Parameter degeneracy and reactor neutrino experiments

Tokyo Metropolitan Univ.
Osamu YASUDA

based on
hep-ph/0211111
$\underbrace{\text{Minakata, Sugiyama, O.Y.,}}_{\text{TMU}}$ $\underbrace{\text{Inoue, Suekane}}_{\text{KamLAND}}$

1. Introduction

2. Reactor measurement of θ_{13}

3-1. Parameter degeneracy in $(s^2_{23}, \sin^2 2\theta_{13})$ plane

3-2. Resolution of $\theta_{23} \Leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy by LBL $\oplus$ reactor

4. Summary

1. Introduction

Oscillation parameters in $N_\nu = 3$ framework

$$(\underbrace{\Delta m^2_{21}, \theta_{12}}_{\Uparrow};\ \underbrace{|\Delta m^2_{32}|, \theta_{23}}_{\Uparrow};\ \underbrace{\text{sign}(\Delta m^2_{32}), \theta_{13}, \delta}_{\Uparrow})$$

$\nu_\odot + \nu_{KamLAND}$

$\begin{pmatrix} \Delta m^2_{21} \sim O(10^{-5} eV^2) \\ \sin^2 2\theta_{12} \sim 0.8 \end{pmatrix}$

$\boxed{\text{KamLAND} \Downarrow \text{LMA}}$

ν_{atm}

$\begin{pmatrix} |\Delta m^2_{32}| \simeq 2.5 \times 10^{-3} eV^2 \\ \sin^2 2\theta_{23} \simeq 1.0 \end{pmatrix}$

things to do in the future

Final goal in ν oscillation physics is measurement of $\cancel{CP}$ (only possible for LMA)

$$\text{Prob}(\cancel{CP}) \propto J = \frac{c_{13}}{8} \underbrace{\sin 2\theta_{12}}_{\sim\sqrt{0.8}} \underbrace{\sin 2\theta_{13}}_{\lesssim\sqrt{0.1}\ \text{unknown}} \underbrace{\sin 2\theta_{23}}_{\simeq 1.0} \underbrace{\sin \delta}_{\text{unknown}}$$

As a first step, we need to know the magnitude of $\sin 2\theta_{13}$

	parameter degeneracy	sensitivity
Long Base Line exp.	some	$\sin^2 2\theta_{13}$(JHF) $\gtrsim O(10^{-3})$
Reactor exp.	none	$\sin^2 2\theta_{13} \gtrsim O(10^{-2})$

2. Reactor measurement of θ_{13}

F. Suekane
K. Inoue) are thinking of the possibity to measure θ_{13} by a reactor experiment at Kashiwazaki - Kariwa Nuclear Power Plant.

Experimental Conditions for Θ_{13}

Optimization of Baseline

SK Result: $\Delta \overline{m}^2_{23} \approx 2.5 \times 10^{-3}\, eV^2$

$$\int f_\nu(E)\sigma(E)\sin^2\frac{\Delta m^2 L}{4E}dE = \max$$

$$\Downarrow$$

$$\boxed{L \sim 1.7\mathrm{km}}$$

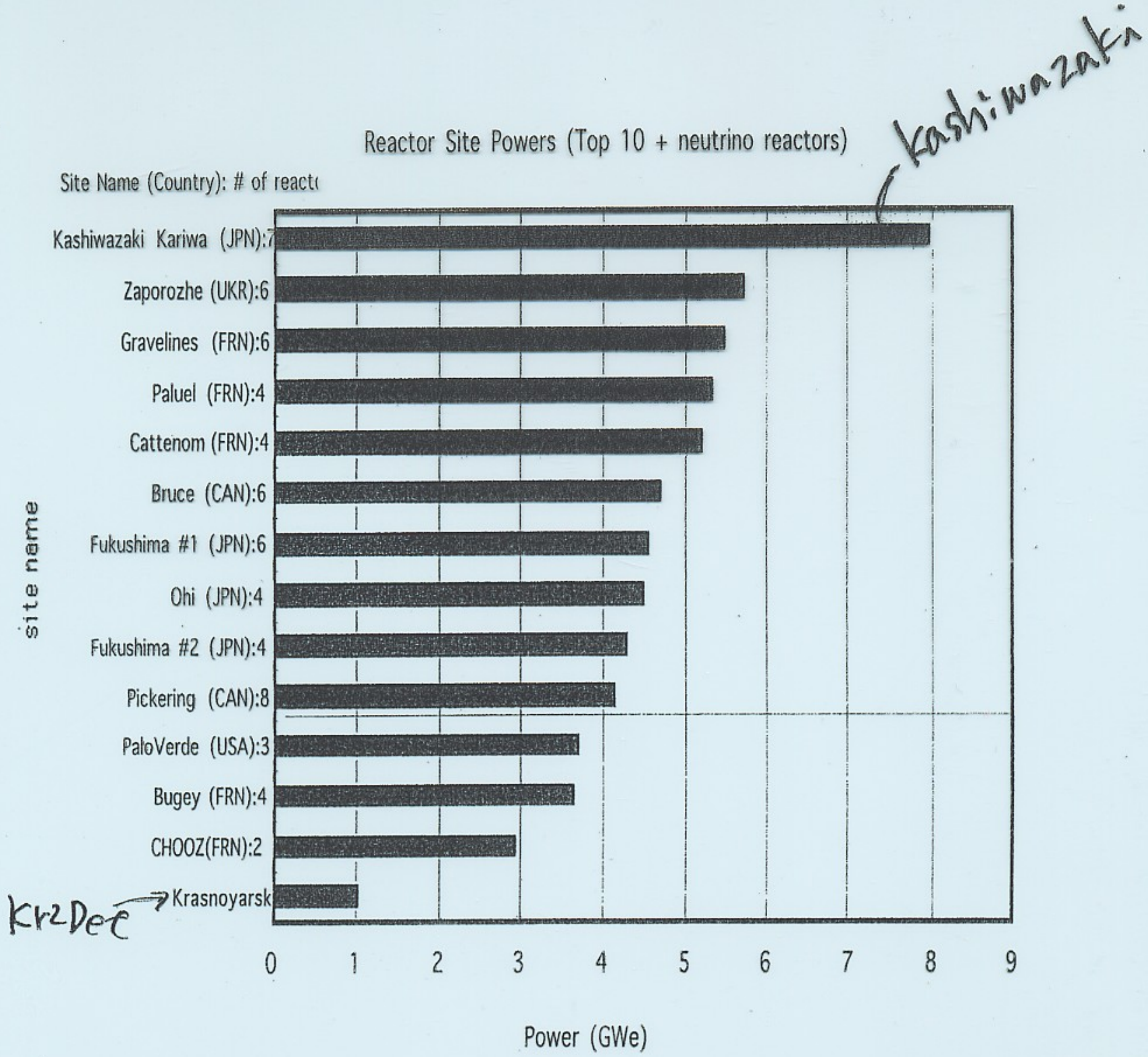
$$\Downarrow$$

$N_\nu \sim 150$/year/target-ton/GW_{th}

1% stat. error/year

$$\Downarrow$$

$M_{Target} * P_{Reactor} = 70[\mathrm{ton} * GW_{th}]$



(Overvies of the World Nuclear Power, Nuclear Training Centre Jozef Stefan Institute (Slovenia); 17.Sept.2001)

Kashiwazaki-Kariwa NPP ($\underline{24.3GW_{th}}$)

$\Downarrow$

Largest Nuclear Reactor Site in the World.

Net M_{Traget}~$\underline{5tons}$ for 80% reactor and 70% detection efficiency (=Just CHOOZ size).

Issues at CHOOZ and solutions

(1) Systematic Error=2.7%

Comes from
- rate prediction: 2.3%
- detection efficiency: 1.5%

<u>Solution</u>:

Identical Front and Far Detectors

$\Downarrow$

most of the systematics cancel out

How good is the cancellation?

Study BUGEY(3 identical detectors) case
Bugey detectors are modular type
(Intrinsically worse systematics than bulk type)

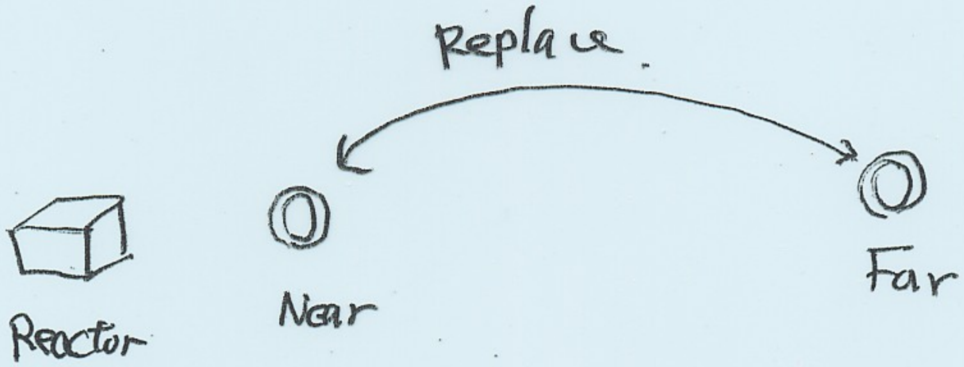
Example,

	BUGEY Case: (modular detectors)	CHOOZ projection: (same fraction assumed)
σf_ν	2.8%→0%	2.1%→0%
N_p	1.9%→0.6%	0.8%→0.3%
L^2	0.5%→0.5%	---
ε	3.5%→1.7%	1.5%→0.7%
Total	4.9% → 2%	2.7%→0.8%

(Kr2Det expects $\sigma_{sys} = 0.5\%$)

CHOOZ detector is (in principle) Movable.

If front and far detectors are exchanged during the experiment, the individualities of the detectors are canceled and it is expected that the systemetic error is further reduced to~ 0.5%.



We assume here :

24.3 GWth

80% operation efficiency

70% detection efficiency @ ($L = 1.7$ km, $L = 0.3$ km)

energ spectrum: 14 bins of 0.5 MeV $\Delta m^2_{32} = 2.5 \times 10^{-3}\,\mathrm{eV}^2$

Results :

in the negative case

Excluded region (analysis w/ d.o.f. = 1)

