

# Study of $U_{e3}$ after KamLAND

Morimitsu TANIMOTO

Niigata University, Niigata, JAPAN

with S. KANEKO

Neutrinos: Data, Cosmos, and Planck Scale

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## 1 Introduction

- **Atmospheric Neutrinos**  $\nu_\mu \rightarrow \nu_\tau$

$$\sin^2 2\theta_{\text{atm}} \geq 0.92, \quad \Delta m_{\text{atm}}^2 = (1.5 \sim 3.9) \times 10^{-3} \text{ eV}^2$$

- **KamLAND + Solar Neutrinos**

**Large Mixing Angle MSW LMA I**

$$\tan^2 \theta_{\text{sol}} = 0.33 \sim 0.67 \\ \Delta m_{\text{sol}}^2 = (6 \sim 8.5) \times 10^{-5} \text{ eV}^2$$

**CHOOZ Exp.:**  $\sin \theta_{\text{Chooz}} \leq 0.2$

- **Bi-Maximal Mixings ?**

$$U_{MNS} \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\sin^2 \theta_{\text{sol}}$  is different from maximum.

Why are  $U_{e2}$  and  $U_{\mu 3}$  so large ?

Another Question: Why is  $U_{e3}$  small ?

## 2 Idea of Large and Small Mixings

Lopsided   Democratic   Pseudo Dirac

$$\begin{pmatrix} \epsilon & 1 \\ \epsilon & 1 \end{pmatrix}_{LR} \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

These textures are reconciled with some flavor symmetries (models).

Is small  $U_{e3}$  always guaranteed? No!

Let me show examples of Textures

**Definition :**  $U_{MNS} = L_E^\dagger L_\nu$

$$L_E^\dagger M_E R_E = M_E^{\text{diag}}, \quad L_\nu^T M_\nu L_\nu = M_\nu^{\text{diag}}$$

**Example 1:** "Anarchy Mass Matrix" leads to large  $U_{e3}$ .

$$M \sim \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}, \quad U_{MNS} \sim \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}$$

### Example 2:

$$U_E = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$U_{MNS} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

### Example 3:

$$U_E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad U_\nu = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_{MNS} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

**Example 4: Democratic**

$$M_E \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1+\epsilon & 1 \\ 1 & 1 & 1+\delta \end{pmatrix}, \quad M_\nu \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_{\text{MNS}} = U_E^\dagger = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$$

**Example 5: Zee Model****Suppose symmetric limit**at which  $U_{e3} = 0$  $L_e - L_\mu - L_\tau, S_3$  Symmetry ...

$\sin^2 2\theta_{\text{sol}} < 1, \sin^2 2\theta_{\text{chooz}} \neq 0$  are due to deviation from symmetric limit.

There should be the relation between  $\sin^2 2\theta_{\text{sol}}$  and  $\sin^2 2\theta_{\text{chooz}}$ .

**Simple Approach in**

C.Giunti, M.Tanimoto, Phys.Rev.D66:113006,2002

$$U_{\text{MNS}} = [U^{(1)\dagger} U^{(0)}]$$

 $U^{(0)}$  Bi-Maximal (symmetric limit); $U^{(1)}$  Symmetry Breaking Effect;

$$U^{(1)} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

where  $\lambda, A, \rho$  and  $\eta$  are independent of ones in the quark sector.

$$|U_{e2}| \simeq \frac{1}{\sqrt{2}} \left( 1 - \frac{1}{\sqrt{2}}\lambda \right), \quad |U_{e3}| \simeq \frac{1}{\sqrt{2}}\lambda$$

$$\tan^2 \theta_{\text{sol}} \sim 1 - 4|U_{e3}| + O(|U_{e3}^2|)$$

**Fig. 1**In the case of  $\lambda = 0.22$ ,

$$|U_{e3}| = 0.15, \quad \tan^2 \theta_{\text{sol}} = 0.45$$

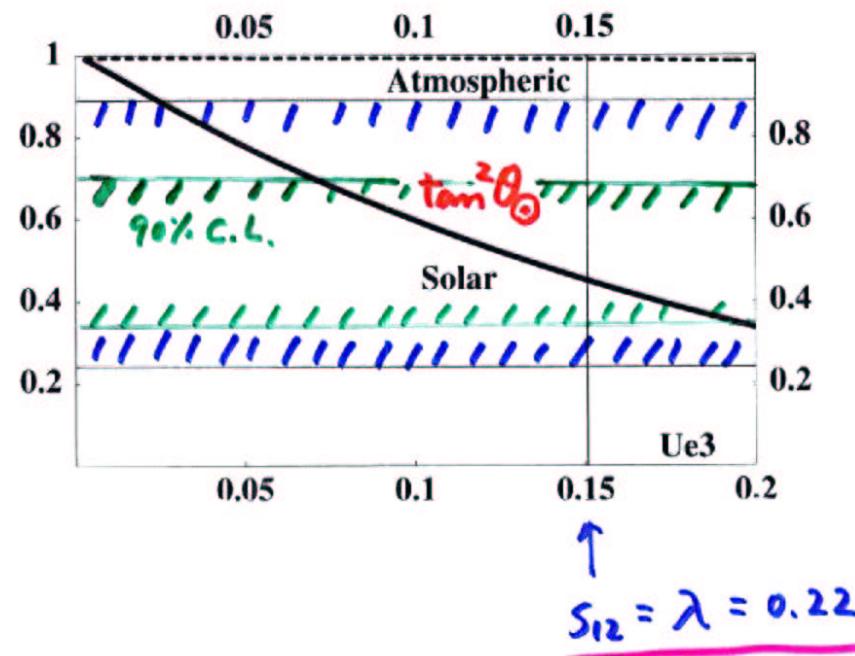


Figure 1:

