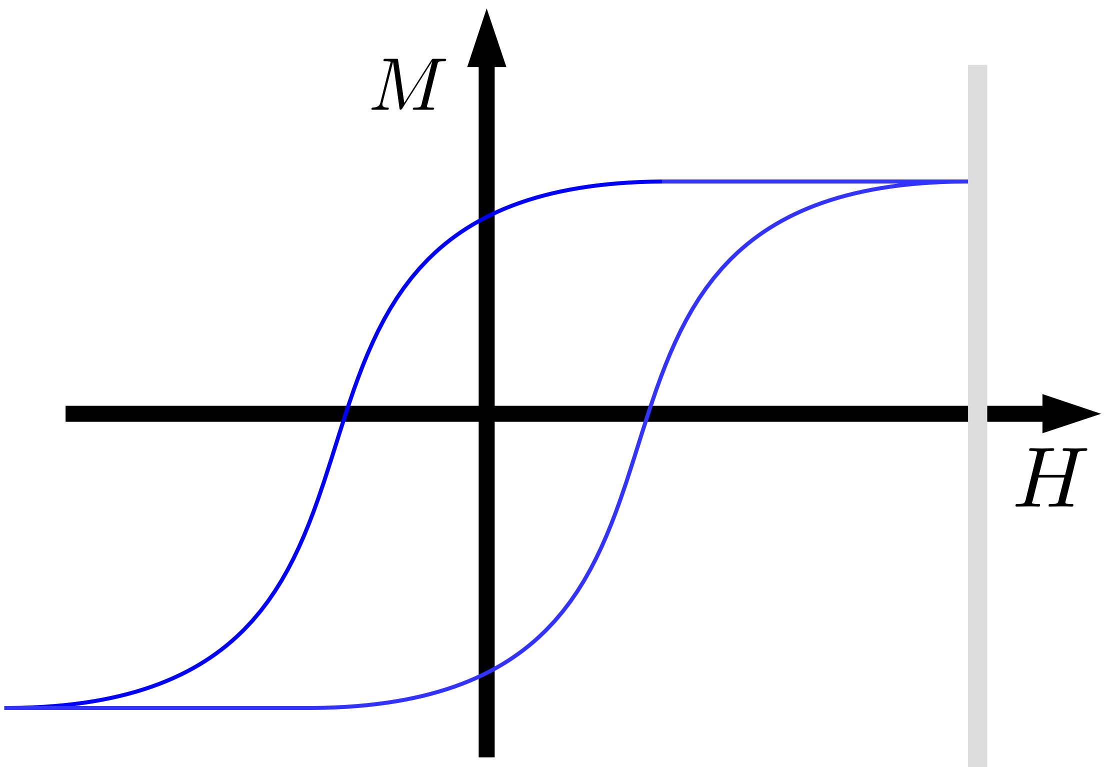
Sherrington-Kirkpatrick model (mean-field of the Edwards-Anderson spin model)

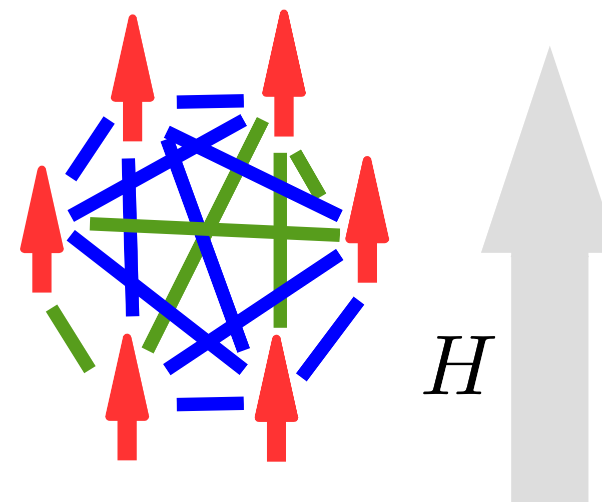
$$\mathcal{H} = -\sum_{i,j} J_{ij} S_i S_j - H \sum_i S_i$$

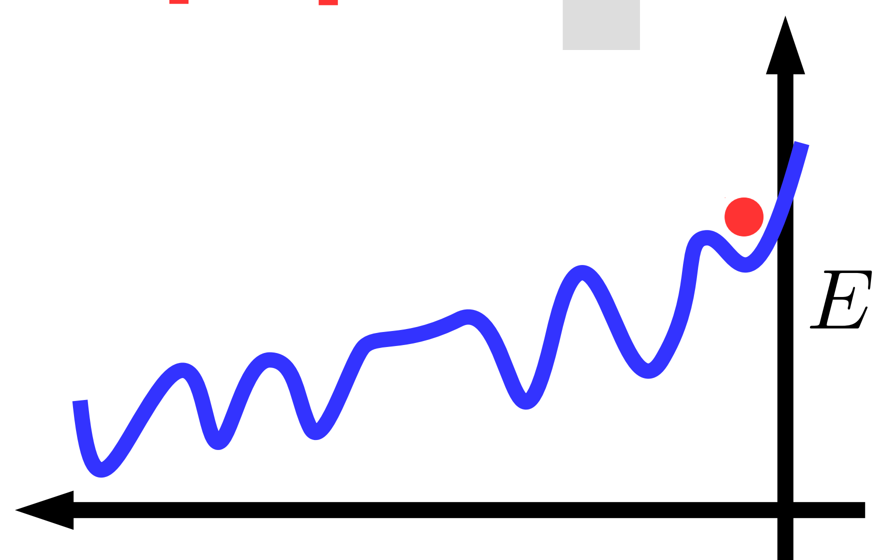
Pazmandi et al. PRL (1999)

Avalanches and self-organized criticality in spin models

single flip dynamics







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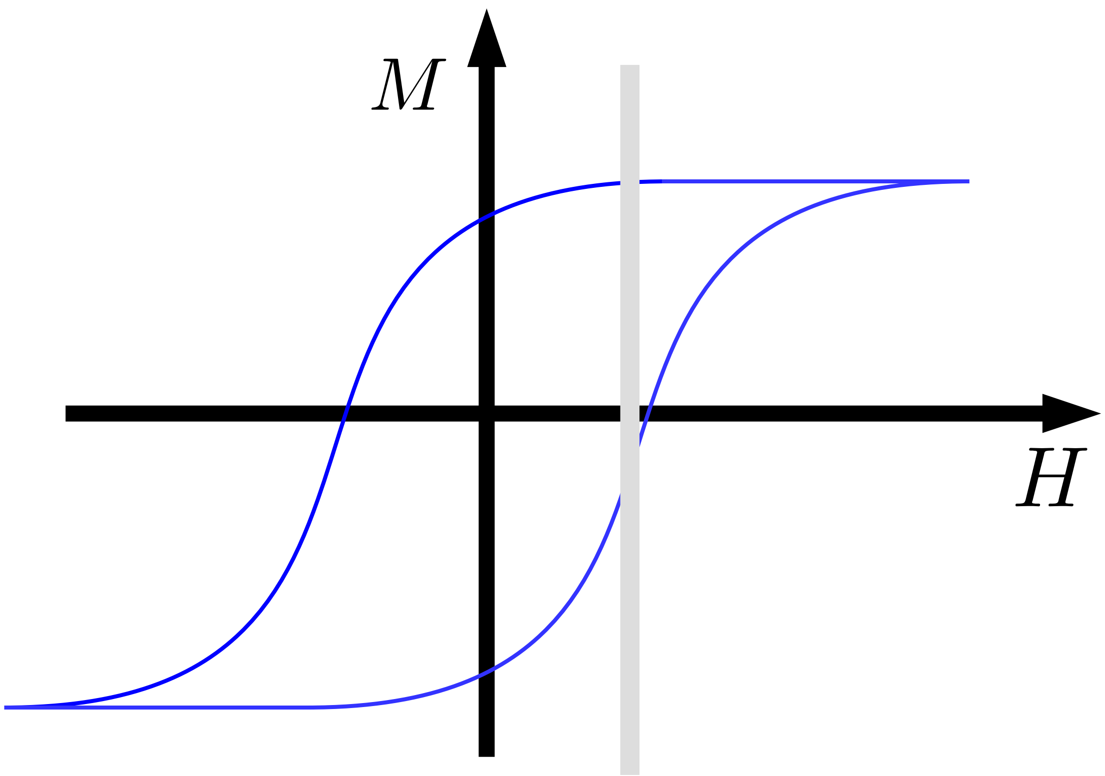

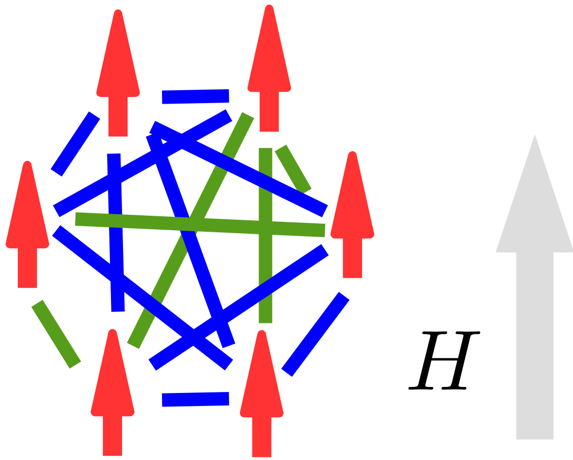

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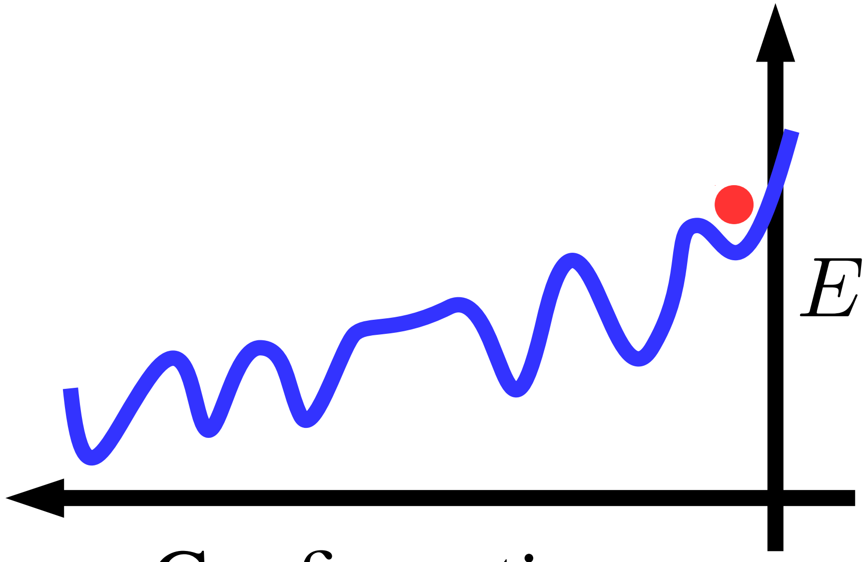
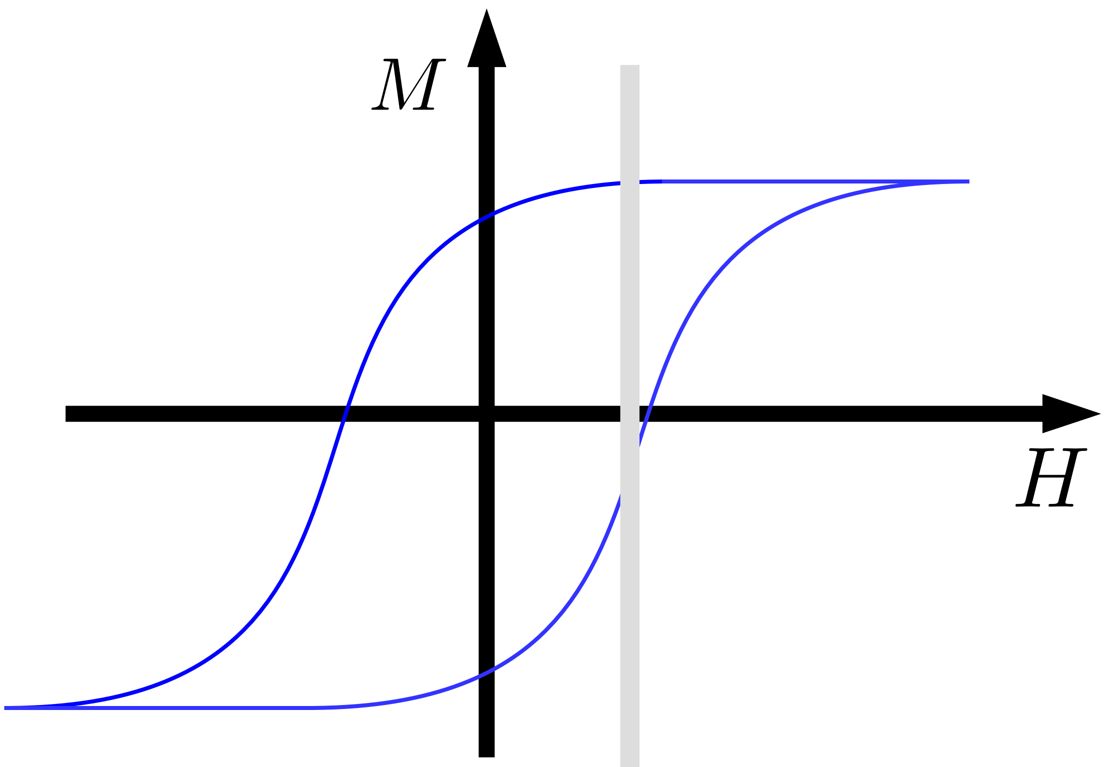
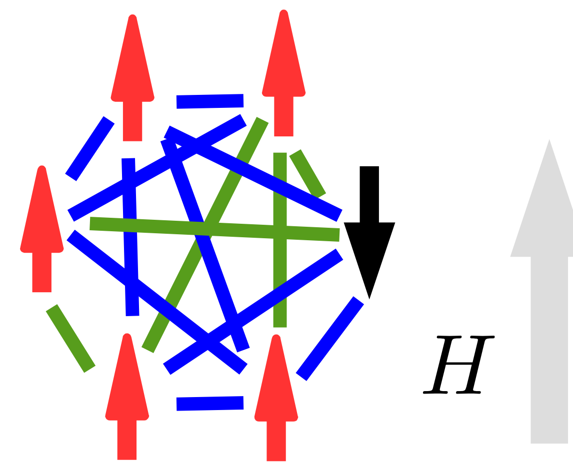
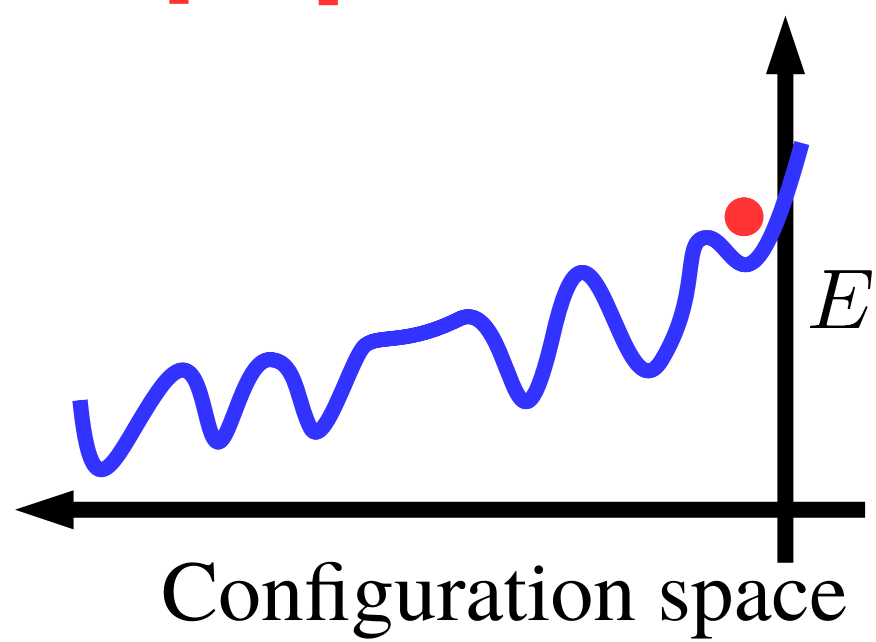