$\sigma_{sys} = 2\%$, 5 t·yr

$\sin^2 2\theta_{13} \lesssim 0.027$

$\sigma_{sys} = 0.8\%$, 20 t·yr

$\sin^2 2\theta_{13} \lesssim 0.013$

in the affirmative case

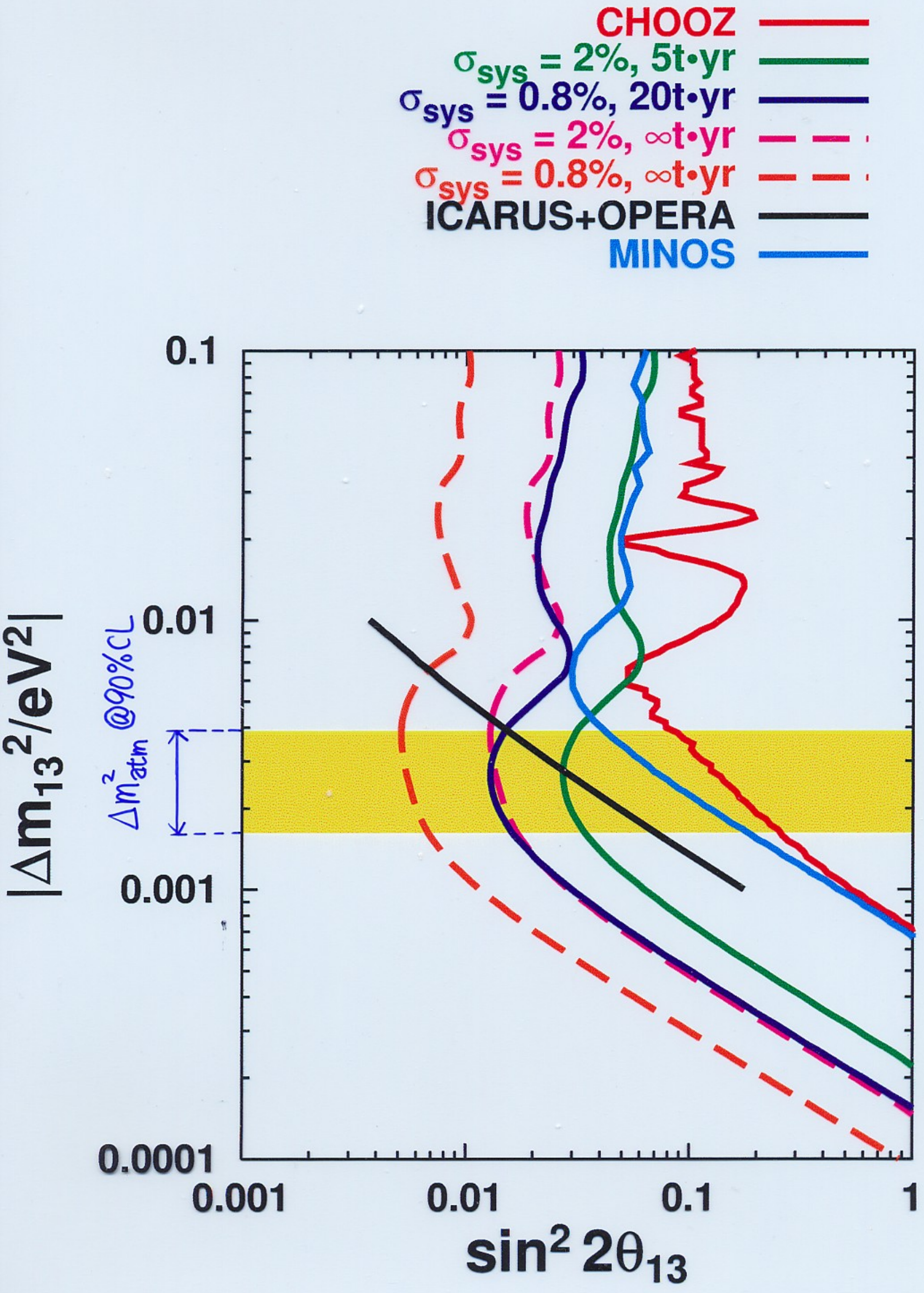
The experimental error in $\sin^2 2\theta_{13}$ is almost independent of the central value

$$\left.\begin{array}{l} \sigma_{sys} = 2\%,\ 5\ \mathrm{t\cdot yr} \\ \quad \delta(\sin^2 2\theta_{13}) = 0.034 \\ \sigma_{sys} = 0.8\%,\ 20\ \mathrm{t\cdot yr} \\ \quad \delta(\sin^2 2\theta_{13}) = 0.015 \end{array}\right\}\ \mathrm{d.o.f.} = 2$$

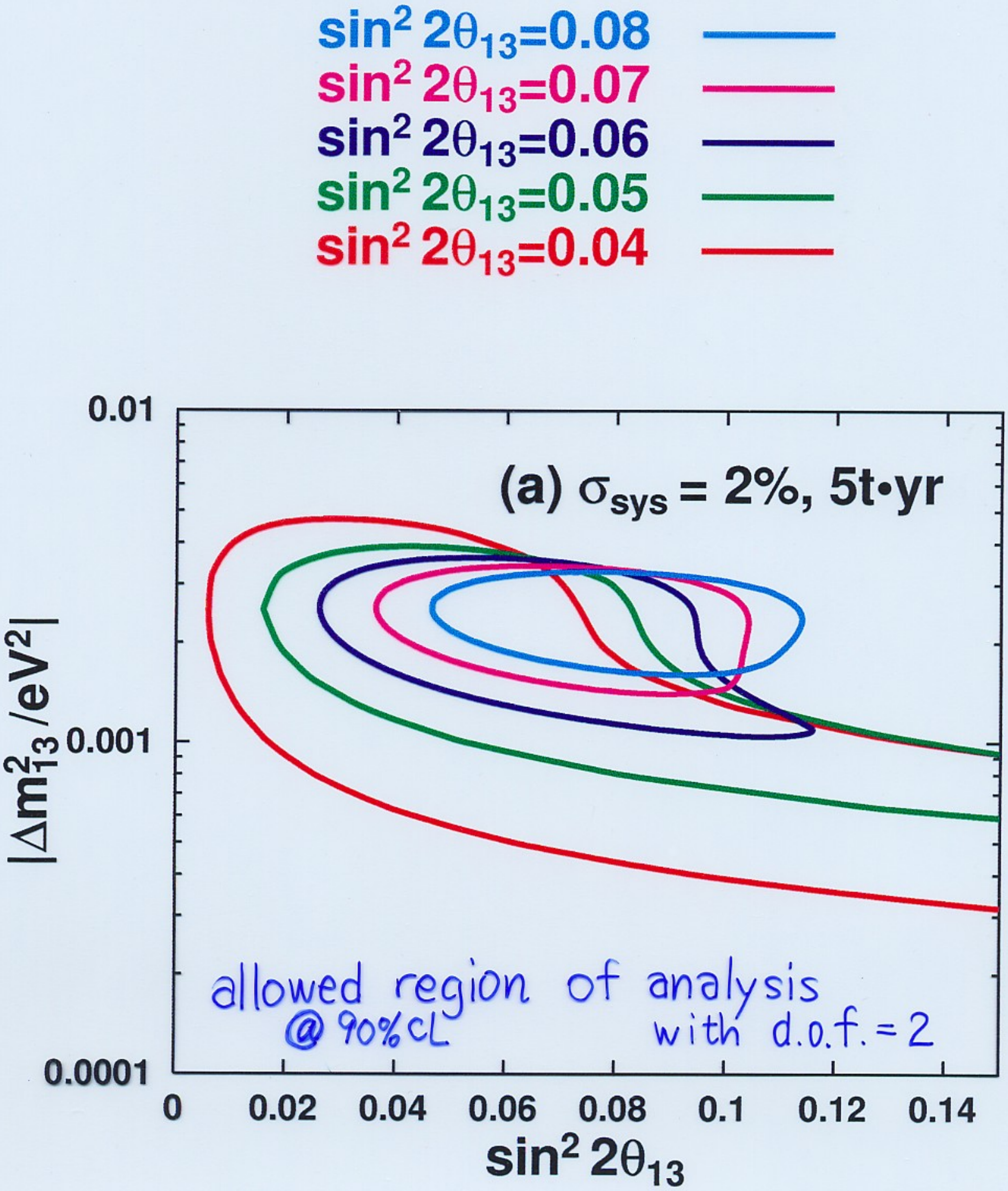
If JHF determines Δm^2_{32} to $10^{-4}\,\mathrm{eV}^2$ then analysis becomes approximately 1-dimensional (w.r.t. $\sin^2 2\theta_{13}$ only)

$\rightarrow \sigma_{sys} = 0.8\%$, 20t·yr

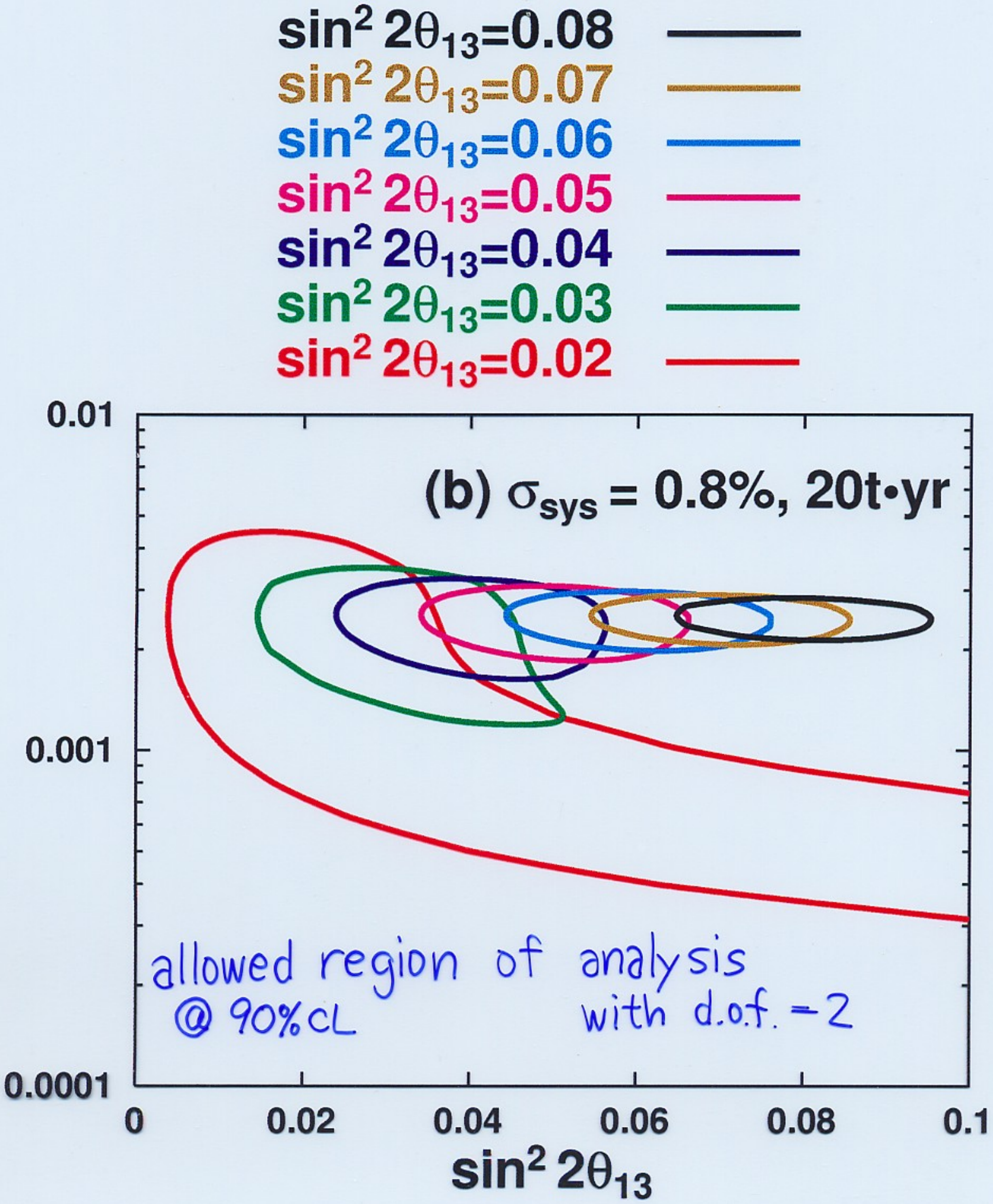
$\delta(\sin^2 2\theta_{13}) = 0.012$ (d.o.f. = 1) excluded region



$\boxed{\delta(\sin^2 2\theta_{13}) = 0.034}$



$\boxed{\delta(\sin^2 2\theta_{13}) = 0.015} \rightarrow 0.012$ (d.o.f.=1)



3-1. Parameter degeneracy in $(s^2_{23}, \sin^2 2\theta_{13})$ plane

Even if $P \equiv P(\nu_\mu \to \nu_e)$ and $\bar{P} \equiv P(\bar{\nu}_\mu \to \bar{\nu}_e)$ are given, there are in general 8 solutions.

