

The Charm and Beauty of Lattice **QCD**

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KITP seminar, Apr. 8, 2003

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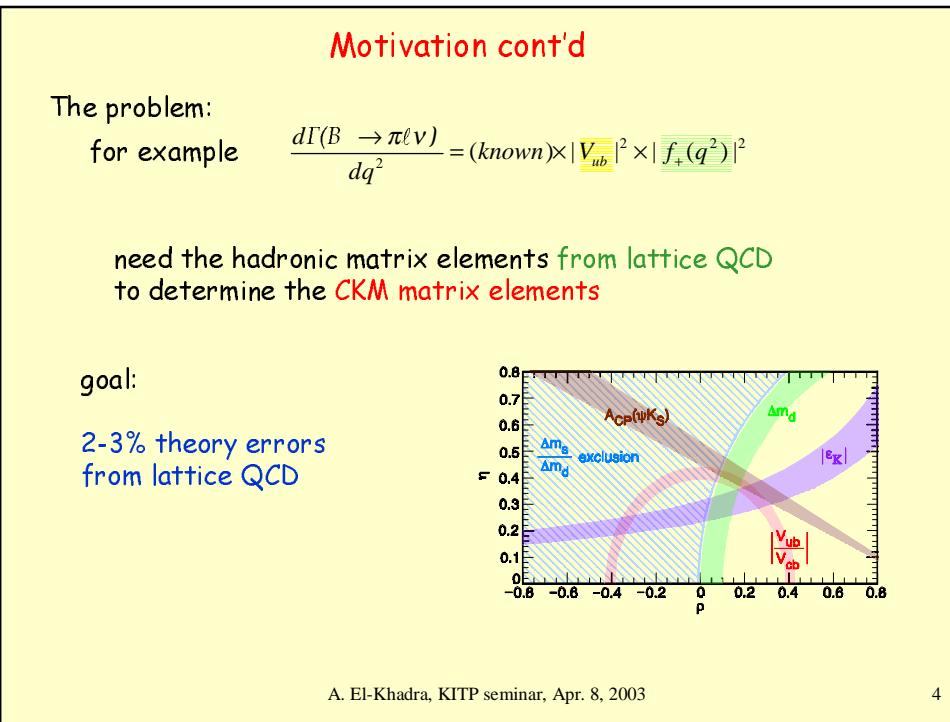
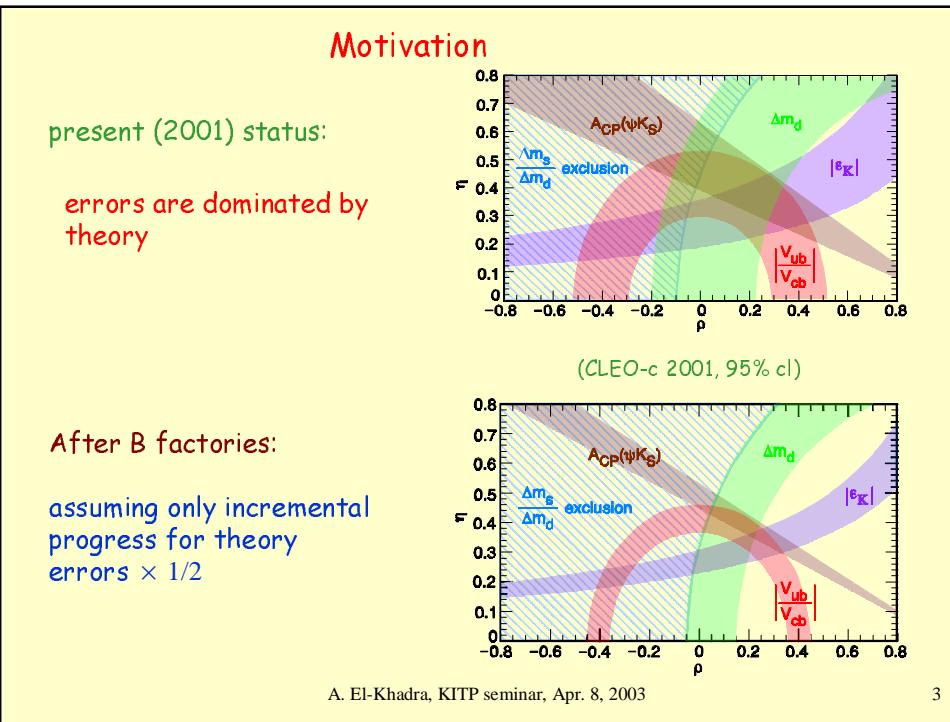
Outline:

- Motivation
- Introduction to lattice QCD
- f_B and B_B
- Semileptonic B meson decays
 - $B \rightarrow D, D^* l\nu$
 - $B \rightarrow \pi l\nu$
- Some recent developments
- Prospects for the near future
- Issues
- Conclusions