Avalanches and self-organized criticality in spin models

single flip dynamics







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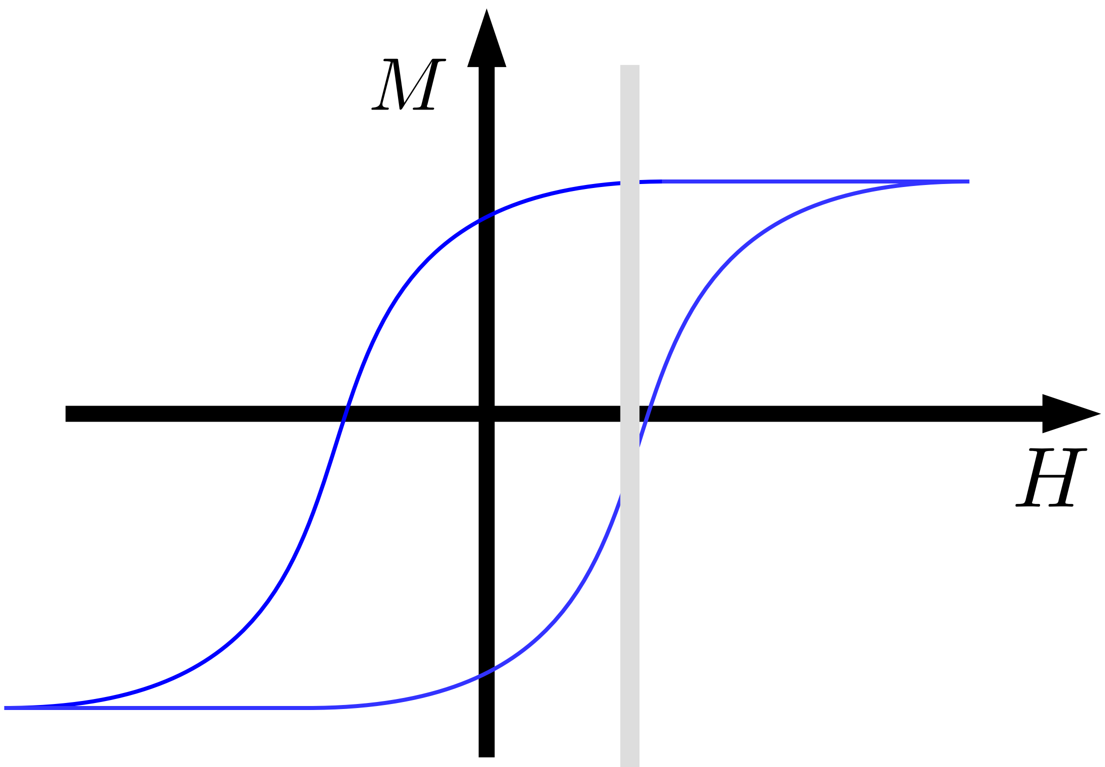

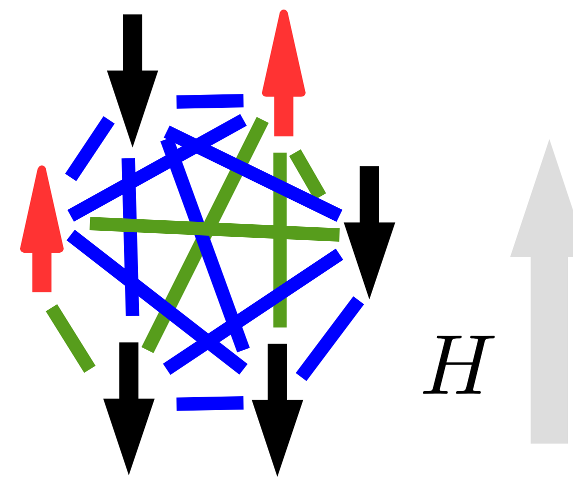

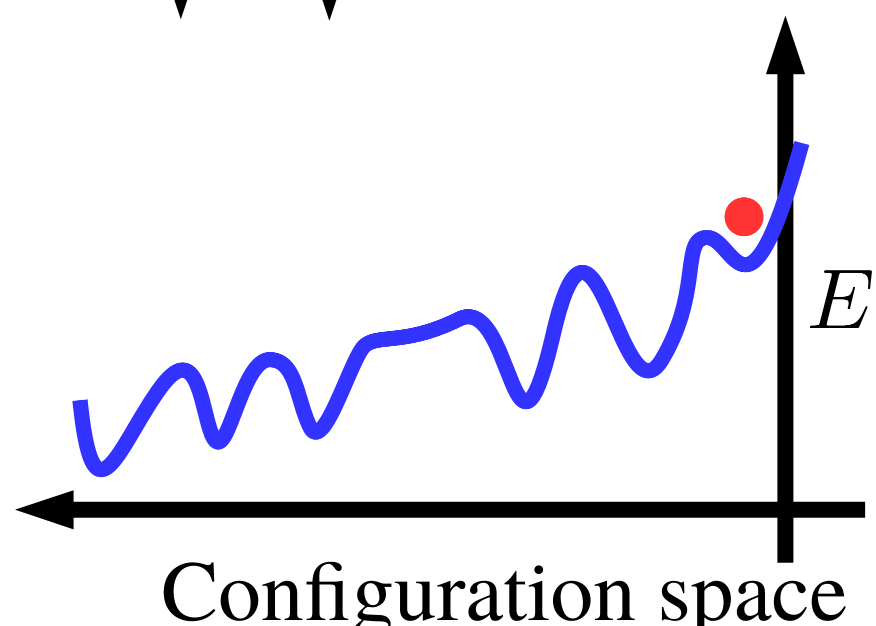

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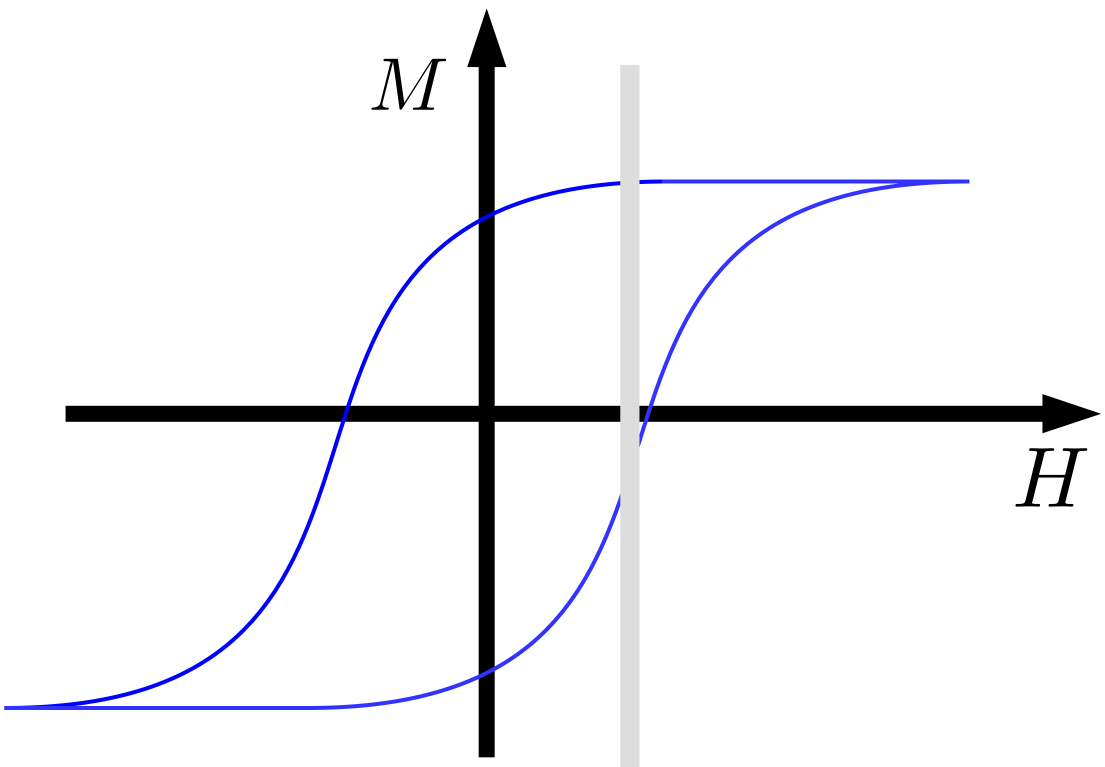

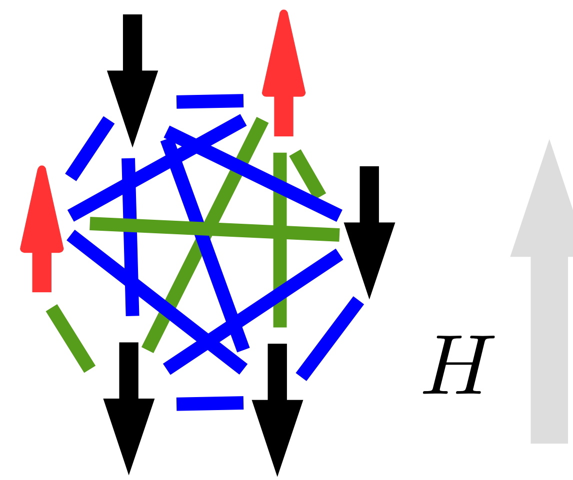

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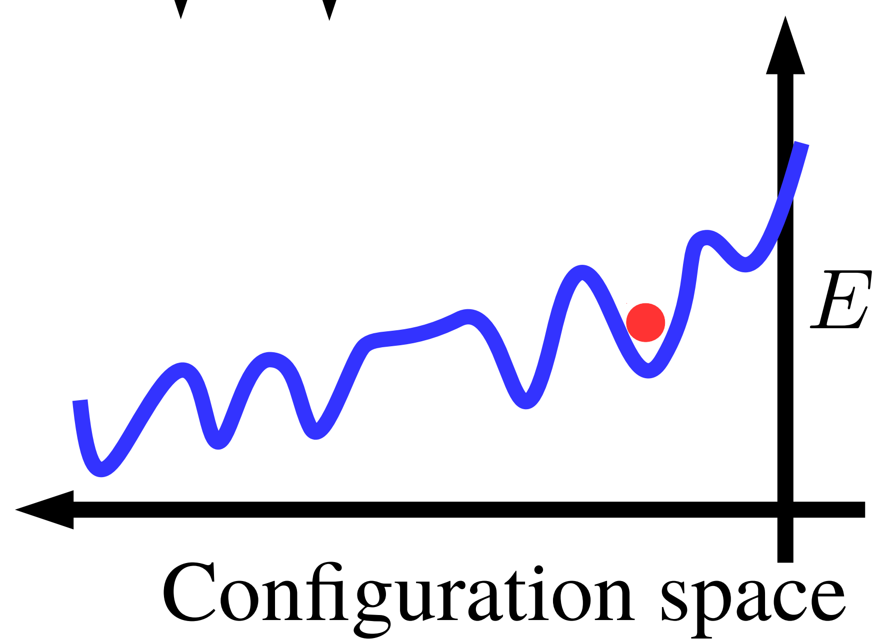