3 kinds of degeneracy

- intrinsic (δ, θ_{13}) — Burguet-Castell et al ('01)
- sign (Δm^2_{31}) — Minakata-Nunokawa ('01)
- $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ — Fogli-Lisi PRD54 ('96) 3667, Barger-Marfatia-Whisnant ('02)

$\Rightarrow$ $\boxed{\text{8-fold degeneracy}}$

Here I assume that accelerator beams are approximately monchromatic.
Experimental errors in long baseline experiments are not taken into account.

I will show how the 8-fold degeneracy is lifted by switching on :

$$\theta_{23} - \frac{\pi}{4}, \qquad \left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right|, \qquad AL$$

$(A \equiv \sqrt{2} G_F N_e)$

(they are all small @ JHF experiment)

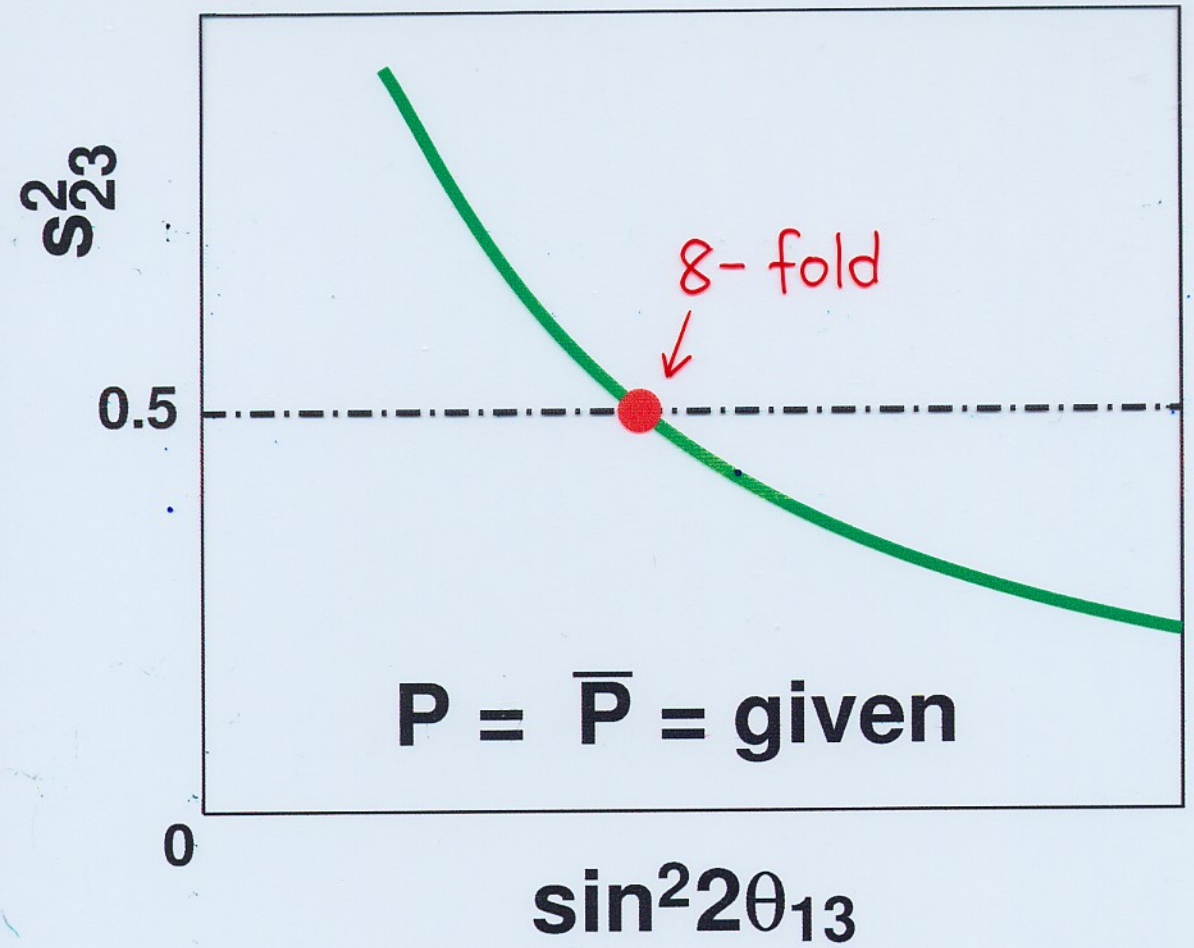
$\sin^2 2\theta_{23} \gtrsim 0.92$ $\qquad \left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right| \sim \frac{1}{35}$ $\qquad \frac{AL}{2} \sim \frac{1}{13}$

Here I visualize the 8-fold degeneracy by using the $(s^2_{23}, \sin^2 2\theta_{13})$ plane step by step.

$\epsilon \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}}$ $\frac{AL}{2}$

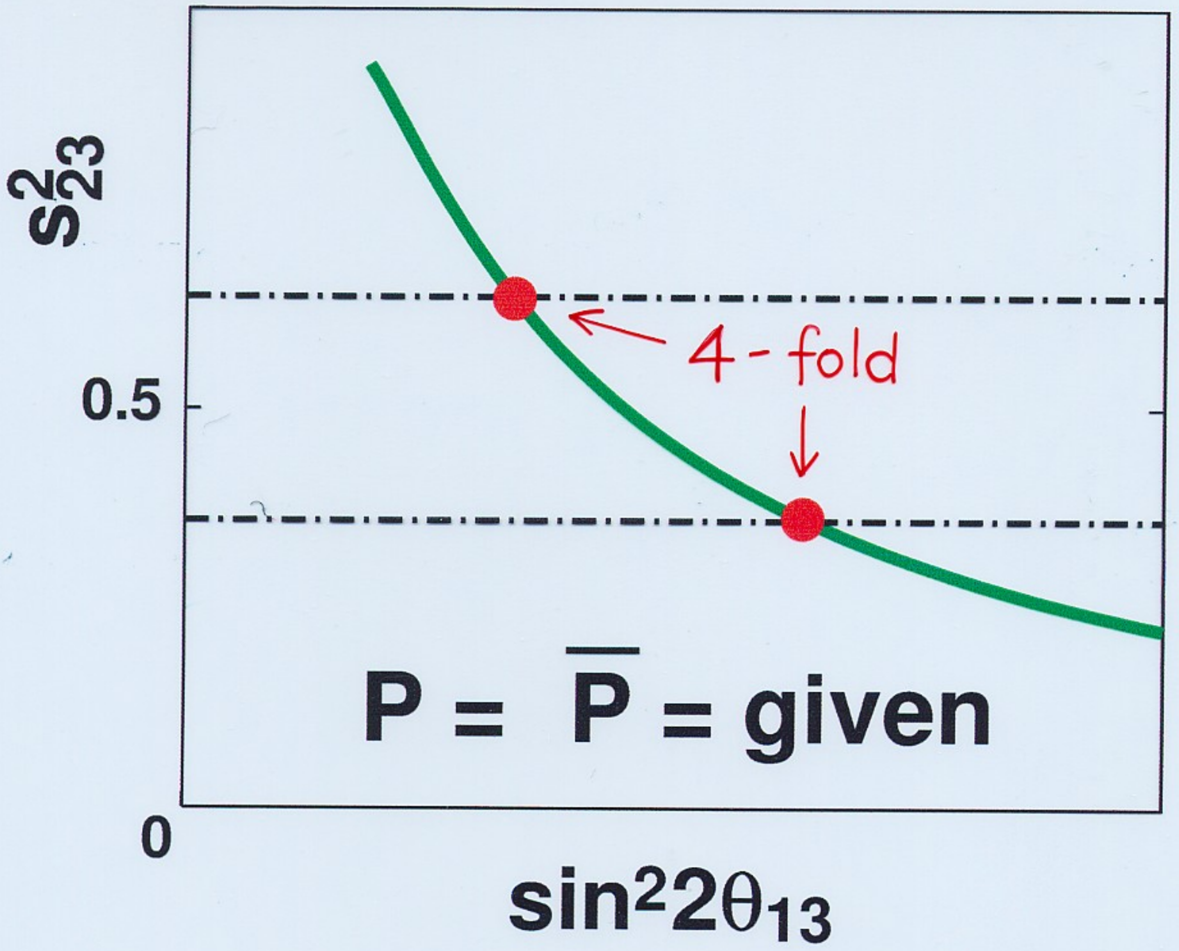
	$\theta_{23} - \frac{\pi}{4}$	Δm^2_{21}	$A \equiv \sqrt{2} G_F N_e$	$\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$	(δ, θ_{13})	$\mathrm{sign}(\Delta m^2_{31})$
(a)	$=0$	$=0$	$=0$	degen.	degen.	degen.
(b)	$\neq 0$	$=0$	$=0$	lifted	degen.	degen.
(c)	$\neq 0$	$\neq 0$	$=0$	lifted	lifted	degen.
(d) off OM	$\neq 0$	$\neq 0$	$\neq 0$	lifted	lifted	lifted
(e) @OM	$\neq 0$	$\neq 0$	$\neq 0$	lifted	degen.	almost degen.

(a) $\theta_{23}=\frac{\pi}{4}$, $\Delta m^2_{21}=0$, A=0



$$P=\overline{P}=\underbrace{s_{23}^2}_{=\frac{1}{2}}\sin^2 2\theta_{13}\underbrace{\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)}_{=\text{const}}$$

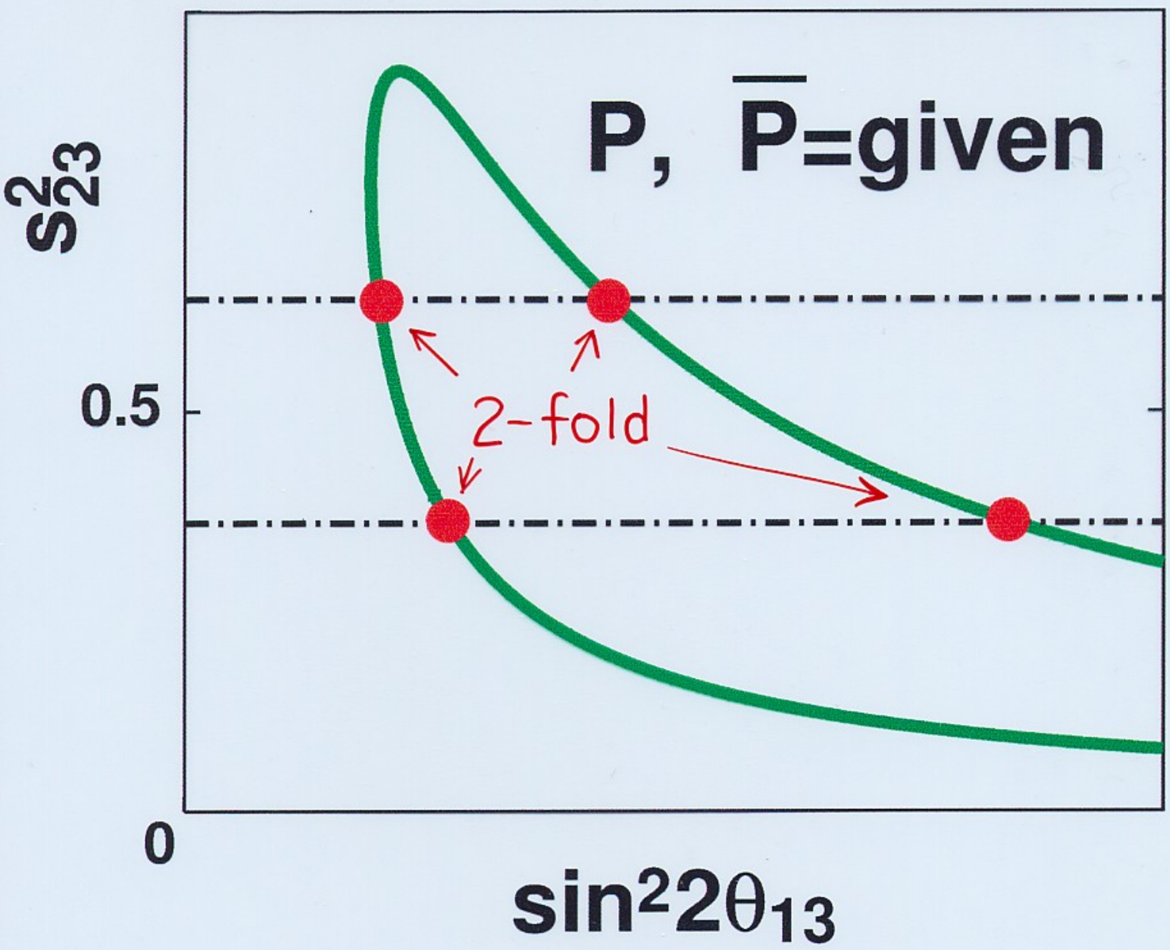
(b) $\theta_{23} \neq \frac{\pi}{4}$, $\Delta m^2_{21}=0$, A=0



$$P = \bar{P} = s_{23}^2 \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$s_{23}^2 = \frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}$$ ← known from $\nu_\mu \to \nu_\mu$