Fig.1: Predictions in the  $|U_{e3}| - \tan^2 \theta_{\text{sol}}$  plane and  $|U_{e3}| - \sin^2 2\theta_{\text{atm}}$  plane. The thick solid curve corresponds to  $\tan^2 \theta_{\text{sol}}$ , while the dashed one to  $\sin^2 2\theta_{\text{atm}}$ . Horizontal lines delimit the experimental allowed regions for solar neutrinos. The parameter  $\lambda$  is varied from 0 to 0.28. The vertical line around  $|U_{e3}| = 0.15$  corresponds to the result in the case of  $\lambda = 0.22$ .

### 3 Texture Zeros in $M_\nu$ Long History of Texture Zeros

7 two-zero textures of neutrino mass matrix in the basis wherein the charged lepton mass matrix is diagonal :

$$\begin{aligned}
 A_1 & \left( \begin{array}{ccc} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{array} \right), \quad A_2 \left( \begin{array}{ccc} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{array} \right) \\
 B_1 & \left( \begin{array}{ccc} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{array} \right), \quad B_2 \left( \begin{array}{ccc} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{array} \right) \\
 B_3 & \left( \begin{array}{ccc} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{array} \right), \quad B_4 \left( \begin{array}{ccc} \times & \times & \times \\ \times & \times & \times \\ 0 & \times & 0 \end{array} \right) \\
 C & \left( \begin{array}{ccc} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{array} \right)
 \end{aligned}$$

x: non-zero entry  
 $9 - 4(2\text{zeros}) = 5$

**Neutrino mass matrix is written as**

$$M_\nu = U \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} U^T ,$$

$$\lambda_1 = m_1 e^{2i\rho}, \quad \lambda_2 = m_2 e^{2i\sigma}, \quad \lambda_3 = m_3$$

Both neutrino mass ratios and Majorana phases are determined if two independent entries of  $M_\nu$  vanish.

$M_\nu$  has six independent complex entries. If two of them vanish, i.e.,  $M_{ab}^\nu = M_{\alpha\beta}^\nu = 0$ , one gets following constraint relations:

$$M_{ab}^\nu = \sum_{i=1}^3 (U_{ai} U_{bi} \lambda_i) = 0, \quad \sum_{i=1}^3 (U_{\alpha i} U_{\beta i} \lambda_i) = 0$$

four subscripts run over  $e, \mu, \tau$ . Solving these condition, one obtains

$$\frac{\lambda_1}{\lambda_3} = \frac{U_{a3} U_{b3} U_{\alpha 2} U_{\beta 2} - U_{a2} U_{b2} U_{\alpha 3} U_{\beta 3}}{U_{a2} U_{b2} U_{\alpha 1} U_{\beta 1} - U_{a1} U_{b1} U_{\alpha 2} U_{\beta 2}}$$

$$\frac{\lambda_2}{\lambda_3} = \frac{U_{a1} U_{b1} U_{\alpha 3} U_{\beta 3} - U_{a3} U_{b3} U_{\alpha 1} U_{\beta 1}}{U_{a2} U_{b2} U_{\alpha 1} U_{\beta 1} - U_{a1} U_{b1} U_{\alpha 2} U_{\beta 2}} .$$

**Therefore, we have**

$$\frac{m_1}{m_3} = \left| \frac{U_{a3} U_{b3} U_{\alpha 2} U_{\beta 2} - U_{a2} U_{b2} U_{\alpha 3} U_{\beta 3}}{U_{a2} U_{b2} U_{\alpha 1} U_{\beta 1} - U_{a1} U_{b1} U_{\alpha 2} U_{\beta 2}} \right|$$

$$\frac{m_2}{m_3} = \left| \frac{U_{a1} U_{b1} U_{\alpha 3} U_{\beta 3} - U_{a3} U_{b3} U_{\alpha 1} U_{\beta 1}}{U_{a2} U_{b2} U_{\alpha 1} U_{\beta 1} - U_{a1} U_{b1} U_{\alpha 2} U_{\beta 2}} \right|$$

$$\rho = \frac{1}{2} \left[ \text{Arg} \frac{\lambda_1}{\lambda_3} \right], \quad \sigma = \frac{1}{2} \left[ \text{Arg} \frac{\lambda_2}{\lambda_3} \right]$$

With inputs of three flavor mixing angles and phase  $\delta$ , three neutrino masses and two Majorana phases are predicted.

