ENVIRONMENTAL DECOHERENCE:

FROM NEUTRINO OSCILLATIONS TO QUANTUM COMPUTATION

or

THE CONFESSIONS OF A PARTICLE PHYSICIST

Raymond R. Volkas
The University of Melbourne

ACKNOWLEDGEMENTS:

- ν cosmology: Robert Foot, Mark Thomson, Bruce McKellar, Pasquale di Bari, Paolo Lipari, Maurizio Lusignoli.
- ν cosmology & beyond: Nicole Bell, Ray Sawyer, Yvonne Wong.

THIS TALK:

Bell, Sawyer, RV: "Entanglement and quantal coherence: study of two limiting cases of rapid system-bath interactions"
Phys. Rev. A 65, 052105 (2002)
www.vjquantuminfo.org (May 2002)

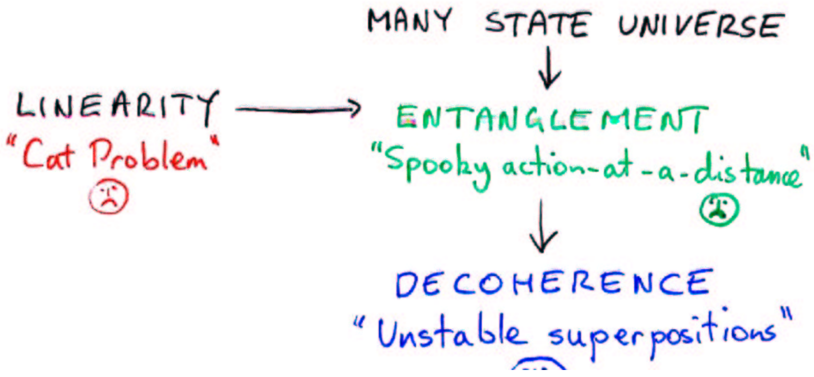
1. Open systems, decoherence, measurements
2. Decoherence and the quantal to classical transition
3. Review of the quantum Zeno effect.
4. Zeno Unbound: environmentally induced synchronisation.
5. Conclusions.

1. Open Systems, Decoherence, Measurements

All systems, apart from the whole universe, are OPEN.

Because of entanglement, this obviously true statement has important implications for quantal dynamics.

Superficially, the world we directly experience seems at odds with the linearity of the Schrödinger Eqn, hence the Schrödinger Cat Paradox.



MANY STATE UNIVERSE

LINEARITY → ENTANGLEMENT

"Cat Problem" ☹

"Spooky action-at-a-distance" ☹

DECOHERENCE

"Unstable superpositions"

reduced density matrix

$$\mathcal{H}_{\text{universe}} = \underset{\substack{\downarrow \\ \{|n\rangle\}}}{\mathcal{H}_{\text{sys}}} \otimes \underset{\substack{\downarrow \\ \{|\alpha\rangle\}}}{\mathcal{H}_{\text{envionment}}}$$

$$|\psi\rangle = \sum_n \sum_\alpha c_{n\alpha} \, |n\rangle \otimes |\alpha\rangle$$

$$\hat{\rho} \equiv |\psi\rangle\langle\psi| = \sum_{n,m,\alpha,\beta} c^*_{m\beta} c_{n\alpha} \big(|n\rangle \otimes |\alpha\rangle\big)\big(\langle m| \otimes \langle\beta|\big)$$

density operator

$$\rho_{n\alpha, m\beta} \equiv c^*_{m\beta} c_{n\alpha}$$

density matrix in $|n\rangle \otimes |\alpha\rangle$ basis

Let $\hat{A}$ be a system observable.

$$\langle \hat{A} \rangle = \langle\psi|\hat{A}|\psi\rangle = \mathrm{Tr}_{S+E}(\hat{\rho}\hat{A})$$

$$= \sum_{n,m,\alpha,\beta} \rho_{n\alpha, m\beta} \langle m|\hat{A}|n\rangle \underbrace{\langle\beta|\alpha\rangle}_{= \delta_{\alpha\beta} \text{ assumed}}$$

$$= \sum_{n,m} \Big(\sum_\alpha \rho_{n\alpha, m\alpha} \Big) A_{mn}$$

$$\equiv \sum_{n,m} \rho^{\text{sys}}_{nm} A_{mn} = \mathrm{Tr}_S(\hat{\rho}^{\text{sys}} \hat{A})$$

$$\boxed{\hat{\rho}^{\text{sys}} \equiv \mathrm{Tr}_{\text{Env}} \, \hat{\rho}}$$

reduced density matrix

encodes state of the system

time evolution

$$i \frac{d}{dt} |\psi\rangle = \hat{H} |\psi\rangle$$ Schrödinger Eqn.

$$\Rightarrow \quad i \dot{\hat{\rho}} = [\hat{H}, \hat{\rho}]$$ von Neumann Eqn.

"unitary evolution"

What is the time-evolution of the reduced density matrix $\hat{\rho}^{\text{sys}}$?

Under some circumstances, it is given by a Master Equation

$$\boxed{\dot{\hat{\rho}}^{\text{sys}} = -i \underset{\substack{\uparrow \\ \text{unitary ev.}}}{\big[\hat{H}^{\text{sys}}, \hat{\rho}^{\text{sys}}\big]} + \underset{\substack{\uparrow \\ \text{non-unitary ev.}}}{L^{\text{sys-env}}\big[\hat{\rho}^{\text{sys}}\big]}}$$

"Every time dependence of a density matrix ρ, for example according to a von Neumann equation, induces a time dependence of its coarse-grained projection* — similar to the motion of a shadow that merely reflects the motion of its source." — H. D. Zeh

"Shadows on the walls of Plato's cave."

---

* reduced density matrix

decoherence

Consider 2 state sys. $|\uparrow\rangle$ & $|\downarrow\rangle$

and a 2 state "env" $|1\rangle$ & $|2\rangle$.

