# Sound waves in granular materials

## the role of disorder, friction, adhesion, ...

Stefan Luding et al. (see below)
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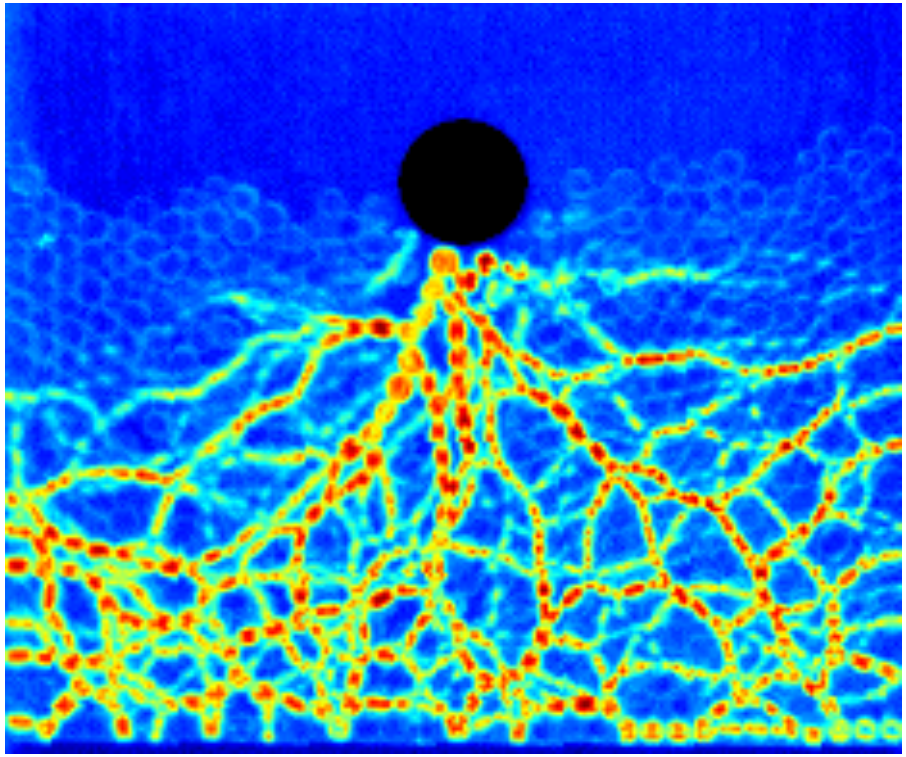
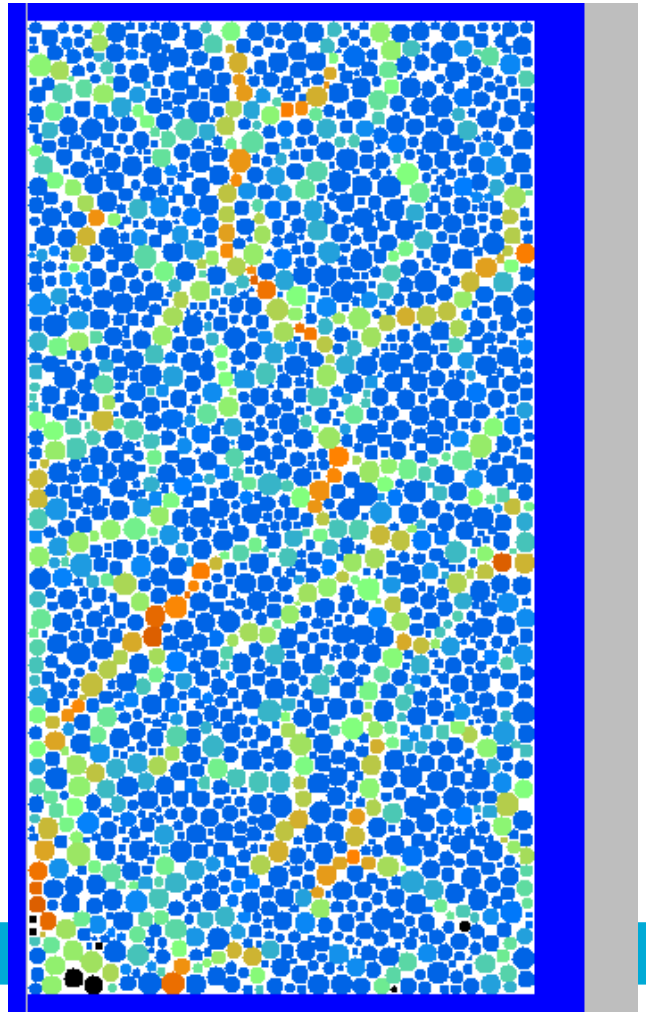
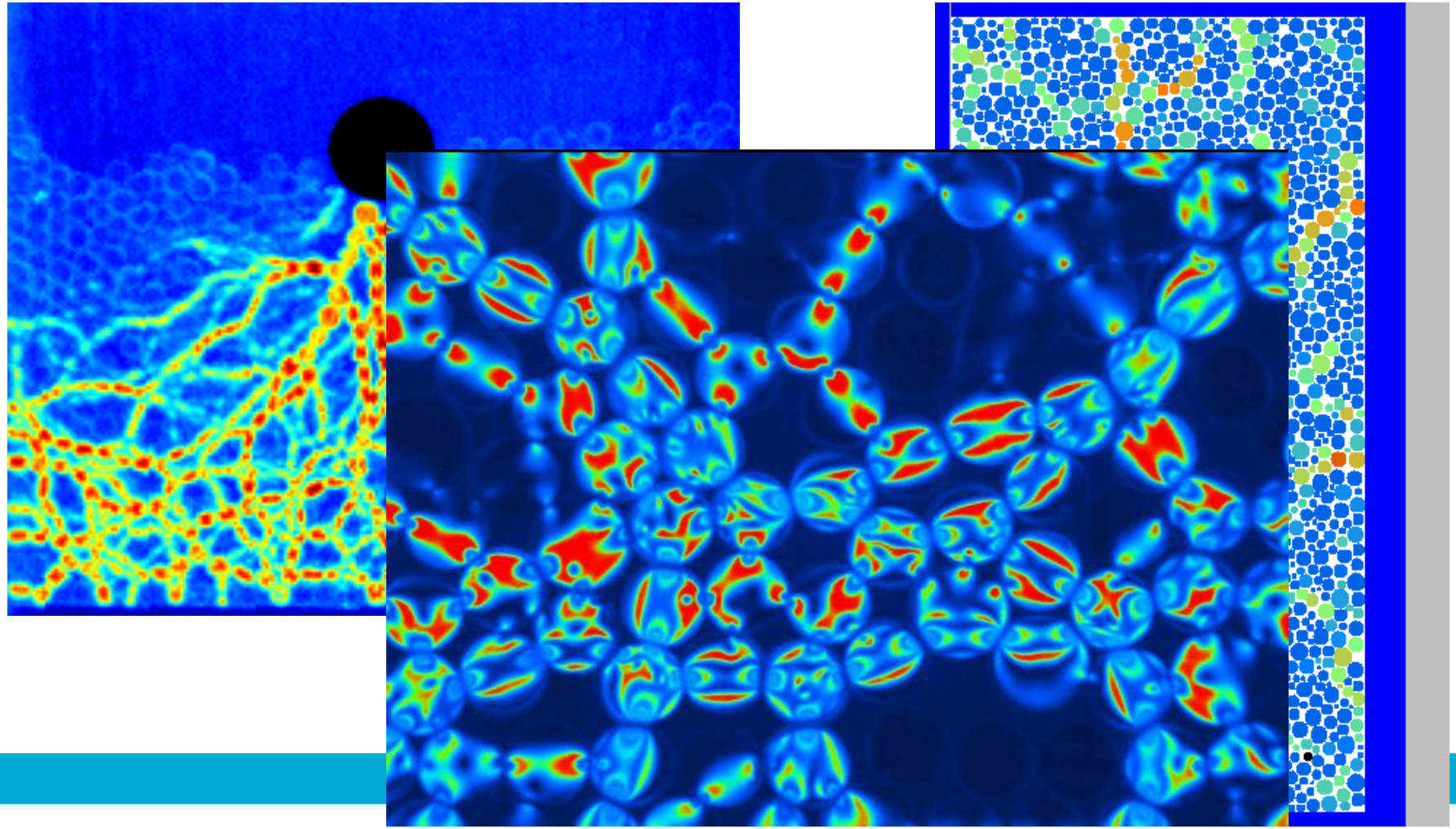
University of Twente
Enschede - The Netherlands

# Force-chains
## experiments - simulations



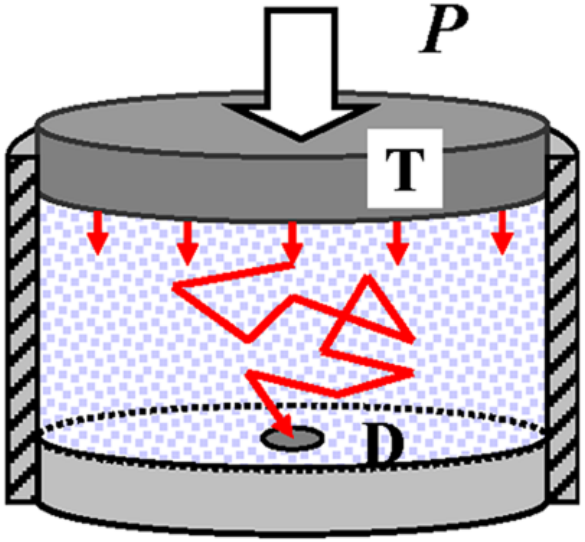


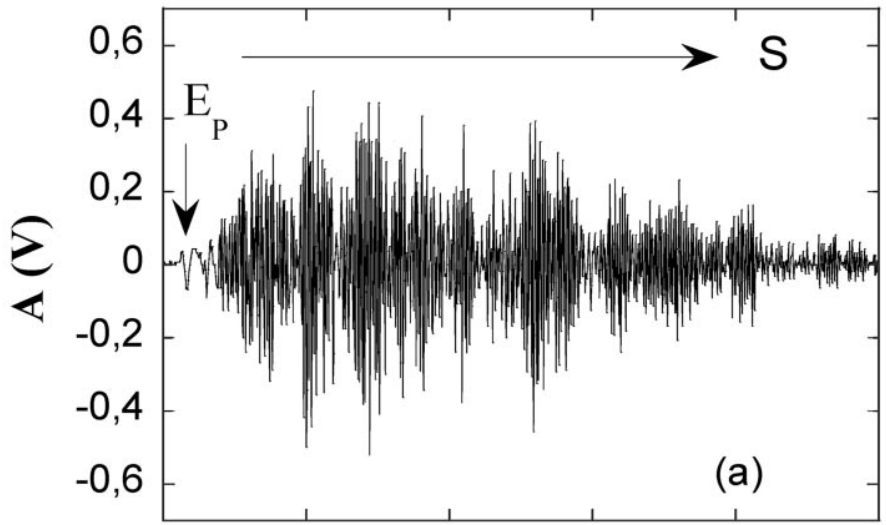
# Force-chains
## experiments - simulations



# Codalike Multiple Scattering of ElasticWaves in Dense Granular Media

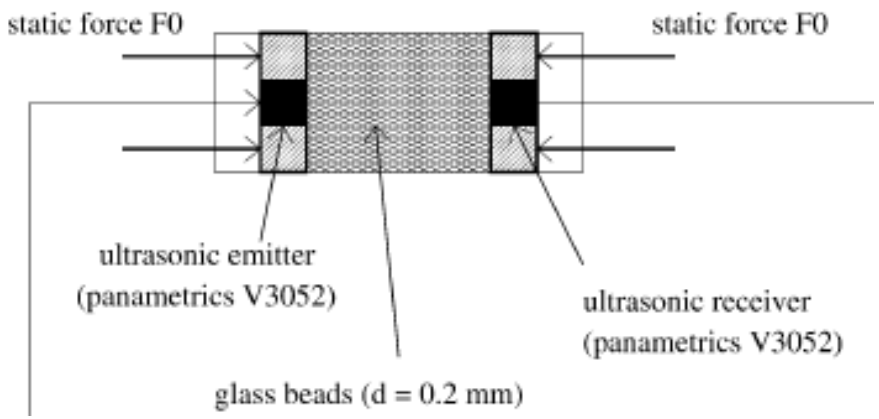
X. Jia, PRL, 2004

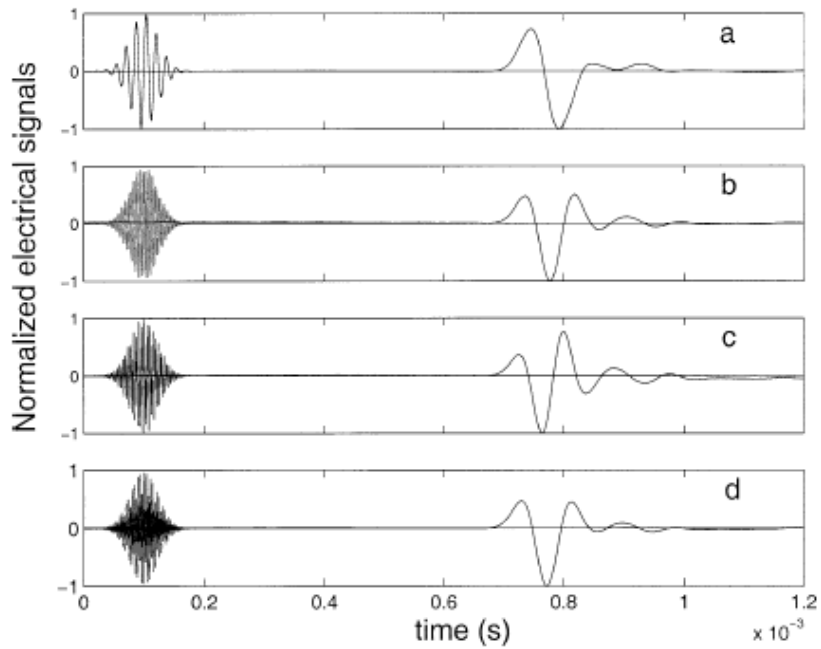


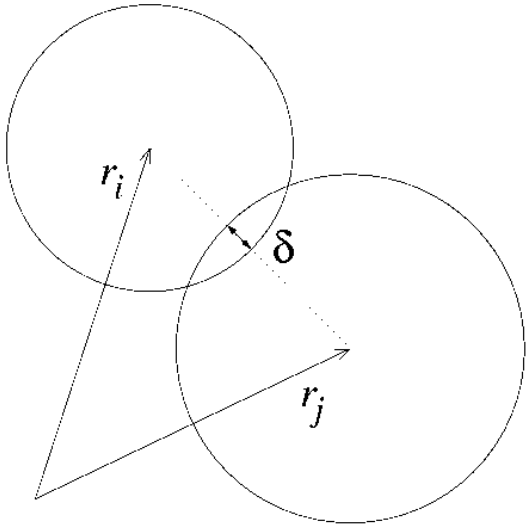
# Self-demodulation acoustic signatures for nonlinear propagation in glass beads

V. Tournat et *al*, C.R.Mecanique, 2002



# Discrete particle model



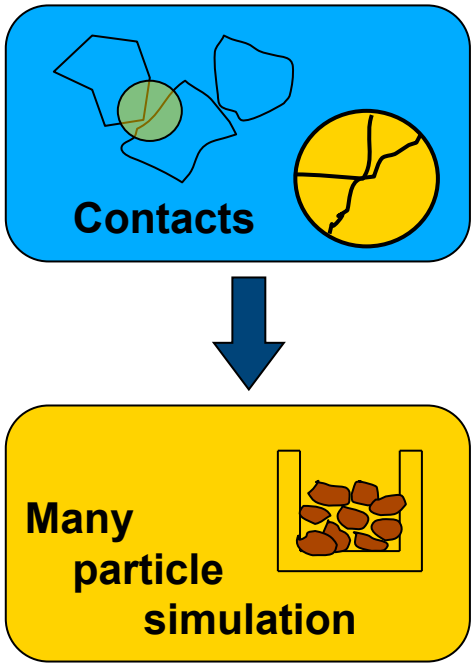

**Equations of motion**

$$m_i \frac{d^2\vec{r}_i}{dt^2} = \vec{f}_i \qquad I_i \frac{d\vec{\omega}_i}{dt} = \vec{t}_i$$