Then, one can test textures in

$$R_\nu \equiv \left| \frac{m_2^2 - m_1^2}{m_3^2 - m_2^2} \right| \approx \frac{\Delta m_{\text{sun}}^2}{\Delta m_{\text{atm}}^2} \ll 1.$$

$\sim 0.02$

Seven acceptable patterns in  $M^\nu$

$A_1, A_2$ : Hierarchical Masses

$B_1, B_2, B_3, B_4, C$ : Degenerate Masses

Parametrizing  $U_{\text{MNS}}$  as follows:

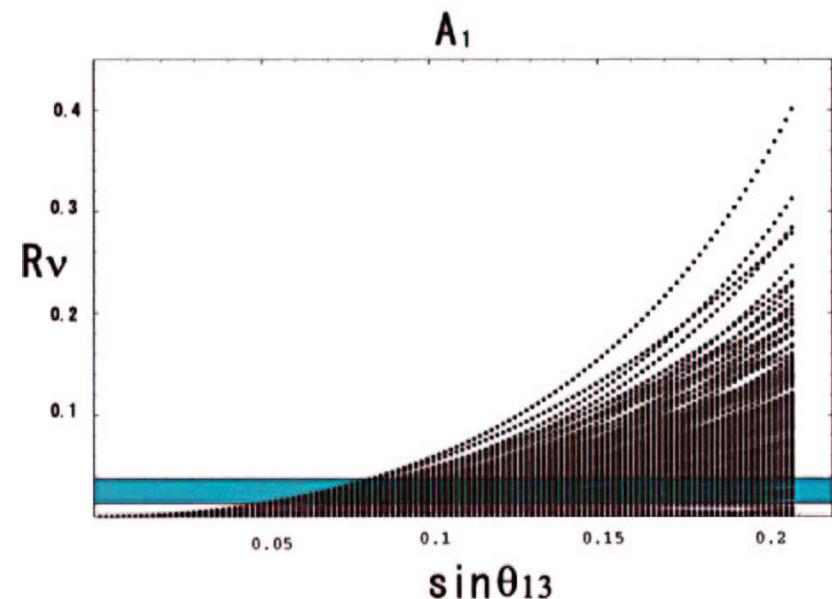
$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}$$

$A_1$  Type     $M_{ee}^\nu = M_{e\mu}^\nu = 0$      $\begin{pmatrix} 0 & 0 & X \\ 0 & X & X \\ X & X & X \end{pmatrix}$

$$\frac{\lambda_1}{\lambda_3} = +\frac{s_{13}}{c_{13}^2} \left( \frac{s_{12}s_{23}}{c_{12}c_{23}} e^{i\delta} - s_{13} \right)$$

$$\frac{\lambda_2}{\lambda_3} = -\frac{s_{13}}{c_{13}^2} \left( \frac{c_{12}s_{23}}{s_{12}c_{23}} e^{i\delta} + s_{13} \right)$$

$$R_\nu = \left| \frac{m_2^2 - m_1^2}{m_3^2 - m_2^2} \right|$$



$$\delta = -\pi \sim \pi$$

$$\theta_{12} = 30^\circ \sim 39^\circ \quad \tan^2 \theta_{\text{sun}} = 0.33 \sim 0.67$$

$$\theta_{23} = 37^\circ \sim 53^\circ \quad \tan^2 \theta_{\text{atm}} = 0.56 \sim 1.76$$

$\text{normal} > \text{inverted}$

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$R_\nu = \left| \frac{m_2^2 - m_1^2}{m_3^2 - m_2^2} \right|$$

$\approx \frac{\Delta m_{SND}^2}{\Delta m_{atm}^2}$

$$A_1 : \quad \frac{m_1}{m_3} = \left| \frac{s_{13}}{c_{13}^2} \left( \frac{s_{12}s_{23}}{c_{12}c_{23}} - s_{13}e^{-i\delta} \right) \right|$$

$$\frac{m_2}{m_3} = \left| \frac{s_{13}}{c_{13}^2} \left( \frac{c_{12}s_{23}}{s_{12}c_{23}} + s_{13}e^{-i\delta} \right) \right|$$

$$A_2 : \quad \frac{m_1}{m_3} = \left| \frac{s_{13}}{c_{13}^2} \left( \frac{s_{12}c_{23}}{c_{12}s_{23}} + s_{13}e^{-i\delta} \right) \right|$$

$$\frac{m_2}{m_3} = \left| \frac{s_{13}}{c_{13}^2} \left( \frac{c_{12}c_{23}}{s_{12}s_{23}} - s_{13}e^{-i\delta} \right) \right|$$

$$A_1 : |U_{e3}| \equiv \sin \theta_{13} \simeq \frac{1}{2} \tan 2\theta_{12} \cot \theta_{23} \sqrt{R_\nu \cos 2\theta_{12}}$$

