

# Parameter degeneracy and reactor neutrino experiments

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based on

hep-ph/0211111

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KamLAND

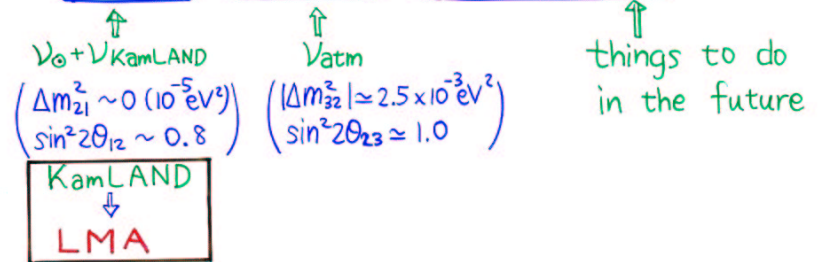
1. Introduction
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4. Summary

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## 1. Introduction

Oscillation parameters in  $N_\nu = 3$  framework

$$(\Delta m_{21}^2, \theta_{12}; |\Delta m_{32}^2|, \theta_{23}; \text{sign}(\Delta m_{32}^2), \theta_{13}, \delta)$$



L2

Final goal in  $\nu$  oscillation physics is measurement of  $\mathcal{CP}$  (only possible for LMA)

$$\text{Prob}(\mathcal{CP}) \propto J = \frac{c_{13}}{8} \underbrace{\sin 2\theta_{12}}_{\sim \sqrt{0.8}} \underbrace{\sin 2\theta_{13}}_{\sim \sqrt{0.1} \text{ unknown}} \underbrace{\sin 2\theta_{23}}_{\approx 1.0} \underbrace{\sin \delta}_{\text{unknown}}$$

As a first step, we need to know the magnitude of  $\sin 2\theta_{13}$

	parameter degeneracy	sensitivity
Long Base Line exp.	some	$\sin^2 2\theta_{13} \gtrsim 20(10^{-3})$
Reactor exp.	none	$\sin^2 2\theta_{13} \gtrsim 0(10^{-2})$

## 2. Reactor measurement of $\theta_{13}$

F. Suekane )  
K. Inoue ) are thinking of the possibility to measure  $\theta_{13}$  by a reactor experiment at Kashiwazaki-Kariwa Nuclear Power Plant.

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## Experimental Conditions for $\theta_{13}$

### Optimization of Baseline

SK Result:  $\Delta\bar{m}_{23}^2 \approx 2.5 \times 10^{-3} eV^2$

$$\int f_\nu(E) \sigma(E) \sin^2 \frac{\Delta m^2 L}{4E} dE = \max$$

↓

$$L \sim 1.7 \text{ km}$$

↓

$$N_\nu \sim 150 / \text{year} / \text{target-ton} / \text{GW}_{\text{th}}$$

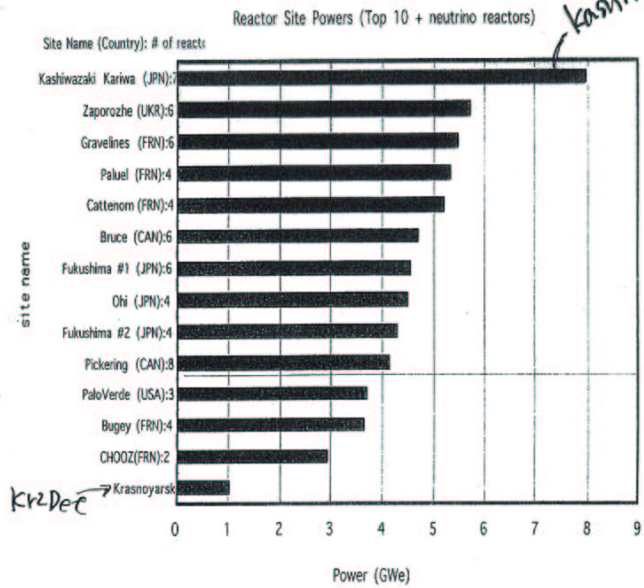
1% stat. error/year

↓

$$M_{\text{Target}} * P_{\text{Reactor}} = 70 [\text{ton} * \text{GW}_{\text{th}}]$$

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(Overviews of the World Nuclear Power, Nuclear Training Centre Jozef Stefan Institute (Slovenia); 17.Sept.2001)

Kashiwazaki-Kariwa NPP ( $24.3\text{GW}_{\text{th}}$ )



Largest Nuclear Reactor Site in the World.

Net  $M_{\text{Target}} \sim 5\text{tons}$  for 80% reactor and 70% detection efficiency (=Just CHOOZ size).

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Issues at CHOOZ and solutions

(1) Systematic Error=2.7%  
 Comes from { rate prediction: 2.3%  
 detection efficiency: 1.5%

Solution:

Identical Front and Far Detectors



most of the systematics cancel out

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How good is the cancellation?

Study BUGEY(3 identical detectors) case  
 Bugey detectors are modular type  
 (Intrinsically worse systematics than bulk type)

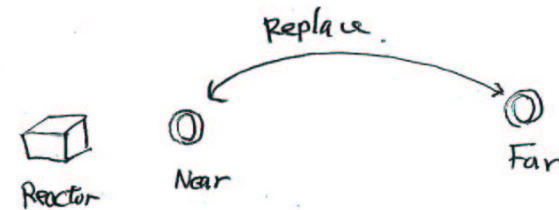
Example,

	BUGEY Case: (modular detectors)	CHOOZ projection: (same fraction assumed)
$\sigma_{\nu}^f$	2.8% → 0%	2.1% → 0%
$N_p$	1.9% → 0.6%	0.8% → 0.3%
$L^2$	0.5% → 0.5%	---
$\varepsilon$	3.5% → 1.7%	1.5% → 0.7%
=====		
Total	4.9% → 2%	2.7% → 0.8%
	(Kr2Det expects $\sigma_{sys} = 0.5\%$ )	

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CHOOZ detector is (in principle) Movable.

If front and far detectors are exchanged during the experiment, the individualities of the detectors are canceled and it is expected that the systematic error is further reduced to ~ 0.5%.