**Forces and torques:**

$$\vec{f}_i = \sum_c \vec{f}_i^{\,c} + \sum_w \vec{f}_i^{\,w} + m_i g$$

$$\vec{t}_i = \sum_c \vec{r}_i^{\,c} \times \vec{f}_i^{\,c}$$

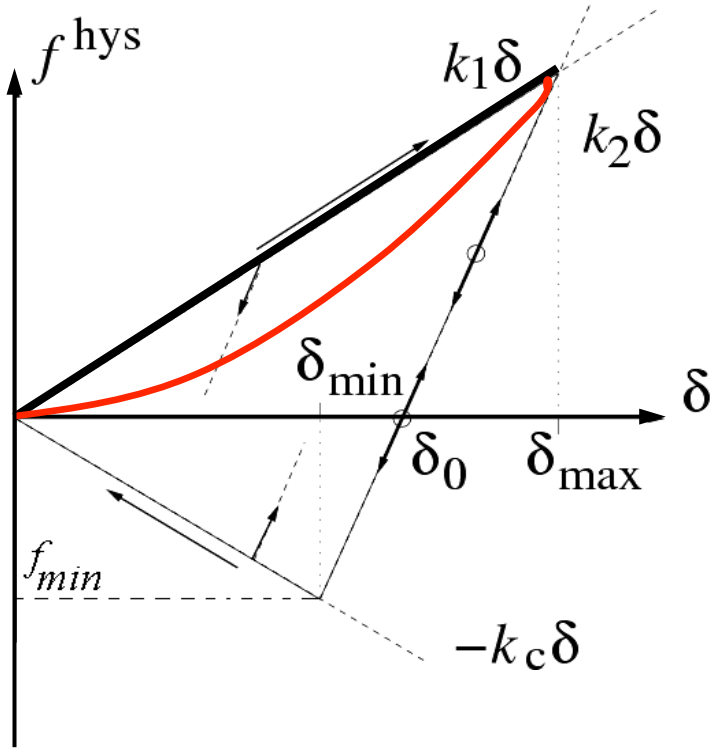



**Overlap** $\delta = \frac{1}{2}\left(d_i + d_j\right) - \left(\vec{r}_i - \vec{r}_j\right) \cdot \vec{n}$

**Normal** $\hat{n} = \vec{n}_{ij} = \frac{\left(\vec{r}_i - \vec{r}_j\right)}{\left|\vec{r}_i - \vec{r}_j\right|}$

# Contact laws

Normal contact model



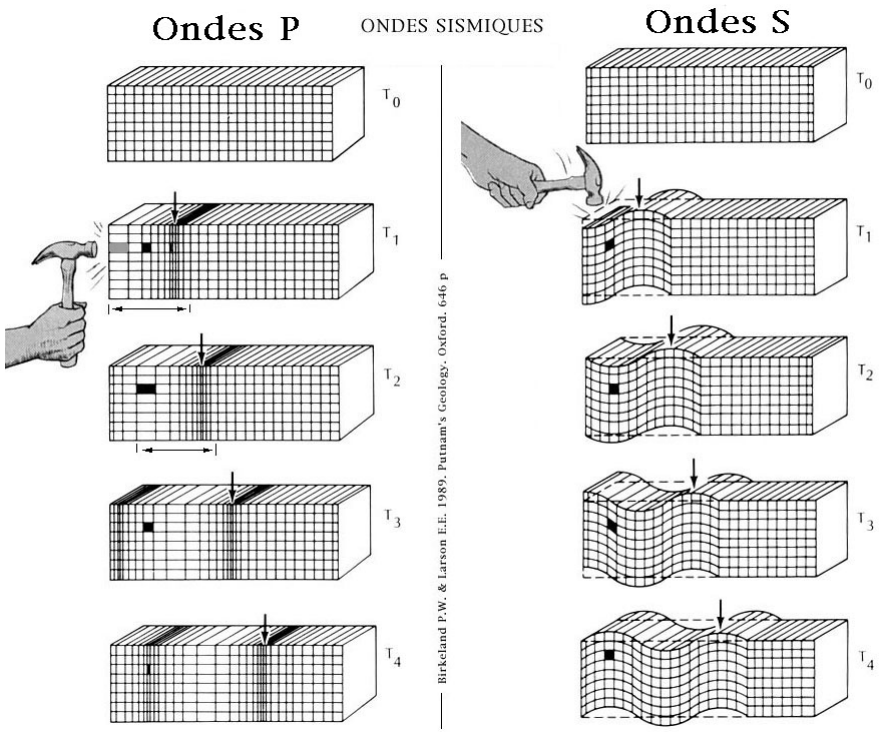

$$f = k_N . \delta$$

$$f = k_N . \delta^{3/2}$$

Tangential contact model → **Sliding**, Rolling, Torsion

# Sound

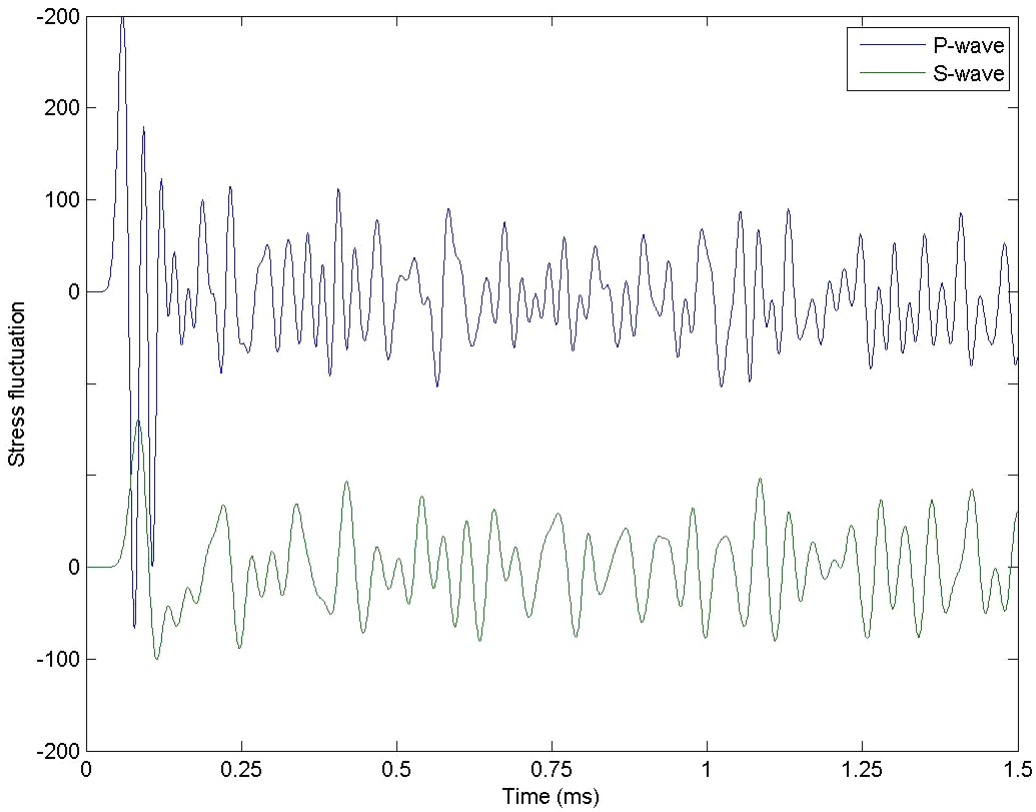
Compressive (P) and  Shear (S) waves



# Time signals

**P-wave** →

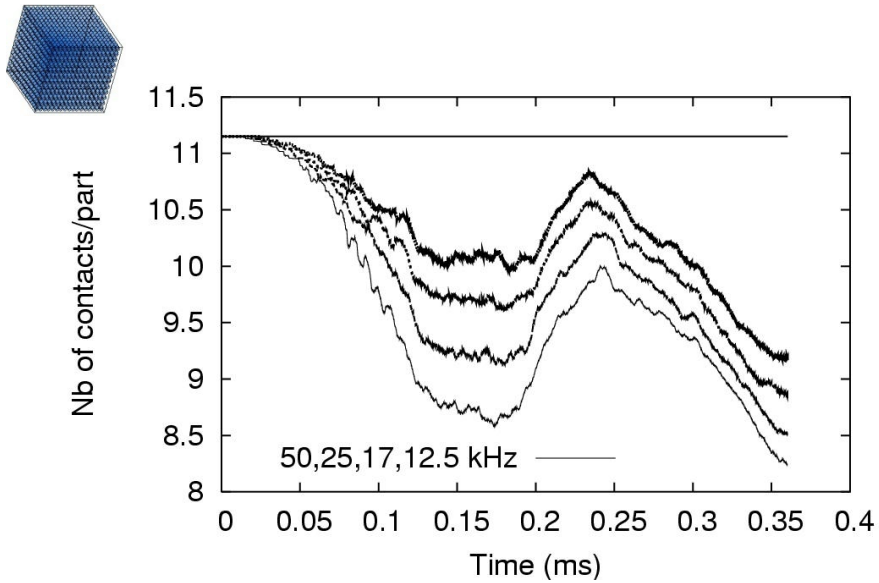
**S-wave** →

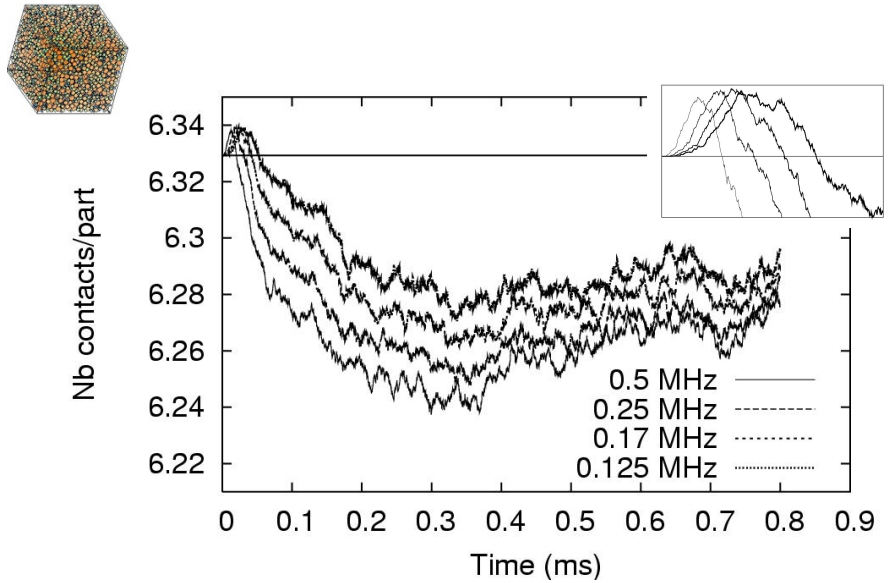


# Frequency dependence

- Compressive wave
- Input frequency
- Large amplitude

Number of contacts in the whole system versus time

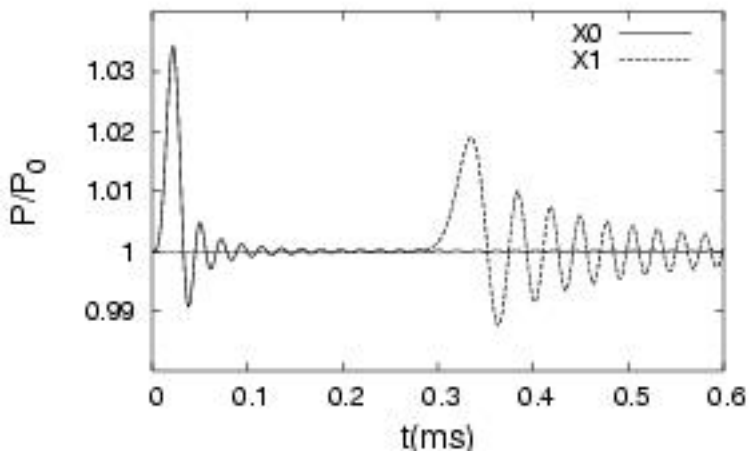


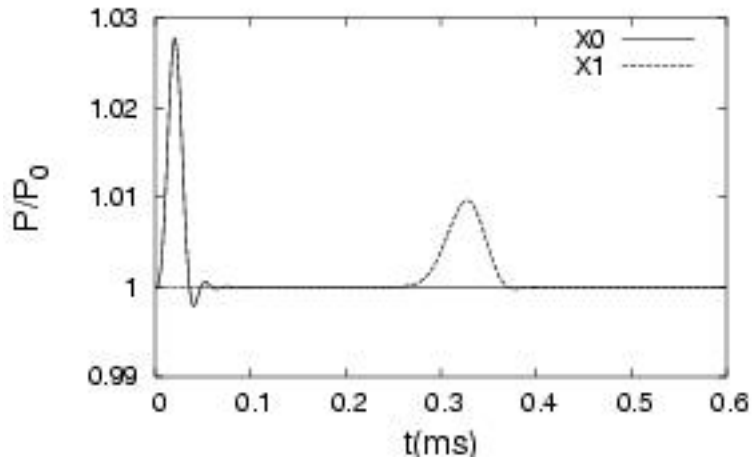
Compressive (P)-wave

# Influence of “micro” properties

Elastic

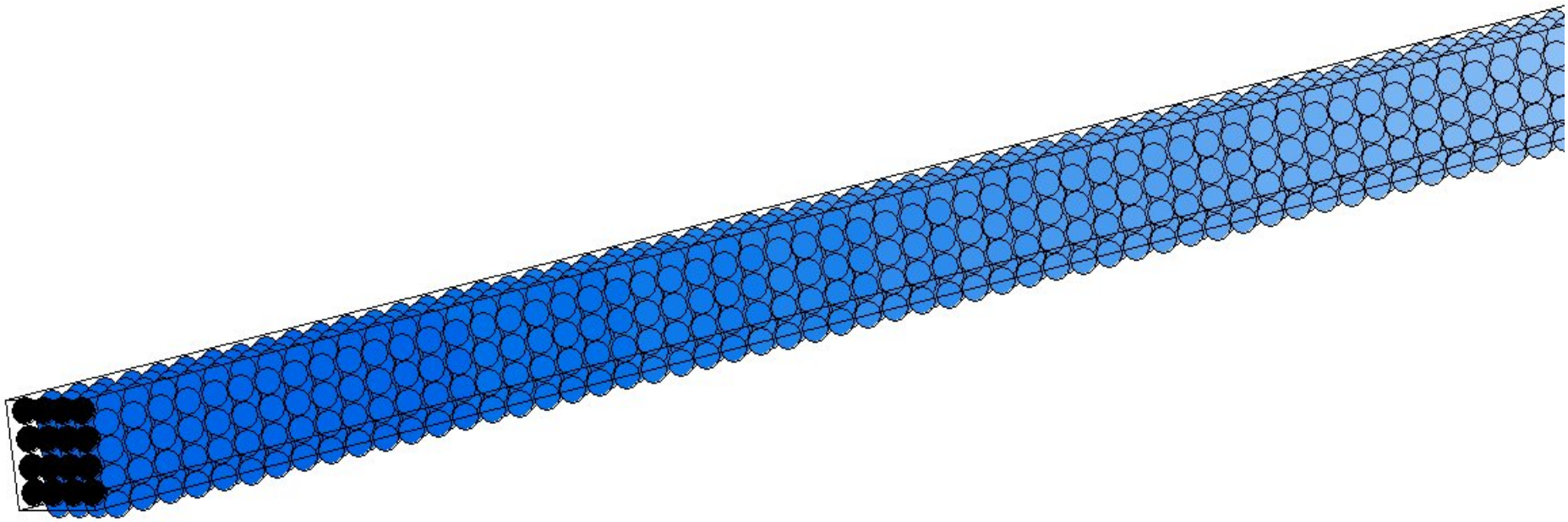



Visco-Elastic

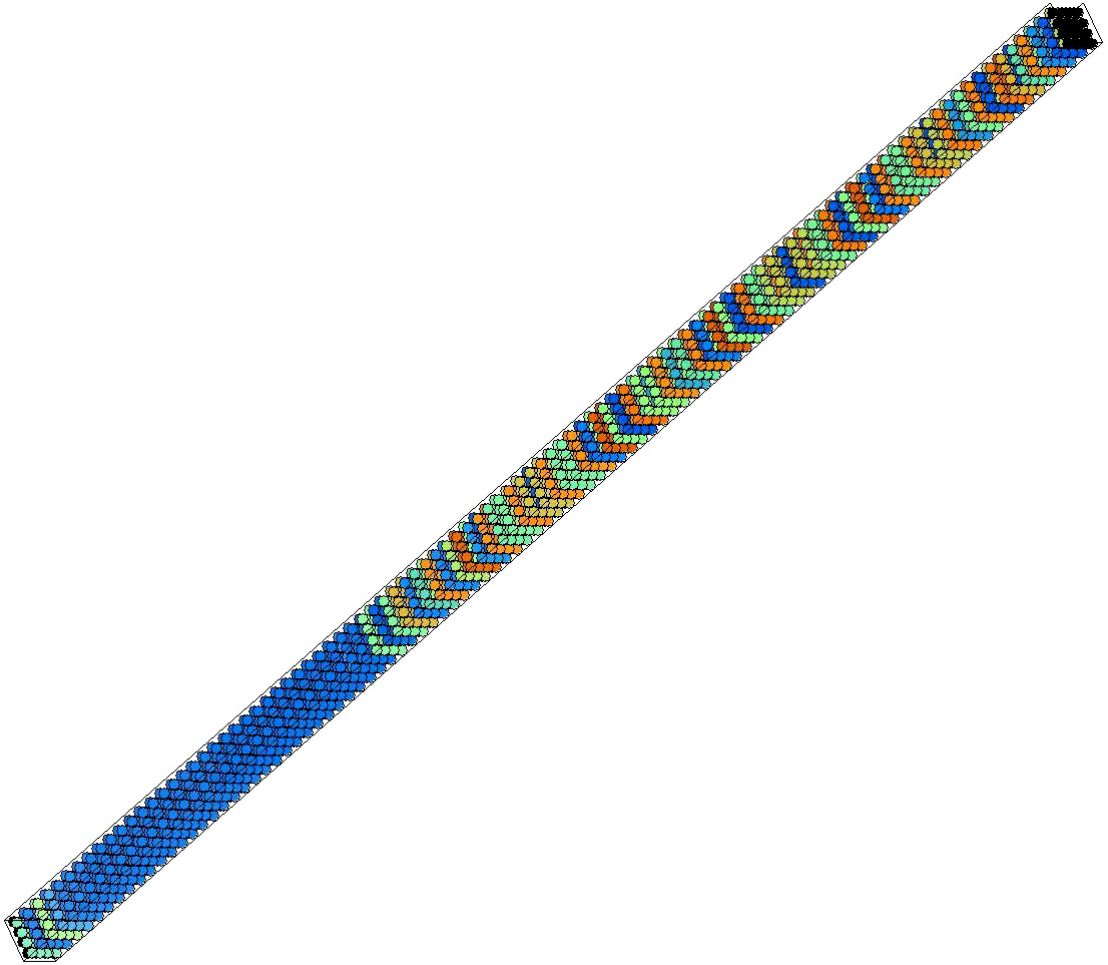



How relevant is the damping coefficient in our model ?

# Model system



# P-wave animation



# Towards complexity

<u>Modes</u>

- P-waves
- S-waves
- R-waves
- …

<u>Micro-Parameters</u>

- Damping
- Friction (Rotations)
- Adhesion
- Contact law
- …

<u>Structure</u>

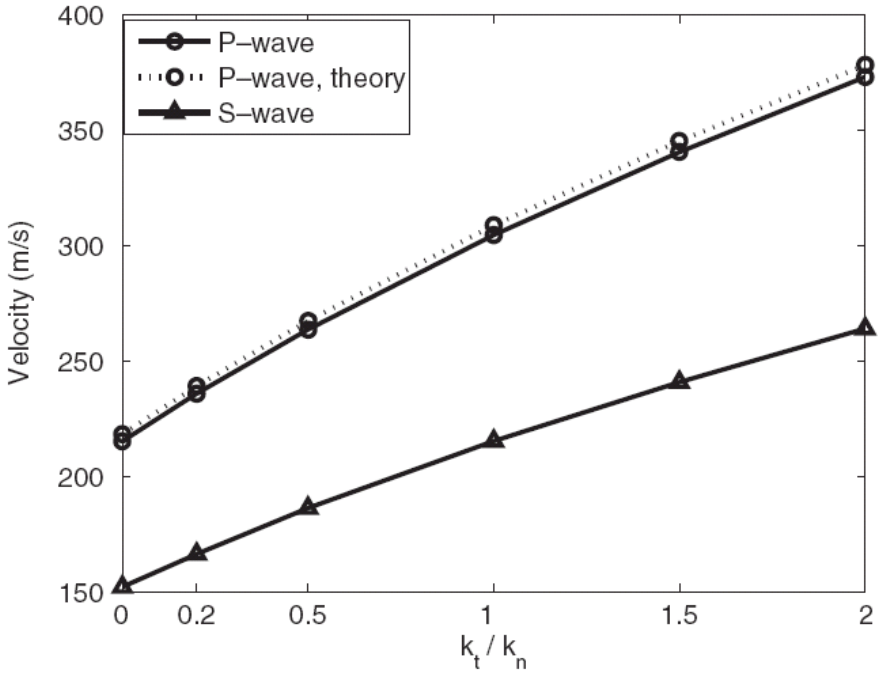
- Mono- poly-disperse
- (Dis-)ordered
- …

# Wave speed from the stiffness tensor

$$C_{\alpha\beta\gamma\phi} = \frac{1}{V}\sum_{p\in V}\frac{a^2}{2}\left(k\sum_{c=1}^{C} n_\alpha^c n_\beta^c n_\gamma^c n_\phi^c + k^t\sum_{c=1}^{C} n_\alpha^c t_\beta^c n_\gamma^c t_\phi^c\right)$$

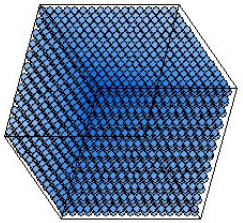
$$V_{pz} = \sqrt{\frac{C_{zzzz}}{\rho}}$$

# Velocities




**Friction**

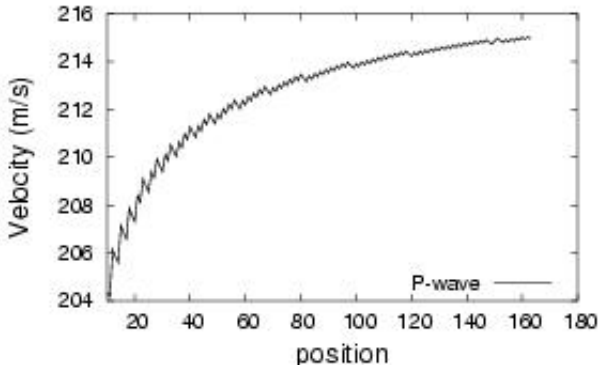
# Waves and stiffnesses



- Components of the stiffness tensor which corresponds to the direction of motion of the particles for both P- and S-wave.

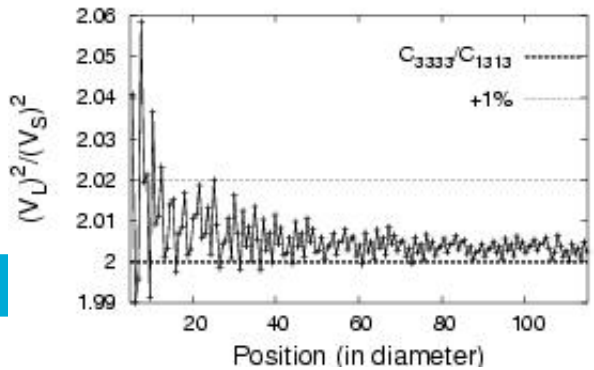
- In this case : $C_{3333}$ for the P-wave and $C_{1313}$ for the S-wave.

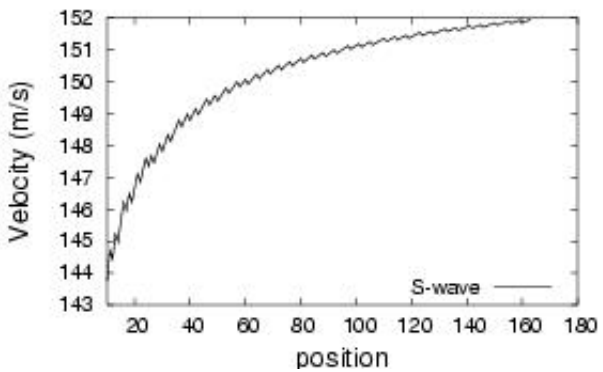
- *$V_p$ and $V_s$ denote the "Peak velocity" .*




$$\text{Theory :} \quad \rho V_p^2 \propto C_{3333} \qquad \rho V_s^2 \propto C_{1313}$$

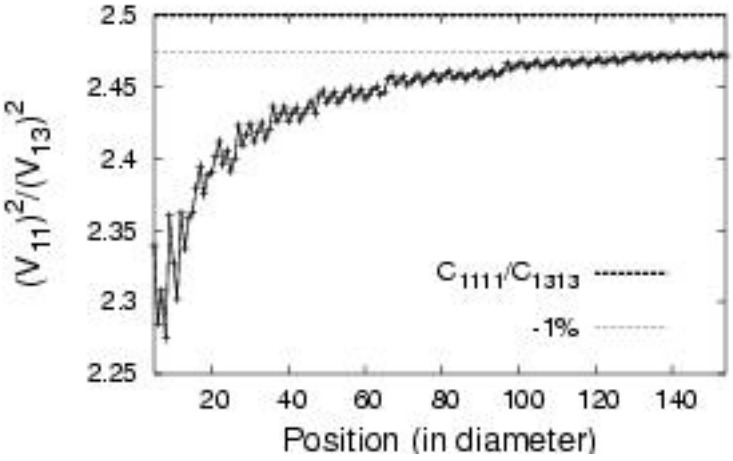
$$\frac{V_p^2}{V_s^2} \rightarrow \frac{C_{3333}}{C_{1313}}$$

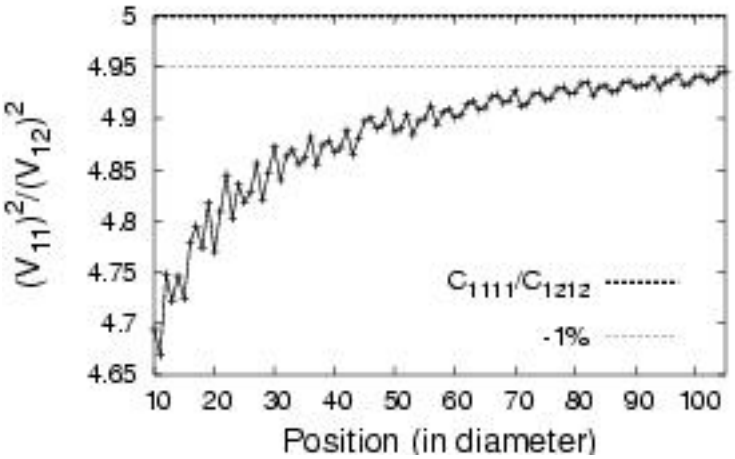


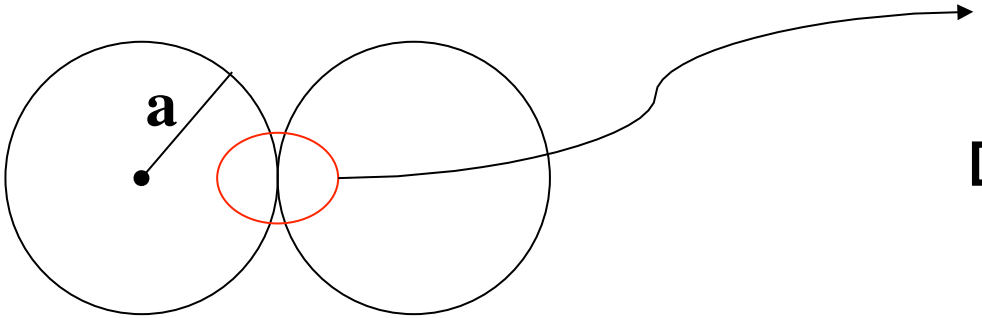
# Waves and stiffnesses

| Ratios of C entries | | Ratios of velocities | |
|---|---|---|---|
| 1111/ 1313 | 2.5 | 11/13 | 2.475 |
| 3333/ 1313 | 2 | 33/13 | 2.005 |
| 1111/ 1212 | 5 | 11/12 | 4.95 |



