ON THE CONNECTION OF LOW ENERGY CPV WITH LEPTOGENESIS & LFV DECAYS

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SUMMARY

1. LOW-ENERGY "OBSERVABLE" CPV:
   δ (oscillations), α_{21} and α_{31} ($\beta\beta_{0\nu}$-decay)
2. THE MINIMAL SEE-SAW MECHANISM
3. LEPTOGENESIS & LFV CHARGED LEPTON DECAYS
4. CONNECTION BETWEEN CPV PHASES IN LEPTOGENESIS, LFV & m_ν
5. TWO EXAMPLES: A NH MODEL AND THE QD CASE
6. CONCLUSIONS

Neutrino oscillations

MASSIVE NEUTRINOS

MIXING

FUNDAMENTAL QUESTION

▹ NATURE of NEUTRINOS

DIRAC particles ? MAJORANA particles

▹ ν-spectrum: hierarchical, inverted hierachical, mass quasi-degeneracy

▹ absolute value of ν-masses

▹ CP-violation in the lepton sector

▹ θ_{13}

CP-Violation in the Lepton Sector

In the case of 3-ν mixing, the lepton mixing matrix can be parametrized as:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

- one universal CPV phase: δ.

It is measurable in LBL experiments if $\sin^2\theta$ is not too small.

- two Majorana CPV phases α_{21} and α_{31}. They are physical only if neutrinos are Majorana particles. They can be present even if $\sin^2\theta = 0$. If CP is conserved we have $\alpha_{21}, \alpha_{31} = 0, \pm\pi$.

They are measurable in principle in $\Delta L = 2$ processes and in particular in $(\beta\beta)_{0\nu}$-decay (e.g., Rodejohann; S.P., Petcov; S.P., Petcov, Rodejohann).

In the lepton sector there exist 3 rephasing invariants : (Nieves, Pal; S.P., Petcov, Bilenky)

- Dirac r.i. J . It is present also in the case of Dirac-ν.

$$J = \mathrm{Im}\left(U_{\mu 2}\, U_{e3}\, U_{\mu 3}^*\, U_{e2}^*\right)$$

It depends only on δ.

- S_1, S_2 related to the Majorana nature of massive ν

$$S_1 = \mathrm{Im}\,(U_{e1}\, U_{e3}^*)(\xi_3^*\, \xi_1)$$
$$S_2 = \mathrm{Im}\,(U_{e2}\, U_{e3}^*)(\xi_3^*\, \xi_2)$$

It is possible to express the CPV phases α_{21}, α_{31} in terms of S_1 and S_2.

$$\cos\alpha_{31} = 1 - \frac{2\, S_1^2}{|U_{e1}|^2 |U_{e3}|^2}$$

$$\cos(\alpha_{31} - \alpha_{21}) = 1 - \frac{2\, S_2^2}{|U_{e2}|^2 |U_{e3}|^2}$$

If CP invariance holds

$$S_1, S_2, J = 0 \quad \text{or} \quad J = 0,\ \mathrm{Re}\,(U_{e1} U_{e3}^*)(\xi_3^*\xi_1) = 0,\ \mathrm{Re}\,(U_{e2} U_{e3}^*)(\xi_3^*\xi_2) = 0$$

CP–violation and ν-oscillations

For 3–nu mixing, under CP:

$$\nu_{a,b} \to \bar{\nu}_{a,b} \;\Rightarrow\; U_{ai} \to U_{ai}^* \quad (\delta \to -\delta)$$

If CPV: $P(\nu_a \to \nu_b, t) \neq P(\bar{\nu}_a \to \bar{\nu}_b, t)$.