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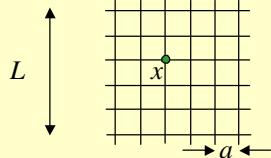
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Introduction

discretize space-time ...



fermion field lives on site: $\psi(x)$

gauge field lives on link: $U_\mu(x) = \exp[iagA_\mu(x)]$

... discretize the QCD action (Wilson)

e.g. discrete derivative $\Delta_\mu \psi = \frac{1}{2a} [\psi(x+a\hat{\mu}) - \psi(x-a\hat{\mu})]$

in QCD Lagrangian $\bar{\psi} \partial_\mu \psi = \bar{\psi} \Delta_\mu \psi + a^2 c(a) \bar{\psi} \Delta_\mu^3 \psi + O(a^4)$

where $c(a) = c(a; \alpha_s, m)$ depends on the QCD parameters
calculable in pert. theory

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Lattice Lagrangian $\mathcal{L}^{\text{lat}} = \sum_i c_i(a; \alpha_s, m) O_i(\psi, \bar{\psi}, U_\mu)$

• in general: $\mathcal{L}^{\text{lat}} = \mathcal{L}^{\text{cont}} + O(a^n) \quad n \geq 1$

errors scale with the typical momenta of the particles,
e.g. $(\Lambda_{\text{QCD}} a)^n$ for gluons and light quarks. keep $1/a \propto \Lambda_{\text{QCD}}$
typical lattice spacing $a \sim 0.1 \text{ fm}$.

• Improvement: add more terms to the action to make n large

• gluons:

Wilson: a^2 errors ($n = 2$)

Lüscher + Weisz: $\alpha_s^2 a^2, a^4$ errors

• light quarks ($am \bar{1}$):

Wilson: a errors ($n = 1$)

Clover (SW): $\alpha_s^2 a$ errors, a^2 errors (Sheikholeslami+Wohlert)

staggered: a^2 errors

improved staggered (Asqtad): $\alpha_s a^2$ errors (Lepage, MILC)

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Lattice Lagrangian $\mathcal{L}^{\text{lat}} = \sum_i c_i(a; \alpha_s, m) O_i(\psi, \bar{\psi}, U_\mu)$

- heavy quarks ($m_Q \gg \pi, \Lambda_{\text{QCD}}$ and $am_Q \ll 1$):

lattice NRQCD (Lepage, et al., Caswell+Lepage):

discretize NRQCD lagrangian: valid when $am_Q > 1$

corresponds to expanding c_i in $1/m_Q$: $c_i = c_i^{(0)} + 1/(am_Q) c_i^{(1)} + \dots$

errors: $\sim (ap)^n, (p/m_Q)^n$

Fermilab (Kronfeld, Mackenzie, AXK):

start with rel. Wilson action (+ improvement)

keep full mass dependence of c_i ,

add time-space asymmetry

smoothly matches heavy and light mass limits: valid for all am_Q

errors: $\sim (ap)^n, (p/m_Q)^n$

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systematic errors

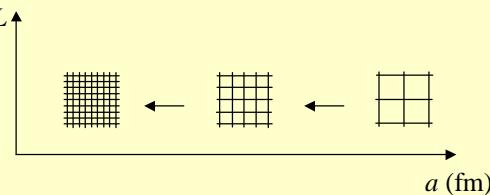
- finite lattice spacing, a :

$$\langle \mathcal{O} \rangle^{\text{lat}} = \langle \mathcal{O} \rangle^{\text{cont}} + O(a^n)$$

take continuum limit:

- by brute force:

computational effort grows like $\sim (L/a)^6$



- by improving the action:

computational effort grows much more slowly

→ improved actions are much better ...

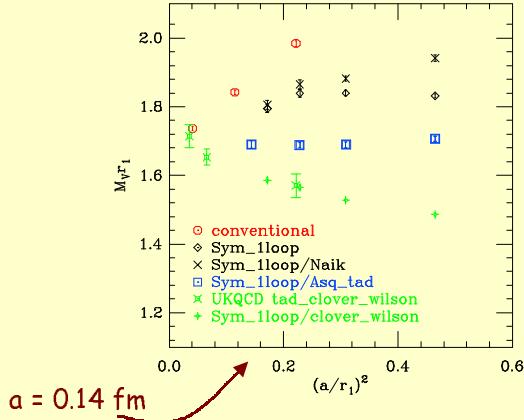
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MILC 1999: compare different light quark actions

example: ρ meson mass vs. a^2



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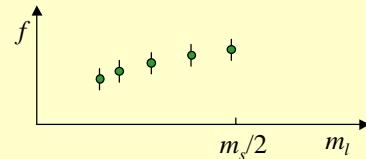
systematic errors, cont'd

- chiral extrapolation, m_l dependence:

In numerical simulations, $m_l > m_{u,d}$ because of the computational cost for small m .

use chiral perturbation theory to extrapolate to $m_{u,d}$

need $m_l < m_s/2$ and several different values for m_l
(easier with staggered than Wilson-type actions)



- finite Volume:

$L \sim 2$ fm okay for B 's.