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We assume here :

24.3 GWth

80% operation efficiency

70% detection efficiency @  $\begin{cases} L = 1.7 \text{ km} \\ L = 0.3 \text{ km} \end{cases}$

energy spectrum: 14 bins of 0.5 MeV  $\Delta m_{32}^2 = 2.5 \times 10^{-3} \text{ eV}^2$

Results :

in the negative case

Excluded region (analysis w/ d.o.f. = 1)

$\sigma_{\text{sys}} = 2\%$ , 5 t.yr

$$\sin^2 2\theta_{13} \lesssim 0.027$$

$\sigma_{\text{sys}} = 0.8\%$ , 20 t.yr

$$\sin^2 2\theta_{13} \lesssim 0.013$$

in the affirmative case

The experimental error in  $\sin^2 2\theta_{13}$  is almost independent of the central value

$\sigma_{\text{sys}} = 2\%$ , 5 t.yr

$$\delta(\sin^2 2\theta_{13}) = 0.034$$

$\sigma_{\text{sys}} = 0.8\%$ , 20 t.yr

$$\delta(\sin^2 2\theta_{13}) = 0.015$$

d.o.f. = 2

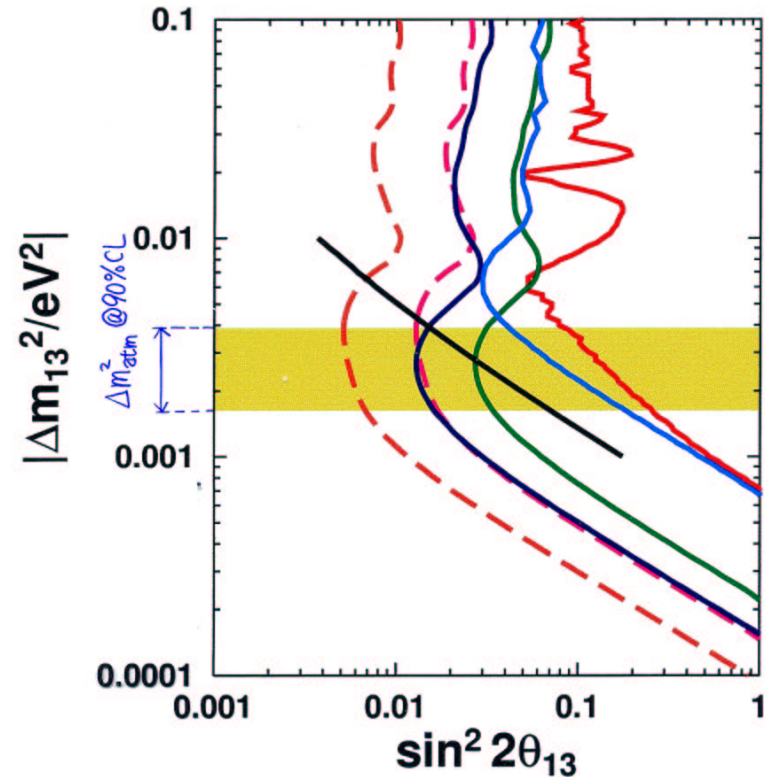
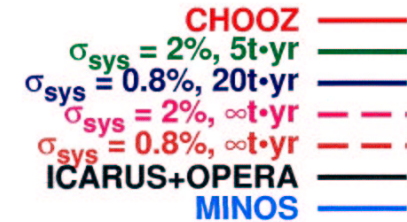
If JHF determines  $\Delta m_{32}^2$  to  $10^{-4} \text{ eV}^2$  then analysis becomes approximately 1-dimensional (w.r.t.  $\sin^2 2\theta_{13}$  only)

$\rightarrow \sigma_{\text{sys}} = 0.8\%$ , 20 t.yr

$$\delta(\sin^2 2\theta_{13}) = 0.012 \quad (\text{d.o.f.} = 1)$$

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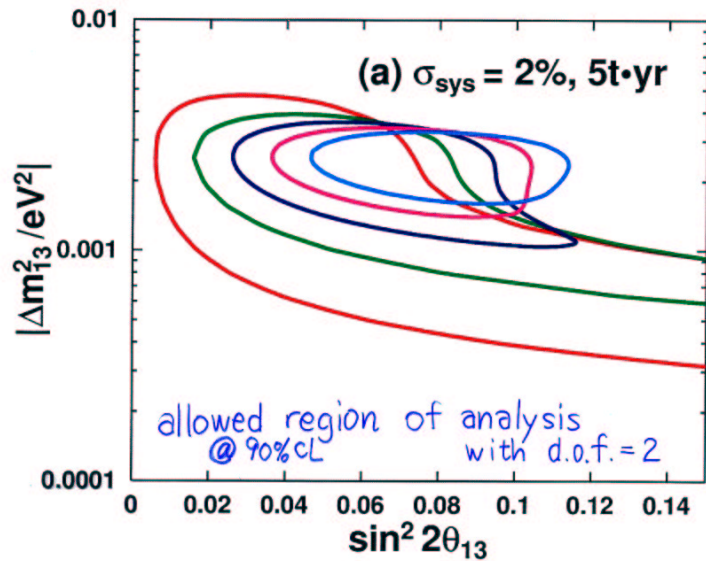
excluded region



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$$\delta(\sin^2 2\theta_{13}) = 0.034$$

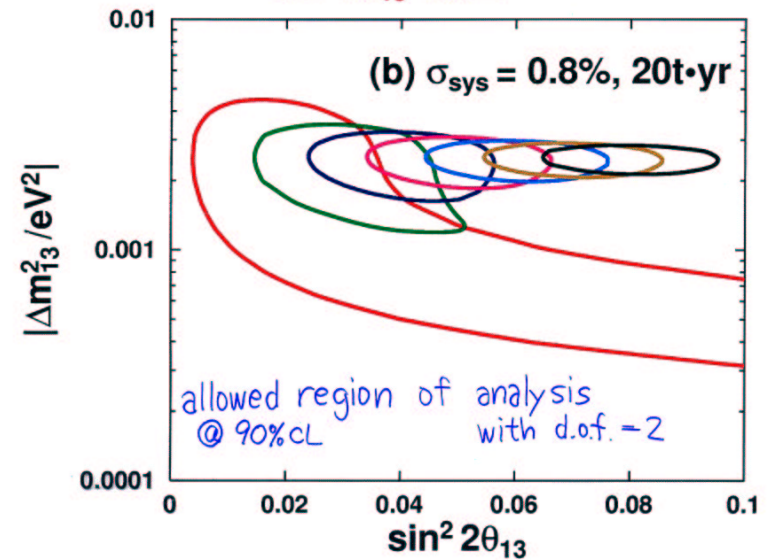
- $\sin^2 2\theta_{13} = 0.08$  — blue line
- $\sin^2 2\theta_{13} = 0.07$  — red line
- $\sin^2 2\theta_{13} = 0.06$  — dark blue line
- $\sin^2 2\theta_{13} = 0.05$  — green line
- $\sin^2 2\theta_{13} = 0.04$  — orange line



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$$\delta(\sin^2 2\theta_{13}) = 0.015 \rightarrow 0.012 \text{ (d.o.f.=1)}$$

- $\sin^2 2\theta_{13} = 0.08$  — black line
- $\sin^2 2\theta_{13} = 0.07$  — brown line
- $\sin^2 2\theta_{13} = 0.06$  — blue line
- $\sin^2 2\theta_{13} = 0.05$  — red line
- $\sin^2 2\theta_{13} = 0.04$  — dark blue line
- $\sin^2 2\theta_{13} = 0.03$  — green line
- $\sin^2 2\theta_{13} = 0.02$  — orange line



3-1. Parameter degeneracy in  $(S_{23}^2, \sin^2 2\theta_{13})$  plane <sup>23</sup>  $\frac{12}{13}$   
 Even if  $P \equiv P(\nu_\mu \rightarrow \nu_e)$  and  $\bar{P} \equiv P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$  are given, there are in general 8 solutions.