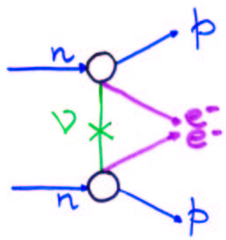
We define the CP-odd oscillation asymmetry in vacuum:

$$\Delta P_{ab} \equiv P(\nu_a \to \nu_b, t) - P(\bar{\nu}_a \to \bar{\nu}_b, t)$$
$$= \sin 2\theta_{12} \sin 2\theta_{23} s_{13} c_{13}^2 \sin\delta$$
$$\left[\sin\left(\frac{\Delta m^2_{21}}{2E} t\right) + \sin\left(\frac{\Delta m^2_{23}}{2E} t\right) + \sin\left(\frac{\Delta m^2_{31}}{2E} t\right)\right]$$

The presence of matter, which violates C, CP and CPT, may induce CPV effects ($P(\nu_a \to \nu_b, t) \neq P(\bar{\nu}_a \to \bar{\nu}_b, t)$) even without intrinsic CPV (e.g. even for $\delta = 0$). To disentangle matter CPV from fundamental CPV it is necessary to measure the energy dependence of oscillated signal, combine signals at different baselines and/or minimize matter effects using low-E beams and not too long baselines.

$(\beta\beta)_{0\nu}$-decay

$(\beta\beta)_{0\nu}$-decay has a special role in the study of neutrino properties. In the simplest case, it procedes through the exchange of light Majorana neutrinos:



The half-life time, $T^{1/2}_{0\nu}$, of the $(\beta\beta)_{0\nu}$-decay can be factorized as:

$$\left[T^{1/2}_{0\nu}(0^+ \to 0^+)\right]^{-1} \propto \left|M_F - g_A^2 M_{GT}\right|^2 |<m>|^2$$

- M_F, M_{GT} are nuclear matrix elements whose computation can be performed within the Nuclear Shell Model (NSM), the QRPA or other approximations. At the moment the uncertainty in the computation of M_F, M_{GT} is still very high, amounting up to a factor of 3 in $|<m>|$.

- $|<m>|$ is the effective Majorana mass parameter:

$$|<m>| \equiv \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}} \right|,$$

in the hypothesis of 3-ν mixing. For light neutrinos, $|<m>|$ contains all the dependence of $T^{1/2}_{0\nu}$ on the neutrino parameters.

In $|<m>|$, U_{ej} are the elements of the lepton mixing matrix U, m_j the masses of the massive neutrinos ν_j, α_{21} and α_{31} the CPV phases.

- Oscillation parameters:

i) $\Delta m^2_\odot$, Δm^2_{atm};

ii) $\theta_\odot$, θ_{13}.

They can be measured with good accuracy.

CRUCIAL: value of $\cos 2\theta_\odot$.

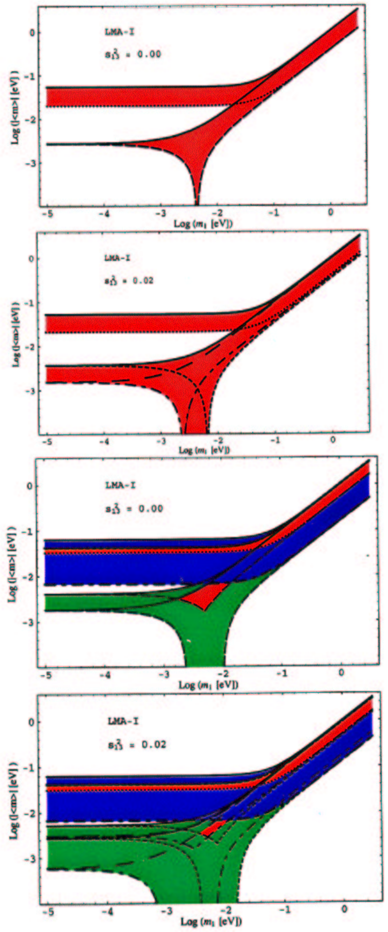
- Non-oscillation parameters:

i) m_1, mass of the lightest neutrino.

CRUCIAL: to determine the correct mass spectrum fixing the value of m_1 and resolving the discrete ambiguity $\Delta m^2_\odot \equiv \Delta m^2_{21}$, Δm^2_{32};

ii) α_{21} and α_{31}, Majorana CPV phases.

After SNO 2002 and KamLAND data (Pascoli, Petcov in prep.)



The red regions denote the "just CPV" regions.