$\delta$ : next leading

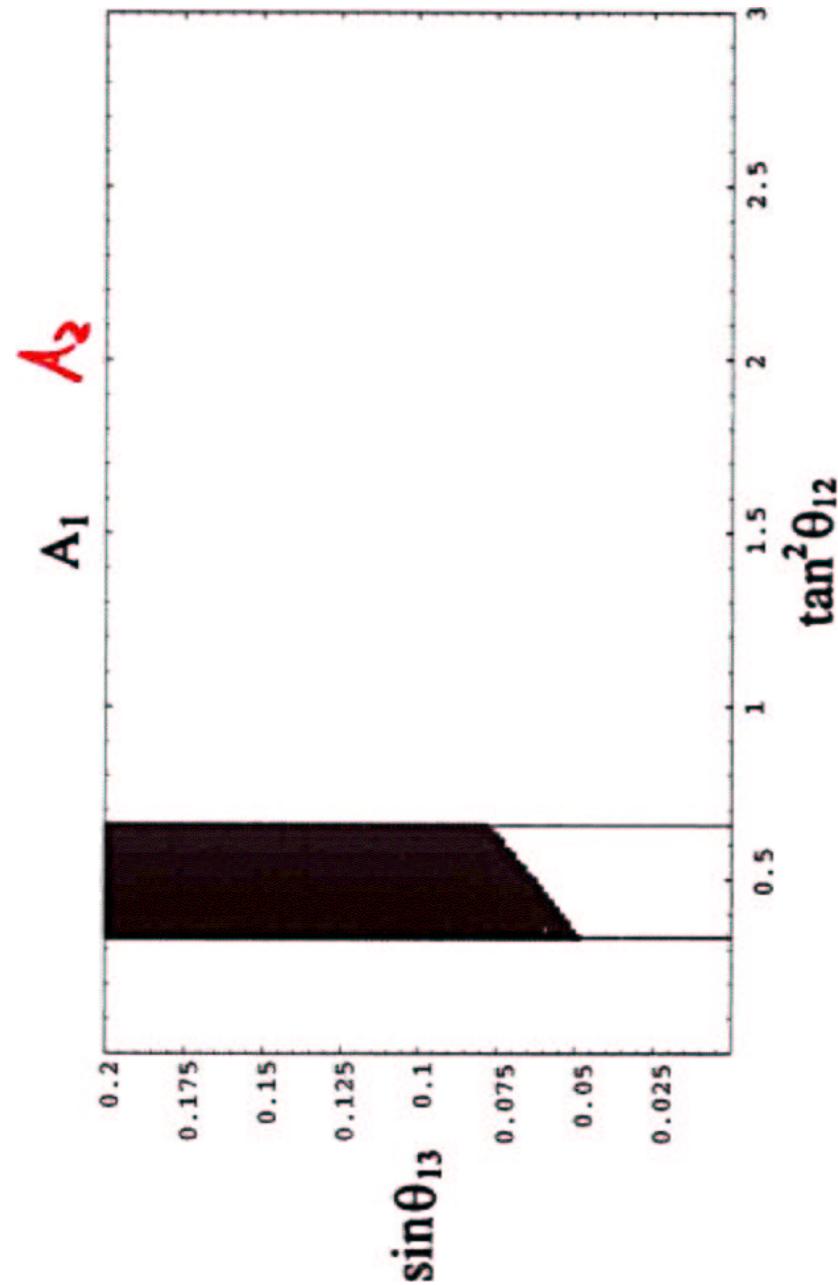
$$A_2 : |U_{e3}| \equiv \sin \theta_{13} \simeq \frac{1}{2} \tan 2\theta_{12} \tan \theta_{23} \sqrt{R_\nu \cos 2\theta_{12}}$$

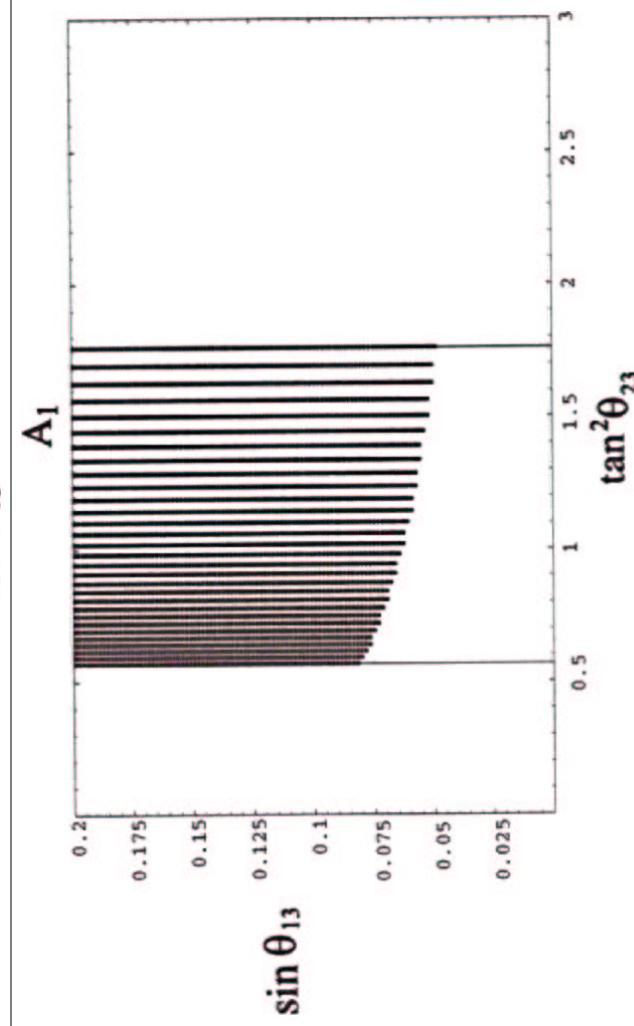
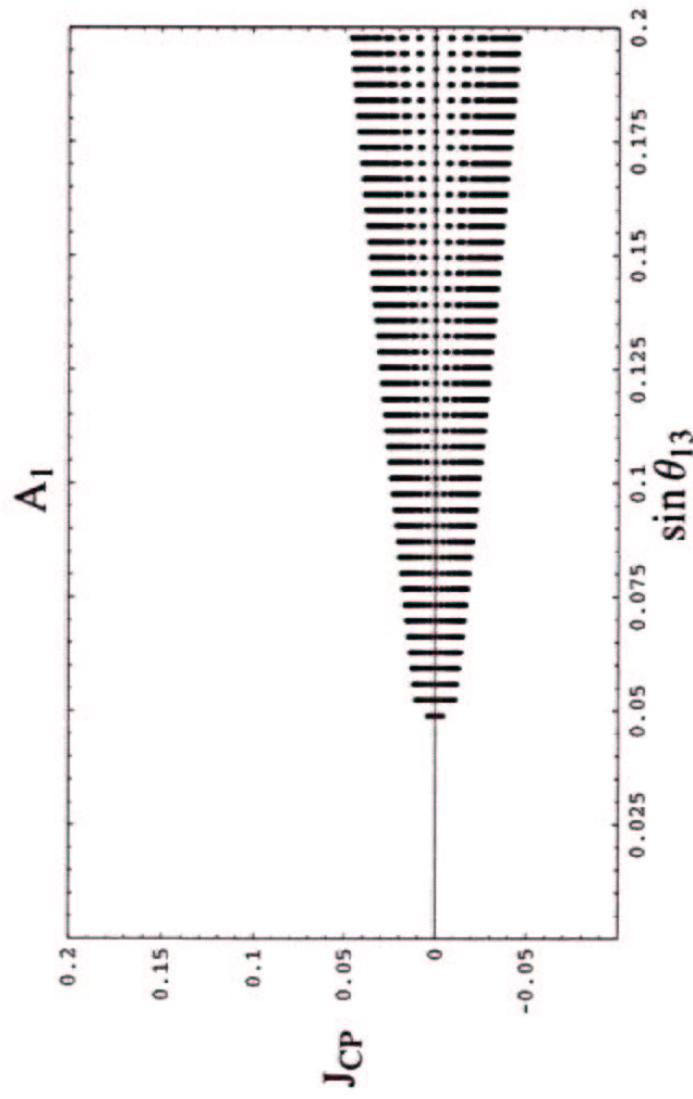
The neutrino mass matrix is roughly given as