The pure state $|a\rangle = c_\uparrow |\uparrow\rangle + c_\downarrow |\downarrow\rangle$

$$\Rightarrow \quad \hat{\rho} = \begin{pmatrix} |c_\uparrow|^2 & c_\downarrow^* c_\uparrow \\ c_\uparrow^* c_\downarrow & |c_\downarrow|^2 \end{pmatrix}$$

probabilities

coherences

$$\boxed{\text{Decoherence} \equiv |\hat{\rho}_{\text{off-diag}}| < |c_\downarrow^* c_\uparrow|}$$

Extreme case: $\hat{\rho} = \begin{pmatrix} P_\uparrow & 0 \\ 0 & P_\downarrow \end{pmatrix}$

"improper ensemble"

statistical mixture

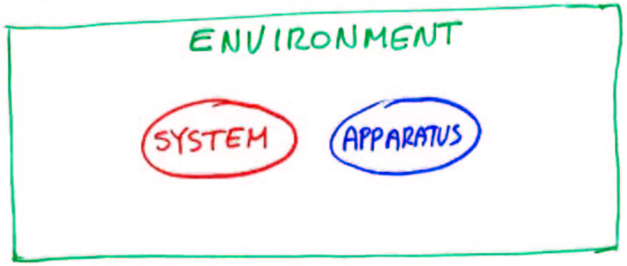
sys-env entanglement ⇒ decoherence

e.g. $[c_\uparrow|\uparrow\rangle + c_\downarrow|\downarrow\rangle] \otimes |1\rangle \longrightarrow c_\uparrow|\uparrow\rangle \otimes |1\rangle + c_\downarrow|\downarrow\rangle \otimes |2\rangle$

$$\hat{\rho}^{sys} = \begin{pmatrix} |c_\uparrow|^2 & c_\downarrow^* c_\uparrow \\ c_\uparrow^* c_\downarrow & |c_\downarrow|^2 \end{pmatrix} \longrightarrow \begin{pmatrix} |c_\uparrow|^2 & 0 \\ 0 & |c_\downarrow|^2 \end{pmatrix}$$

"quantal correlations diffused through the environment"

measurement



initial state: $(c_\uparrow|\uparrow\rangle + c_\downarrow|\downarrow\rangle)\otimes|a_\uparrow\rangle\otimes|e_\uparrow\rangle$

$$\downarrow$$

pre-measurement: $(c_\uparrow|\uparrow\rangle\otimes|a_\uparrow\rangle + c_\downarrow|\downarrow\rangle\otimes|a_\downarrow\rangle)\otimes|e_\uparrow\rangle$

$$\downarrow$$

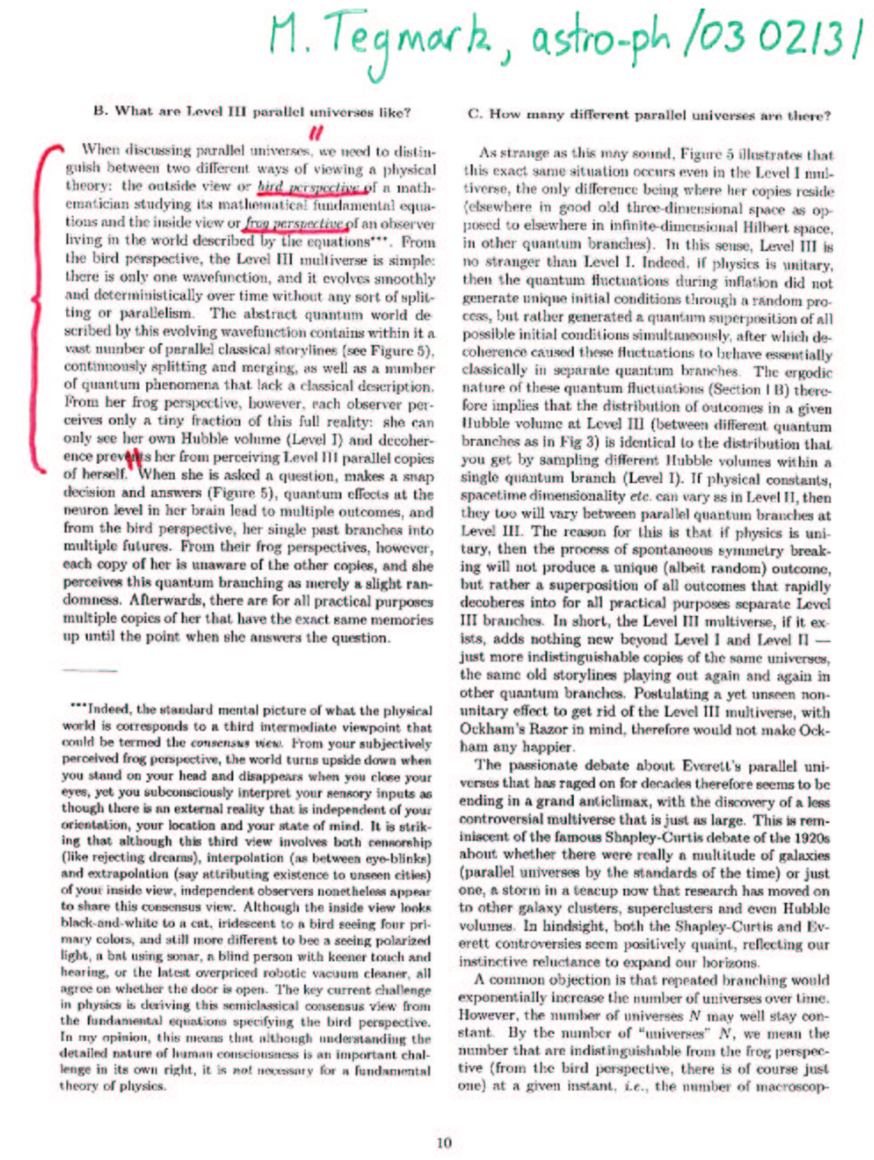
measurement : $c_\uparrow|\uparrow\rangle\otimes|a_\uparrow\rangle\otimes|e_\uparrow\rangle + c_\downarrow|\downarrow\rangle\otimes|a_\downarrow\rangle\otimes|e_\downarrow\rangle$

$$\Downarrow$$

$$\hat{\rho}^{sys+app} = \mathrm{Tr}_{Env}\,\hat{\rho} = \langle e_\uparrow|\hat{\rho}|e_\uparrow\rangle + \langle e_\downarrow|\hat{\rho}|e_\downarrow\rangle$$

$$= |c_\uparrow|^2\left(|\uparrow\rangle\otimes|a_\uparrow\rangle\right)\left(\langle\uparrow|\otimes\langle a_\uparrow|\right) + |c_\downarrow|^2\left(|\downarrow\rangle\otimes|a_\downarrow\rangle\right)\left(\langle\downarrow|\otimes\langle a_\downarrow|\right)$$

outcomes not superposed.



