Enhancing Mechanisms of Neutrino Transitions in the Earth and Atmospheric Neutrino Oscillations

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2002 was exceptional for the studies of $\nu$'s:
- SNO: NC DATA $\rightarrow$ $\nu_{\text{e}}$, $\nu_{\text{m}}$, $\nu_{\text{e}}$ in $\Phi(\nu)$
- KamLAND:
  - first evidence for $\nu$-oscillations in an experiment with terrestrial $\nu$'s
  - evidence for $\nu$-mixing in vacuum
- $\nu_0$: LMA solution (CPT)
- KamLAND "massacre": SMA, LOW, QV0, VO, RSFP, FCNC, ...
- Determines the priorities of the future research
- SK is operational again
- MiniBOONE started
- The achievements in the field ($\nu_0$ - astronomy, SN $\nu$'s detection)
  and the fundamental contributions made by R. Davis and M. Koshiba
  honored by the Nobel Prize for Physics.

UCLA, 1 April, 2003
EVIDENCES FOR $\nu$-OSCILLATIONS:

- $\nu_{\text{ATM}}$: SK  
  UP-DOWN ASYMMETRY  
  (ZENITH ANGLE DEPENDENCE) 
  MULTI-GeV $\mu$-LIKE SAMPLE  
  K2K; MINOS; CNGS.

- $\nu_{\odot}$: 
  HOMESTAKE; KAMIOKANDE;  
  SAGE; GALLEX/GNO; 
  SUPER-KAMIOKANDE;  
  SNO

- LSND  
  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  
  MINIBOONE

$\nu_{\ell L} = \sum_{j=1}^{3} U_{\ell j} \nu_{\ell j}$;  
$\ell = \mu, e$.

$\nu$-FACTORIES: 3-$\nu$ MIXING, LMA-NSW 
$L \sim (3000 - 7000)$ km.
Allowed region
(FC + PC + UP-thru + UP-stop)

\[
\begin{align*}
\nu_\mu & \quad \nu_e \\
79.3 \text{ kt. yrs}
\end{align*}
\]

Best fit:
\[\Delta m^2 = 2.5 \times 10^{-3} \text{eV}^2, \sin^2 2\theta = 1.00\]
\[(\chi^2 = 142.1 / 152 \text{ d.o.f.})\]

SK combined result
\[\Delta m^2 = (1.7-4) \times 10^{-3} \text{eV}^2\]
\[\sin^2 2\theta > 0.89 \quad (90\% \text{ C.L.})\]

\text{m_1 < m_2 < m_3 - NH}

\text{CHOOZ: } \bar{\nu}_e \rightarrow \bar{\nu}_e
\sim \frac{1 \text{ km}}{E_{\nu}} \sim 2 \text{ MeV}

Analysis A

\text{90\% CL Kamiokande (multi-GeV)}
\text{90\% CL Kamiokande (sub-multi-GeV)}

95\% CL

90\% CL

Figure 9: Exclusion plot for the oscillation parameters based on the absolute comparison of measured vs. expected positron yields.
3-

J MIXING:

\[
\begin{pmatrix}
\mathcal{J}_{eL} \\
\mathcal{J}_{\mu L} \\
\mathcal{J}_{\tau L}
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix}
\begin{pmatrix}
\mathcal{J}_{eL} \\
\mathcal{J}_{\mu L} \\
\mathcal{J}_{\tau L}
\end{pmatrix}
\]

UPMNS

PARAMETERS: UPMNS - n x n

\[
\begin{array}{cccc}
2 & 3 & 4 \\
\end{array}
\]

ANGLES

\[
\frac{n(n-1)}{2}
\]

CP-VIOLATING PHASES:

\[
\begin{array}{c}
\text{\textit{J}}
\end{array}
\]

- \textit{J} = \text{DIRAC}

\[
\frac{(n-1)(n-2)}{2}
\]

- \textit{J} = \text{MAJORANA}

\[
\frac{n(n-1)}{2}
\]