Due to the experimental errors on the parameters and nuclear matrix elements uncertainties, determining that CP is violated in the lepton sector due to Majorana CPV phases is very difficult. (Barger et al.; S.P., Petcov, Rodejohann)

However it is possible if: (S.P., Petcov, Rodejohann)

- an experimental error on $|<m>|$ $< 15\%$;
- a large value of $\tan^2\theta_\odot \gtrsim 0.55$;
- in the QD spectrum, a high value of neutrino masses: $m_{\bar{\nu}_e} \gtrsim 0.70$ eV or $\Sigma \gtrsim 1.5$ eV and an experimental error smaller than $(10 \div 15)\%$;
- $\alpha_{21,(32)} \sim (\pi/2 - 3\pi/4)$ or $\alpha_{21,(32)} \sim (5\pi/4 - 3\pi/2)$.
- an uncertainty in the nuclear matrix elements which accounts to a factor ζ in $|<m>|$, $\zeta < 2$:

$$|<m>| \equiv \xi \left(|<m>|_{exp} \right)_{MIN}$$

THE SEE-SAW MECHANISM

We assume the ∃ of heavy right-handed Majorana neutrinos, ν_R singlets under $SU(3) \times SU(2) \times U(1)$, with Majorana mass M_R.

$$\mathcal{L}_\nu = - Y_\nu \overline{\nu_R} L \cdot H - \frac{1}{2} \overline{\nu_R^c} M_R \nu_R + h.c.$$

The vev of H^0, v, provides a Dirac mass term: $m_D = Y_\nu v$.

mass matrix : $\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$

which is diagonalized: $\mathcal{D}_{M+m} = V \mathcal{M} V^T$

Eigenstates: $\left.\begin{array}{l} \nu = U^T \nu_L + \nu_L^c U^* \\ N \cong \nu_R + \nu_R^c \end{array}\right\}$ Majorana fields

Masses: $d_m \cong - U^T Y_\nu^T M_R^{-1} Y_\nu U v^2$

$D_M \cong M_R$

In the case of 3 ν_L and 3 ν_R:

"high energy parameters"

D_M	3 real
Y_ν	9 real
	$9-3=6$ phases
	18 parameters

"parameters accessible at low energy"

D_m	3 real	:	$\Delta m^2_\odot$, Δm^2_{atm}, m_1, discrete ambiguity
U_{PMNS}	3 angles	:	$\theta_\odot$, θ_{atm}, θ_{13}
	3 phases	:	δ, α_{21}, α_{31}
	9 parameters		

9 parameters are missing of which 3 phases.

PARAMETRIZATIONS OF Y_ν

BIUNITARY PARAMETRIZATION:

Davidson-Ibarra
Ellis et al.
P., Petcov, Rodejohann

$$Y_\nu = V_R^\dagger D_Y V_L$$

$$= \begin{pmatrix} 1 & & \\ & e^{i\alpha_1'} & \\ & & e^{i\alpha_2'} \end{pmatrix} \tilde{V}_R^\dagger D_Y W \tilde{V}_L \qquad W = \begin{pmatrix} 1 & & \\ & e^{i\alpha_W} & \\ & & e^{i\beta_W} \end{pmatrix}$$

2 1 2 1 $\Rightarrow$ 6 phases

ORTHOGONAL PARAMETRIZATION

Casas-Ibarra
Ellis et al.
Branco et al.
P., Petcov, Yaguna

$$d_m^{\frac{1}{2}} d_m^{\frac{1}{2}} \simeq -U^T Y_\nu^T D_M^{-\frac{1}{2}} \underbrace{\,}_{RR^T} D_M^{-\frac{1}{2}} Y_\nu U v^2$$

$$\Rightarrow Y_\nu \cong i \frac{D_M^{\frac{1}{2}}}{v} R\, d_m^{\frac{1}{2}} U^\dagger$$

R is an orthogonal complex matrix: $R = O e^{iA}$ (PPY)

with $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$ $\qquad e^{iA} = 1\!\!1 - \frac{\cosh r - 1}{r^2} A^2 + i \frac{\sinh r}{r} A$, $r = \sqrt{a^2+b^2+c^2}$

TRIANGULAR PARAMETRIZATION

Branco et al.