3 kinds of degeneracy

- intrinsic  $(\delta, \theta_{13})$  Burguet-Castell et al ('01)
- sign  $(\Delta m_{31}^2)$  Minakata-Nunokawa ('01)
- $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$  Fogli-Lisi PRD54 ('96) 3667
- 8-fold degeneracy Barger-Marfatia-Whisnant ('02)

Here I assume that accelerator beams are approximately monochromatic.

Experimental errors in long baseline experiments are not taken into account.

I will show how the 8-fold degeneracy is lifted by switching on :

$$\theta_{23} - \frac{\pi}{4}, \quad \left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right|, \quad \frac{AL}{2} \quad (A \equiv \sqrt{2} G_F N_e)$$

(they are all small @ JHF experiment)

$$\sin^2 2\theta_{23} \geq 0.92 \quad \left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right| \sim \frac{1}{35} \quad \frac{AL}{2} \sim \frac{1}{13}$$

Here I visualize the 8-fold degeneracy by using the  $(S_{23}^2, \sin^2 2\theta_{13})$  plane step by step.

$$\epsilon \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \quad \frac{AL}{2}$$

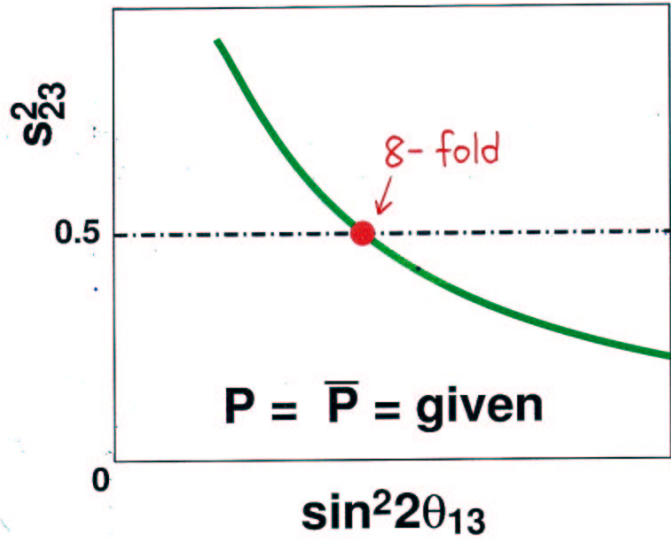
	$\theta_{23} - \frac{\pi}{4}$	$\Delta m_{21}^2$	$A \equiv \sqrt{2} G_F N_e$	$\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$	$(\delta, \theta_{13})$	$\text{sign}(\Delta m_{31}^2)$
(a)	= 0	= 0	= 0	degen.	degen.	degen.
(b)	≠ 0	= 0	= 0	lifted	degen.	degen.
(c)	≠ 0	≠ 0	= 0	lifted	lifted	degen.
(d) off OM	≠ 0	≠ 0	≠ 0	lifted	lifted	lifted
(e) @ OM	≠ 0	≠ 0	≠ 0	lifted	degen.	almost degen.

$\frac{13}{14}$



24  $\frac{4}{15}$

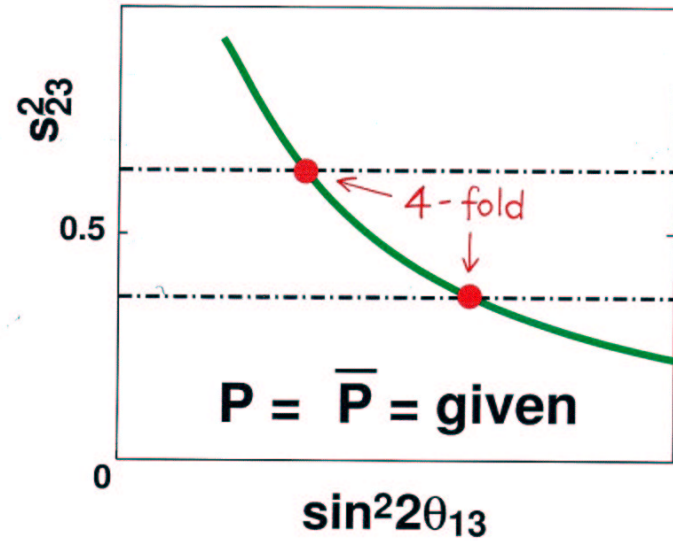
(a)  $\theta_{23} = \frac{\pi}{4}$ ,  $\Delta m_{21}^2 = 0$ ,  $A = 0$



$$P = \bar{P} = \underbrace{s_{23}^2}_{\frac{1}{2}} \sin^2 2\theta_{13} \underbrace{\sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)}_{\text{const}}$$

25  $\frac{5}{16}$

(b)  $\theta_{23} \neq \frac{\pi}{4}$ ,  $\Delta m_{21}^2 = 0$ ,  $A = 0$



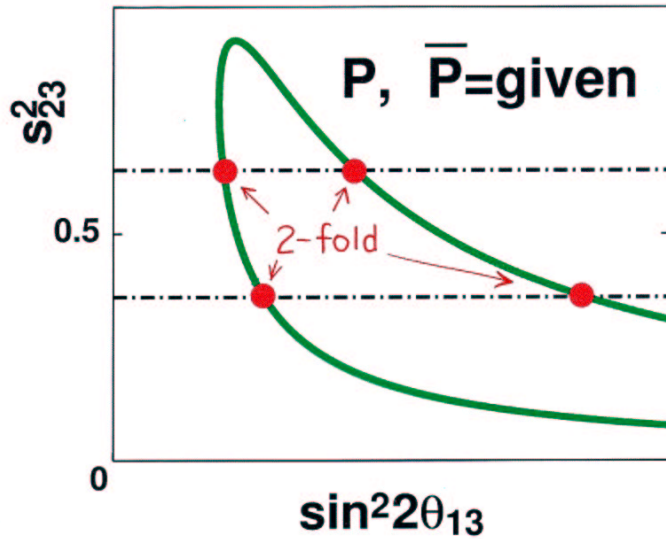
$$P = \bar{P} = s_{23}^2 \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

$$s_{23}^2 = \frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2} \leftarrow \text{known from } \nu_{\mu} \rightarrow \nu_{\mu}$$