STANDARD PARAMETRIZATION:

\[
U_{PMNS} =
\begin{pmatrix}
C_{12}C_{13} & s_{12}s_{23}e^{i\Delta_{21}/2} & U_{e3}e^{i\delta_{13}} \\
-s_{12}C_{13} - C_{12}S_{23}S_{13} & C_{12}C_{23} - S_{12}S_{23}S_{13} & S_{12}S_{23} - S_{12}S_{23}S_{13} \\
S_{12}S_{23}C_{13} - S_{12}S_{23}S_{13} & C_{12}C_{23}S_{13} - S_{12}S_{23}S_{13} & C_{12}C_{23}S_{13} - S_{12}S_{23}S_{13}
\end{pmatrix}
\]

\[
c_{ij} = \cos \theta_{ij}, \ s_{ij} = \sin \theta_{ij}, \ 0 = \theta_{12}, \theta_{13}, \theta_{23} \leq \frac{\pi}{2}
\]

\[
U_{e3} = s_{13}e^{-i\delta_{13}}, \ S_{\delta} \in [0,2\pi] - \text{DIRAC CP-VIOLATING PHASES}
\]

\[
\delta_{21}, \delta_{31} - \text{MAJORANA CP-VIOLATING PHASES}
\]

IF J ARE MAJORANA PARTICLES, \ S.M.Bilenky, J.Hosak, S.T.B.'80

CP-SYMMETRY CAN BE VIOLATED EVEN IN THE CASE OF n = 2 FAMILIES OF LEPTONS:

\[
N_{CP}^M = \frac{n(n+1)}{2} - n = \frac{n(n-1)}{2}
\]

\[
\delta_{21}, \delta_{31} \text{ DO NOT APPEAR IN } \mathcal{P}(\frac{\mathcal{O}}{2} \rightarrow \frac{\mathcal{O}}{2})
\]

- AFFECT THE (BB)_{00} - DECAY RATE

- IN SUSY SEE-SAW MODELS, THE RATES OF THE LFV DECAYS.
Dr. Serguey Petcov, KITP & SISSA-Trieste (KITP Neutrinos 4-03-03) Prospects for No-Nu Double Beta Decay

- \( \beta_2, \beta_3, \beta_2, \beta_3 \) : CHOOSE \( \theta = \frac{1}{2} \) or \( \frac{3}{2} \)
  - \( \theta = \frac{1}{2} \)
  - \( \theta = \frac{3}{2} \)

**PARAMETERS:**

- \( \theta_{12}, \theta_{13}, \theta_{23} \)
- \( \delta, \delta_2 \)
- \( m_4, m_2, m_3 \)

\( m_{1,2,3} \) : MEASURED IN \( \nu \)-OSCILLATION EXPERIMENTS

\[ \Delta m_{12}^2 = \Delta m_{21}^2 > 0, \quad |\Delta m_{12}^2| = |\Delta m_{31}^2| \]

- \( m_1 < m_2 < m_3 \) or \( m_3 < m_1 < m_2 \)

- \( m_4 < m_2 < m_3 \)

\[ m_4, m_2, m_3 \Rightarrow m_4, \Delta m_{21}^2 > 0, \Delta m_{32}^2 > 0 \]

- \( m_2 = \sqrt{m_1^2 + \Delta m_{21}^2} \)
- \( m_3 = \sqrt{m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2} \)

\[ \Delta m_{\text{ATM}}^2 = \Delta m_{31}^2 = \Delta m_{21}^2 + \Delta m_{32}^2 \]

**TWO POSSIBILITIES:**

- \( \Delta m_{\odot}^2 = \Delta m_{21}^2 - NH \)
- \( \Delta m_{\odot}^2 = \Delta m_{32}^2 - IH \) - 'DISCRETE' PARAMETER
Further studies of $\Delta^a$-oscillations by SK (or other water-\(\nu\) detectors) can produce information on

- $\sin^2 \theta_{13}$
- $\sin^2 2 \theta_{23}$ ($\sin^2 2 \theta_{23} \neq 1, \cos 2 \theta_{23} > 0$ or $\cos 2 \theta_{23} < 0$)
- Sign of $\Delta m^2_{31}$, i.e., type of the neutrino mass spectrum
  \(NH\) vs \(IH\)
Atmospheric neutrinos