M. Tegmark, astro-ph/0302131

B. What are Level III parallel universes like?

When discussing parallel universes, we need to distinguish between two different ways of viewing a physical theory: the outside view or *bird perspective* of a mathematician studying its mathematical fundamental equations and the inside view or *frog perspective* of an observer living in the world described by the equations***. From the bird perspective, the Level III multiverse is simple: there is only one wavefunction, and it evolves smoothly and deterministically over time without any sort of splitting or parallelism. The abstract quantum world described by this evolving wavefunction contains within it a vast number of parallel classical storylines (see Figure 5), continuously splitting and merging, as well as a number of quantum phenomena that lack a classical description. From her frog perspective, however, each observer perceives only a tiny fraction of this full reality: she can only see her own Hubble volume (Level I) and decoherence prevents her from perceiving Level III parallel copies of herself. When she is asked a question, makes a snap decision and answers (Figure 5), quantum effects at the neuron level in her brain lead to multiple outcomes, and from the bird perspective, her single past branches into multiple futures. From their frog perspectives, however, each copy of her is unaware of the other copies, and she perceives this quantum branching as merely a slight randomness. Afterwards, there are for all practical purposes multiple copies of her that have the exact same memories up until the point when she answers the question.

C. How many different parallel universes are there?

As strange as this may sound, Figure 5 illustrates that this exact same situation occurs even in the Level I multiverse, the only difference being where her copies reside (elsewhere in good old three-dimensional space as opposed to elsewhere in infinite-dimensional Hilbert space, in other quantum branches). In this sense, Level III is no stranger than Level I. Indeed, if physics is unitary, then the quantum fluctuations during inflation did not generate unique initial conditions through a random process, but rather generated a quantum superposition of all possible initial conditions simultaneously, after which decoherence caused these fluctuations to behave essentially classically in separate quantum branches. The ergodic nature of these quantum fluctuations (Section I B) therefore implies that the distribution of outcomes in a given Hubble volume at Level III (between different quantum branches as in Fig 3) is identical to the distribution that you get by sampling different Hubble volumes within a single quantum branch (Level I). If physical constants, spacetime dimensionality *etc.* can vary as in Level II, then they too will vary between parallel quantum branches at Level III. The reason for this is that if physics is unitary, then the process of spontaneous symmetry breaking will not produce a unique (albeit random) outcome, but rather a superposition of all outcomes that rapidly decoheres into for all practical purposes separate Level III branches. In short, the Level III multiverse, if it exists, adds nothing new beyond Level I and Level II — just more indistinguishable copies of the same universes, the same old storylines playing out again and again in other quantum branches. Postulating a yet unseen non-unitary effect to get rid of the Level III multiverse, with Ockham's Razor in mind, therefore would not make Ockham any happier.

The passionate debate about Everett's parallel universes that has raged on for decades therefore seems to be ending in a grand anticlimax, with the discovery of a less controversial multiverse that is just as large. This is reminiscent of the famous Shapley-Curtis debate of the 1920s about whether there were really a multitude of galaxies (parallel universes by the standards of the time) or just one, a storm in a teacup now that research has moved on to other galaxy clusters, superclusters and even Hubble volumes. In hindsight, both the Shapley-Curtis and Everett controversies seem positively quaint, reflecting our instinctive reluctance to expand our horizons.

A common objection is that repeated branching would exponentially increase the number of universes over time. However, the number of universes N may well stay constant. By the number of "universes" N, we mean the number that are indistinguishable from the frog perspective (from the bird perspective, there is of course just one) at a given instant, *i.e.*, the number of macroscop-

---

***Indeed, the standard mental picture of what the physical world is corresponds to a third intermediate viewpoint that could be termed the *consensus view*. From your subjectively perceived frog perspective, the world turns upside down when you stand on your head and disappears when you close your eyes, yet you subconsciously interpret your sensory inputs as though there is an external reality that is independent of your orientation, your location and your state of mind. It is striking that although this third view involves both censorship (like rejecting dreams), interpolation (as between eye-blinks) and extrapolation (say attributing existence to unseen cities) of your inside view, independent observers nonetheless appear to share this consensus view. Although the inside view looks black-and-white to a cat, iridescent to a bird seeing four primary colors, and still more different to bee a seeing polarized light, a bat using sonar, a blind person with keener touch and hearing, or the latest overpriced robotic vacuum cleaner, all agree on whether the door is open. The key current challenge in physics is deriving this semiclassical consensus view from the fundamental equations specifying the bird perspective. In my opinion, this means that although understanding the detailed nature of human consciousness is an important challenge in its own right, it is *not* necessary for a fundamental theory of physics.