Avalanches and self-organized criticality in spin models

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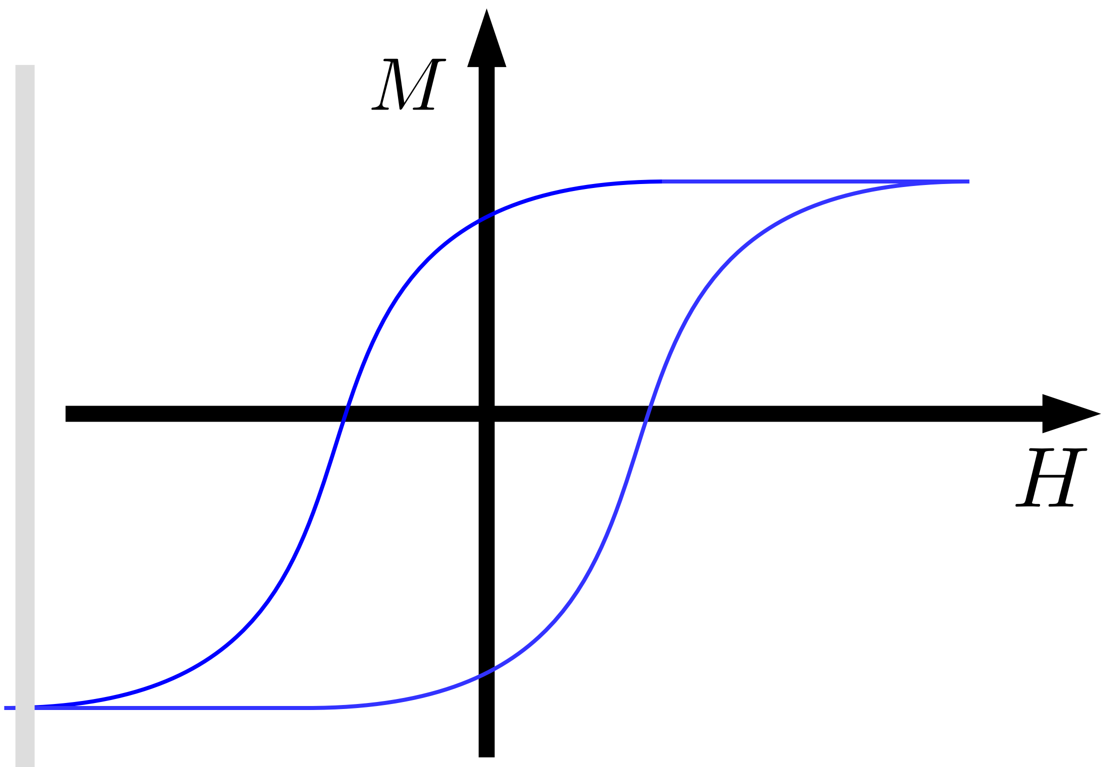

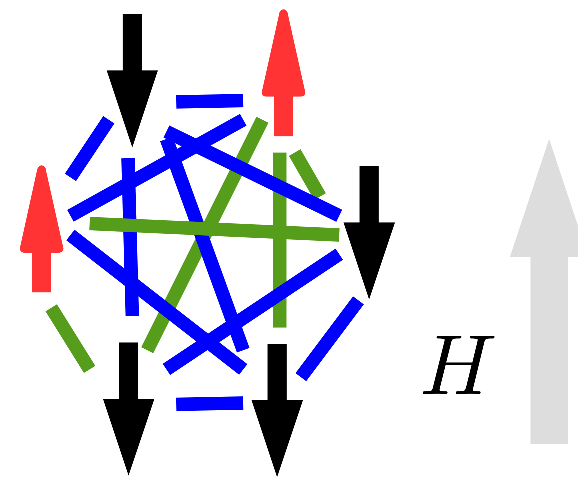

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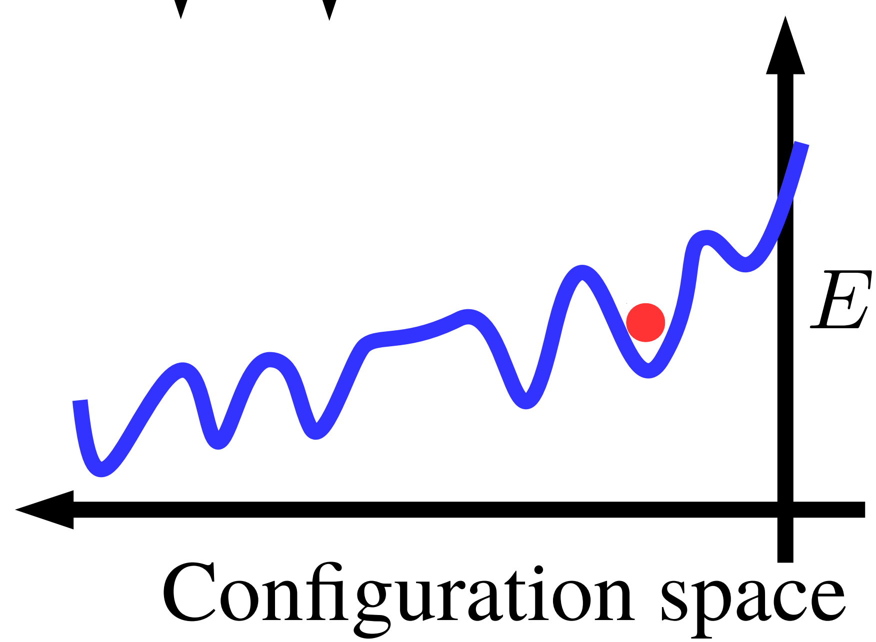

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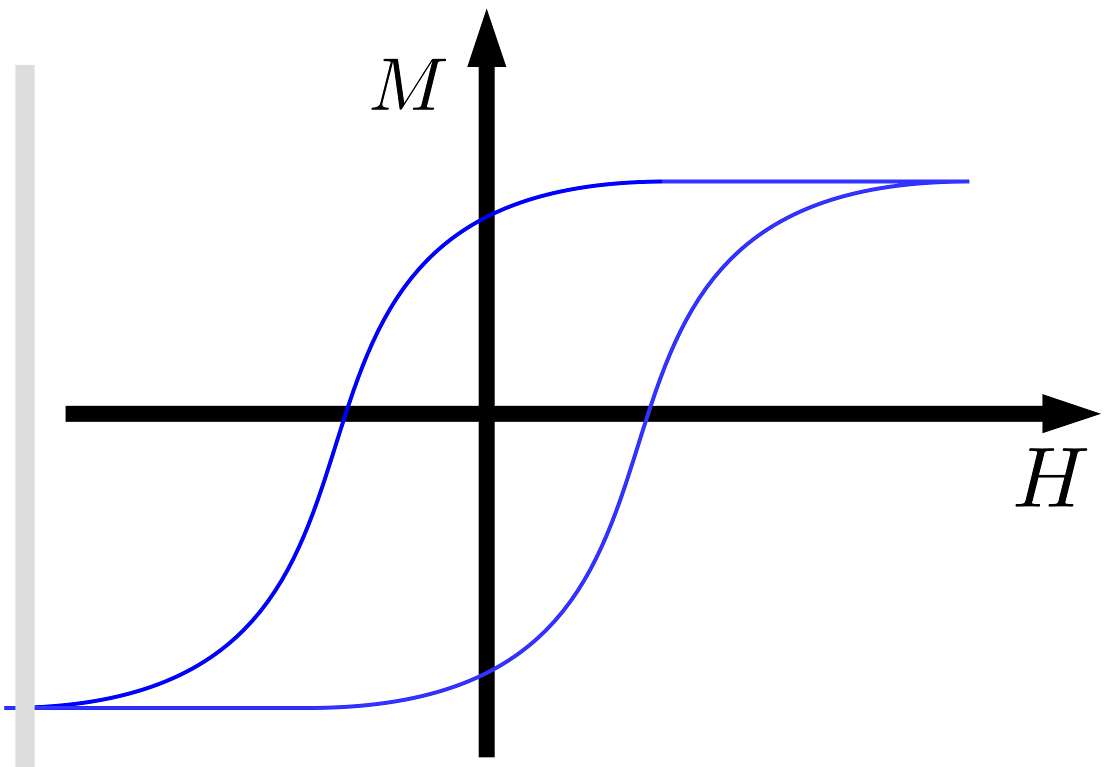

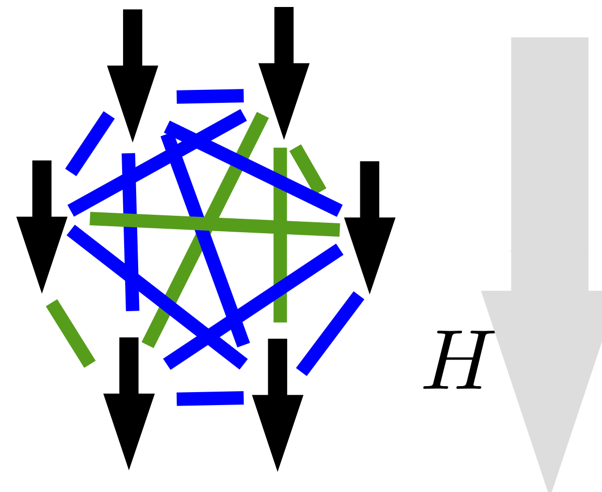

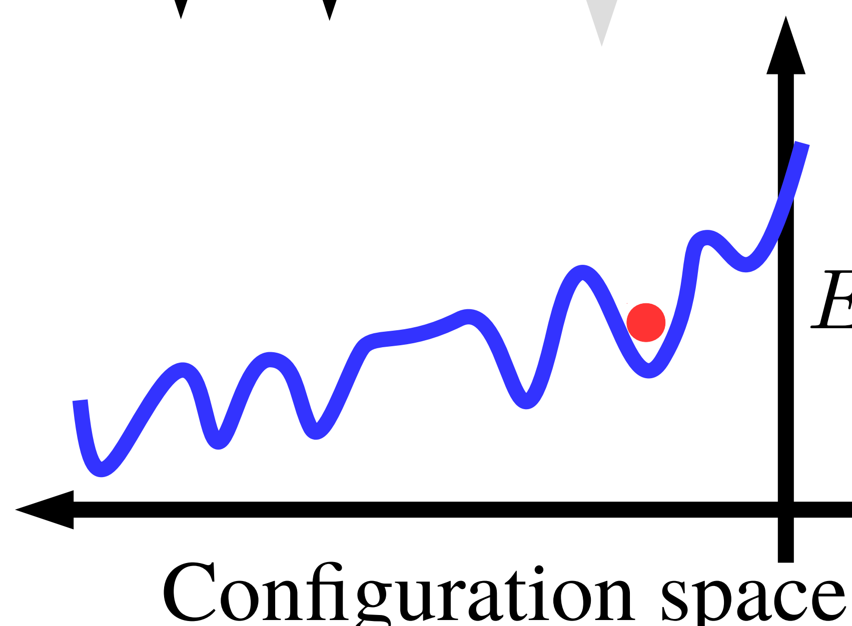

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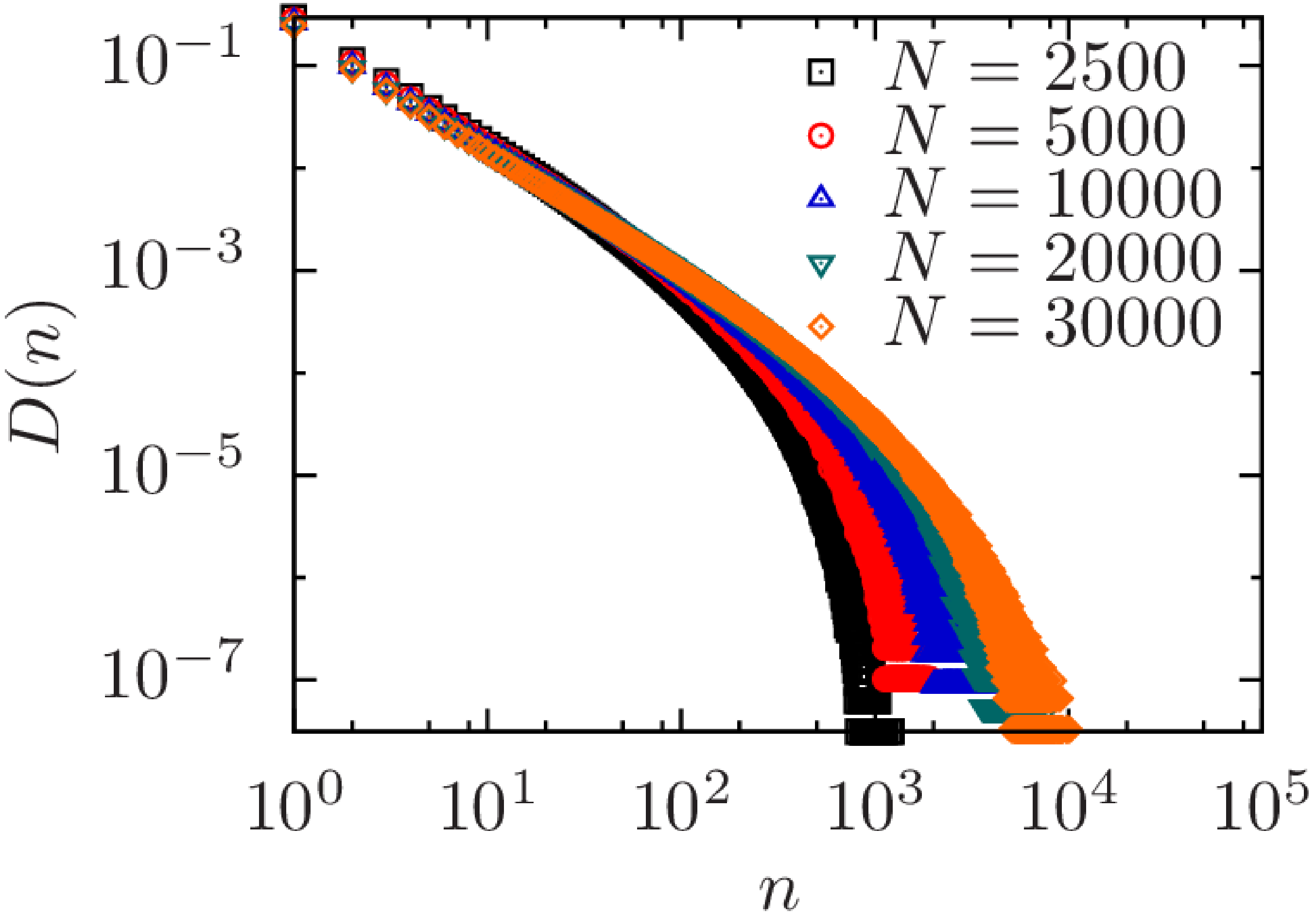
Self-organized criticality in the SK model