$$M_\nu \sim \begin{pmatrix} 0 & 0 & \lambda \\ 0 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} \quad \text{for } A_1 \quad \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad \text{for } A_2$$

$\lambda = 0.2$

However, texture zeros are not be preserved to all orders. Moreover, zeros of the neutrino are realized while the charged lepton mass matrix may has off-diagonal's. We need to investigate the stability of predictions !!





A<sub>2</sub> case  
 $\tan^2 \theta_{23} \rightarrow \cot^2 \theta_{23}$

Why does this texture give two large mixings and one small mixing  $U_{e3}$ ?

$$A_1 \quad A_2$$

$$M_\nu = \begin{pmatrix} 0 & 0 & \epsilon \\ 0 & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & \epsilon & 0 \\ \epsilon & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

After (2-3) maximal rotation

$$M_\nu = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}\epsilon & -\frac{1}{\sqrt{2}}\epsilon \\ \frac{1}{\sqrt{2}}\epsilon & \epsilon' & 0 \\ -\frac{1}{\sqrt{2}}\epsilon & 0 & 2 \end{pmatrix}, \quad \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}}\epsilon & \frac{1}{\sqrt{2}}\epsilon \\ -\frac{1}{\sqrt{2}}\epsilon & \epsilon' & 0 \\ \frac{1}{\sqrt{2}}\epsilon & 0 & 2 \end{pmatrix}$$

$\epsilon \sim \epsilon'$        $\epsilon' \rightarrow 0$  limit  $\sim$  Bi-maximal

$$\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} \simeq \epsilon^2, \quad \epsilon = 0.1 \sim 0.2$$

$\tan^2 \theta_{\text{sol}} \simeq O(1)$  depends on  $\epsilon/\epsilon'$

$$U_{e3} = O(\epsilon) \quad \epsilon \rightarrow 0 \quad m_1 = 0 \quad m_2 = 0 \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$U(1)$  Flavour Symmetry

Diagonal Basis of  $M_E$

$$M_\nu \sim \begin{pmatrix} \epsilon^2 & \epsilon & \bar{\epsilon} \\ \bar{\epsilon} & 1 & 1 \\ \bar{\epsilon} & 1 & 1 \end{pmatrix} \quad \epsilon \sim \bar{\epsilon}$$

After (2-3) rotation

$$M_\nu \sim \begin{pmatrix} \epsilon^2 & \epsilon_1 & \epsilon_2 \\ \epsilon_1 & \epsilon' & 0 \\ \epsilon_2 & 0 & 2 \end{pmatrix} \quad \epsilon_1 = \frac{1}{\sqrt{2}}(\bar{\epsilon} - \epsilon) \quad \epsilon_2 = \frac{1}{\sqrt{2}}(\bar{\epsilon} + \epsilon)$$

$\tan^2 \theta_{\text{sol}}$  : LMA, SMA are allowed.

$$U_{e3} \simeq O(\epsilon_2) \simeq 0 \sim \epsilon$$

Less Predictive

J. Seto and  
T. Yanagida

$$\epsilon, \omega \ll \lambda \quad \begin{pmatrix} 2\bar{\epsilon} & 2\bar{\omega} & \lambda \\ 2\bar{\omega} & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

The two conditions turn to

$$(M_\nu)_{ab} = \sum_{i=1}^3 U_{ai} U_{bi} \lambda_i = \epsilon, \quad (M_\nu)_{\alpha\beta} = \sum_{i=1}^3 U_{\alpha i} U_{\beta i} \lambda_i = \omega$$