(c) $\theta_{23} \neq \frac{\pi}{4}$, $\Delta m^2_{21} \neq 0$, A=0



$$\frac{1}{\cos^2\Delta}\left(\frac{P+\bar{P}}{2} - x^2\sin^2\Delta - y^2\Delta^2\right)^2 + \frac{1}{\sin^2\Delta}\left(\frac{P-\bar{P}}{2}\right)^2 = (2xy\Delta\sin\Delta)^2$$

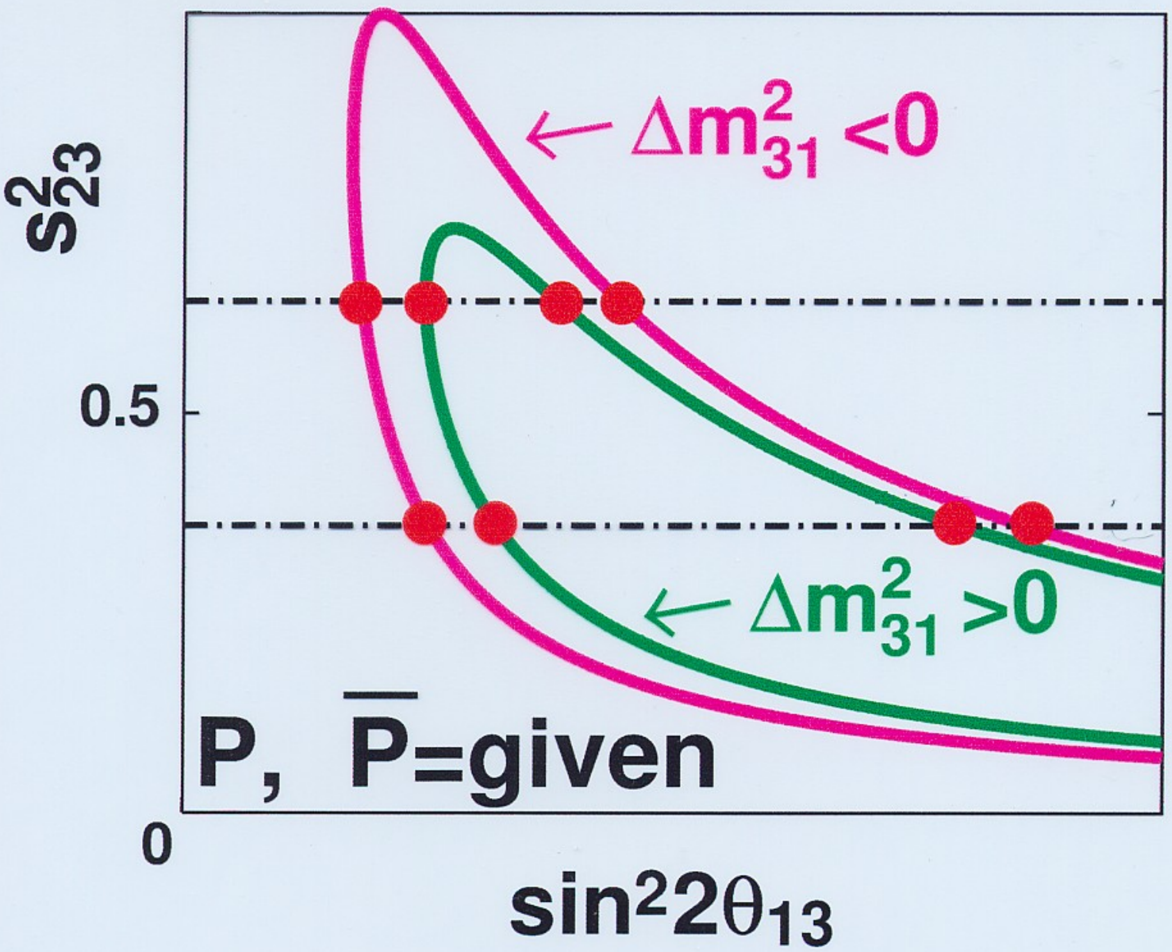
quadratic eq. in x^2

$$\left(\begin{aligned} x &\equiv s_{23}\sin 2\theta_{13} \\ y &\equiv \left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right| c_{23}\sin 2\theta_{12} \\ \Delta &\equiv \frac{\Delta m^2_{31} L}{4E} \end{aligned}\right)$$

$$s_{23} = \sqrt{\frac{1 \pm \sqrt{1-\sin^2 2\theta_{23}}}{2}}$$

(d) $\theta_{23} \neq \frac{\pi}{4}$, $\Delta m^2_{21} \neq 0$, $A \neq 0$

off OM



$$\frac{1}{4\cos^2\Delta}\left(\frac{P - x^2 f^{(\mp)2} - y^2 g^2}{f^{(\mp)}} + \frac{\bar{P} - x^2 f^{(\pm)2} - y^2 g^2}{f^{(\pm)}}\right)^2$$

$$+\frac{1}{4\sin^2\Delta}\left(\frac{P - x^2 f^{(\mp)2} - y^2 g^2}{f^{(\mp)}} - \frac{\bar{P} - x^2 f^{(\pm)2} - y^2 g^2}{f^{(\pm)}}\right)^2 = (2xyg)^2$$

for $\Delta m^2_{31} \begin{Bmatrix} >0 \\ <0 \end{Bmatrix}$

quadratic in x^2

$$x \equiv s_{23} \sin 2\theta_{13}$$

$$y \equiv \left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right| c_{23} \sin 2\theta_{12}$$

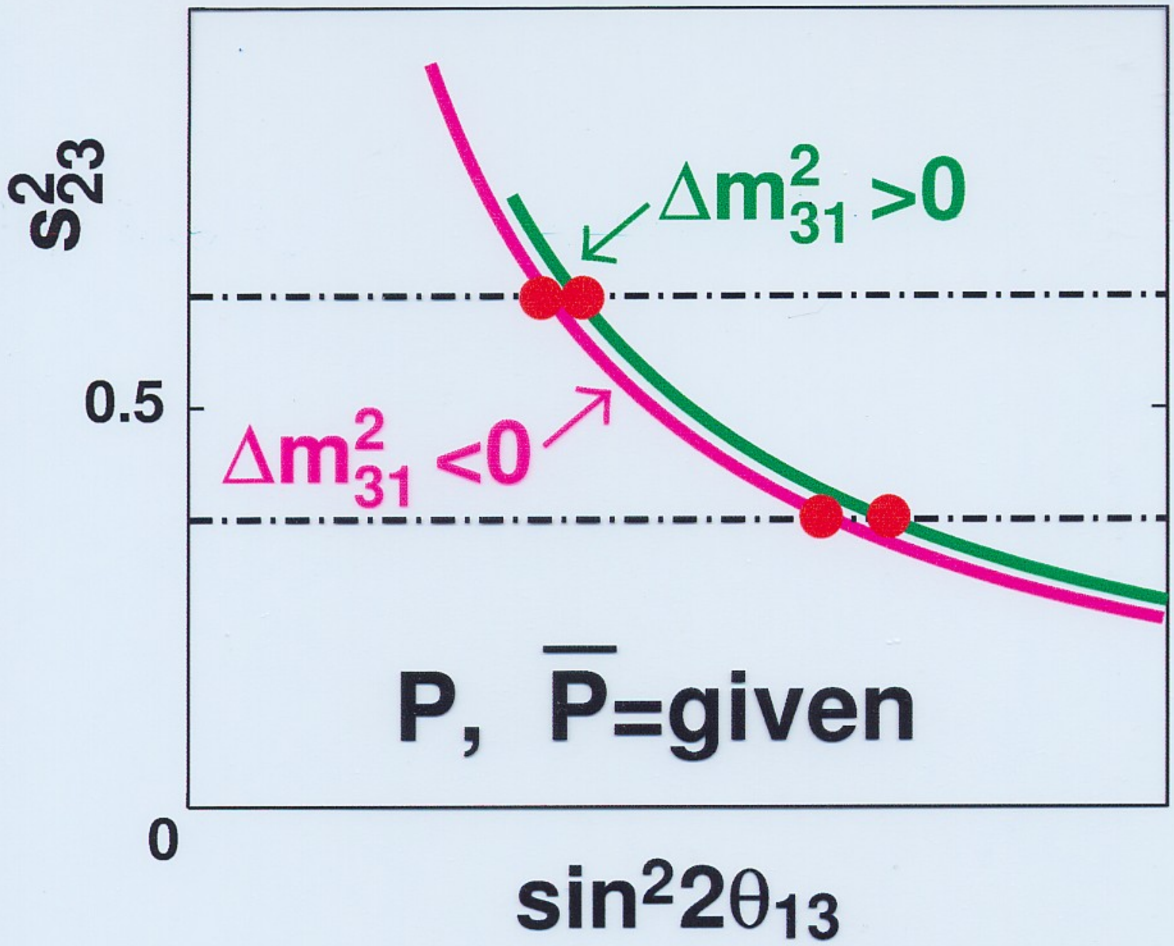
$$\Delta \equiv \frac{\Delta m^2_{31} L}{4E}$$

$$f^{(\pm)} \equiv \frac{\sin(\Delta \pm AL/2)}{1 \mp AL/2\Delta}$$

$$g \equiv \frac{\sin(AL/2)}{AL/2\Delta}$$

$$s_{23} = \sqrt{\frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}}$$

(e) $\theta_{23}\neq\frac{\pi}{4}$, $\Delta m^2_{21}\neq 0$, $A\neq 0$ @OM $\left(\frac{\Delta m^2_{31}L}{4E}=\frac{\pi}{2}\right)$



$$\frac{P-x^2f^{(\mp)2}-y^2g^2}{f^{(\mp)}}=-\frac{\bar{P}-x^2f^{(\pm)2}-y^2g^2}{f^{(\pm)}} \quad \text{for } \Delta m^2_{31}\begin{Bmatrix}>0\\<0\end{Bmatrix}$$