$\nu_\mu + \bar{\nu}_\mu \sim 2$ @ low energy ($E_\nu < 1$ GeV)

$\nu_e + \bar{\nu}_e$

$\nu_\mu + \bar{\nu}_\mu$ @ high energy

Error in absolute flux $\sim 20\%$, but $\nu_\mu/\nu_e$ ratio $\sim 5\%$

Neutrino oscillations:

$\left( \frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e} \right)_{\text{data}} / \left( \frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e} \right)_{\text{MC}} < 1$

Atmospheric neutrino spectrum

MODEL dependence of ENERGY spectrum (P. Lipari)

$\nu_\mu$, $\mu^+ \nu_\mu$, $e^+ \nu_e$
Zenith Angle Distribution (1D)

Calculated zenith angle distribution

For $E_{\nu} > \text{a few GeV}$,
Upward / downward = 1 (within a few %)

Up/Down asymmetry for neutrino oscillations

Zenith Angle Asymmetry

$A = \frac{(U-D)}{(U+D)}$

$A_{\text{multi-GeV}\mu} = -0.311 \pm 0.043 \pm 0.01 > 7\sigma \text{ from 0}$
3-7 Oscillations of $\theta_{13}$:

\[ \Delta m_{21}^2 \equiv \Delta m_{12}^2 \ll \Delta m_{31}^2 \equiv \Delta m_{13}^2 \]

\[ P_{30} (\nu_e \rightarrow \nu_e) = P_{30} (\nu_\mu \rightarrow \nu_e) \approx S_{23}^2 P_{20} (\Delta m_{31}^2, \theta_{13}^2, \nu_\mu, \nu_e) \]

\[ P_{30} (\nu_e \rightarrow \nu_\tau) = P_{30} (\nu_\tau \rightarrow \nu_e) \approx C_{23}^2 P_{20} (\Delta m_{31}^2, \theta_{13}^2, \nu_\tau, \nu_e) \]

\[ P_{30} (\nu_e \rightarrow \nu_\tau) = 1 - P_{20} (\nu_\tau \rightarrow \nu_e) \]

\[ P_{30} (\nu_\mu \rightarrow \nu_\mu) \approx 1 - S_{23}^4 P_{20} - 2C_{23}^2 S_{23} \left[ 1 - \text{Re}(e^{-i\alpha} A_{20} (\nu_\tau \rightarrow \nu_\tau)) \right] \]

\[ P_{32} (\nu_\mu \rightarrow \nu_\mu) = 1 - P_{30} (\nu_\tau \rightarrow \nu_\mu) - P_{30} (\nu_\tau \rightarrow \nu_\tau) \]

\[ P_{20} (\Delta m_{31}^2, \theta_{13}^2, \nu_\tau, \nu_e) \text{ - 2-D Oscillation Probability} \]

\[ P_{20} = P_{20} (\nu_e \rightarrow \nu_\tau'), \nu_\tau' = S_{23}^2 \nu_\mu + C_{23}^2 \nu_\tau \]

\[ \alpha, A_{20} (\nu_\tau \rightarrow \nu_\tau) \text{ - Known} \]

**Similar for Anti-\(\nu\)'s:**

\[ P_{20} \rightarrow P_{\overline{20}}, \ \overline{\nu_e} \rightarrow \overline{\nu_e}, \ A_{20} \rightarrow \overline{A_{20}} \]

\[ \phi_{e, \mu} (E, \theta_n), \ \phi_{e, \mu} (E, \theta_n): \]

\[ \phi_{e, \mu} = \phi_{e, \mu}^0 \left[ 1 + (S_{23}^2 \tau (E, \theta_n) - 1) P_{20} \right] \]

\[ \phi_{e, \mu} = \phi_{e, \mu}^0 \left[ 1 + S_{23}^4 \left( \frac{1}{S_{23}^2 \tau} - 1 \right) P_{20} - 2C_{23}^2 S_{23}^2 (1 - \text{Re}(e^{-i\alpha} A_{20})) \right] \]

\[ \tau = \frac{\phi_{e, \mu} (E, \theta_n)}{\phi_{e, \mu}^0 (E, \theta_n)} \]