# Weak polydispersity



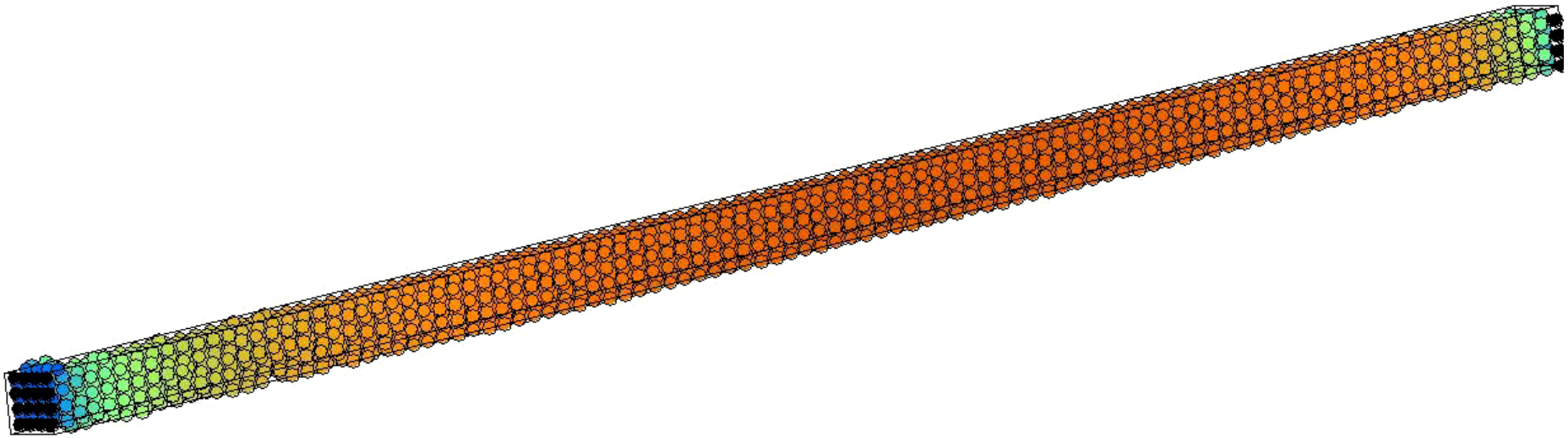

**$\delta = a/1000$**

**$Da = 0, \delta/2, \delta$ and $2\delta$**

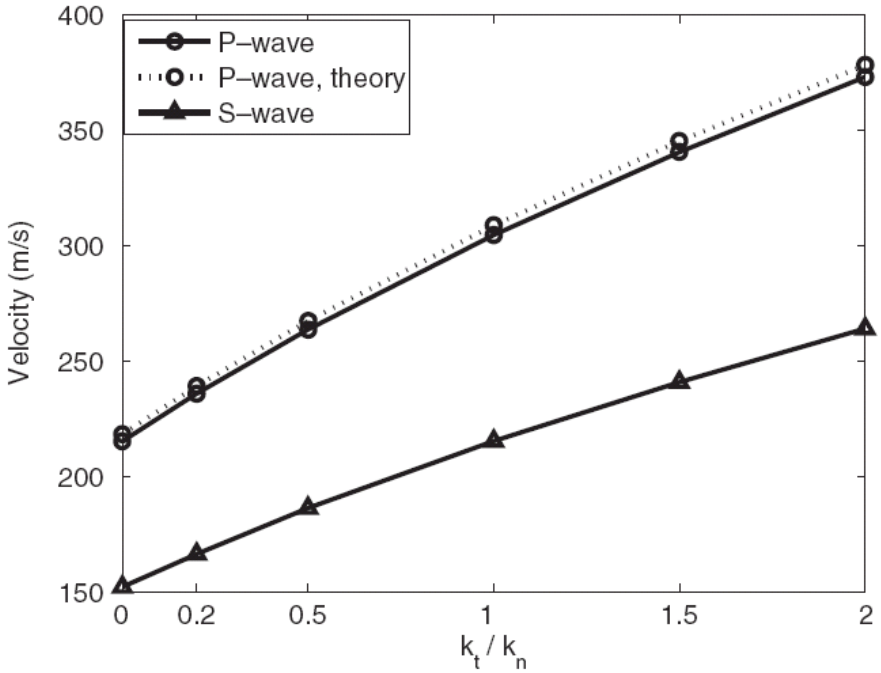
- The system is practically unchanged at the structure level

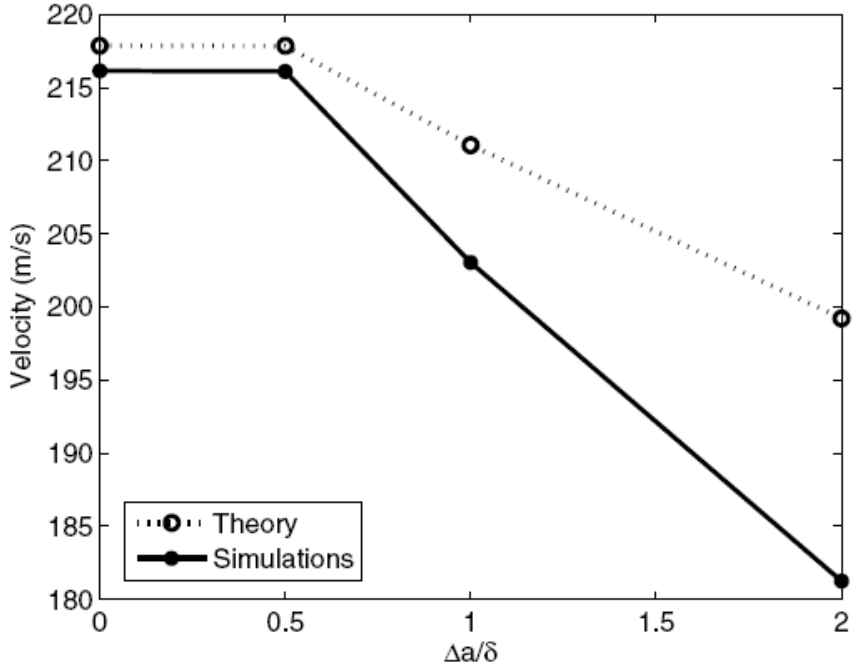
- **Wide** distribution of weak and strong contacts
**and <u>most important opening of contacts</u>**