linear in x^2

$$x\equiv s_{23}\sin 2\theta_{13}$$

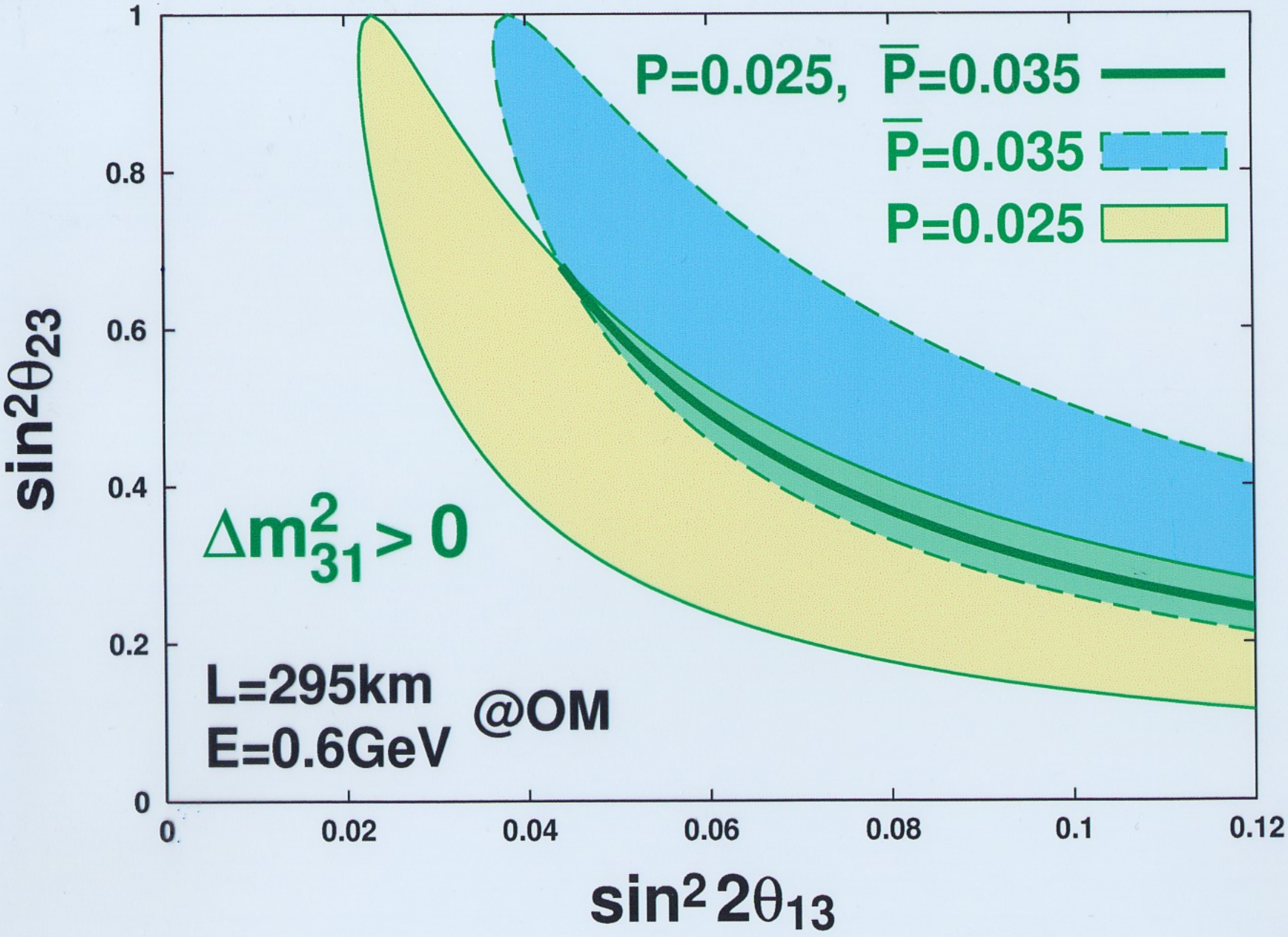
$$y\equiv\left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right|c_{23}\sin 2\theta_{12}$$

$$f^{(\pm)}\equiv\frac{\cos(AL/2)}{1\mp AL/\pi}$$

$$g\equiv\frac{\sin(AL/2)}{AL/\pi}$$

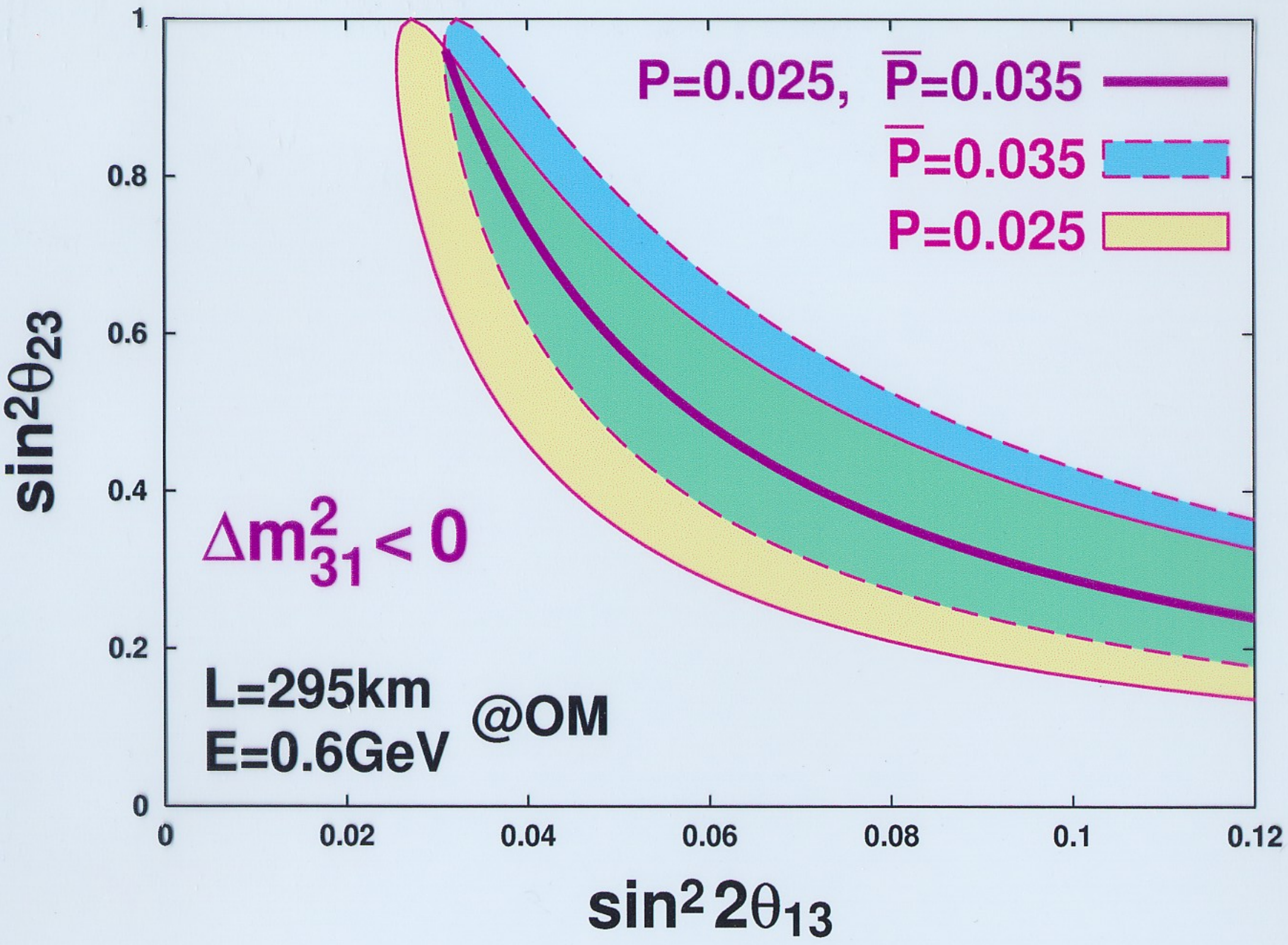
$$s_{23}=\sqrt{\frac{1\pm\sqrt{1-\sin^2 2\theta_{23}}}{2}}$$

L9
20



P=0.025, $\bar{P}$=0.035
$\bar{P}$=0.035
P=0.025
$\Delta m^2_{31} > 0$
L=295km
E=0.6GeV
@OM
$\sin^2\theta_{23}$
$\sin^2 2\theta_{13}$
0
0.2
0.4
0.6
0.8
1
0.02
0.04
0.06
0.08
0.1
0.12

10
21



P=0.025, $\bar{P}$=0.035
$\bar{P}$=0.035
P=0.025
$\Delta m^2_{31} < 0$
L=295km
E=0.6GeV
@OM
$\sin^2\theta_{23}$
$\sin^2 2\theta_{13}$
0
0.2
0.4
0.6
0.8
1
0.02
0.04
0.06
0.08
0.1
0.12

@ Oscillation Maximum $\left(\frac{\Delta m_{31}^2 L}{4E} = \frac{\pi}{2}\right)$

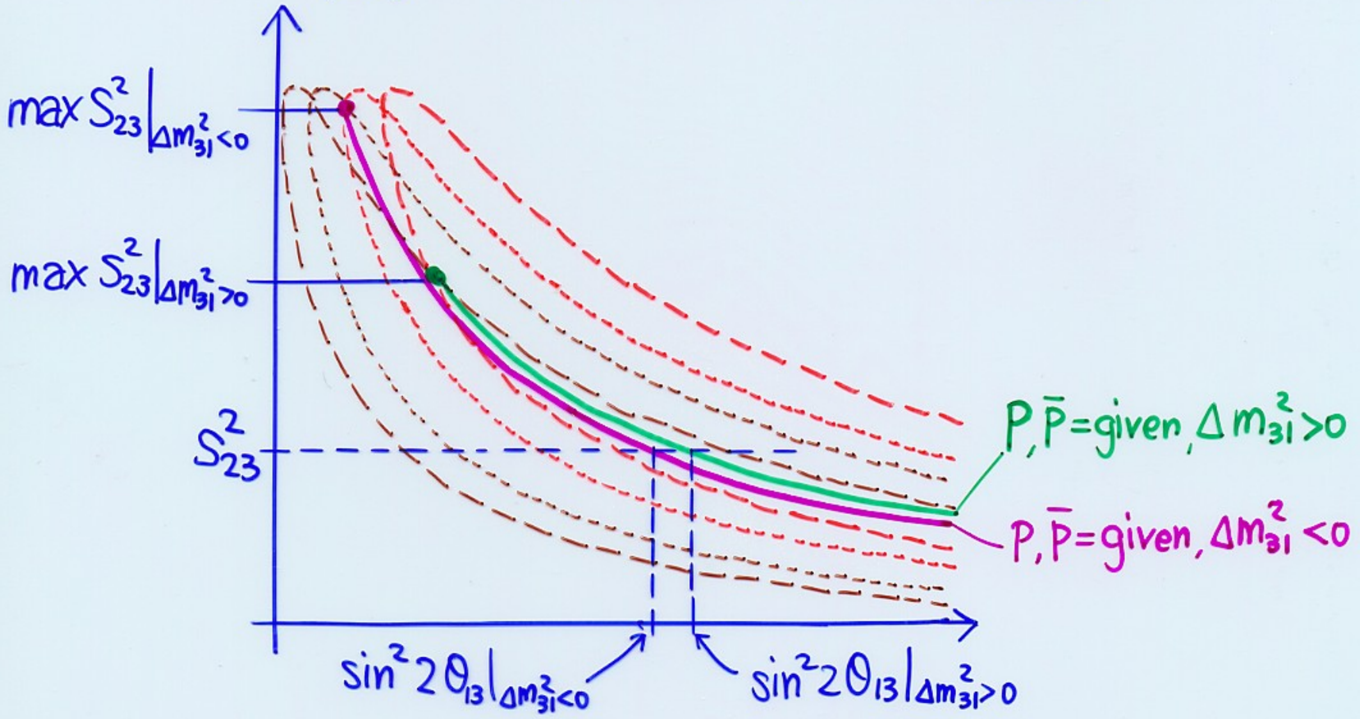
$$P \equiv P(\nu_\mu \to \nu_e) = x^2 f^2 - 2xy\, fg \sin\delta + y^2 g^2$$

$$\bar{P} \equiv P(\bar{\nu}_\mu \to \bar{\nu}_e) = x^2 \bar{f}^2 + 2xy\, \bar{f} g \sin\delta + y^2 g^2$$

where $x \equiv S_{23} \sin 2\theta_{13}$

$y \equiv \epsilon\, C_{23} \sin 2\theta_{12}$, $\epsilon \equiv \left|\frac{\Delta m_{21}^2}{\Delta m_{31}^2}\right|$

$$\left\{\begin{matrix} f \\ \bar{f} \end{matrix}\right\} \equiv \frac{\cos(AL/2)}{1 \mp AL/\pi} , \quad g \equiv \frac{\sin(AL/2)}{AL/\pi} , \quad A \equiv \sqrt{2}\, G_F N_e$$