26  $\frac{16}{17}$

(c)  $\theta_{23} \neq \frac{\pi}{4}$ ,  $\Delta m_{21}^2 \neq 0$ ,  $A=0$



$$\frac{1}{\cos^2 \Delta} \left( \frac{P+\bar{P}}{2} - x^2 \sin^2 \Delta - y^2 g^2 \right)^2 + \frac{1}{\sin^2 \Delta} \left( \frac{P-\bar{P}}{2} \right)^2 = (2xy\Delta \sin \Delta)^2$$

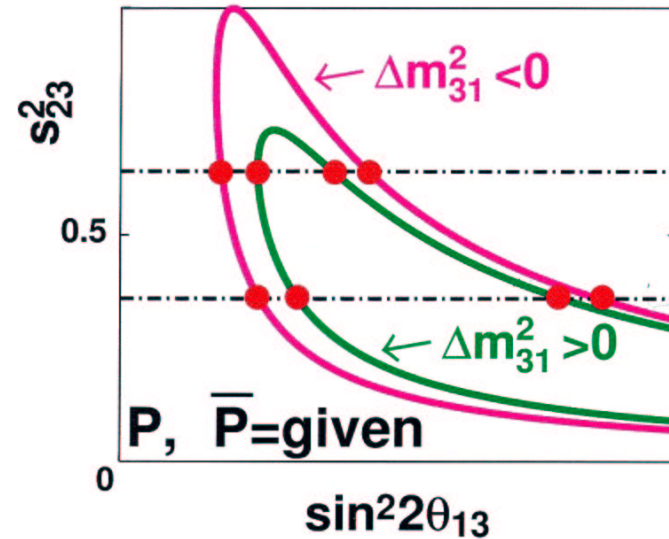
quadratic eq. in  $x^2$

$$\left( \begin{array}{l} x \equiv S_{23} \sin 2\theta_{13} \\ y \equiv \left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right| c_{23} \sin 2\theta_{12} \\ \Delta \equiv \frac{\Delta m_{31}^2 L}{4E} \end{array} \right)$$

$$S_{23} = \sqrt{\frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}}$$

27  $\frac{17}{18}$

(d)  $\theta_{23} \neq \frac{\pi}{4}$ ,  $\Delta m_{21}^2 \neq 0$ ,  $A \neq 0$   
off OM



$$\frac{1}{4\cos^2 \Delta} \left( \frac{P-x^2 f^{(\mp)} - y^2 g^2}{f^{(\mp)}} + \frac{\bar{P}-x^2 f^{(\pm)} - y^2 g^2}{f^{(\pm)}} \right)^2 + \frac{1}{4\sin^2 \Delta} \left( \frac{P-x^2 f^{(\mp)} - y^2 g^2}{f^{(\mp)}} - \frac{\bar{P}-x^2 f^{(\pm)} - y^2 g^2}{f^{(\pm)}} \right)^2 = (2xyg)^2$$

quadratic in  $x^2$

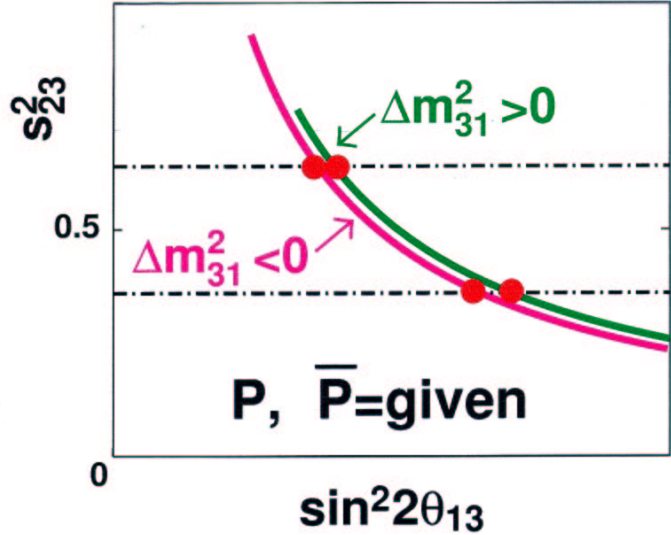
for  $\Delta m_{31}^2 \begin{cases} > 0 \\ < 0 \end{cases}$

$$S_{23} = \sqrt{\frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}}$$

$$\begin{array}{l} x \equiv S_{23} \sin 2\theta_{13} \\ y \equiv \left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right| c_{23} \sin 2\theta_{12} \\ \Delta \equiv \frac{\Delta m_{31}^2 L}{4E} \\ f^{(\pm)} \equiv \frac{\sin(\Delta \pm AL/2)}{1 \mp AL/2\Delta} \\ g \equiv \frac{\sin(AL/2)}{AL/2\Delta} \end{array}$$

28 | 8  
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(e)  $\theta_{23} \neq \frac{\pi}{4}$ ,  $\Delta m_{21}^2 \neq 0$ ,  $A \neq 0$   
 @OM ( $\frac{\Delta m_{31}^2 L}{4E} = \frac{\pi}{2}$ )



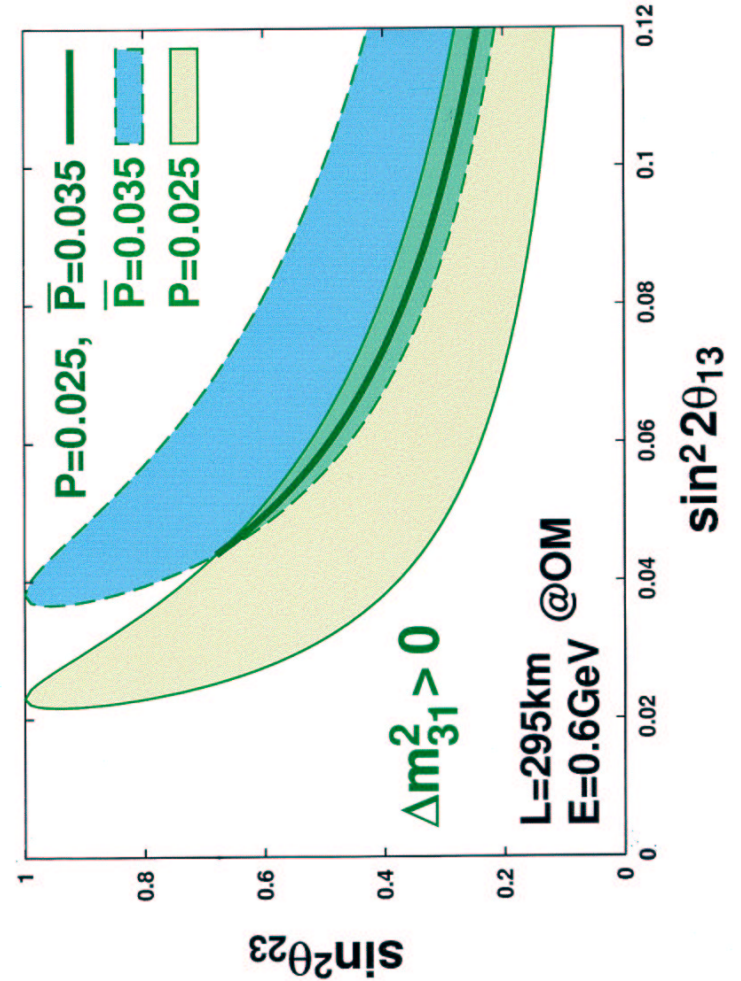
$$\frac{P - x^2 f^{(+)} - y^2 g^2}{f^{(+)}} = - \frac{\bar{P} - x^2 f^{(-)} - y^2 g^2}{f^{(-)}} \quad \text{for } \Delta m_{31}^2 \begin{cases} > 0 \\ < 0 \end{cases}$$