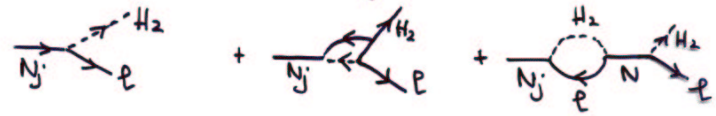
$$Y_\nu = Y_\Delta U_Y \qquad , \quad Y_\Delta = \begin{pmatrix} Y_{\Delta_1} & Y_{12} & Y_{13} \\ 0 & Y_{\Delta_2} & Y_{23} \\ 0 & 0 & Y_{\Delta_3} \end{pmatrix}$$

Y_1, Y_2, Y_3 real

LEPTOGENESIS

Fukugita, Yanagida
Covi, Roulet, Vissani
Buchmuller, Plumacher

The out-of-equilibrium decay of N_R can generate a lepton asymmetry which is then converted into Y_B.
The decay asymmetry is given by the interference of tree-level and one-loop diagrams:



$$\varepsilon_i \equiv \frac{\Gamma(N_i \to \ell H) - \Gamma(N_i \to \ell^c H^c)}{\Gamma(N_i \to \ell H) + \Gamma(N_i \to \ell^c H^c)}$$

$$\simeq -\frac{1}{8\pi} \frac{1}{(Y_\nu Y_\nu^\dagger)_{ii}} \sum_{i \neq j} \mathrm{Im}(Y_\nu Y_\nu^\dagger)_{ij}^2 \left[f\left(\frac{M_j^2}{M_i^2}\right) + g\left(\frac{M_j^2}{M_i^2}\right) \right]$$

$f(x) = \sqrt{x} \left[1 - (1+x) \ln\left(\frac{1+x}{x}\right) \right]$ and $g(x) = \frac{\sqrt{x}}{1-x}$

The lepton asymmetry is

$$Y_L = \frac{n_L - \bar{n}_L}{s} \propto \varepsilon_i$$

$Y_B = C\, Y_L$ where $C \sim O(1)$ ($C = -0.35$ in MSSM).

LFV DECAYS: $\ell_i \to \ell_j \gamma$

Hisano et al.
Casas, Ibarra
Tanimoto et al.

In the SM, they are very much suppressed.

We consider the supersymmetrization of the minimal see-saw model:

$$W = W_0 - \frac{1}{2} \nu_R^{cT} M \nu_R^c + \nu_R^c Y_\nu L \cdot H_2$$

$$-\mathcal{L}_{soft} = (m_L^2)_{ij} \tilde{L}_i^\dagger \tilde{L}_j + (m_{eR}^2)_{ij} \tilde{e}_{Ri}^* \tilde{e}_{Rj} + \ldots + (A_{ij}^e H_d e_{Ri}^* \tilde{L}_j + \ldots + h.c.$$

We assume universality at M_X scale:

$$(m_L^2)_{ij} = (m_{e_R}^2)_{ij} = (m_{\tilde{\nu}}^2)_{ij} = \delta_{ij}\, m_0^2$$

$$A^e = Y_e\, a_0\, m_0$$

$\vdots$

In the leading-log approximation, the off-diagonal soft terms at low energy induced by the RGE are given by:

$$(m_{\tilde{L}}^2)_{ij} \cong -\frac{1}{8\pi}(3m_0^2 + A_0^2)\,(Y_\nu^\dagger Y_\nu)_{ij} \log\frac{M_X}{M_R}$$

The BR for $\ell_i \to \ell_j \gamma$ reads:

$$BR(\ell_i \to \ell_j \gamma) \sim \frac{\alpha^3}{G_F^2 m_S^8} \left| -\frac{1}{8\pi}(3m_0^2 + A_0^2) \log\frac{M_X}{M_R} \right|^2 \left|(Y_\nu^\dagger Y_\nu)_{ij}\right|^2 \tan^2\beta$$

① LEPTOGENESIS

$$Y_\nu Y_\nu^\dagger = \phi^* \tilde{V}_R^\dagger D_Y^2 \tilde{V}_R \phi$$

$$\cong D_M^{\frac{1}{2}} R \frac{D_m}{v^2} R^\dagger D_M^{\frac{1}{2}}$$

$$= Y_\Delta Y_\Delta^\dagger$$

depends on 3 phases

U disappears !?