M. Tegmark, astro-ph/0302131

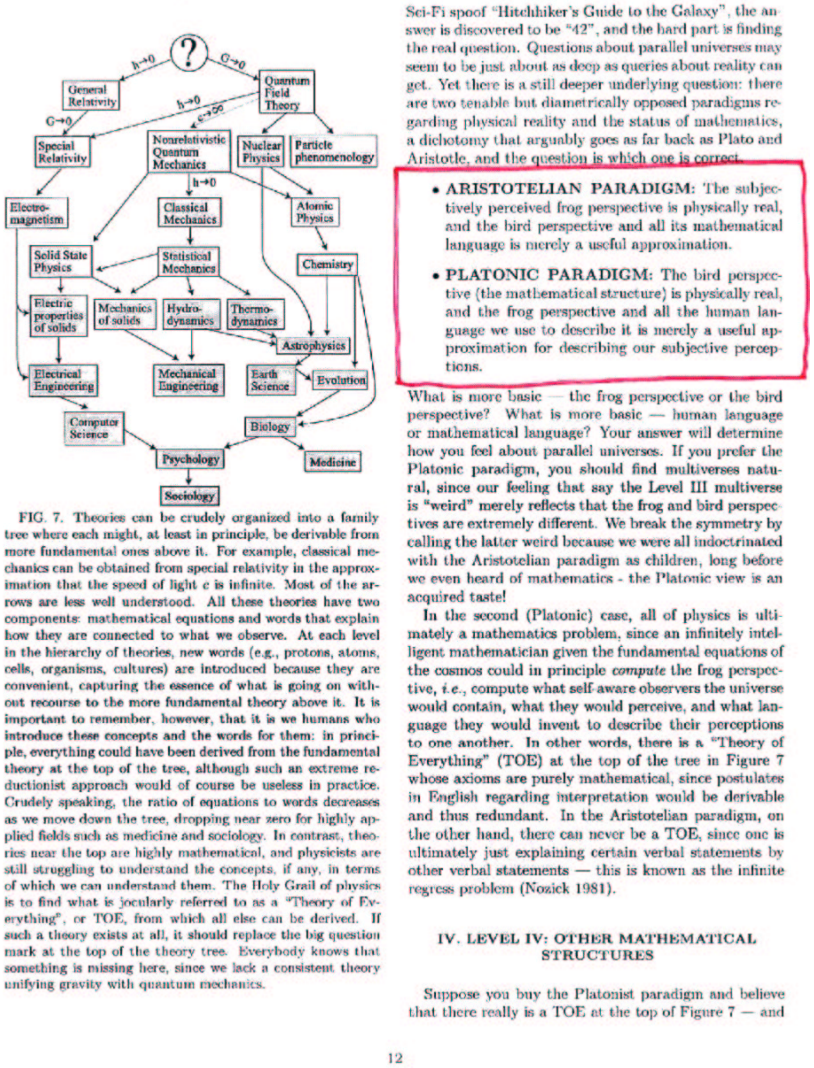


FIG. 7. Theories can be crudely organized into a family tree where each might, at least in principle, be derivable from more fundamental ones above it. For example, classical mechanics can be obtained from special relativity in the approximation that the speed of light c is infinite. Most of the arrows are less well understood. All these theories have two components: mathematical equations and words that explain how they are connected to what we observe. At each level in the hierarchy of theories, new words (e.g., protons, atoms, cells, organisms, cultures) are introduced because they are convenient, capturing the essence of what is going on without recourse to the more fundamental theory above it. It is important to remember, however, that it is we humans who introduce these concepts and the words for them: in principle, everything could have been derived from the fundamental theory at the top of the tree, although such an extreme reductionist approach would of course be useless in practice. Crudely speaking, the ratio of equations to words decreases as we move down the tree, dropping near zero for highly applied fields such as medicine and sociology. In contrast, theories near the top are highly mathematical, and physicists are still struggling to understand the concepts, if any, in terms of which we can understand them. The Holy Grail of physics is to find what is jocularly referred to as a "Theory of Everything", or TOE, from which all else can be derived. If such a theory exists at all, it should replace the big question mark at the top of the theory tree. Everybody knows that something is missing here, since we lack a consistent theory unifying gravity with quantum mechanics.

Sci-Fi spoof "Hitchhiker's Guide to the Galaxy", the answer is discovered to be "42", and the hard part is finding the real question. Questions about parallel universes may seem to be just about as deep as queries about reality can get. Yet there is a still deeper underlying question: there are two tenable but diametrically opposed paradigms regarding physical reality and the status of mathematics, a dichotomy that arguably goes as far back as Plato and Aristotle, and the question is which one is correct.

- **ARISTOTELIAN PARADIGM:** The subjectively perceived frog perspective is physically real, and the bird perspective and all its mathematical language is merely a useful approximation.
- **PLATONIC PARADIGM:** The bird perspective (the mathematical structure) is physically real, and the frog perspective and all the human language we use to describe it is merely a useful approximation for describing our subjective perceptions.

What is more basic — the frog perspective or the bird perspective? What is more basic — human language or mathematical language? Your answer will determine how you feel about parallel universes. If you prefer the Platonic paradigm, you should find multiverses natural, since our feeling that say the Level III multiverse is "weird" merely reflects that the frog and bird perspectives are extremely different. We break the symmetry by calling the latter weird because we were all indoctrinated with the Aristotelian paradigm as children, long before we even heard of mathematics - the Platonic view is an acquired taste!

In the second (Platonic) case, all of physics is ultimately a mathematics problem, since an infinitely intelligent mathematician given the fundamental equations of the cosmos could in principle *compute* the frog perspective, *i.e.*, compute what self-aware observers the universe would contain, what they would perceive, and what language they would invent to describe their perceptions to one another. In other words, there is a "Theory of Everything" (TOE) at the top of the tree in Figure 7 whose axioms are purely mathematical, since postulates in English regarding interpretation would be derivable and thus redundant. In the Aristotelian paradigm, on the other hand, there can never be a TOE, since one is ultimately just explaining certain verbal statements by other verbal statements — this is known as the infinite regress problem (Nozick 1981).

IV. LEVEL IV: OTHER MATHEMATICAL STRUCTURES

Suppose you buy the Platonist paradigm and believe that there really is a TOE at the top of Figure 7 — and

12

8.

2. Decoherence and the quantal to classical transition.

$$\hat{H} = \hat{H}^{sys} + \hat{H}^{env} + \hat{H}^{s-e}$$

May affect $\hat{H}^{sys}$
e.g. refractive index

non-unitary
term in the
Master Eqn.

Tractable model:

see Zurek, Physics Today
Oct. 91
Joos & Zeh 1985
Caldeira, Leggett 1983

system = particle moving in x-direction

env. = scalar field $\varphi(q,t)$

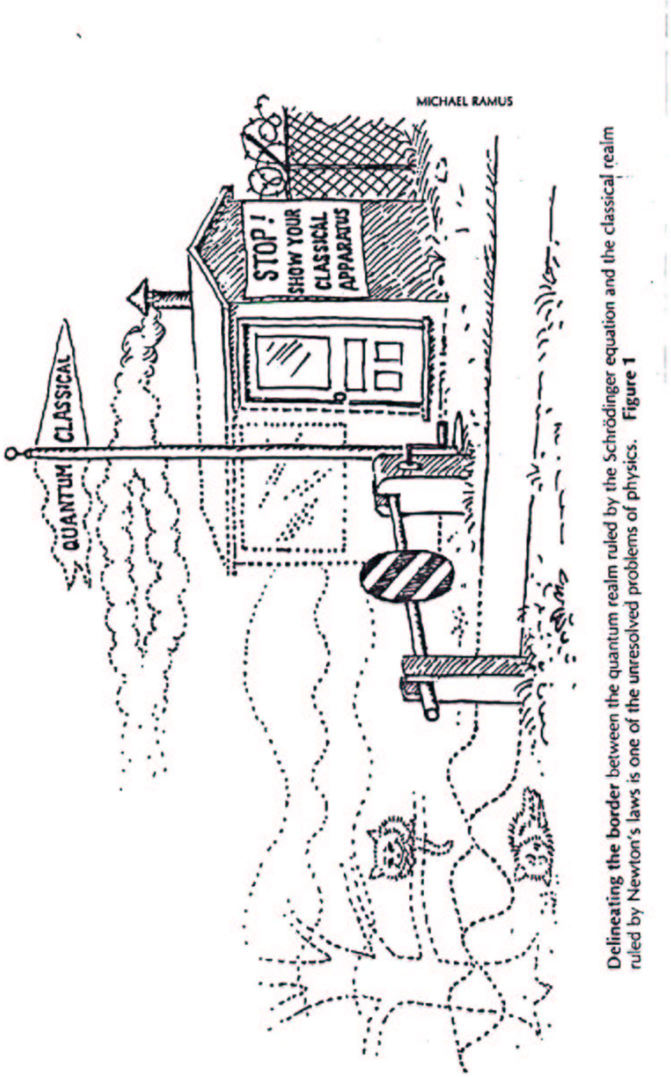
perp. to x

$$\hat{H}^{s-e} = \varepsilon x \dot{\varphi} \Rightarrow$$

$$\dot{\hat{\rho}}^{s} = -\frac{i}{\hbar}[\hat{H}^{s}, \hat{\rho}^{s}] - \gamma(x-x')\left(\frac{\partial\rho}{\partial x} - \frac{\partial\rho}{\partial x'}\right) - \frac{2m\gamma k_B T}{\hbar^2}(x-x')^2\rho$$