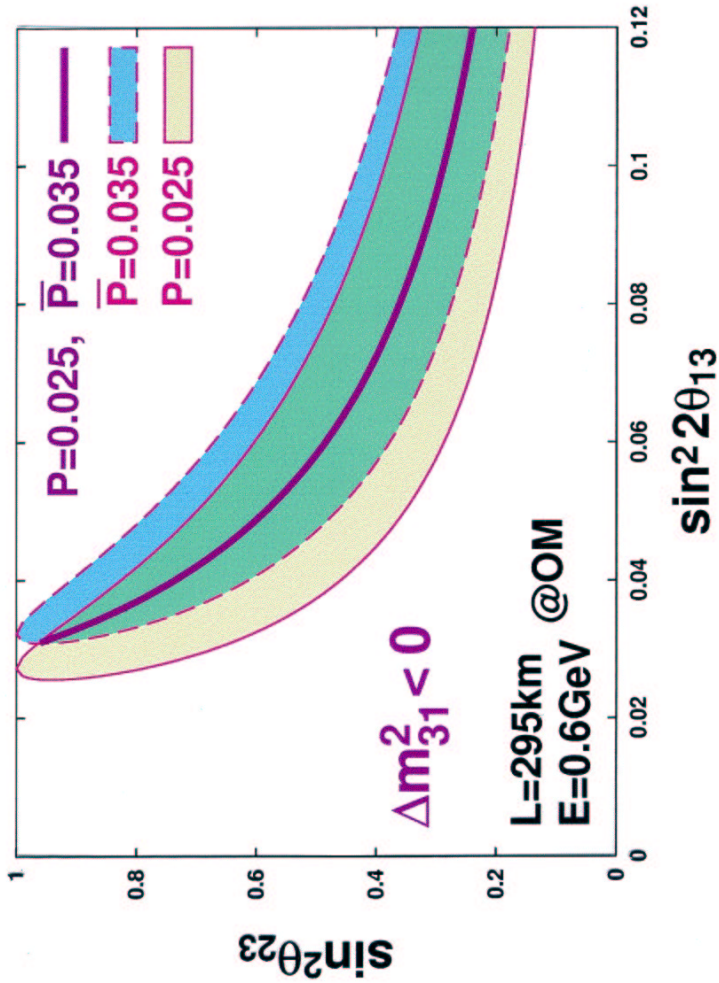
linear in  $x^2$

$$S_{23} = \sqrt{\frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}}$$

$$\begin{aligned} x &\equiv S_{23} \sin 2\theta_{13} \\ y &\equiv \left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right| c_{23} \sin 2\theta_{12} \\ f^{(\pm)} &\equiv \frac{\cos(AL/2)}{1 \mp AL/\pi} \\ g &\equiv \frac{\sin(AL/2)}{AL/\pi} \end{aligned}$$

20 | 19





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@ Oscillation Maximum ( $\frac{\Delta m_{31}^2 L}{4E} = \frac{\pi}{2}$ )

11/22

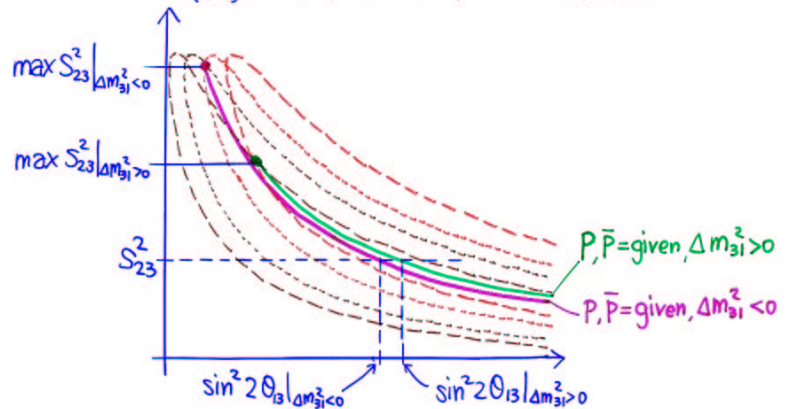
$$P \equiv P(\nu_\mu \rightarrow \nu_e) = x^2 f^2 - 2xyfg \sin \delta + y^2 g^2$$

$$\bar{P} \equiv P(\bar{\nu}_\mu \rightarrow \nu_e) = x^2 \bar{f}^2 + 2xy\bar{f}g \sin \delta + y^2 g^2$$

where  $x \equiv S_{23} \sin 2\theta_{13}$

$$y \equiv \epsilon C_{23} \sin 2\theta_{12}, \quad \epsilon \equiv \left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right|$$

$$\left\{ \frac{f}{\bar{f}} \right\} \equiv \frac{\cos(AL/2)}{1 \mp AL/\pi}, \quad g \equiv \frac{\sin(AL/2)}{AL/\pi}, \quad A = \sqrt{2} G_F N_e$$



When  $P$  &  $\bar{P}$  are given, one can show

$$\bullet \sin^2 2\theta_{13} \Big|_{\Delta m_{31}^2 > 0} - \sin^2 2\theta_{13} \Big|_{\Delta m_{31}^2 < 0} = \frac{1}{S_{23}^2} \frac{1}{f\bar{f}} \frac{f-\bar{f}}{f+\bar{f}} (\bar{P}-P) \approx \frac{AL}{S_{23}^2 \pi} (\bar{P}-P) \sim 5 \times 10^{-4}$$

for any  $\theta_{23}$  if  $|AL/2| \ll 1$

$$\bullet \max S_{23}^2 \Big|_{\Delta m_{31}^2 < 0} - \max S_{23}^2 \Big|_{\Delta m_{31}^2 > 0} = \frac{1}{\epsilon^2} \frac{1}{\sin^2 2\theta_{12}} \frac{1}{g^2} \frac{f-\bar{f}}{f+\bar{f}} (\bar{P}-P) \approx \frac{1}{\epsilon^2} \frac{(2/\pi)^2}{\sin^2 2\theta_{12}} \frac{AL}{\pi} (\bar{P}-P) \sim 0.3$$

$$\text{if } P=0.025, \bar{P}=0.035, L=295 \text{ km}, \epsilon = \frac{7 \times 10^{-5} \text{ eV}^2}{2.5 \times 10^{-3} \text{ eV}^2} \approx \frac{1}{53}$$

$$A \approx \frac{1}{1900 \text{ km}}$$



3-2. Resolution of  $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$  degeneracy  
 by LBL  $\oplus$  reactor 29 112  
23  
 cf. Fogli-Lisi PRD54('96)3667  
 Barenboim-de Grourea ('02)