# P-wave animation



# Velocities



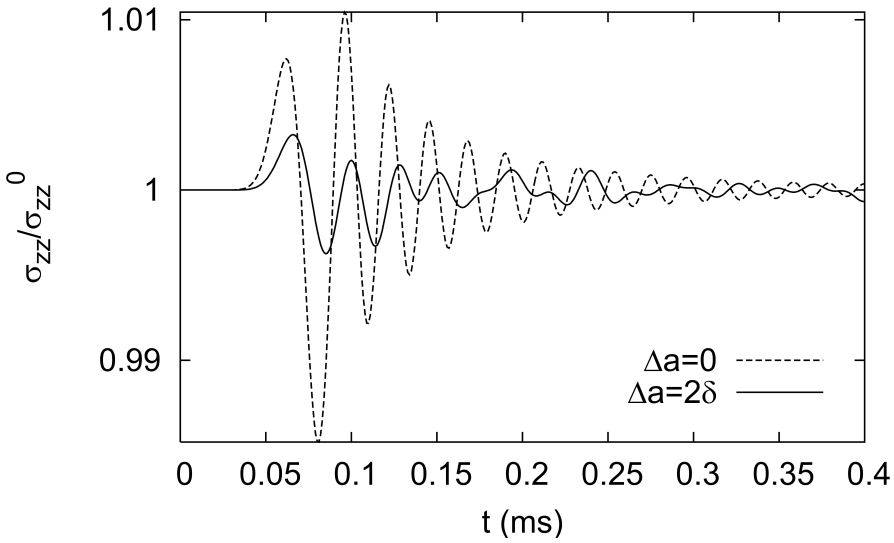




**Friction+rotation**

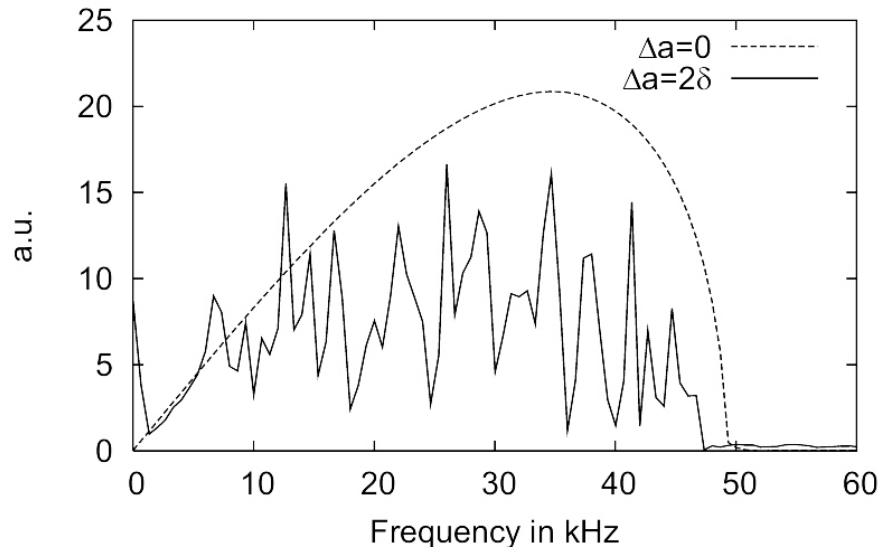
**Weak disorder**

# Signal Analysis

time-FFT

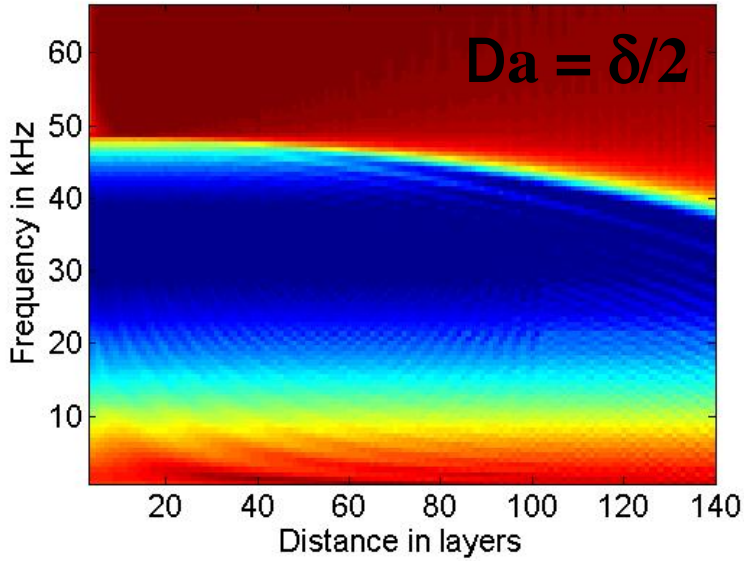


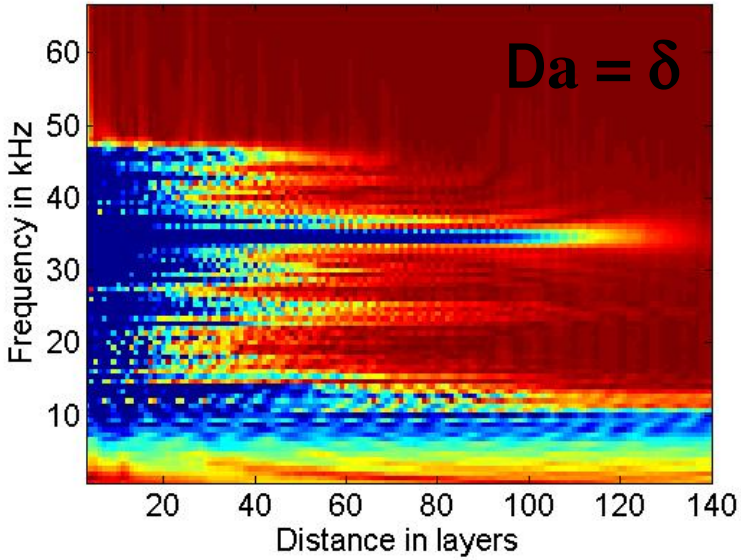

Stress-time signal

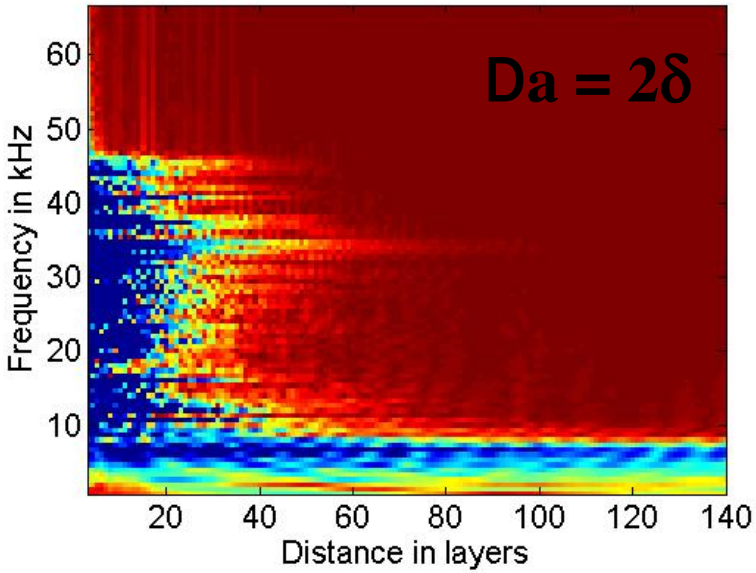



Power-spectrum

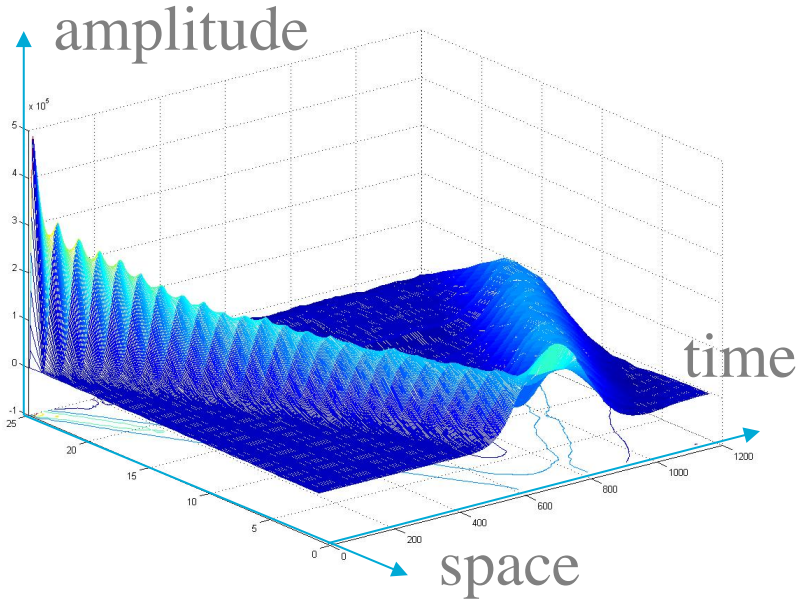
# Frequency-space Diagrams

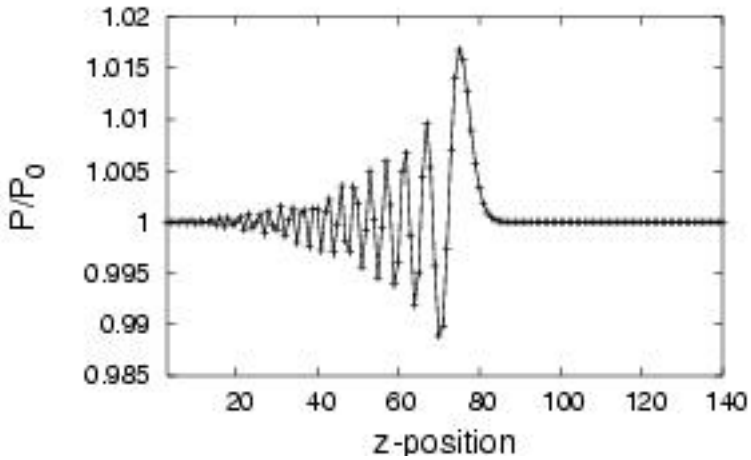


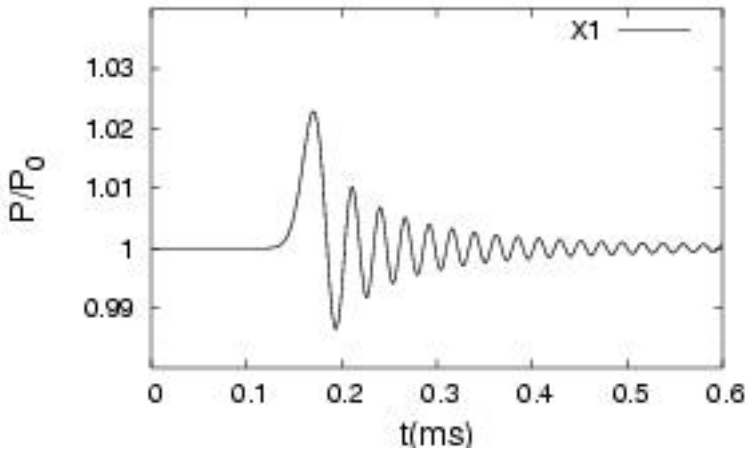



# Space-Time analysis

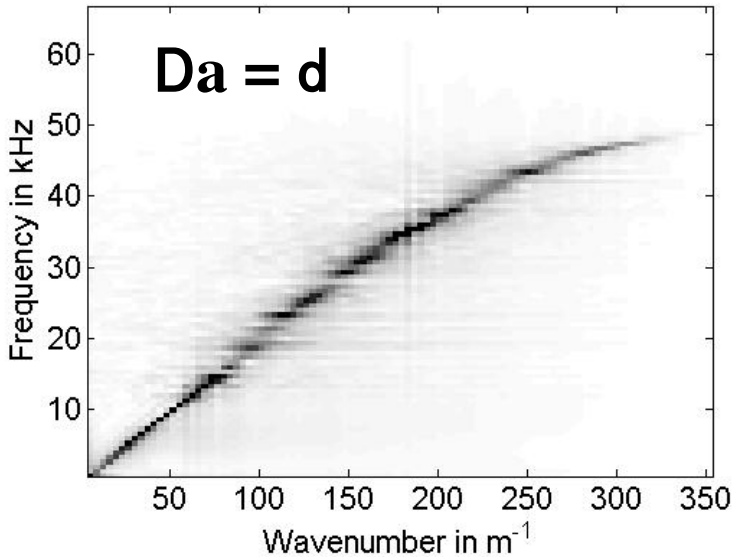


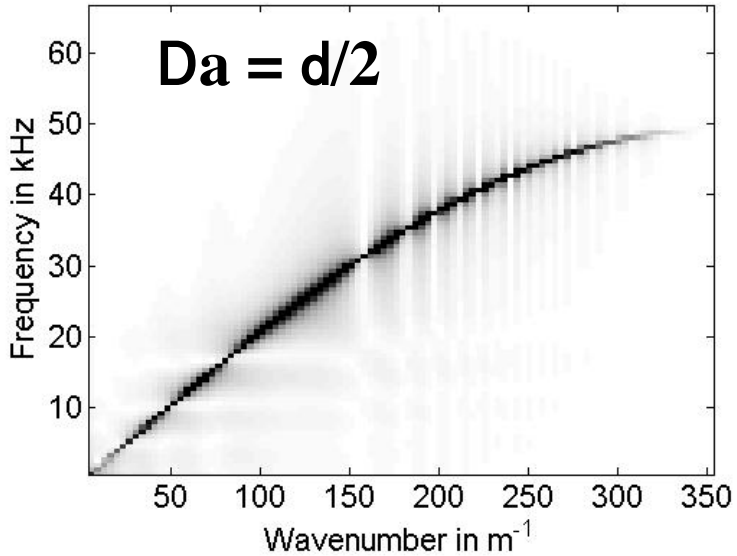



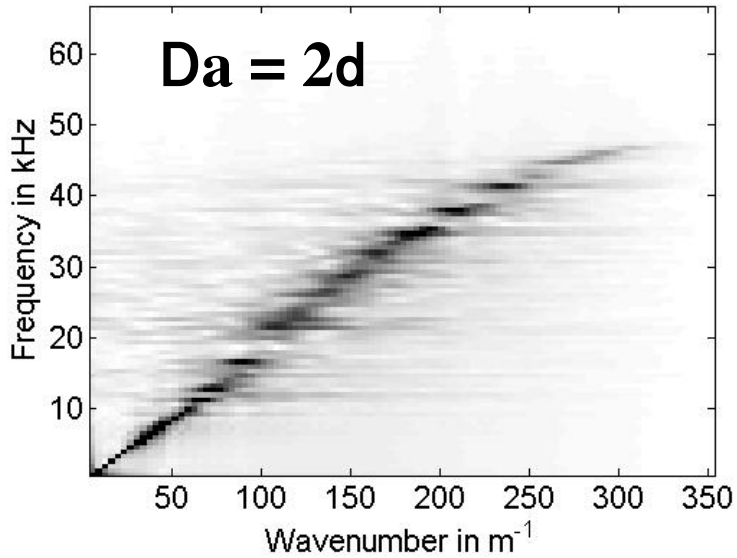



# Dispersion relations

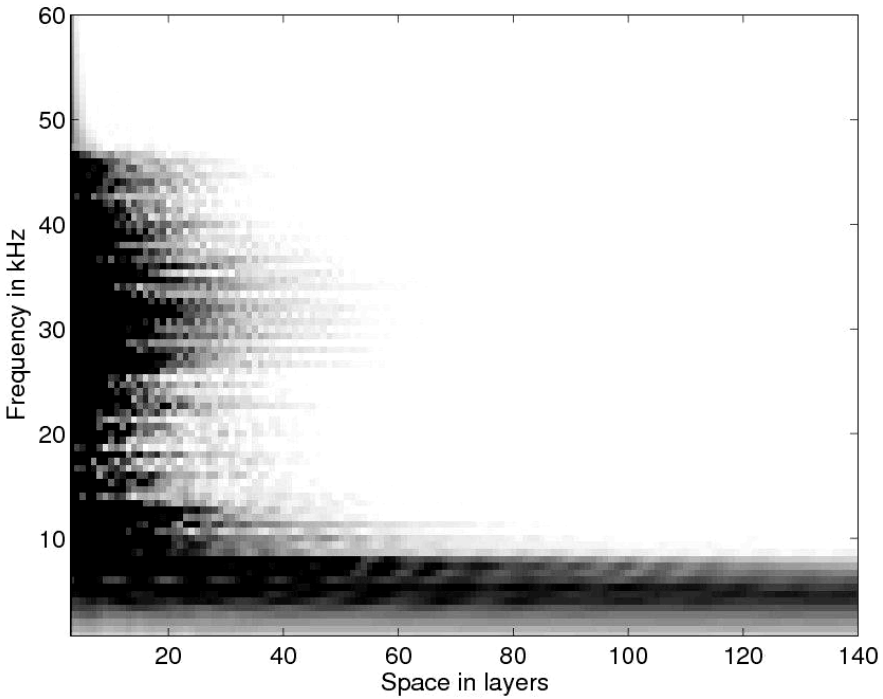
space-time-FFT

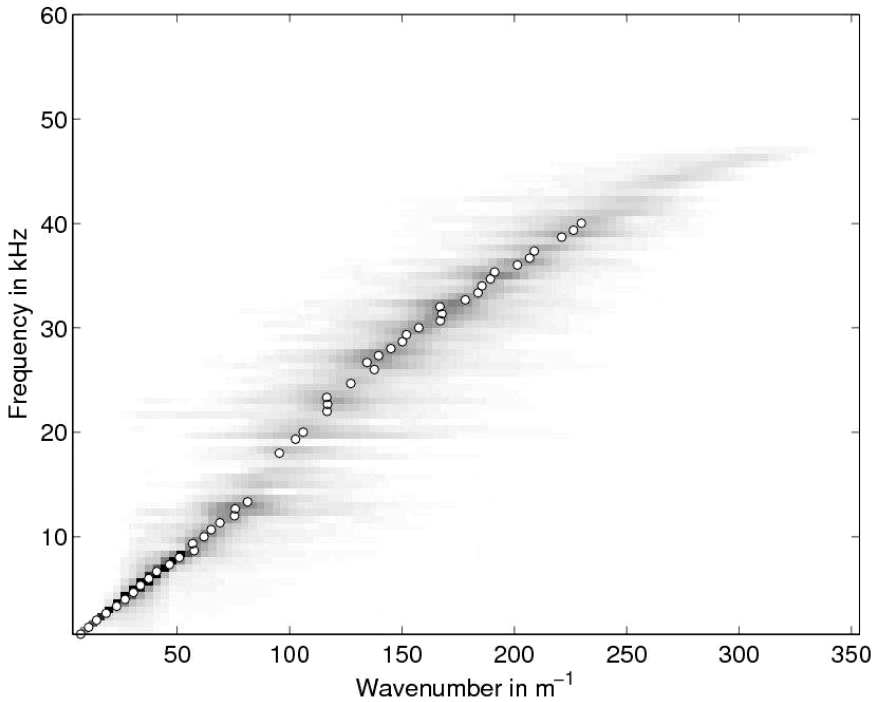


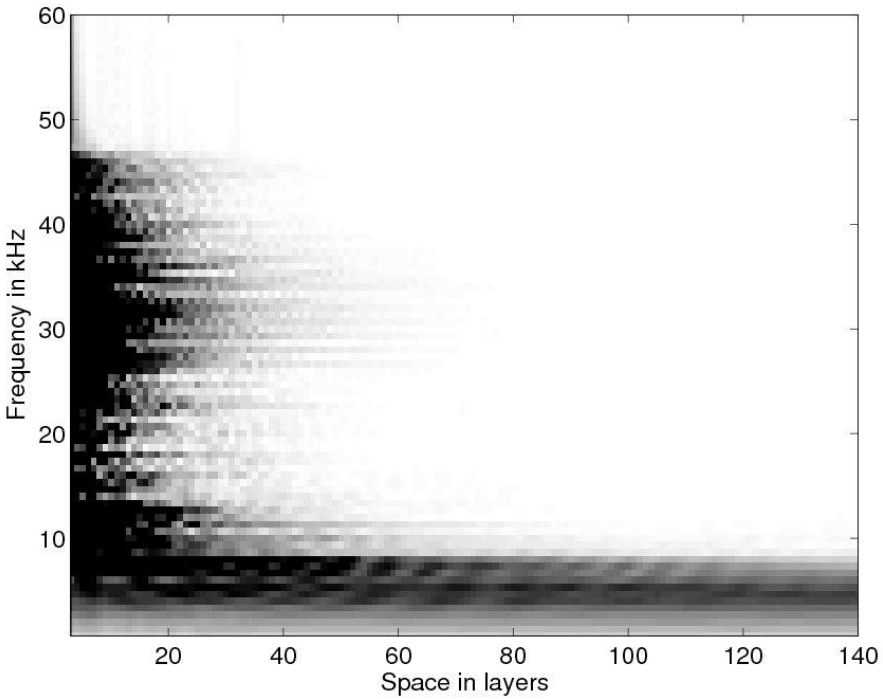



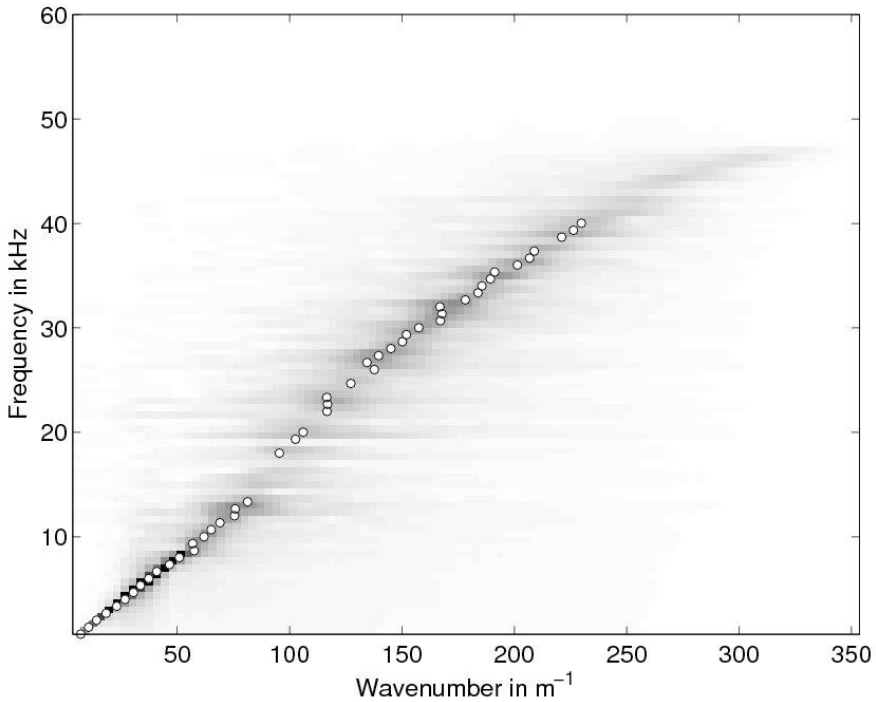
# Frequency analysis



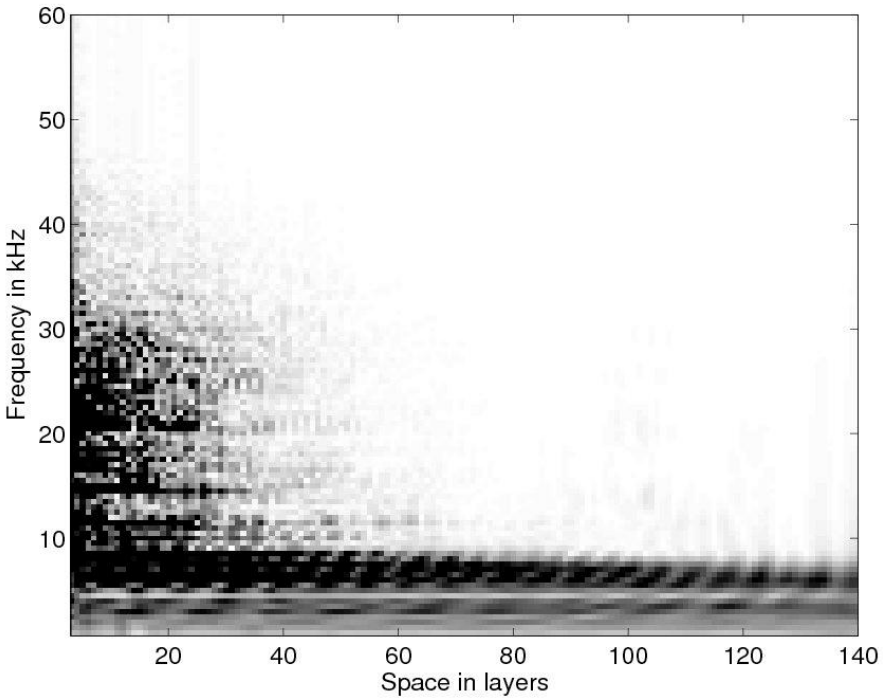



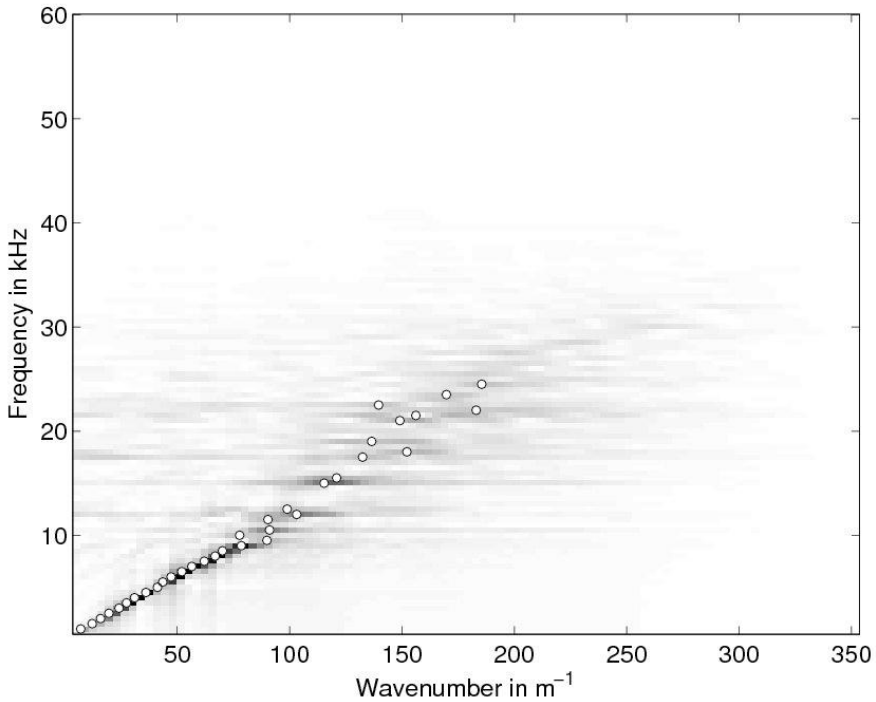
# Larger system



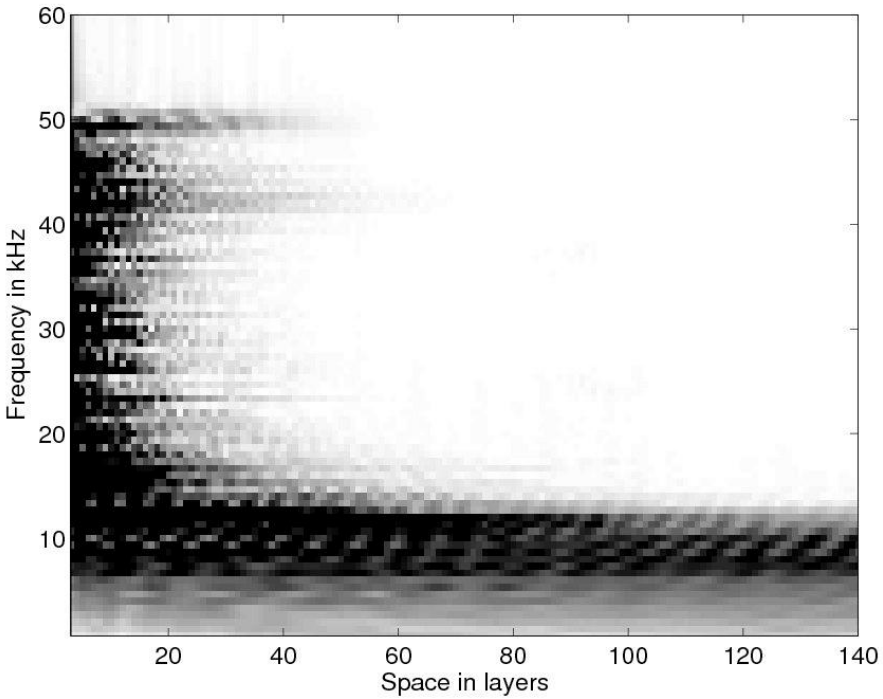



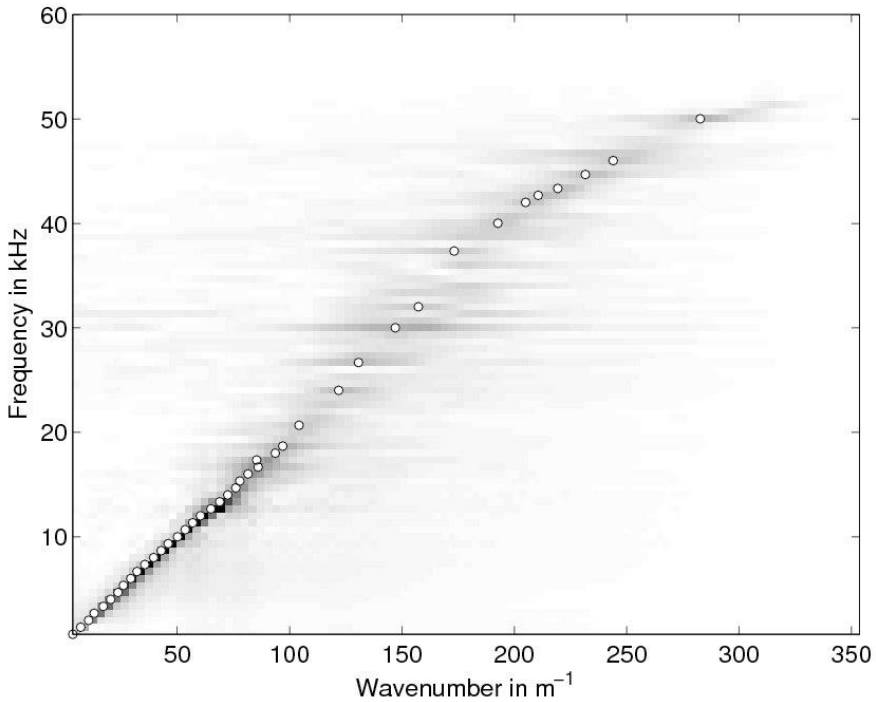
# Shear wave


Frequency in kHz
60
50
40
30
20
10
20
40
60
80
100
120
140
Space in layers

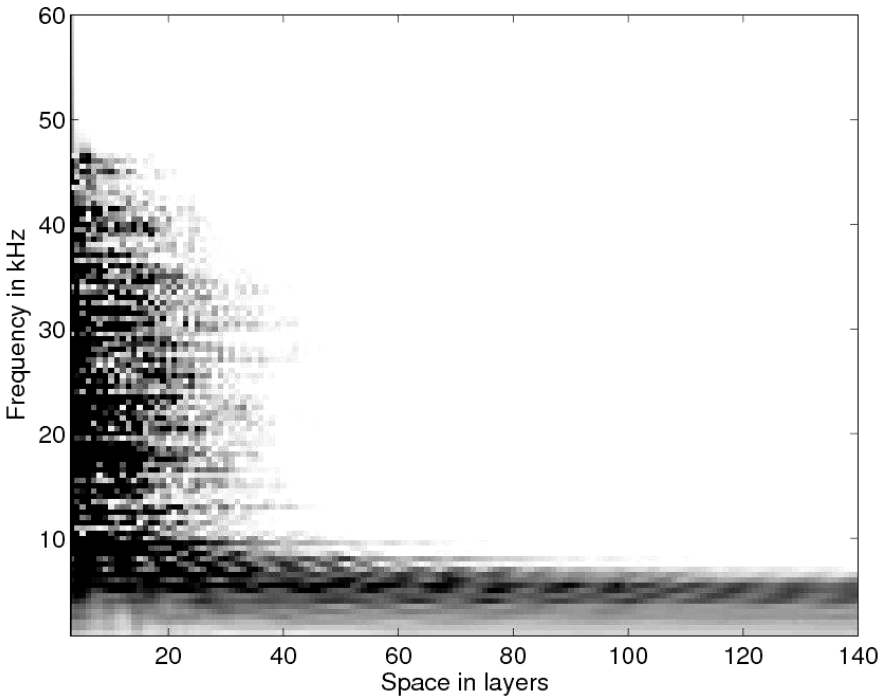

Frequency in kHz
60
50
40
30
20
10
50
100
150
200
250
300
350
Wavenumber in m$^{-1}$

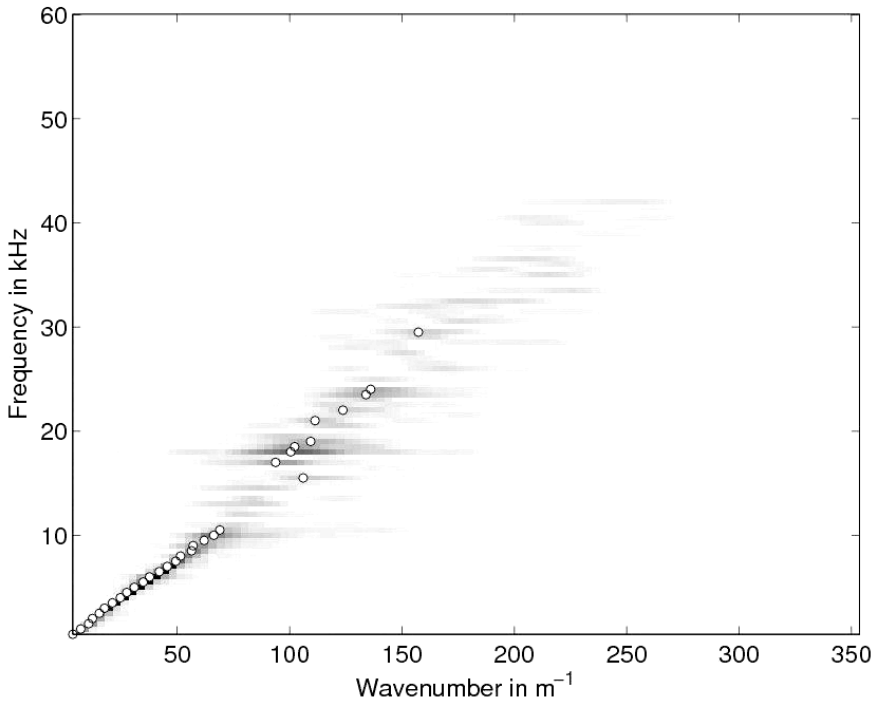
# P-wave with friction



# Hertz Contact Law



## Question

How does sound propagation depend on

- structure?

Lattice+tiny disorder => enormous effect

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How does sound propagation depend on

- structure?