relaxation
$\gamma = \frac{\varepsilon}{4m}$

decoherence



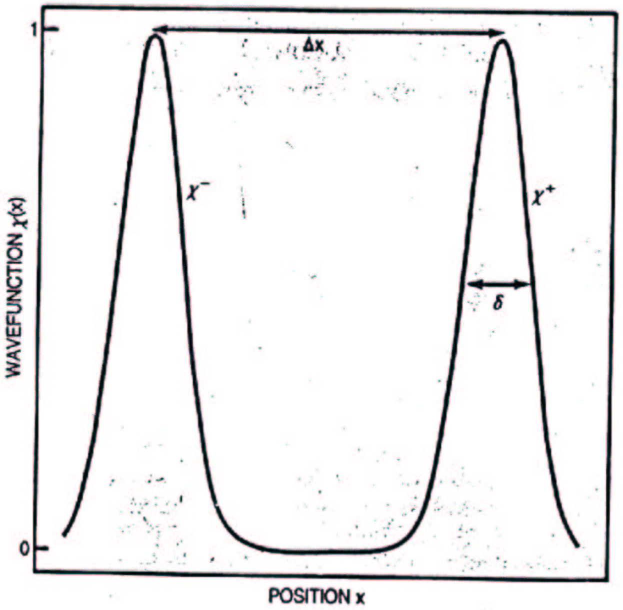
Delineating the border between the quantum realm ruled by the Schrödinger equation and the classical realm ruled by Newton's laws is one of the unresolved problems of physics. **Figure 1**

Zureh, Physics Today, Oct. 91

9.

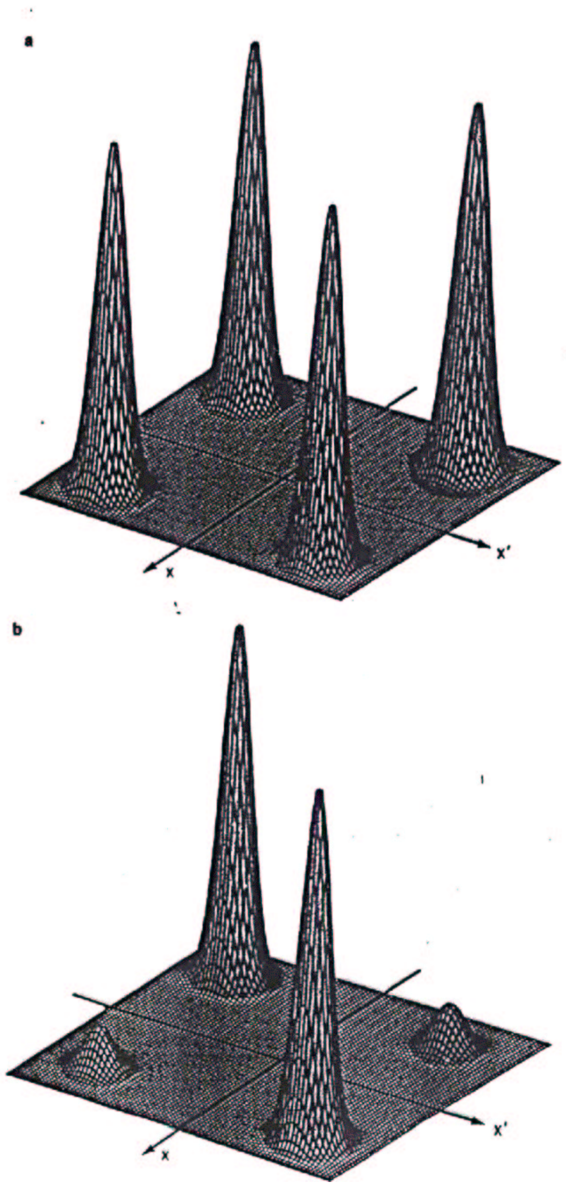
Zurek, Physics Today, Oct. 91

10.



Coherent superposition of two Gaussian wavepackets. Such a wavefunction may describe a particle inside a Stern–Gerlach apparatus (figure 2) or may develop in the course of a double-slit experiment. The phase between the two components has been chosen to be zero. **Figure 3**

Zurek, Physics Today, Oct. 91



$$\frac{\tau_{decoh}}{\tau_{relax}} \sim \frac{\hbar^2}{2mk_BT(\Delta x)^2} = \left(\frac{\lambda_T}{\Delta x}\right)^2$$

separation of peaks

thermal de-Broglie wavelength $\frac{\hbar}{\sqrt{2mk_BT}}$

e.g. $T = 300\,K$, $m = 1g$, $\Delta x = 1cm$

$$\frac{\tau_{decoh}}{\tau_{relax}} \sim 10^{-40}$$

Decoherence is typically very quick.

Cat states are VERY unstable.

3 Review of the quantum Zeno effect

$$\hat{H} = \hat{H}^{sys} + \hat{H}^{env} + \hat{H}^{s-e}$$

suppose that $\hat{H}^{s-e} = \zeta V$

operates on $\mathcal{H}_{sys}$

operates on $\mathcal{H}_{env}$

Let $\mathcal{H}_{sys}$ be 2-d e.g. $|\uparrow\rangle$ & $|\downarrow\rangle$
$|L\rangle$ & $|R\rangle$
$|\nu_\alpha\rangle$ & $|\nu_\beta\rangle$

ζ is a 2×2 Hermitian matrix.

We will consider diagonal matrices only:

$$\boxed{\zeta = 1 + b\,\sigma_3 = \begin{pmatrix} 1+b & 0 \\ 0 & 1-b \end{pmatrix}}$$

The parameter "b" specifies the strength of the coupling of the env. to each of the system basis states.

Two extreme cases: $b = 1$ or -1

$$\zeta = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$$

env. couples to one basis state only

e.g active - sterile ν system in a collisional environment (early universe, SN)
(∞ mass scatterers only here)

Stodolsky
Enqvist & Kainulainen
Dolgov

and $b = 0 \Rightarrow \zeta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

env. couples equally to the two basis states.