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systematic errors, cont'd

- n_f dependence

$n_f = 0$:  quenched approximation

introduces systematic error $\sim 10 - 30 \%$

- $n_f \neq 0$: computationally difficult

keep a large, $a \approx 0.1$ fm  need improved actions

so far: $n_f = 2$ with staggered and SW fermions (a^2 errors)

new MILC (2002):

$n_f = 3$ with $m_s \neq m_{light}$ and $m_{light} = m_s/8, m_s/4, \dots, m_s/2, \dots, m_s$

using an improved staggered action ($\alpha_s a^2$ errors)

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systematic errors, cont'd

- perturbation theory:

for example $\langle J_\mu^{\text{cont}} \rangle = Z^{\text{lat}} \langle J_\mu^{\text{lat}} \rangle$

Renormalization: from $p \sim \pi/a$
calculable in perturbation theory if a is small enough.

For $a \sim 0.1$ fm, $\alpha_s \sim 0.25$:
with 1-loop pert. thy, errors $\sim O(\alpha_s^2) \sim 5\%$

Need 2-loop lattice perturbation theory for \sim few% errors
difficult, especially with improved actions!

use:

- automated perturbation theory (Lüscher+Weisz, Lepage, et al)
- computational methods (di Renzo, et al)
- numerical methods (Lepage, Mackenzie, Trottier, ...)
- nonperturbative methods (Alpha, APE, ...)

example: static self energy to 3-loops (Trottier, et al, di Renzo+Scorzato)
 α_s to 2-loops (in progress)

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What are the "easy" lattice calculations ?

For stable (or almost stable) hadrons, masses and amplitudes
with no more than one initial (final) state hadron,
for example:

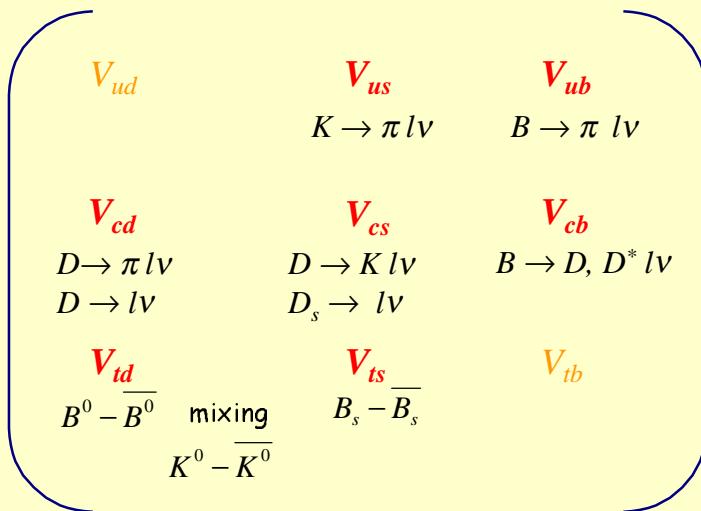
- π, K, D, D_s, B, B_s mesons
masses, decay constants, weak matrix elements for mixing,
semileptonic and rare decays
- charmonium and bottomonium ($\eta_c, J/\psi, h_c, \dots, \eta_b, Y(1S), Y(2S), \dots$)
states below open D/B threshold
masses, leptonic widths, electromagnetic matrix elements

This list includes most of the important quantities for CKM
physics. Excluded are ρ mesons and other resonances.