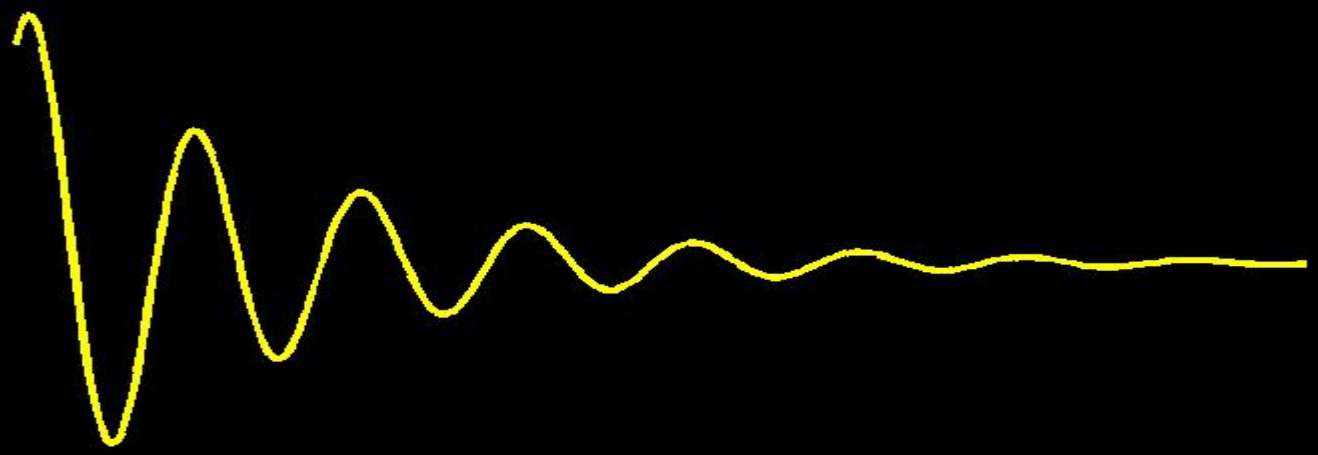
Lattice+tiny disorder => enormous effect

3D effect? or 1D?

UNIVERSITEIT TWENTE.

# Sound Propagation in Particulate Systems with Disorder and Rotations ...

Lisa de Mol,S. Emmerich, Brian Lawney, Vanessa Magnanimo, Stefan Luding
Multi-Scale Mechanics, UTwente, Netherlands



L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

# OVERVIEW

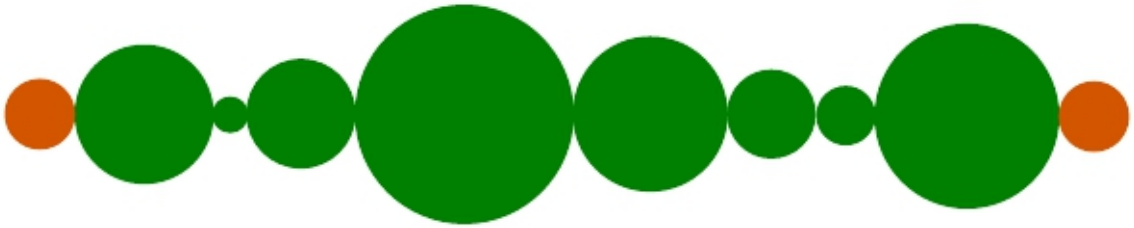
Introduction: Frequency filtering and high-f bands

**One–dimensional chains**
•Model system and equations
•Introduction of mass–disorder (most simple)
•Model for frequency (low-pass) filtering

**Two- and three-dimensional regular packings with rotations**
•Dispersion relation
- Smooth (frictionless)
- With friction/rotations

# ONE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

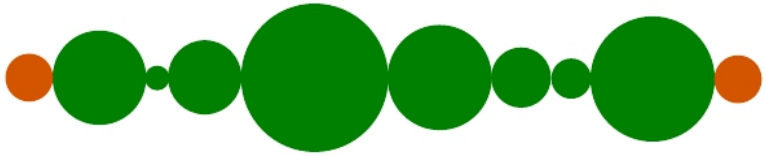


- Convenient model
  - Analytically accessible
  - Isolation of mass–disorder
- Significant attention in literature
  - Nonlinear oscillators
  - Soliton–like waves

# ONE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- Force Displacement Model



$$\tilde{F}_{(i,j)} = \tilde{\kappa}_{(i,j)} \tilde{\delta}^{1+\beta}$$

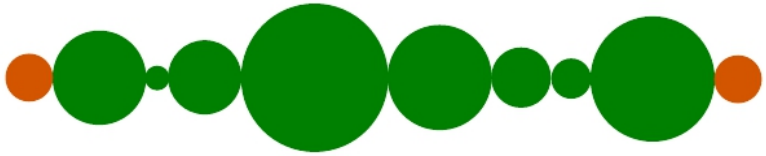
$$\tilde{m}^{(i)} \frac{d^2 \tilde{x}^{(i)}}{d\tilde{t}^2} = \tilde{F}_{(i,i-1)} + \tilde{F}_{(i,i+1)}$$

- Scaling
  - Mass $\tilde{m}_o$
  - Length $\tilde{\ell}$
  - Time $\tilde{t}_c = \frac{1}{\tilde{\ell}^{\beta/2}} \sqrt{\frac{\tilde{m}_o}{\tilde{\kappa}_o}}$

# ONE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- General equation of motion



$$b^{(i)}\frac{d^2u^{(i)}}{d\tau^2} = \kappa_{(i-1,i)}\left[\Delta_{(i-1,i)} - u^{(i)} + u^{(i-1)}\right]^{1+\beta}$$
$$-\kappa_{(i+1,i)}\left[\Delta_{(i+1,i)} + u^{(i)} - u^{(i+1)}\right]^{1+\beta}$$

$$b \equiv \tilde{m}^{(i)}/\tilde{m}_o$$
$$\tau \equiv \tilde{t}/\tilde{t}_c$$
$$\kappa_{(i,j)} \equiv \tilde{\kappa}_{(i,j)}/\tilde{\kappa}_o$$

# ONE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- General (nonlinear) equation of motion:

$$b^{(i)}\frac{d^2u^{(i)}}{d\tau^2} = \kappa_{(i-1,i)}\left[\Delta_{(i-1,i)} - u^{(i)} + u^{(i-1)}\right]^{1+\beta} - \kappa_{(i+1,i)}\left[\Delta_{(i+1,i)} + u^{(i)} - u^{(i+1)}\right]^{1+\beta}$$

- Linearized model:

$$\mathbf{M}\frac{\mathrm{d}^2\mathbf{u}}{\mathrm{d}\tau^2} = \mathbf{K}\mathbf{u}$$

# ONE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- General (nonlinear) equation of motion:

$$b^{(i)}\frac{d^2u^{(i)}}{d\tau^2} = \kappa_{(i-1,i)}\left[\Delta_{(i-1,i)} - u^{(i)} + u^{(i-1)}\right]^{1+\beta} - \kappa_{(i+1,i)}\left[\Delta_{(i+1,i)} + u^{(i)} - u^{(i+1)}\right]^{1+\beta}$$

- Linearized model:

$$\mathbf{M}\frac{\mathrm{d}^2\mathbf{u}}{\mathrm{d}\tau^2} = \mathbf{K}\mathbf{u}$$

- Analytical Solution: Sine-agitation

$$u^{(p)}(\tau) = \sum_{j=1}^{N}\frac{S_{pj}S_{1j}}{\left(\omega_j^2 - \omega_o^2\right)}\left(\sin\omega_o\tau - \frac{\omega_o}{\omega_j}\sin\omega_j\tau\right)$$

# ONE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- Harmonically driven:


- Mass–disorder: Normal (Gaussian) distribution
  - Mean mass → b = 1
  - Standard deviation → σ = ξ
- Pre–stress → equilibrium overlap → NOT sonic vacuum

# DISORDERED CHAINS AS A FREQUENCY FILTER?

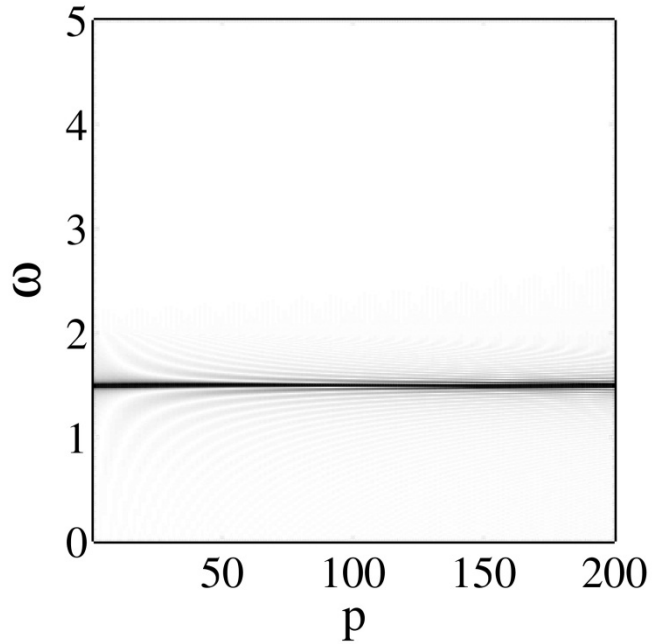
L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

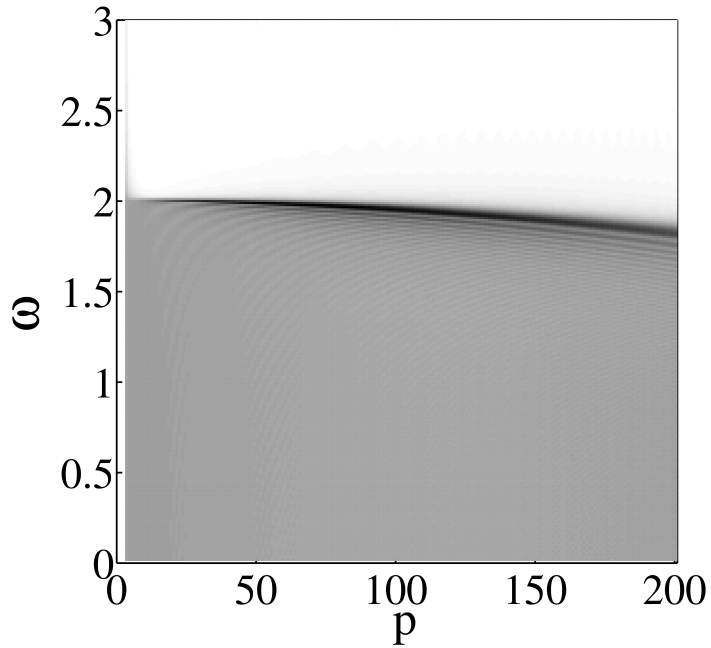

How do signals propagate in such systems?

- Disorder $\xi$
- Input frequency $\omega_0$
- Mass distribution
- Contact order/disorder
- Linear vs. nonlinear

# BASE CASE – PERFECT CHAIN

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

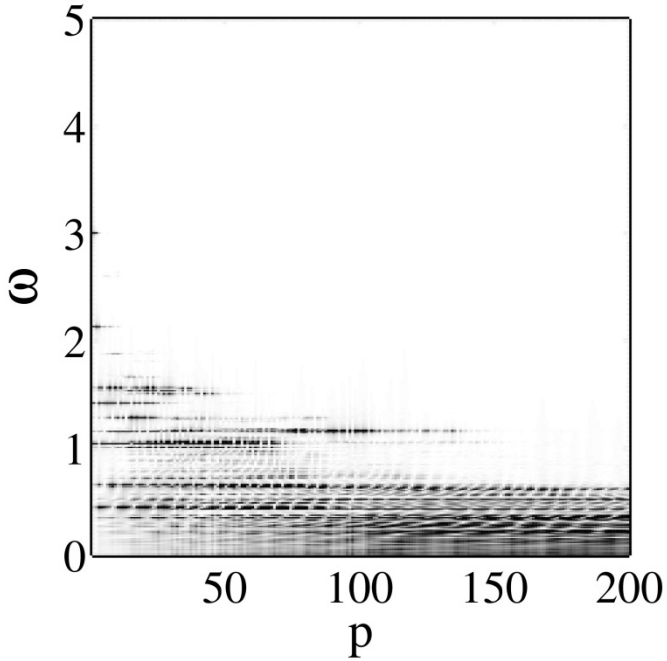


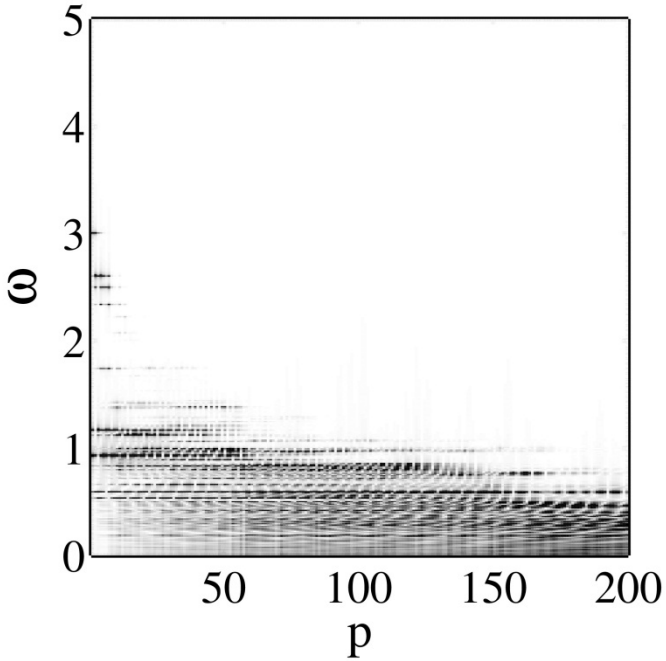




- Linear
- Uniform stiffness κ(i,j) = kn
- ξ = 0.0

# DISORDERED CHAINS AS A FREQUENCY FILTER?

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding



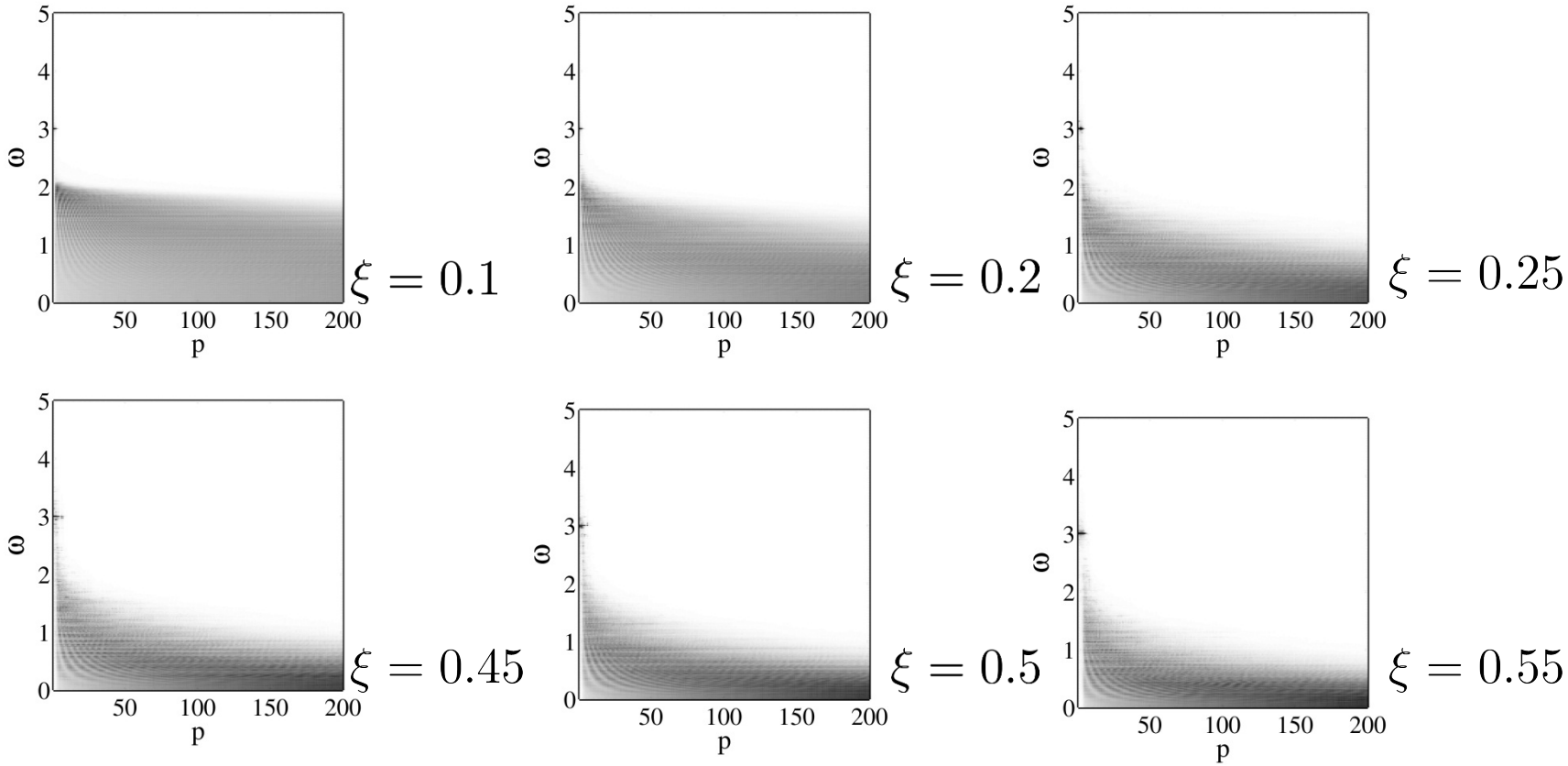




- Linear
- Uniform stiffness $\kappa(i,j) = k_n$
- $\omega_o = 3.0$
- $\xi = 0.5$

# DISORDERED CHAINS AS A FREQUENCY FILTER?

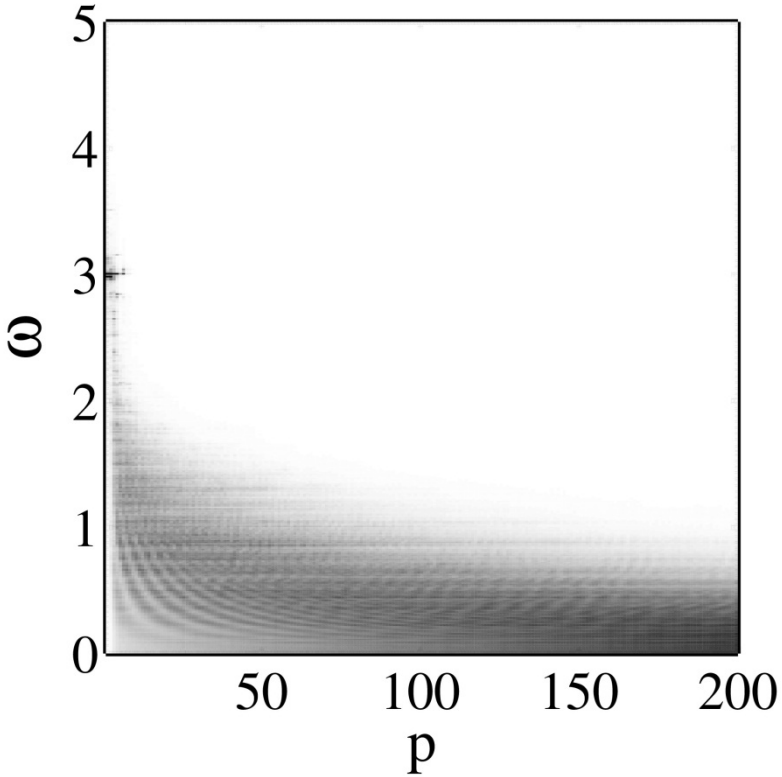
L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- Consider an ensemble of chains, fix $\omega_o$ = 3.0, and vary ξ:



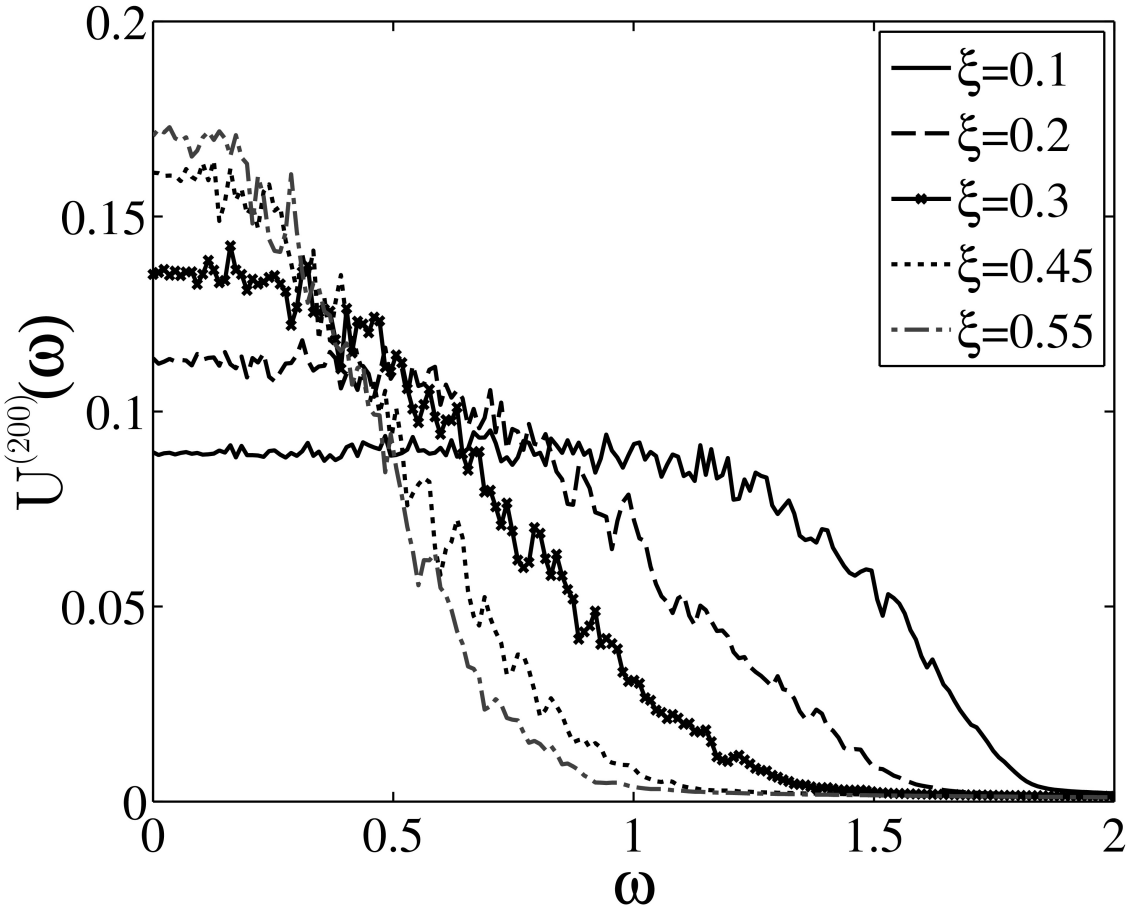
# DISORDERED CHAINS AS A FREQUENCY FILTER?