Characteristic avalanche size $n_N^* \sim exp(-n/n_N^*)$



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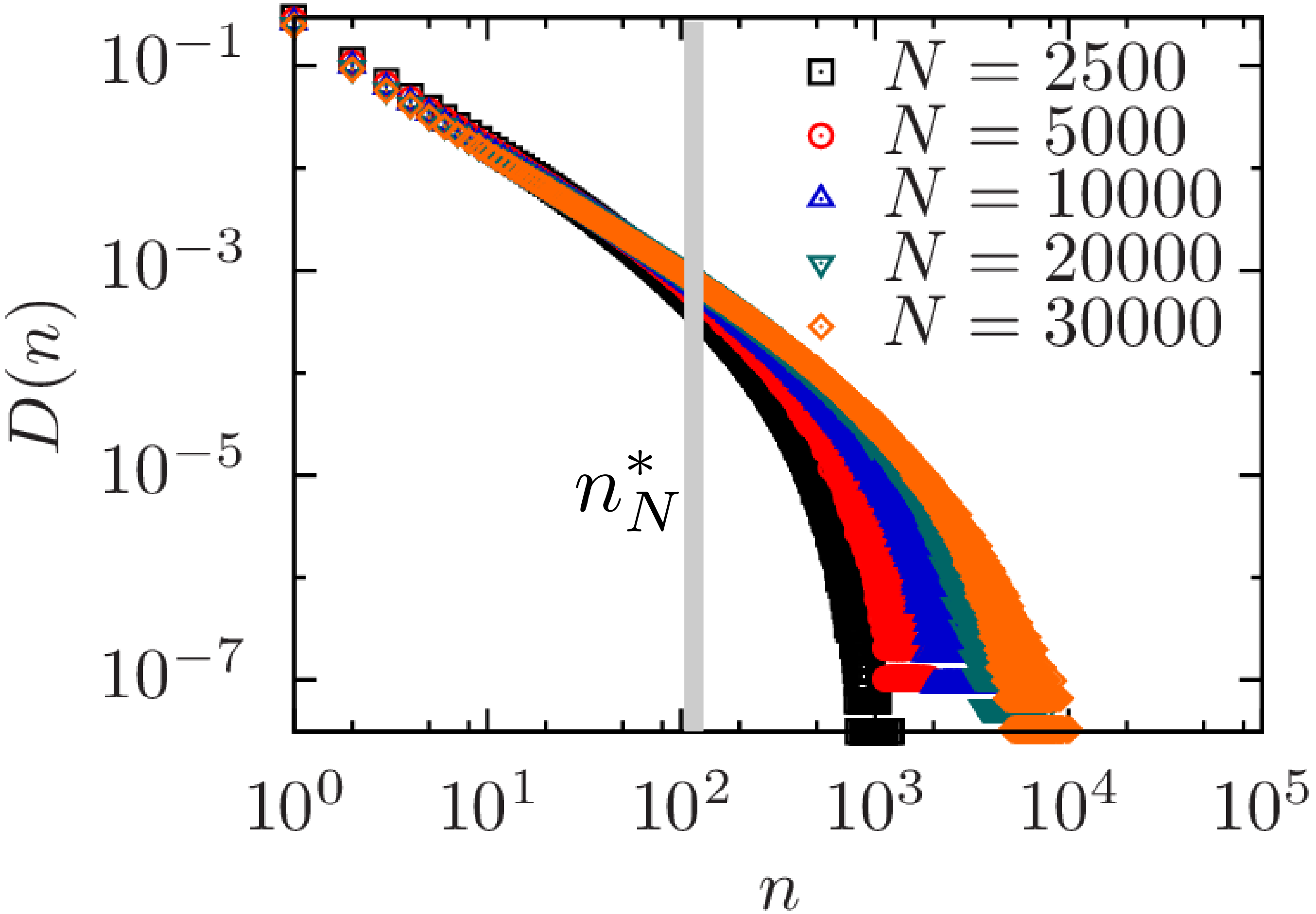

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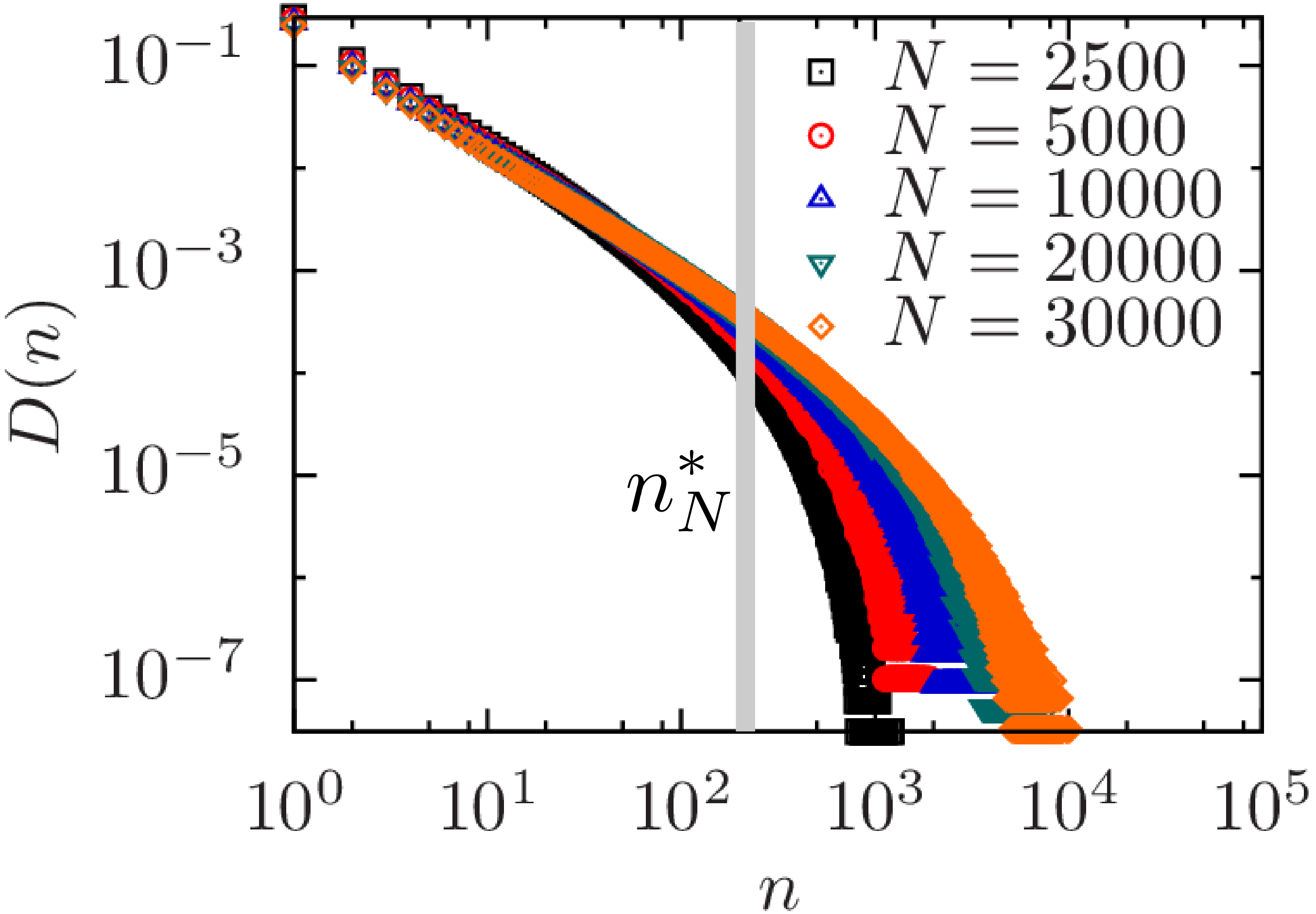

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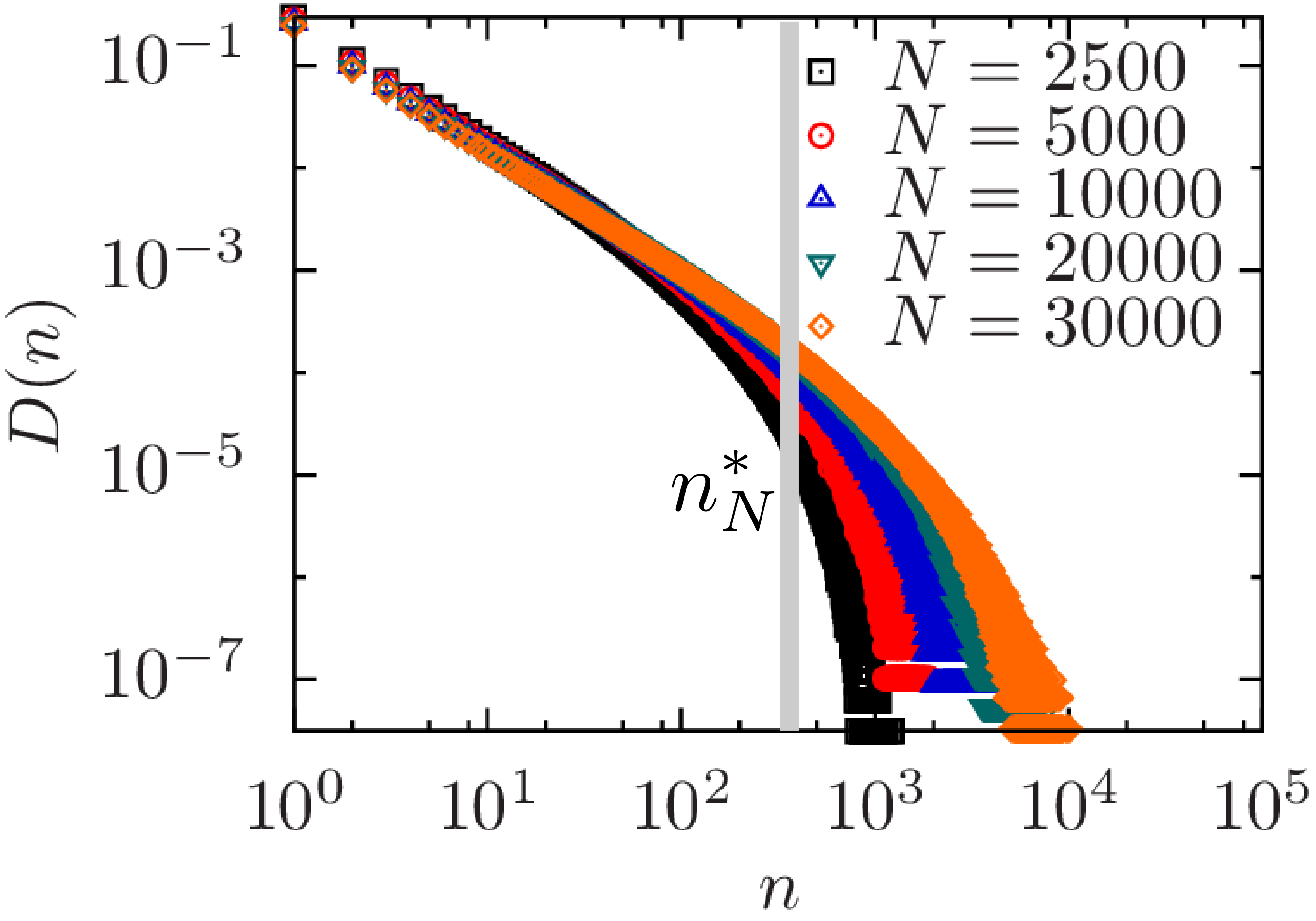
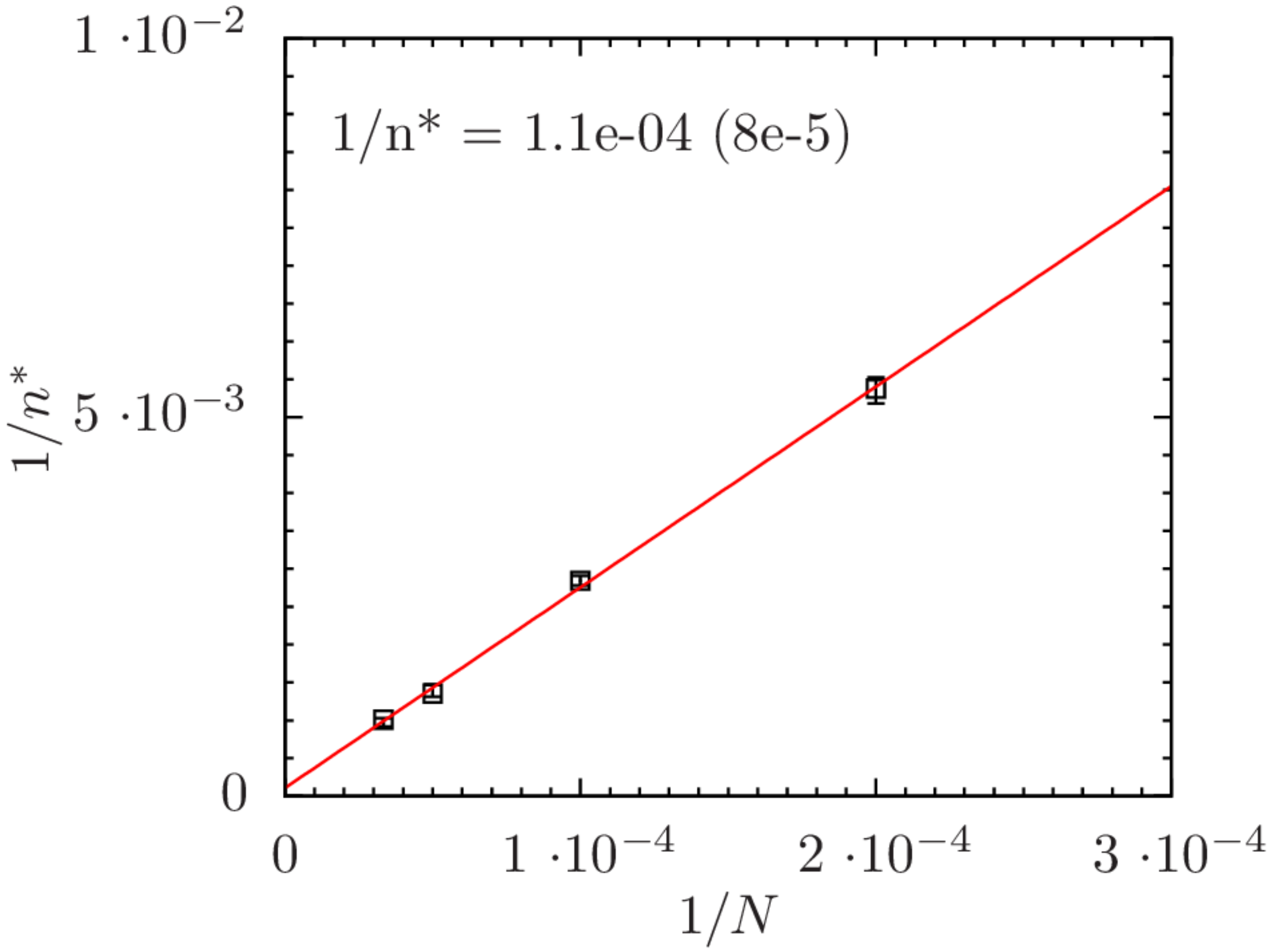