When P & $\bar{P}$ are given, one can show

- $\sin^2 2\theta_{13}|_{\Delta m_{31}^2>0} - \sin^2 2\theta_{13}|_{\Delta m_{31}^2<0} = \frac{1}{S_{23}^2} \cdot \frac{1}{f\bar{f}} \cdot \frac{f-\bar{f}}{f+\bar{f}} (\bar{P}-P) \simeq \frac{AL}{S_{23}^2 \pi}(\bar{P}-P) \sim 5\times10^{-4}$

  for any θ_{23} if $|AL/2| \ll 1$

- $\max S_{23}^2|_{\Delta m_{31}^2<0} - \max S_{23}^2|_{\Delta m_{31}^2>0} = \frac{1}{\epsilon^2} \cdot \frac{1}{\sin^2 2\theta_{12}} \cdot \frac{1}{g^2} \frac{f-\bar{f}}{f+\bar{f}} (\bar{P}-P) \simeq \frac{1}{\epsilon^2} \frac{(2/\pi)^2}{\sin^2 2\theta_{12}} \cdot \frac{AL}{\pi}(\bar{P}-P)$

  ~ 0.3

if $P=0.025$, $\bar{P}=0.035$, $L=295$ km, $\epsilon = \frac{7\times10^{-5}\text{eV}^2}{2.5\times10^{-3}\text{eV}^2} \simeq \frac{1}{35}$

$A \simeq \frac{1}{1900\text{ km}}$

3-2. Resolution of $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy by LBL ⊕ reactor

cf. Fogli-Lisi PRD54('96)3667
Barenboim-de Gourea ('02)

Our scenario

- JHF $\nu \oplus \bar{\nu}$ @ Oscillation Maximum

⊕

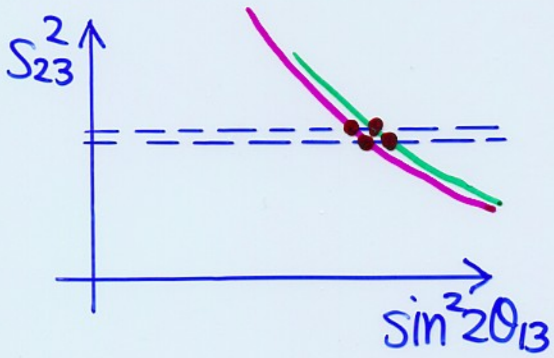
- reactor experiment (@ Kashiwazaki ?)

From $\nu_\mu \leftrightarrow \nu_\mu$ @JHF we will know that θ_{23} satisfies either of the followings:

(A) $|1 - \sin^2 2\theta_{23}| <$ a few $\times 10^{-2}$

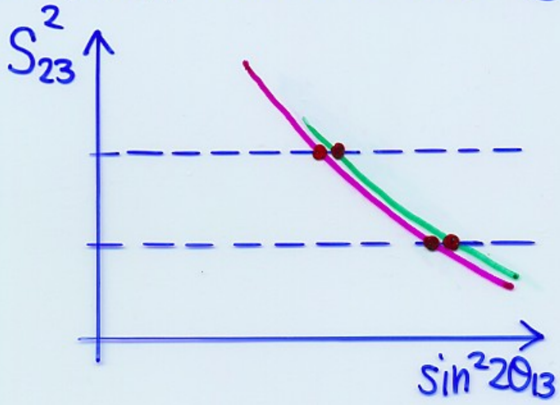
(B) $|1 - \sin^2 2\theta_{23}| \geq$ a few $\times 10^{-2}$

(A) with JHF $\nu \oplus \bar{\nu}$ @OM



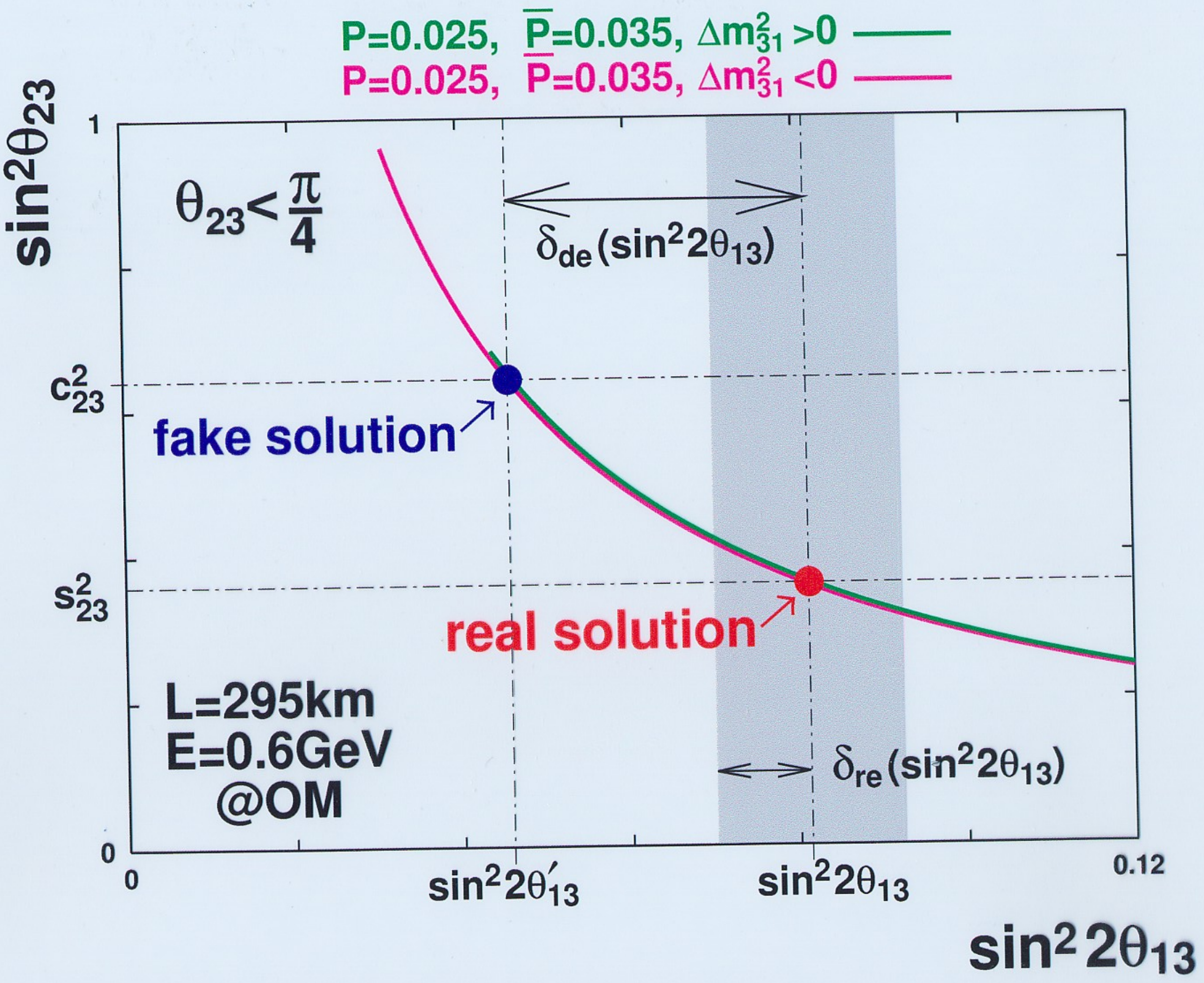
The situation looks like the upper figure.
The precise determination of true $\sin^2 2\theta_{13}$ is difficult, but the values of $\sin^2 2\theta_{13}$ for the 4 solutions are approximately the same.

(B) with JHF $\nu \oplus \bar{\nu}$ @OM

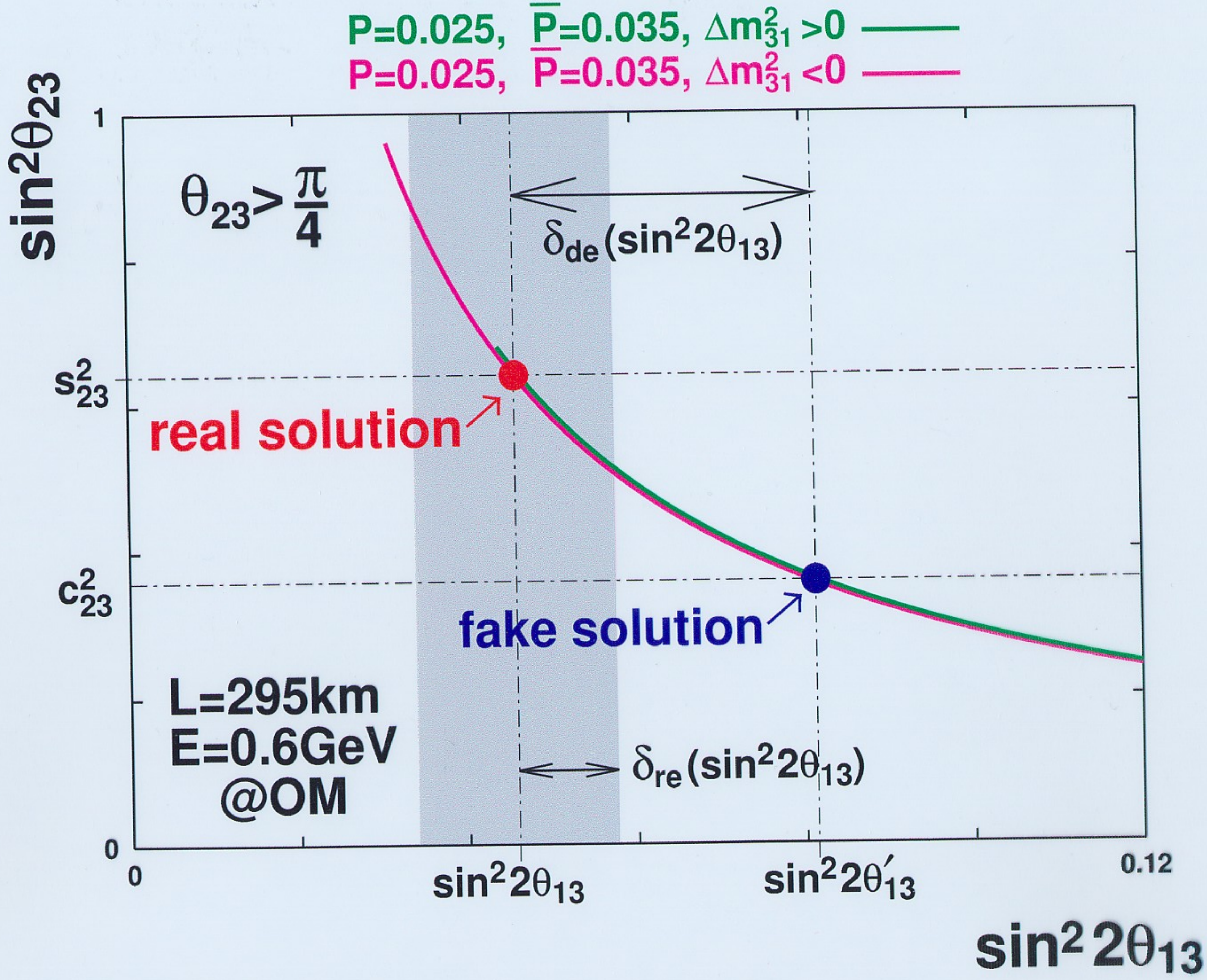


The values of $\sin^2 2\theta_{13}$ for $\theta_{23} < \frac{\pi}{4}$ and $\theta_{23} > \frac{\pi}{4}$ are quite different and it may be possible to determine the true value of $\sin^2 2\theta_{13}$ if the error $\delta_{re}(\sin^2 2\theta_{13})$ of the reactor exp. is smaller than the ambiguity $\delta_{de}(\sin^2 2\theta_{13})$ due to the degeneracy.