A<sub>1</sub>  
case

$$\frac{m_1}{m_3} = \frac{U_{13}U_{13}U_{12}U_{22} - U_{12}U_{12}U_{13}U_{23} - U_{12}U_{22}\bar{\epsilon} + U_{12}U_{12}\bar{\omega}}{U_{12}U_{12}U_{11}U_{21} - U_{11}U_{11}U_{12}U_{22}}$$

$$\frac{m_2}{m_3} = \frac{U_{11}U_{11}U_{13}U_{23} - U_{13}U_{13}U_{11}U_{21} - U_{11}U_{21}\bar{\epsilon} + U_{11}U_{11}\bar{\omega}}{U_{12}U_{12}U_{11}U_{21} - U_{11}U_{11}U_{12}U_{22}}$$

$$\bar{\epsilon} = \epsilon/\lambda_3, \bar{\omega} = \omega/\lambda_3$$

$$\frac{m_1}{m_3} \simeq s_{13}t_{12}t_{23} - \frac{t_{12}}{c_{23}}\bar{\omega} + \bar{\epsilon}$$

$$\frac{m_2}{m_3} \simeq -s_{13}\frac{1}{t_{12}}t_{23} - \frac{1}{t_{12}c_{23}}\bar{\omega} - \bar{\epsilon}$$

The  $|U_{e3}| = \sin \theta_{13}$  is given approximately as

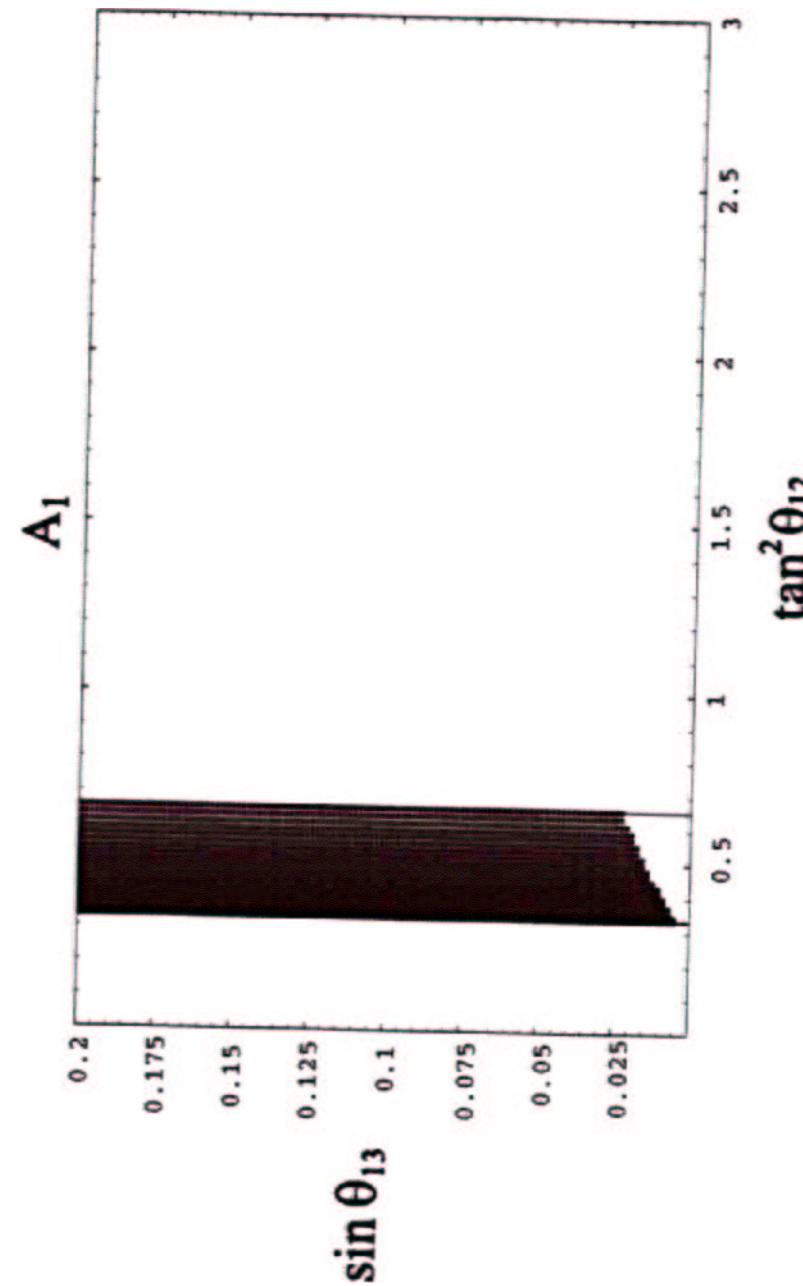
$$\frac{1}{2} \tan 2\theta_{12} \cot \theta_{23} \sqrt{R_\nu \cos 2\theta_{12}} - \frac{\bar{\omega}}{\sin \theta_{23}} \frac{1 + \tan^4 \theta_{12}}{1 - \tan^4 \theta_{12}} - \frac{t_{12}}{t_{23} 1 + \tan^2 \theta_{12}} \bar{\epsilon}$$