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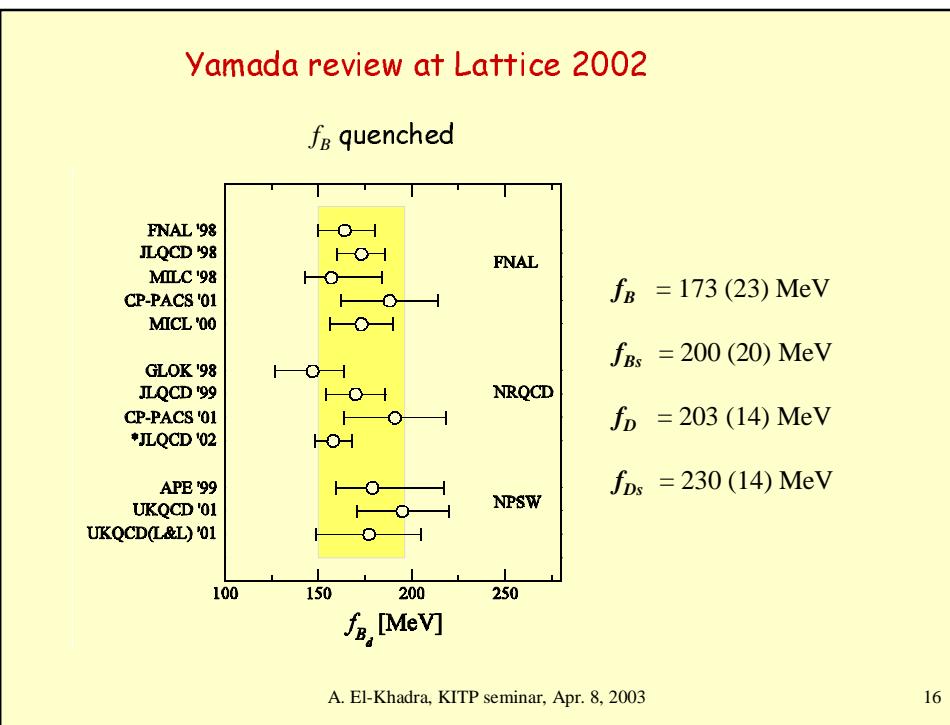
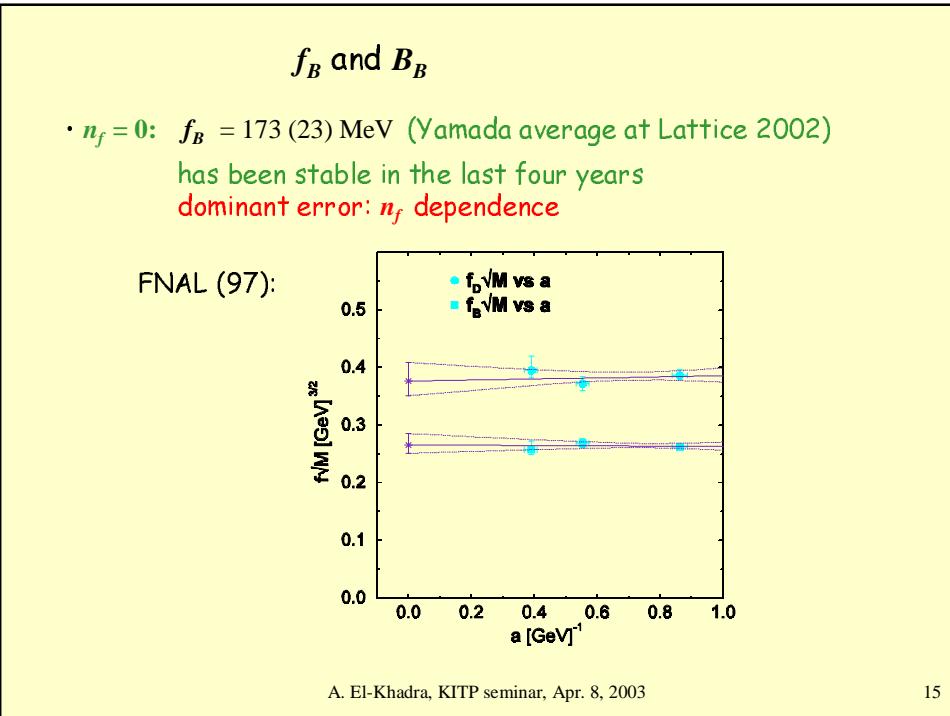
"easy" quantities for most CKM elements ...