31 13/24



P=0.025, $\overline{P}$=0.035, Δm^2_{31} >0
P=0.025, $\overline{P}$=0.035, Δm^2_{31} <0
$\sin^2\theta_{23}$
1
$\theta_{23} < \frac{\pi}{4}$
$\delta_{de}(\sin^2 2\theta_{13})$
c^2_{23}
fake solution
s^2_{23}
real solution
L=295km
E=0.6GeV
@OM
$\delta_{re}(\sin^2 2\theta_{13})$
0
0
$\sin^2 2\theta'_{13}$
$\sin^2 2\theta_{13}$
0.12
$\sin^2 2\theta_{13}$



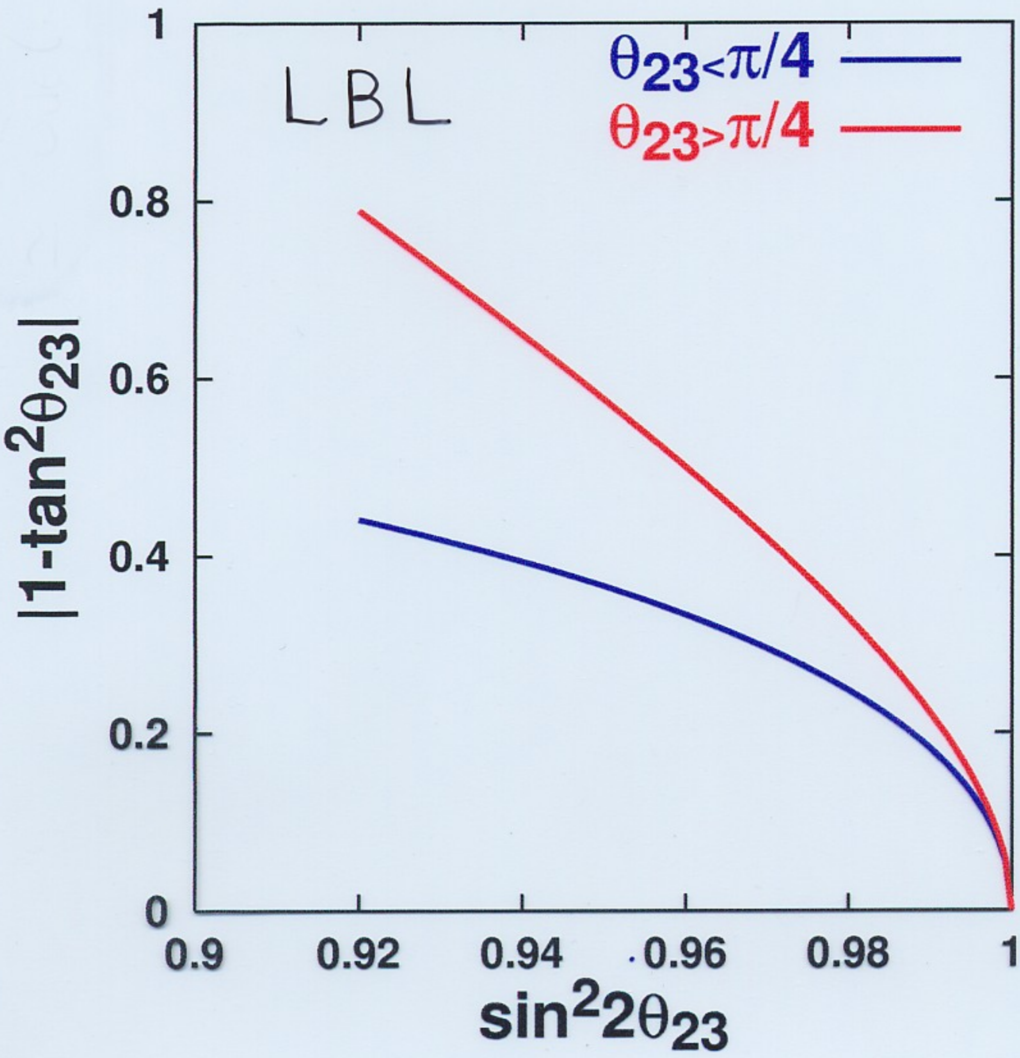
P=0.025, $\overline{P}$=0.035, $\Delta m^2_{31}>0$
P=0.025, $\overline{P}$=0.035, $\Delta m^2_{31}<0$
$\sin^2\theta_{23}$
1
$\theta_{23}>\frac{\pi}{4}$
$\delta_{de}(\sin^2 2\theta_{13})$
s^2_{23}
real solution
c^2_{23}
fake solution
L=295km
E=0.6GeV
@OM
$\delta_{re}(\sin^2 2\theta_{13})$
0
0
$\sin^2 2\theta_{13}$
$\sin^2 2\theta'_{13}$
0.12
$\sin^2 2\theta_{13}$

$$\frac{\delta_{de}(\sin^2 2\theta_{13})}{\sin^2 2\theta_{13}} \equiv \frac{|\sin^2 2\theta_{13}' - \sin^2 2\theta_{13}|}{\sin^2 2\theta_{13}}$$

$$= |1-\tan^2\theta_{23}| \cdot \left\{ 1 + \left(\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right)^2 \frac{\tan^2(AL/2)}{AL/\pi} \left[1-\left(\frac{AL}{\pi}\right)^2\right] \sin^2 2\theta_{12} \right\}$$

$$\simeq |1-\tan^2\theta_{23}|$$

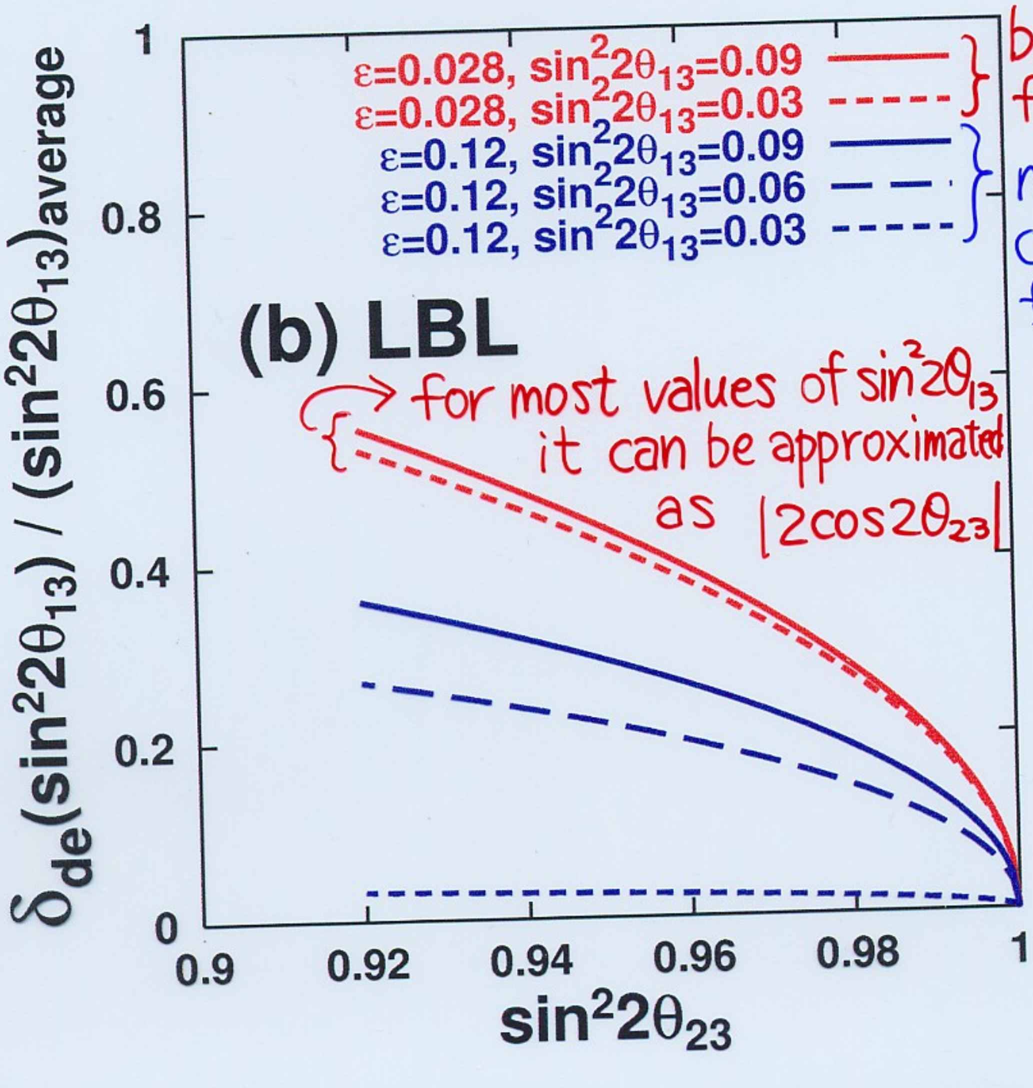
$\delta_{de}(\sin^2 2\theta_{13})$: ambiguity due to the $\theta_{23} \Leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy



ambiguity due to the degeneracy

$$\delta_{de}(\sin^2 2\theta_{13}) \equiv |\sin^2 2\theta_{13} - \sin^2 2\theta_{13}'|$$