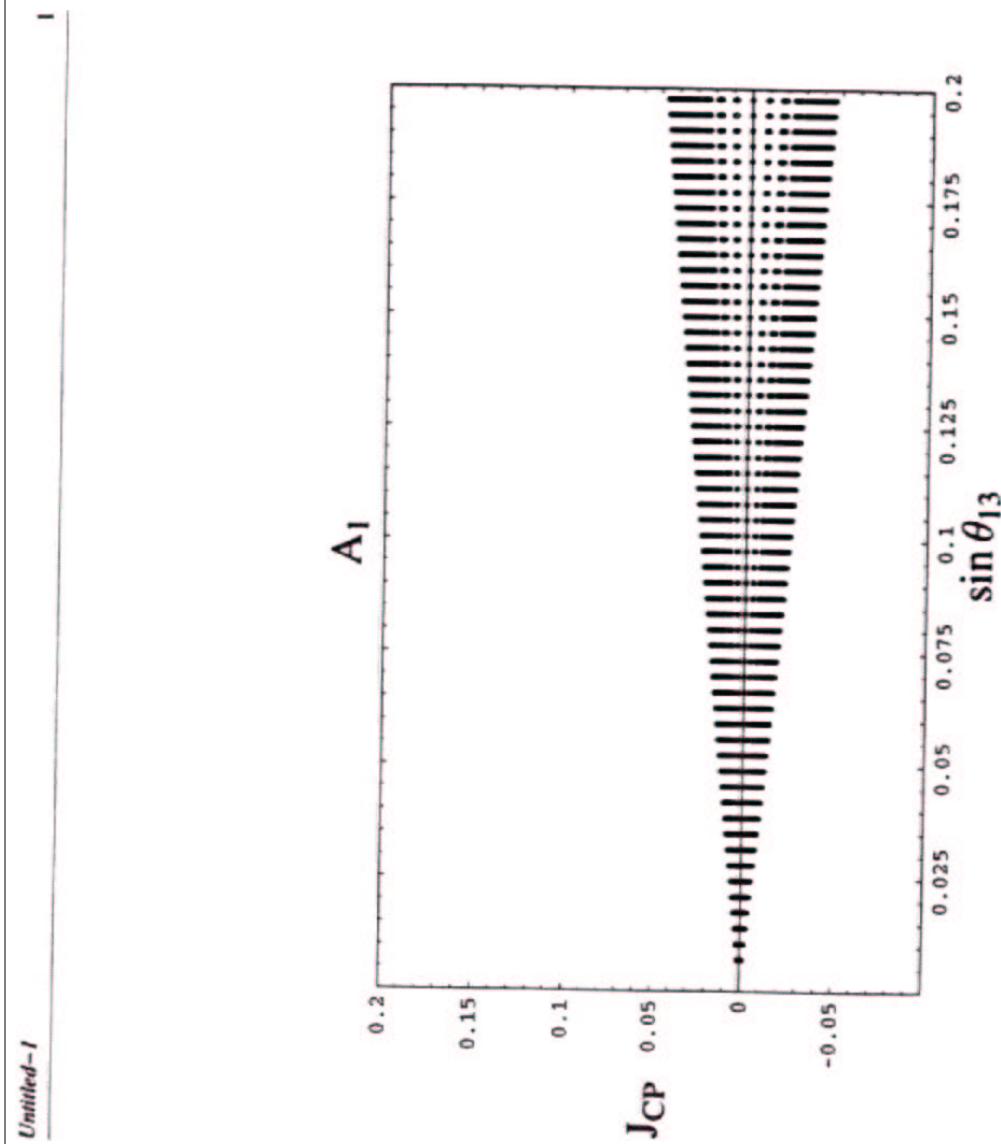
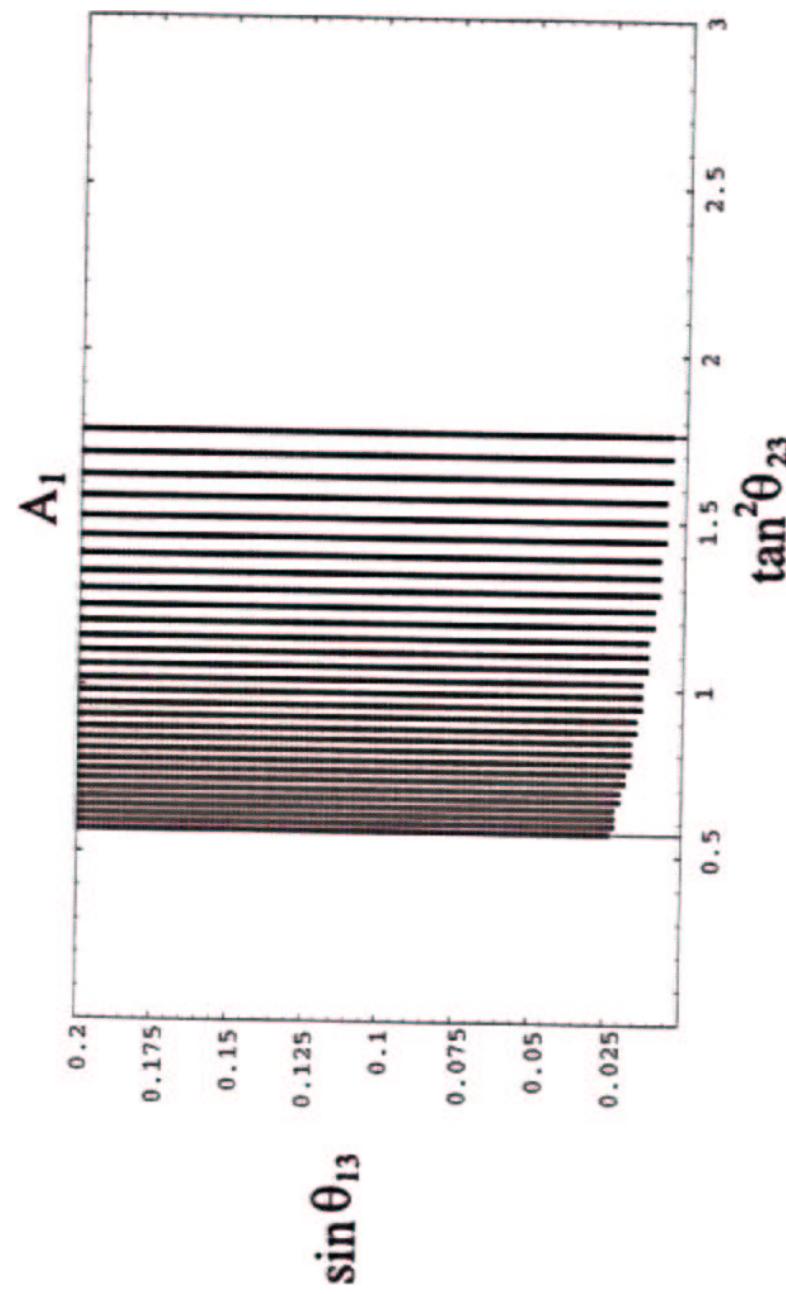
Example: Georgi-Jarlskog Texture