Bell, Sawyer, RV

e.g. $\nu_\mu + \nu_\tau$ in nuclear matter (protoneutron star)

The master equation is

$$\boxed{\frac{\partial \hat{\rho}^s}{\partial t} = -i[\hat{H}^s, \hat{\rho}^s] - \frac{\Gamma}{2}[\zeta, [\zeta, \hat{\rho}^s]]}$$

where Γ is a sys-env. interaction rate that can be computed from V.

$$-\frac{\Gamma}{2}\left[\zeta, \left[\zeta, \hat{\rho}^s\right]\right] = -2b^2\Gamma \begin{pmatrix} 0 & \rho_{12} \\ \rho_{21} & 0 \end{pmatrix}$$

So, for any $b \neq 0$ this term tries to exponentially damp ρ_{12} & ρ_{21}

$\longrightarrow$ decoherence.

But, $\dot{\rho}_{11} = -i\left(H_{12}\rho_{21} - H_{21}\rho_{12}\right)$

$\dot{\rho}_{22} = -i\left(H_{21}\rho_{12} - H_{12}\rho_{21}\right)$

so $\dot{\rho}_{11} = \dot{\rho}_{22} = 0$ if $\rho_{12} = \rho_{21} = 0$.

$\Gamma = \infty \Rightarrow \rho_{12} = \rho_{21} = 0$ through decoherence.

The probabilities ρ_{11} & ρ_{22} are

FROZEN

in this limit.

If $\rho_{11} = 1$, $\rho_{22} = 0$ or vice-versa, then all evolution is frozen (quantum Zeno effect)

Notes:

(i) apparent similarity to measurement: $\Gamma \to \infty$ like "constant monitoring" of system by the environment.

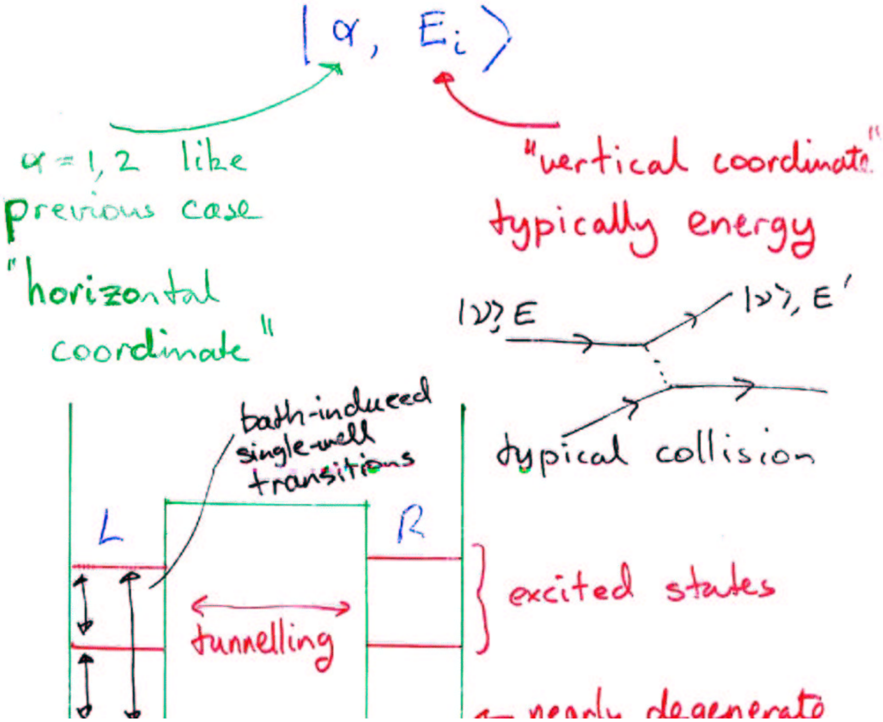
For $b \neq 0$, the environment can tell the difference between the basis states.

(ii) For $b = 0$ ie. $\zeta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, the non-unitary-evolution term is zero.

Env. has no effect on the system, & the system evolution is unitary (decoherence-free)

4. Zeno Unbound : environmentally induced synchronisation.

$\hat{\zeta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ gives non-trivial effects in a well-motivated generalisation of the previous two-state system case.



Master eqn. is now:

$$\boxed{\begin{aligned} \frac{\partial}{\partial t}\tilde{\rho}(E_i, t) &= -i\left[\hat{H}^s(E_i), \hat{\rho}^s(E_i, t)\right] \\ &+ \sum_j \hat{\zeta}\hat{\rho}^s(E_j, t)\hat{\zeta}\,\Gamma(E_j, E_i) \\ &- \frac{1}{2}\left[\hat{\zeta}^2\hat{\rho}^s(E_i, t) + \hat{\rho}^s(E_i, t)\hat{\zeta}^2\right]\sum_j \Gamma(E_j, E_i) \end{aligned}}$$

$$\Gamma(E_j, E_i) = e^{\frac{E_j - E_i}{T}}\,\Gamma(E_i, E_j)$$

T = bath temp.

bath-induced rate for $E_j \to E_i$ single-well transition.

1ST term = unitary evolution

2ND term = scattering from other vertical states into E_i

3RD term = scattering out of E_i

Notes:

(i) Reduces to $-\frac{\Gamma}{2}[\hat{\eta},[\hat{\eta},\hat{\rho}^s]]$ in the absence of vertical structure.

(ii) $\hat{\eta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ case is now

NON-TRIVIAL

(iii) For times $t \gg (\Delta E)^{-1}$, superpos$^{\underline{n}}$

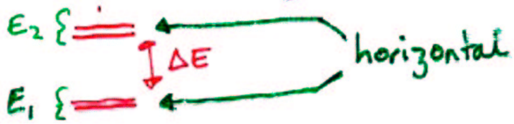
$\uparrow$ single-well spacing

in vertical space quickly decohered.

$$\hat{H}^s = \sum_{i,\alpha,\beta} \left[E_i \delta_{\alpha\beta} + \lambda_{\alpha\beta}(E_i) \right] |E_i,\alpha\rangle\langle E_i,\beta|$$

$$\lambda_{\alpha\beta}(E_i) = \underset{\substack{\uparrow \\ L \leftrightarrow R \text{ tunnelling rate}}}{g(E_i)} [\sigma_1]_{\alpha\beta}$$

Take $\lambda \ll E$ $\longrightarrow$



(iv) For ν case, generalisations to Fermi statistics (Raffelt, Sigl & Stodolsky) and ν-$\bar{\nu}$ annihilation (McKellar & Thomson) have been derived (though not USED very much!).

Active-active ν oscillations in a collisional medium are very complicated; collisions not properly analysed yet.