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding



# DISORDERED CHAINS AS A FREQUENCY FILTER?

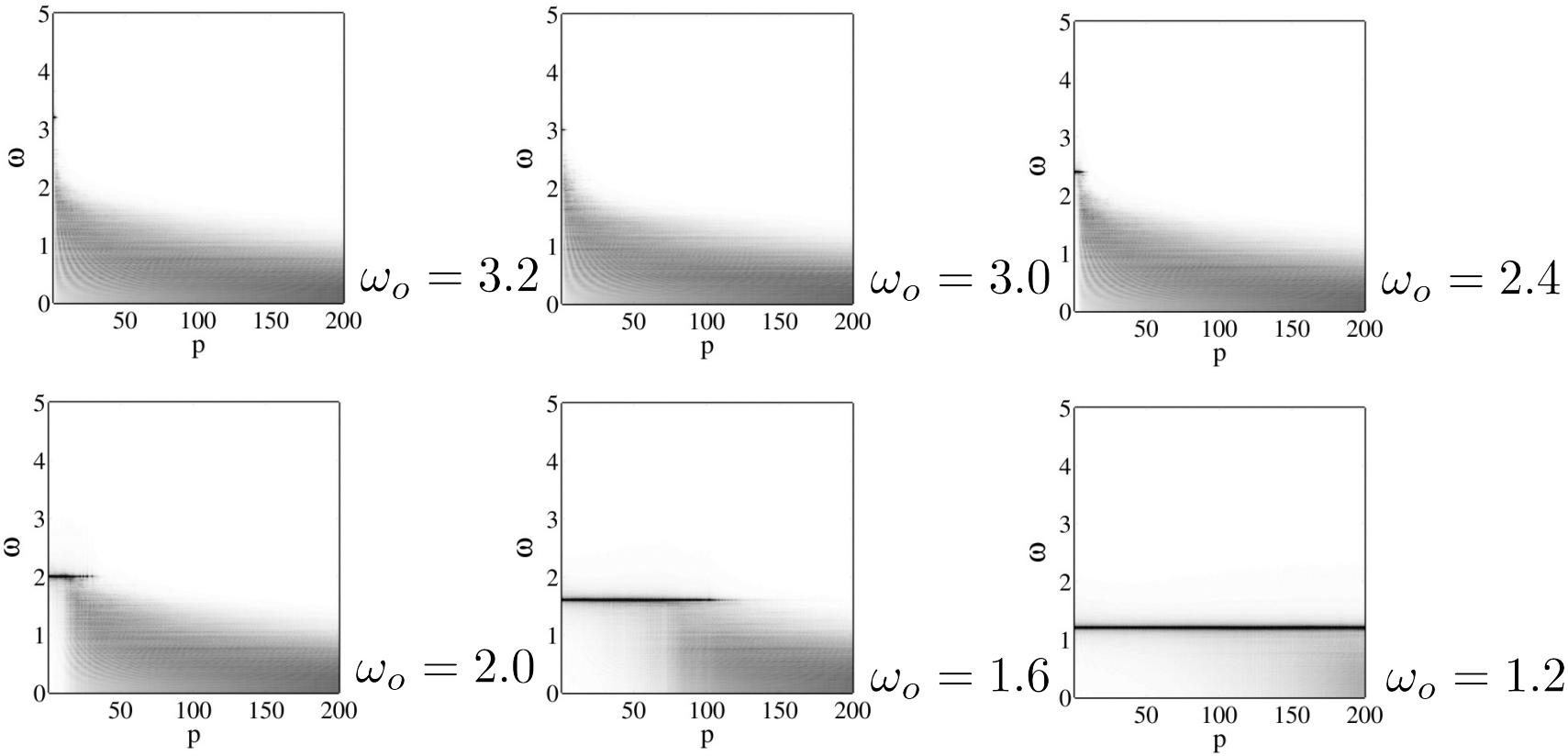
L.de Mol, B.Lawney,
V. Magnanimo, S. Luding



# DISORDERED CHAINS AS A FREQUENCY FILTER?

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- Fix ξ, vary $\omega_o$:



L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

# CONCLUSIONS 1

- ↑ disorder ξ, ↓ transmission bandwidth.
  - Threshold value of ξ
- Lower input $\omega_o$ , improved transmission
  - Low frequencies less sensitive to mass arrangements
- Mass–distribution: only moments matter …

**QUALITATIVELY THE SAME AS IN 3D**
**=> PROCEED ANALYTICAL 1D**

# ONE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- Linearized model:

$$\mathbf{M}\frac{\mathrm{d}^2\mathbf{u}}{\mathrm{d}\tau^2} = \mathbf{K}\mathbf{u}$$

$$u^{(p)}(\tau) = \sum_{j=1}^{N} \frac{S_{pj}S_{1j}}{\left(\omega_j^2 - \omega_o^2\right)} \left(\sin\omega_o\tau - \frac{\omega_o}{\omega_j}\sin\omega_j\tau\right)$$

# ONE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

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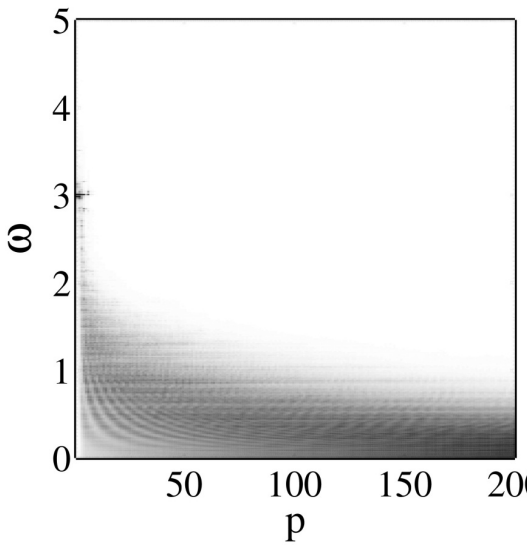
- Evolution (Master) Equation in k-space or w-space

$$\frac{d}{dt}\Psi = \mathbf{Q}\Psi \qquad \frac{d}{dx}\Psi = \mathbf{Q}\Psi$$



# ONE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- Linearized model: $$\mathbf{M}\frac{\mathrm{d}^2\mathbf{u}}{\mathrm{d}\tau^2} = \mathbf{K}\mathbf{u}$$
- Evolution (Master) Equations in f-space or k-space

$$\frac{d}{dt}\Psi = \mathbf{Q}\Psi$$

$$\frac{\mathrm{d}}{\mathrm{d}t} q^{\downarrow}(f_i) = -b_i\, q(f_i) + \sum_{1\le j<N-i} b^i_{i+j}\, q(f_{i+j})$$

$$b_i = \sum_{j\ge 1} b^i_{i+j}$$

# ONE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- Linearized model: $$\mathbf{M}\frac{\mathrm{d}^2\mathbf{u}}{\mathrm{d}\tau^2} = \mathbf{K}\mathbf{u}$$
- Evolution (Master) Equations in f-space or k-space

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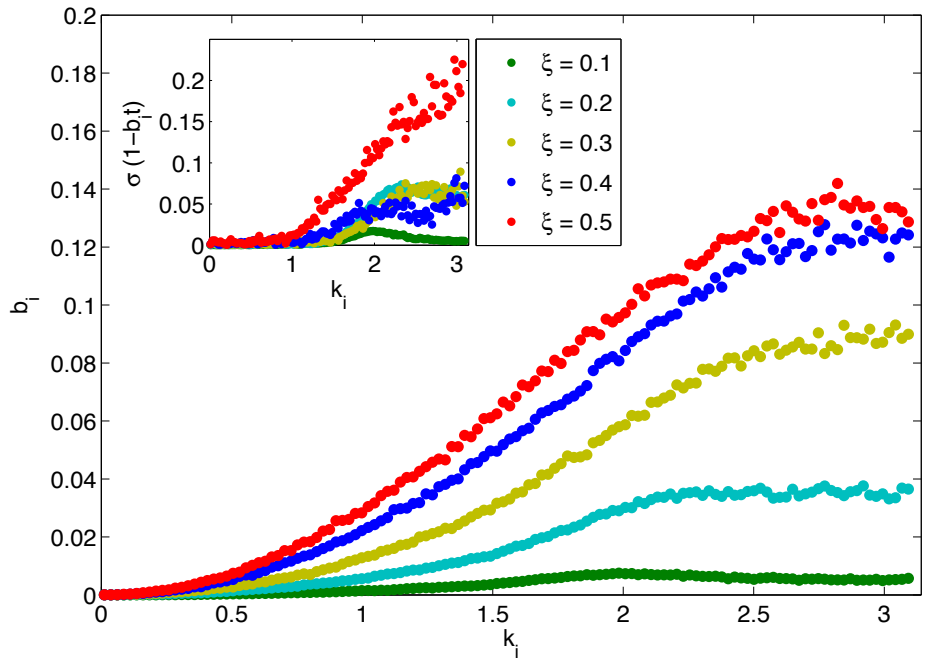
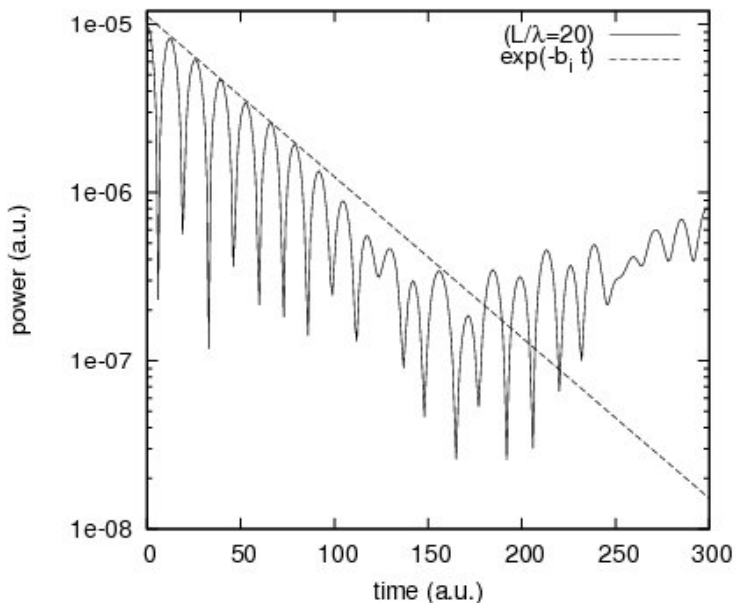


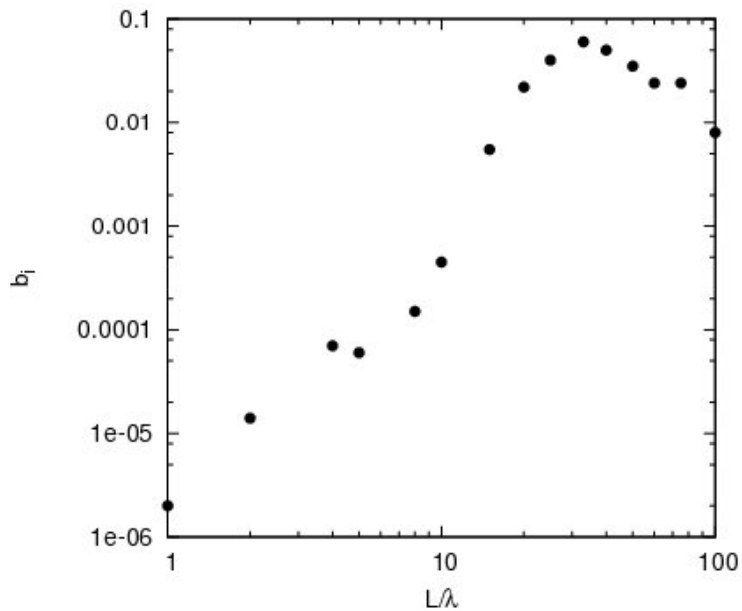
# THREE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- Linearized model: $$\mathbf{M}\frac{\mathrm{d}^2\mathbf{u}}{\mathrm{d}\tau^2} = \mathbf{K}\mathbf{u}$$
- Evolution (Master) Equations in f-space or k-space

$$\frac{\mathrm{d}}{\mathrm{d}t} q^{\downarrow}(f_i) = -b_i\, q(f_i) + \sum_{1\le j<N-i} b^i_{i+j}\, q(f_{i+j}) \qquad b_i = \sum_{j\ge 1} b^i_{i+j}$$

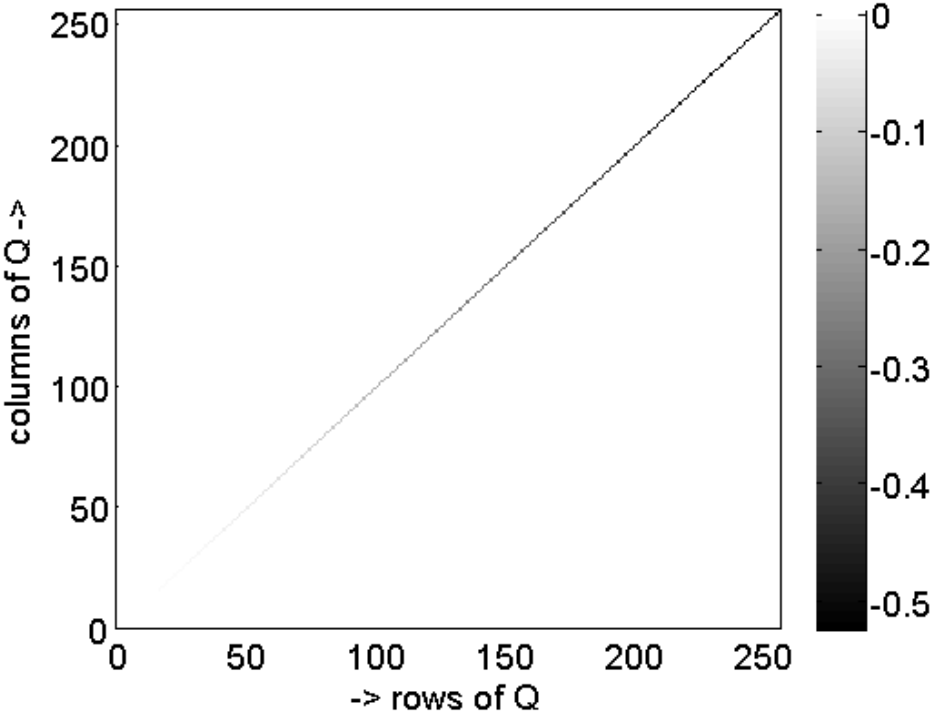


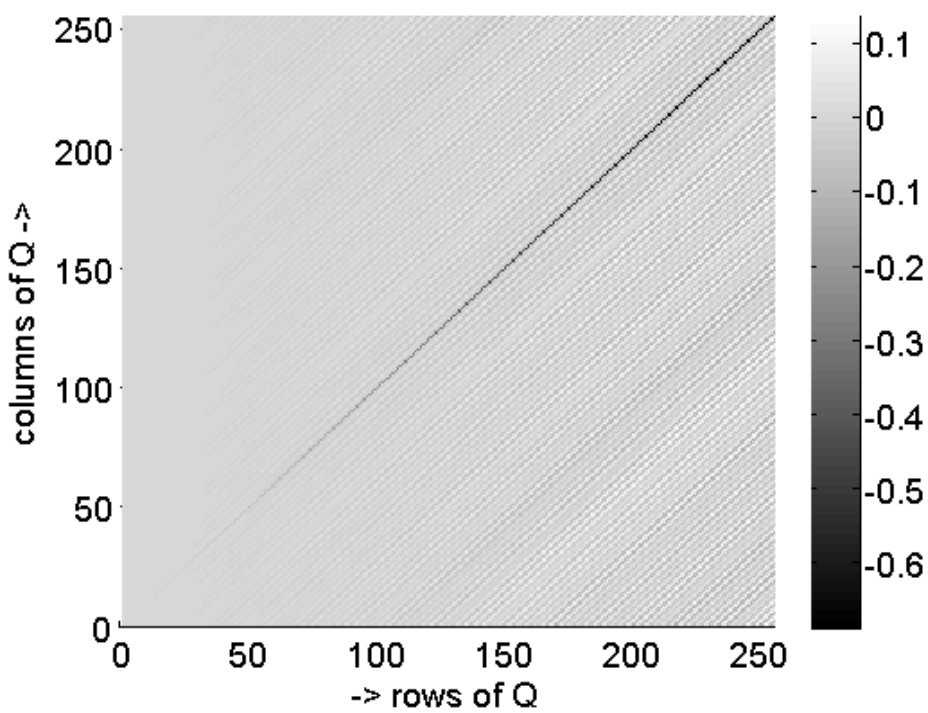
# ONE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- Linearized model: $$\mathbf{M}\frac{\mathrm{d}^2\mathbf{u}}{\mathrm{d}\tau^2} = \mathbf{K}\mathbf{u}$$
- Evolution (Master) Equations in f-space or k-space $$\frac{d}{dt}\Psi = \mathbf{Q}\Psi$$




(a) $\xi = 0.1$




(b) $\xi = 0.4$

# ONE DIMENSIONAL CHAINS

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- Linearized model: $$\mathbf{M}\frac{\mathrm{d}^2\mathbf{u}}{\mathrm{d}\tau^2} = \mathbf{K}\mathbf{u}$$
- Evolution (Master) Equations in f-space or k-space

$$\frac{d}{dt}\Psi = \mathbf{Q}\Psi$$

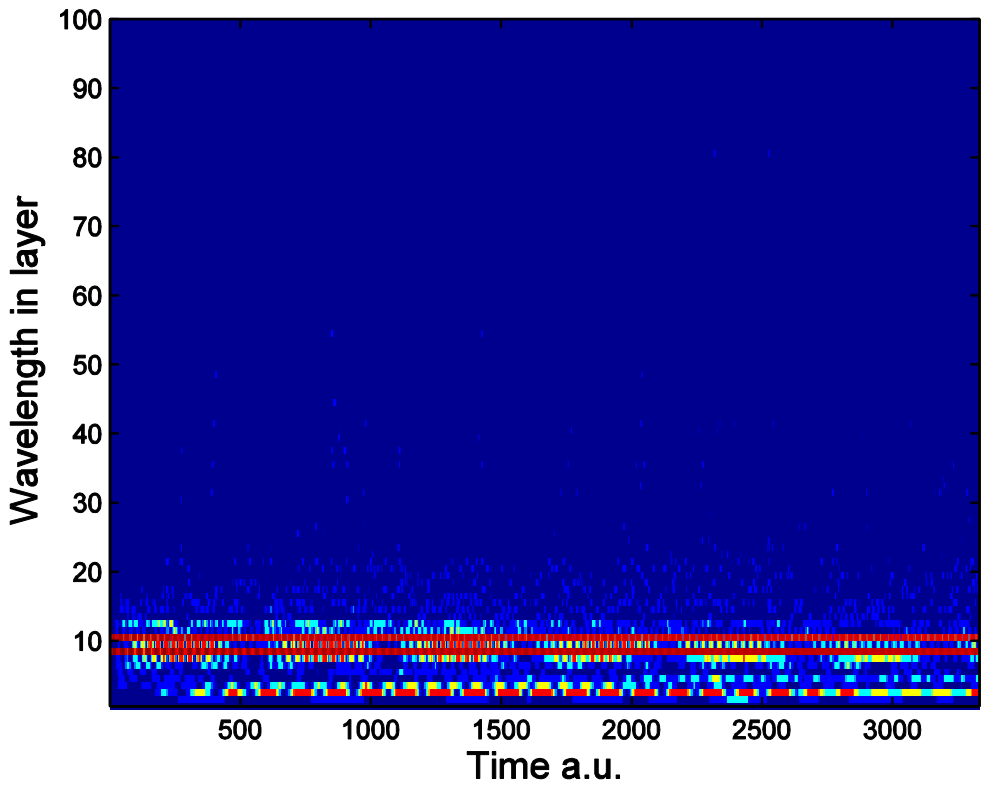
$$\frac{\mathrm{d}}{\mathrm{d}t}q^{\downarrow}(f_i) = -b_i\, q(f_i) + \sum_{1\le j<N-i} b^i_{i+j}\, q(f_{i+j})$$

$$b_i = \sum_{j\ge 1} b^i_{i+j}$$

- Nonlinear …

$$\frac{\mathrm{d}}{\mathrm{d}t}q^{\uparrow}(f_i) = -\sum_j a_{i,j}\, q(f_i)q(f_j) + \sum_{j<i} a_{i-j,j}\, q(f_{i-j})q(f_j)$$

# Self-demodulation ? higher f/k-generation



# OVERVIEW

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Introduction: Frequency filtering and high-f bands

**One–dimensional chains**
•Model system and equations
•Introduction of mass–disorder (most simple)
•Model for frequency (low-pass) filtering

**Two- and three-dimensional regular packings with rotations**
•Dispersion relation
- Smooth (frictionless)
- With friction/rotations

**Experiments (in water) ...**

# HOMOGENEOUS & STRUCTURED SAMPLES
## MSc thesis S. Emmerich

- Solid sample and porous sample (PCL) with $\Phi = 0.7$
- Discrete particle radii of 0.01 mm
- Stratified model used to build the porous sample
- Wave velocity needs to be calculated:

$$c_p = \sqrt{\left(\frac{E(1-\rho)}{\rho(1+\nu)(1-2\nu)}\right)}$$

$$\rho = 1145\ \mathrm{kg/m^3}$$
$$E = 204\ \mathrm{MPa}$$
$$\nu = 0.47$$
$$c_p = 1034.7\ \mathrm{m/s}$$