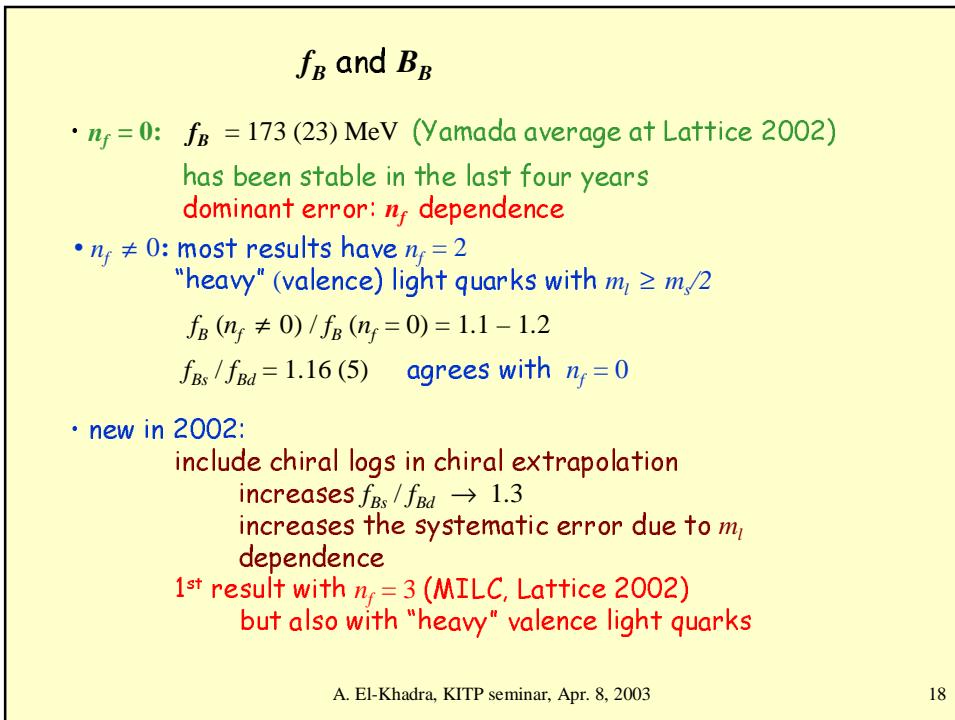
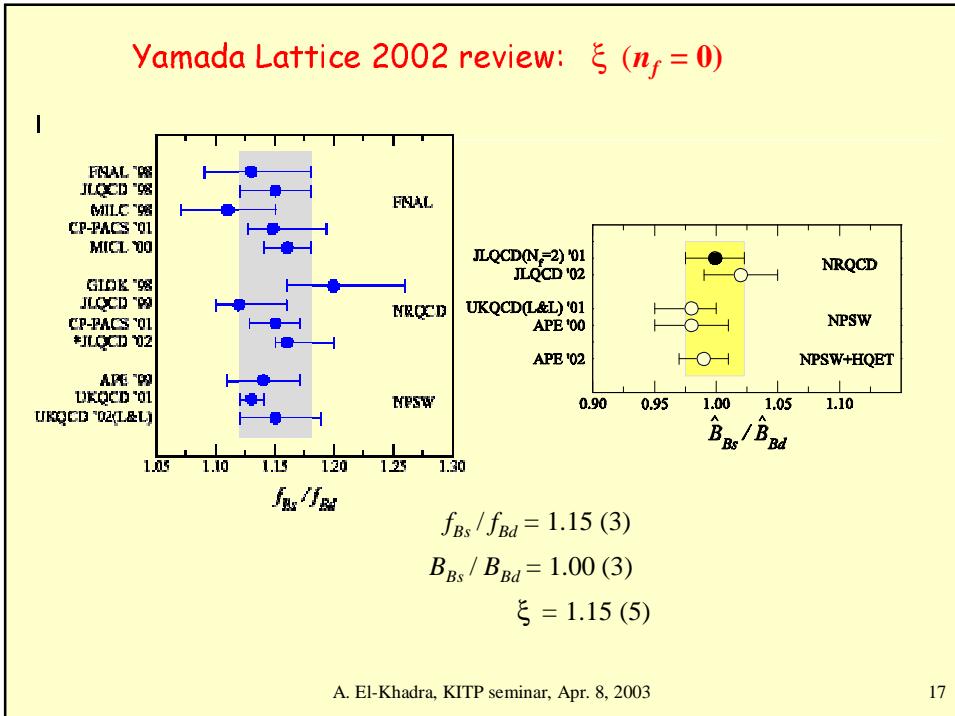
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m_l dependence: chiral logarithms

- When $m_l > m_{u,d}$, use ChPT to extrapolate $f_\pi(m_l)$ to the physical u,d quark masses (Gasser+Leutwyler, Sharpe, Bernard, Golterman, Shoreish):

$$f_\pi(m_l) = f(1 + ax_l + bx_l \log x_l + cx_l^2 \dots)$$

where $x_l = 2 B_0 m_l / (4\pi f)^2 \rightarrow 1$, and b is known.

chiral log becomes important for small $m_l \rightarrow$ need $m_l < m_s/2$

- for f_B (Grinstein, et al, Booth, Sharpe, Zhang):

$$f_B(m_l) = f(1 + ax_l + b(1 + 3g^2)x_l \log x_l \dots)$$

where $g \sim g_{B^*B\pi}$ is the $B^*B\pi$ coupling, which is poorly known.

from the D^* width (CLEO) $g^2 \sim 0.35$.

- the problem:

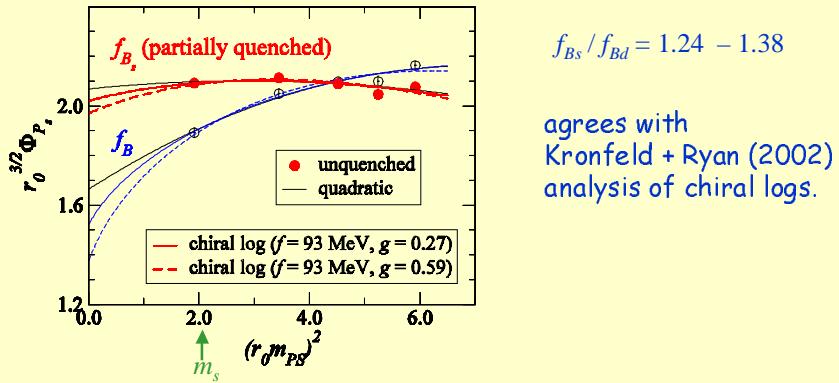
(valence) $m_l > m_s/2$ in most simulations to date

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chiral logarithms cont'd

Hashimoto (Lattice 2002)



$$f_{B_s}/f_{B_d} = 1.24 - 1.38$$

agrees with
Kronfeld + Ryan (2002)
analysis of chiral logs.