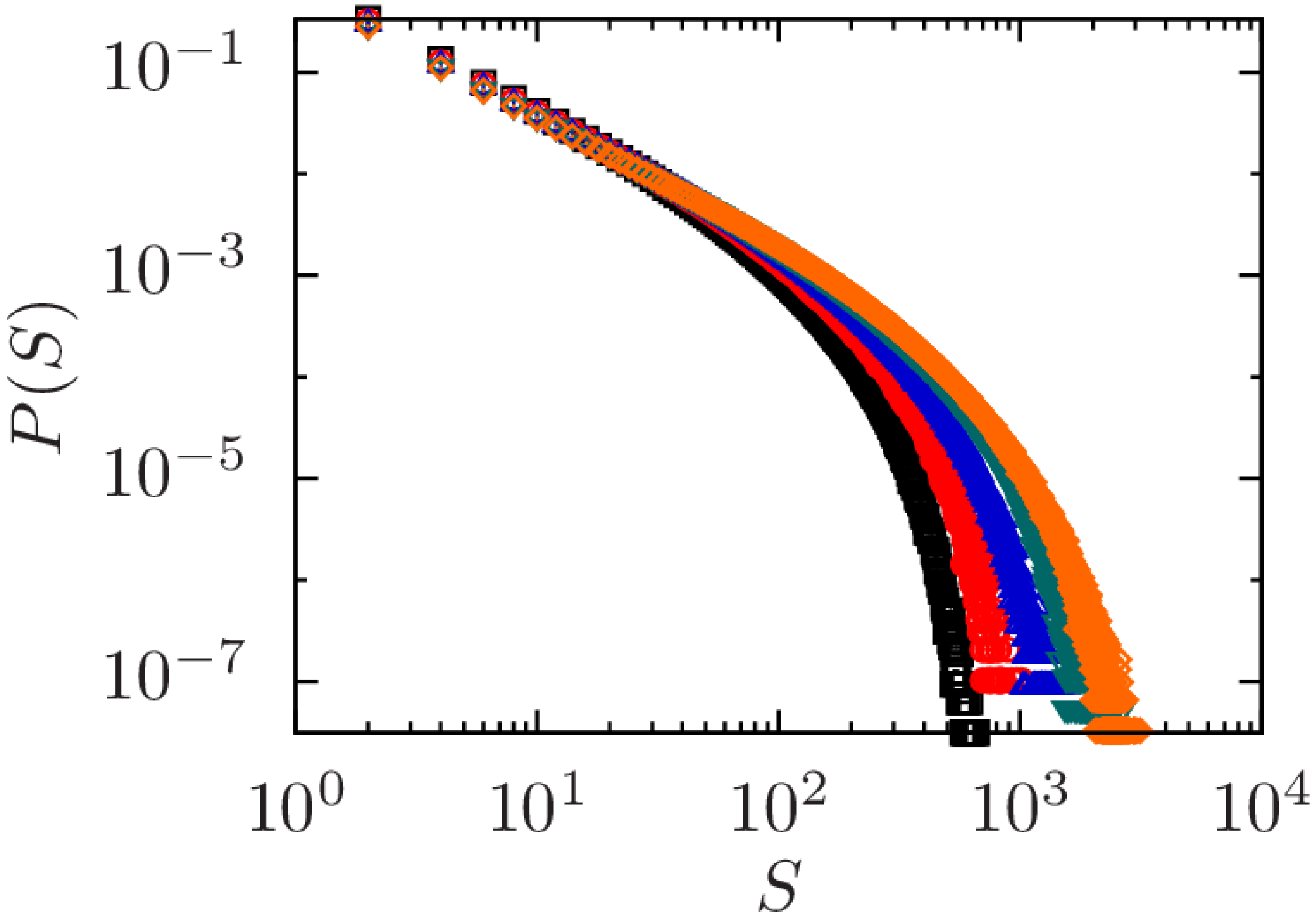
Self-organized criticality in the SK model

$N \to \infty$ extrapolation of n^*_N



Self-organized criticality in the SK model

Similar resutls for avalanches magnetization jumps



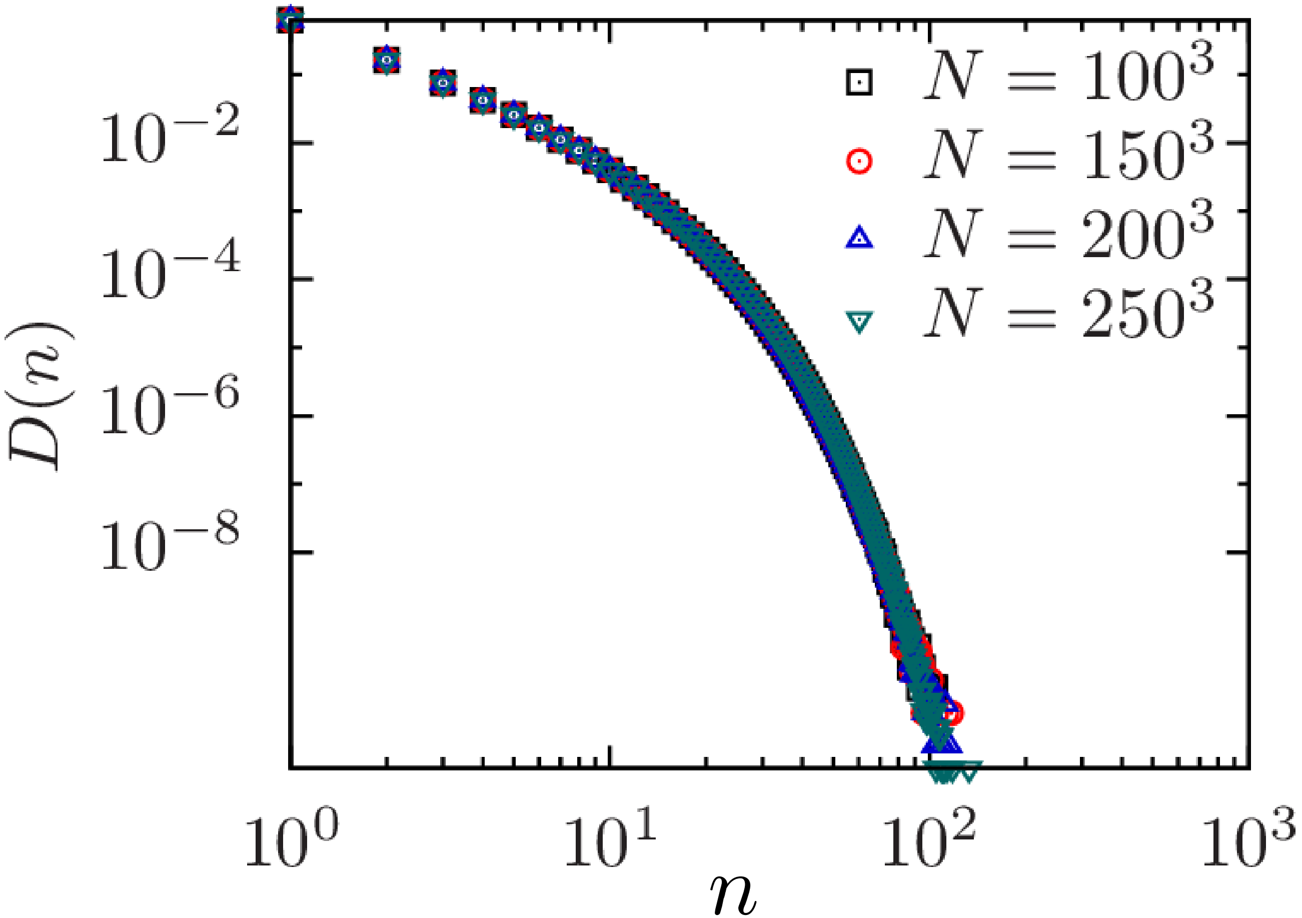
What about the EA model?

- SK: Similarities out-of-equilibrium avalanches with static calculations [Le Doussal et al. PRB (2012)]
- Previous work predic SOC in equilibrium avalanches of the 3-dimensional spin-glass model [Le Doussal et al. PRB (2012)]
- What about out-of-equilibrium avalanches of Edward-Anderson model?

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j - H \sum_i S_i$$

What about the EA model?

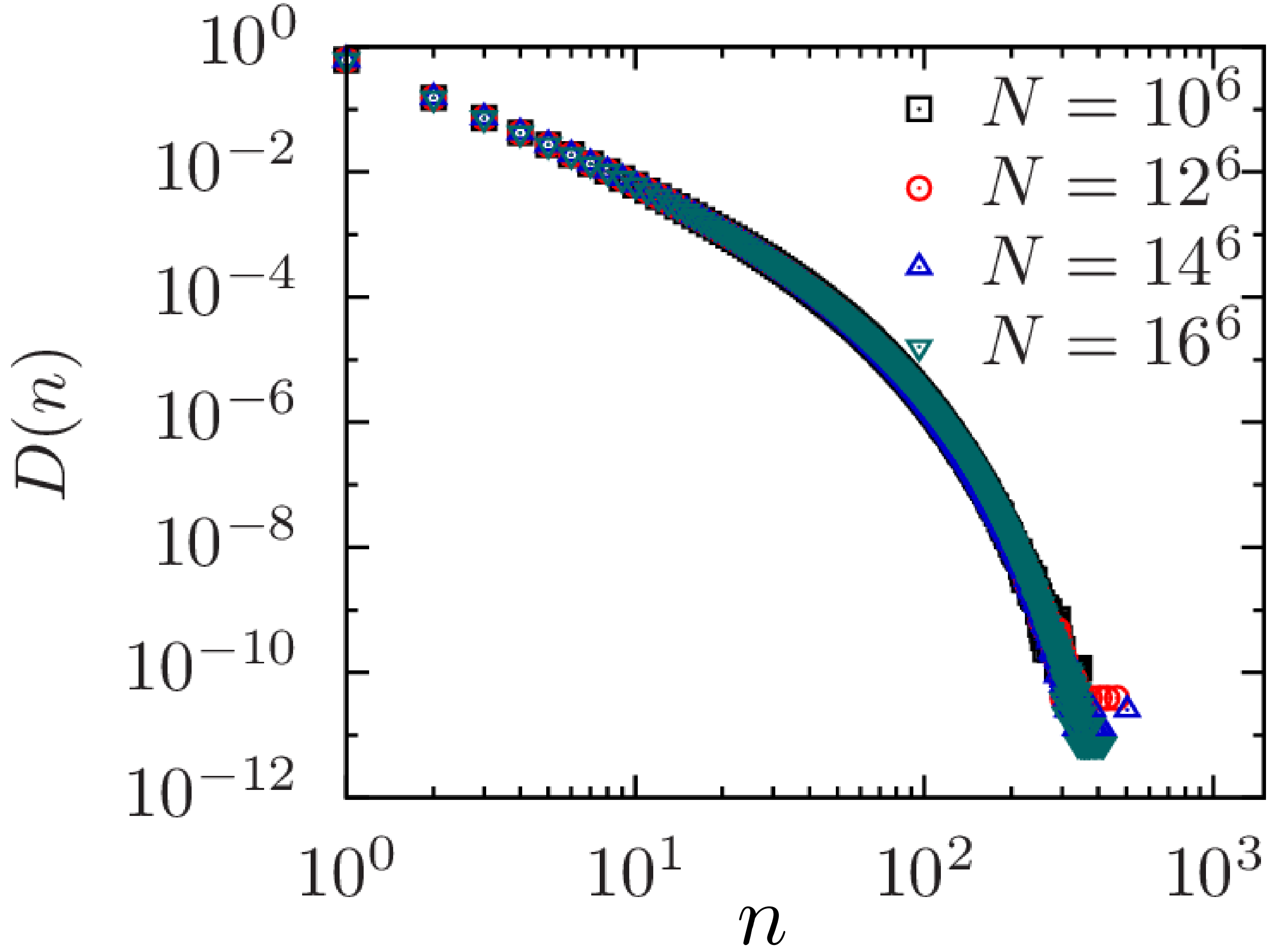
Same procedure as for the SK model for the 3-dimensional EA model



What about the EA model?