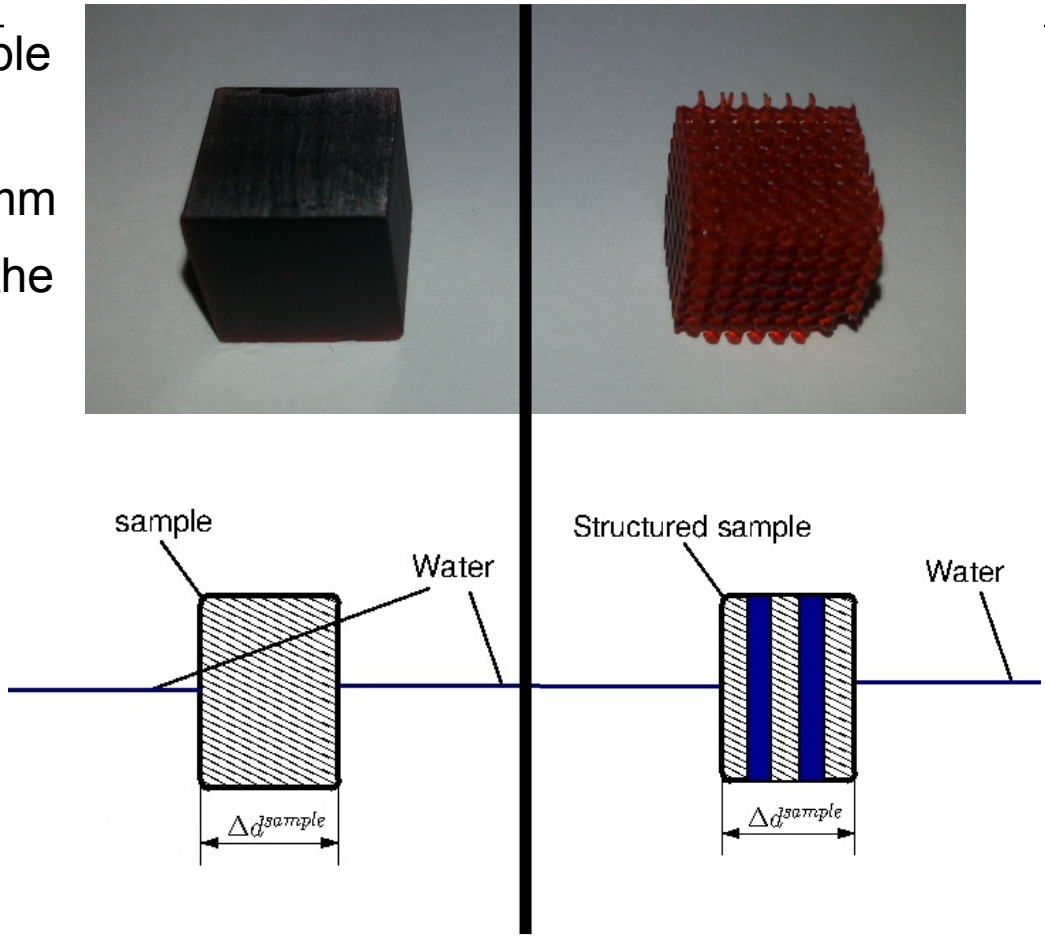
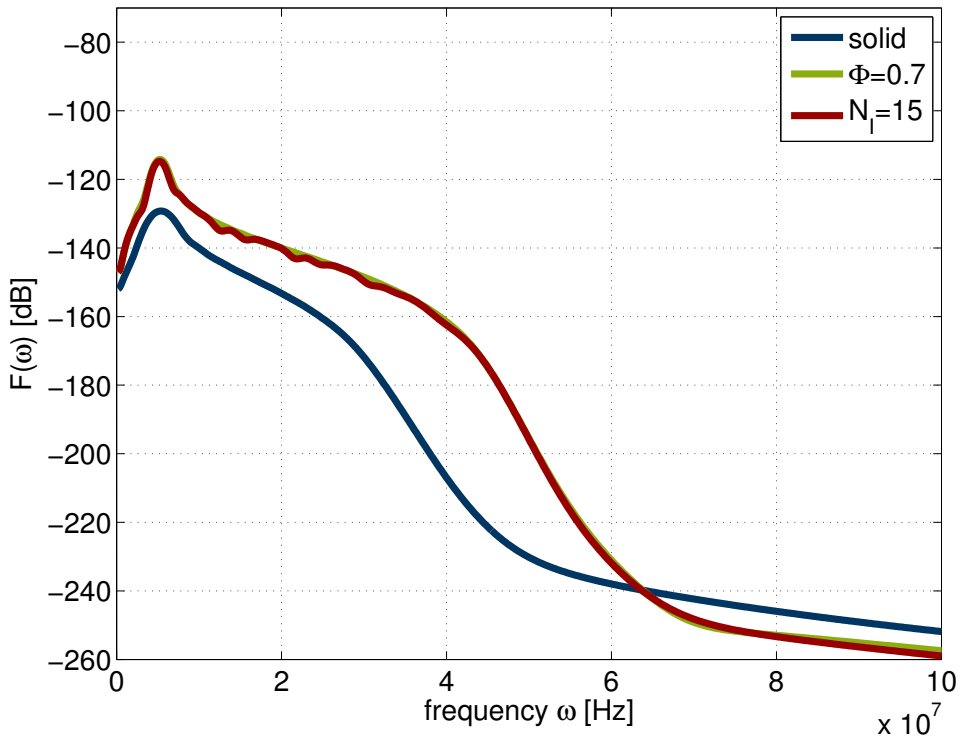
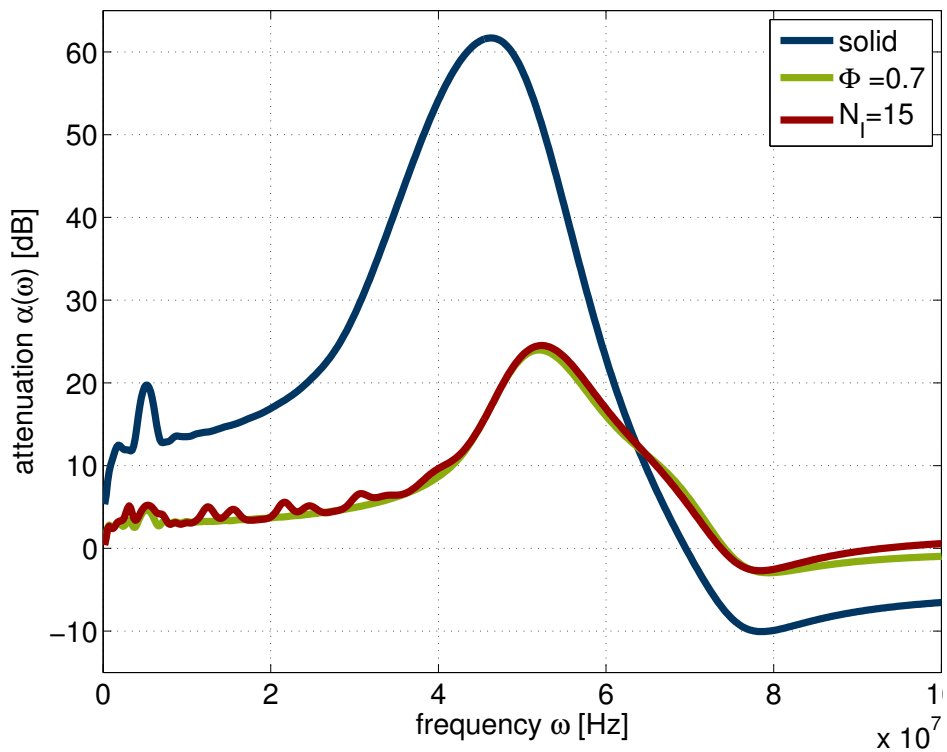


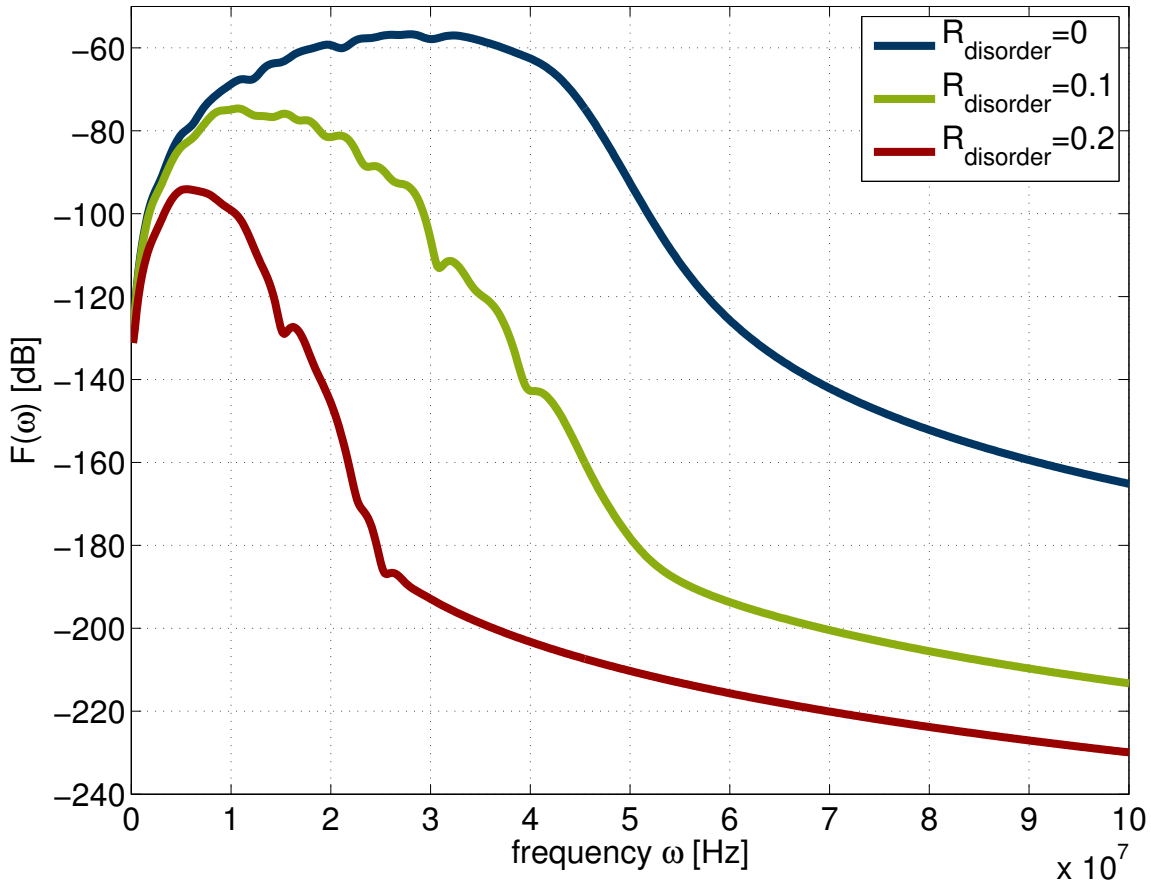

**Fig. 13: Modeling differences between solid and porous samples**

# DISORDERED SAMPLES
# MSc thesis S. Emmerich



# DISORDERED SAMPLES
# MSc thesis S. Emmerich



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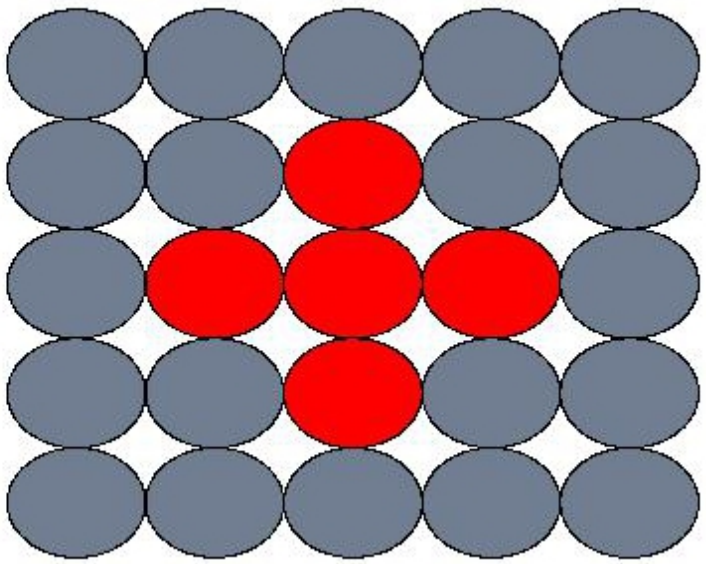
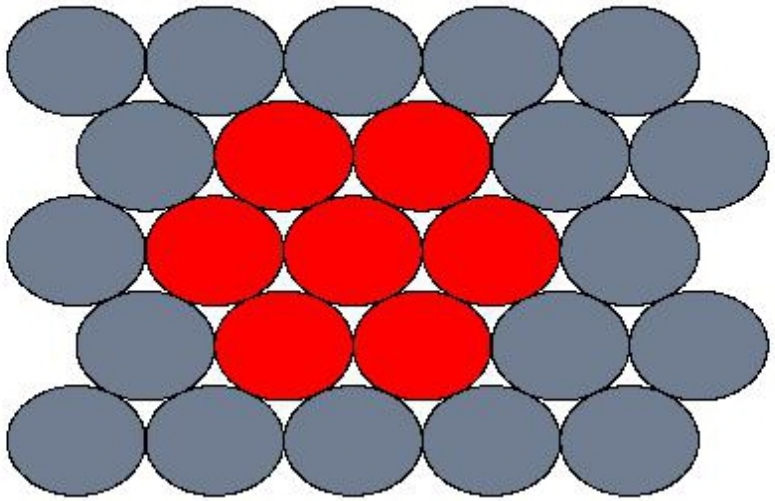
# REGULAR SYSTEMS

- Cubic lattice



- Hexagonal lattice



# MD APPROACH

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V. Magnanimo, S. Luding

- Equations of motion

$$\mathbf{M}^p \cdot \ddot{\mathbf{U}}^p = \sum_q \mathbf{F}^{pq}$$

$$\mathbf{M}^p \cdot \ddot{\mathbf{U}}^p = \sum_c \mathbf{S}^c \Delta^c$$

where

- $U^p$ is the **generalized** coordinate vector (including rotations)
- $\mathbf{S}^c$ is the contact stiffness matrix
- $\Delta^c$ contact overlap (linear spring model)

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# EIGENVALUE PROBLEM

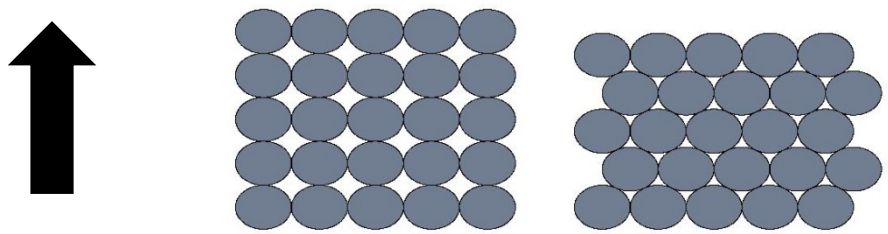
- Harmonic wave solution

$$\mathbf{U} = \hat{\mathbf{U}} e^{i(\omega t - \mathbf{k}\mathbf{x})}$$

- Inserted into the equation of motion

$$\left(\bar{\mathbf{K}} - \omega^2 \mathbf{M}\right) \hat{\mathbf{U}} = 0$$

- Solve Eigenvalue problem for a wave excitation in vertical direction



# DISPERSION

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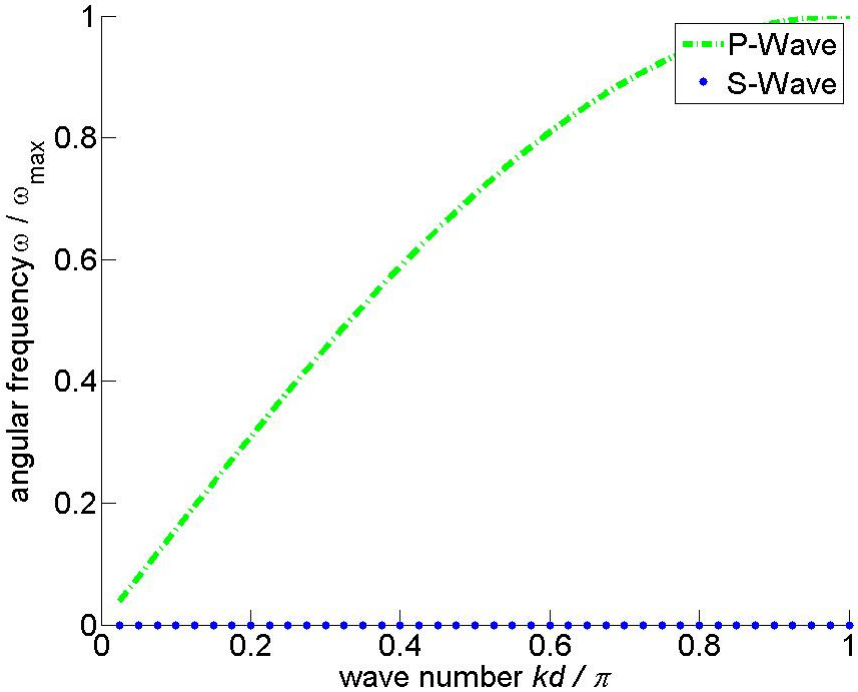
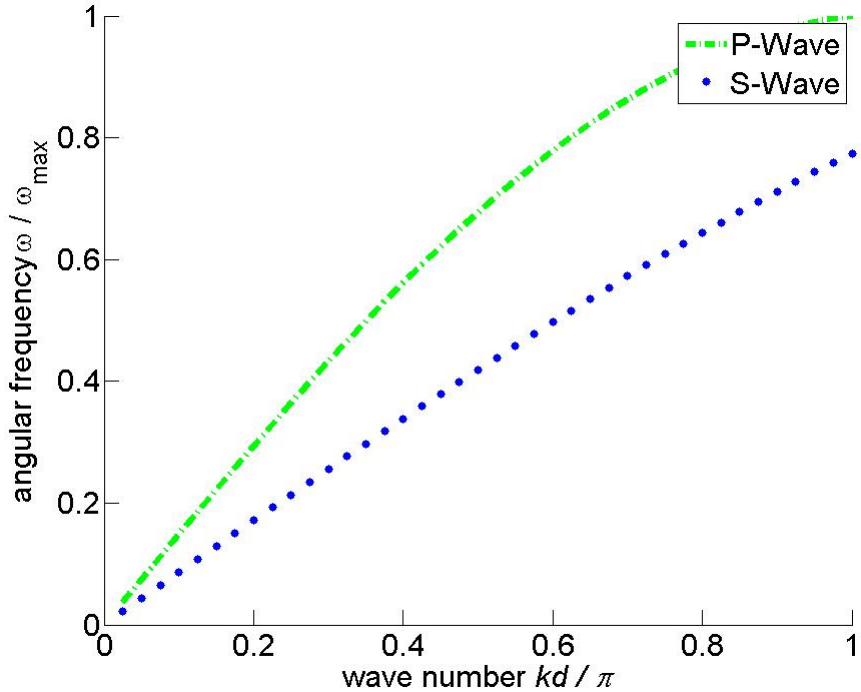
V. Magnanimo, S. Luding

- Frictionless

Cubic lattice



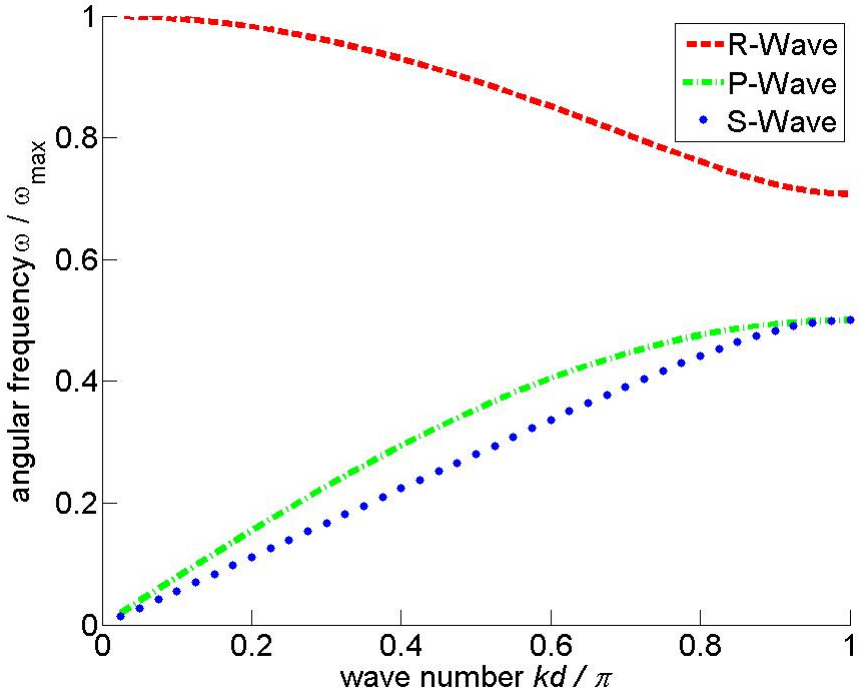

Hexagonal lattice



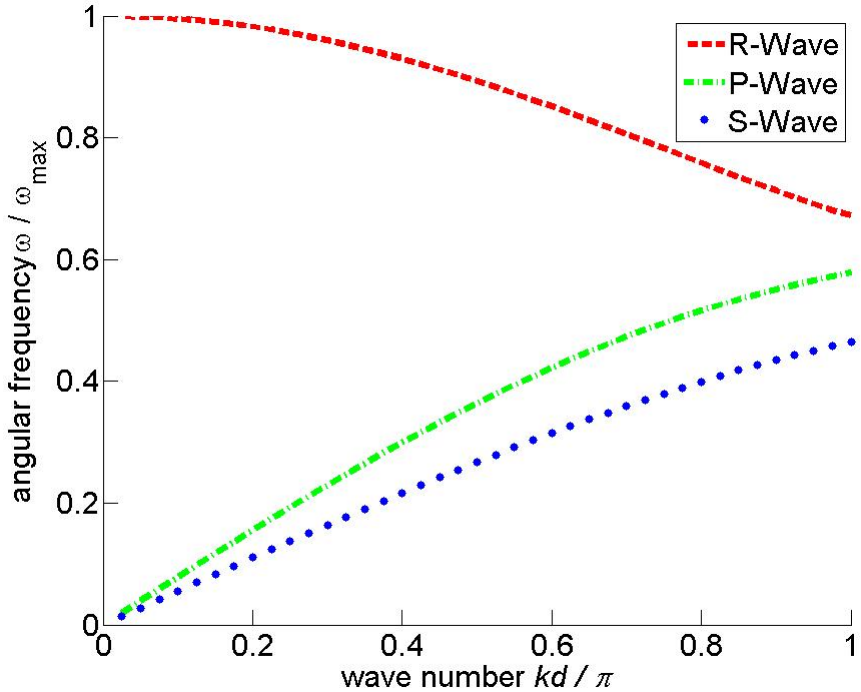
# DISPERSION

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- Friction: $k_t = k_n$
- Cubic lattice