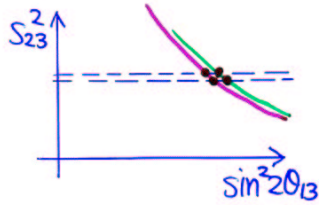
Our scenario

- JHF  $\nu \oplus \bar{\nu}$  @ Oscillation Maximum  $\oplus$
- reactor experiment (@ Kashiwazaki?)

From  $\nu_\mu \leftrightarrow \nu_\mu$  @ JHF we will know that  $\theta_{23}$  satisfies either of the followings:

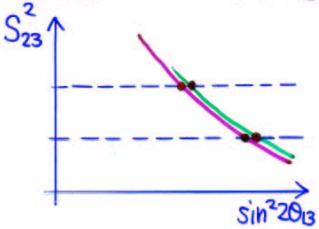
- (A)  $|\sin^2 2\theta_{23}| < \text{a few} \times 10^{-2}$
- (B)  $|\sin^2 2\theta_{23}| \geq \text{a few} \times 10^{-2}$

(A) with JHF  $\nu \oplus \bar{\nu}$  @ OM

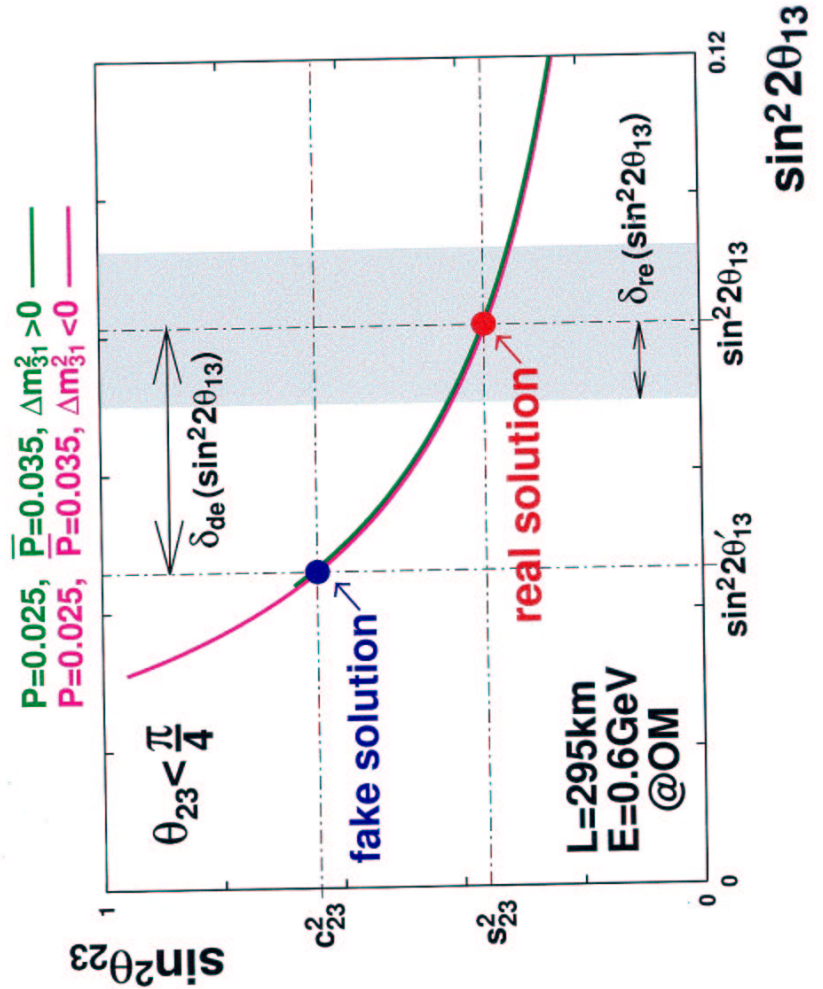


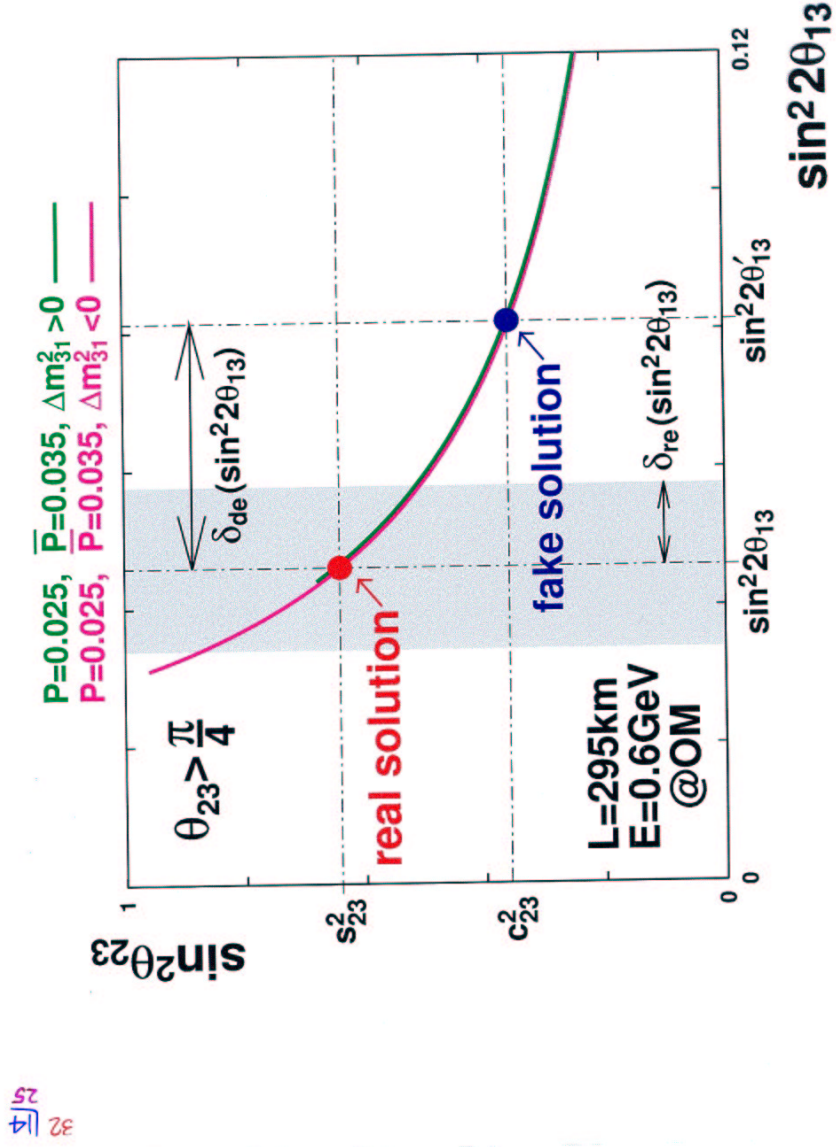
The situation looks like the upper figure. The precise determination of true  $\sin^2 2\theta_{13}$  is difficult, but the values of  $\sin^2 2\theta_{13}$  for the 4 solutions are approximately the same.