- Kronfeld+Ryan: chiral logs are small for B_B
- Becirevic, et al: use double ratios: $(f_{B_s}/f_{B_d})/(f_K/f_\pi)$
- solution: simulations with $m_l < m_s/2$

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Semileptonic B meson decays

$B \rightarrow D, D^* l \nu$:

e.g. $\frac{d\Gamma(B \rightarrow D^* l \nu)}{d\omega} = (\text{known}) \times (\omega^2 - 1)^{1/2} \times |\mathbf{V}_{cb}|^2 |\mathcal{F}_{B \rightarrow D^*}(\omega)|^2$

- calculate $\mathcal{F}(1)$ in lattice QCD from double ratios,

e.g.

$$R_+ = \frac{\langle D | V_0 | \bar{B} \rangle \langle \bar{B} | V_0 | D \rangle}{\langle D | V_0 | D \rangle \langle \bar{B} | V_0 | \bar{B} \rangle} = |h_+(1)|^2$$

- when $m_b = m_c$: $R \equiv 1$.
- systematic errors scale with $R - 1$, not R .
- with $O(a)$ $O(1/m)$ improved action R is correct through $1/m^2$ (Kronfeld)

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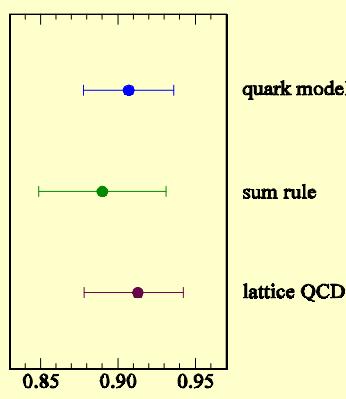
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$B \rightarrow D, D^* l \nu$ cont'd

Hashimoto, et al
(FNAL):

$$\mathcal{F}_{B \rightarrow D}(1) = 1.058 \pm 0.016^{+0.014}_{-0.005}$$

$$\mathcal{F}_{B \rightarrow D^*}(1) = 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003+0.000+0.006}_{-0.014-0.016-0.014}$$



stat.	pert.	a	m_l	n_f
	thy			

- FNAL result is obtained at $n_f=0$
- $n_f=3$ will be done soon
- chiral extrapolation error will be reduced by having $m_l < m_s/2$

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$B \rightarrow \pi l \nu$

- $p_\pi(q^2)$ dependence: $\bar{p}_\pi \neq 0$

$$\langle \pi | V_\mu | B \rangle^{\text{lat}} = \langle \pi | V_\mu | B \rangle^{\text{cont}} + O(ap_\pi)^n$$

$$\Rightarrow p_\pi \delta 1 \text{ GeV}$$

improved actions help (keep n large)

- experiment: measure $d\Gamma/dp_\pi$ for $0 < \bar{p}_\pi < m_B/2$

✗ old solution:

extrapolate to $p_\pi < m_B/2$ ($q^2 = 0$) by assuming shape
(pole dominance) — introduces model dependence!

✓ better solution:

limit recoil momentum range, e.g.

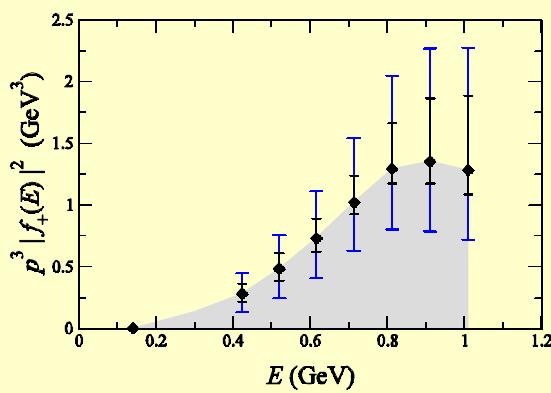
$$400 \text{ MeV} < p_\pi < 800 \text{ MeV}$$