② LFV DECAYS

$$Y_\nu^\dagger Y_\nu = \tilde{V}_L^\dagger D_Y^2 \tilde{V}_L$$

$$\cong U D_m^{\frac{1}{2}} R^\dagger D_M R D_m^{\frac{1}{2}} U^\dagger v^{-2}$$

$$= U^\dagger Y_\Delta^\dagger Y_\Delta U_Y$$

1 phase

③ m_ν

$$m_\nu \cong -V_L^T D_Y V_R^* D_M^{-1} V_R^\dagger D_Y V_L$$

$$= U^* d_m U^\dagger$$

$$= U_Y^T Y_\Delta^T D_M^{-1} Y_\Delta U_Y$$

We can draw the conclusions:

① $\Longrightarrow$ R enters in leptogenesis $R(\alpha_R)$

U does not enter "esplicitly" in leptogenesis

③ $\Longrightarrow$ $U(\alpha_R, \alpha_L, \alpha_W)$

So in general, if $\alpha_R \neq 0 \Rightarrow U$ complex

Es. $D_M \propto D_m$, $R = e^{iA}$

$$② \Rightarrow \quad v^2 \tilde{V}_L^\dagger D_Y^2 \tilde{V}_L = -U D_m^{\frac{1}{2}} R^\dagger D_M R D_m^{\frac{1}{2}} U^\dagger \stackrel{!}{=} -U \Phi_R^* \tilde{V}_R^\dagger D_Y^2 \tilde{V}_R \Phi_R U^\dagger$$

$$\Longrightarrow \quad U = i \tilde{V}_L W \tilde{V}_R \Phi$$

Note: α_R denote the phases in $\Phi, \tilde{V}_R$;

α_L . . . . $\tilde{V}_L$

α_W . . . . W.

Can we have leptogenesis without CPV in U?

We need V_R complex $(R \neq 0)$

U real

$U(\alpha_R, \alpha_L, \alpha_W) \Longrightarrow$ the phases in $\tilde{V}_L(\alpha_L)$ and $W(\alpha_W)$ combined with the angles in $\tilde{V}_L$ and $\tilde{V}_R$ and D_Y, need to cancel the dependence on α_R.

$$J_{CP} \propto - \frac{\mathrm{Im}\,(h_{12}\, h_{23}\, h_{31})}{\Delta m^2_{21}\ \Delta m^2_{31}\ \Delta m^2_{32}} \qquad h = m_\nu^\dagger m_\nu$$

$$\propto -\mathrm{Im}\Big[\big(\tilde{V}_L^\dagger W^* D_Y \tilde{V}_R \Phi^* M_R^{-1} \Phi^* \tilde{V}_R^T D_Y^2 \tilde{V}_R^* M_R^{-1} \Phi^2 \tilde{V}_R^\dagger D_Y \tilde{V}_L \big)_{12} \big(\quad \cdots \quad \big)_{23} \big(\quad \cdots \quad \big)_{31} \Big]$$

Can we have NO leptogenesis but CPV in U ?

We need V_R real
U complex

As $U(\alpha_R, \alpha_L, \alpha_W)$, it is sufficient to have
$V_R = O_R$ and $\tilde{V}_L$ or W complex.

Example

$Y_\nu = W D_Y \tilde{V}_L$ 3 phases

NO LEPTOGENESIS $V_R = 1\!\!1 \implies R = 1\!\!1$

LFV: $Y_\nu^\dagger Y_\nu = \tilde{V}_L^\dagger D_Y^2 \tilde{V}_L$

m_ν: $-U^* d_m U^\dagger \cong \tilde{V}_L^T W D_Y^2 D_M^{-1} W \tilde{V}_L$

$U = \tilde{V}_L^\dagger W^*$

$\delta_{DIRAC} = \delta(\alpha_L)$

$\alpha_{21}, \alpha_{31} = f(\alpha_W)$

Can we have a "direct" connection between leptogenesis and CPV in U ?