(B) with JHF  $\nu \oplus \bar{\nu}$  @ OM



The values of  $\sin^2 2\theta_{13}$  for  $\theta_{23} < \frac{\pi}{4}$  and  $\theta_{23} > \frac{\pi}{4}$  are quite different and it may be possible to determine the true value of  $\sin^2 2\theta_{13}$  if the error  $\delta_{re}(\sin^2 2\theta_{13})$  of the reactor exp. is smaller than the ambiguity  $\delta_{de}(\sin^2 2\theta_{13})$  due to the degeneracy.





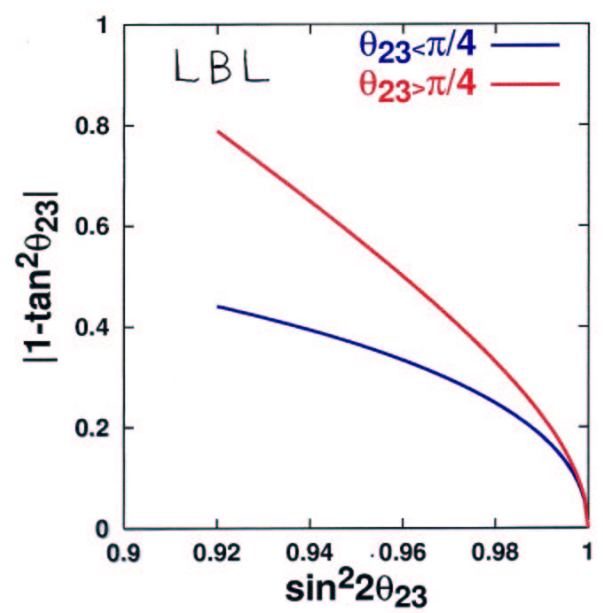
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$$\frac{\delta_{de}(\sin^2 2\theta_{13})}{\sin^2 2\theta_{13}} \equiv \frac{|\sin^2 2\theta'_{13} - \sin^2 2\theta_{13}|}{\sin^2 2\theta_{13}}$$

$$= |1 - \tan^2 \theta_{23}| \cdot \left\{ 1 + \frac{(\Delta m_{21}^2)^2 \tan^2(AL/2)}{\Delta m_{31}^2} \left[ 1 - \left(\frac{AL}{\pi}\right)^2 \right] \sin^2 2\theta_{12} \right\}$$

$$\approx |1 - \tan^2 \theta_{23}|$$

$\delta_{de}(\sin^2 2\theta_{13})$ : ambiguity due to the  
 $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$  degeneracy

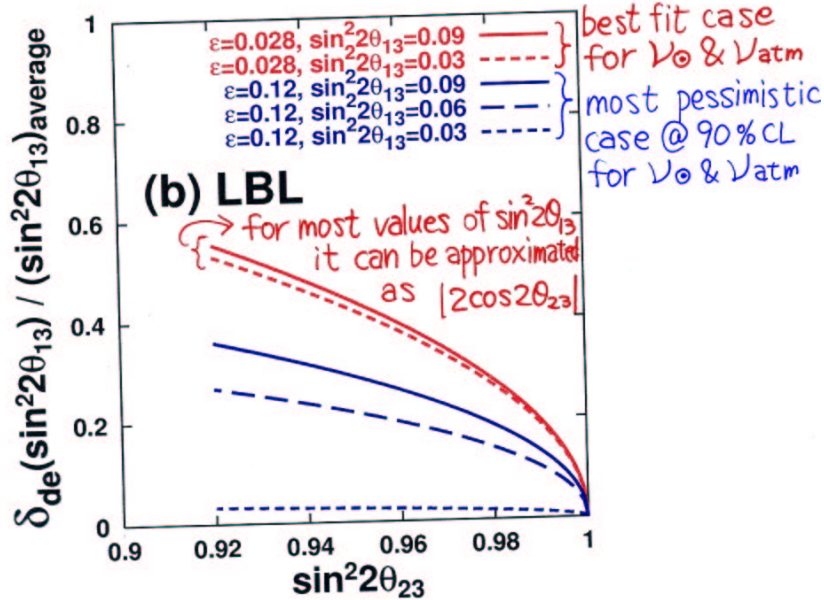


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ambiguity due to the degeneracy

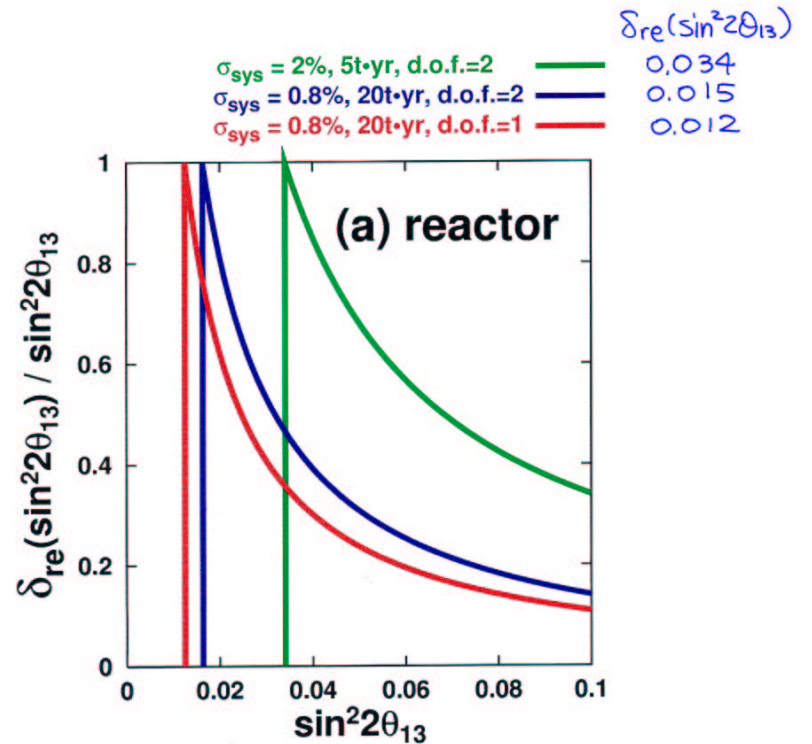
$$\delta_{de}(\sin^2 2\theta_{13}) \equiv |\sin^2 2\theta_{13} - \sin^2 2\theta'_{13}|$$

$$(\sin^2 2\theta_{13})_{\text{average}} \equiv \frac{1}{2}(\sin^2 2\theta_{13} + \sin^2 2\theta'_{13})$$