See recent papers:

Dolgov et al, NPB (in press), hep-ph/0201287

Abazajian, Beacom, Bell, PRD (in press), astro-ph/0203442

Y. Wong, Phys. Rev. D (in press), hep-ph/0203180

BATH - INDUCED SYNCHRONISATION

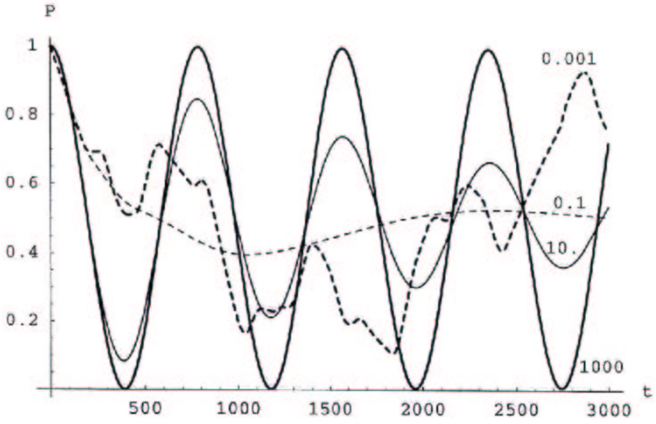


FIG. 1. The probability P for a particle to be found in the left hand well, for a bath temperature $T = 5$, in units of the ground state energy of the single infinite well. The initial condition is that the particle is on the left with a thermal distribution of energies. The curves are labelled by values of the rate-ratio parameter Q that range from 0.001 to 1000.

$$Q \equiv \frac{\langle \Gamma \rangle}{\langle g \rangle}$$

$\langle \cdots \rangle$ = thermal average

$Q \to \infty$ $\Rightarrow$ sys-bath int. rate $\gg$ osc. rate.

→ make the barrier width bigger

The frequency of the synchronised oscillation is given in terms of thermal averages of the $\hat{H}^{sys}$ matrix elements.

See Bell, Sawyer, Volkas PLB 500, 16 (2001) for a derivation in the neutrino context.

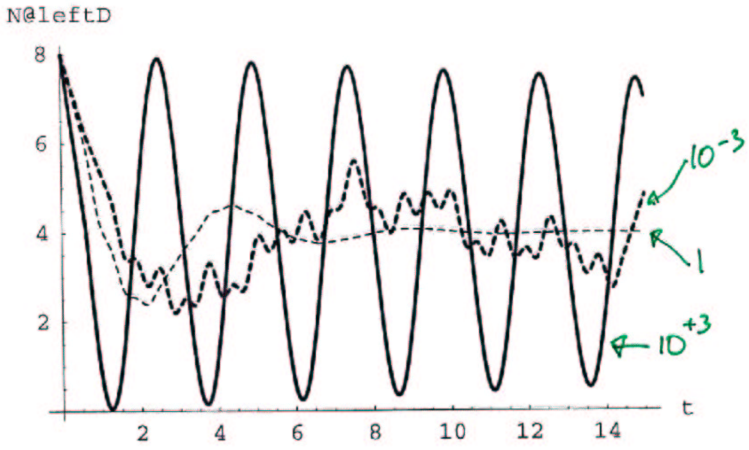


FIG. 3. The time development of the average number of particles in the left hand well. The initial conditions are: the first sixteen levels of the left well each occupied with 50% probability. The heavy dashed curve gives the behavior with extremely small coupling to the bath, the lighter dashed curve the behavior with an intermediate value of the coupling, and the heavy solid curve the behavior with the largest coupling. The respective couplings are in the ratios of 10^{-3}, 1, and 10^3.

With Fermi statistics for the system particles, employing the GCE.

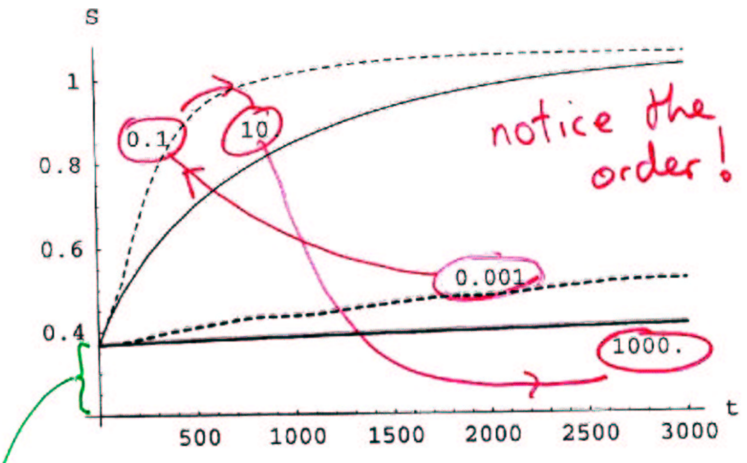


FIG. 2. The entropy S as a function of time for the same values of Q as used in fig.1.

$$S = - \sum_j \mathrm{Tr} \left\{ \hat{\rho}^s(E_j) \ln \left[\hat{\rho}^s(E_j) \right] \right\}$$

offset due to initial thermal distribution of particles in LH well → "vertical entropy" if you like

movements from this offset quantify

Sharpening of adiabatic inversion

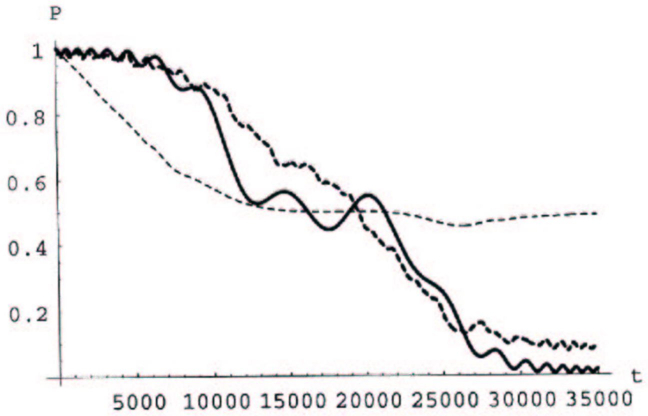
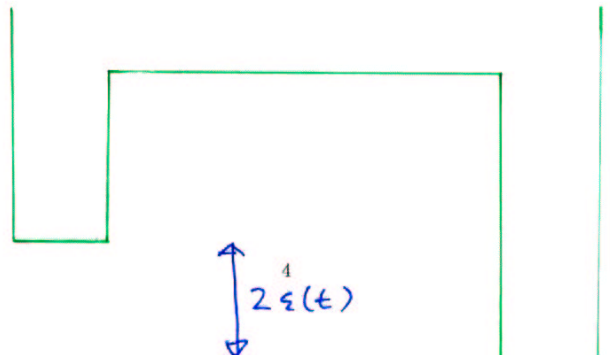