**Sub-GeV:** \( \tau \approx 2.0 \)

\[ (S_{23}^2 \tau - 1) \approx \begin{cases} 0, & S_{23}^2 = 0.5 \\ 0.28, & S_{23}^2 = 0.64 \end{cases} \]

**Multi-GeV:** \( \tau \approx (2.6 - 4.5) \)

\[ (S_{23}^2 \tau - 1) \approx \begin{cases} 0.3, & \tau = 2.6 \\ 1.25, & \tau = 4.5 \end{cases} \]

\[ S_{23}^2 = 0.5 \]
Oscillation effects larger for larger S<sub>23</sub> than for the sub-GeV sample (μ multi > τ sub). Larger in the multi-GeV sample should lead to increase of φ<sub>0</sub> (Ne) decrease of φ<sub>3,4</sub> (Ne).}

All the above results can be derived using the CPT invariance. CPT-transformation properties of the processes:

\[ \langle \phi (\mu, \tau) \rangle = \langle \phi (\mu, \tau) \rangle = \langle \phi (\mu, \tau) \rangle \]

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\[ \langle \phi (\mu, \tau) \rangle = \langle \phi (\mu, \tau) \rangle = \langle \phi (\mu, \tau) \rangle \]
When the D's cross only the Earth Mantle:

\[ L = 2 R_E \cos \theta, \quad R_E = 6371 \text{ km} \]
\[ \theta = 33^\circ, \quad L \leq 10600 \text{ km} \]

\[ L' = 10^5 - 10^4 \text{ km}; \quad \bar{\rho} = (2.9-4.8) \text{ g/cm}^3 \]
\[ \gamma_e = 0.494 \]

\[ \rho_c = (10-13) \text{ g/cm}^3 \quad \text{over a distance of } R_c = 3486 \text{ km} \]
\[ \rho_m = (5.3-5.5) \text{ g/cm}^3 \quad \text{over a distance of } \sim 2885 \text{ km} \]

Figure 1: The Earth's core and mantle structure.

Figura 5.1: Distribuzione di densità della Terra (Stacey, 1977).
**Neutrino Oscillations in Matter**

\[ \nu_e > = \nu_1 > \cos \theta + \nu_2 > \sin \theta \]
\[ \nu_\mu > = - \nu_1 > \sin \theta + \nu_2 > \cos \theta \]

\[ H_m = H_{vac} + H_{int} \]

\[ \sum_{e,\mu} \] 

\[ \pm \nu_e, \pm \nu_\mu \]

\[ \Delta m^2 = \Delta m^2_{12} + \Delta m^2_{23} \]

\[ \theta_{12} \]

\[ \sin^2 2\theta_{12} = \frac{1}{2} \sum_{\nu_e,\nu_\mu} \]

\[ \rho_{12} = \frac{\sin^2 2\theta_{12}}{2\Delta m^2_{12} G_F} \]

\[ \rho_{13} = \frac{\sin^2 2\theta_{13}}{2\Delta m^2_{13} G_F} \]

For \( \sin \theta << 1 \):
\begin{align*}
1 \nu_e > & \approx 1 \nu_\mu >, \text{ if } N_e << N_e^{res} \\
1 \nu_\mu > & \approx 1 \nu_e >, \text{ if } N_e >> N_e^{res} \\
\end{align*}

\[ P(\nu_e \rightarrow \nu_\mu) = \frac{\Delta m^2_{12}}{2 \pi \rho_{12} \theta_{12}} \sin^2 2\theta_{12} \left( 1 - \cos 2\theta_{12} \right) \]

\[ L_m = \frac{L_{\nu e}}{\sqrt{\left( 1 - \frac{L_{\nu e}}{L_{\nu e}^{res}} \right)^2 - \sin^2 \theta_{12}^2}} \]

\[ L_{\nu e}^{res} = \frac{L_{\nu e}}{\sin \theta_{12}^2} \]