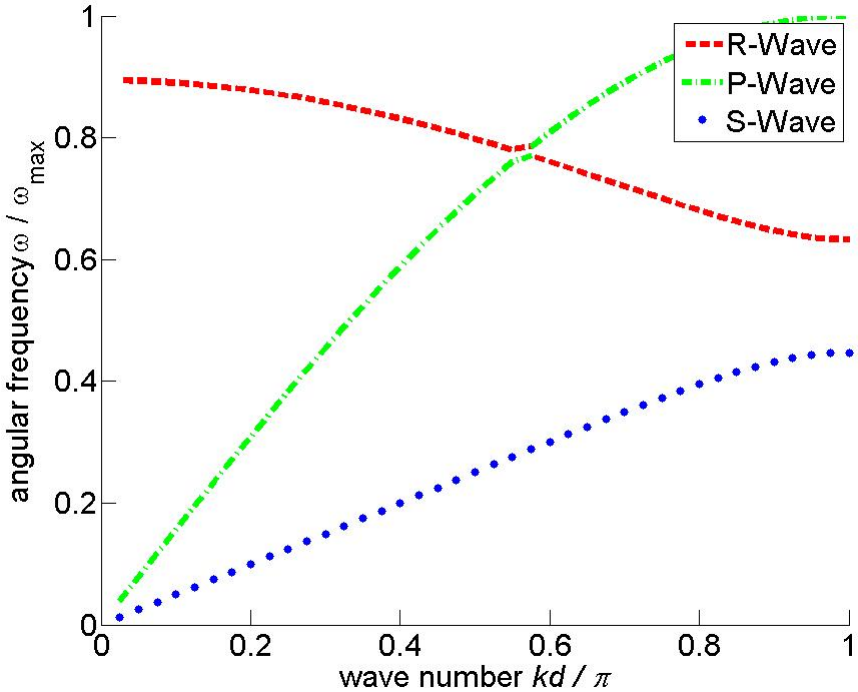

Hexagonal lattice



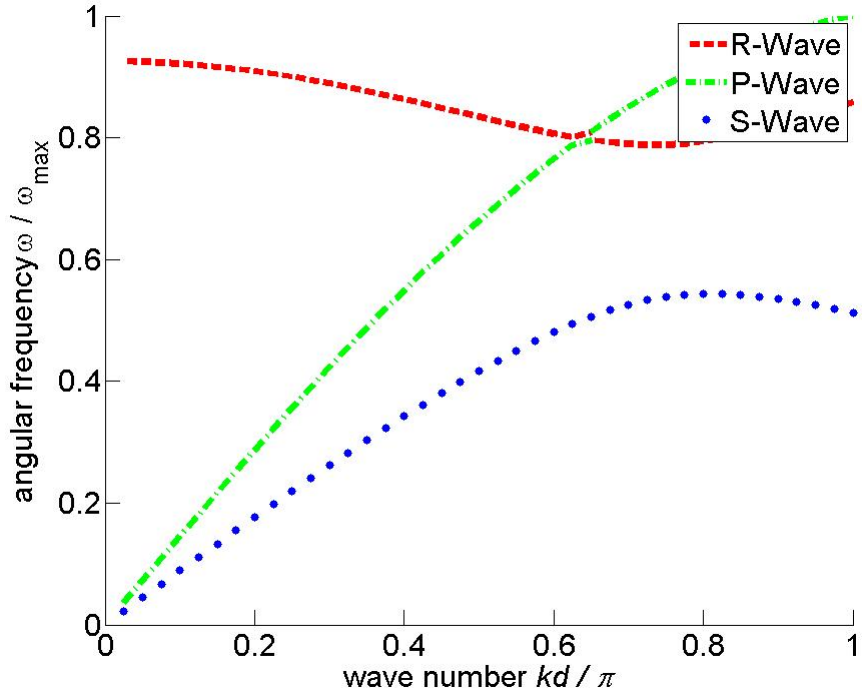
# DISPERSION

L.de Mol, B.Lawney,
V. Magnanimo, S. Luding

- Friction: $k_t = \frac{1}{5} k_n$
- Cubic lattice

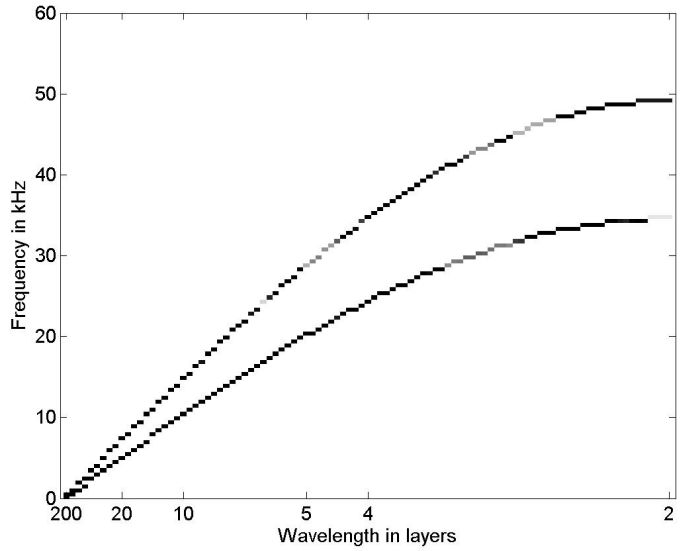



Hexagonal lattice

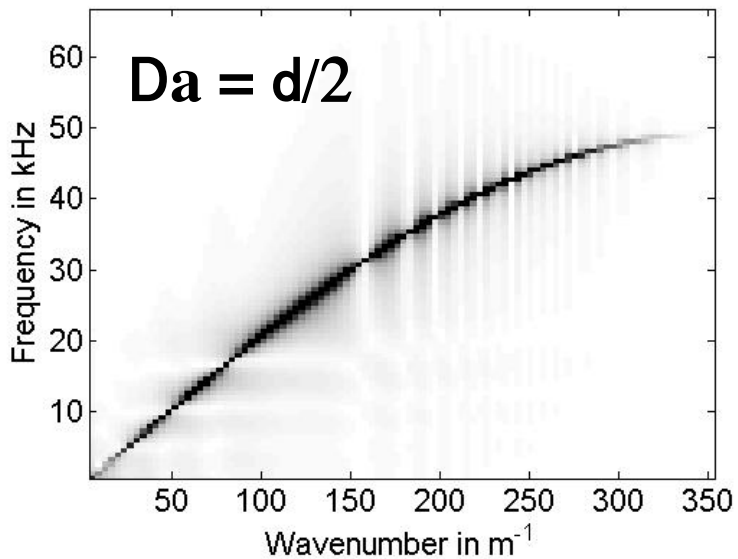


# Dispersion relations

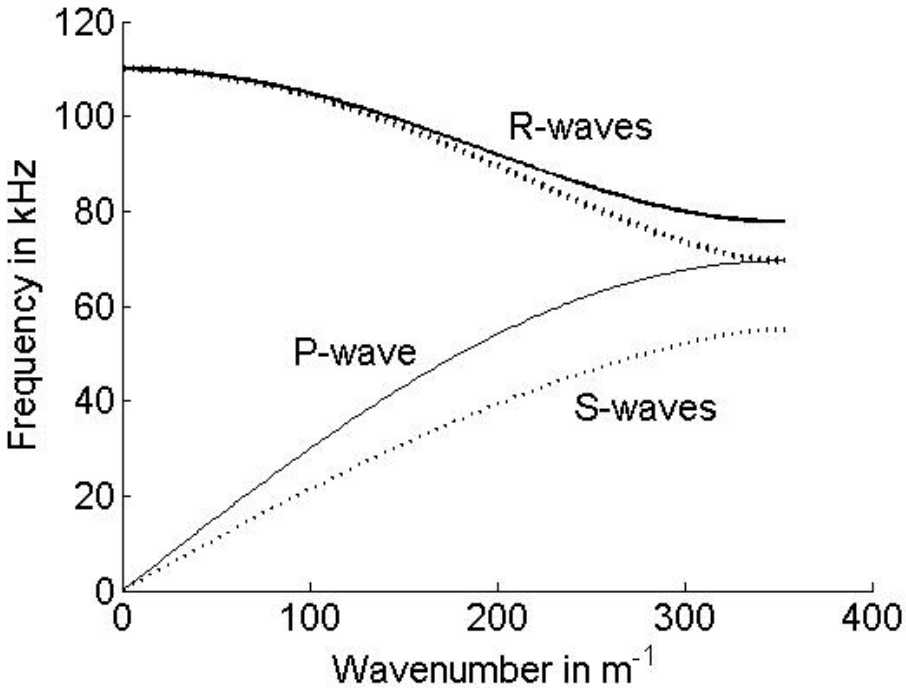
<u>from eigenvalue calc.</u>

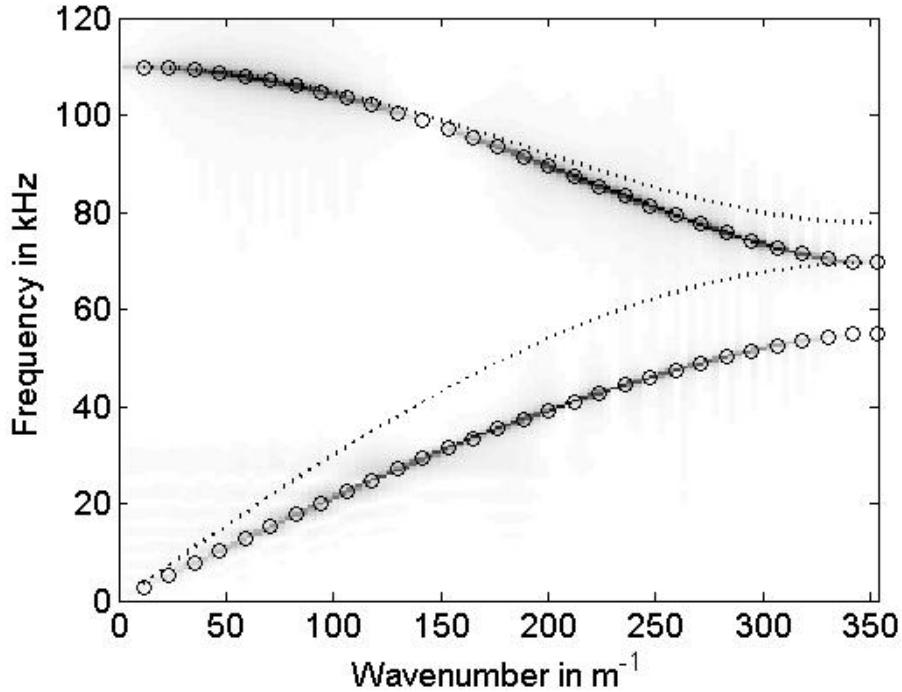



space-time-FFT



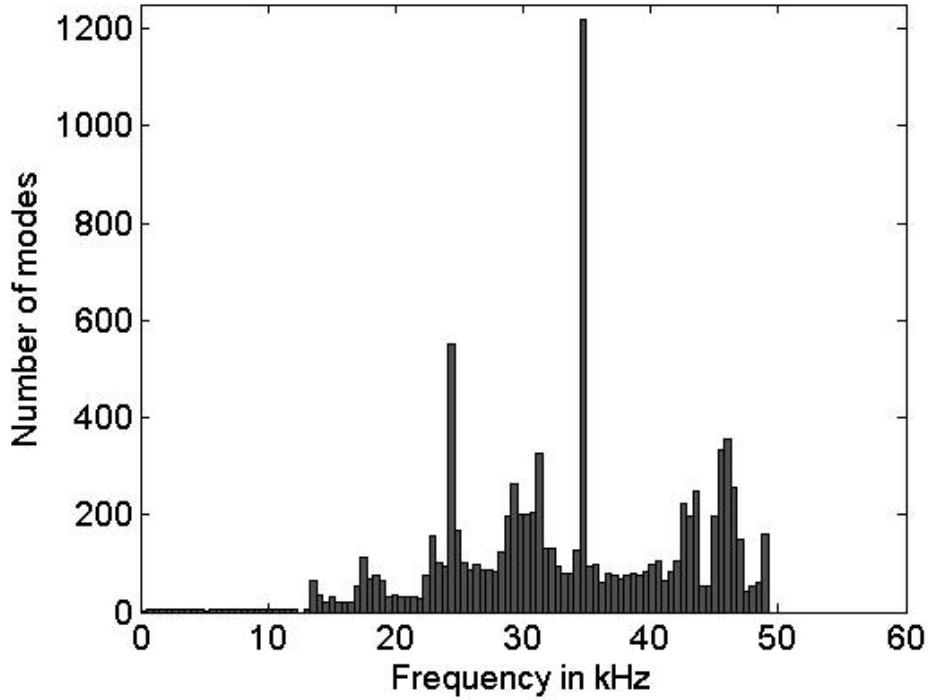
# Dispersion relations with tangential elasticity



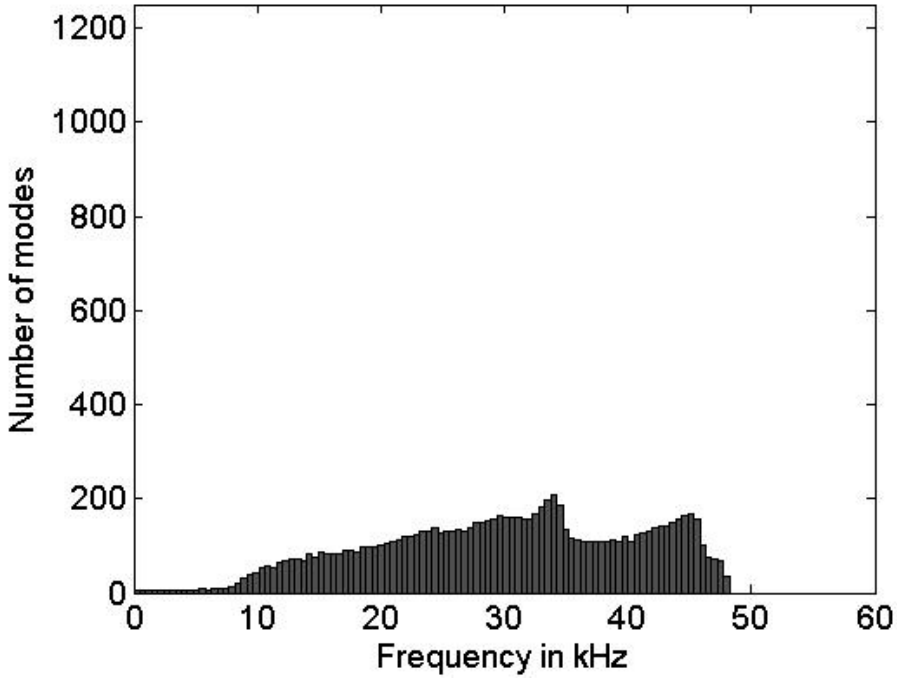



# Density of states

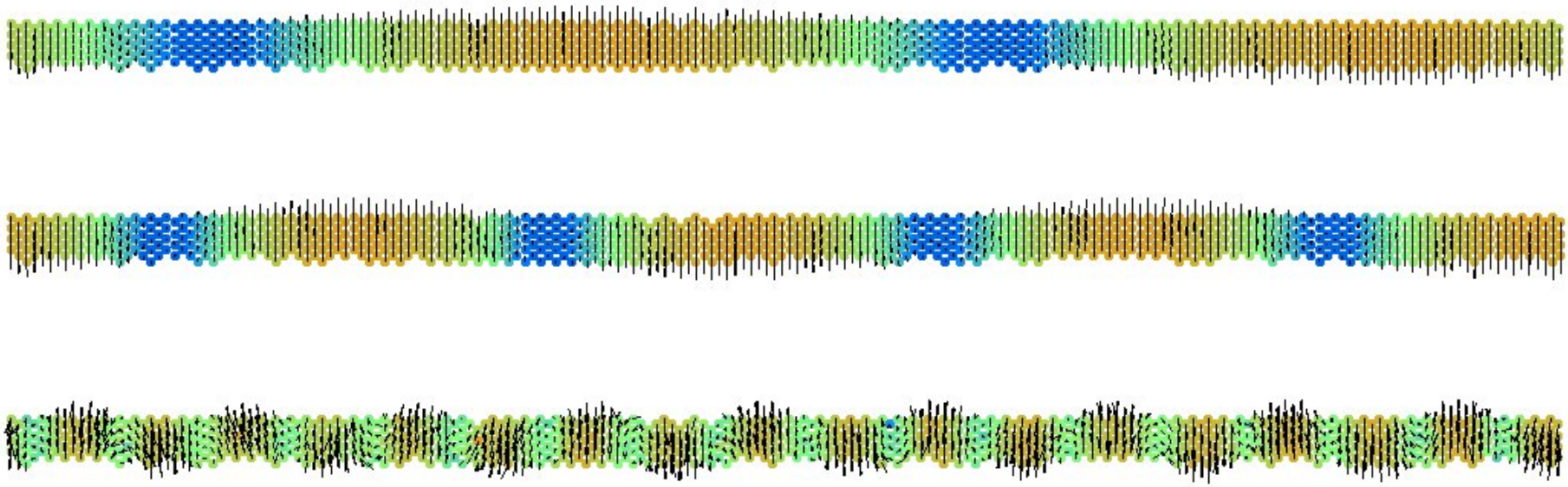
ordered



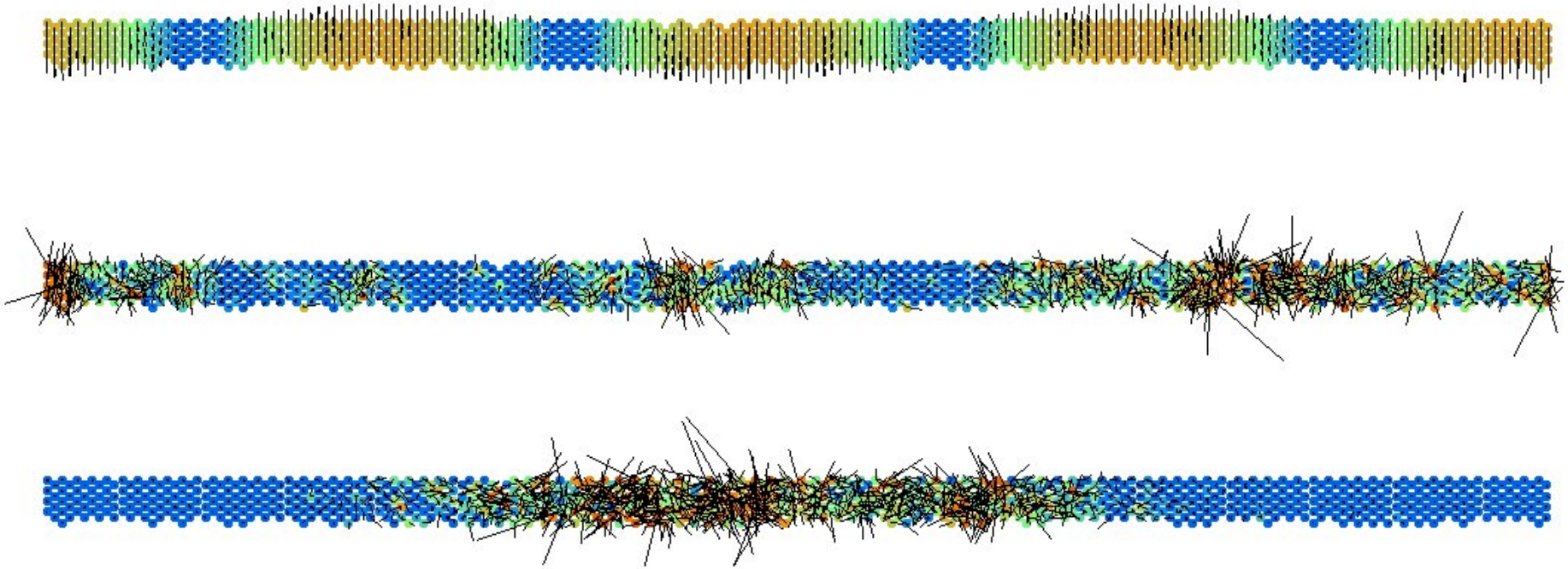

disordered



# Eigenmodes



# Eigenmodes



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# SUMMARY AND OUTLOOK

- 2D+3D dispersion relations
  - Frictionless lattices
  - Lattices with rotation (tang. elasticity)

WORK IN PROGRESS
- Fully random Systems and
- Comparison with Experiments