FIG. 4. The probability that the particle is on the left, where the well bias is reversed. The heavy dashed curve is for zero system-bath coupling, the light dashed curve for moderate coupling and the solid curve for large coupling. (The dependence of ϵ on t has been taken to be proportional to $(t-t_0)^3$, where t_0 is the zero-bias time.)

$$\varepsilon(t) \sim (t-t_0)^3$$

as an example

$$\hat{H}^s = \dots + \varepsilon(t)\sigma_3$$



Preservation of entanglement

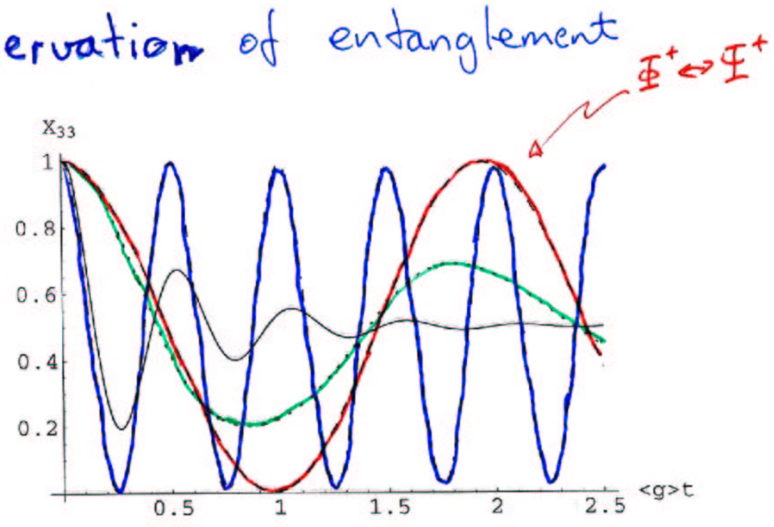


FIG. 5. The evolution of X_{33} as a function of time, for the initial state $|\Phi^+(E_1,E_1)\rangle$. The dashed, dotted, solid and dot-dashed curves are for $Q = 0.0003$, 0.03, 3 and 300 respectively.

$$|\Phi^\pm\rangle \equiv \frac{|E_i,L\rangle\otimes|E_j,L\rangle \pm |E_i,R\rangle\otimes|E_j,R\rangle}{\sqrt{2}}$$

Bell States

$X_{33} \equiv$ Prob. that if one of the pcles. is on the left (right), the other pcle. is also on the left (right).

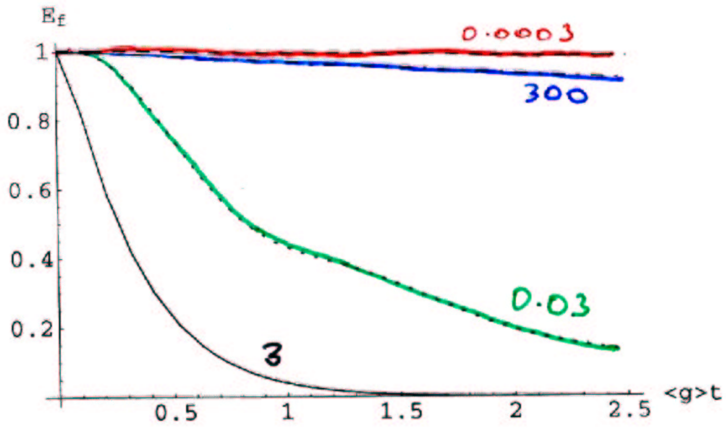


FIG. 6. The entanglement of formation, E_f, for the same parameters as Fig.5.

The study of "measures of entanglement" is now pursued actively in the literature.

The technical definition of E_f can be found in

Hill & Wootters, PRL 78, 5022 (1997)
Wootters, PRL 80, 2245 (1998)

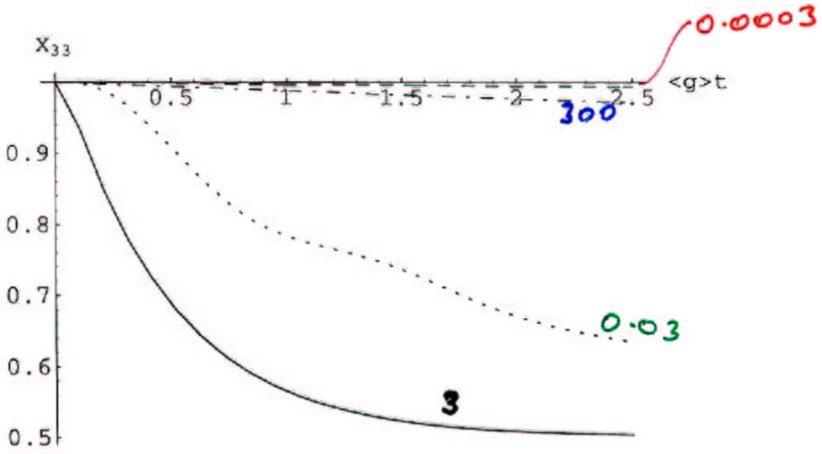


FIG. 7. The evolution of X_{33} as a function of time, for the initial state $|\Phi^-(E_1, E_1)\rangle$. The values of the parameter Q are as in Fig.5.

Φ^- is a const. (up to phase) in the absence of bath coupling.

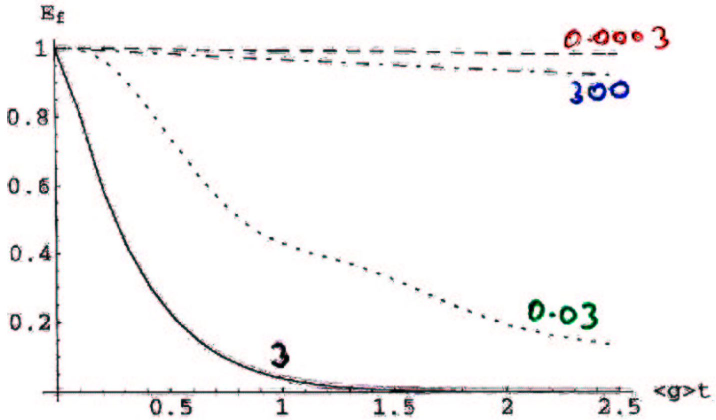


FIG. 8. The entanglement of formation, E_f, for the same parameters as in Fig.7.

5. Conclusion

The study of open quantum system dynamics is very interesting and important, from both a fundamental and practical point of view.

- Environments which couple differently to horizontal states freeze the evolution in the large Γ limit (q. Zeno effect)

- Environments that couple identically induce synchronisation or motional narrowing in the same limit. (preservation of quantal coherence despite strong sys-env. coupling)