As $U(\alpha_R, \alpha_L, \alpha_W)$, one of the conditions must be satisfied:

① $\alpha_L = 0$;
② $\alpha_L = \alpha_L(\alpha_R)$;
and the same for α_W;
③ the dependence on α_L, α_W is suppressed by the other parameters or hidden in the unphysical phases in U.

Many examples in the literature:

- Branco et al.; Davidson & Ibarra : $V_L = 1\!\!1$, $Y_\nu = V_R^\dagger D_Y$
  ~~LFV~~, leptogenesis o.k.
  $\alpha_R(\delta, \phi)$
- Branco et al.; Rebelo; Ellis et al.; Tanimoto et al.; Lisbon-Saclay Coll.; Davidson-Ibarra; Frampton, Glashow, Yanagida; King; P., Petcov, Rodejohann; P., Petcov, Yaguna; Rodejohann; Paschos et al.; Nezri & Orloff...

THE CASE OF HIERARCHICAL SPECTRUM

P., Petcov, Rodejohann
hep-ph/0302054

Assumptions: $M_1 \ll M_2 \ll M_3$

$dy_1 \ll dy_2 \ll dy_3$

$$V_L^\dagger = \begin{pmatrix} c_{1L} c_{3L} & s_{1L} c_{3L} & s_{3L} e^{-i\delta} \\ -s_{1L} c_{2L} - s_{1L} s_{2L} s_{3L} e^{i\delta} & c_{1L} c_{2L} - s_{1L} s_{2L} s_{3L} e^{i\delta} & s_{2L} c_{3L} \\ s_{1L} s_{2L} - c_{1L} c_{2L} s_{3L} e^{i\delta} & -c_{1L} s_{2L} - s_{1L} c_{2L} s_3 e^{i\delta} & c_{2L} c_{3L} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha_w} & \\ & & e^{i\beta_w} \end{pmatrix}$$

$s_{1L} \sim 10^{-1} \gg s_{2L} \sim 10^{-2} \gg s_{3L}$
and analogously for V_R.

$$Y_\nu^\dagger = (V_R^\dagger D_Y V_L)^\dagger \simeq \begin{pmatrix} -dy_2' s_{1L} s_{1R} & e^{i\alpha_R} dy_2' s_{1L} & e^{i(\beta_R+\delta_R-\delta_L)} dy_3' s_{3L} \\ -dy_2' s_{1R} & e^{i\alpha_R} dy_2' & e^{i(\beta_R+\delta_R)} dy_3' s_{2L} \\ dy_3' (s_{1R} s_{2R} - e^{i\delta_R} s_{3R}) & -e^{i\alpha_R} dy_3' s_{2R} & e^{i(\beta_R+\delta_R)} dy_3' \end{pmatrix}$$

with $dy_1' = dy_1$, $dy_2' = dy_2 e^{i\alpha_w}$, $dy_3' = dy_3 e^{i\beta_w}$

LEPTOGENESIS:

$$\mathrm{Im}\,(Y_\nu Y_\nu^\dagger)_{12}^2 \cong (y_2^2 + y_3^2 s_{2R}^2)^2 s_{1R}^2 \quad \sin 2\alpha_R$$

$$\mathrm{Im}\,(Y_\nu Y_\nu^\dagger)_{13}^2 \cong (y_3^2)\, s_{1R}^2 s_{2R}^2 \quad \sin 2(\beta_R+\delta_R)$$

$$\varepsilon_1 \simeq -\frac{3}{16\pi}\left((y_2^2 + s_{2R}^2 y_3^2) \sin 2\alpha_R \frac{M_1}{M_2} + \frac{y_3^4 s_{2R}^2}{y_2^2 + y_3^2 s_{2R}^2} \sin 2(\beta_R+\delta_R) \frac{M_1}{M_3}\right)$$

$Y_B \sim 10^{-10}$ implies

$y_3 v = m_{D_3} \sim 10^2$ GeV ; $M_1 \sim 10^9$ GeV , $M_2 \sim 10^{10}$ GeV $\ll M_3$.