SOC mean-field universality class property?

$d_c = 6$

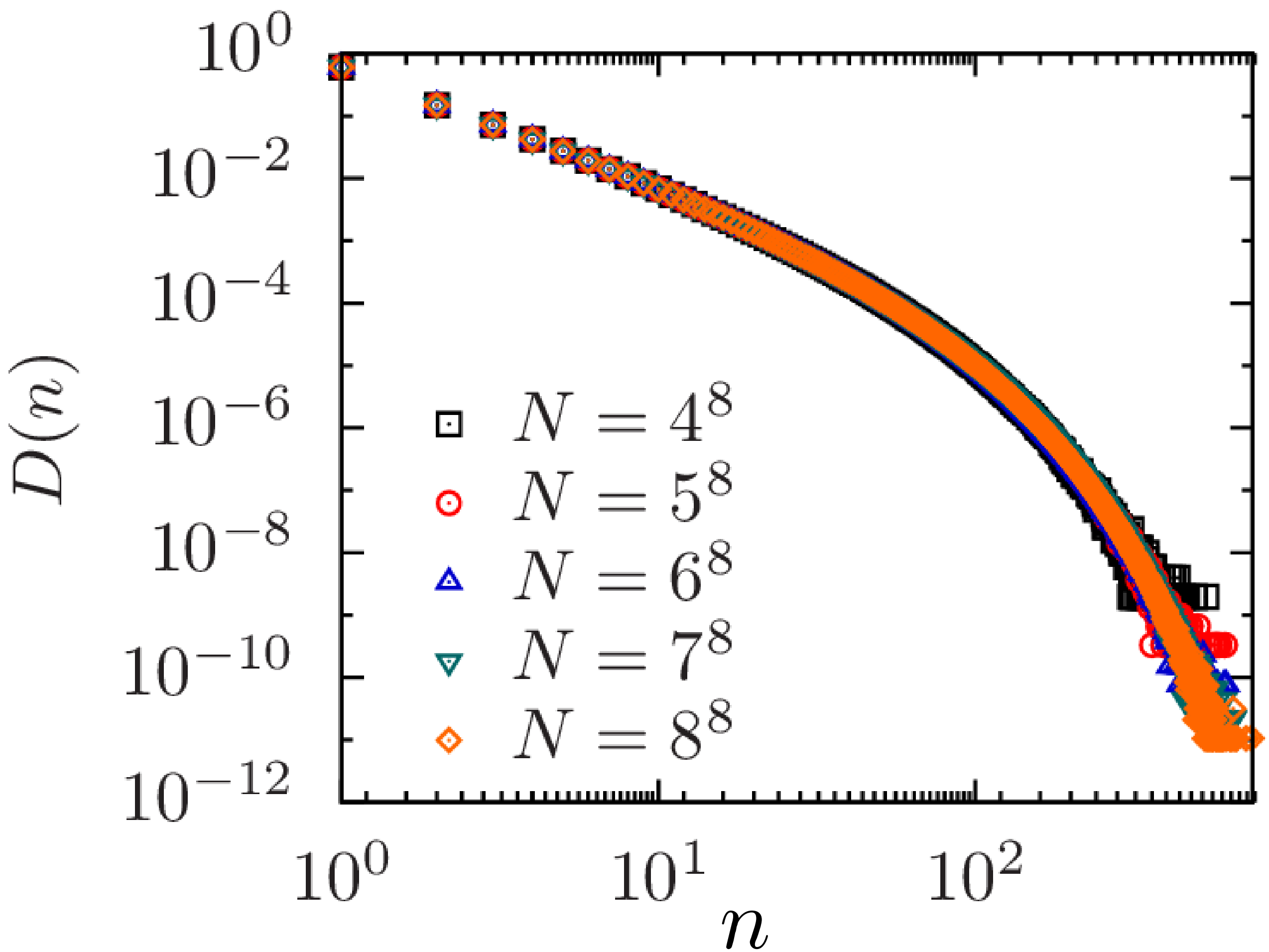


$d = \infty \to z = \infty$

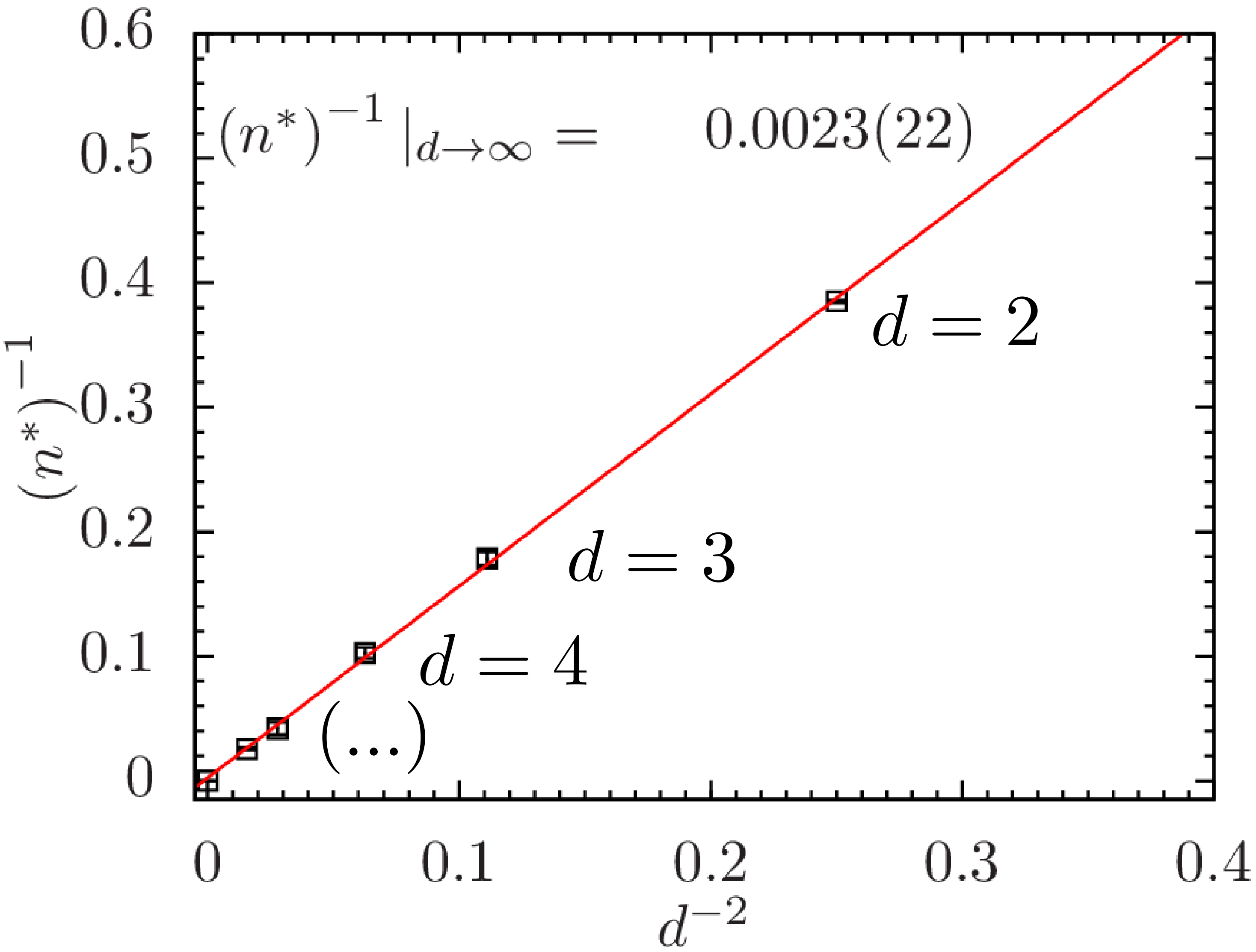
What about the EA model?

SOC mean-field universality class property?

$d = 8 > d_c$



Avalanches: from EA to SK



Does SOC come from the infinite range property?

- Long-range nature of the SK model?
- So far $z = 2d$, such that $d = \infty \rightarrow z = \infty$
- We want to test $d = \infty$, with $z = \mathrm{const.}$

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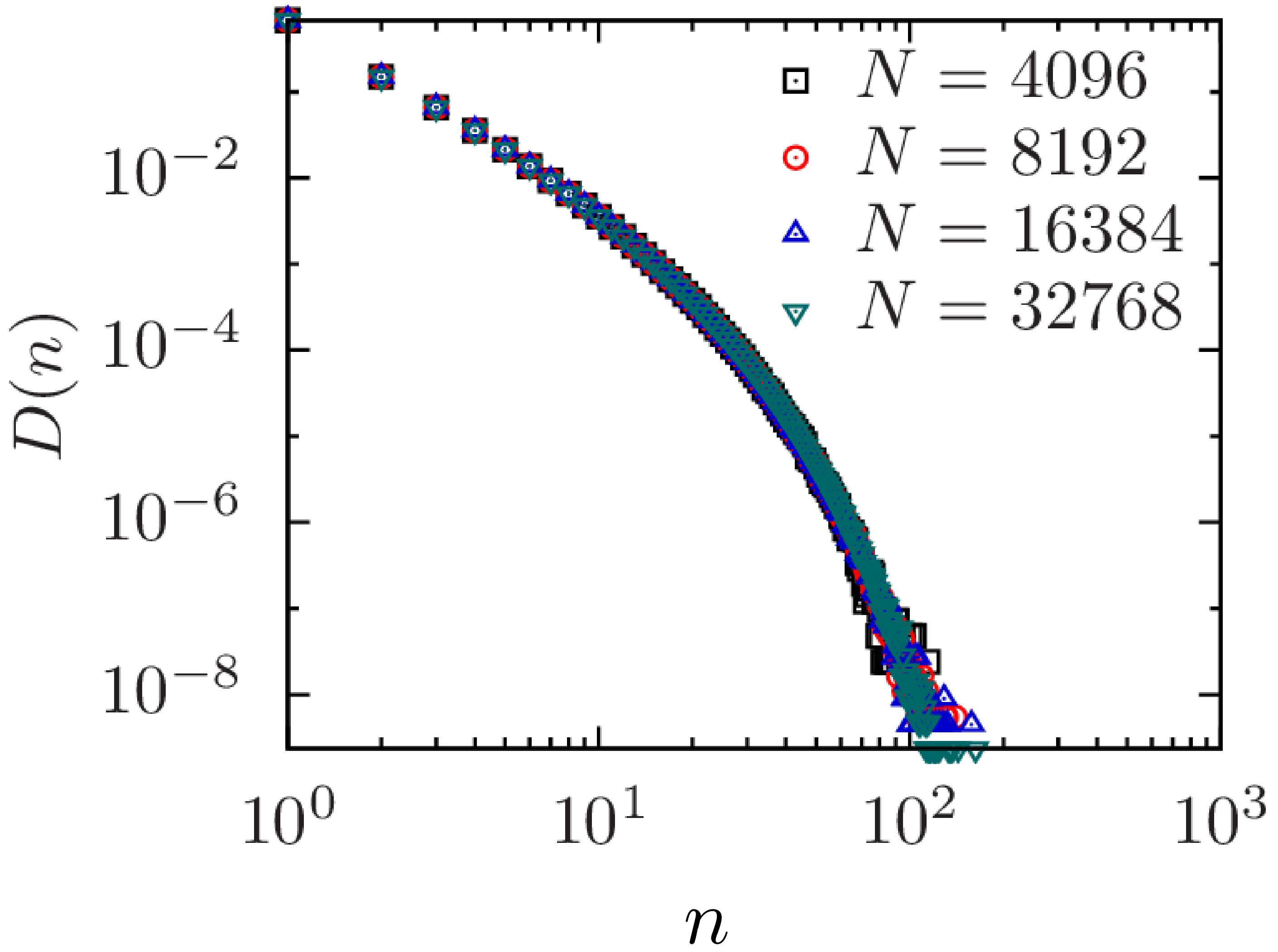
Viana-Bray Model!

$$\mathcal{H} = -\sum_{i,j} J_{ij} S_i S_j - H \sum_i S_i$$

Fixed coordination number z, random neighbors

Does SOC come from the infinite range property?