So, for \( N_e << N_e^{res} \) - \( \nu_e \rightarrow \nu_\mu \) like in vacuum

\( N_e >> N_e^{res} \) - \( \nu_e \rightarrow \nu_\mu \) damped

\( N_e \approx N_e^{res} \) - the oscillations can be resonantly enhanced

\[ \sin^2 2\theta_{13} = \frac{\sin^2 2\theta_{13}}{\left( 1 - \frac{N_e}{N_e^{res}} \right)^2 + \rho_{13}^2 \theta_{13}^2} \]

\[ \rho_{13} = \frac{\Delta m^2_{13}}{2 \pi \rho_{13} \theta_{13}} \]

\[ \sin^2 2\theta_{13} = \frac{\sin^2 2\theta_{13}}{\left( 1 - \frac{N_e}{N_e^{res}} \right)^2 + \rho_{13}^2 \theta_{13}^2} \]

\[ \max P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta_{13} \Delta E' R \]

\[ \max P(\bar{\nu_e} \rightarrow \bar{\nu_\mu}) \approx 1 \left( \sin^2 \theta_{13} \right) \Delta E R \]
$\bar{\nu}_e$ passing through the Earth mantle:

$N_{e}^{\text{MAN}} \approx \text{const.} \times (2.0 - 2.4) N_A \text{cm}^{-3}$

$(\cos \theta_{13} \approx 0.4)$

$\Delta m_{31}^2 \approx 3 \times 10^{-3} \text{eV}^2$

$\sin^2 2\theta_{13} < 0.2$

$\bar{\nu}_e \rightarrow \nu_\mu(\tau), \quad \nu_\mu \rightarrow \nu_e$:

$E_R \approx 6.6 \frac{\Delta m_{31}^2}{10^{-3} \text{eV}^2} \frac{1}{N_A \text{cm}^{-3}} \cos^2 \theta_{13} \text{ GeV}$

$\approx 10 \text{ GeV}$.

$\cos \Delta E' L = -1$:

$\Delta E'_{\text{RES}} L = 1.23 \tan 2\theta_{13} \frac{N_{e}^{\text{MAN}}}{N_A \text{cm}^{-3}} \frac{L}{10^4 \text{ km}}$

$\sin^2 \theta_{13} = 0.05 : \quad L \approx 8000 \text{ km}$

$= 0.025 : \quad L \approx 10000 \text{ km}$

Diagram:

- Neutrinos
- Vacuum
- Antineutrinos

$L = 7330 \text{ km}$

$\sin^2 (2\theta_{33}) = 1.0$

$\sin^2 (2\theta_{13}) = 0.1$
SK Multi-GeV:

\[ N_e(\mu) = \frac{2}{3} N_e(\mu) + \frac{1}{3} N_e(\mu) \]

Due to $\bar{\nu}_e(\bar{\mu})$

Due to $\bar{\nu}_e(\bar{\mu})$

\[ \cos 2\theta_{13} > 0 \]

\[ \Delta m_{31}^2 > 0 \] RESONANTLY ENHANCED

\[ P_{2e} \approx 1 \]

AFFECTS $\frac{2}{3} N_e(\mu)$

\[ \Delta m_{31}^2 < 0 \] RESONANTLY ENHANCED

\[ P_{2e} \approx 1 \]

AFFECTS $\frac{1}{3} N_e(\mu)$

The effects of the oscillations are larger for $\Delta m_{31}^2 > 0$ than for $\Delta m_{31}^2 < 0$. Sensitivity to the type of $\nu$-mass spectrum.
The new effect or resonance-like enhancement of 
\[ P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu, \tau), \quad P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e), \] etc.

- Exhibits strong dependence on \( E \)
- Is sufficiently wide (it is wider than the MSW resonance)

\[ \frac{\Delta E}{E_{\text{max}}} \approx (0.3-0.4) \text{ and is } \sin^2 2\theta \text{ independent for } \sin^2 2\theta \leq 0.05 \]

The "resonance" takes place in the \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) and \( \bar{\nu}_e \rightarrow \bar{\nu}_\mu, \tau \) transitions of atmospheric neutrinos:

For \( \Delta m^2 = 3 \times 10^{-3} \text{eV}^2 \), \( \sin^2 2\theta \approx (0.01-0.10) \)
\[ \text{Max} \ (P(\bar{\nu}_\mu(e) \rightarrow \bar{\nu}_e(\mu))) \text{ occurs at } E \approx 4.8 \text{GeV} \]
Based on: S.T.P., PL B434 (98) 321
HEP-PH/9809587 (98)
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Applications:

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See also: M. Maris, S.T.P., PR D56, 7444
HEP-PH/
PR D58,
HEP-PH/

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PR D56, 5394
HEP-PH/

Three transitions in a two constant-density layer medium

θ_m', θ_m'', 2ϕ_m' = ΔE'x', 2ϕ_m'' = ΔE''x''

Earth: mantle-core-mantle

0 < ν_{dP} < ν_{uP}
are provided by the solutions of eq. (52), which can be found explicitly:

$$P_{\alpha\beta} = 1$$

**solution A:**

$$\tan \phi' = \pm \frac{1}{\cos(2\theta_m')} \left| \frac{-\cos 2\theta_m'}{\cos(2\theta_m' - 4\theta_m')} \right|$$
$$\tan \phi'' = \pm \frac{1}{\cos(2\theta_m')} \left| \frac{\cos 2\theta_m'}{\cos(2\theta_m' - 4\theta_m')} \right|$$

where the signs are correlated.

The probability $P_{\alpha\beta}$ exhibits a system of of maxima which is similar to that in the two layer case. Under the conditions (5), (18) and if $V_{23}^* > 0$ (i.e., for the $\nu_e \rightarrow \nu_e$ (n.nu), $\nu_x \rightarrow \nu_x$ and $\nu_x \rightarrow \nu_x$ transitions in the Earth), solutions (54) are realized in the region $A$ (Figs. 3 - 5),

**region A:**

$\cos(2\theta_m') \leq 0$, $\cos(2\theta_m' - 4\theta_m') \geq 0$.

On the line belonging to region $A$, we have

**case B:** $\max P_{\alpha\beta} = \sin^2 2\theta_m' = 1$,

provided

$$\cos 2\phi' = 0, \text{ or } 2\phi' = \frac{\pi}{2}(2k' + 1), \text{ or } k' = 0, 1, ...$$
$$\sin 2\phi'' = 0, \text{ or } 2\phi'' = 2\pi k'', \text{ or } k'' = 0, 1, ...$$

Besides these absolute maxima, there exist two regions,

**region C:** $\cos(2\theta_m') \geq 0$,

and

**region D:** $\cos(2\theta_m' - 4\theta_m') \leq 0$.

with maxima

**case C:** $\max P_{\alpha\beta} = \sin^2 2\theta_m'$

and

**case D:** $\max P_{\alpha\beta} = \sin^2(2\theta_m' - 4\theta_m')$.

which correspond to the solutions

**solution C:**

$$\sin \phi' = 0, \text{ or } 2\phi' = 2\pi k', \text{ or } k' = 0, 1, ...$$
$$\cos \phi'' = 0, \text{ or } 2\phi'' = 2\pi (k'' + 1), \text{ or } k'' = 0, 1, ...$$

and

**solution D:**

$$\cos \phi' = 0, \text{ or } 2\phi' = \pi (2k' + 1), \text{ or } k' = 0, 1, ...$$
$$\cos \phi'' = 0, \text{ or } 2\phi'' = \pi (2k'' + 1), \text{ or } k'' = 0, 1, ...$$
$\nu_e \rightarrow \nu_\mu : \theta = 0^\circ$

$\frac{\Delta m^2}{E} \left[ 10^{-2} \text{eV}^2 \right]$

$\sin^2 2\theta$

$0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1$

$0 \quad 5 \quad 10 \quad 15 \quad 20$

$0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1$

$0 \quad 5 \quad 10 \quad 15 \quad 20$
Figure 1: Muon- to electron-like events ratio $\frac{N_e}{N_\mu}$ in different scenarios: two neutrino oscillation case (solid line), $N_{\nu}^e/N_{\nu}$, three neutrino oscillation case for normal hierarchy (dashed line), $N_{\nu}^e/N_{\nu}^{\nu e}$NR, three neutrino oscillation case for inverted hierarchy (dotted line), $N_{\nu}^e/N_{\nu}^{\nu e}$IH, and three neutrino oscillation case for vacuum (dot-dashed line), $N_{\nu}^e/N_{\nu}^{\nu e}$vac. They are depicted as a function of the cosine of the nadir angle, $\cos \theta_n$, for different values of $\theta_{13}$ and $\theta_{23}$: $\sin^2 2\theta_{13} = 0.05$ (left panels), 0.10 (right panels); $\sin^2 \theta_{23} = 0.36, 0.50, 0.64$ (from top to bottom).