LFV DECAYS

$\mu \to e\gamma$: $(Y_\nu^\dagger Y_\nu)_{12} \simeq y_2^2 s_{1L}$

$\tau \to e\gamma$: $(Y_\nu^\dagger Y_\nu)_{13} \simeq y_3^2 s_{3L} e^{i\delta_L}$

$\tau \to \mu\gamma$: $(Y_\nu^\dagger Y_\nu)_{32} \simeq y_3^2 s_{2L} \frac{v^2}{v^2}$

$\Rightarrow$ $BR(\tau \to \mu\gamma) \simeq 10^2 BR(\tau \to e\gamma) \simeq 10^6 BR(\mu \to e\gamma)$

LOW-ENERGY CPV: δ, α_{21}, α_{31}.

$(\beta\beta)_{0\nu}$: $$\langle m \rangle \simeq \left(y_2^2 s_{1L}^2 e^{2i\alpha_W} \left(\frac{s_{1R}^2}{M_1} + \frac{e^{2i\alpha_R}}{M_2} \right) \right) v^2 + O\left(\frac{m_3^2}{M_3} \right)$$

Neglecting M_3^{-1}, m_ν can be taken real.

$|\langle m \rangle|$ depends on α_R, the leptogenesis phase, unless $\frac{s_{1R}^2}{M_1} \ll\gg \frac{1}{M_2}$.

$$|\langle m \rangle| \sim (0.001 \div 0.01) \text{ eV}$$

δ:

$$J_{CP} \propto 10^{-2} \sin\left(\alpha_W - (\beta_W + \delta_L) + 2\alpha_R - 2(\beta_R + \delta_R) \right)$$

No direct connection between the leptogenesis phases α_R, $\beta_R + \delta_R$ and δ can be established.

A fine tuning is required in order to have leptogenesis and $J_{CP} \simeq 0$.

QUASI-DEGENERATE ν MASS SPECTRUM

P., Petcov, Yaguna
hep-ph/0301095

Assumptions: $m_1 \simeq m_2 \simeq m_3 \gg \Delta m^2_\odot, \Delta m^2_{atm}$

$D_M \simeq M_R \cdot \mathbb{1}$ + small deviations

We use the orthogonal parametrization:

$$Y_\nu = i \frac{1}{v} D_M^{\frac{1}{2}} R D_m^{\frac{1}{2}} U^\dagger = i \frac{1}{v_u} D_M^{\frac{1}{2}} e^{iA} D_m^{\frac{1}{2}} U^\dagger$$

LEPTOGENESIS

$$Y_\nu Y_\nu^\dagger = \frac{D_M d_m}{v^2} e^{2iA}$$

$$\epsilon_1 = \epsilon_2 \simeq \frac{1}{\pi} \frac{D_M d_m}{v^2} \frac{abc}{M_1 - M_2}$$

$$\epsilon_3 \simeq -\frac{2}{\pi} \frac{D_M d_m}{v^2} \frac{abc}{M_1 - M_3}$$

$$\frac{n_B}{s} \simeq 1.4 \times 10^{-8} \left(\frac{m_\nu}{0.1\,\text{eV}} \right) \left(-\frac{abc}{M_1 - M_2} \right) \left(Br^{(1)} + Br^{(2)} \right)$$

$$\Longrightarrow \quad |abc| \simeq 10^{-5}$$

LFV DECAYS:

$$Y_\nu^\dagger Y_\nu \sim \frac{1}{v^2} D_M d_m \left(U e^{2iA} U^\dagger \right)$$

For real R, $(A=0)$, the BR are strongly suppressed (Casas & Ibarra, Tanimoto et al.) for QD neutrinos.

$$(Y_\nu^\dagger Y_\nu)_R = \frac{D_M}{v^2} \left(U_{e2} U_{e'2}^* \frac{\Delta m^2_\odot}{2 d_m} + U_{e3} U_{e'3}^* \frac{\Delta m^2_{atm}}{2 d_m} \right)$$