Note: okay for D decays

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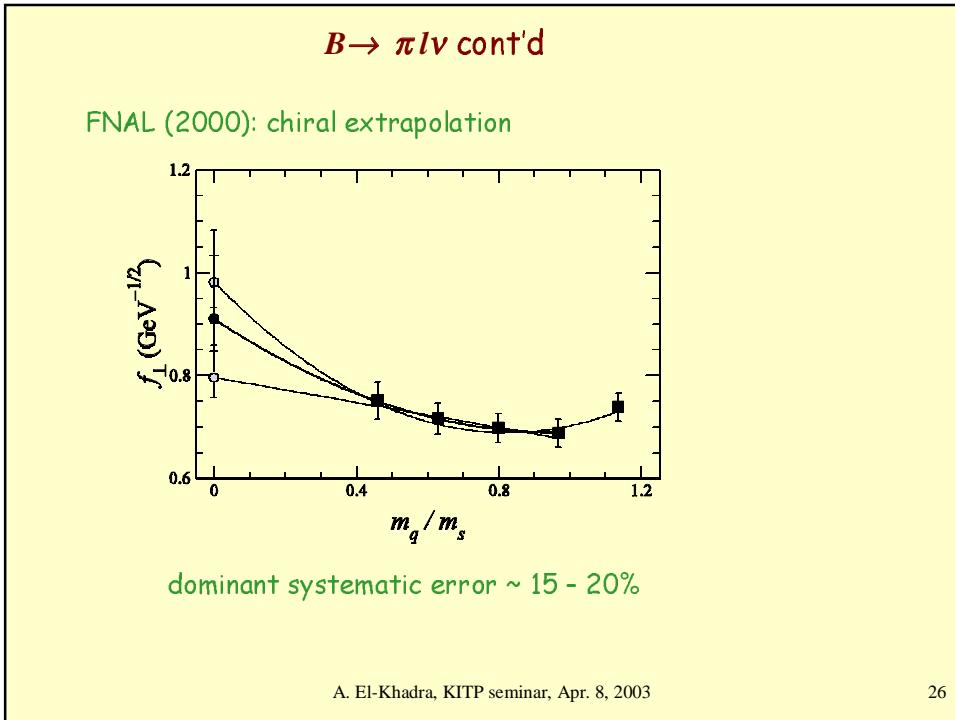
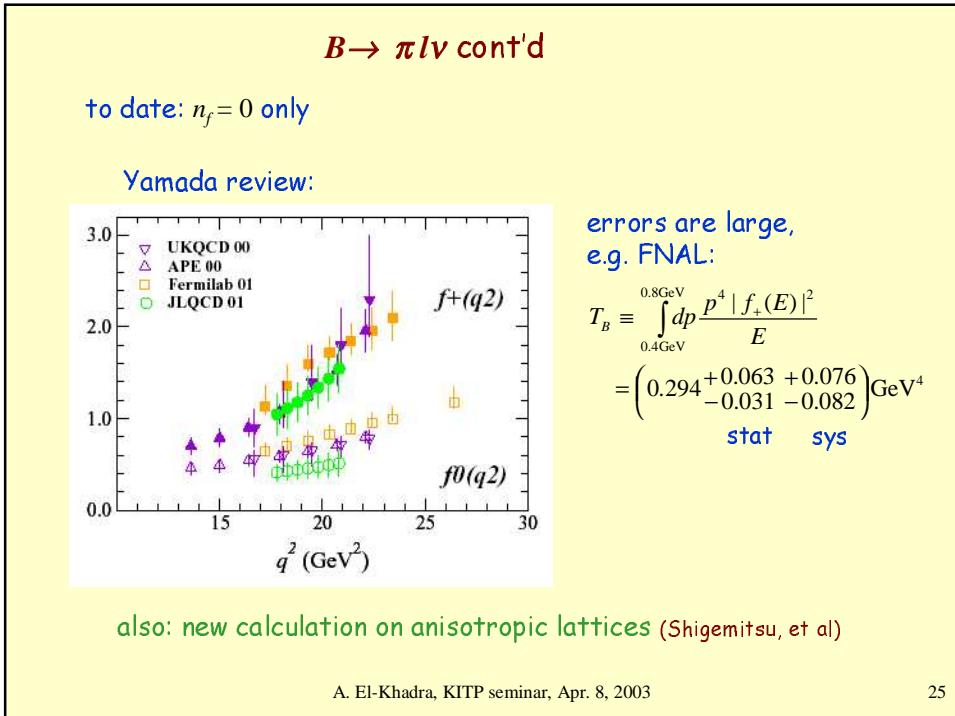
FNAL (2000): partial differential decay rate



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Some Recent Developments

- ✓ highly improved actions:

light quarks:

improved staggered action: correct through $\sim O(a^2)$
computationally affordable $n_f = 3$ feasible!