$$M_E \simeq \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & 0 \\ \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_e m_\tau} \\ 0 & \sqrt{m_e m_\tau} & m_\tau \end{pmatrix}, \quad M_\nu \sim \begin{pmatrix} 0 & 0 & \lambda \\ 0 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

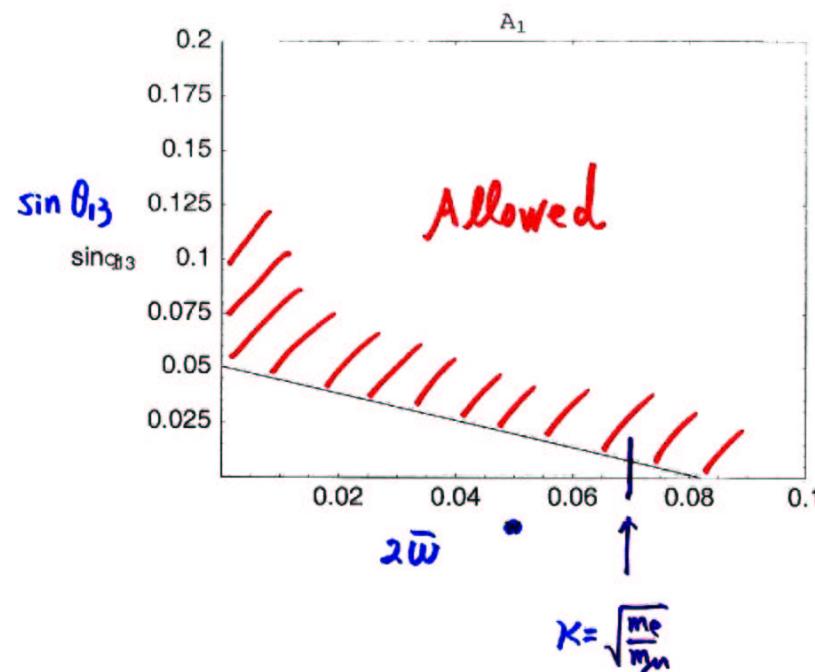
$$\dots \rightarrow M_E \simeq \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad M_\nu \sim \begin{pmatrix} \kappa^2 & \kappa & \lambda \\ \kappa & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

$$\kappa = \sqrt{\frac{M_E}{M_\nu}} = 0.07$$





Untitled-2



#### 4 Discussions

Texture Zeros are reconciled with GUT model.

Bando, Obara hep-ph/0302034 S0(10) GUT

$$M_D = \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 10 \\ 0 & 10 & 10 \end{pmatrix}, \quad M_U = \begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix}$$

$$M_{\nu D} = m_t \begin{pmatrix} 0 & *a_u & 0 \\ *a_u & *b_u & *c_u \\ 0 & *c_u & *d_u \end{pmatrix}, \quad M_R = m_R \begin{pmatrix} 0 & r & 0 \\ r & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

\* = 1, -3 for 10, 126

$$a = -3 \frac{\sqrt{m_\nu m_c}}{m_t} \quad b = \frac{m_c}{m_t} \quad c = \sqrt{\frac{m_u}{m_t}} \quad d = -3$$

$$M_\nu = \frac{m_t^2}{m_R} \begin{pmatrix} 0 & \frac{a^2}{r} & 0 \\ \frac{a^2}{r} & 2\frac{ab}{r} + c^2 & c(\frac{a}{r} + 1) \\ 0 & c(\frac{a}{r} + 1) & d^2 \end{pmatrix}$$

A<sub>2</sub> type

$r \simeq 10^{-7}$  Prediction:  $U_{e3} \simeq 0.01 \sim 0.06$

See-Saw  
realization

## 4 Summary

**Texture Two Zeros are consistent with the data after KamLAND.**

$$A_1 : \begin{pmatrix} 0 & 0 & \lambda \\ 0 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} \quad A_2 : \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\Delta m_{\text{sol}}^2, \tan^2 \theta_{\text{sol}}, U_{e3}$$

**are correlated in many models**

We need precise determination of

$$\Delta m_{\text{sol}}^2, \tan^2 \theta_{\text{sol}}, U_{e3}$$

**in Long Baseline Experiments.**