For R complex, there is an enhancement of the value of the BR:

$$(Y_\nu^\dagger Y_\nu)_{12} \propto \frac{D_M d_m}{v^2} \left[-a \left(c_{12}^2 e^{i\alpha_{21}} + s_{12}^2 e^{-i\alpha_{21}} \right) c_{23} - e^{i\alpha_{31}} s_{23} \left(b c_{12} + c s_{12} e^{-i\alpha_{21}} \right) \right]$$

$$(Y_\nu^\dagger Y_\nu)_{13} \propto \frac{D_M d_m}{v^2} \left[a \left(c_{12}^2 e^{i\alpha_{21}} + s_{12}^2 e^{-i\alpha_{21}} \right) s_{23} - e^{i\alpha_{31}} s_{23} \left(b c_{12} + c s_{12} e^{-i\alpha_{21}} \right) \right]$$

$$(Y_\nu^\dagger Y_\nu)_{23} \propto \frac{D_M d_m}{v^2} \left[-2ia\, s_{12} c_{12} s_{23} c_{23} \sin\alpha_{12} + \left(b s_{12} - c c_{12} e^{i\alpha_{21}} \right) \left(s_{23}^2 e^{i\alpha_{31}} + c_{23}^2 e^{-i\alpha_{31}} \right) \right]$$

$$|(Y_\nu^\dagger Y_\nu)_{21}|^2 \sim |(Y_\nu Y_\nu)_{31}|^2_R \simeq \frac{M_R^2 m_\nu^2}{v^4} \begin{cases} 6 \times 10^{-6} & s_{13} = 0.2 \\ 1.4 \times 10^{-8} & s_{13} = 0 \end{cases}$$

$$|(Y_\nu^\dagger Y_\nu)_{12}|^2_{\mathbb{C}} \simeq \frac{M_R^2 m_\nu^2}{v^4} \begin{cases} 0.34 & (a,b,c) = (0.2, -0.4, 0.5) \\ 0.81 & (a,b,c) = (0.4, 0.3, 0.2) \end{cases}$$

CONCLUSIONS

- CPV IN THE LEPTON SECTOR IS PARAMETRIZED BY THE δ PHASE AND 2 MAJORANA PHASES. THEY ARE MEASURABLE, IN PRINCIPLE, IN ν-OSCILLATIONS (δ) AND $(\beta\beta)_{0\nu}$-DECAY (α_{21}, α_{31}). IT WILL BE A CHALLENGING TASK.
- IN THE CONTEXT OF THE MINIMAL SEE-SAW MECHANISM, LEPTOGENESIS LFV ℓ-DECAYS MAY PROVIDE ADDITIONAL INFORMATION ON THE SEE-SAW PARAMETERS.
- PARAMETRIZATIONS OF Y_ν: $Y_\nu = V_R^\dagger D_Y V_L$, $Y_\nu \simeq \frac{i}{v} D_M^{\frac{1}{2}} R D_m^{\frac{1}{2}} U^\dagger$, $Y_\nu = Y_\Delta$
  LEPTOGENESIS DEPENDS ON V_R, R, Y_Δ.
  LFV-ℓ DECAYS DEPENDS ON V_L, R & U, Y_Δ & U_Y.
  m_ν DEPENDS ON V_R & V_L, U, Y_Δ & U_Y.
  THIS IMPLIES $R(\alpha_R)$, $U(\alpha_R, \alpha_L, \alpha_W)$.
  THEREFORE THERE IS NOT AN "EXPLICIT" CONNECTION BETWEEN δ, α_{21}, α_{31} and α_R. HAVING U REAL (NO CPV IN m_ν) WHILE LEPTOGENESIS REQUIRES SPECIAL CANCELLATIONS OF THE PARAMETERS. A "DIRECT CONNECTION" BETWEEN δ, α_{21}, α_{31} and α_R MAY BE OBTAINED: i) $\alpha_L, \alpha_W = 0$ OR $f(\alpha_R)$; ii) α_L, α_W ARE SUPPRESSED BY PARAMETERS