MILC (2002): $n_f = 3$ configurations at $a = 0.09, 0.13$ fm
with $m_s \neq m_{light}$ and $m_{light} = m_s/8, m_s/4 \dots, m_s/2, \dots, m_s$

heavy quarks:

NRQCD: correct through $\sim O(a^2), O(v^4)$
 $O(v^6)$ in progress

Fermilab: correct through $\sim O(\alpha_s a), O(p/m)$
 $O(a^2), O(p^2/m^2), O(v^4)$ in progress

- ✓ automated perturbation theory

get pert. results for new actions quickly
two-loop calculations in progress

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Recent Developments cont'd

- the new MILC configurations include realistic sea quark effects.

strategy:

the only free parameters in lattice QCD lagrangian:
quark masses and α_s

tune the lattice QCD parameters using experiment:
 $m_{u,d}, m_s, m_c, m_b$ using $\pi, K, J/\psi, Y$ meson masses
 α_s using 1P-1S splitting in J/ψ and/or Y system

all other quantities should agree with experiment ...
try this for some easy quantities ...

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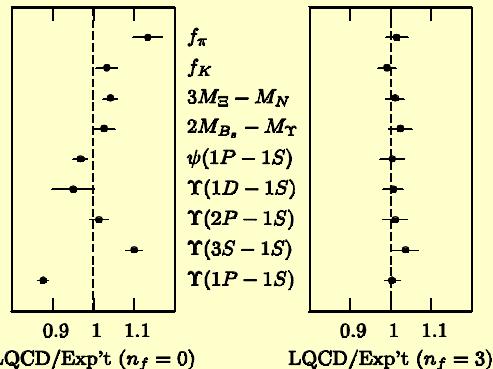
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Recent Developments cont'd

HPQCD (NRQCD+MILC+FNAL), compiled by P. Lepage

lattice QCD/experiment before now



works quite well!!

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Prospects for the near future

work currently in progress using the MILC configurations
within the next year we can expect first results for ...

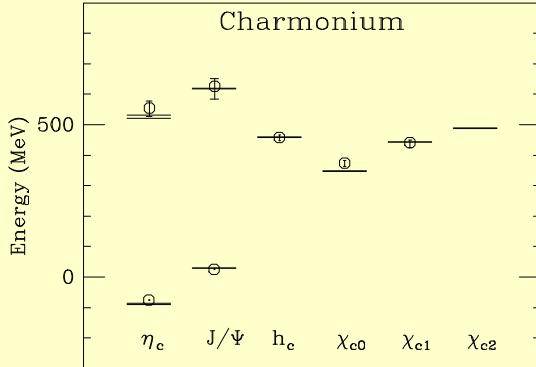
- ✓ Y and J/ψ systems using NRQCD and Fermilab actions
- test the new heavy quark actions
- $\Rightarrow \alpha_s, m_b, m_c$

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Fermilab 2003 (preliminary)



result at $m_{light} = m_s/4$, $a = 0.12 \text{ fm}$ with $O(a)$ improved action

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Prospects for the near future

work currently in progress using the MILC configurations
within the next year we can expect first results for ...

- ✓ Y and J/ψ systems using NRQCD and Fermilab actions
test the new heavy quark actions
 $\Rightarrow \alpha_s, m_b, m_c$
- ✓ π, K meson systems
using improved staggered light quarks with $m_l < m_s/2$
masses, decay constants, mixing, SL form factors

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Prospects for the near future cont'd

... and for ...

- ✓ D, D_s, B, B_s meson systems
 - using improved staggered light quarks with $m_l < m_s/2$
 - masses (splittings), decay constants, mixing, SL form factors
 - ⇒ comparison with CLEO-c essential to test lattice results

expect initial accuracy of < 10% errors
with an ultimate goal of 2-3% errors.

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Issues

- test the highly improved light and heavy quark actions
 - NRQCD vs. Fermilab
 - staggered vs. Wilson ?
- for $n_f = 2+1$ staggered fermions one has to take the $\sqrt{\det}$ - a nonlocal operation:
 - staggered LQCD = QCD ?
 - short distance: ok
 - nonpert. quark loop structure:
 χ PT: okay, a^2 corrections known
(Bernard, ...)
 - test staggered LQCD against experiment
- semileptonic B decays at high recoil
⇒ moving NRQCD (Foley, Lepage)

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Conclusions

- ✓ lattice QCD calculations are an important component of the physics program of the B factories.
- ✓ current status:
 $f_B, B_B, f_{B \rightarrow \pi}(E)$ to 10-30% accuracy
 $\mathcal{F}_{B \rightarrow D^*}(1)$ to few % accuracy
- ✓ lattice results with realistic sea quark effects are here! expect to see a growing number of results within the next year
- ✓ made possible with improved staggered action
- ✓ improved heavy quark actions NRQCD/Fermilab
 - 2-3% accuracy requires 2-loop pert. matching
need to redo pert. calculations for the new actions
automated pert. theory methods help
 - CLEO-c experiment is important for testing lattice QCD

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