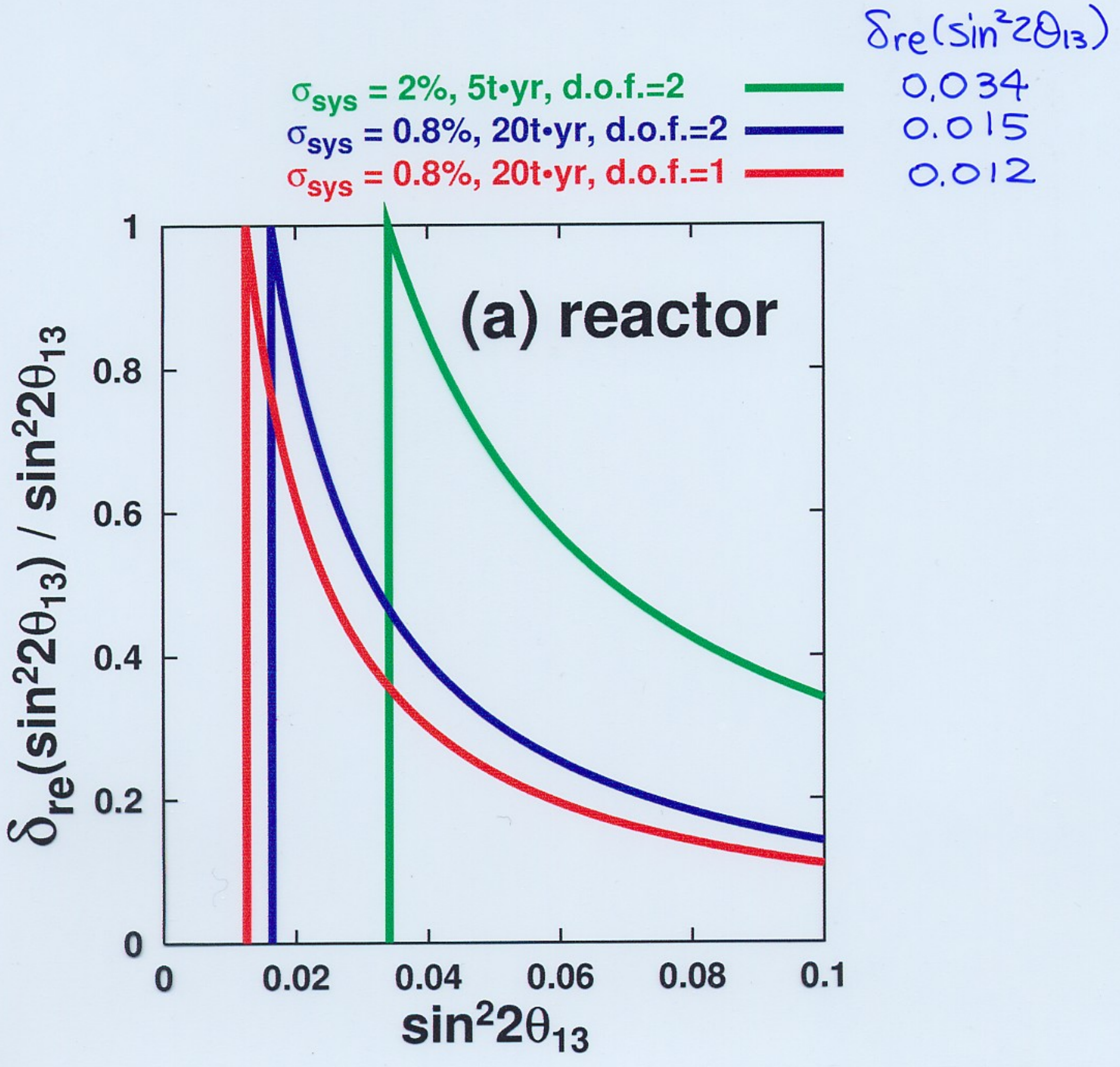
$$(\sin^2 2\theta_{13})_{\text{average}} \equiv \frac{1}{2}(\sin^2 2\theta_{13} + \sin^2 2\theta_{13}')$$



best fit case for $\nu_\odot$ & ν_{atm}

most pessimistic case @ 90% CL for $\nu_\odot$ & ν_{atm}

error in the reactor experiment
$\delta_{re}(\sin^2 2\theta_{13})$



4. Summary

Reactor experiment on θ_{13}

* much cheaper / may be done earlier } than JHF

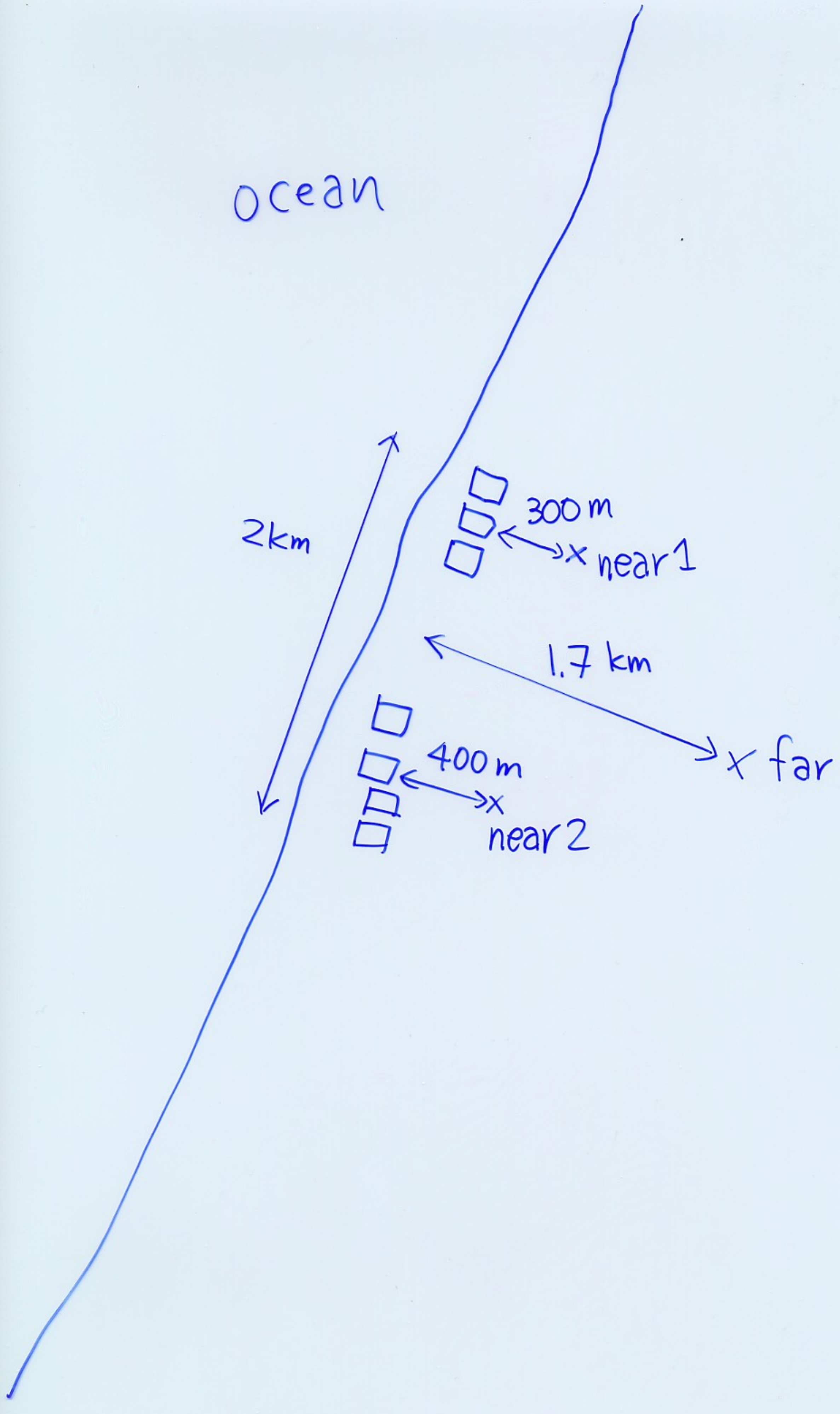
* sensitivity

$\sin^2 2\theta_{13} \gtrsim 0.013$ for $\sigma_{sys} = 0.8\%$, 20 ton·yr (d.o.f.=1)
@ KK-NPP

* free from degeneracy

if $\sin^2 2\theta_{23} \lesssim 0.96$ & $\sin^2 2\theta_{13} \gtrsim 0.06$

then a reactor experiment may be able to determine $\sin^2 2\theta_{13}$ & s_{23}^2 for the true solution.


ocean
2km
300m
near1
1.7 km
far
400m
near2

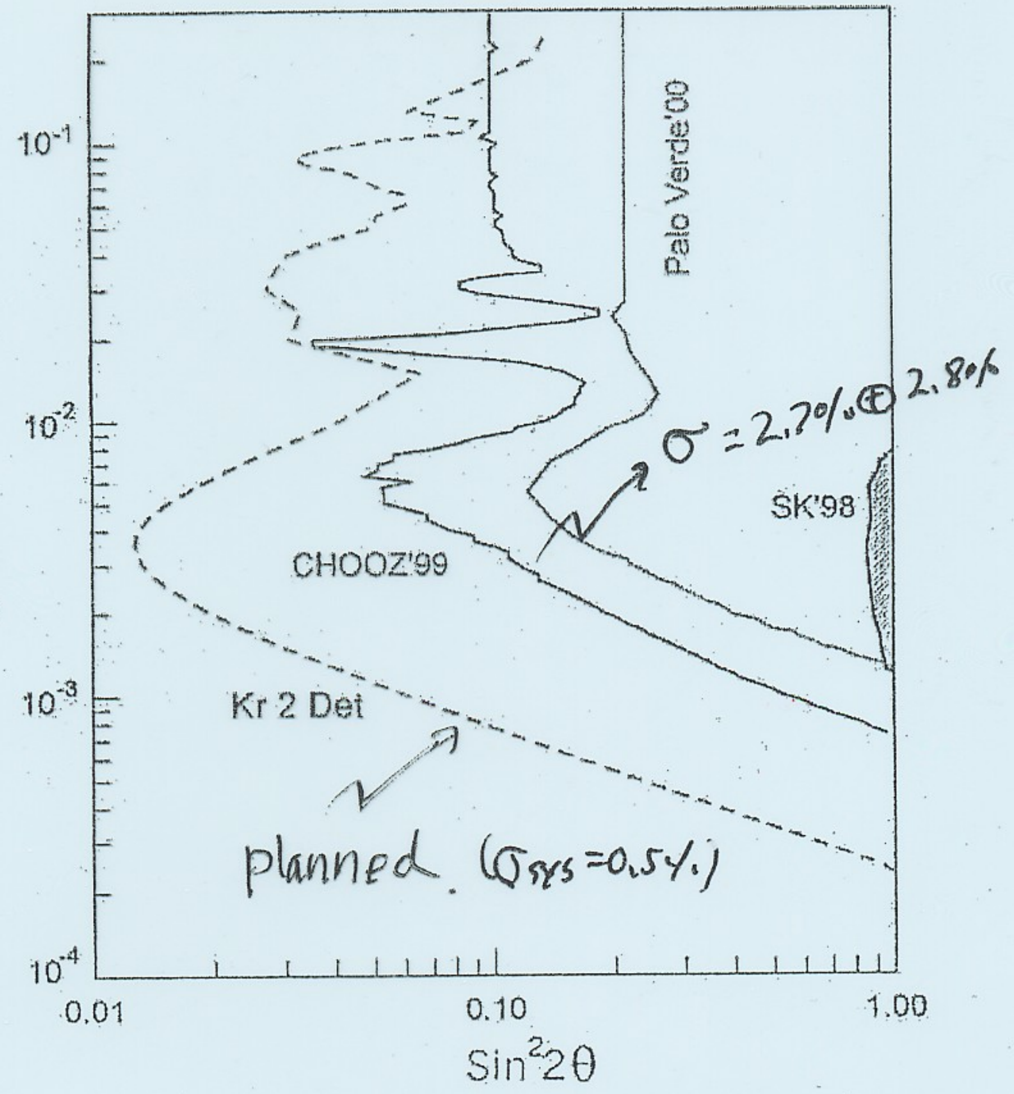
Reactor θ_{13} Status

(2)

Mikael- - 02

OSCILLATION LIMITS

Mikaelian



For reactor $\tilde{\nu}_e$ at 1 km:

$$\text{Sin}^2 2\theta = 4U_{e3}^2(1 - U_{e3}^2) \approx 4U_{e3}^2$$

$\nu_e = U_{e1}\,\nu_1 + U_{e2}\,\nu_2 + U_{e3}\,\nu_3$ $\qquad$ $U_{e3} \equiv \sin\theta_{13}$