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error in the reactor experiment  
 $\delta_{re}(\sin^2 2\theta_{13})$





4. Summary

Reactor experiment on  $\theta_{13}$

\* { much cheaper  
may be done earlier } than JHF

\* sensitivity

$$\sin^2 2\theta_{13} \geq 0.013 \quad \text{for } \sigma_{\text{sys}} = 0.8\%, 20 \text{ ton}\cdot\text{yr (dof.=1)}$$

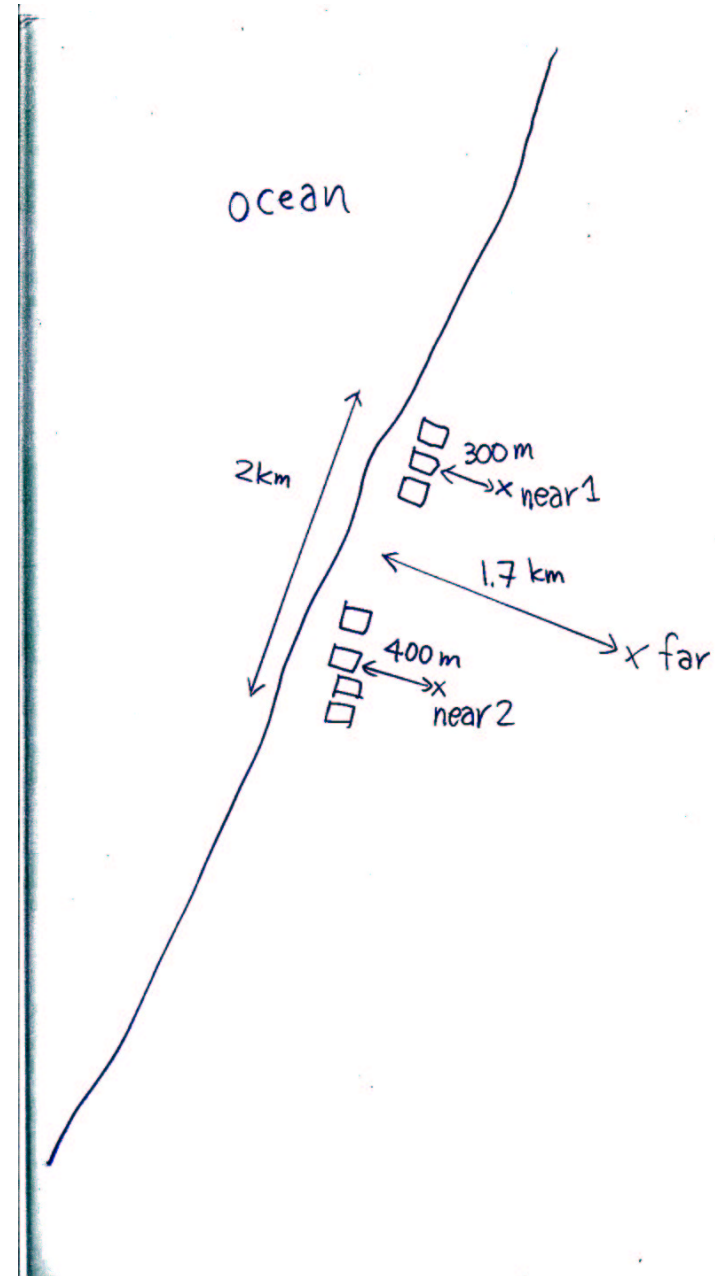
@ KK - NPP

\* free from degeneracy

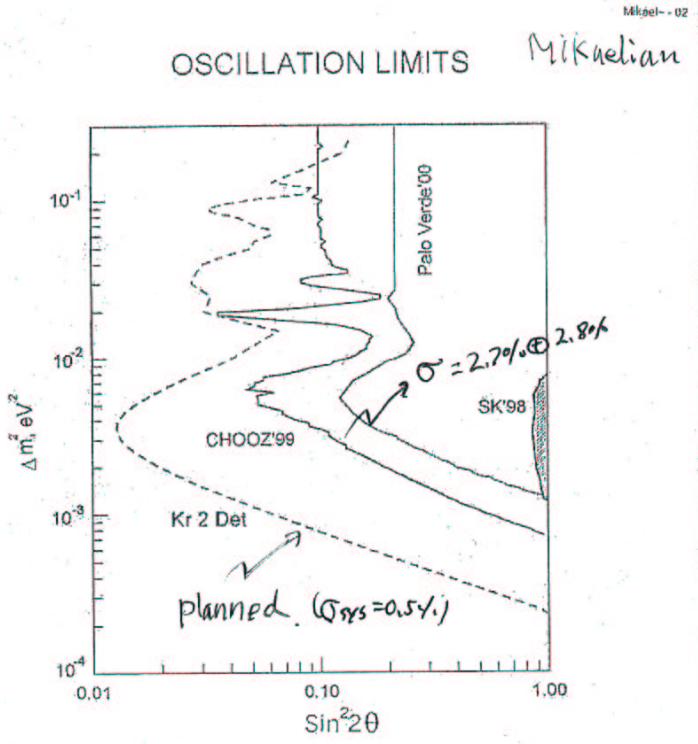
$$\text{if } \sin^2 2\theta_{23} \leq 0.96 \quad \& \quad \sin^2 2\theta_{13} \geq 0.06$$

then a reactor experiment may be able to determine  $\sin^2 2\theta_{13}$  &  $s_{23}^2$  for the true solution.

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Reactor  $\theta_{13}$  Status.



For reactor  $\tilde{\nu}_e$  at 1 km:

$$\text{Sin}^2 2\theta = 4U_{e3}^2 (1 - U_{e3}^2) \approx 4U_{e3}^2$$

$$\nu_e = U_{e1} \nu_1 + U_{e2} \nu_2 + U_{e3} \nu_3 \quad U_{e3} \equiv \sin \theta_{13}$$