Self-organized criticality in spin-glasses

Condition for self-organized criticality:
Diverging number of neighbors
in the thermodynamic limit

Andresen et al. PRL (2013)

Other possible spin-models showing scale-free avalanches

Coulomb-glass model

$$\mathcal{H} = \frac{1}{2}\sum_{i,j}\frac{q_i q_j}{|r_i - r_j|}\left(n_i - \frac{1}{2}\right)\left(n_j - \frac{1}{2}\right) + \sum_i(\phi_i + V_i)n_i$$

r_i position of charge q_i, V_i external potential,

ϕ_i random potential, $n_i \in \{0, 1\}$

and is a charge neutral system (half filled)

Coulomb glass mapping

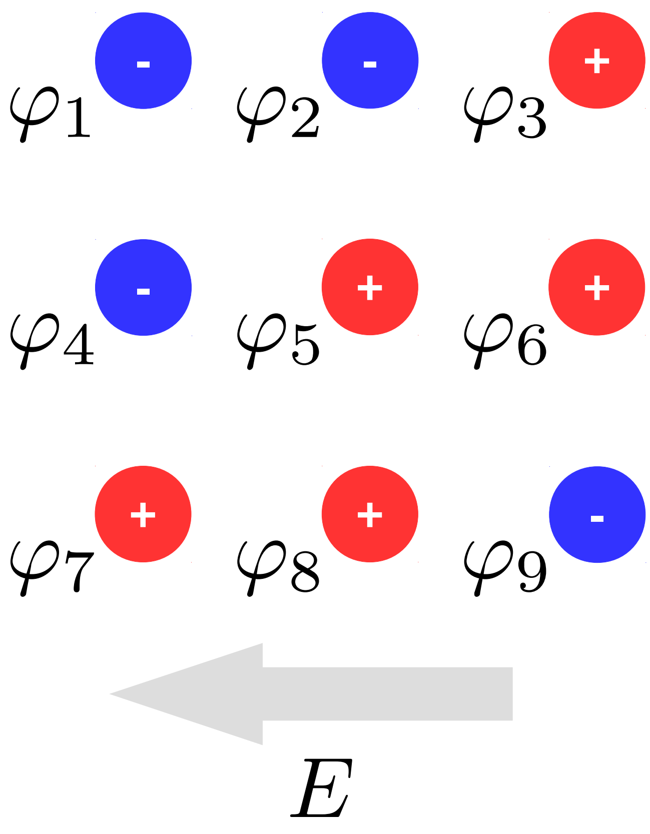
Mapped onto an Ising spin model with magnetization conserving dynamics:

$$\mathcal{H} = \sum_{ij} J_{ij} S_i S_j + \sum_i (\varphi_i + V_i) S_i$$

- Long-range interactions
- Disorder
- Out of equilibrium through an external potential

Avalanches in the Coulomb-glass model

magn. cons. dynamics



- Adiabatically increase the external potential
- Observables
  - Total electron hop
  - Net charge displacement

Avalanches in the Coulomb-glass model

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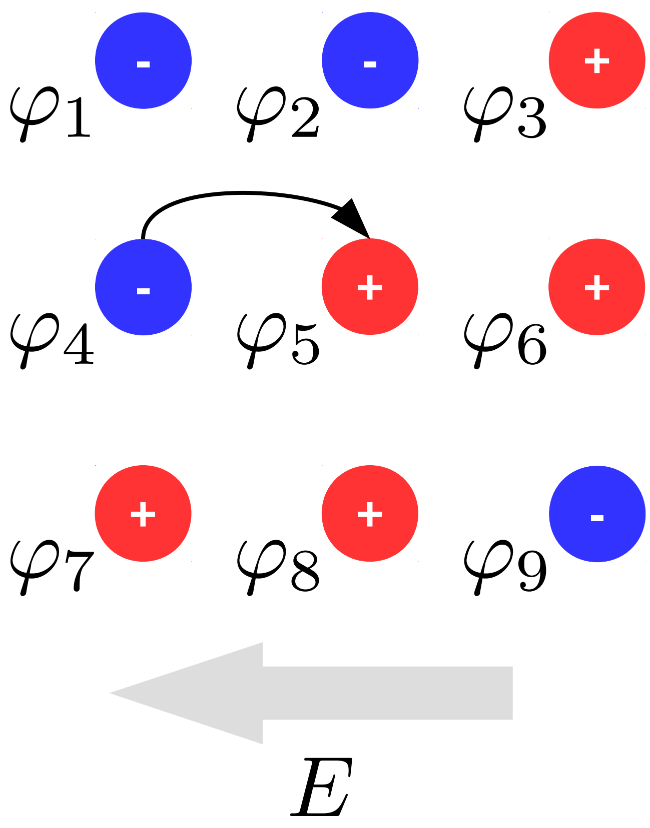

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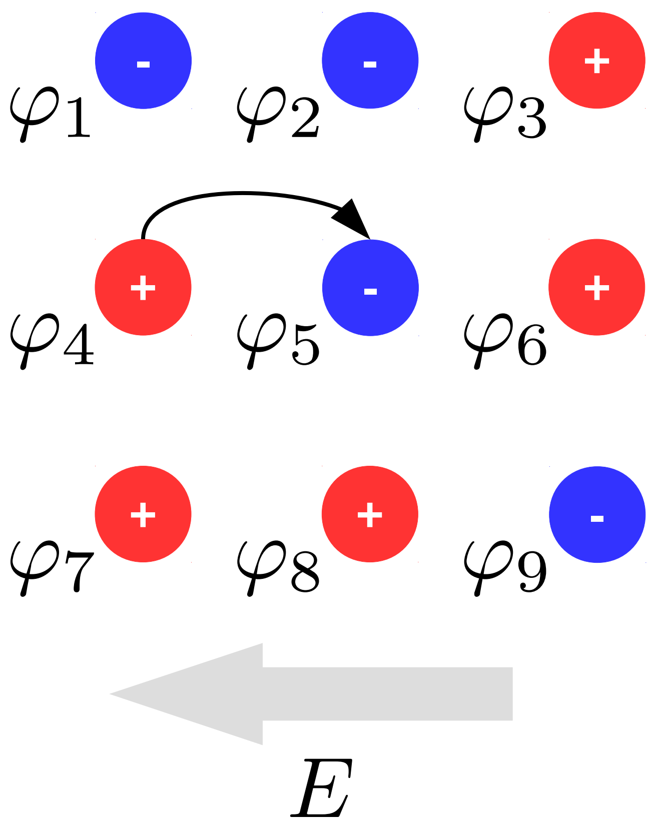

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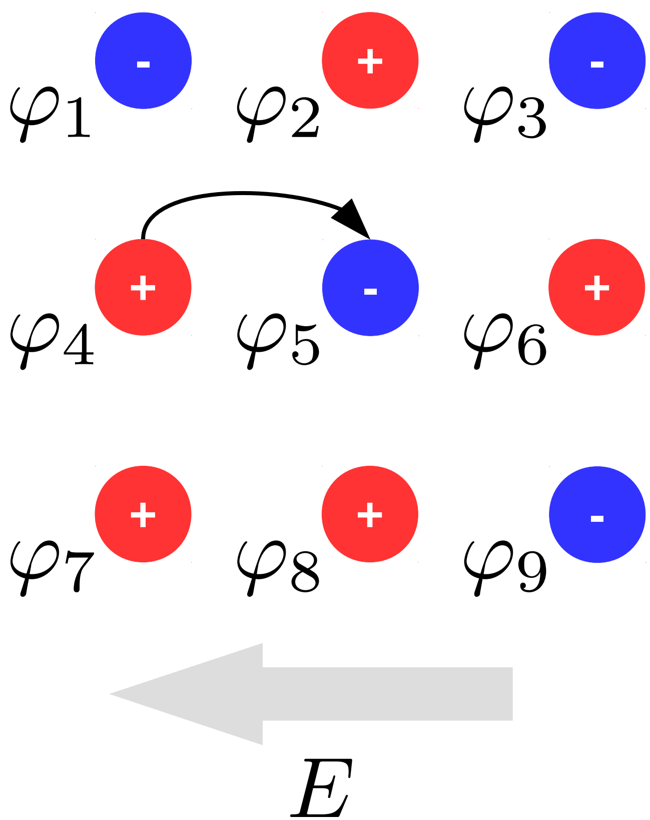
Avalanches in the Coulomb-glass model

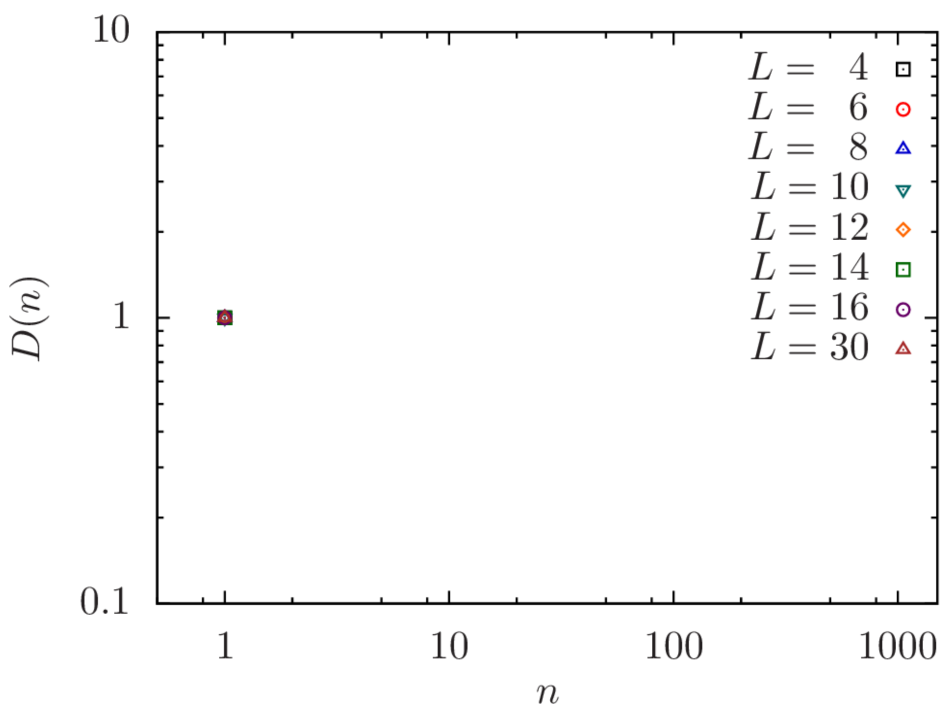
magn. cons. dynamics



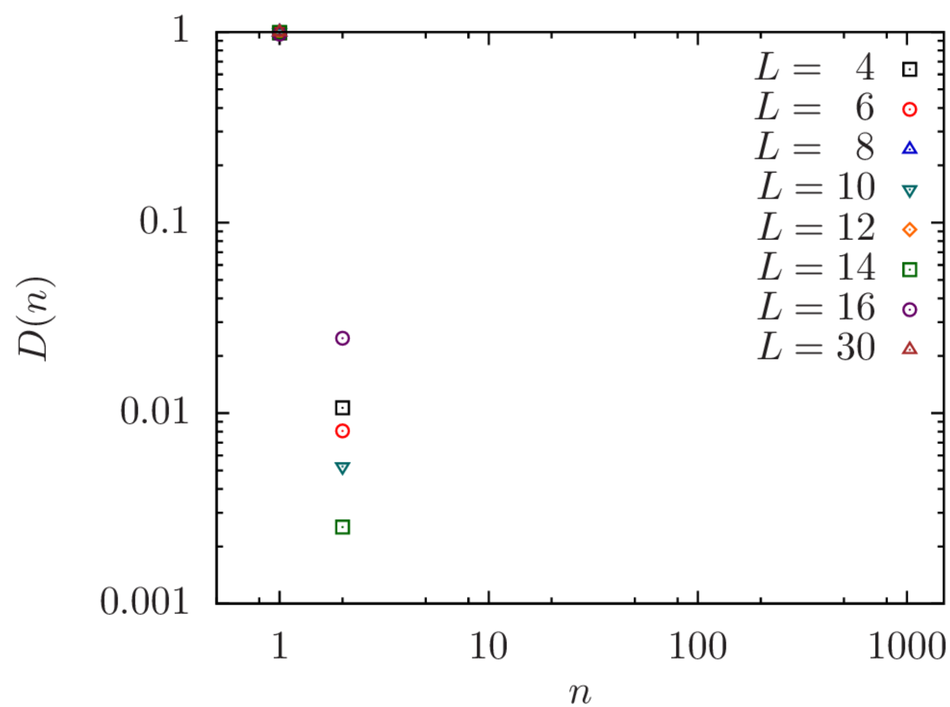
- Adiabatically increase the external potential
- Observables
  - Total electron hop
  - Net charge displacement