Figure 2: Muon- to electron-like events ratio $\frac{N_e}{N_\mu}$ in different scenarios: two neutrino oscillation case (solid lines), $N_{\nu}^e/N_{\nu}$, three neutrino oscillation case for normal hierarchy (dashed lines), $(N_{\nu}^e/N_{\nu}^{\nu e})_{NR}$, three neutrino oscillation case for inverted hierarchy (dotted lines), $(N_{\nu}^e/N_{\nu}^{\nu e})_{IH}$, and three neutrino oscillation case for vacuum (dot-dashed lines), $(N_{\nu}^e/N_{\nu}^{\nu e})_{vac}$. They are depicted as a function of the cosine of the nadir angle, $\cos \theta_n$, for $E = 4 - 10$ GeV (left panel) and $2 - 100$ GeV for also PC events (right panel).
Figure 3: Muon- to electron-like events ratio, $N_\mu / N_e$, integrated for $\cos \theta_e > 0.4$, for different scenarios: two neutrino oscillation case (solid lines), $N_{\mu}^e / N_{\mu}^\nu$, three neutrino oscillation case for normal hierarchy (dashed lines), $N_{\mu}^e / N_{\mu}^\nu$, three neutrino oscillation case for inverted hierarchy (dotted lines), $N_{\mu}^e / N_{\mu}^\nu_{1e}$, and three neutrino oscillation case for vacuum (dot-dashed lines), $N_{\mu}^e / N_{\mu}^\nu_{1e}$. Left panel: as a function of $\sin^2 2\theta_{13}$ for $|\Delta m^2_{31}| = 3 \times 10^{-3} \text{eV}^2$. Right panel: as a function of $|\Delta m^2_{31}|$ for $\sin^2 2\theta_{13} = 0.10$. From top to bottom: $\sin^2 2\theta_{13} = 0.36, 0.50, 0.64$. 

Figure 4: $N_\mu^e / N_{\mu}^\nu$ as a function of $\cos \theta_e$, for $\sin^2 2\theta_{13} = 0.10$ (solid lines) and $\sin^2 2\theta_{13} = 0.05$ (dashed lines) for normal (upper lines) and inverted (lower lines) hierarchy, for $\sin^2 \theta_{23} = 0.5$ (left panel) and $\sin^2 \theta_{23} = 0.64$ (right panel). $N_{e}^{\nu}$ and $N_{\mu}^{\nu}$ are the e-like events in the case of three neutrino oscillation and in the case of no oscillations, respectively.
CPT Violation and the Nature of Neutrinos

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Abstract

In order to accommodate the neutrino oscillation signals from the solar, atmospheric, and LSND data, a sterile fourth neutrino is generally invoked, though the fits to the data are becoming more and more constrained. However, it has recently been shown that the data can be explained with only three neutrinos, if one invokes CPT violation to allow different masses and mixing angles for neutrinos and antineutrinos. We explore the nature of neutrinos in such CPT-violating scenarios. Majorana neutrino masses are allowed, but in general, there are no longer Majorana neutrinos in the conventional sense. However, CPT-violating models still have interesting consequences for neutrinoless double beta decay. Compared to the usual case, while the larger mass scale (from LSND) may appear, a greater degree of suppression can also occur.

Key words: Neutrino mass and mixing, double beta decay
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1 Introduction

In recent years, stronger and stronger experimental evidence for neutrino oscillations has been accumulating. As is well-known, this evidence would extend the Standard Model by requiring neutrino masses and mixings. While knowing the values of the mass and mixing parameters may be an important clue to physics beyond the Standard Model, more information is needed. For example,
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