Avalanches in the Coulomb-glass model

For small fields $\mathcal{E}$ no more than one or two electron hops



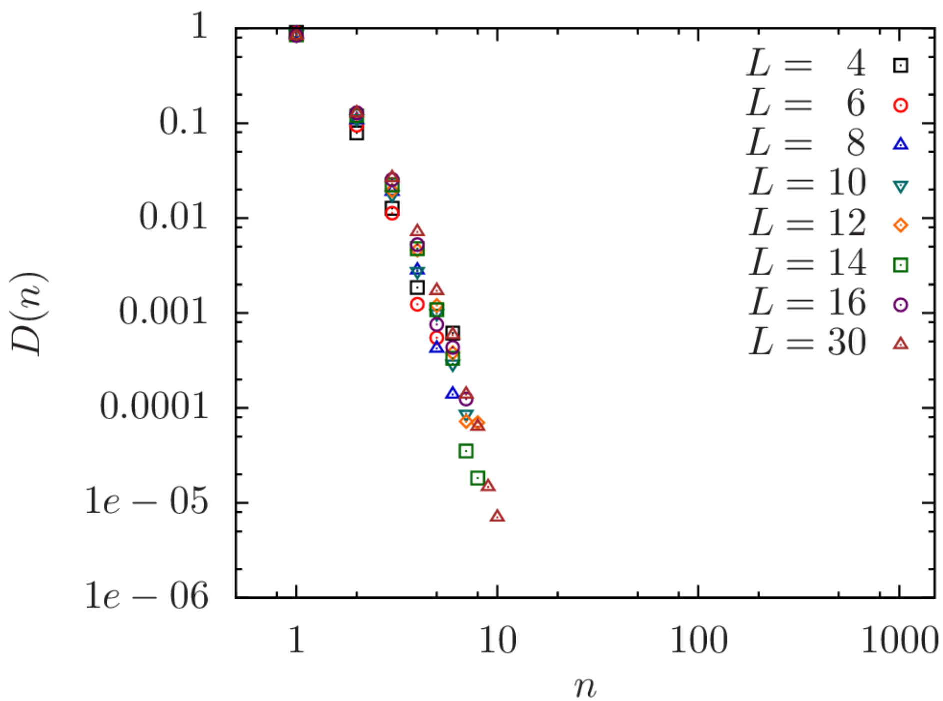
$0 < \mathcal{E} < 0.1$



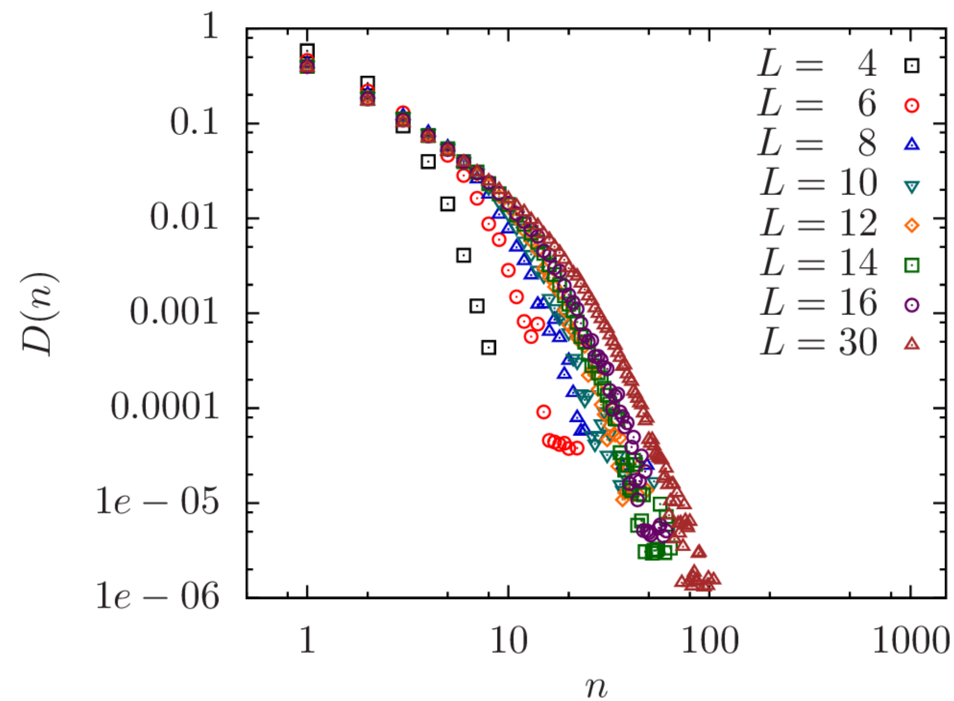
$0.1 < \mathcal{E} < 0.2$

Avalanches in the Coulomb-glass model

For intermediate fields an avalanche size dependence emerges



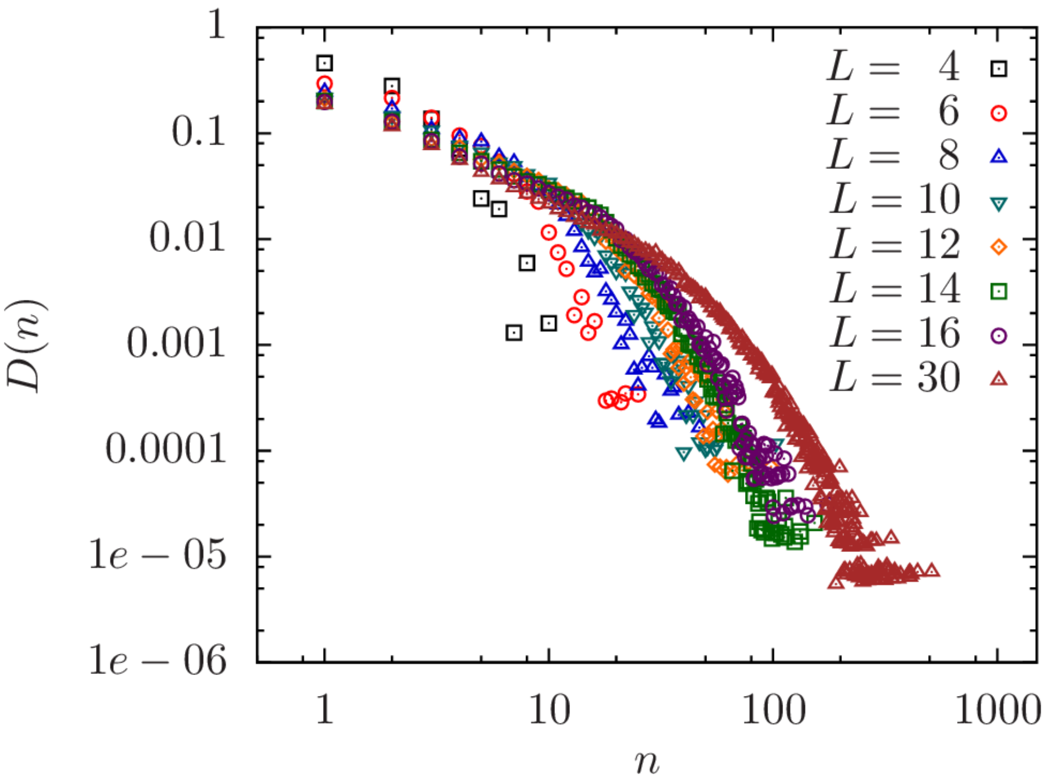
$0.2 < \mathcal{E} < 0.3$



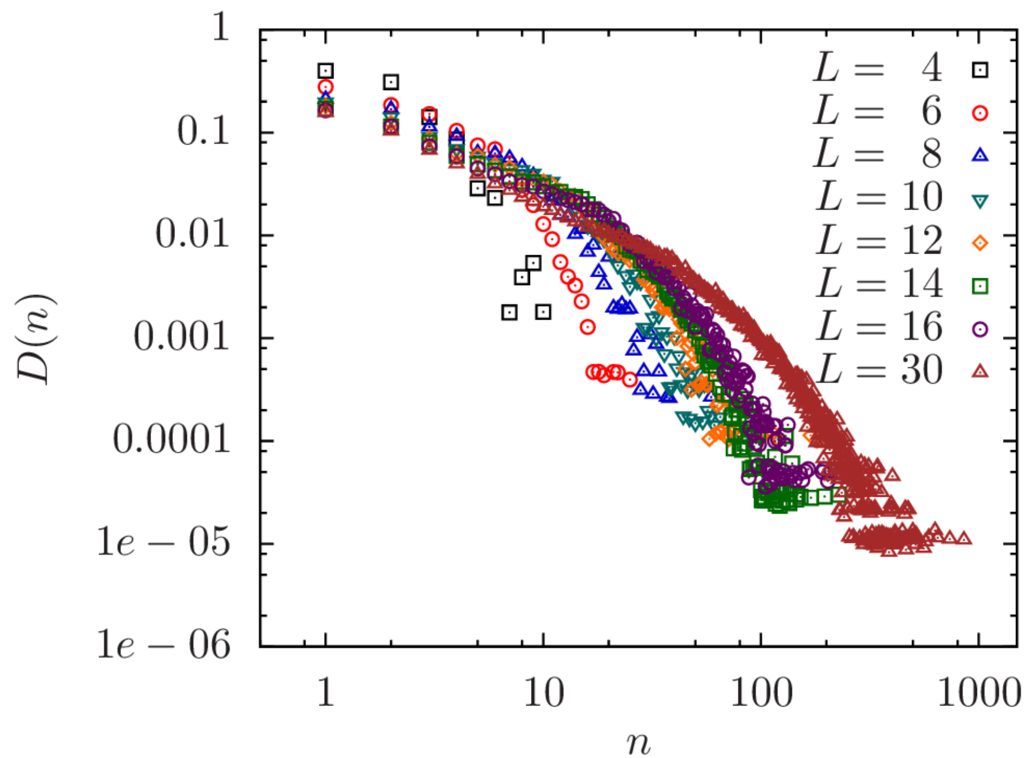
$0.3 < E < 0.4$

Avalanches in the Coulomb-glass model

Close to the depinning field $\mathcal{E}_{dp} = 0.6$



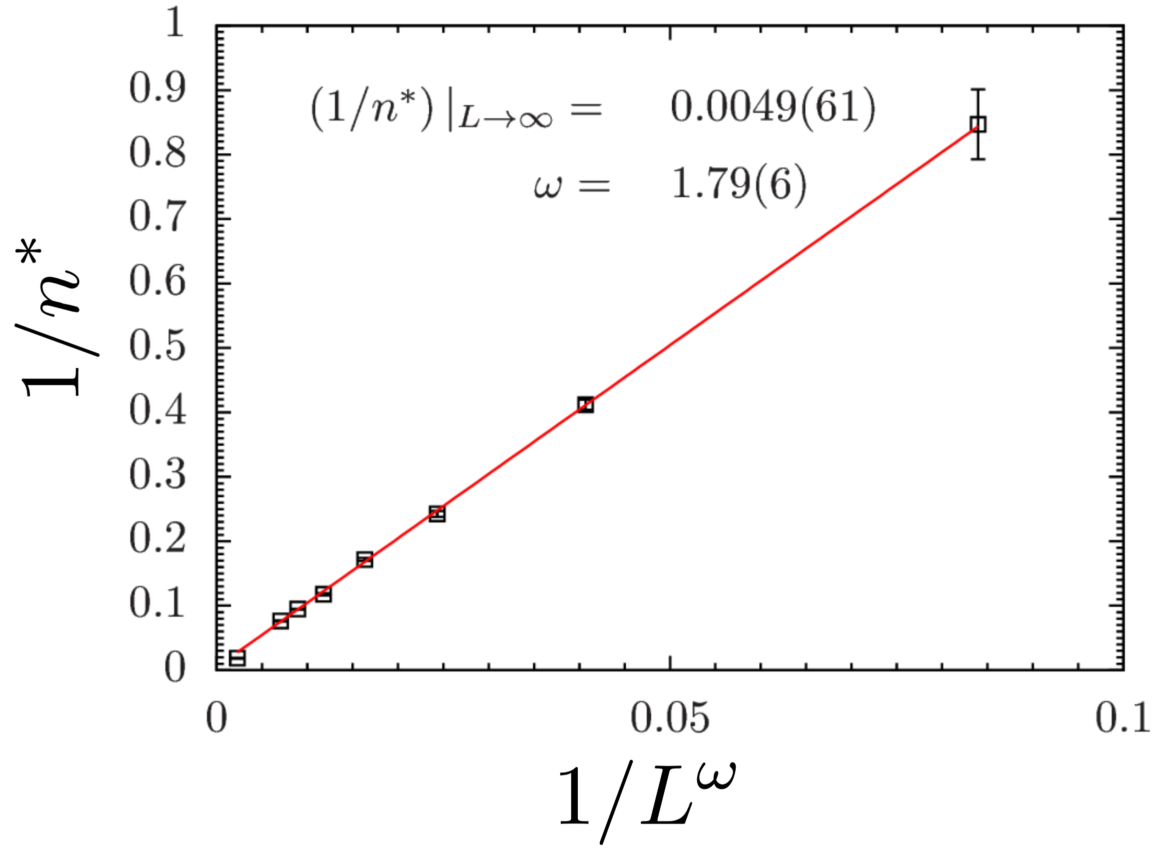
$0.55 < \mathcal{E} < 0.575$



$0.575 < E < 0.6$

Avalanches in the Coulomb-glass model

Extrapolation of the characteristic avalanche size



Andresen et al. arXiv:1309.2887

$$0.5 < E < 0.6$$

Summary: single flip vs. magnetization conserving?

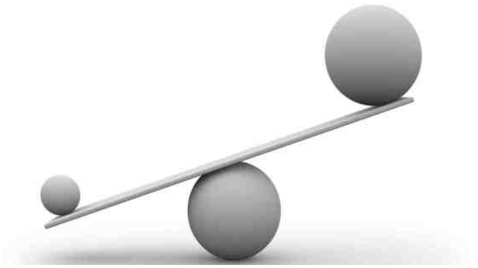
- Single flip: SOC for spin-glasses when number of neighbors diverge
- Magnetiztion conserving dynamics: scale-free avalanches only close to the depinning transition

- Is this due to the different spin flip dynamics applied?

Work in progress: single flip vs. magn. cons. dynamics

- Work in progress ...

- Does the dynamic imposed to the system change its behavior out of equilibrium? E.g. presence or absence of self-organized criticality?



Work in progress: single flip vs. magn. cons. dynamics

- Single flip:

  Diluted dipolar systems driven by an external magnetic field

  SOC when $z \rightarrow$ infty?

- Magn. cons. dynamics:

  SK model with zero magnetic field constrain driven by an site-alternating magnetic field

  No SOC even when $z \rightarrow$ infty?

Work in progress: single flip vs. magn. cons. dynamics

THANK YOU!

- Single flip:

  Diluted dipolar systems driven by an external magnetic field

  SOC when z ➞ infty?

- Magn. cons. dynamics:

  SK model with zero magnetic field constrain driven by an site-alternating magnetic field

  No SOC even when z ➞ infty?